# Behaviour of Reinforced Concrete Conical Tanks under Hydrostatic Loading 

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A thesis submitted in partial fulfillment of the requirements for the degree in Master of Engineering Science
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# BEHAVIOUR OF REINFORCED CONCRETE CONICAL TANKS UNDER HYDROSTATIC LOADING 

(Thesis format: Monograph)

By

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Graduate Program in Engineering Science Department of Civil and Environmental Engineering

A thesis submitted in partial fulfillment of the requirements for the degree of Master of Engineering Science

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#### Abstract

Reinforced concrete conical tanks are used in municipalities and industrial applications as liquid containing vessels. Such tanks can be ground supported tanks or elevated on a supporting shaft. Although most design codes provide guidelines for rectangular and cylindrical tanks, no guidance is provided in such codes for conical tanks. Therefore, this thesis is motivated to study the behaviour and design of this type of tanks. In the current study, the accuracy of a design approach based on the provisions of Portland Cement Association (PCA-CCTWP) code for cylindrical tanks combined with an equivalent cylindrical approach provided by the American Water Works Association AWWA-D100 (2005) is assessed. This assessment is done by comparing the internal forces resulting from this method with those obtained from a linear finite element analysis model built inhouse. It is noticed that in some of the studied tanks, the PCA-CCTWP approach combined with the equivalent cylinder method is found to be unsafe. As such, and due to the complexity of analysing these conical tanks, a simplified design approach in the form of design charts is provided in this study. This set of charts can be easily used for the analysis and design of reinforced concrete conical tanks subjected to hydrostatic pressure and having a constant wall thickness. This approach is developed using the results obtained from finite element analysis of a wide range of reinforced concrete conical tanks having different configurations combined with code requirements. This simplified approach is then utilized to investigate the economics of reinforced concrete conical tanks versus steel counterparts. A cost analysis is conducted for several conical tanks having different capacities and different construction materials by including both construction and life-cycle costs. In addition to the cost analysis, a general study of the effect of tank dimensions on its cost is illustrated.


KEYWORDS: Conical Tank, Hydrostatic Pressure, Finite Element Analysis, Wall Thickness, Hoop Tension, Meridional Moment, Meridional Compression, Cost Analysis.

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## LIST OF ABBREVIATIONS

ACI

API
AWWA

PCA
PCA-CCTWP

FEA
ANOVA

American Concrete Institute

American Petroleum Institute
American Water Works Association

Portland Cement Association
Portland Cement Association - Circular Concrete Tanks without Pre-stressing

Finite Element Analysis
Analysis of Variance

## LIST OF SYMBOLS

| $u, v, w$ | Displacement Degrees of Freedom |
| :---: | :---: |
| $\boldsymbol{A}_{\boldsymbol{c}}$ | Area of Concrete Section |
| $\boldsymbol{A}_{\boldsymbol{g}}$ | Concrete Gross Area |
| $\boldsymbol{A}_{\boldsymbol{s}}$ | Area of Reinforcement Steel |
| $b$ | Wall Section Width |
| C | Shrinkage Coefficient |
| $C_{H}$ | Hoop Coefficient |
| $C_{M}$ | Moment Coefficient |
| $d_{b}$ | Steel Bar Diameter |
| $\boldsymbol{D}_{\text {cy }}$ | Cylindrical Tank Diameter |
| $\boldsymbol{E}_{\boldsymbol{c}}$ | Elastic Modulus of Concrete |
| $\boldsymbol{E}_{\boldsymbol{s}}$ | Elastic Modulus of Steel |
| $f_{c}$ | The Applied Tensile Strength |
| $f_{c}{ }^{\prime}$ | Concrete Compressive Strength |
| $f_{c r}$ | Concrete Tensile Strength |
| $\boldsymbol{f}_{s}$ | Stress in Reinforcement Steel at Service Loads |
| $\boldsymbol{f}_{\text {smax }}$ | Steel Maximum Allowable Stress |
| $\boldsymbol{f}_{\boldsymbol{y}}$ | Reinforcement Steel Yield Strength |
| H | Tank Height |
| $\boldsymbol{H}_{\text {cy }}$ | Cylindrical Tank Height |
| M | Service Meridional Moment |


| $n$ | Modular Ratio |
| :---: | :---: |
| $\boldsymbol{R}_{\boldsymbol{b}}$ | Base Radius of the Tank |
| S | Steel Bar Spacing |
| $S_{d}$ | Environmental Durability Factor |
| $T$ | Service Hoop Tension |
| $t$ | Wall Thickness |
| $\boldsymbol{t}_{\text {cy }}$ | Cylindrical Wall Thickness |
| $\boldsymbol{t}_{\text {min }}$ | Steel Wall Thickness |
| $t_{s}$ | Minimum Wall Thickness |
| $T_{u}$ | Factored Hoop Tension |
| $\boldsymbol{U}$ | Factored Loads |
| $x, y, z$ | Global Coordinates |
| $x^{\prime}, y^{\prime}, z^{\prime}$ | Local Cartesian Coordination |
| $\alpha_{a}$ | Significance Level for Analysis of Variables |
| $\alpha, \beta, \phi, \psi$ | Rotational Degrees of Freedom |
| $\beta$ | Magnification Function |
| $\beta$ | Strain Gradient Amplification |
| $\gamma$ | Ratio between Factored Load and Unfactored Load |
| $\gamma_{w}$ | Liquid Unit Weight |
| $v$ | Poisson Ratio |
| $\varnothing$ | Strength Reduction Factor |
| $\boldsymbol{\theta}_{v}$ | Tank Wall Inclination Angle |


| $\rho_{s h}$ | Horizontal Steel Ratio |
| :--- | :--- |
| $\rho_{s v}$ | Vertical Steel Ratio |
| $\sigma_{\boldsymbol{t}}{ }^{\text {th }}$ | Theoretical Tensile Hoop Stress |
| $\sigma_{l}$ | Total Actual Stress |
| $\sigma_{l}^{\text {th }}$ | Theoretical Maximum Effective Membrane Stresses |
| $\sigma_{\boldsymbol{m}}{ }^{\text {th }}$ | Theoretical Meridional Compression Stress |
| $\sigma_{y}$ | Steel Yield Strength |

## CHAPTER 1

INTRODUCTION

### 1.1 Background

During the last few decades, above ground tanks were extensively constructed around the world. These tanks play an important role to store different liquids in functional and safe manners. The above ground tanks are categorized into ground, standpipe and elevated tanks. Ground tanks, which are also known as reservoirs, can take different shapes (e.g. rectangular, cylindrical, and cylindrical with conical base). Although ground tanks have a high storage capacity due to their large diameter, they have a low operation head pressure. Stand pipe tanks are cylindrical shape tanks that have a height up to 46 m and a diameter ranging between 7 m and 9 m . They are characterized by high storage capacity and high internal hydrostatic pressure. On the other hand, elevated tanks have smaller capacity compared to standpipes and ground tanks. However, they provide high operation pressure with relatively low internal liquid height, up to 10 m , (Grieve et al. 1987).

Many water supply systems widely utilized elevated tanks because of their advantages that include: functional, economical and aesthetical aspects. Elevated tanks are used in districts with high elevation since this type of tanks provides sufficient head pressure during peak hours or even after power outages. Also, they provide lower energy cost since the water can be pumped during off peak times. Elevated tanks are considered as an economical solution for upgrading existing water supplying systems to satisfy the increasing demand of water supply. Moreover, the supporting tower (i.e., supporting shaft) of an elevated tank can be utilized as a multipurpose structure, especially for regions with lack of space below ground. Elevated tanks present aesthetical pleasure and
they are considered as visible landmarks for the surrounding areas. It should be mentioned that elevated tanks are essentially required to remain functional even during and after disaster events to meet the emergency requirements such as firefighting and public water demands. The damage of a storage tank containing hazardous materials (e.g. chemicals, and fuels) can adversely affect the environment causing significant economic loses.

Nowadays, there is an extensive need to increase the storage capacities of elevated tanks. Therefore, elevated tanks have been built using different construction materials (e.g. reinforced concrete, partially pre-stressed concrete, or steel), and different shapes (e.g. rectangular, cylindrical, and conical) in order to obtain the optimum capacity in a safe and economical manner. Figure 1-1 shows different shapes of elevated tanks.


Figure 1-1 Different Configurations of Elevated Tanks

A typical elevated conical tank consists of a tower that supports the superstructure (i.e., storage conical vessel). This tower usually has a shape of cylindrical shaft constructed of reinforced concrete. The geometry of the conical vessel can take two configurations, including pure conical and combined conical cylindrical shapes. A pure conical tank is defined as a vessel that has a pure truncated conical geometry (Figure 1-2), while a combined conical tank refers to a conical vessel that has a superimposed top cylindrical cap (Figure 1-3).


Elevated conical tanks are considered as one of the most popular constructions since they provide greater liquid retaining capacity for the same base radius of a cylindrical counterpart. These tanks require also lower height of water for the same containing volume of the cylindrical shape. Consequently, the hydrostatic pressure acting on the vessel base is minimized, leading to an increase in its structural efficiency. Moreover, a large containing volume can be achieved without having the base over hanged and cantilevered from the supporting tower as in the case of elevated cylindrical tanks.

Conical tanks are more economical than cylindrical tanks that have the same capacity. The total cost of reinforced concrete cylindrical tanks is $18 \%$ to $40 \%$ more than that of conical tanks having the same capacity, as stated by Barakat and Altoubat (2009).

As mentioned earlier, different materials can be used in the construction of such storage tanks. Selecting the proper construction material depends mainly on various criteria, including; required storage capacity, service life, structural performance, construction and operation cost. According to Meier (2002), steel is widely used as the construction material for tanks built in Canada and USA over the last 25 years. This is related to the fact that such tanks provide high tension resistance and lighter own weight compared to those constructed from reinforced concrete. The drawback of using steel as a construction material for liquid-filled tanks is that steel vessels might suffer from corrosion, buckling and geometric imperfections. On the other hand, reinforced concrete tanks provide high resistance to compression stresses and they have long service life (i.e., up to 50 years) compared to steel tanks (i.e., up to 20 years). However, the main concern about reinforced concrete tanks is related to the low tensile strength and the large required wall thickness which leads to a significant own weight (Cheremisinoff, 1996).

### 1.2 Objectives of the Study

The main objectives of the present research are as follows:

1. Investigate the applicability of the available design provisions when applied to design reinforced concrete conical tanks
2. Develop a simple procedure in the form of design charts for analysing and designing liquid-filled reinforced concrete conical tanks. These charts are developed based on
coupling code requirements with a set of data obtained from finite element analysis of a number of tanks covering a wide practical range of geometrical parameters.
3. Utilize the simplified approach developed in step 2 to perform cost analysis for reinforced concrete tanks and investigate the economics of these concrete tanks versus steel counterparts.

### 1.3 Scope of the Thesis

This thesis has been prepared in "Monograph" format. This chapter introduces the general background and the main objectives of this research. In the next chapter, a review of previous researches and current available design codes as well as the motivation for the study are presented. The following three chapters address the objectives of this research. Chapter 6 presents relevant conclusions of the study together with suggestions for further research.

### 1.3.1 Analysis and Design of Reinforced Concrete Conical Tanks

In chapter 3, several reinforced concrete conical tanks subjected to hydrostatic pressure are analyzed and designed. Two different analysis methods are utilized to evaluate the internal forces for each tank. The first method follows a simplified approach provided by Portland Cement Association for concrete circular tanks combined with equivalent cylinder method to transfer conical shape tanks to equivalent cylinders. The second method is a linear Finite Element Analysis model built in-house and is based on a degenerated consistent sub-parametric shell element. A parametric study is conducted for a wide range of conical tanks with different configurations in order to compare the results
obtained from these two analysis methods. The results of this parametric study are utilized to assess the adequacy of code provisions available for cylindrical tanks when applied on conical shape vessels.

### 1.3.2 Simplified Design Charts for Reinforced Concrete Conical Tanks under

## Hydrostatic Loading

The objective of chapter 4 is to develop simplified design charts in order to design reinforced concrete conical tanks under the effect of hydrostatic pressure. A number of conical tanks are analyzed using a built in-house finite element model that is based on a degenerated consistent shell element. These tanks are initially designed to comply with the recommendations of both American Concrete Institute for liquid retaining structures (ACI350-06), and Portland Cement Association guidelines for the analysis and design of circular concrete tanks (PCA-CCTWP, 1993). Finally, a comparison between the finite element model and design charts is conducted to validate the accuracy of the developed charts. Useful conclusions are achieved from this study.

### 1.3.3 Cost Analysis of Conical Tanks; Comparison between Reinforced

## Concrete and Steel

In chapter 5, the economics of reinforced concrete conical tanks are investigated by comparing the cost of reinforced concrete and steel as construction materials used for such tanks. The design charts, which are introduced in chapter 4, are employed to design a number of reinforced concrete conical tanks having different capacities. The steel conical tanks are designed by using a simplified approach that was developed in a previous investigation. The cost analysis is implemented for each of the concrete and
steel tanks. This analysis includes the cost of materials, formwork, labour and life-cycle cost. Also, a general study of the effect of tank dimensions is presented.

## CHAPTER 2 <br> LITERATURE REVIEW ON CONICAL TANKS AND DESIGN PROVISIONS

### 2.1 Introduction

This chapter presents a review of the available literature regarding conical tanks and code provisions for design of reinforced concrete tanks.

### 2.2 Conical Tanks

Design of axisymmetric structures (e.g. conical tanks) depends on the concept of surface of revolution that is developed by the rotation of the curved surface (i.e. conical tank wall) about the vertical axis lying in the same plane. Based on this concept, Ghali (1979) presented an analytical method for the evaluation of circular cylindrical tanks subjected to hydrostatic pressure. According to Ghali (1979), it is sufficient to consider an element strip of one meter along the circumference of the wall and parallel to the cylinder axis. Under the effect of axisymmetric loading, the wall strip is assumed to deflect as a beam on elastic foundation. Therefore, Ghali presents the general elastic solution for circular tanks that is based on finite difference method. This method has been applied to conical shape tanks but without taking into account the effect of vertical components of the hydrostatic pressure. Hilal (1988) utilized the theory of plates and shells to design a paste tank having a funnel shape by presenting a set of equations to determine the hoop and meridional moments at different heights of the tank. This method is based on static analysis and is found to lead to conservative design. In 2000, Ghali extended his work to include a one dimensional straight finite element represented as a conical frustra and can
be generalized for any shape. However, this element does not account for the spurious shear modes and locking phenomenon.

Intensive studies on liquid containing conical tanks started after a catastrophic failure of a steel conical water tank in Belgium in 1978. One of these studies was initiated at Ghent University by Vandepitte et al. (1982). The research was mainly conducted experimentally. A large number of small-scale conical vessel models were constructed. The models had different dimensions and were made of different materials. The experiments were conducted by gradually increasing the height of water inside the models. The water height at which each model buckled was detected. The experimental results were employed to develop a set of equations that can be used to assess the stability of conical tanks. Later on, Bornscheuer et al. (1983) studied the elasto-plastic behaviour of conical vessels using a degenerated shell element. The results of their study showed that the buckling strength of the studied tanks is significantly reduced by the presence of axisymmetric imperfections.

In 1990, another catastrophic failure of an elevated conical tank occurred in Fredericton, Canada. Vandepitte (1999) related this failure to the inappropriate thickness of the lower part of the tank. The miscalculation of the thickness was attributed to the reason that the designer used buckling formulae that are valid for aerospace applications where the quality of the manufacturing is much higher than that in civil projects. Another investigation that was conducted by El Damatty et al. (1997) studied the elastic stability of a conical vessel under hydrostatic load assuming perfect shells. In their study, the imperfection shape, which leads to the lowest limit load, was determined by conducting elastic analyses of conical tanks with different imperfections. The study indicated that the
limit load for hydrostatically loaded conical vessels is reduced by the presence of an axisymmetric imperfection shape resulting from welding of curved steel panels. Furthermore, it was noticed that the smaller the thickness of a conical tank, the more the structure is sensitive to geometric imperfections. Lagae et al. (2007) conducted a numerical simulation of steel conical tanks with large axisymmetric imperfections. This study showed that circumferential stresses are increased by axisymmetric imperfections causing local yielding to precipitate the buckling failure. All of these previous studies focused mainly on the behaviour of steel conical tanks which shows complexity in the analysis and design of these storage vessels. This complexity motivates Sweedan and El Damatty (2009) to develop a simplified procedure for the design of hydrostatically loaded combined conical tanks. In this procedure, a magnification function was provided in order to relate the maximum overall stresses developed in the tank walls to the theoretical membrane stresses resulting from static equilibrium of the shell under internal hydrostatic pressure.

Despite all these previous studies related to conical tanks, which focused on steel as a construction material, there is a lack in the literature regarding reinforced concrete conical tanks. The literature shows only few records for collapses of reinforced concrete elevated conical tanks occurred mainly during past earthquakes. For example, in 1997, the Jabalpur earthquake caused failure of elevated conical water tank having a capacity of $2270 \mathrm{~m}^{3}$. Another three reinforced concrete conical tanks collapsed during the Bhuj earthquake in 2001 (Rai, 2002). These failures grabbed the attention of a number of researchers to study the behaviour of such tanks. Most of these studies focused on the
supporting shafts and did not investigate the conical vessel itself (Rai 2003, Sezen et al. 2008, Dutta et al. 2009).

Barakat and Salah (2009) introduced an application of optimization techniques, which were combined with the finite element method, in the analysis and design of reinforced concrete conical and cylindrical water tanks. The finite element model was based on a 4node axisymmetric quadrilateral shell element. In addition, they illustrated the effect of different parameters on the optimum design. This study concluded that shear strength and crack width are the governing criteria that determine the optimum design based on working stress design method while crack width is the governing requirement in the strength-based formulation. According to Barakat and Altoubat (2009), the total cost for cylindrical tanks is found to be more than that for conical water tanks of the same volume by ( $20 \%$ to $30 \%$ ) and by ( $18 \%$ to $40 \%$ ) when working stress design method and strength design method are used, respectively.

Based on the information presented above, it can be concluded that most of the published literature studied the structural behaviour of steel conical tanks. However, there is scant data available for the design and performance of reinforced concrete tanks. Moreover, the effect of different loads, e.g. dead, hydrostatic, and earthquake loads, on the behaviour of the supporting shaft of a reinforced concrete tank was investigated using finite element methods, while there is no clear understanding of the behaviour of the conical vessel of these tanks.

### 2.3 Design Codes for Reinforced Concrete Tanks

Since 1940, reinforced concrete liquid storage tanks and their properties have long been studied. Earlier studies done by Slater (1940), Gray (1948), Timoshenko and WoinowskiKrieger (1959), and Wilby (1977) provided the basis for the design of such tanks. Those researchers proposed that the analysis of reinforced concrete tanks can be based on a linear approach.

The theory of plates and shells, which was introduced by Timoshenko and WoinowskiKrieger (1959), indicated that all problems of symmetrical deformation of cylindrical shells under uniformly distributed load can be expressed as a function of radial displacement at an arbitrary height. Latterly, Ghali (1979) presented an analytical method for the evaluation of circular cylindrical tanks subjected to hydrostatic pressure. According to Ghali (1979), it is sufficient to consider an element strip of one meter along the circumference of the wall and parallel to the cylinder axis. He proposed that such a wall strip behaves as a beam on elastic foundation. Due to the inclination of the tank's wall, the behaviour of reinforced concrete conical tanks under hydrostatic loading differs from that of circular cylindrical tanks.

In addition to structural requirements, reinforced concrete tanks have to satisfy the durability needs, leading to functional success during the service life. Grieve et al. (1987) presented a report to investigate and inspect several above ground reinforced concrete tanks in Ontario. According to this report, 53 above ground tanks, which were constructed during the period of 1956 to 1980, suffered from deteriorations and cracks, and functionally failed although they were structurally accepted. This report reflects the importance of durability and service life of reinforced concrete tanks. During that time
and later on, many committees have been established to analyze and design liquid retaining structures, including: Portland Cement Association Circular Concrete Tanks without Pre-stressing (PCA-CCTWP), American Concrete Institute Design: Considerations for Environmental Engineering Concrete Structures (ACI350-06), British Standard Institute: Code of Practice for the Design of Concrete Structures for Retaining Aqueous Liquids (BS 8007), and Eurocode: Design of Concrete Structures - Part 3 Liquid Retaining and Containment Structures (EN 1992-3). The PCA- CCTWP and ACI350-06 are globally used while BS 8007 and Eurocode are the predominated choice in Europe.

