Modeling Traffic Through Intersections

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Abstract

This paper analyzes the effects traffic flow controllers, such as stop signs and stop lights, have on the flow of traffic through a 4-way intersection. In particular, we investigate which controller (either using 2 stop signs, 4 stop signs, or stop lights) to use depending on the number of cars in a lane. We use a method that accounts for a set number of cars in each lane and a car's velocity based on the distance to the stop line or to the car in front. We also account for how a car will turn once it reaches the stop line. When a car stops at an intersection, we change our model depending on the traffic flow controller being used. With stop signs, a car must first stop and can only proceed when certain cars have cleared the intersection. With lights, a car must wait for the appropriate light to turn green before the car can continue moving. In the end, we found it best to use 2 stop signs when there are less than 20 cars in each lane, stop lights when there are more than 30 cars in each lane, and 4 stop signs otherwise.

Contents

1   Introduction

Throughout the world, vehicles travel along roads to get from one location to another, but in order to get to a destination, a car will likely need to travel through at least one intersection of two or more roadways. However, in order to avoid collisions or confusion about which vehicle should travel through the intersection first, traffic flow controllers such as stop signs and stoplights are implemented to control the flow of traffic through the intersection. Some intersections use a minimum number of stop signs (such as 2 stop signs at a four-way intersection) due to a low amount of traffic at that intersection. Other intersections have a enough cars flowing through the intersection to require a stop light to only allow cars traveling from certain directions to move through the intersection at a certain time. Our goal is to analyze the traffic flow caused by different types of traffic flow controllers for an intersection of two, 2-way streets in order to determine which controller should be utilized at this particular intersection.

In order to analyze the effects of traffic flow controllers, we need to generate a method to model the flow of traffic along a street or at an intersection. One method, used by Ruskin and Wang, has cars moving through discrete sections of the road that are a certain distance apart [1]. Once a car passes through the intersection, the model maps the cars movement through the intersection using the discrete sections. In this particular model, the authors were investigating intersections without stop lights, so they implemented a system that forced any car at the stop line (location where a vehicle must stop before the intersection) to check a certain number of these segments for occupying cars before entering the intersection. The model also incorporates two different types of lanes, major and minor, signifying different traffic densities in each direction.

One other report created by the U.S. Department of Transportation explains how to investigate the effectiveness of stoplights. In particular, they discuss a method in Chapter 7.4 called Quick Estimation Method (QEM) that analyzes the effectiveness of a stoplight using the volume of traffic and the number of lanes at that intersection per hour [2]. This method determines the types of allowable turns for each lane, calculates the greatest number of cars allowed in each lane in order to derive a significant result, and finds how many cars actually move through the intersection. Ultimately, QEM is a relatively simple method that determines whether an intersection is operating with more traffic than it can safely handle [2].

Finally, a short article by Alberto Bressan of Penn State University discusses two different models that can be used to simulate traffic flow, microscopic particle models and macroscopic models. Microscopic models analyze each specific car on a road and sets up ordinary differential equations to "specify how each driver adjusts his velocity depending on the distance and the velocity of the vehicle ahead" [3]. Macroscopic models analyze the number of cars for a specific length of road and utilizes partial differential equations and conservation laws. Bressan mentions the microscopic method are "simple to simulate" and was more interested in the mathematics behind the macroscopic model since they "yield a better qualitative understanding of traffic patterns" [3].

Therefore, we will adopt portions of these three methods into a model for analyzing the traffic flow through each intersection. The vehicles in our simulation will move using a discrete method similar to a microscopic particle model, adjusting a vehicle's velocity based on the distance to the car in front or the distance to the stop line, and when a vehicle reaches an intersection, the vehicle will only move once certain cars have cleared the intersection. In the end, our simulations will produce a traffic flow through the intersection for a constant number of cars in a set distance of one kilometer, so we can compare the different traffic flow controllers using the traffic flow through each one.

2   Assumptions and Goals

When constructing a model that will simulate traffic flow, we first need to define the goal of our model, the constraints on driving set by the law, and our assumptions behind the model. All of these ideas apply to our three types of traffic flow controllers unless otherwise specified.

2.1   General Ideas Behind the Models

We created a mathematical model that simulates traffic at and around an intersection in order to analyze the effects of each type of traffic flow controller. We decided to use a discrete numerical

method that uses a set number of cars for each lane and their distance from the stop line (location where a car will stop in the lane).

This model will calculate each car's position at different steps in time depending on certain circumstances, such as the position of the car directly ahead of them or the distance to a stop line. Then, once a car reaches a stop line, the model will determine the car's motion through the intersection based on the traffic flow controller being used.

For example, if the intersection includes a stop sign and a car is close enough to the stop line, the car must stop first, and then check to see if anyone else is waiting at a stop line or passing through the intersection. If no one else is waiting, the car can safely travel through the intersection. However, if the car was at an intersection using a stop light, it can go through the intersection if the light is green, but the driver does not need to check the other lanes to see if they can go first.

2.2   Rules of the Road

While at an intersection, drivers must follow certain rules and procedures to decide whether or not they can continue driving through the intersection. For this particular project, we used the traffic laws for the state of Montana, although the laws we implemented in our model are generally accepted throughout the United States of America [4]. The key rules we focused on include:

- Coming to a full stop at a stop sign

- Checking other lanes for oncoming traffic before entering the intersection

- Allowing other cars to clear an intersection before moving through the intersection

- The first driver to stop at an intersection is the first driver to travel through the intersection

- If two or more drivers arrive at an intersection at the same time, the right-most driver must go first

2.3   Assumptions

When generating our model, we made the following assumptions to simplify our model and :

- Cars travel on the right side of the road

- We will ignore the effects of pedestrians and abnormal human behavior of drivers, such as aggressive driving.