Recently, many researchers investigated the behaviour and design of reinforced concrete tanks, especially for rectangular and cylindrical shape tanks. To the best of the author's knowledge, there are no particular provisions or standards that are available for concrete conical tanks. However, it is important to review current design codes that provide recommendations and standards for cylindrical and rectangular reinforced concrete tanks. This can help in establishing future procedures for reinforced concrete conical tanks. The majority of designers use strength design method, yet some still use working stress design approach. In the working stress method, stresses are kept at fairly low levels to minimize cracking, which leads to prevention of leakage. On the other hand, strength design method deals with cracked section analysis, which may not sufficiently address the leakage problem in liquid-filled structures.

### 2.3.1 American Concrete Institute Guidelines

In 1964, the American Concrete Institute (ACI) established a committee (ACI350-64) to provide guidelines for the design of liquid retaining reinforced concrete structures. The
most recent version of this guideline is the ACI 350-06. The basic design philosophy of this code is to reduce the stresses on the reinforcement under the effect of applied working stress. This code (ACI 350-06) provides an expression to calculate the maximum stress in the steel, which should not be exceeded, in order to keep the crack width less than the allowable width.

ACI 350-06 limits the flexural cracks to 0.27 mm and 0.23 mm for normal and sever environmental exposures, respectively. It should be mentioned that ACI 350-06 refers to another code ACI 224R-01 to control the cracking in environmental engineering concrete structures such as elevated conical tanks. Conservatively, ACI 224R-01 specifies 0.1 mm as the maximum allowable crack width in order to protect the steel reinforcement from corrosion. For resisting the shrinkage and temperature effects, ACI 350-06 provides minimum reinforcement ratio to be ranging between $0.3 \%$ and $0.6 \%$. Moreover, ACI 350-06 limits the minimum thickness to 300 mm for walls equal to or higher than 3 m height. ACI 350-06 refers to the Portland Cement Association code of practice (PCACCTWP, 1993) for the analysis and design of cylindrical concrete tanks while it does not refer to any codes for the design of conical shaped tanks.

### 2.3.2 Portland Cement Association Guidelines

The Portland Cement Association (PCA) provided guidelines for analysis and design of rectangular and cylindrical reinforced concrete tanks (PCA 1942, 1963, 1981, and 1993). However, PCA does not specify any recommendations for conical shape tanks. The most widely used code for design of cylindrical concrete tanks is PCA-CCTWP (1993). The advantage of this code over others is that it provides guidelines for the carrying capacity
of the stresses in concrete resulting from ring tension. It includes coefficients for evaluating the ring tensions, moments and shears in cylindrical tank walls.

Several studies were published to investigate the discrepancy between PCA-CCTWP and other design guidance. Godbout et al. (2003) evaluated analytically the internal forces in a cylindrical tank wall and compared them with to those obtained from PCA-CCTWP-93. It was concluded that the estimated internal forces developed in the cylindrical walls (i.e., circumferential tensions and vertical bending moments) agreed well with the code results. Bruder (2011) evaluated the internal forces of the walls of cylindrical concrete tanks with a conical base. He used two different analysis methods; PCA-CCTWP and finite element analysis (FEA). It was reported that there was a disagreement between these two methods. Also, Bruder (2011) concluded that FEA should be employed if the tank parameters (e.g. shape, load cases, and boundary conditions) are not covered by PCACCTWP. However, this study did not present any information about the analysis of the conical part of the tank.

### 2.4 Conclusions

Elevated reinforced concrete conical tanks are widely used for storage of different liquids since they provide greater capacity with lower liquid height. Although most codes provide guidelines for analysis and design of reinforced concrete rectangular and cylindrical tanks, no guidance is provided in such codes for conical shape tanks. The literature shows that most of the previous studies focused on the structural behaviour of steel conical tanks. However, there is scant data available for the design and behaviour of reinforced concrete tanks. It is important to review the current design codes that provide
recommendations and standards for reinforced concrete tanks. These codes which mainly provide guidelines for cylindrical tanks might be employed to design reinforced concrete conical tanks.

FEA is a predominate choice for analysis of conical tanks due to the complexity of the analysis of these conical shaped tanks. The parameters that lead to such complexity include the angle of inclination of the tank wall with the vertical axis, total height of the tank and the base radius. Therefore, there is an extensive need to establish a simplified method for design and analysis of such tanks that can be used separately or in conjunction with FEA models, leading to an economical and safe design.

## CHAPTER 3

## ANALYSIS AND DESIGN OF REINFORCED CONCRETE CONICAL TANKS

### 3.1 Introduction

Elevated tanks are playing very important role in municipal systems. Their roles are to contain different liquids at sufficient head pressure, and to satisfy the emergency requirements after any disaster that might happen. Elevated tanks are made from steel, reinforced concrete, or partially pre-stressed concrete. Moreover, they can take different shapes such as rectangular, cylindrical or conical. Reinforced concrete is used as a construction material for elevated conical tanks because of its advantages such as strength, durability, low maintenance cost and high buckling resistance compared to steel counterparts.

The structural design of reinforced concrete conical tanks includes selection of adequate wall thickness, circumferential and longitudinal reinforcement steel, and related detailing. Serviceability is the most important design requirement for such a type of structures and mainly governs the design. Moreover, the design of conical tanks has to satisfy the general requirements of environmental engineering concrete structures specified by ACI350 (2006). Although most design codes provide guidelines for rectangular and cylindrical tanks, no guidance is provided in such codes for conical tanks. The analysis and design of conical tanks under the effect of hydrostatic pressure incorporates the presence of many parameters, including the angle of inclination of the tank wall with the vertical axis, total height of the tank and the base radius. The evaluation of the internal
forces in a conical tank wall is not an easy task because of the complicated state of stresses that includes bending stresses as well as membrane stresses. In addition, current available codes do not provide any guidelines to evaluate these internal forces.

Consequently, two different analysis methods are presented in this study. The first method follows a simplified approach given in the Portland Cement Association for Concrete Circular Tanks without Pre-stressing (PCA-CCTWP, 1993). Although, this approach is specific for cylindrical tanks, an attempt is made in this study to extend it to conical tanks. This is done by combining this approach with a procedure to transform the geometry of a conical tank to an equivalent cylinder based on the information provided in the American Water Works Association AWWA-D100 (2005). The second method is based on a Finite Element Analysis (FEA) model built in-house using a degenerated consistent sub-parametric shell element developed by Koziey and Mirza (1997). In this study, both methods are used to investigate the behaviour of reinforced concrete conical tanks.

Elevated conical tanks can be subjected to different types of loading such as hydrostatic pressure, earthquake and wind loads. The current study only focuses on the effect of axisymmetric hydrostatic pressure. The study proceeds by first analyzing a number of conical tanks having different practical geometric parameters using the PCA-CCTWP procedure. This involves trials that are carried out in order to obtain the required thickness and the amount of reinforcement that comply with the recommendations of ACI350-06. Second, the designed thickness, which is obtained from PCA-CCTWP, is used in the FEA to model the chosen conical tanks. Third, a comparison is conducted between the internal forces obtained from PCA-CCTWP procedure and those predicted
by the FEA. Finally, the wall section, which was first designed by PCA-CCTWP approach, is checked for ultimate strength requirements of ACI350-06 under the internal forces determined by FEA.

### 3.2 Forces Due to Hydrostatic Pressure

The weight of the contained liquid exerts an internal hydrostatic pressure on the tank walls. The hydrostatic pressure varies linearly along the wall height while it is constant along the circumferential direction of the wall as shown in Figure 3-1. In cylindrical tanks, the horizontal hydrostatic pressure results in outward displacement that is prevented due to the symmetry of the tank vessel, leading to both hoop tension force and meridional moment. The hoop tension acts on the vertical segment of the circular wall (i.e., circumferential direction) while the meridional moment acts along the horizontal segment (i.e., longitudinal direction). In case of conical vessels, the inclination of the wall of the tank complicates the state of stresses. In such conical shaped tanks, the horizontal segment of the wall is subjected to an additional meridional axial compression that is constant along the circumferential direction but varies along the longitudinal direction of the wall (Figure 3-2). The magnitude and distribution of these stresses are based on several parameters, including the tank height, tank base diameter, angle of inclination with the vertical, and wall thickness.


Figure 3-1 Axisymmetric Loading Conditions


Figure 3-2 Wall Segments of a Conical Tank
Vertical segment in the circumferential direction and Horizontal segment in the longitudinal direction

### 3.3 Analysis Approach

This study employs the two different methods described earlier for the analysis and design of conical tanks. The first method is based on using PCA-CCTWP provisions for cylindrical concrete tanks combined with a procedure to transform conical tanks into equivalent cylinders. The second method is based on a FEA model, which is recommended for tank shapes that fall outside the parameters outlined by PCA-CCTWP (Bruder 2011).

### 3.3.1 PCA-CCTWP Analysis Method

Portland Cement Association provides a commonly used publication known as circular concrete tanks without pre-stressing (PCA-CCTWP). In this publication, tabled coefficients, including $C_{H}$, and $C_{M}$, are presented in order to simplify the evaluation of different forces in the walls of a liquid-filled circular tank under different support conditions. The provided coefficients are based on theory of plates and shells (Kamara 2010). PCA-CCTWP does not account for the meridional compression resulting from the self-weight of the circular walls. The tabled hoop coefficient $C_{H}$ is function of the ratio $\frac{H_{c y}{ }^{2}}{D_{c y} \cdot t}$, where $H_{c y}$ is the total height of the cylindrical tank, $D_{c y}$ is the diameter of the circular tank, and $t$ is the wall thickness. Based on the equations provided by PCACCTWP, the internal forces acting at different heights of the circular wall can be calculated by applying Equations 3.1 and 3.2 for evaluating the hoop tension $(T)$ and meridional moment $(M)$, respectively.

$$
\begin{equation*}
T=C_{H} \gamma_{w} H_{c y} R_{c y} \tag{3.1}
\end{equation*}
$$

$M=C_{M} \gamma_{w} H_{c y}{ }^{3}$

Where, $T$ is the service hoop tension force per unit length acting on a vertical segment of the wall, $C_{H}$ is the hoop coefficient at different heights of the wall, which depends on the value of $\frac{H_{c y}{ }^{2}}{D_{c y}, t}$ as shown in Table A. 1 in appendix A, $\gamma_{\mathrm{w}}$ is the liquid unit weight, $M$ is the service meridional moment per unit length acting on a horizontal segment of the wall, and $C_{M}$ is the moment coefficient at different heights of the wall. Table A. 2 presents the values of $C_{M}$, according to the ratio $\frac{H_{c y}{ }^{2}}{D_{c y} .}$.

In order to apply this procedure on conical tanks, such vessels have to be transformed to equivalent cylinders. The procedure provided by AWWA-D100 (2005) recommendations for such transformation is used in the current study. The AWWA-D100 predicts that the behaviour of the steel conical tank is simulated by an equivalent geometry of a cylinder having the same thickness and projected perpendicular to the longitudinal axis of the cone. The equivalent diameter of the cylinder is taken as the average of the top and bottom diameter of the original conical tank.

This transformation approach can be applied using the following equations.
$H_{c y}=\frac{H}{\operatorname{Cos} \theta_{v}}$
$R_{c y}=\frac{2 R_{b}+H \tan \theta_{v}}{2 \operatorname{Cos} \theta_{v}}$
$t_{c y}=t$

Where, $H_{c y}$ and $R_{c y}$ are the height and the radius of the equivalent cylinder, respectively. $H$ is the total height of the conical tank, $\theta_{V}$ is the angle of inclination of the meridian with the vertical, $R_{b}$ is the base radius of the conical tank, $t_{c y}$ is the wall thickness of the equivalent cylindrical tank, and $t$ is the wall thickness of the conical tank.

### 3.3.2 Finite Element Analysis Method

In this study, a finite element model based on a degenerated consistent sub-parametric shell element is used. The consistent shell element is an excellent tool to analyze plates and shell structures. It was successfully used in several previous studies and was validated versus many experimental and numerical results (e.g., Koziey and Mirza 1997; El Damatty et al. 1997, 1998; Sweedan and El Damatty 2002). This element was developed by Koziey and Mirza (1997) and extended by El Damatty et al. (1997) to include the geometric nonlinear effects. The consistent shell element has two main advantages. First, it eliminates the spurious shear modes and locking phenomenon observed when many isoparametric elements are used to model shell structures. Second, its formulation includes special rotational degree of freedom that lead to cubic variation of the displacement through the thickness. As such, quadratic transverse shear strain and shear stress can be predicted by this element. This feature is very useful in analyzing thick plates and shell structures such as reinforced concrete conical tanks.

The formulation of the element, which has 13 nodes, as shown in Figure 3-3, includes three displacement degrees of freedom; $u, v$, and $w$ along the global $x, y$, and $z$ coordinates, respectively, and four rotational degrees of freedom $\alpha, \beta, \phi$, and $\psi$ acting at the corner and mid-side nodes. Both $\alpha$ and $\phi$ are about local axis $y^{\prime}$, and $\beta$ and $\psi$ are
about local axis $x^{\prime}$, where the local axes $y^{\prime}$ and $x^{\prime}$ are located in a plane tangent to the surface. Rotations $\alpha$ and $\beta$ are constant through the depth of the element, while rotations $\phi$ and $\psi$ vary quadratically. Thus, $\alpha$ and $\beta$ provide a linear variation of displacements $u, v$, and $w$ along the thickness representing bending deformations, while $\phi$ and $\psi$ lead to a cubic variation of displacements $u, v$, and $w$, simulating transverse shear deformations. The shape functions of the consistent shell element are given in Appendix B.


Figure 3-3 Consistent Shell Element Coordinate System and Nodal Degrees of Freedom

Both the load acting on the tank walls resulting from hydrostatic pressure and the tank geometry are symmetric about two perpendicular axes located in the cross sectional plan of the tanks. Accordingly, one quarter of the tank is modeled in the analysis. The vertical projection of a typical finite element mesh for one quarter of a conical vessel is shown in

Figure 3-4. As shown in this figure, the mesh is developed using 256 triangular elements, with 8 and 16 rectangular divisions along the circumferential and longitudinal directions, respectively. A finer mesh is applied at the bottom region of the vessel where stress concentration is anticipated near the tank base. The mentioned mesh size is selected based on a sensitivity analysis that is conducted for one of the tanks $\left(\theta_{\mathrm{v}}=45^{\circ}, \mathrm{H}=7 \mathrm{~m}\right.$, $\mathrm{R}=3.5 \mathrm{~m}, \mathrm{t}=200 \mathrm{~mm}$ ) using different mesh sizes under the same hydrostatic pressure as presented in Table 3-1. This optimum mesh size which yields to accurate radial displacements has been used for all other studied tanks. At the base of the vessel, the boundary conditions are assumed to be simply supported. The tank wall is assumed to be hinged at the base since the hoop tension predicted following this assumption is greater than that in case of fixed bottom edge of the wall (CPA-CCTWP, 1993). A free edge boundary condition is assumed at the top of the conical vessel. This assumption is valid since the hydrostatic pressure at the top is negligible and the radial displacement at the top is so small (El Damatty et al. 1997).


Figure 3-4 Finite Element Mesh of a Quarter Cone

Table 3-1 Mesh Sensitivity Analysis

| Mesh Size ${ }^{(*)}$ | Max Radial <br> Displacement <br> $(\mathbf{m m})$ | Max Hoop <br> $(\mathbf{k N} / \mathbf{m})$ | Max <br> Meridional <br> Meridional <br> $(\mathbf{k N} . \mathbf{m} / \mathbf{m})$ | Max <br> Meridioanl <br> Compression <br> $(\mathbf{k N} / \mathbf{m})$ |
| :---: | :---: | :---: | :---: | :---: |
| $4 \times 8$ | 1.1390 | 411.06 | 8.32 | 509.60 |
| $6 \times 12$ | 1.1330 | 412.19 | 8.52 | 521.98 |
| $8 \times 16$ | 1.1320 | 414.12 | 8.85 | 528.06 |
| $10 \times 20$ | 1.1310 | 414.15 | 8.84 | 527.86 |

(*) For tank dimensions of $\theta_{\mathrm{v}}=45^{\circ}, \mathrm{H}=7 \mathrm{~m}, \mathrm{R}=3.5 \mathrm{~m}, \mathrm{t}=200 \mathrm{~mm}$.

### 3.4 Cracking Mechanisms

Cracking in liquid retaining structures is of significant importance since cracks adversely affect the required serviceability, leading to functional failure (i.e. liquid leakage). In order to achieve a durable and safe design of concrete conical tanks, it is essential to understand the mechanism of cracking that may occur due to the effect of hydrostatic pressure. PCA-CCTWP presents two main cracking mechanisms, which occur due to hydrostatic pressure, including pure tension cracks and flexural cracks.

### 3.4.1 Pure Tension Cracking

The vertical segment of the tank wall resists pure tension stresses resulting from hoop forces. Since ACI 350-06 does not provide any explicit measurement to control the direct tension cracks, PCA-CCTWP prevents any direct tension cracks to form. The main reason behind this assumption is that direct tension cracks are normally full depth cracks, leading to water seepage even at low range of cracks' widths. These cracks lead to leakage, reinforcement corrosion and functional failure. A study that was conducted by

Ziari and Kianoush (2009) showed that leakage can begin at a very low crack width of 0.04 mm . Also, a reduction in the rate of leakage is observed over time due to selfhealing of cracks. The self-healing can be defined as the ability of concrete to heal its cracks which have a width less than 0.2 mm . When the water flows in the micro-cracks, it reacts with non-hydrated cement molecules to produce further limestone, which fills these cracks. The ACI350-06 code does not account for self-healing of cracks when exposed to water flow in case of water tanks although it was found that 0.2 mm width cracks can be sealed after seven weeks of continuous exposure to water (Ziari and Kianoush 2009).

Based on the requirements of PCA-CCTWP and ACI350-06, the applied tension stresses should not exceed the allowable tensile stress of concrete, which is considered as $10 \%$ of the compressive strength. The cracks are only governed by the tensile strength of concrete while the reinforcements control the crack width and do not prevent occurring of cracks. The applied tension stress includes the combined hoop tension and shrinkage effect acting on the area of the wall section transformed to concrete.

The tension stress in concrete can be calculated from Equation 3.6 that takes into account an explicit allowance for shrinkage.

$$
\begin{equation*}
f_{c}=\frac{C E_{s} A_{s}+T}{A_{c}+n A_{s}} \tag{3.6}
\end{equation*}
$$

Where $f_{c}$ is the applied tensile strength acting on the ring that should be less than the tensile strength of concrete $\left(0.1 \times f_{c}{ }^{\prime}\right), f_{c}{ }^{\prime}$ is the concrete compressive strength, $C$ is the shrinkage coefficient (i.e., $C=0.0003$ ), $E_{S}$ is the modulus of elasticity of horizontal
reinforcement steel, $A_{s}$ is the area of horizontal reinforcement per 1000 mm height section, $T$ is the non-factored ring hoop force per 1000 mm length resulting from the hydrostatic pressure, $A_{c}$ is the area of concrete for 1000 mm height section (i.e., $A_{c}=$ $1000 t-A_{s}$ ), $t$ is the wall thickness, $n$ is the modular ratio (i.e., $n=\frac{E_{s}}{E_{c}}$ ), and $E_{c}$ is the concrete modulus of elasticity.

### 3.4.2 Flexural Cracking

Although PCA-CCTWP and ACI350-06 don't allow direct tension cracks to occur, flexural cracking is allowed to be formed. This is justified by the fact that flexural cracks are less severe than the direct tension cracks. ACI350-06 limits the flexural cracks to 0.27 mm and 0.23 mm for normal and severe environmental exposures, respectively. The normal environmental exposure is defined as exposure to a liquid with a pH value greater than 5 or sulfate solutions of 1000 ppm or less, while the severe exposures are considered when these limits are exceeded (Kamara 2010). Moreover, ACI350-06 refers to other code ACI 224R (2001) to control the cracking in environmental engineering concrete structures, such as elevated conical tanks. Conservatively, ACI 224R specifies 0.1 mm as the maximum allowable crack width in order to protect the reinforcement from corrosion. This limit is followed by most of designers for this type of structures. ACI350-06 presents a special method to control the width of flexural cracks. This method is based on the Frosch model for predicting flexural cracking (Frosch, 1999). Frosch's model specifies the maximum crack spacing to be twice the controlling cover distance. ACI35006 specifies rules for the spacing of flexural reinforcement and for the allowable stresses that can be achieved by preventing the tensile stresses in the steel reinforcement from
exceeding the maximum allowable stresses specified by ACI350-06 (i.e., $\left.\left(\mathrm{f}_{\mathrm{s}}\right)_{\text {Applied Loads }} \ngtr\left(\mathrm{f}_{\text {s max }}\right)_{\text {specified by ACI350-06 }}\right)$.

These maximum allowable stresses are provided for non-compression controlled sections and are presented by Equations 3.7a and 3.7b.

For normal exposures, $f_{S \max }=\frac{5600}{\grave{\beta} \sqrt{S^{2}+4\left(50+d_{b} / 2\right)^{2}}}$
$140 \mathrm{MPa} \leq f_{\text {smax }} \leq 250 \mathrm{MPa}$, for one-way members
$165 \mathrm{MPa} \leq f_{\text {smax }} \leq 250 \mathrm{MPa}$, for two-way members

For severe exposures, $f_{\text {smax }}=\frac{45500}{\grave{\beta} \sqrt{S^{2}+4\left(50+d_{b} / 2\right)^{2}}}$
$115 \mathrm{MPa} \leq f_{\text {smax }} \leq 250 \mathrm{MPa}$, for one-way members
$140 \mathrm{MPa} \leq f_{\text {smax }} \leq 250 \mathrm{MPa}$, for two-way members

Where, $f_{\text {smax }}$ is the maximum allowable steel stress (MPa), $S$ is the bar spacing $(\mathrm{mm}), d_{b}$ is the bar diameter (mm), $\grave{\beta}$ is the strain gradient amplification factor $\grave{\beta}=1.2$ for a wall thickness $\geq 400 \mathrm{~mm}$ and 1.35 for a wall thickness $<400 \mathrm{~mm}$, and $f_{s}$ is the calculated stress in reinforcement at service loads ( MPa ), it can be calculated as the service moment divided by the product of steel area and internal moment arm, as shown in Equation 3.8. The steps to calculate the stress in reinforcement are shown in Equations 3.8 to 3.12.
$f_{s}=\frac{M}{A_{s} j d}$

$$
\begin{align*}
& j=1-\frac{K}{3}  \tag{3.9}\\
& K=\sqrt{2 \rho n+(\rho n)^{2}}-\rho n  \tag{3.10}\\
& d=t-\operatorname{cover}-\frac{d_{b}}{2}  \tag{3.11}\\
& \rho=\frac{A_{s}}{b d} \tag{3.12}
\end{align*}
$$

Where, $M$ is the service moment resulting from the applied loads (i.e., unfactored moment), $A_{s}$ is area of flexural reinforcement, $\rho$ is steel ratio, $b$ section width ( $b=$ $1000 \mathrm{~mm}), n$ is the modular ratio $n=\frac{E_{S}}{E_{c}}, E_{S}$ is the modulus of elasticity of flexural reinforcing steel, $E_{c}$ is the concrete modulus of elasticity, and $d_{b}$ is bar diameter.

### 3.5 Design Approach

The design of liquid containing tanks is generally governed by the serviceability requirements, including durability and leakage. Basically, the philosophy of serviceability limit state is to minimize the stresses applied on the reinforcing steel. This can be achieved by using the working stress approach. However, most of recent design codes are based on the ultimate strength approach. Consequently, ACI350-06 introduced the environmental durability factor as an additional load factor used for liquid retaining structures. This factor enables the designers to achieve the serviceability requirements through providing a sufficient amount of reinforcement steel.

### 3.5.1 Environmental Durability Factor

The last publication of PCA-CCTWP, which complies with the requirements of the old version of ACL350 (1989), requires that the lateral liquid pressure shall be multiplied by a load factor of 1.7. Moreover, sanitary durability factor (i.e., 1.65 for axial tension and 1.3 for flexural) should be provided to reduce the cracks, leading to a more conservative design. However, the last publication of ACI350-06, which is applied in this research, uses a load factor of 1.4 instead of 1.7 for both hydrostatic and dead loads. In addition, an environmental durability factor $S_{d}$ should be utilized. This factor is essential to reduce the stresses in the reinforcement steel, leading to fewer cracks. Consequently, durability and long term service life required for reinforced concrete conical tanks can be achieved. On the other hand, ACI350-06 does not recommend to apply the environmental durability factor for compression controlled sections since compression controlled members are subjected to lower tensile stress and associated low strain (i.e., less than or equal to 0.002), and the cracks are in minor concern. According to the ACI350-06, the strength of concrete should be greater than $S_{d} \cdot U$, where $U$ is the factored loads. The environmental durability factor $S_{d}$ can be calculated from Equation 3.13.

$$
\begin{equation*}
S_{d}=\frac{\varnothing f_{y}}{\gamma f_{s}} \tag{3.13}
\end{equation*}
$$

Where $\varnothing$ is the strength reduction factor, $(\varnothing=0.9$ for both hoop tension and flexural members), $f_{y}$ is the steel yield strength, $f_{s}=140 \mathrm{MPa}$ is the allowable stress in normal environment, and $\gamma=\frac{\text { factored load }}{\text { unfactored load }}=1.4$, in case of hydrostatic pressure and dead loads.

### 3.5.2 Wall Thickness

A reasonable wall thickness is required to satisfy the strength requirements, the allowed crack width, concrete cover and ease of construction. Based on the recommendations of PCA-CCTWP, which prevent forming of direct tension cracks, the wall thickness can be estimated using Equation 3.14, considering 1000 mm width of the wall.
$t=\frac{C E_{s}+f_{s}-n f_{c r}}{1000 f_{c r} f_{s}} \times T$

Where $f_{c}$ is the allowable concrete tensile strength $\left(f_{c r}=0.1 \times f_{c}{ }^{\prime}\right), f_{c}{ }^{\prime}$ is the concrete compressive strength, $f_{s}$ is the allowable stress in hoop tension $\left(f_{s}=140 \mathrm{MPa}\right), C$ is the coefficient of shrinkage $(C=0.0003), E_{S}$ is the modulus of elasticity of horizontal reinforcing steel, $T$ is the non-factored ring hoop force per 1000 mm length resulting from the hydrostatic pressure, $n$ is the modular ratio $n=\frac{E_{s}}{E_{c}}$, and $E_{c}$ is the concrete modulus of elasticity, (PCA-CCTWP, 1993).

Although several codes, such as BS2007, do not specify a minimum wall thickness, ACI350-06 provides a minimum thickness of 300 mm for walls equal to or higher than 3 m . The required concrete volume and the overweight of the wall depend on the minimum wall thickness required for constructability.

### 3.5.3 Wall Reinforcement

Since the tank walls are subjected to two types of stresses, including hoop tension stresses and meridional stresses, it is required to provide sufficient reinforcing steel for
both circumferential and longitudinal directions. Moreover, selection of bar diameter and distribution of reinforcements is important to design a leak free tank.

The horizontal reinforcement steel (i.e., circumferential reinforcement) is required to resist all hoop tension forces resulting from the hydrostatic pressure. According to PCACCTWP, the area of circumferential steel can be specified from the following expression ( $A_{S}=\frac{T_{u}}{0.9 \times f_{y}}$ ), where $T_{u}$ is the maximum factored hoop tension force magnified by the environmental durability factor $S_{d},\left(T_{u}=1.4 \times S_{d} \times T\right)$, where $T$ is the service hoop tension obtained from the analysis method, and $f_{y}$ is the steel yielding strength. It should be noted that the required area of horizontal steel should not be less than the minimum specified area (i.e. $0.06 \times A_{g}$ ), for walls without joints (ACI350, 2006).

Moreover, vertical reinforcement (i.e., longitudinal reinforcement) is provided to carry the forces applied on the horizontal segment of the wall. In cylindrical tanks, PCACCTWP specifies the vertical reinforcement to resist the flexural moment resulting from the hydrostatic pressure ignoring the compression normal force due to wall self-weight. In case of conical tanks, the vertical reinforcement resists both flexural moment and compression normal force, which has a large value compared to cylindrical tanks. That is related to the inclination of the vessel wall. Hence, the circumferential segment of the conical shaped tanks is to be designed as a compression member. These compression normal forces have a confinement effect which leads to a reduction in the crack width initiated in the circumferential segment. In such cases, check for crack width can be neglected except if the section is under large flexural moment compared to a small normal compression force. ACI350-06 requires a minimum vertical reinforcement for the
wall to be $\left(0.01 \times A_{g}\right)$, where $A_{g}$ is the concrete gross area and can be simplified to $(1000 \times t)$ for 1000 mm width section.

### 3.6 Parametric Study

Selection of an appropriate method for the analysis of liquid-filled reinforced concrete conical tanks is extremely important in order to determine the internal forces acting on the tank's wall. Two analysis tools, as previously presented, are available; the first is PCA-CCTWP and the second is FEA. The simplified approach presented by PCACCTWP can be used exclusively or as a preliminary design in order to determine the initial wall thickness and the internal forces developed in the tank wall due to hydrostatic pressure. The initial thickness obtained from PCA-CCTWP can be utilized in FEA models to predict the actual behaviour. This section presents a parametric study that is conducted to compare the two methods and to evaluate the discrepancy that may exist due to analysis assumptions and approximations in the approach used to transfer from a conical shape to an equivalent cylinder. The findings of this parametric study assist to understand the behaviour of reinforced concrete conical tanks under hydrostatic pressure.

In the first step of this parametric study, all studied conical tanks are transformed to equivalent cylinders using the AWWA-D100 (2005) procedure. The wall thickness of each equivalent cylindrical tank is then designed to comply with the requirements of PCA-CCTWP. The maximum forces (i.e., hoop tension and meridional moment) are obtained. In the second step, the designed thickness of the equivalent cylinder is used in the finite element analysis to model the conical tank in order to predict the maximum internal forces which are compared with those obtained from PCA-CCTWP. Finally, the
wall sections which are designed according to the forces resulting from the PCACCTWP, are checked under the effect of the internal forces obtained from the finite element analysis. For more illustration, Figure 3-5 presents a flowchart for the steps followed in this parametric study.


Figure 3-5 Flow Chart for the Steps of the Parametric Study

### 3.6.1 Assumptions for Analysis and Design

The study focuses on the behaviour of reinforced concrete tanks that have a pure conical shape. The walls of these tanks are assumed to be simply supported at the base while they are free on the top. The wall thickness is considered as constant along its height and designed according to the requirements of PCA-CCTWP. The tanks are analyzed under the hydrostatic loads resulting from the liquid weight. This liquid is assumed to be water and fully fill the tank vessel. The own weight of the tank vessel is not considered in the FEA model since it is ignored by the PCA-CCTWP method. Also, linear elastic behaviour of the material is assumed in both cases as the design of liquid tanks has to satisfy serviceability requirements by preventing cracks from initiating at any location of the concrete section. The dimensions and properties of the tanks in concern are chosen to cover a wide practical range of reinforced concrete conical tanks (Table 3-2).

Table 3-2 Tank Properties

| Property |  |
| :--- | :--- |
|  |  |
| Inner volume (Capacity) | $100 \sim 2000 \mathrm{~m}^{3}$ |
| Tank height $(H)$ | $3 \sim 8 \mathrm{~m},(1 \mathrm{~m}$ increment $)$ |
| Base radius $\left(R_{b}\right)$ | $3 \sim 5 \mathrm{~m},(1 \mathrm{~m}$ increment $)$ |
| Inclination angle $\left(\theta_{v}\right)$ | $15^{\circ} \sim 60^{\circ},\left(15^{\circ}\right.$ increment $)$ |
| Wall thickness $(t)$ | $300 \sim 700 \mathrm{~mm}$ |
| Concrete compressive strength $\left(f_{c}{ }^{\prime}\right)$ | 30 MPa |
| Concrete tensile strength $\left(f_{c r}\right)$ | 3 MPa |
| Steel yield strength $\left(f_{y}\right)$ | 400 MPa |
| Concrete modulus of elasticity $\left(E_{c}\right)$ | 24647 MPa |
| Steel modulus of elasticity $\left(E_{s}\right)$ | 200000 MPa |
| Poission ratio $(v)$ | 0.3 |
| Modular ratio $(n)$ | 8.1 |
| Liquid specific weight $\left(\gamma_{w}\right)$ | $10000 \mathrm{~N} / \mathrm{m}^{3}$ |
|  |  |

### 3.6.2 Parametric Study Results

In this section, the results of the parametric study are presented. Such study is conducted by considering 66 reinforced concrete conical tanks covering practical geometries with different vessel capacities. This parametric study is used to assess the adequacy of utilizing the code provisions used for cylindrical tanks when applied on conical shaped tanks. This is done by checking the wall thickness as well as reinforcements in both directions; horizontal and vertical that are obtained following code guidelines. Table 3-3 shows the maximum forces (i.e., hoop tension forces and meridional moments), required thickness, and the designed section based on PCA-CCTWP method. It should be noted that the thickness of the wall is mainly governed by the maximum hoop tension. Also, the minimum flexural reinforcement satisfies the ultimate strength requirement.

Table 3-3 Results of PCA-CCTWP Analysis Method

$$
\left(\theta_{v}=15^{\circ}\right)
$$

| No. | $\begin{aligned} & \mathbf{R}_{\mathbf{b}} \\ & (\mathbf{m}) \end{aligned}$ | $\begin{gathered} \mathbf{H} \\ (\mathbf{m}) \end{gathered}$ | $\begin{gathered} \text { Volume } \\ \left(\mathbf{m}^{3}\right) \end{gathered}$ | Equivalent Cylindrical Tank |  | Max. Hoop Tension (N/m) | Max. <br> Meridional Moment (N.m/m) | Section Design(PCA-CCTWP / ACI350-06) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\begin{gathered} \mathbf{t} \\ (\mathbf{m m}) \end{gathered}$ |  | $\rho_{\text {s }}$ |  |
|  |  |  |  | $\mathrm{H}_{\mathrm{cy}}(\mathrm{m})$ | $\mathrm{D}_{\mathrm{cr}}(\mathrm{m})$ |  |  | Horizontal Reinforcement | Vertical Reinforcement |
| 1 | 3 | 3 | 110 | 3.1 | 7.0 | 65682 | 3129 | 300 | 0.006 | 0.0075 |
| 2 |  | 4 | 158 | 4.1 | 7.3 | 104872 | 4191 | 300 | 0.006 | 0.0075 |
| 3 |  | 5 | 214 | 5.2 | 7.6 | 147018 | 5512 | 300 | 0.006 | 0.0075 |
| 4 |  | 6 | 277 | 6.2 | 7.9 | 190791 | 6792 | 300 | 0.006 | 0.0075 |
| 5 |  | 7 | 347 | 7.2 | 8.2 | 243763 | 8575 | 300 | 0.006 | 0.0075 |
| 6 |  | 8 | 426 | 8.3 | 8.4 | 295917 | 10033 | 300 | 0.007 | 0.0075 |
| 7 | 4 | 3 | 183 | 3.1 | 9.1 | 77941 | 4030 | 300 | 0.006 | 0.0075 |
| 8 |  | 4 | 260 | 4.1 | 9.4 | 125493 | 5475 | 300 | 0.006 | 0.0075 |
| 9 |  | 5 | 345 | 5.2 | 9.7 | 179538 | 6718 | 300 | 0.006 | 0.0075 |
| 10 |  | 6 | 439 | 6.2 | 9.9 | 233275 | 8678 | 300 | 0.006 | 0.0075 |
| 11 |  | 7 | 543 | 7.2 | 10.2 | 292470 | 10183 | 300 | 0.007 | 0.0075 |
| 12 |  | 8 | 656 | 8.3 | 10.5 | 359534 | 12627 | 300 | 0.009 | 0.0075 |
| 13 | 5 | 3 | 276 | 3.1 | 11.2 | 87971 | 4805 | 300 | 0.006 | 0.0075 |
| 14 |  | 4 | 386 | 4.1 | 11.5 | 146318 | 6698 | 300 | 0.006 | 0.0075 |
| 15 |  | 5 | 507 | 5.2 | 11.7 | 208566 | 8477 | 300 | 0.006 | 0.0075 |
| 16 |  | 6 | 639 | 6.2 | 12.0 | 275082 | 10280 | 300 | 0.007 | 0.0075 |
| 17 |  | 7 | 782 | 7.2 | 12.3 | 339177 | 12378 | 300 | 0.008 | 0.0075 |
| 18 |  | 8 | 936 | 8.3 | 12.6 | 417665 | 13990 | 300 | 0.010 | 0.0075 |

Table 3-3 (continued)
$\left(\theta_{\mathrm{v}}=30^{\circ}\right)$

| No. | $\begin{gathered} \mathbf{R}_{\mathbf{b}} \\ (\mathbf{m}) \end{gathered}$ | $\begin{gathered} \mathbf{H} \\ (\mathbf{m}) \end{gathered}$ | Volume ( $\mathrm{m}^{3}$ ) | Equivalent Cylindrical Tank |  | Max. Hoop <br> Tension ( $\mathbf{N} / \mathbf{m}$ ) | Max. <br> Meridional Moment (N.m/m) | Section Design(PCA-CCTWP / ACI350-06) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\begin{gathered} \mathbf{t} \\ (\mathbf{m m}) \end{gathered}$ |  | $\rho_{\text {s }}$ |  |
|  |  |  |  | $\mathrm{H}_{\mathrm{cv}}(\mathrm{m})$ | $\mathrm{D}_{\mathrm{cv}}(\mathrm{m})$ |  |  | $\begin{gathered} \text { Horizontal } \\ \text { Reinforcement } \\ \hline \end{gathered}$ | Vertical <br> Reinforcement |
| 19 | 3 | 3 | 143 | 3.5 | 8.9 | 92359 | 4426 | 300 | 0.006 | 0.0075 |
| 20 |  | 4 | 222 | 4.6 | 9.6 | 150922 | 6225 | 300 | 0.006 | 0.0075 |
| 21 |  | 5 | 321 | 5.8 | 10.3 | 218697 | 8182 | 300 | 0.006 | 0.0075 |
| 22 |  | 6 | 441 | 6.9 | 10.9 | 288451 | 10548 | 300 | 0.007 | 0.0075 |
| 23 |  | 7 | 584 | 8.1 | 11.6 | 379933 | 13317 | 300 | 0.009 | 0.0075 |
| 24 |  | 8 | 753 | 9.2 | 12.3 | 473907 | 16240 | 300 | 0.011 | 0.0075 |
| 25 | 4 | 3 | 226 | 3.5 | 11.2 | 107553 | 5546 | 300 | 0.006 | 0.0075 |
| 26 |  | 4 | 339 | 4.6 | 11.9 | 176503 | 7727 | 300 | 0.006 | 0.0075 |
| 27 |  | 5 | 476 | 5.8 | 12.6 | 257958 | 9840 | 300 | 0.006 | 0.0075 |
| 28 |  | 6 | 638 | 6.9 | 13.2 | 344142 | 12883 | 300 | 0.008 | 0.0075 |
| 29 |  | 7 | 827 | 8.1 | 13.9 | 433786 | 15671 | 300 | 0.010 | 0.0075 |
| 30 |  | 8 | 1045 | 9.2 | 14.6 | 550689 | 17308 | 300 | 0.013 | 0.0075 |
| 31 | 5 | 3 | 327 | 3.5 | 13.5 | 120668 | 6462 | 300 | 0.006 | 0.0075 |
| 32 |  | 4 | 482 | 4.6 | 14.2 | 202552 | 9257 | 300 | 0.006 | 0.0075 |
| 33 |  | 5 | 663 | 5.8 | 14.9 | 293220 | 12047 | 300 | 0.007 | 0.0075 |
| 34 |  | 6 | 873 | 6.9 | 15.5 | 394721 | 14674 | 300 | 0.009 | 0.0075 |
| 35 |  | 7 | 1114 | 8.1 | 16.2 | 496611 | 18327 | 300 | 0.012 | 0.0075 |
| 36 |  | 8 | 1387 | 9.2 | 16.9 | 603129 | 23179 | 300 | 0.013 | 0.0076 |