- All drivers will follow the rules of the road, including staying at or below the speed limit.

- No accidents or other random events will occur that would stop the flow of traffic.

- No right turns are allowed when a car is stopped at a stop light.

- All vehicles are the same size and the distance recorded is from the front of the vehicle to the stop line.

- All intersections are 4-way intersections composed of two perpendicular streets, one horizontal lane and one vertical lane.

- All cars will be a minimum safe distance from the car directly in front of them.

- No u-turns are allowed.

- Each street has 2 lanes, allowing cars to travel two different directions down the same street.

- The simulation occurs at the busiest time of day to ensure we model the largest number of cars the intersection will contain.

3   Types of Intersections

There are a variety of intersections we can investigate, including 3-way intersections and traffic circles, but we decided to focus on three main types of 4-way intersection traffic flow controllers:

1. Two lanes with stop signs and two lanes with no signs or lights (TSS)

2. All four lanes with stop signs (FSS)

3. Timed stop light (TSL)

We decided to focus on these three types of controllers since they are among the most common controllers used at a 4-way intersection, and the mathematical models for these controllers are very similar and can be compared easily.

4   General Behavior of Roadways and Intersections

For each type of intersection, the separate models had multiple commonalities between them. These shared traits include how a lane is composed, how to determine the movement of vehicles prior to the intersection, and how each car turns.

4.1   Composition of Lanes

When generating our model, we used the four compass direction (North, East, South, and West) as labels for the four streets that intersect. North and South are considered the vertical lanes, and East and West are the horizontal lanes (see Figure 1). Since each street has 2 lanes, cars entering from one direction may proceed into any of the other 3 directions.
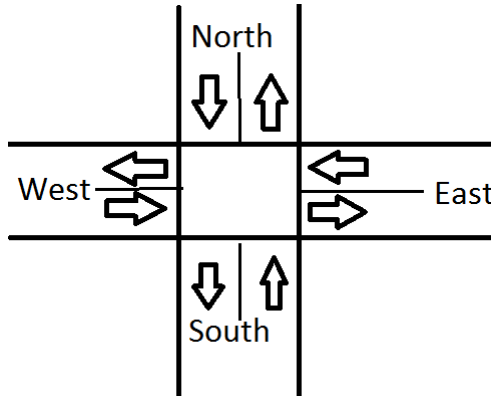

Figure 1: Composition of Four Streets and Directions of Lanes

4.2   Driving Prior to the Intersection

Since all of our models analyze cars position and motion prior to the intersection, we generated a method to describe each car's motion before the stop line. This general method applies to each of the types of intersections we will analyze and applies to all cars in the simulation.

For each of the cars behind the front car (the car closest to the stop line), we analyzed the distance between each car and the car in front of them (Δx) to determine the velocity v at each time step Δt. The highest speed a car can travel in a single time step (v_{max}) will be the speed limit on each particular road. If Δx for a car is greater than the possible distance it can travel in a time step ($v_{max} \cdot \Delta t$), the car will travel at the speed limit for the duration of the time step. If Δx is between $v_{max} \cdot \Delta t$ and a minimum safe distance d_{min}, the car will adjust its maximum speed to travel to the minimum distance in that time step. If $\Delta x \leq d_{min}$, it will not travel forward in that time step. This process is outlined in Equation 1 and Figure 4.2.

$$v = \begin{cases} v_{max}, & \Delta x > v_{max} \cdot \Delta t \\ v_{max}\left(\frac{\Delta x - d_{min}}{v_{max} \cdot \Delta t - d_{min}}\right), & d_{min} < \Delta x \leq v_{max} \cdot \Delta t \\ 0, & \Delta x \leq d_{min} \end{cases} \qquad (1)$$
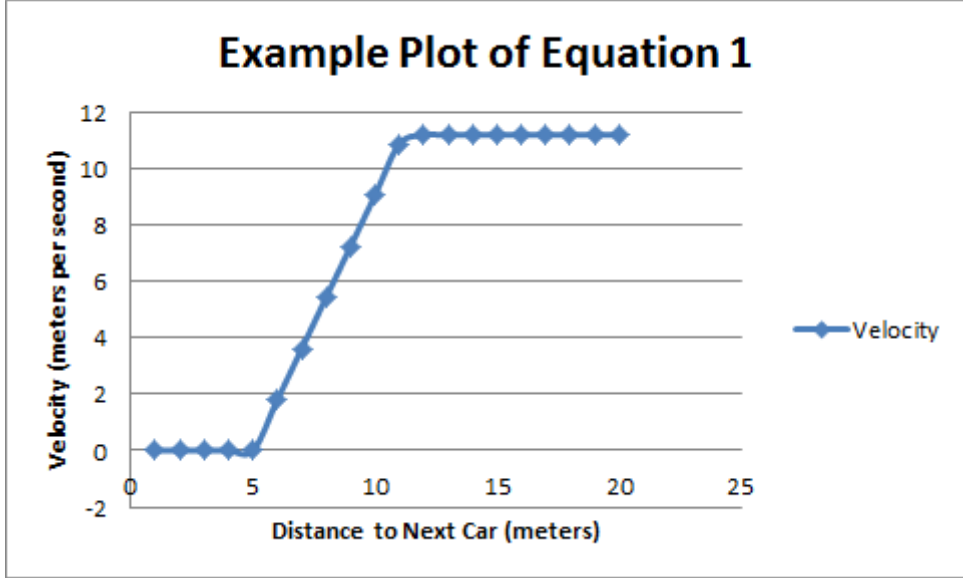

Figure 2: Plot of Equation 1 for $v_{max} = 25$mph and $d_{min} = 5$

For the front car, we only looked at its current position x from the stop line. If $x \geq v_{max}\Delta t$, then $v = v_{max}$. If $x < v_{max}\Delta t$, then the car's current velocity will adjust to allow the car to travel to the stop line (for intersections with stop signs or at red lights) or to travel through the stop line (if there is a green light), similar to the second part of Equation 1.