Table 3-3 (continued)

$$
\left(\theta_{v}=45^{\circ}\right)
$$

| No. | $\begin{aligned} & \mathbf{R}_{\mathrm{b}} \\ & (\mathbf{m}) \end{aligned}$ | $\begin{gathered} \mathbf{H} \\ (\mathbf{m}) \end{gathered}$ | $\begin{gathered} \text { Volume } \\ \left(\mathrm{m}^{3}\right) \end{gathered}$ | Equivalent Cylindrical Tank |  | Max. Hoop <br> Tension (N/m) | Max. <br> Meridional Moment (N.m/m) | Section Design(PCA-CCTWP / ACI350-06) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\begin{gathered} \mathbf{t} \\ (\mathbf{m m}) \end{gathered}$ |  | $\rho_{\text {s }}$ |  |
|  |  |  |  | $\mathrm{H}_{\mathrm{cy}}(\mathrm{m})$ | $\mathrm{D}_{\mathrm{cy}}(\mathrm{m})$ |  |  | Horizontal Reinforcement | Vertical Reinforcement |
| 37 | 3 | 3 | 198 | 4.2 | 12.7 | 163656 | 7703 | 300 | 0.006 | 0.0075 |
| 38 |  | 4 | 331 | 5.7 | 14.1 | 273859 | 11188 | 300 | 0.007 | 0.0075 |
| 39 |  | 5 | 508 | 7.1 | 15.6 | 405426 | 15153 | 300 | 0.010 | 0.0075 |
| 40 |  | 6 | 735 | 8.5 | 17.0 | 548074 | 19987 | 300 | 0.013 | 0.0075 |
| 41 |  | 7 | 1019 | 9.9 | 18.4 | 697752 | 29628 | 350 | 0.016 | 0.0078 |
| 42 |  | 8 | 1366 | 11.3 | 19.8 | 858525 | 44280 | 425 | 0.014 | 0.0081 |
| 43 | 4 | 3 | 292 | 4.2 | 15.6 | 188238 | 9394 | 300 | 0.006 | 0.0075 |
| 44 |  | 4 | 469 | 5.7 | 17 | 312339 | 13577 | 300 | 0.007 | 0.0075 |
| 45 |  | 5 | 696 | 7.1 | 18.4 | 464477 | 17516 | 300 | 0.011 | 0.0075 |
| 46 |  | 6 | 980 | 8.5 | 19.8 | 624659 | 24992 | 320 | 0.014 | 0.0076 |
| 47 |  | 7 | 1327 | 9.9 | 21.2 | 788408 | 37378 | 380 | 0.015 | 0.0079 |
| 48 |  | 8 | 1743 | 11.3 | 22.6 | 957249 | 57412 | 480 | 0.014 | 0.0082 |
| 49 | 5 | 3 | 405 | 4.2 | 18.4 | 208578 | 10980 | 300 | 0.006 | 0.0075 |
| 50 |  | 4 | 633 | 5.7 | 19.8 | 350294 | 15894 | 300 | 0.008 | 0.0075 |
| 51 |  | 5 | 916 | 7.1 | 21.2 | 514717 | 20153 | 310 | 0.012 | 0.0076 |
| 52 |  | 6 | 1263 | 8.5 | 22.6 | 675038 | 33226 | 380 | 0.013 | 0.0079 |
| 53 |  | 7 | 1679 | 9.9 | 24 | 839095 | 51956 | 480 | 0.012 | 0.0082 |
| 54 |  | 8 | 2170 | 11.3 | 25.5 | 1030823 | 70964 | 550 | 0.013 | 0.0083 |

Table 3-3 (continued)

$$
\left(\theta_{\mathrm{v}}=60^{\circ}\right)
$$

| No. | $\begin{gathered} \mathbf{R}_{\mathrm{b}} \\ (\mathbf{m}) \end{gathered}$ | $\underset{(\mathbf{m})}{\mathbf{H}}$ | $\begin{gathered} \text { Volume } \\ \left(\mathbf{m}^{3}\right) \end{gathered}$ | Equivalent Cylindrical Tank |  | Max. Hoop Tension ( $\mathbf{N} / \mathbf{m}$ ) | Max. <br> Meridional Moment (N.m/m) | Section Design(PCA-CCTWP / ACI350-06) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  | $\underset{(\mathrm{mm})}{\mathbf{t}}$ |  | $\rho_{\text {s }}$ |  |
|  |  |  |  | $\mathrm{H}_{\mathrm{cy}}(\mathrm{m})$ | $\mathrm{D}_{\mathrm{cy}}(\mathrm{m})$ |  |  | Horizontal Reinforcement | Vertical <br> Reinforcement |
| 55 | 3 | 3 | 317 | 6 | 22.4 | 419787 | 19063 | 300 | 0.010 | 0.0075 |
| 56 |  | 4 | 575 | 8 | 25.9 | 679372 | 37174 | 380 | 0.013 | 0.0079 |
| 57 |  | 5 | 942 | 10 | 29.3 | 992392 | 64806 | 470 | 0.015 | 0.0082 |
| 58 |  | 6 | 1436 | 12 | 32.8 | 1316450 | 117150 | 630 | 0.015 | 0.0084 |
| 59 | 4 | 3 | 432 | 6 | 26.4 | 474885 | 22653 | 300 | 0.011 | 0.0075 |
| 60 |  | 4 | 750 | 8 | 29.9 | 746288 | 45187 | 400 | 0.013 | 0.0080 |
| 61 |  | 5 | 1188 | 10 | 33.3 | 1057166 | 81657 | 520 | 0.015 | 0.0083 |
| 62 |  | 6 | 1764 | 12 | 36.8 | 1413543 | 136682 | 660 | 0.015 | 0.0085 |
| 63 | 5 | 3 | 565 | 6 | 30.4 | 511590 | 27744 | 320 | 0.011 | 0.0076 |
| 64 |  | 4 | 951 | 8 | 33.9 | 804500 | 55548 | 430 | 0.013 | 0.0081 |
| 65 |  | 5 | 1466 | 10 | 37.3 | 1113460 | 106820 | 600 | 0.013 | 0.0084 |
| 66 |  | 6 | 2129 | 12 | 40.8 | 1512211 | 161218 | 700 | 0.015 | 0.0085 |

In the FEA method, the selected conical tanks are modeled using the consistent shell element described in the previous section. The wall thickness is assumed following PCACCTWP (1993) code design provisions. The internal forces acting on the wall are obtained, including hoop tension force, meridional moment and meridional normal compression force. The tank wall, which was previously designed by the PCA-CCTWP, is checked under the forces predicted by the FEA. Table 3-4 summarizes the maximum internal forces obtained from FEA. Moreover, it shows the results of the checked wall sections for both serviceability and ultimate strength requirements.

Table 3-4 Results of FEA Method

$$
\left(\theta_{\mathrm{v}}={15^{\circ}}^{\circ}\right)
$$

| No. | $\begin{gathered} \mathbf{R}_{\mathbf{b}} \\ (\mathbf{m}) \end{gathered}$ | $\begin{gathered} \mathbf{H} \\ (\mathbf{m}) \end{gathered}$ | Designed Section (PCA-CCTWP / ACI350-06) (From Table 3-2) |  |  | Мах. <br> Hoop Tension (N/m) | Max. <br> Meridional Moment (N.m/m) | Max. <br> Meridional Compression ( $\mathrm{N} / \mathrm{m}$ ) | Check Designed Sections along Wall Height |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\begin{gathered} \mathbf{t} \\ (\mathbf{m m}) \end{gathered}$ | $\rho_{\text {s }}$ |  |  |  |  |  |  |
|  |  |  |  | Horizontal | Vertical |  |  |  | Horizontal | Vertical |
| 1 | 3 | 3 | 300 | 0.006 | 0.0075 | 62395 | 18542 | 10410 | safe | safe |
| 2 |  | 4 | 300 | 0.006 | 0.0075 | 97056 | 24584 | 20524 | safe | safe |
| 3 |  | 5 | 300 | 0.006 | 0.0075 | 132480 | 30451 | 34311 | safe | safe |
| 4 |  | 6 | 300 | 0.006 | 0.0075 | 168121 | 37290 | 52026 | safe | unsafe |
| 5 |  | 7 | 300 | 0.006 | 0.0075 | 205422 | 43439 | 73915 | safe | unsafe |
| 6 |  | 8 | 300 | 0.007 | 0.0075 | 241638 | 48493 | 100179 | safe | unsafe |
| 7 | 4 | 3 | 300 | 0.006 | 0.0075 | 74133 | 24810 | 9662 | safe | safe |
| 8 |  | 4 | 300 | 0.006 | 0.0075 | 118840 | 32966 | 19245 | safe | safe |
| 9 |  | 5 | 300 | 0.006 | 0.0075 | 165283 | 41119 | 32321 | safe | unsafe |
| 10 |  | 6 | 300 | 0.006 | 0.0075 | 212343 | 48676 | 49021 | safe | unsafe |
| 11 |  | 7 | 300 | 0.007 | 0.0075 | 259149 | 57975 | 69518 | safe | unsafe |
| 12 |  | 8 | 300 | 0.009 | 0.0075 | 307959 | 66087 | 94007 | safe | unsafe |
| 13 | 5 | 3 | 300 | 0.006 | 0.0075 | 84247 | 30694 | 9127 | safe | safe |
| 14 |  | 4 | 300 | 0.006 | 0.0075 | 139174 | 41178 | 18328 | safe | unsafe |
| 15 |  | 5 | 300 | 0.006 | 0.0075 | 195366 | 51061 | 30927 | safe | unsafe |
| 16 |  | 6 | 300 | 0.007 | 0.0075 | 251917 | 61467 | 46985 | safe | unsafe |
| 17 |  | 7 | 300 | 0.008 | 0.0075 | 311230 | 71501 | 66592 | safe | unsafe |
| 18 |  | 8 | 300 | 0.010 | 0.0075 | 369749 | 82997 | 89997 | safe | unsafe |

Table 3-4 (continued)

$$
\left(\theta_{\mathrm{v}}=\mathbf{3 0}^{\circ}\right)
$$

| No. | $\begin{gathered} \mathbf{R}_{\mathbf{b}} \\ (\mathbf{m}) \end{gathered}$ | $\underset{(\mathbf{m})}{\mathbf{H}}$ | $\begin{gathered} \text { Designed Section } \\ \text { (PCA-CCTWP / ACI350-06) } \\ \text { (From Table 3-2) } \end{gathered}$ |  |  | Max. <br> Hoop Tension (N/m) | Max. <br> Meridional Moment (N.m/m) | Max. <br> Meridional Compression ( $\mathrm{N} / \mathrm{m}$ ) | Check Designed Sections along Wall Height |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\begin{gathered} \mathbf{t} \\ (\mathbf{m m}) \end{gathered}$ | $\rho_{\text {s }}$ |  |  |  |  |  |  |
|  |  |  |  | Horizontal | Vertical |  |  |  | Horizontal | Vertical |
| 19 | 3 | 3 | 300 | 0.006 | 0.0075 | 76842 | 22204 | 27886 | safe | safe |
| 20 |  | 4 | 300 | 0.006 | 0.0075 | 120848 | 29407 | 56124 | safe | safe |
| 21 |  | 5 | 300 | 0.006 | 0.0075 | 165731 | 36611 | 95676 | safe | safe |
| 22 |  | 6 | 300 | 0.007 | 0.0075 | 212595 | 45487 | 147754 | safe | unsafe |
| 23 |  | 7 | 300 | 0.009 | 0.0075 | 259751 | 53448 | 213479 | safe | unsafe |
| 24 |  | 8 | 300 | 0.011 | 0.0075 | 308316 | 62771 | 294076 | safe | unsafe |
| 25 | 4 | 3 | 300 | 0.006 | 0.0075 | 90307 | 29456 | 25486 | safe | safe |
| 26 |  | 4 | 300 | 0.006 | 0.0075 | 145870 | 38858 | 51560 | safe | unsafe |
| 27 |  | 5 | 300 | 0.006 | 0.0075 | 204159 | 48505 | 87820 | safe | unsafe |
| 28 |  | 6 | 300 | 0.008 | 0.0075 | 262847 | 58374 | 135166 | safe | unsafe |
| 29 |  | 7 | 300 | 0.010 | 0.0075 | 322450 | 69705 | 194450 | safe | unsafe |
| 30 |  | 8 | 300 | 0.013 | 0.0075 | 383702 | 79838 | 266569 | safe | unsafe |
| 31 | 5 | 3 | 300 | 0.006 | 0.0075 | 101987 | 36165 | 23781 | safe | safe |
| 32 |  | 4 | 300 | 0.006 | 0.0075 | 169263 | 48476 | 48405 | safe | unsafe |
| 33 |  | 5 | 300 | 0.007 | 0.0075 | 238816 | 60068 | 82584 | safe | unsafe |
| 34 |  | 6 | 300 | 0.009 | 0.0075 | 309138 | 71786 | 126971 | safe | unsafe |
| 35 |  | 7 | 300 | 0.012 | 0.0075 | 382426 | 85054 | 182272 | safe | unsafe |
| 36 |  | 8 | 300 | 0.013 | 0.0076 | 454066 | 98587 | 249173 | safe | Unsafe |

Table 3-4 (continued)

$$
\left(\theta_{\mathrm{v}}=45^{\circ}\right)
$$

| No. | $\begin{aligned} & \mathbf{R}_{\mathrm{b}} \\ & (\mathbf{m}) \end{aligned}$ | $\underset{(\mathbf{m})}{\mathbf{H}}$ | $\begin{gathered} \text { Designed Section } \\ \text { (PCA-CCTWP / ACI350-06) } \\ \text { (From Table 3-2) } \\ \hline \end{gathered}$ |  |  | Max. <br> Hoop Tension ( $\mathrm{N} / \mathrm{m}$ ) | Max. Meridional Moment (N.m/m) | Max. <br> Meridional Compression ( $\mathrm{N} / \mathrm{m}$ ) | Check Designed Sections along Wall Height |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\stackrel{\mathbf{t}}{(\mathbf{m m})}$ | $\rho_{\text {s }}$ |  |  |  |  |  |  |
|  |  |  |  | Horizontal | Vertical |  |  |  | Horizontal | Vertical |
| 37 | 3 | 3 | 300 | 0.006 | 0.0075 | 107817 | 29632 | 67842 | safe | safe |
| 38 |  | 4 | 300 | 0.007 | 0.0075 | 170599 | 39284 | 138909 | safe | unsafe |
| 39 |  | 5 | 300 | 0.010 | 0.0075 | 235697 | 50533 | 240966 | safe | unsafe |
| 40 |  | 6 | 300 | 0.013 | 0.0075 | 303569 | 63353 | 378091 | safe | unsafe |
| 41 |  | 7 | 350 | 0.016 | 0.0078 | 373990 | 76796 | 550687 | safe | unsafe |
| 42 |  | 8 | 425 | 0.014 | 0.0081 | 449513 | 91773 | 763268 | safe | unsafe |
| 43 | 4 | 3 | 300 | 0.006 | 0.0075 | 126177 | 38692 | 60996 | safe | unsafe |
| 44 |  | 4 | 300 | 0.007 | 0.0075 | 203264 | 50738 | 124951 | safe | unsafe |
| 45 |  | 5 | 300 | 0.011 | 0.0075 | 285049 | 63498 | 215839 | safe | unsafe |
| 46 |  | 6 | 320 | 0.014 | 0.0076 | 365711 | 79086 | 335396 | safe | unsafe |
| 47 |  | 7 | 380 | 0.015 | 0.0079 | 449286 | 95623 | 484670 | safe | unsafe |
| 48 |  | 8 | 480 | 0.014 | 0.0082 | 536551 | 113172 | 665784 | safe | unsafe |
| 49 | 5 | 3 | 300 | 0.006 | 0.0075 | 141350 | 47424 | 56367 | safe | unsafe |
| 50 |  | 4 | 300 | 0.008 | 0.0075 | 233672 | 62975 | 115748 | safe | unsafe |
| 51 |  | 5 | 310 | 0.012 | 0.0076 | 328765 | 78595 | 199103 | safe | unsafe |
| 52 |  | 6 | 380 | 0.013 | 0.0079 | 419228 | 95694 | 305021 | safe | unsafe |
| 53 |  | 7 | 480 | 0.012 | 0.0082 | 512256 | 113861 | 436950 | safe | unsafe |
| 54 |  | 8 | 550 | 0.013 | 0.0083 | 614622 | 132554 | 601382 | safe | safe |

Table 3-4 (continued)
$\left(\theta_{\mathrm{v}}=60^{\circ}\right)$

| $\left(\theta_{\mathrm{v}}=60^{\circ}\right.$ ) |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. | $\begin{gathered} \mathbf{R}_{\mathrm{b}} \\ (\mathbf{m}) \end{gathered}$ | $\begin{gathered} \mathbf{H} \\ (\mathbf{m}) \end{gathered}$ | $\begin{gathered} \text { Designed Section } \\ \text { (PCA-CCTWP / ACI350-06) } \\ \text { (From Table 3-2) } \\ \hline \end{gathered}$ |  |  | Max. <br> Hoop Tension (N/m) | Max. <br> Meridional Moment (N.m/m) | Max. <br> Meridional Compression ( $\mathbf{N} / \mathbf{m}$ ) | Check Designed Sections along Wall Height |  |
|  |  |  | $\begin{gathered} \mathbf{t} \\ (\mathbf{m m}) \end{gathered}$ | $\rho_{\text {s }}$ |  |  |  |  |  |  |
|  |  |  |  | Horizontal | Vertical |  |  |  | Horizontal | Vertical |
| 55 | 3 | 3 | 300 | 0.010 | 0.0075 | 186661 | 46498 | 202472 | safe | unsafe |
| 56 |  | 4 | 380 | 0.013 | 0.0079 | 285820 | 66338 | 411399 | safe | unsafe |
| 57 |  | 5 | 470 | 0.015 | 0.0082 | 403030 | 87671 | 721953 | safe | safe |
| 58 |  | 6 | 630 | 0.015 | 0.0084 | 527755 | 115271 | 1106827 | safe | safe |
| 59 | 4 | 3 | 300 | 0.011 | 0.0075 | 217343 | 59926 | 178817 | safe | unsafe |
| 60 |  | 4 | 400 | 0.013 | 0.0080 | 331843 | 82641 | 360029 | safe | unsafe |
| 61 |  | 5 | 520 | 0.015 | 0.0083 | 458726 | 109392 | 620716 | safe | safe |
| 62 |  | 6 | 660 | 0.015 | 0.0085 | 598605 | 139231 | 970910 | safe | safe |
| 63 | 5 | 3 | 320 | 0.011 | 0.0076 | 238606 | 73545 | 161541 | safe | unsafe |
| 64 |  | 4 | 430 | 0.013 | 0.0081 | 366588 | 101832 | 323433 | safe | unsafe |
| 65 |  | 5 | 600 | 0.013 | 0.0084 | 497659 | 134417 | 549217 | safe | safe |
| 66 |  | 6 | 700 | 0.015 | 0.0085 | 659872 | 165342 | 864731 | safe | safe |

For more illustration, the vertical section is checked under the effect of hoop tension that is predicted by the FEA model. In order to achieve the adequate wall thickness, the following conditions have to be satisfied;
$\left(A_{s}\right)_{F E A} \leq\left(A_{s}\right)_{P C A}$ (i.e., $T_{F E A} \leq T_{P C A}$ ), and
$\left(f_{c}\right)_{F E A}=\frac{C E_{s}\left(A_{s}\right)_{P C A}+T_{F E A}}{\left(A_{c}\right)_{P C A}+n\left(A_{s}\right)_{P C A}} \leq\left(10 \% f_{c}{ }^{\prime}\right)$.

Where $\left(A_{S}\right)_{F E A}$ is the area of horizontal reinforcement required to resist the hoop tension predicted by FEA, $\left(A_{S}\right)_{F E A}=\frac{T_{u_{F E A}}}{0.9 \times f_{y}},\left(\mathrm{~T}_{\mathrm{u}}\right)_{\text {FEA }}$ is the factored hoop tension, and $\mathrm{f}_{\mathrm{y}}$ is the steel yield strength, $\left(\mathrm{A}_{\mathrm{S}}\right)_{\text {PCA }}$ is the horizontal reinforcing steel required to resist the hoop tension obtained from PCA-CCTWP.

The design requirements (i.e., ultimate strength and serviceability) for different horizontal sections along the wall are checked. Interaction diagrams are developed for the designed sections which are subjected to a combined bending and high axial load, while sectional analysis is performed for wall sections that are mainly governed by flexural moments. Based on the constructability aspects, the section is assumed to have the same vertical reinforcement for both sides (i.e., external and internal faces of the tank's wall). Serviceability is controlled by preventing the tensile stresses in the steel reinforcement from exceeding the maximum allowable stresses specified by ACI350-06. Figure 3-6 presents a typical distribution for meridional moment and meridional normal compression along the wall height of tank No. 52 while Figure 3-7 shows the related interaction diagram.

Meridional Compression Force ( $\mathbf{k N} / \mathbf{m}$ )


Figure 3-6 Distribution diagram of the meridional moment and axial compression for tank \#52 ( $\left.\theta_{\mathrm{v}}=45, \mathrm{R}_{\mathrm{b}}=5 \mathrm{~m}, \mathrm{H}=6 \mathrm{~m}\right)$


Figure 3-7 Interaction Diagram for tank \#52 $\left(\theta_{\mathrm{v}}=45, \mathrm{R}_{\mathrm{b}}=5 \mathrm{~m}, \mathrm{H}=6 \mathrm{~m}\right)$ $\rho_{\mathrm{s}}=0.79 \%$ is the ratio of vertical reinforcement

### 3.6.2.1 Comparison between PCA-CCTWP and FEA Results

This subsection provides a comparison between the two analysis methods used in the current study (i.e., PCA-CCTWP, and FEA). This comparison is presented in order to evaluate the adequacy of applying code provisions in case of conical tanks.

Based on the results obtained from both PCA-CCTWP and FEA, it is noticed that the internal forces (i.e., hoop tension, meridional moment, and meridional normal force) increase as the angle of wall inclination increases, leading to a larger required wall thickness. For example, the thickness of broad conical tanks (i.e., $\theta_{v} \geq 45^{\circ}$ ) ranging between 400 mm to 600 mm , and reaches up to 700 mm for $\operatorname{tank}$ No. 66 . On the other hand, the minimum wall thickness specified by PCA-CCTWP and ACI350-06 (i.e., 300 mm ) is found to be sufficient for serviceability requirements for a wide range of narrow conical tanks (i.e., $\theta_{v}<45^{\circ}$ ). The same trend is observed by increasing the tank height and base radius. It can be also noticed that the meridional compression force obtained from FEA decreases as the base radius increases because the hydrostatic pressure is distributed over a larger circumferential area.

The internal forces obtained from PCA-CCTWP are compared with FEA in order to evaluate the range of discrepancies. Table 3-5 presents the differences between the maximum forces, including hoop tension forces and meridional moments for the all 66 studied tanks. It should be mentioned that the meridional compression is excluded from the comparison because PCA-CCTWP ignores the compression developed by the wall
own weight in case of cylindrical tanks. It is noticed that PCA-CCTWP method results in maximum hoop tension forces that are greater than those obtained from FEA. The range of discrepancy between the two approaches depends on the angle of inclination of the wall, base radius and tank height. Typically, the maximum hoop obtained from both methods is located between the bottom third and bottom fifth region of the wall height. For hoop tension distribution along wall height, a typical agreement is found for shortnarrow tanks having small inclination angles (i.e., $\theta_{v}<45^{\circ}, \mathrm{H}<5 \mathrm{~m}$ ). On the other hand, there are disagreements for the hoop values along wall height of tall-wide conical tanks. This can be related to the fact that as the inclination angle decreases; the behaviour of the conical tank is closer to be cylindrical. Figure 3-8 shows a typical distribution diagram for hoop tension predicted by the two methods for a short-narrow conical tank while Figure 3-9 presents the hoop diagram of a tall-wide tank.

Table 3-5 Comparison between PCA-CCTWP and FEA Maximum Internal Forces

$$
\left(\theta_{\mathrm{v}}=15^{\circ}\right)
$$

| No. | $\mathbf{R}_{\text {b }}(\mathrm{m})$ | H (m) | t (mm) | Maximum Hoop Tension (N/m) |  |  | Maximum Meridional Moment (N.m/m) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\begin{gathered} \text { PCA- } \\ \text { CCTWP } \end{gathered}$ | FEA | $\begin{gathered} \hline \text { Diff } \\ (\%)^{\mathbf{1})} \end{gathered}$ | $\begin{gathered} \text { PCA- } \\ \text { CCTWP } \end{gathered}$ | FEA | $\begin{gathered} \text { Diff } \\ (\%)_{(1)}^{(i)} \end{gathered}$ |
| 1 | 3 | 3 | 300 | 65682 | 62395 | -5 | 3129 | 18542 | 83 |
| 2 |  | 4 | 300 | 104872 | 97056 | -8 | 4191 | 24584 | 83 |
| 3 |  | 5 | 300 | 147018 | 132480 | -11 | 5512 | 30451 | 82 |
| 4 |  | 6 | 300 | 190791 | 168121 | -13 | 6792 | 37290 | 82 |
| 5 |  | 7 | 300 | 243763 | 205422 | -19 | 8575 | 43439 | 80 |
| 6 |  | 8 | 300 | 295917 | 241638 | -22 | 10033 | 48493 | 79 |
| 7 | 4 | 3 | 300 | 77941 | 74133 | -5 | 4030 | 24810 | 84 |
| 8 |  | 4 | 300 | 125493 | 118840 | -6 | 5475 | 32966 | 83 |
| 9 |  | 5 | 300 | 179538 | 165283 | -9 | 6718 | 41119 | 84 |
| 10 |  | 6 | 300 | 233275 | 212343 | -10 | 8678 | 48676 | 82 |
| 11 |  | 7 | 300 | 292470 | 259149 | -13 | 10183 | 57975 | 82 |
| 12 |  | 8 | 300 | 359534 | 307959 | -17 | 12627 | 66087 | 81 |
| 13 | 5 | 3 | 300 | 87971 | 84247 | -4 | 4805 | 30694 | 84 |
| 14 |  | 4 | 300 | 146318 | 139174 | -5 | 6698 | 41178 | 84 |
| 15 |  | 5 | 300 | 208566 | 195366 | -7 | 8477 | 51061 | 83 |
| 16 |  | 6 | 300 | 275082 | 251917 | -9 | 10280 | 61467 | 83 |
| 17 |  | 7 | 300 | 339177 | 311230 | -9 | 12378 | 71501 | 83 |
| 18 |  | 8 | 300 | 417665 | 369749 | -13 | 13990 | 82997 | 83 |

1): Difference $=\frac{(\text { Maximum } F E A-\text { Maximum PCA-CCTWP })}{\text { Maximum } F E A} \%$

Table 3-5 (continued)
( $\theta_{\mathrm{v}}=3 \mathbf{3 0}^{\circ}$ )

| No. | $\mathbf{R}_{\mathrm{b}}(\mathrm{m})$ | H (m) | t (mm) | Maximum Hoop Tension ( $\mathrm{N} / \mathrm{m}$ ) |  |  | Maximum Meridional Moment (N.m/m) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\begin{gathered} \text { PCA- } \\ \text { CCTWP } \end{gathered}$ | FEA | $\begin{gathered} \hline \text { Diff } \\ (\%)^{\mathbf{( 1 )}} \\ \hline \end{gathered}$ | $\begin{gathered} \text { PCA- } \\ \text { CCTWP } \end{gathered}$ | FEA | $\begin{gathered} \text { Diff. } \\ (\%)^{(1)} \\ \hline \end{gathered}$ |
| 19 | 3 | 3 | 300 | 92359 | 76842 | -20 | 4426 | 22204 | 80 |
| 20 |  | 4 | 300 | 150922 | 120848 | -25 | 6225 | 29407 | 79 |
| 21 |  | 5 | 300 | 218697 | 165731 | -32 | 8182 | 36611 | 78 |
| 22 |  | 6 | 300 | 288451 | 212595 | -36 | 10548 | 45487 | 77 |
| 23 |  | 7 | 300 | 379933 | 259751 | -46 | 13317 | 53448 | 75 |
| 24 |  | 8 | 300 | 473907 | 308316 | -54 | 16240 | 62771 | 74 |
| 25 | 4 | 3 | 300 | 107553 | 90307 | -19 | 5546 | 29456 | 81 |
| 26 |  | 4 | 300 | 176503 | 145870 | -21 | 7727 | 38858 | 80 |
| 27 |  | 5 | 300 | 257958 | 204159 | -26 | 9840 | 48505 | 80 |
| 28 |  | 6 | 300 | 344142 | 262847 | -31 | 12883 | 58374 | 78 |
| 29 |  | 7 | 300 | 433786 | 322450 | -35 | 15671 | 69705 | 78 |
| 30 |  | 8 | 300 | 550689 | 383702 | -44 | 17308 | 79838 | 78 |
| 31 | 5 | 3 | 300 | 120668 | 101987 | -18 | 6462 | 36165 | 82 |
| 32 |  | 4 | 300 | 202552 | 169263 | -20 | 9257 | 48476 | 81 |
| 33 |  | 5 | 300 | 293220 | 238816 | -23 | 12047 | 60068 | 80 |
| 34 |  | 6 | 300 | 394721 | 309138 | -28 | 14674 | 71786 | 80 |
| 35 |  | 7 | 300 | 496611 | 382426 | -30 | 18327 | 85054 | 78 |
| 36 |  | 8 | 300 | 603129 | 454066 | -33 | 23179 | 98587 | 76 |

1): Difference $=\frac{(\text { Maximum FEA-Maximum PCA-CCTWP })}{\text { Maximum FEA }} \%$

Table 3-5 (continued)
( $\theta_{\mathrm{v}}=45^{\circ}$ )

| No. | $\mathbf{R}_{\mathrm{b}}(\mathrm{m})$ | H (m) | t (mm) | Maximum Hoop Tension ( $\mathrm{N} / \mathrm{m}$ ) |  |  | Maximum Meridional Moment (N.m/m) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\begin{gathered} \text { PCA- } \\ \text { CCTWP } \end{gathered}$ | FEA | $\begin{gathered} \hline \text { Diff } \\ (\%)^{\mathbf{( 1 )}} \\ \hline \end{gathered}$ | $\begin{gathered} \text { PCA- } \\ \text { CCTWP } \end{gathered}$ | FEA | $\begin{gathered} \text { Diff. } \\ (\%)^{(1)} \\ \hline \end{gathered}$ |
| 37 | 3 | 3 | 300 | 163656 | 107817 | -52 | 7703 | 29632 | 74 |
| 38 |  | 4 | 300 | 273859 | 170599 | -61 | 11188 | 39284 | 72 |
| 39 |  | 5 | 300 | 405426 | 235697 | -72 | 15153 | 50533 | 70 |
| 40 |  | 6 | 300 | 548074 | 303569 | -81 | 19987 | 63353 | 68 |
| 41 |  | 7 | 350 | 697752 | 373990 | -87 | 29628 | 76796 | 61 |
| 42 |  | 8 | 425 | 858525 | 449513 | -91 | 44280 | 91773 | 52 |
| 43 | 4 | 3 | 300 | 188238 | 126177 | -49 | 9394 | 38692 | 76 |
| 44 |  | 4 | 300 | 312339 | 203264 | -54 | 13577 | 50738 | 73 |
| 45 |  | 5 | 300 | 464477 | 285049 | -63 | 17516 | 63498 | 72 |
| 46 |  | 6 | 320 | 624659 | 365711 | -71 | 24992 | 79086 | 68 |
| 47 |  | 7 | 380 | 788408 | 449286 | -75 | 37378 | 95623 | 61 |
| 48 |  | 8 | 480 | 957249 | 536551 | -78 | 57412 | 113172 | 49 |
| 49 | 5 | 3 | 300 | 208578 | 141350 | -48 | 10980 | 47424 | 77 |
| 50 |  | 4 | 300 | 350294 | 233672 | -50 | 15894 | 62975 | 75 |
| 51 |  | 5 | 310 | 514717 | 328765 | -57 | 20153 | 78595 | 74 |
| 52 |  | 6 | 380 | 675038 | 419228 | -61 | 33226 | 95694 | 65 |
| 53 |  | 7 | 480 | 839095 | 512256 | -64 | 51956 | 113861 | 54 |
| 54 |  | 8 | 550 | 1030823 | 614622 | -68 | 70964 | 132554 | 46 |

1): Difference $=\frac{(\text { Maximum FEA-Maximum PCA-CCTWP })}{\text { Maximum FEA }} \%$

Table 3-5 (continued)
$\left(\theta_{\mathrm{v}}=60^{\circ}\right)$

| No. | $\mathbf{R}_{\mathrm{b}}(\mathrm{m})$ | H (m) | $t(\mathrm{~mm})$ | Maximum Hoop Tension ( $\mathbf{N} / \mathrm{m}$ ) |  |  | Maximum Meridional Moment (N.m/m) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\begin{aligned} & \text { PCA- } \\ & \text { CCTWP } \end{aligned}$ | FEA | $\begin{gathered} \text { Diff } \\ (\%)^{(1)} \\ \hline \end{gathered}$ | PCACCTWP | FEA | $\begin{gathered} \text { Diff. } \\ (\%)^{\mathbf{1})} \end{gathered}$ |
| 55 | 3 | 3 | 300 | 419787 | 186661 | -125 | 19063 | 46498 | 59 |
| 56 |  | 4 | 380 | 679372 | 285820 | -135 | 37174 | 66338 | 41 |
| 57 |  | 5 | 470 | 992392 | 403030 | -146 | 64806 | 87671 | 26 |
| 58 |  | 6 | 630 | 1316450 | 527755 | -149 | 117150 | 115271 | -2 |
| 59 | 4 | 3 | 300 | 474885 | 217343 | -118 | 22653 | 59926 | 62 |
| 60 |  | 4 | 400 | 746288 | 331843 | -125 | 45187 | 82641 | 45 |
| 61 |  | 5 | 520 | 1057166 | 458726 | -130 | 81657 | 109392 | 25 |
| 62 |  | 6 | 660 | 1413543 | 598605 | -136 | 136682 | 139231 | 2 |
| 63 | 5 | 3 | 320 | 511590 | 238606 | -114 | 27744 | 73545 | 62 |
| 64 |  | 4 | 430 | 804500 | 366588 | -119 | 55548 | 101832 | 45 |
| 65 |  | 5 | 600 | 1113460 | 497659 | -124 | 106820 | 134417 | 21 |
| 66 |  | 6 | 700 | 1512211 | 659872 | -129 | 161218 | 165342 | 2 |



Figure 3-8 Hoop Tension Distribution Diagram in the Wall of tank \#9 ( $\left.\theta_{\mathrm{v}}=15^{\circ}, \mathrm{R}_{\mathrm{b}}=4 \mathrm{~m}, \mathrm{H}=5 \mathrm{~m}\right)$,
CPA-CCWTWP compared to FEA - Case of Good Agreement


Figure 3-9 Hoop Tension Distribution Diagram in the Wall of $\operatorname{tank} \# 45\left(\theta_{\mathrm{v}}=45^{\circ}, \mathrm{R}_{\mathrm{b}}=4 \mathrm{~m}, \mathrm{H}=5 \mathrm{~m}\right)$, CPA-CCWTWP compared to FEA - Case of Disagreement

### 3.6.2.2 Effect of Changing Geometric Parameters on Results

This part of the study discusses the effect of changing the dimensions of conical tanks (i.e., radius, height, wall inclination angle, and wall thickness) on the maximum internal forces due to hydrostatic pressure. In addition, it is found that increasing the tank height leads to more difference in the maximum hoop tension forces obtained from the two analysis methods (i.e., PCA-CCTWP and FEA). Figure 3-10 shows this trend for conical tanks having base radius $\left(\mathrm{R}_{\mathrm{b}}=4 \mathrm{~m}\right)$; the same trend is found for other base radiuses. This figure shows that increasing the wall inclination angle leads to more discrepancies in the values of the maximum hoop tension. Moreover, for relatively tall tanks having a ratio of $\left(\frac{H_{c y}{ }^{2}}{D_{c y} . t} \geq 16\right)$, PCA-CCTWP neglects the effect of hydrostatic pressure on the upper part of tall walls of the equivalent cylindrical tanks compared to a significant great hoop


Figure 3-10 Ratio of Discrepancies of Maximum Hoop Tension for $R_{b}=4 m$ CPA-CCWTWP compared to FEA

$$
\text { Difference }=\frac{(\text { Maximum FEA-Maximum PCA-CCTWP })}{\text { Maximum FEA }}
$$

tension at the lower third part of the wall. As illustrated in Figure 3-11, significant disagreement is observed for such cases (i.e., $\frac{H_{c y}{ }^{2}}{D_{c y}{ }^{2}} \geq 16$ ).


Figure 3-11 Hoop tension distribution diagram in the wall of tank \#23 ( $\left.\theta_{\mathrm{v}}=30, \mathrm{R}_{\mathrm{b}}=3 \mathrm{~m}, \mathrm{H}=7 \mathrm{~m}\right),\left(H_{c y}{ }^{2} / D_{c y} \cdot t=18.8\right)$ CPA-CCWTWP compared to FEA

Although PCA-CCTWP approach results in higher values of maximum hoop tension compared to the FEA method, the maximum meridional moment obtained from PCACCTWP is found to be less than that from the FEA. This is because the equivalent cylindrical approach does not present the vertical component of the hydrostatic pressure acting on the inclined walls of the conical tanks. The average difference is noticed to be $83 \%, 79 \%, 66 \%$, and $32 \%$ for $\theta_{V}=15^{\circ}, \theta_{V}=30^{\circ}, \theta_{V}=45^{\circ}$, and $\theta_{V}=60^{\circ}$, respectively. The results also show that increasing the wall height leads to a reduction in the difference between the maximum meridional moments obtained from the two analysis methods as
illustrated in Figure 3-12. This figure presents the ratio of discrepancies of maximum meridional moment for a tank having 4 m base radius. The same trend is observed for tanks having 3 m and 5 m . This is due to the fact that for large conical tanks, the equivalent cylinder approach leads to deep cylindrical tanks which results in meridional moments higher than that of shallow tanks. Figure 3-13 shows the moment distribution diagram for a typical short-narrow conical tank while Figure 3-14 presents the diagram for a tall-wide one. As concluded from these figures, moment discrepancies between the two analysis methods are reduced by increasing the angle of inclination of the wall with the vertical.


Figure 3-12 Ratio of Discrepancies of Maximum Meridional Moment for $\mathrm{R}_{\mathrm{b}}=4 \mathrm{~m}$ CPA-CCWTWP compared to FEA

$$
\text { Difference }=\frac{(\text { Maximum FEA-Maximum PCA-CCTWP })}{\text { Maximum FEA }}
$$



Figure 3-13 Meridional moment distribution diagram in the wall of tank \#26 $\left(\theta_{\mathrm{v}}=30, \mathrm{R}_{\mathrm{b}}=4 \mathrm{~m}, \mathrm{H}=4 \mathrm{~m}\right)$

CPA-CCTWP compared to FEA


Figure 3-14 Meridional moment distribution diagram in the wall of tank \#62 $\left(\theta_{\mathrm{v}}=60, \mathrm{R}_{\mathrm{b}}=4 \mathrm{~m}, \mathrm{H}=6 \mathrm{~m}\right)$

CPA-CCTWP compared to FEA

The compression force that acts on the meridional direction of the wall is neglected by PCA-CCTWP while FEA method predicts the meridional compression force developed by the effect of hydrostatic pressure. The maximum compression is located at the base of the wall and it exhibits a high compression stresses acting on the horizontal section of the tank wall. However, such compression force is expected to increase the compression zone of the wall and enhance its cracking resistance.

As mentioned earlier, the wall of the pure conical tank requires two segments to be designed; vertical section in the circumferential direction to resist hoop forces, and horizontal section in the longitudinal direction to resist the dual effect of both moments and compression forces. For the vertical section of the wall, PCA-CCTWP provides sufficient concrete thickness that can resist concrete cracking but it requires area of horizontal reinforcement more than that required by the FEA method. Therefore, the hoop tension predicted by PCA-CCTWP is more conservative than that obtained from the FEA. It is found that the area of horizontal steel estimated by PCA-CCTWP approach is more than that predicted according to the FEA results by $10 \%, 30 \%, 65 \%$, and $129 \%$ for angles of inclination $15^{\circ}, 30^{\circ}, 45^{\circ}$, and $60^{\circ}$, respectively.

On the other hand, PCA-CCTWP neglects the resultant meridional compression force and designs the horizontal section of the wall to only resist the flexural moment. Based on the ultimate strength and serviceability requirements provided by the design code ACI35006 , the minimum flexural reinforcement is sufficient. In contrast, it is found that the designed vertical reinforcement does not satisfy the ultimate strength requirements when using the FEA method. This is attributed to the reason that FEA provides meridional
compression forces and significant moments compared with those obtained from PCACCTWP. Consequently, the horizontal section requires ( $2 \%$ to $180 \%$ ) additional vertical reinforcement compared to the PCA-CCTWP. This main conclusion proves the inadequacy of applying code provisions used for cylindrical tanks on conical storage vessels.