Once a car's velocity at that time step is determined, that car will move forward $v \cdot \Delta t$ meters.

4.3   Traveling Through the Intersection

A vehicle can only travel through an intersection once certain conditions are met, and these conditions change depending on the type of intersection being modeled. However, once a car can travel through an intersection, each model uses a similar method to track the car in the intersection.

First, the model records the specific time when a car entered the intersection and in the specific street it entered from. This time is then used to determine how long the intersection has been occupied, and we estimate it takes approximately 4 seconds for a car to leave the intersection. If a car is in the intersection, then certain cars cannot enter the intersection to avoid collisions.

After the time step is recorded, the car in the intersection is then removed from our data and replaced by another vehicle that enters at a certain position before the intersection, so that the total number of cars in our simulation remains constant. By conserving the number of cars in a lane, we can analyze the intersection when there is a constant maximum number of cars that we can expect at that intersection.

4.4   Implementing Different Types of Turns

When a car reaches an intersection, it can choose to either turn to the left, continue going straight, or turn to the right. At a stop light, the car must wait for the green light that allows its desired turn, and at a stop sign, the car must wait for certain vehicles to exit the intersection for each type of turn.

To implement this aspect into our model, we assigned each vehicle one of three random integers: 0 (left), 1 (straight), or 2 (right). Then, once the car is at the stop line, the model will

analyze the locations of the other cars in the lane or the stop light. If the right conditions are met, the car can safely travel through the intersection and be removed from our simulation. A random number is then assigned to the car that enters the lane once the other car leaves the lane.

5 Stop Sign Behavior at Intersections

When an intersection involves a stop sign, each car that stops at the stop line must check for two factors. First, they must let other cars already at the other stop lines enter the intersection first. Second, they must wait for certain cars to exit the intersection, depending on the direction of the vehicle or vehicles in the intersection and the desired direction of the car at the stop line.

5.1 Going Straight

If a car wishes to continue straight, it must wait for the following vehicles to exit the intersection:

- All vehicles from the street to the right
- Vehicles in the left street that are going straight or to the left
- Vehicles from the parallel street that are turning left

For example, if a car in the East lane wishes to go straight, it must wait for all cars from the North lane in the intersection turning left, right, or straight and all South cars that are going straight or left. Since West is also a horizontal lane, the East car would have to wait for any West cars turning left. See Figure 5.1 for a diagram of this scenario.
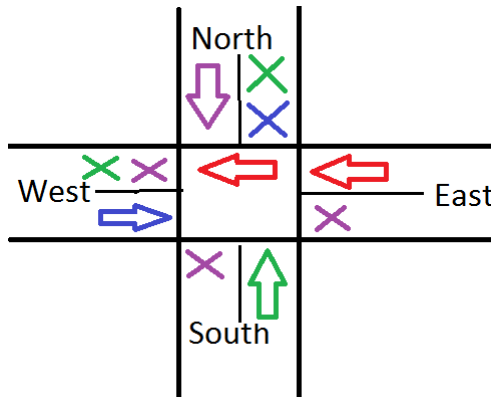

Figure 3: Scenario When East Goes Straight. "X" Means Directions Other Lanes (With Matching Colors) are Turning that East Must Wait For

5.2 Left Turns

Since a left turn requires crossing the largest section of the road, a left-turn car must wait for more vehicles to clear the intersection before it can drive through the intersection. If a car wishes to turn left, it must wait for the following vehicles:

- All vehicles from the parallel street
- Vehicles from the right street turning left or going straight
- Vehicles from the left street turning left or going straight

For example, a car in the East street must wait for vehicles from the West street and any vehicle from the North or South streets that are not turning right. See Figure 4 for a diagram of this scenario.
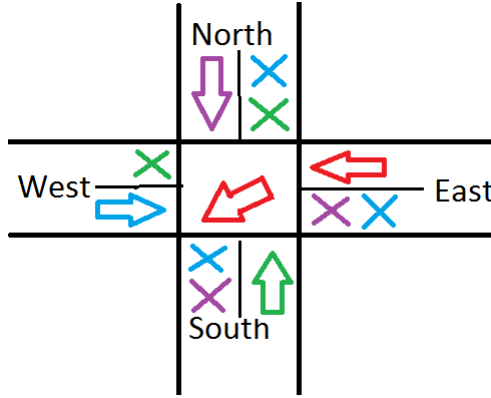
Figure 4: Scenario When East Goes Left. "X" Means Directions Other Lanes (With Matching Colors) Are Turning that East Must Wait For

5.3   Right Turns

Due to the cars traveling on the right side of the road, a driver wishing to turn right only has to worry about other vehicles turning into the same lane, namely

- Vehicles from parallel streets turning left
- Vehicles in the left street going straight

For example, a car from the East only has to worry about cars from the South going straight and cars from the West turning left. See Figure 5 for a diagram of this scenario.
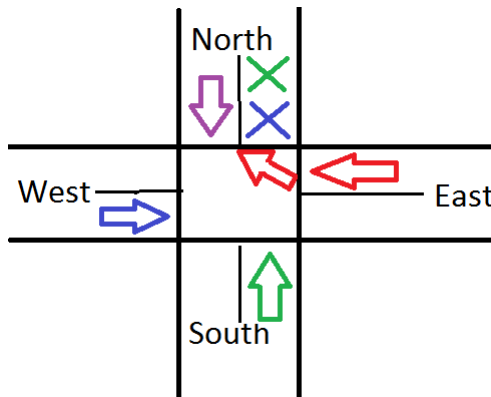

Figure 5: Scenario When East Goes Right. "X" Means Directions Other Lanes (With Matching Colors) Are Turning that East Must Wait For

6   Behavior at Stop Lights

If stop lights are used, cars can only travel past the stop line if the light for their particular lane and direction is green. A stop light changes color on regular time intervals or, in special cases, when a car triggers the lights to change for a short period of time. However, we are only analyzing the use of timed stop lights in this model.