### 3.7 Conclusions

Based on the available codes of design, there are currently no guidelines to evaluate the internal forces acting on the walls of elevated reinforced concrete conical tanks. These codes provide guidelines only for rectangular and cylindrical tanks. PCA-CCTWP (1993) provides a simplified approach for concrete cylindrical tanks. This approach is applied for the selected conical tanks by using a special procedure specified by AWWA-D100 (2005) to convert these conical tanks to equivalent cylindrical tanks. Moreover, a built inhouse finite element model, which is based on a degenerated consistent sub-parametric shell element, is employed to model the conical tanks. In this study, several reinforced concrete conical tanks subjected to hydrostatic pressure are analyzed and designed. Based on a parametric study conducted on 66 conical tanks having different configurations and by comparing the internal forces in the walls of these tanks that are obtained from PCACCTWP and FEA, the following conclusions can be drawn:

- PCA-CCTWP provides higher maximum hoop tension than that obtained from FEA models. The average difference is $10 \%, 30 \%, 65 \%$, and $129 \%$ for wall inclination angles $\theta_{v}=15^{\circ}, \theta_{v}=30^{\circ}, \theta_{v}=45^{\circ}$, and $\theta_{v}=60^{\circ}$, respectively. However,
the study shows that the results of hoop tension from PCA-CCTWP and FEA have a good agreement for short-narrow tanks.
- The maximum meridional moments obtained from PCA-CCTWP are less than that predicted by FEA. The average range of disagreement is $32 \%$ to $83 \%$ depending on the inclination angle and the wall height. It is noticed that as the inclination angle and wall height increase the range of disagreement decreases. Therefore, there is an agreement in maximum meridional moments obtained from PCA-CCTWP and FEA in case of tall-wide conical tanks.
- The wall minimum thickness specified by ACI350-06 (i.e., 300 mm ) is found to be sufficient for serviceability requirements for conical tanks having $\theta_{v}<45^{\circ}$. However, increasing wall inclination angle and tank height, especially for $\theta_{v} \geq$ $45^{\circ}$, leads to an increase in the required wall thickness in order to achieve serviceability requirements.
- For tall-wide reinforced concrete conical tanks (i.e., $\theta_{v} \geq 45^{\circ}, \mathrm{H}>5 \mathrm{~m}$ ), the designed wall thickness using PCA-CCTWP method is found to be overestimated. This is attributed to that PCA-CCTWP method provides higher hoop tension, leading to an over conservative design. Consequently, wall thickness can be reduced without affecting the serviceability requirement.
- PCA-CCTWP method is mainly used to design tank walls to carry only hoop tension and meridional moment. In case of conical tanks, a larger meridional moment combined with high meridional compression, which is not presented in PCA-CCTWP, are caused by the vertical component of the hydrostatic pressure
acting on the inclined walls of the tanks. This may lead to an inadequate design by following the PCA-CCTWP approach.


## CHAPTER 4 <br> SIMPLIFIED DESIGN CHARTS FOR REINFORCED CONCRETE CONICAL TANKS UNDER HYDROSTATIC LOADING

### 4.1 Introduction

Structural engineers usually seek a simple approach that satisfies all design requirements to be utilized while designing complex structures. The design of a conical shaped tank is considered as a challenging task because of the complication in the state of stresses and the lack of direct guidelines for hydrostatically loaded reinforced concrete conical tanks in codes of design. The design of steel liquid-storage structures in North America is usually based on the specifications provided by either the American Water Works Association AWWA-D100 (2005) or the American Petroleum Institute API 650 (2006). Both specifications adopt an equivalent cylindrical tank approach in which the conical segment is replaced by a cylindrical part. The main drawback of this approximation is that it does not accurately simulate the state of stresses induced in the inclined walls of the conical segment.

In chapter 3, the accuracy of a design approach for reinforced concrete conical tanks which combines PCA-CCTWP provisions with the equivalent cylinder approach defined by AWWA-D100 (2005) has been assessed. This assessment was done by conducting a parametric study covering a practical range of conical tanks having different dimensions and capacities. It was concluded that the proposed PCA-CCTWP approach leads to an inadequate design if applied to conical tanks. It was also noticed that such approach
provides higher hoop tension and less meridional moment compared to those obtained from FEA results.

In order to achieve the most economical and safe design of conical tanks, a comprehensive finite element analysis combined with detailed design specifications is essentially required. In performing the analysis of these tanks using the finite element method, the resulting internal forces are predicted along both the meridional and circumferential directions of the tanks' walls. Shell elements are used to predict such behaviour, which requires knowledge and expertise in sophisticated 3-D finite element analysis. Accordingly, it is not an easy task for the designer to choose the adequate thickness as well as to provide sufficient reinforcement for both circumferential direction for tension hoop, and longitudinal direction for meridional moment and concurrent compression. Several trials should be done to achieve an optimum design.

This is obviously a complex procedure and most likely not applicable in the routine practice applications. As such, simple approaches can be usefully employed for a preliminary design phase and for cost estimation. To the best of the author's knowledge, no previous studies have been conducted to provide a simple approach for analysing and designing reinforced concrete conical tanks. The only available studies focused on steel conical vessels.

In a previous study that was conducted by El Damatty et al. (1999), a simple design approach for hydrostatically loaded conical steel vessels was developed based on a linear regression analysis of the buckling strength values determined numerically using finite element analysis. The suggested procedure is limited to pure steel conical tanks having wall-inclination angle of $45^{\circ}$ and yield strength of 300 MPa . Later on, Sweedan and El

Damatty (2009) extended this simplified procedure to include combined conical steel tanks taking into account variation of both the angle of inclination of the conical part of the vessel as well as the yield strength of steel. Another simplified procedure was developed by El Damatty and Marroquin (2001) to design stiffened liquid-filled steel conical tanks. This procedure depends on the combination of the orthotropic theory and design formulae for unstiffened conical tanks.

Based on the above, and according to the main conclusions achieved in the previous chapter, the current study is motivated to provide practitioners with a simplified design approach based on a set of design charts for practical applications. The present study is confined to linear elastic analysis of reinforced concrete pure conical tanks having a constant wall thickness and subjected to an axisymmetric hydrostatic pressure. The proposed design charts take into account the fundamental requirements of the Portland Cement Association PCA-CCTWP (1993) and the American Concrete Institute ACI35006 (2006). These charts are developed to assist the designer in determining the required minimum wall thickness, and the associated maximum internal forces acting on the tank's walls. In order to achieve the main goal of the research, this chapter starts by reviewing the available code provisions for reinforced concrete liquid storage tanks. This is followed by describing the finite element model. The methodology of developing the proposed design charts is then presented. Finally, a number of numerical case studies that are used to validate the accuracy of the developed charts are presented.

### 4.2 Finite Element Analysis

The conical tanks considered in the current study are simulated numerically in a threedimensional continuum approach. The numerical model is based on a degenerated consistent shell element that was developed by Koziey and Mirza (1997) and was extended by El Damatty et al. (1997) to account for the geometric nonlinear effects. This element was successfully used in several previous studies and was validated versus many experimental and numerical results (e.g., El Damatty et al. 1997, 1998; Hafeez et al. 2010, 2011).

The formulation of this element, which has 13 nodes, as shown in Figure 4-1, includes three displacement degrees of freedom; $u, v$, and $w$ along the global $x, y$, and $z$ coordinates, respectively, and four rotational degrees of freedom $\alpha, \beta, \phi$, and $\psi$ acting at the corner and mid-side nodes. Both $\alpha$ and $\phi$ are about a local axis $y^{\prime}$, and $\beta$ and $\psi$ are about local axis $x^{\prime}$, where the local axes $y^{\prime}$ and $x^{\prime}$ are located in a plane tangent to the surface. Rotations $\alpha$ and $\beta$ are constant through the depth of the element, while rotations $\phi$ and $\psi$ vary quadratically. Thus, $\alpha$ and $\beta$ provide a linear variation of displacements $u, v$, and $w$ along the thickness representing bending deformations, while $\phi$ and $\psi$ lead to a cubic variation of displacements $u, v$, and $w$, simulating transverse shear deformations. These special rotational degrees of freedom are important when modeling thick plates or shells, where the shear deformation is significant.


Figure 4-1 Consistent Shell Element Coordinate System and Nodal Degrees of Freedom

### 4.2.1 Modelling and Assumptions

Due to the double symmetry in geometry, loading, and boundary conditions, only one quarter of the vessel is modeled in the analysis, following the same procedure used in chapter 3. A number of analyses with different mesh sizes are performed for one of the studied tanks under the same hydrostatic loading. The maximum radial displacement in the tank walls are obtained for various mesh sizes until the mesh size that yields to accurate results has been reached. From the analyses, it is shown that a finite element mesh consisting of 256 triangular elements can predict the behaviour with good accuracy.

The vertical projection of a typical finite element mesh for one quarter of the concrete vessel used for all studied tanks is shown in Figure 4-2. It can be noticed from this figure
that the mesh consists of 8 and 16 rectangular divisions in both the circumferential and longitudinal directions, respectively. A finer mesh is applied at the bottom region of the vessel where stress concentration is anticipated near the tank base. In this numerical model, the walls of the vessels are assumed to be simply supported at the base and free at the top. This assumption is valid since the hydrostatic pressure is negligible at the top and the radial displacement at the top is so small (El Damatty et al. 1997). The wall thickness is considered to be constant along the height and is designed according to the general requirements of PCA-CCTWP (1993) and ACI 350-06 (2006) that prevent forming of pure tension cracks and limit flexural crack width. All tanks are analyzed under the effect of hydrostatic loading assuming that the liquid is filling the whole vessel. Table 4-1 shows the dimensions and material properties of the selected tanks that are chosen to cover a wide practical range of reinforced concrete conical tanks.


Figure 4-2 Finite Element Mesh of a Quarter Cone

Table 4-1 Tank Properties

| Property | Assumption |
| :--- | :--- |
|  |  |
| Tank height $(H)$ | $4-12 \mathrm{~m},(1 \mathrm{~m}$ increment $)$ |
| Base radius $\left(R_{b}\right)$ | $3-6 \mathrm{~m},(1 \mathrm{~m} \mathrm{increment})$ |
| Inclination angle $\left(\theta_{v}\right)$ | $15^{\circ}-60^{\circ},\left(15^{\circ} \mathrm{increment}\right)$ |
| Minimum wall thickness $\left(t_{\text {min }}\right)$ | 200 mm |
| Concrete compressive strength $\left(f_{c}{ }^{\prime}\right)$ | 30 MPa |
| Concrete tensile strength $\left(f_{c r}\right)$ | 3 MPa |
| Steel yield strength $\left(f_{y}\right)$ | 400 MPa |
| Concrete modulus of elasticity $\left(E_{c}\right)$ | 24647 MPa |
| Steel modulus of elasticity $\left(E_{s}\right)$ | 200000 MPa |
| Poisson ratio $(v)$ | 0.3 |
| Modular ratio $(n)$ | 8.1 |
| Liquid specific weight $\left(\gamma_{w}\right)$ | $10000 \mathrm{~N} / \mathrm{m}^{3}$ |
| Vertical reinforcement ratio | $1 \%$ of concrete gross area |
|  |  |

### 4.3 Methodology for Developing Simplified Design Charts

In order to achieve the main objective of this study (i.e., a set of simplified design charts), 144 tanks with different geometries and capacities covering a wide range of tanks in practice and designed following PCA-CCTWP provisions are analyzed using the finite element model described in the previous section.

For the first analysis and design, the wall thickness is assumed to be less than that was obtained by using PCA-CCTWP. This thickness is then utilized in FEA model and the internal forces obtained for each tank are then implemented in the design equations provided by PCA-CCTWP. The designed wall thickness and the required reinforcement
in both the circumferential and longitudinal directions are achieved by repeating this procedure.

Following the above approach, all chosen tanks are designed to satisfy both serviceability and ultimate strength requirements. A minimum thickness for each tank is predicted to satisfy both requirements and to achieve minimum weight of the vessel. It should be mentioned that ACI350-06 and PCA-CCTWP provide a minimum thickness of 300 mm for walls equal to or higher than 3 m to satisfy the constructability aspects. However, other standards (e.g. BS2007 and EN 1992-3), do not provide a specific minimum wall thickness and it is left for the designer and constructability requirements. In the current study, a theoretical minimum thickness of 200 mm for the tanks' walls is assumed following the provisions of standards that do not provide restrictions on the minimum wall thickness. This assumption is considered such that the simplified design charts provide the designer with an optimum thickness that satisfies serviceability and strength requirements only without taking into account the constructability aspects. The minimum thickness that will be predicted from these design charts could be increased to satisfy constructability requirements according to the designer judgment which might differ based on the techniques used in construction.

The following steps summarize the methodology for developing the proposed design charts.

### 4.3.1 Step 1: Design for the Minimum Wall Thickness

The main conclusion drawn from chapter 3 is that the PCA-CCTWP provisions combined with the equivalent cylinder approach leads to an overdesigned thickness of the tanks'
walls. The reason behind this inaccurate design is that the hoop tension predicted by the PCA-CCTWP approach is much higher than that obtained from the finite element analysis. This discrepancy is related to the approximation in the equivalent cylinder approach used to transfer the conical tank to an equivalent cylinder. As such, the designed wall thickness following PCA-CCTWP provisions combined with the equivalent cylinder approach can be reduced without affecting the serviceability requirements.

In order to reach the reduced thickness (i.e., optimum thickness), the wall's thickness is first assumed to be less than that obtained in the first step by PCA-CCTWP. The reduced thickness is then utilized in the FEA model to predict the internal forces. The tensile capacity of the wall section is checked by Equation 4.1 to satisfy the cracking criteria. This step is repeated many times for each tank until the applied tensile stresses equals to the concrete tensile strength (i.e., $10 \% f_{c}{ }^{\prime}$ ).
$f_{c}=\frac{C E_{s} A_{s}+T}{A_{c}+n A_{s}} \cong f_{c r}=\left(10 \% f_{c}{ }^{\prime}\right)$

Where $f_{c}$ is the applied tensile strength, $f_{c}{ }^{\prime}$ is concrete compressive strength, $C$ is the shrinkage coefficient (i.e., $C=0.0003$ ), $E_{S}$ is the modulus of elasticity of horizontal reinforced steel, $A_{c}$ is area of concrete for 1000 mm height section (i.e., $A_{c}=1000 t-$ $A_{s}$ ), $t$ is the wall thickness, $n$ is the modular ratio (i.e., $n=\frac{E_{S}}{E_{c}}$ ), and $E_{c}$ is the concrete modulus of elasticity. $A_{s}$ is the area of horizontal reinforcement per 1000 mm height section, and can be estimate from Equation 4.2, $T$ is the non factored ring hoop force per 1000 mm length resulting from the hydrostatic pressure,
$A_{S}=\frac{T_{u}}{0.9 \times F_{y}}$

Where $T_{u}$ is the maximum factored hoop tension $T_{u}=1.4 \times S_{d} \times T$ ), where $T$ is the service hoop tension, $F_{y}$ is the steel yielding strength, and $S_{d}$ is the environmental durability factor that can be calculated from Equation 4.3.
$S_{d}=\frac{\varnothing f_{y}}{\gamma f_{s}}$

Where $\varnothing$ is the strength reduction factor, $(\varnothing=0.9$ for both hoop tension and flexural members), $f_{y}$ is the steel yield strength, $f_{s}=140 \mathrm{MPa}$ is the allowable stress in normal environment, and $\gamma=\frac{\text { factored load }}{\text { unfactorred load }}=1.4$, in case of hydrostatic pressure and dead loads.

In order to assess the applicability of the proposed methodology, the tanks that were designed in chapter 3 following PCA-CCTWP approach are analyzed using the finite element model. This is followed by redesigning these tanks by implementing the internal forces obtained from the finite element analysis in PCA-CCTWP equations to predict the wall thickness. It is noticed that the designed thicknesses, which comply with the requirements of ACI350-06, are decreased significantly. Table 4-2 shows a comparison between the walls` thicknesses designed by the FEA and those by the PCA-CCTWP as previously presented in chapter 3. It is observed from this comparison that the optimum design can lead to a reduction in the wall thicknesses ranging from $43 \%$ to $57 \%$ which leads to a reduction in the cost of the construction material.

Table 4-2 Designed Thickness (PCA-CCTWP and FEA)

| $\boldsymbol{\theta}_{\mathbf{v}}$ | $\mathbf{R}_{\mathbf{b}}(\mathbf{m})$ | $\mathbf{H}(\mathbf{m})$ | $\mathbf{t}_{\min }(\mathbf{m m})$ |  |
| :--- | :---: | :---: | :---: | :---: |
|  |  | PCA-CCTWP ${ }^{(*)}$ | FEA |  |
| 45 | 3 | 7 | 350 | 200 |
| 45 | 3 | 8 | 425 | 200 |
| 45 | 4 | 7 | 380 | 204 |
| 45 | 4 | 8 | 480 | 240 |
| 45 | 5 | 7 | 480 | 238 |
| 45 | 5 | 8 | 550 | 281 |
| 60 | 3 | 5 | 470 | 200 |
| 60 | 3 | 6 | 630 | 241 |
| 60 | 4 | 5 | 520 | 250 |
| 60 | 4 | 6 | 660 | 285 |
| 60 | 5 | 5 | 600 | 257 |
| 60 | 5 | 6 | 700 | 327 |
|  |  |  |  |  |

(*): Thickness is designed based on the analysis of PCA-CCTWP combined with the equivalent $_{\text {( }}$ cylinder approach, which is presented in chapter 3.

After proving the feasibility of this methodology, the analysis is extended by conducting a parametric study on the 144 chosen tanks. Table 4-3 summarizes the proposed minimum thickness for each tank which will be checked in the next steps for ultimate strength and serviceability requirements.

Table 4-3 Required Minimum Thickness

|  | $\mathbf{H}(\mathbf{m})$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ | $\mathbf{1 1}$ | $\mathbf{1 2}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{\theta}_{\mathbf{v}}$ | $\mathbf{R}_{\mathbf{b}}$ |  |  |  | $\left.\mathbf{t}_{\mathbf{m i n}} \mathbf{( m m}\right)$ |  |  |  |  |  |  |  |
| $\mathbf{1 5}^{\circ}$ | $\mathbf{3}$ | 200 | 200 | 200 | 200 | 200 | 200 | 200 | 200 | 200 |  |  |
|  | $\mathbf{4}$ | 200 | 200 | 200 | 200 | 200 | 200 | 200 | 202 | 222 |  |  |
|  | $\mathbf{5}$ | 200 | 200 | 200 | 200 | 200 | 200 | 218 | 243 | 266 |  |  |
|  | $\mathbf{6}$ | 200 | 200 | 200 | 200 | 200 | 223 | 251 | 280 | 310 |  |  |
| $\mathbf{3 0}^{\circ}$ | $\mathbf{3}$ | 200 | 200 | 200 | 200 | 200 | 200 | 200 | 200 | 221 |  |  |
|  | $\mathbf{4}$ | 200 | 200 | 200 | 200 | 200 | 200 | 224 | 251 | 278 |  |  |
|  | $\mathbf{5}$ | 200 | 200 | 200 | 200 | 205 | 235 | 266 | 298 | 331 |  |  |
|  | $\mathbf{6}$ | 200 | 200 | 200 | 201 | 235 | 270 | 306 | 343 | 380 |  |  |
| $\mathbf{4 5}^{\circ}$ | $\mathbf{3}$ | 200 | 200 | 200 | 200 | 200 | 227 | 263 | 304 | 349 |  |  |
|  | $\mathbf{4}$ | 200 | 200 | 200 | 204 | 240 | 279 | 320 | 362 | 407 |  |  |
|  | $\mathbf{5}$ | 200 | 200 | 200 | 238 | 281 | 326 | 373 | 421 | 472 |  |  |
|  | $\mathbf{6}$ | 200 | 200 | 224 | 271 | 320 | 370 | 422 | 477 | 534 |  |  |
| $\mathbf{6 0}^{\circ}$ | $\mathbf{3}$ | 200 | 200 | 241 | 302 | 373 | 450 | 535 | 630 | 731 |  |  |
|  | $\mathbf{4}$ | 200 | 240 | 285 | 355 | 430 | 510 | 599 | 697 | 803 |  |  |
|  | $\mathbf{5}$ | 200 | 256 | 327 | 402 | 485 | 572 | 666 | 768 | 876 |  |  |
|  | $\mathbf{6}$ | 216 | 286 | 363 | 446 | 534 | 629 | 730 | 837 | 950 |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |

### 4.3.2 Step 2: Design for the Ultimate Strength and Serviceability

Based on the previous step, the minimum required thickness is obtained for each tank. Design steps proceed by checking to a 1 m wall section having the minimum required designed thickness for ultimate strength and serviceability requirements. Two different types of wall section should be designed along the meridional direction. The first section is subjected to bending and high axial where an interaction diagram is developed for this section and is checked for strength requirements. The other type is a section governed by flexural moments where sectional analysis is performed. The minimum vertical
reinforcement is assumed to be $(1 \% \mathrm{Ag})$ and is layered into two layers $(0.5 \%$ for each face).

The previous mentioned steps are repeated for all selected tanks, leading to a comprehensive data that are employed to develop the required design charts. These charts provide a simple approach to adequately estimate the minimum required thickness and the associated state of stresses that varies according to the tank's geometry (i.e., tank height H , base radius $\mathrm{R}_{\mathrm{b}}$, and angle of inclination $\theta_{v}$ ). It should be mentioned that the proposed design charts are limited to the chosen range for each geometric parameter.

### 4.4 Simplified Design Charts

Based on the obtained data, two sets of charts are developed. The first one provides the minimum required thickness associated with specific tank geometry. The charts are arranged for each inclination angle and depend on both the tank height and its base radius. The second set enables the designer to predict the maximum internal forces developed in the tank's wall. The maximum internal forces are illustrated according to the inclination angle. For each angle, the internal forces are evaluated according to the tank base radius and the ratio of tank height to wall thickness as will be explained in the next sub-sections.

### 4.4.1 Minimum wall thickness determination

Using the charts provided in Figures 4-3, 4-4, 4-5, and 4-6, the minimum required wall thickness is determined for each specific tank. These charts are categorized according to inclination angles of $\left(\theta v=15^{\circ}, 30^{\circ}, 45^{\circ}\right.$, and $\left.60^{\circ}\right)$. For each specific angle of inclination, the design chart gives different values of the required thickness according to the tank
height as well as the base radius. As an example, the required thickness for a conical tank that have $\left(\theta_{v}=30^{\circ}, \mathrm{H}=9 \mathrm{~m}\right.$, and $\left.\mathrm{R}_{\mathrm{b}}=6 \mathrm{~m}\right)$ is 270 mm . it is important to note that this study limits the minimum thickness to 200 mm . By observing the design charts for wall thicknesses, it is noticed that the design charts are almost linear. This is expected since this study deals with a non-cracked section and performs a linear analysis. The minimum thickness of 200 mm satisfies the design requirements for most of the tanks having inclination angle of $15^{\circ}$ and height less than 9 m . it also is the designed thickness for angles of 30 and 45, and a height less than 6 m . It is noticed that increasing the inclination angle from $15^{\circ}$ to $60^{\circ}$ increased the required wall thickness about $10 \%$ to $320 \%$ depending on tank dimensions. The design charts also show that as the wall height is increased from 3 m to 12 m , the ratio of increasing wall thickness ranges from $3 \%$ to 57\%.


Figure 4-3 Minimum Required Thickness $\left(\theta_{v}=15^{\circ}\right)$


Figure 4-4 Minimum Required Thickness ( $\theta_{v}=30^{\circ}$ )


Figure 4-5 Minimum Required Thickness ( $\theta_{V}=45^{\circ}$ )


Figure 4-6 Minimum Required Thickness ( $\theta_{v}=60^{\circ}$ )

### 4.4.2 Determination of internal forces

This sub-section presents the design charts developed in this study to determine the maximum internal forces acting on the walls of reinforced concrete conical tanks under hydrostatic pressure, as shown in Figures 4-7, 4-8, 4-9, and 4-10. Each figure presents the maximum internal forces for each angle of inclination (i.e., $\theta_{v}=15^{\circ}, 30^{\circ}, 45^{\circ}$, and $60^{\circ}$ ). The maximum values of hoop tension, meridional moment and meridional compression can be obtained from these charts depending on the tank dimensions as well as the designed thickness, which is obtained from Figures 4-3, 4-4, 4-5, and 4-6. In order to relate the tank dimensions to its thickness, a factor $G_{f}$ is presented in the design charts. This factor $\mathrm{G}_{\mathrm{f}}$ depends on the wall thickness, tank height and inclination angle, and can be calculated from Equation (4.4).
$\left(\mathrm{G}_{\mathrm{f}}\right)=\frac{H^{2}}{t_{\text {min. }} \cdot\left(\operatorname{Cos} \theta_{v}\right)^{2}}$

Where, $H$ is the total height of the conical tank, $\theta_{V}$ is the angle of inclination of the meridian with the vertical, $R_{b}$ is the base radius of the conical tank, $t_{\text {min }}$ is the wall minimum thickness that was determined in sub-section 4.5.1.

It is noticed that the factor $\left(G_{f}\right)$ and the associated maximum forces display two different trends: linear and non-linear trend. The linear trend is due to that the tanks are designed for the limited thickness (i.e., 200 mm ). On the other hand, for nonlinear trend, the designed thickness is variable depending on the tank dimensions.


Figure 4-7 Design Charts for Maximum Internal Forces $\left(\theta_{\mathrm{v}}=15^{\circ}\right)$


Figure 4-8 Design Charts for Maximum Internal Forces $\left(\theta_{\mathrm{v}}=30^{\circ}\right)$




Figure 4-9 Design Charts for Maximum Internal Forces $\left(\theta_{\mathrm{v}}=45^{\circ}\right)$


Figure 4-10 Design Charts for Maximum Internal Forces $\left(\theta_{\mathrm{v}}=60^{\circ}\right)$

### 4.5 Case Studies

A number of case studies are used to assess the accuracy of the proposed simplified design approach. These case studies include nine conical tanks having geometric parameters different from those used in developing the design charts. The dimensions of these tanks are presented in Table 4-4.

Table 4-4 Dimensions of Case Studies Tanks

| Case Study | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{\theta}_{\mathbf{v}}$ | 30 | 30 | 30 | 45 | 45 | 45 | 60 | 60 | 60 |
| $\mathbf{R}_{\mathbf{b}}(\mathbf{m})$ | 3.75 | 4.75 | 6 | 3.25 | 5.25 | 5.75 | 4.5 | 3.6 | 3.4 |
| $\mathbf{H}(\mathbf{m})$ | 10.25 | 9.5 | 8.5 | 11.55 | 7.85 | 6.35 | 4.5 | 7.85 | 11.85 |

These tanks are analyzed and designed utilizing the simplified charts. A linear interpolation is done to predict the optimum wall thickness and the associated internal forces in each tank. In order to assess the accuracy of the proposed charts, all nine tanks are analyzed using the finite element model which provides the accurate behaviour of conical shaped tanks. The comparison between the two methods is shown in Table 4-5. This table gives the designed thicknesses associated with finite element analysis as well as those that are determined by the design charts. The discrepancy between the internal forces (hoop, moment and compression) is also presented. By comparing the internal forces obtained from both the simplified charts and finite element analysis, it is notices that there is a very good agreement with a maximum of $7 \%$ difference.

Table 4-5 Comparison between FEA and Design Charts

| Case Study ${ }^{*}$ | $\mathbf{t}_{\text {min }}(\mathrm{mm})$ |  |  | $\mathbf{G}_{\mathrm{f}}{ }^{\left({ }^{*}\right)}$ | Max. Hoop Tension (kN/m) |  |  | Max. Meridional Moment (kN.m/m) |  |  | Max. Meridional Compression (kN/m) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | FEA | Charts | $\begin{gathered} \hline \text { Diff. } \\ (\%) \\ \hline \end{gathered}$ |  | FEA | Charts | $\begin{aligned} & \text { Diff. } \\ & (\%) \\ & \hline \end{aligned}$ | FEA | Charts | $\begin{gathered} \hline \text { Diff. } \\ (\%) \\ \hline \end{gathered}$ | FEA | Charts | $\begin{aligned} & \hline \text { Diff. } \\ & (\%) \\ & \hline \end{aligned}$ |
| 1 | 219 | 223 | -1.8 | 629 | 508.3 | 520 | -2.3 | 12.1 | 12 | 0.8 | 551.7 | 580 | -5.1 |
| 2 | 241 | 241 | 0 | 499 | 559.8 | 566 | -1.1 | 14.20 | 14.65 | -3.2 | 428.0 | 428.1 | 0 |
| 3 | 253 | 252 | 0.4 | 382 | 587.4 | 585 | 0.4 | 16.2 | 16 | 1.2 | 312.3 | 312.5 | -0.1 |
| 4 | 341 | 343 | -0.6 | 777 | 791.8 | 775 | 2.1 | 32.8 | 31.6 | 3.7 | 2013 | 1896 | 5.8 |
| 5 | 284 | 284 | 0 | 434 | 660.0 | 659.5 | 0.1 | 20.1 | 20.6 | -2.5 | 646.9 | 647.5 | -0.1 |
| 6 | 234 | 233 | 0.4 | 346 | 542.6 | 555 | -2.3 | 14.2 | 14.45 | -1.8 | 389.0 | 403.75 | -3.8 |
| 7 | 211 | 224 | -6.2 | 361 | 488.9 | 476 | 2.6 | 11.4 | 11.5 | -0.9 | 559.1 | 540 | 3.4 |
| 8 | 394 | 396 | -0.5 | 622 | 916.4 | 916.4 | 0 | 41.1 | 44 | -7.1 | 2320.6 | 2274 | 2 |
| 9 | 741 | 744 | -0.4 | 755 | 1722.2 | 1638 | 4.9 | 163.9 | 152.24 | 7.1 | 6646.1 | 6130 | 7.8 |

(*) for tanks dimensions of each case study, refer to Table 4-2.
$\left(^{*}\right) \mathrm{G}_{\mathrm{f}}=\frac{H^{2}}{t_{\text {min. }} \cdot\left(\operatorname{Cos} \theta_{v}\right)^{2}}$,

### 4.5.1 Numerical Example

For more illustration, the general procedure used to design tank No. 5 is presented as an example in the following steps. These steps are considered as a typical procedure that can be repeated to analyze and design any conical tank with geometry lying within the range specified in this study.

- Tank dimensions $\left(\theta_{v}=30^{\circ}, H=6 \mathrm{~m}, R_{b}=8.5 \mathrm{~m}\right)$
- In order to determine the minimum required thickness, Figure 4-4 is used for $\theta_{v}=$ $30^{\circ}$, and height $H=6 \mathrm{~m}$. Wall thicknesses for base radiuses $R_{b}=8 \mathrm{~m}$, and $R_{b}=9$ m are determined. A linear interpolation is then used to determine the required thickness of $R_{b}=8.5 \mathrm{~m} .\left(t_{\text {min }}=252 \mathrm{~mm}\right)$.
- Calculate the factor $G_{f}=\frac{H^{2}}{t_{\min .} \cdot\left(\operatorname{Cos} \theta_{v}\right)^{2}}=\frac{8.5^{2}}{252 .(\operatorname{Cos} 30)^{2}} \times 1000=382 \mathrm{~m}$.
- Figure 4-8 is used for $\theta_{v}=30^{\circ}$, where the internal forces corresponding to factors $G_{f}=300 \mathrm{~m}$ and $G_{f}=400 \mathrm{~m}$ are obtained. A linear interpolation is applied to determine the maximum internal forces for $G_{f}=382 \mathrm{~m}($ Hoop $=520 \mathrm{kN} / \mathrm{m}$, Moment $=12 \mathrm{kN} . \mathrm{m} / \mathrm{m}$, Compression $=580 \mathrm{kN} / \mathrm{m})$.


### 4.6 Conclusions

The current study presents a simplified procedure to analyze and design reinforced concrete conical tanks under the effect of hydrostatic pressure. This procedure depends on different design charts that are categorised according to the tank dimensions. The required wall thickness and the associated maximum internal forces (hoop, moment and
compression) can be determined by using the proposed charts. In order to develop these design charts, a wide range of conical tanks having various dimensions is analyzed and designed by using a linear finite element analysis method. The design is repeated for each tank until the minimum required thickness is achieved. The serviceability and strength requirements of both ACI350-06 and PCA-CCTWP are satisfied in the design. The accuracy of the developed charts is validated by a number of numerical case studies. The following main conclusion can be drawn from the current study.

- The design charts provided in this study enable the designers to determine the minimum required thickness for tanks' walls. This thickness satisfies serviceability and strength requirements, leading to a reduction in the vessel own weight as well as the construction cost.
- It is noticed that all tanks having walls inclined with an angle $15^{\circ}$ to the vertical axis are governed by a minimum thickness of 200 mm . Also, this thickness (i.e., 200 mm ) satisfies the design requirements of all conical tanks with height ranges from 4 m to 5 m regardless the variation in other geometric parameters. On the other hand, significant increase in the required thickness is noticed for tanks higher than 7 m height.
- The design charts provide simple and accurate procedure to determine the maximum internal forces (hoop, meridional moment and meridional compression). The proposed charts are presented graphically according to the angle of inclination and the base radius. For each specific angle of inclination and base radius, the predicted maximum internal forces are provided according to the ratio between the tank height and the minimum designed thickness. The obtained forces can be
successfully utilized to design the wall section for ultimate strength design requirements and the related reinforcements.


## CHAPTER 5 <br> COST ANALYSIS OF CONICAL TANKS; COMPARISON BETWEEN REINFORCED CONCRETE AND STEEL

### 5.1 Introduction

The vessels used for liquid storage containers are commonly built in a conical shape, including pure conical tanks and conical cylindrical combined tanks. The construction of conical tanks is dominated by using either steel, conventional reinforced concrete or partially pre-stressed concrete. The decision to select the most proper construction material for such tanks depends on various factors: structural performance, material cost, life service, material availability and cost of labour works (Barry 2011).

The main advantages of reinforced concrete tanks over steel tanks are that they provide high resistance to compression stresses and have long service life (i.e., up to 50 years) compared to steel tanks (i.e., up to 20 years) (Cheremisinoff 1996). On the other hand, the main disadvantages of reinforced concrete tanks are related to the low tensile strength and the large thickness required to satisfy design requirements which leads to a significant increase in the own weight. Despite the advantage of using reinforced concrete as a construction material for storage tanks, steel tanks are widely used in Canada and USA over the last 25 years. This is due to the fact that steel storage tanks are leak-free structures and they also provide high tension resistance and lighter own weight compared to reinforced concrete counterparts. The only concern about steel as a construction material is that it is sensitive to geometric imperfections, buckling, and corrosion problems. Most of structural optimization techniques of conical tanks deal with
minimizing the weight of the structure by achieving the minimum thickness which satisfies design requirements. (Kamal, 1998; El Ansary et al. 2010, 2011).

Choosing the most proper construction material which leads to an economical design is not an easy task as it involves many parameters. These parameters are: type of the structure, construction techniques, and life-cycle cost of construction material. The literature shows that very few studies are concerned about comparing the cost of reinforced concrete conical tanks to that of steel counterparts.

In this regard, few researches presented trials to minimize the cost of storage tanks; Saxena et al. (1987) presented a cost function which includes the cost of different construction materials (e.g. concrete and steel) and the cost of formwork. It was concluded in their study that mores savings in cost can be achieved for water tanks having large storage capacities. Later, Copley et al. (2002) presented the design and cost analysis of a partially pre-stressed concrete conical tank having a storage capacity of 2 Million Gallons. In their cost analysis, they showed that the cost of construction of a steel tank is more economical than a pre-stressed concrete counterpart. However, the life-cycle cost analysis, which was implemented in Copley`s work, showed that pre-stressed concrete is a better alternative in terms of long service life.

Barakat and Altoubat (2009) introduced optimization techniques which were coupled with the finite element method in the analysis and design of reinforced concrete conical and cylindrical water tanks. They illustrated the effect of different parameters, which include the wall thickness at the base and at the top of the tank, the base thickness, the tank height, the inclination angle, and concrete compressive strength, on the optimum design. Barakat and Altoubat (2009) concluded that the total cost of cylindrical tanks is
more than that of conical water tanks having the same volume. This increase in cost has been estimated by $20 \%$ to $30 \%$ and by $18 \%$ to $40 \%$ when working stress design method and ultimate strength design method were used, respectively.

The main objective of this chapter is to investigate the economics of reinforced concrete conical tanks versus steel counterparts. This study considers only conical vessels having a constant thickness and subjected to hydrostatic loading. The design of concrete tanks is conducted following the simplified approach presented in the previous chapter, which complies with the requirements of the ACI350-06. On the other hand, the design of steel tanks is obtained by using the simplified approach provided by Sweedan and El Damatty (2009).

According to the American Public Works Association, it is essential to include the lifecycle costing procedures in the project bidding (Ross, 2001). As such, this research includes an estimation of a 50 years life-cycle cost for both concrete and steel tanks in addition to the construction cost (i.e., construction material and labour works). The same service life of 50 years is selected for both reinforced concrete conical tanks and steel tanks for the purpose of comparison between the two options.

Moreover, this study presents an average unit prices for contractors working in Canada. It should be noted that these unit prices are variable depending on various factors such as site location, material availability, energy cost and others. A total of 52 tanks are chosen to cover a wide range of practical tank dimensions and are categorized into three capacities; $500 \mathrm{~m}^{3}, 1750 \mathrm{~m}^{3}$, and $3000 \mathrm{~m}^{3}$. These tanks are designed first as reinforced concrete tanks then as steel tanks. The cost of each tank is estimated and a comparison is then conducted to analyze the economics of using the two construction materials (i.e.,
reinforced concrete and steel) for these tanks. Statistical analyses are also performed in order to evaluate the factors having the most significant effect on the cost of conical tanks.

### 5.2 Design of Reinforced Concrete Conical Tanks under Hydrostatic Load

Design of reinforced concrete conical tanks includes many parameters; angle of inclination of tank's wall, tank height, base radius, and wall thickness. In order to achieve an adequate design, it is essential to predict the maximum internal forces that include hoop tension acting in the circumferential direction and the meridional moment combined with the axial compression force acting in the longitudinal direction. Conducting this analysis needs modeling experience and knowledge about design steps. An alternative way is to rely on simplified design procedures which satisfy code provisions. In this study, a reliable simplified procedure proposed in the previous chapter was utilized in the design of reinforced concrete tanks. This approach includes certain design charts that were developed by modelling a wide practical range of conical tanks having different dimensions. All analyzed tanks were modelled using a degenerated consistent subparametric shell element developed in-house (Koziey and Mirza 1997; El Damatty et al. 1997, 1998).

The simplified design charts enable the designers to easily evaluate the required minimum thickness and the associated internal forces in both circumferential and longitudinal directions. Consequently, the cost of the required construction material can be estimated.

The steps of the procedure involved in the design can be explained as follows:

1. The tank dimensions (angle of inclination $\theta_{v}$, base radius $R_{b}$, and tank height $H$ ) are chosen according to the required tank volume (i.e., capacity). It should be mentioned that specific capacity ranges are assumed in this study covering a practical range starting from $500 \mathrm{~m}^{3}$ up to $3000 \mathrm{~m}^{3}$.
2. Figures $4-5,4-6,4-7$, or $4-8$, which were presented in chapter 4 , are then used to determine the minimum required thickness. By knowing the values of the base radius and the tank height, linear interpolation is applied to predict the minimum required thickness.
3. A factor $\left(G_{f}\right)$, which relates the tank dimensions to the internal forces that are developed in the tank wall due to hydrostatic pressure, is calculated using Equation 5.1. This factor is then used in the charts illustrated in Figures 4-9, 4-10, 4-11 or 4-12 to estimate the internal forces developed in tanks' walls due to unfactored hydrostatic pressure. The outputs of these charts include hoop tension, meridional moment and meridional compression.

$$
\begin{equation*}
\left(\mathrm{G}_{\mathrm{f}}\right)=\frac{H^{2}}{t_{\min .} \cdot\left(\operatorname{Cos} \theta_{v}\right)^{2}} \tag{5.1}
\end{equation*}
$$

4. The required circumferential (horizontal reinforcement) $\left(A_{S}\right)$ is then calculated using Equation 5.2.

$$
\begin{equation*}
A_{S}=\frac{T_{u}}{0.9 \times F_{y}} \tag{5.2}
\end{equation*}
$$

Where $T_{u}$ is the maximum factored hoop tension force magnified by the environmental durability factor $S_{d},\left(T_{u}=1.4 \times S_{d} \times T\right)$, where $T$ is the service hoop tension obtained from step $3, f_{y}$ is the steel yielding strength, and $S_{d}$ is the environmental durability factor calculated from Equation 5.3.
$S_{d}=\frac{\varnothing f_{y}}{\gamma f_{s}}$

In Equation $5.3, \varnothing$ is the strength reduction factor, $(\varnothing=0.9$ for both hoop tension and flexural members), $f_{y}=400 \mathrm{MPa}$ is the steel yield strength, $f_{s}=140 \mathrm{MPa}$ is the allowable stress in normal environment, and $\gamma=\frac{\text { factored load }}{\text { unfactorred load }}=1.4$, in case of hydrostatic pressure and dead loads.