At a timed stop light (TSL), the light changes colors after a fixed amount of time, and there are four different green lights that allows the following cars to turn without running into another car:

- Cars turning right or going straight from vertical lanes
- Cars turning right or going straight from horizontal lanes

- Cars turning left from vertical lanes

- Cars turning left from horizontal lanes

We used equation 2, below, to regulate which light is green, where T is the time interval the light is green (or light time), t is the current time, and n is an integer between 0 and 3. The floor operation rounds the number in parentheses to the next lowest integer. Each of the four scenarios was assigned a different integer between 0 through 3, so when n equals a certain number, that scenario has the green light.

$$n = \text{mod}_3 \left(\text{floor} \left(\frac{t}{T} \right) \right) \tag{2}$$

7   Collecting Data

To determine when each traffic flow controller should be used, we counted the number of cars that pass by a single stop line (C) and then divided this value by the total amount of time our simulation covered (t_{total}). The resulting value is the flow of cars from a single street (F) (see equation 3).

$$F = \frac{C}{t_{total}} \tag{3}$$

We recorded the flow through each street and then summed the four flow values from each direction to determine the total flow of cars through the intersection.

To analyze the effects the density of traffic has on the flow, we held the length of the road (i.e. where a new car enters the lane) at a fixed 1000 meters. We then altered the number of cars in each lane (N) between 2 and 80 cars in 2 car increments (for $N < 20$) and 4 car increments (for $N > 20$).

We also had the car in front start at the same initial distance from the stop line, and all other cars were initially spaced 10 meters apart. We then kept all other variables in our model constant to determine the effect of the total number of cars N on the flow of cars past the stop line. For the numerical values of the constants, see Table 1.

Δt (seconds)	1
v_{max} (miles per hour)	25
d_{min} (meters)	5
t_{total} (seconds)	4,000

Table 1: List of Constants Used in Model and Respective Values

Recall our simulation uses random numbers to determine the direction each car will turn when it reaches the stop line. Therefore, we ran multiple simulations and found the average traffic flow and standard deviation for each lane . We found the standard deviation was fairly large (occasionally equal to or greater than the average), which was caused by outliers in the data. Thus, we removed any flow rates more than 2 of these large standard deviations from the mean (theoretically removing the top or bottom 2.5% of the data), and recalculated the average and standard deviation from the remaining values.

8   Determining Light Time

Before running the TSL model, we first had to determine a light time value (T) that would maximize traffic flow at an intersection to model the best possible stop light. To do so, we ran the TSL simulation for 10 to 30 cars in the lane (N) with five T values (15, 20, 25, 30, and 35) and recorded the total traffic flow for all lanes, producing the results in Figure 6.

Observe when $T = 25$, the traffic flow rates are overall greater than other T values for low values of N, and the flows are equal for larger values of N. Since $T = 25$ provides the maximum flow for all N, we will use this value for our light time.
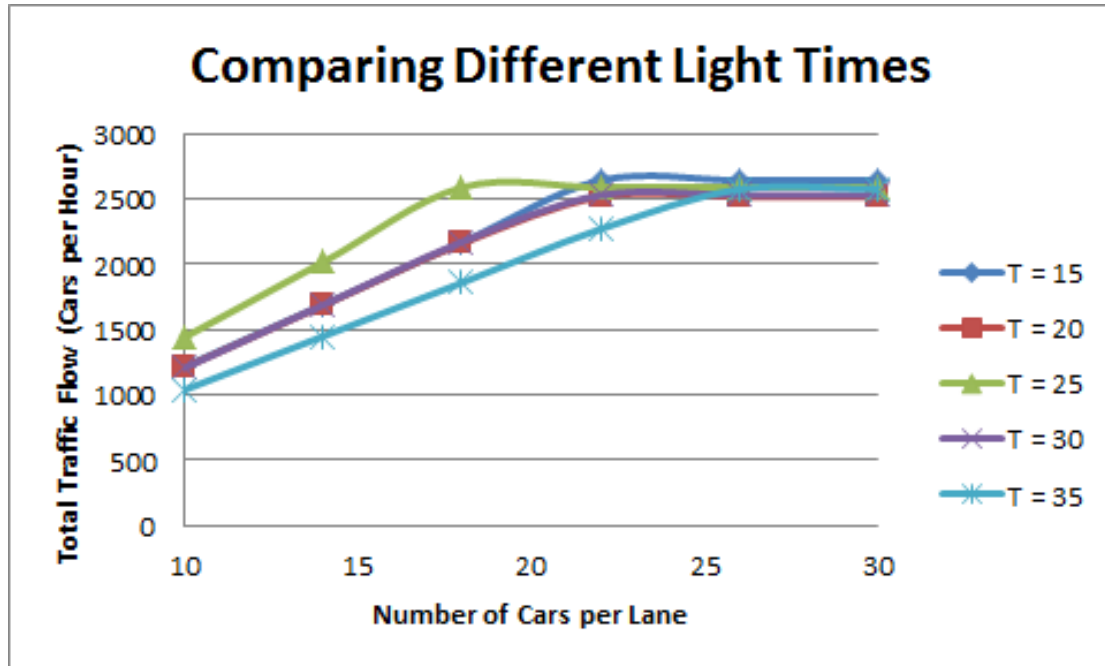
Figure 6: Results of Varying Light Time and Number of Cars

9 Results of Simulations

For each model, we monitored the traffic flow in cars per hour through each of the four lanes and the total traffic flow through the intersection. We varied N from 2 to 50 cars and ran 50 simulations for each N, recording the traffic flow through each lane for each simulation. We then found the average traffic flow and the standard deviation for each lane and for the total flow for each traffic flow controller. Our results include the average traffic flow for each of the directions and the two standard errors from the mean (represented by error bars).

9.1 Two Stop Signs

For the TSS simulation, we let the North and South lanes have no stop signs while the East and West lanes each have a stop sign. The results for the four directions of this simulation are found in Figure 7.

Observe the North and South lanes contain most of the traffic flow through the intersection while the East and West have comparatively little flow for most values of N. This phenomenon is caused by the presence of stop signs forcing East and West to stop and wait before entering the intersection. The overall traffic flow continues to increase as the number of cars increase, but the East and West lanes flow rates through the intersection decrease as more cars occupy the lanes because the intersection is usually occupied by North and South cars.

Also, notice East and West have nearly identical flow rates as well as North and South. The model compares these two pairs of lanes almost the same, such as changing which lanes to wait for when turning but not how many lanes to wait for, so the values should be similar. Overall, we find the effectiveness of using two stop signs decreases as the number of cars per lane increases past roughly 10 cars since the East and West flow rates start decreasing after that point.

9.2 Four Stop Signs

The results for the FSS simulation can be found in Figure 8.

As we increase N, all four lanes have roughly the same traffic flow for each value of N, and the flow levels off at approximately 10 cars and occasionally fluctuates from an average traffic flow of roughly 225 cars per hour. As in the TSS simulation, all four lanes are mitigated by a stop sign and have very similar models to determine motion, so all four lanes will have nearly identical traffic flow rates.
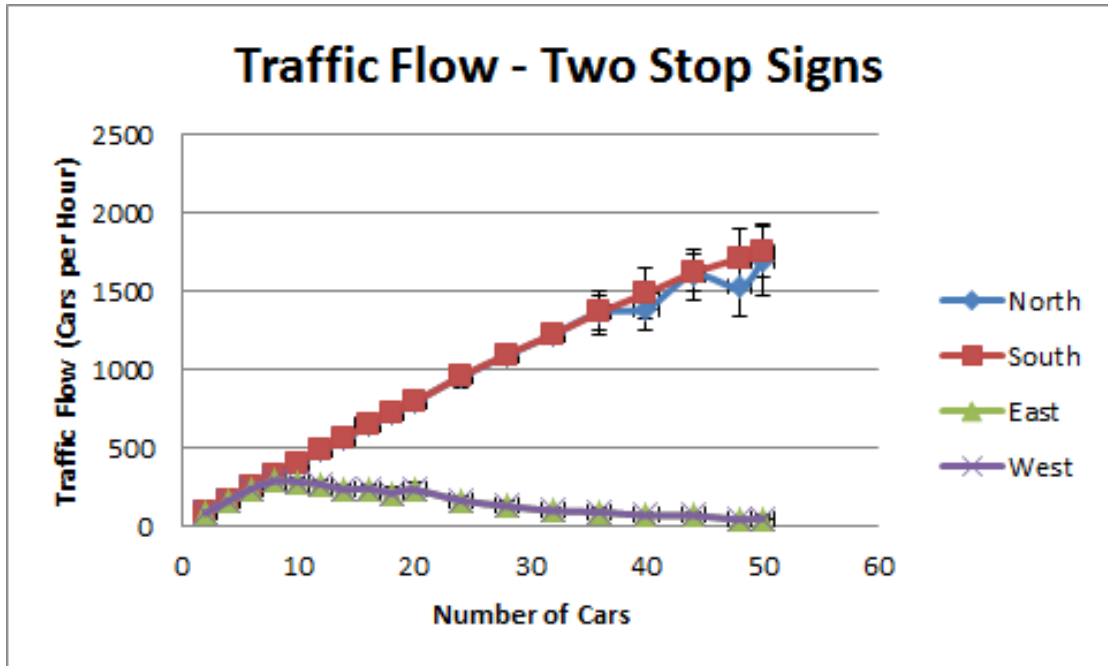
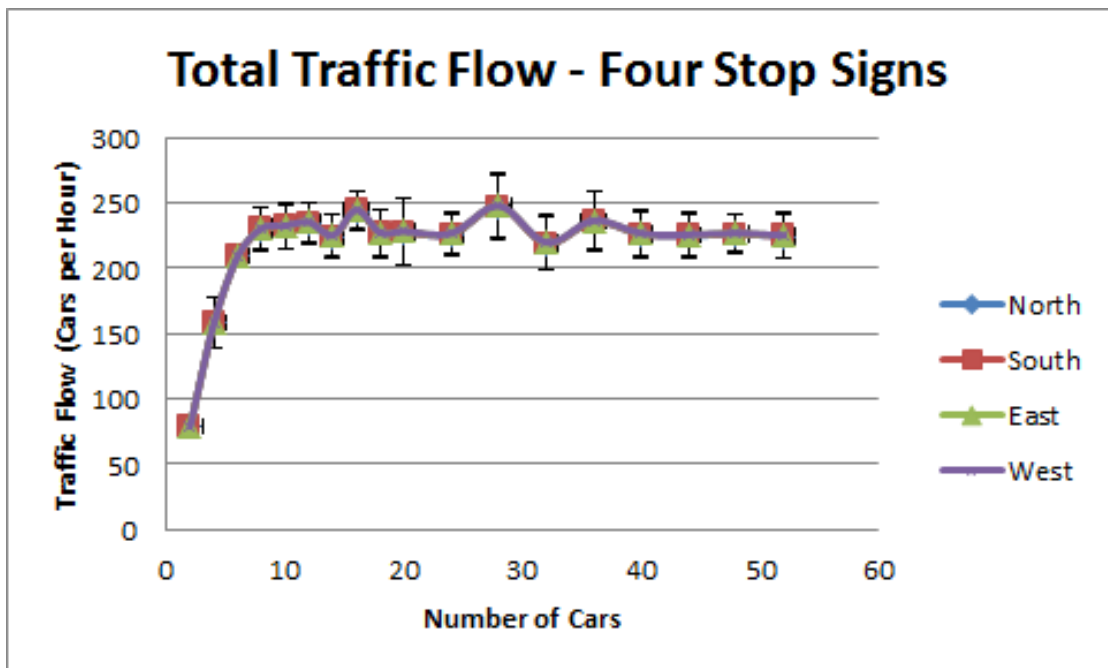Figure 7: Traffic Flow for 2 Stop Sign Intersection vs. Number of Cars

Figure 8: Traffic Flow for 4 Stop Signs vs. Number of Cars

9.3   Timed Stop Light

For the TSL simulation, we let the horizontal, straight light be green initially and switch after T seconds. Using our derived value of $T = 25$, we produced the results found in Figure 9.

All four directions have roughly the same flow rates for all N (with virtually no deviation from the mean), and the flow increases linearly from $N = 0$ to about $N = 18$. Afterwards, the traffic flow stays a constant rate for the remaining values of N since the amount of cars that can travel through an intersection during a green light cannot exceed the number of cars that can travel past a point in 25 seconds. Thus, more cars will be waiting in line as we increase the number of cars in a lane since only so many can travel through the intersection in per green
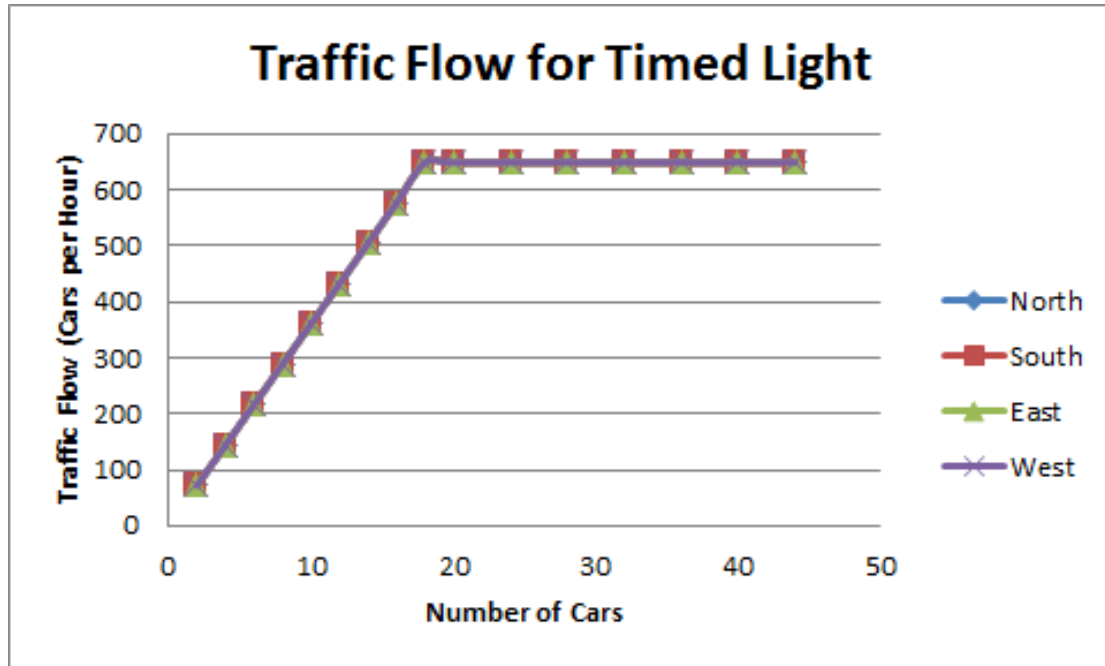
Figure 9: Traffic Flow for Timed Stop Light vs. Number of Cars

light.

10 Comparisons

We know the behavior of each of the three types of intersections, but we need to determine when each model should be used. By construction, there should be transition between the TSS and FSS intersections and the FSS and SL intersections, so we compared the flow rates of these intersections to each other.

10.1 Two Stops Signs vs. Four Stop Signs

First, we will compare the TSS and FSS simulations. Figure 10 shows the total flow rates for both types of intersections.

At relatively low N values, the two intersections have roughly the same total flow. However, for the rest of the N values, TSS provides a greater total flow than FSS due to the vertical lanes having unrestricted flow. This unrestricted flow does decrease the flow of the East and West lanes, as seen in Figure 7, so we then compared the East flow (which also equals the West flow) for both TSS and FSS simulations (Figure 11).

With this figure, we see FSS allows more traffic from the East lane to flow through the intersection than TSS after $N = 20$ cars. Therefore, once there are more than 20 cars in each direction per 1000 meters, the intersection should use four stop signs rather than two.

10.2 Four Stop Signs vs. Timed Stop Light

See Figure 12 for a comparison of the FSS and TSL total flows. We chose to compare total flow rates rather than a particular direction since the four directions had nearly identical average flow rates for all N.

The Timed Light traffic flow is constantly greater than or equal to the 4 Stop Signs flow, which seems plausible given cars always have to stop at a stop sign and wait for traffic in the intersection. At a stop light, as long as the light is green, the cars can continue driving without stopping or checking for traffic since the the only moving vehicles in the intersection are designed to never collide in the intersection.
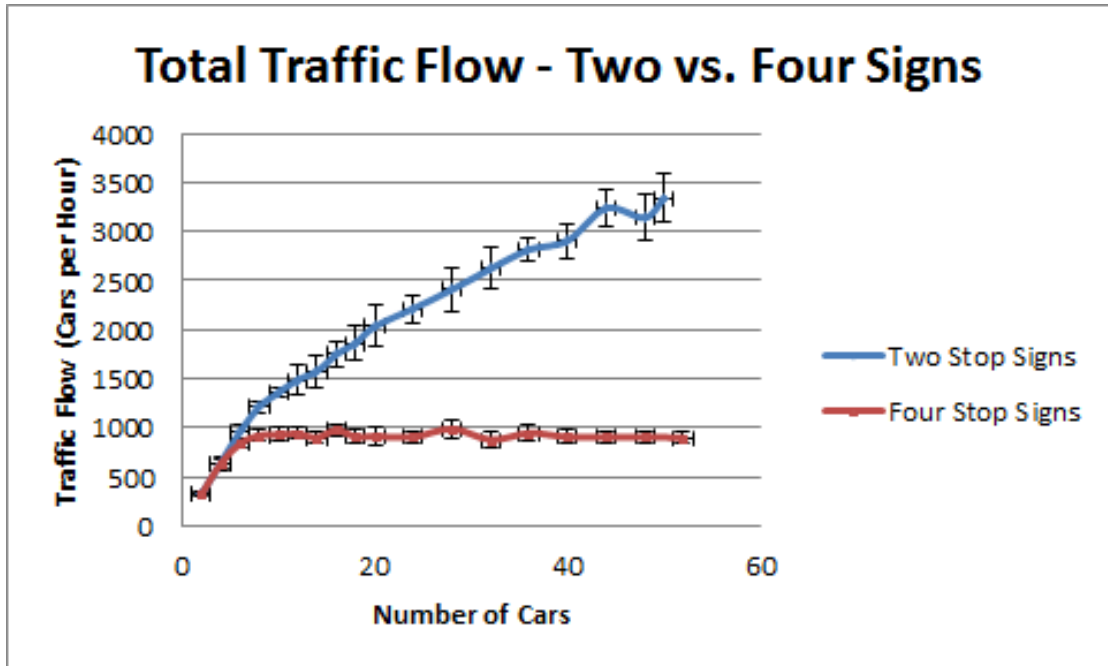
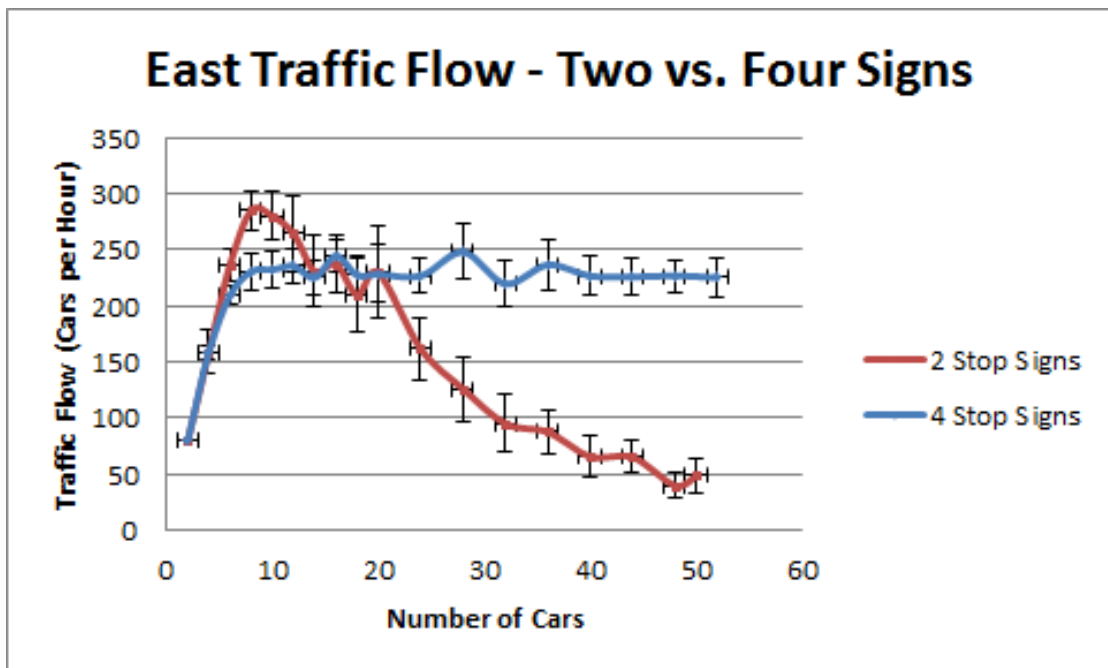Figure 10: Total Traffic Flow for Two Stop Signs vs. Four Stop Signs

Figure 11: East Traffic Flow for Two Stop Signs vs. Four Stop Signs

Therefore, based on our simulation alone, 4 stop signs could be used for less than 10 cars, but stop lights are better when there are more than 10 cars per kilometer. Coupled with our comparison of 2 and 4 stop signs, our simulation shows 4 stop signs should never be used since it is always better to use 2 stop signs or stop lights.

However, other factors outside of our simulation, such as cost of installing a stop light, could influence when a stop light should be used instead of 4 stop signs. For example, a small town with a limited budget by a single relatively busy intersection would install stop signs rather than a stop light to save on money. In these cases, it is more practical to install 4 stop signs than a stop light, so stop lights are not always strictly better than 4 stop signs.
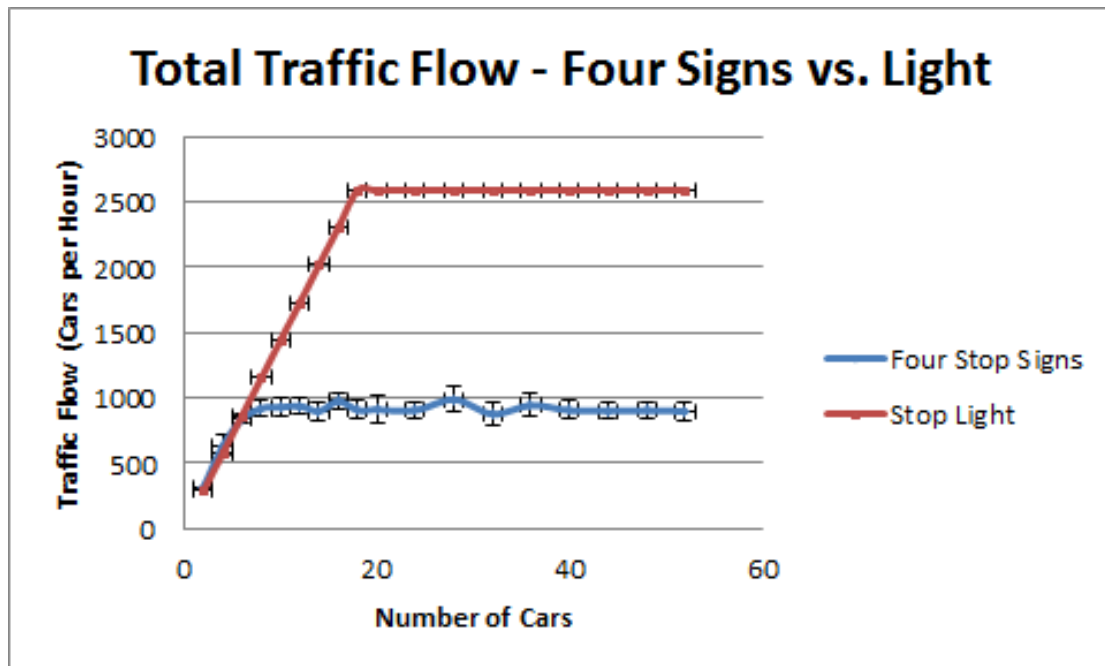
Figure 12: Traffic Flow of 4 Stop Signs vs. Timed Stop Lights

11 Summary

In conclusion, we find a 4-way intersection should use two stop signs (either for vertical or horizontal lanes) if each lane has less than 20 cars per kilometer and stop lights whenever there are more than 20 cars per kilometer. Four stop sign intersections are a practical, cheaper substitute for stop lights if stop lights are too expensive to install at an intersection.

11.1 Strengths and Weaknesses

Some of the strengths of our model includes:

- Accounting for all possible turns and which cars to wait for based on the turn being made

- Implementing rules of the road, such as the right of way, into our simulation to make it applicable to real world scenarios

- Relatively small standard errors means our data is consistent

The weaknesses of our model include the following:

- Lacking a right turn on red in the stop light code potentially underestimates the traffic flow for stop lights.

- Only using one speed limit rather than analyzing the effect of multiple speed limits would have on the traffic flow.

- Removing the human element in driving which could affect the final outcome of our traffic flow rates.

11.2 Future Work

One variable we did not change was the speed limit, which is one of the factors that is most likely to be different for different intersections. Logically, a greater speed limit would allow more cars to flow through an intersection since cars are not traveling faster, but we would want to investigate the effects of changing the speed limit just in case.

We also mentioned a stop light that changes based on a trigger. These types of lights would be used if there is a busy street and a not-so-busy street, such as a car from an off-road trying to

turn into a busy street. There are also intersections called roundabouts that have cars yield to traffic moving around in a circle. We want to generate a simulation for these types of intersections in the future to compare to our results.

Finally, one of our key assumptions was all of our streets were two way streets with only one lane per direction. We would want to expand our model to monitor a four or six lane street instead of two lanes to analyze the traffic flow for different types of roadways.

Appendices

A List of Variables and Definitions

N	Number of cars in each lane per kilometer
v_{max}	Speed limit of the roadway (meters per second)
v	Velocity of a car for a particular time (meters per second)
Δt	Time step (seconds)
Δx	Distance between the front of two adjacent cars (meters)
d_{min}	Minimum safe distance between the front of adjacent cars (meters)
n	Integer used for determining which light is green for stop lights
T	Length of time a light is green (seconds)
t	Current time (seconds)
F	Traffic flow rate (cars per second)
C	Number of cars that pass by a stop line
t_{total}	Total amount of time a simulation covers (seconds)
TSL	Timed Stop Light
FSS	Four Stop Signs
TSS	Two Stop Signs

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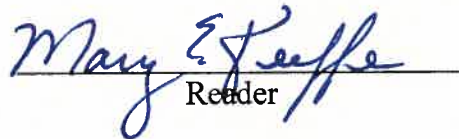
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