### 5.3 Design of Steel Conical Tanks under Hydrostatic Load

Similar to reinforced concrete tanks, hydrostatic pressure acting on the walls of steel tanks leads to tension hoop stresses $\left(\sigma_{h}\right)$ that are acting in the circumferential direction and vary along the wall height. In addition, meridional compressive stresses $\left(\sigma_{m}\right)$ that reach their maximum value at the base of the wall are acting in the meridional direction. Those stresses are magnified due to the effect of boundary conditions as well as geometric imperfections. As such, a magnification factor should be provided to relate the theoretical membrane stresses, which can be evaluated from static equilibrium of the shell to the actual maximum stresses acting on the wall. Sweedan and El Damatty (2009) developed a simplified procedure that can evaluate this magnification factor associated with the maximum stresses developed in the tank's wall. Consequently, the wall thickness can be designed to prevent steel yielding. This simplified procedure is utilized in the current study to design the steel conical tanks under consideration according to the following steps:

1. The tanks' dimensions (angle of inclination $\theta_{v}$, base radius $R_{b}$, and tank height $H$ ) are chosen to be similar to the concrete tanks designed in section 5.2 to keep
storage capacities the same. For each tank, an initial value of the wall thickness $\left(t_{s}\right)$ is assumed taking into account that the minimum thickness is 6.4 mm according to AWWA-D100 (2005) code provisions.
2. From static equilibrium, the theoretical tensile hoop stress $\left(\sigma_{h}{ }^{\text {th }}\right)$ and the theoretical meridional compression stress $\left(\sigma_{m}{ }^{t h}\right)$ are calculated from Equations 5.4, and 5.5, respectively.

$$
\begin{equation*}
\sigma_{h}{ }^{t h}=\frac{\gamma H R_{b}}{t_{s} \cos \theta_{v}} \tag{5.4}
\end{equation*}
$$

$\sigma_{m}{ }^{\text {th }}=\frac{\gamma H \tan \theta_{v}}{2 R_{b} t_{s} \cos \theta_{v}}\left[R_{b} H+H \tan \theta_{v}(1 / 3 H)\right]$
3. Based on the Von Mises yield criterion, the theoretical maximum effective membrane stresses $\left(\sigma_{l}^{t h}\right)$ is calculated from Equations ( 5.6 to 5.9 )
$\sigma_{l}^{t h}=\sqrt{\frac{3}{2}\left[\left(\bar{\sigma}_{1}\right)^{2}+\left(\bar{\sigma}_{2}\right)^{2}+\left(\bar{\sigma}_{3}\right)^{2}\right]}$
in which
$\bar{\sigma}_{1}=\sigma_{m}{ }^{t h}-\frac{\sigma_{m}{ }^{t h}-\sigma_{h}{ }^{t h}}{3}$
$\bar{\sigma}_{2}=\sigma_{h}{ }^{t h}-\frac{\sigma_{m}^{t h}-\sigma_{h}{ }^{t h}}{3}$
$\bar{\sigma}_{3}=-\frac{\sigma_{m}^{t h}-\sigma_{h}{ }^{t h}}{3}$
4. The magnification factor $(\beta)$ is then calculated from Equation 5.10.

$$
\begin{equation*}
\beta=a\left(\frac{R_{b}}{H}\right)^{b}+c\left(\frac{t_{s}}{H}\right)^{d}+e\left(\frac{R_{b}}{H}\right)^{f}\left(\frac{t_{s}}{H}\right)^{g}\left(\theta_{v}\right)^{h} \tag{5.10}
\end{equation*}
$$

Where, the regression factors ( $\mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{e}, \mathrm{f}, \mathrm{g}, \mathrm{h}$ ) are given in Table B1 in Appendix B. It should be mentioned that a good quality of welding of steel panels is assumed in the current study. As such, regression factors for good conical shells are used in Equation 5.10. A yield stress of 300 MPa is assumed for all studied tanks.
5. The total actual stress $\left(\sigma_{l}\right)$ is then calculated by multiplying the magnification factor $(\beta)$ by the theoretical maximum effective membrane stresses $\left(\sigma_{l}^{\text {th }}\right)$ as shown in Equation (5.11).
$\sigma_{l}=\beta \sigma_{l}{ }^{\text {th }}$
6. The actual total stress is then compared to the yield strength of steel ( $\sigma_{y}=300$ MPa ). The yield strength should be greater than the actual total stress. The procedure is repeated until the optimum thickness is achieved (i.e., $\sigma_{l} \cong \sigma_{y}$ ).

### 5.4 Cost Estimation

The total cost of the storage vessel of a tank is the summation of the cost of different parameters. This study focuses on the construction costs, which includes material, labours, and erection costs as well as the life-cycle cost. This section presents the details and methodology of analyzing the cost of each tank using two different construction materials (reinforced concrete and steel).

### 5.4.1 Construction Cost Estimation

In this sub-section, the cost of construction using each material (i.e., reinforced concrete, and steel) is estimated to identify the most cost effective construction material for conical tanks. The cost of reinforced concrete, which includes labour works, is measured by concrete volume, the weight of steel rebar and the surface area of the formwork. For the cost of steel tanks, the martial unit prices are presented by unit weight. The prices assumed in the current study are based on average prices collected from the local construction industry.

### 5.4.1.1 Construction Cost Estimation for Concrete Tanks

- The cost of materials and construction is estimated according to the volume of concrete and the reinforcing ratio of circumferential (i.e., horizontal) and longitudinal (i.e., vertical) steel as well as the surface area for the formwork (EL Reedy, 2011). Table 5-1 shows the unit prices for concrete used in the construction of the tanks considered in this study. The construction cost function is presented as the summation of the following parameters:
- Cost of concrete $=($ tank's surface area $\times$ wall thickness $) \times$ cost of cubic meter of concrete
- Cost of reinforcement steel $=$ concrete volume $\times 7.85 \frac{t o n}{m^{3}} \times\left(\rho_{\text {sh }}+\rho_{\text {sv }}\right) \times$ cost of steel $\frac{C A D}{\text { ton }}$

Where $\rho_{s h}$ is the ratio of circumferential steel $\left(\rho_{s h}=\frac{A_{s}}{A_{c}}\right), \mathrm{A}_{\mathrm{s}}$ is the area of circumferential reinforcement that is determined by using the simplified design charts. Referring to Equation (5.2), the area of the circumferential reinforcement can
be calculated which is then used to determine the ratio of circumferential steel. The ratio of vertical steel $\rho_{\mathrm{sv}}$ is always taken as ( $\rho_{\mathrm{sv}}=1 \%$ of gross area of concrete).

- Cost of formwork $=$ tank's surface area $\times$ cost of double face of formwork

Table 5-1 Unit Price for Reinforced Concrete Conical Tanks

| Item Description | Unit | Price <br> (CAD/Unit) |
| :--- | :--- | :--- |

1. Cost of Material

Pumped Concrete with $\mathrm{m}^{3} 255$
admixtures and air entraining
agents
Reinforcement steel M16/20 ton 1324
Impermeable plywood $\mathrm{m}^{2} \quad 266$
formwork double face

## 2. Cost of Labour

Fabrication of wood and $\mathrm{m}^{3} \quad 45$
reinforcing steel and pouring concrete (per concrete volume)

### 5.4.1.2 Construction Cost Estimation for Steel Tanks

In this sub-section, the cost of the designed steel tanks is estimated assuming the material unit cost for steel to be $3000 \frac{\mathrm{CAD}}{\text { ton }}$. The construction and erection unit cost is taken as $30 \%$ of the total material cost, as stated by (EL Reedy, 2011). The construction cost function is calculated as the summation of the cost of the following parameters:

- Material cost $=$ Material weight $\times$ Material unit cost
$=\left(\right.$ Steel unit weight; $\left.7.850 \frac{\text { ton }}{\mathrm{m}^{3}}\right) \times$ wall thickness; $\left.t_{s}\right) \times($ Tank surface area $) \times\left(\right.$ material unit cost; $\left.3000 \frac{\mathrm{CAD}}{\text { ton }}\right)$
- Total cost (material + Construction) per volume $\left(\frac{C A D}{m^{3}}\right)=\frac{(\text { Material cost } \times 1.3)}{\text { volume }}$


### 5.4.2 Life-Cycle Costs

In order to estimate the current cost of future maintenance and rehabilitation works, the present value analysis method is performed for all concrete and steel tanks for a service life of 50 years (EL Reedy, 2011). This method is widely used in construction applications and it also presents the future costs in today's monetary taking in considerations the inflation and interest rates. It should be mentioned that for comparison purposes, the same period of life-cycle (i.e., 50 years) is chosen for both steel and concrete tanks. EL Reedy (2011) provided an expression to calculate the value of maintenance and repairs required, as shown in Equation (5.12).

Present Value $=$ Repair Cost $\times(1+m)^{(-n)}$
Where; $m$ is the discount rate ( $m=4 \%$ ), and $n$ is the number of years of each maintenance period.

Based on the data collected from the local market, the maintenance cost of concrete tanks is assumed in the current study to be $89 \frac{C A D}{m^{2}}$ every 5 years while in case of steel, it is recommended to cost $40 \frac{C A D}{m^{2}}$ at a period of 3 years. It is worth to mention that the operating cost is not taken as part of this study.

### 5.5 Results and Discussion

This study includes 52 conical tanks having wide range of dimensions with different capacities; $500 \mathrm{~m}^{3}, 1750 \mathrm{~m}^{3}$, and $3000 \mathrm{~m}^{3}$. The dimensions of these tanks are presented in Table 5-2. The considered tanks are first designed as reinforced concrete and then as steel tanks according to the simplified design procedures mentioned in the previous sections. The cost analysis is then conducted for all designed tanks as presented in Table 5-2. This table shows the design outputs and the total cost described as price per unit volume (i.e., CAD per $\mathrm{m}^{3}$ ) for each tank.

The comparison between the cost of reinforced concrete conical tanks and steel tanks is displayed in Figures 5-1, 5-2, and 5-3 for tanks with volumes of $500 \mathrm{~m}^{3}, 1750 \mathrm{~m}^{3}$, $3000 \mathrm{~m}^{3}$, respectively. Also, each figure categorizes the tank cost according to the base radiuses, where $R_{b}$ is varying from 3 m to 6 m with an increment of 0.5 m . Table 5-2 and Figure 5-1 show that steel tanks are more cost-effective than reinforced concrete for small capacity tanks, i.e., $500 \mathrm{~m}^{3}$ tanks. The average total cost of reinforced concrete conical tanks are estimated to be $338 \frac{C A D}{\mathrm{~m}^{3}}$, which is approximately 1.7 times the cost of steel counterparts.

For conical tanks having a volume of $1750 \mathrm{~m}^{3}$, it is concluded that steel tanks are more economical than reinforced concrete tanks. Figure 5-2 shows that the total cost of steel tanks is less than that of reinforced concrete tanks having the same dimensions. In general, steel tanks show less cost compared to reinforced concrete counterparts with a percentage of reduction varying between $4 \%$ and $39 \%$. It can be noticed from the figure that in only two cases the cost of steel tanks is found to be greater than that of reinforced concrete tanks. The reported percentage of increase for these two cases are $9 \%$ and $2 \%$
for tanks having walls inclined to the vertical with an angle of $60^{\circ}$ and having base radiuses of 3 m , and 3.5 m , respectively.

Based on the cost analysis of large capacity $\left(3000 \mathrm{~m}^{3}\right)$ tanks as presented in Figure 5-3, it can be noticed that in some cases concrete as a construction material is a more economical choice. Figure 5-3 shows that the cost of concrete tanks is less than steel for the case of wide conical tanks having walls inclined to the vertical with an angle greater than $45^{\circ}$ and a base radius less than 4 m . Otherwise, steel provides a more economical choice for all conical tanks having $30^{\circ}$ inclination angle and tanks with $45^{\circ}$ walls and having a base radiuses of $(4 \mathrm{~m}$ to 6 m$)$. Based on the results reported for large capacity tanks, no clear trend can be reached in order to decide which construction material is the most cost effective one.

The results obtained from the cost analysis are evaluated statistically by using one way analysis of variance ANOVA for a single factor (Stamatis 2002). This analysis is conducted to assess the significance in the change of the cost from one case to another. Two different case studies are performed using ANOVA. The first case is conducted for reinforced concrete tanks and steel counterparts in order to study the variance in the cost function with the change of material type. In the second case of this study, ANOVA is employed to evaluate the effect of tank dimensions on its cost for each type of the studied tanks.

Table 5-2 Design and Cost of Conical Tanks
$\left(\right.$ Capacity $\left.=500 \mathrm{~m}^{3}\right)$

| Tank \# | $\begin{aligned} & \mathbf{R}_{\mathrm{b}} \\ & (\mathbf{m}) \end{aligned}$ | 日v | H (m) | Section Design |  |  | Cost (CAD/m ${ }^{\text {3 }}$ ) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Concrete |  | Steel | Concrete |  |  | Steel |  |  |
|  |  |  |  | $\begin{gathered} \mathbf{t}_{\mathbf{c}} \\ (\mathbf{m m}) \\ \hline \end{gathered}$ | $\begin{gathered} \rho_{\mathrm{sh}} \\ (\%) \\ \hline\left(\begin{array}{l} \text { en } \end{array}\right. \\ \hline \end{gathered}$ | $\mathrm{t}_{\mathrm{s}}(\mathrm{mm})$ | Construction | Life-Cycle | Sum | Construction | Life-Cycle | Sum |
| 1 | 3 | 15 | 8.85 | 200 | 1.01 | 6.4 | 177 | 214 | 391 | 94 | 134 | 228 |
| 2 | 3 | 30 | 6.43 | 200 | 0.88 | 6.4 | 165 | 201 | 366 | 89 | 126 | 215 |
| 3 | 3 | 45 | 4.96 | 200 | 0.9 | 6.4 | 177 | 215 | 392 | 95 | 134 | 229 |
| 4 | 3.5 | 15 | 7.65 | 200 | 0.98 | 6.4 | 165 | 200 | 365 | 88 | 125 | 213 |
| 5 | 3.5 | 30 | 5.77 | 200 | 0.87 | 6.4 | 158 | 192 | 350 | 85 | 120 | 205 |
| 6 | 3.5 | 45 | 4.55 | 200 | 0.91 | 6.4 | 171 | 207 | 378 | 91 | 129 | 220 |
| 7 | 4 | 15 | 6.6 | 200 | 0.92 | 6.4 | 153 | 186 | 339 | 82 | 116 | 198 |
| 8 | 4 | 30 | 5.16 | 200 | 0.86 | 6.4 | 150 | 182 | 332 | 80 | 114 | 194 |
| 9 | 4 | 45 | 4.15 | 200 | 0.91 | 6.4 | 164 | 199 | 363 | 88 | 124 | 212 |
| 10 | 4.5 | 15 | 5.7 | 200 | 0.88 | 6.4 | 142 | 173 | 315 | 76 | 108 | 184 |
| 11 | 4.5 | 30 | 4.61 | 200 | 0.84 | 6.4 | 142 | 173 | 315 | 76 | 108 | 184 |
| 12 | 5 | 15 | 4.94 | 200 | 0.86 | 6.4 | 133 | 161 | 294 | 71 | 101 | 172 |
| 13 | 5.5 | 15 | 4.3 | 200 | 0.82 | 6.4 | 124 | 151 | 275 | 67 | 94 | 161 |
| 14 | 6 | 15 | 3.76 | 200 | 0.79 | 6.4 | 115 | 141 | 256 | 62 | 88 | 150 |

Table 5-2 (Continued)
(Capacity $=1750 \mathrm{~m}^{3}$ )

| Tank \# | $\begin{gathered} \mathbf{R}_{\mathrm{b}} \\ (\mathrm{~m}) \end{gathered}$ | $\theta \mathrm{v}$ | H (m) | Section Design |  |  | Cost (CAD/m ${ }^{\text {3 }}$ ) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Concrete |  | Steel | Concrete |  |  | Steel |  |  |
|  |  |  |  | $\begin{gathered} \mathbf{t}_{\mathbf{c}} \\ (\mathbf{m m}) \\ \hline \end{gathered}$ | $\begin{gathered} \rho_{\text {sh }} \\ (\%) \\ \hline \end{gathered}$ | $\mathrm{t}_{5}(\mathrm{~mm})$ | Construction | Life-Cycle | Sum | Construction | Life-Cycle | Sum |
| 15 | 3 | 45 | 8.93 | 226 | 1.66 | 16 | 134 | 150 | 284 | 166 | 94 | 260 |
| 16 | 3 | 60 | 6.52 | 273 | 1.67 | 22 | 172 | 180 | 352 | 273 | 112 | 385 |
| 17 | 3.5 | 30 | 11.3 | 233 | 1.76 | 11.5 | 128 | 141 | 269 | 112 | 88 | 200 |
| 18 | 3.5 | 45 | 8.47 | 236 | 1.75 | 14.5 | 134 | 147 | 281 | 148 | 92 | 240 |
| 19 | 3.5 | 60 | 6.25 | 280 | 1.75 | 20 | 172 | 177 | 349 | 245 | 111 | 356 |
| 20 | 4 | 30 | 10.56 | 240 | 1.66 | 11 | 125 | 137 | 262 | 104 | 85 | 189 |
| 21 | 4 | 45 | 8.02 | 241 | 1.69 | 13 | 132 | 144 | 276 | 130 | 90 | 220 |
| 22 | 4 | 60 | 5.98 | 285 | 1.66 | 18.5 | 170 | 175 | 345 | 223 | 109 | 332 |
| 23 | 4.5 | 30 | 9.85 | 241 | 1.71 | 10 | 122 | 133 | 255 | 92 | 83 | 175 |
| 24 | 4.5 | 45 | 7.58 | 244 | 1.52 | 12 | 129 | 141 | 270 | 117 | 88 | 205 |
| 25 | 4.5 | 60 | 5.72 | 230 | 3.5 | 17 | 172 | 172 | 344 | 202 | 107 | 309 |
| 26 | 5 | 30 | 9.17 | 241 | 1.66 | 9 | 118 | 129 | 247 | 80 | 80 | 160 |
| 27 | 5 | 45 | 7.16 | 245 | 1.67 | 11.5 | 127 | 138 | 265 | 110 | 86 | 196 |
| 28 | 5 | 60 | 5.46 | 230 | 3.39 | 16 | 168 | 169 | 337 | 187 | 106 | 293 |
| 29 | 5.5 | 15 | 11.2 | 267 | 1.61 | 8.5 | 122 | 129 | 251 | 76 | 81 | 157 |
| 30 | 5.5 | 30 | 8.52 | 258 | 1.42 | 8.5 | 115 | 125 | 240 | 73 | 78 | 151 |
| 31 | 5.5 | 45 | 6.75 | 244 | 1.7 | 10.5 | 124 | 135 | 259 | 98 | 84 | 182 |
| 32 | 5.5 | 60 | 5.21 | 287 | 2.14 | 15 | 167 | 166 | 333 | 172 | 104 | 276 |
| 33 | 6 | 15 | 10.17 | 256 | 1.68 | 8 | 115 | 123 | 238 | 68 | 77 | 145 |
| 34 | 6 | 30 | 7.92 | 233 | 1.66 | 8 | 109 | 121 | 230 | 67 | 75 | 142 |
| 35 | 6 | 45 | 6.36 | 241 | 1.67 | 10 | 120 | 131 | 251 | 91 | 82 | 173 |
| 36 | 6 | 60 | 4.96 | 284 | 2.46 | 14.5 | 166 | 163 | 329 | 163 | 102 | 265 |

Table 5-2 (Continued)
(Capacity $=3000 \mathrm{~m}^{3}$ )

| Tank \# | $\begin{aligned} & \mathbf{R}_{\mathrm{b}} \\ & (\mathbf{m}) \end{aligned}$ | $\theta \mathrm{v}$ | H (m) | Section Design |  |  | Cost (CAD/m ${ }^{\text {3 }}$ ) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Concrete |  | Steel | Concrete |  |  | Steel |  |  |
|  |  |  |  | $\begin{gathered} \mathbf{t}_{\mathbf{c}} \\ (\mathbf{m m}) \\ \hline \end{gathered}$ | $\begin{gathered} \hline \rho_{\text {sh }} \\ (\%) \\ (\%) \end{gathered}$ | $\mathrm{t}_{\mathrm{s}}(\mathrm{mm})$ | Construction | Life-Cycle | Sum | Construction | Life-Cycle | Sum |
| 37 | 3 | 45 | 11.25 | 316 | 1.66 | 24.5 | 129 | 127 | 256 | 215 | 80 | 295 |
| 38 | 3 | 60 | 8.13 | 384 | 1.66 | 34 | 167 | 152 | 319 | 356 | 95 | 451 |
| 39 | 3.5 | 45 | 10.8 | 325 | 1.68 | 22 | 129 | 126 | 255 | 191 | 79 | 270 |
| 40 | 3.5 | 60 | 7.86 | 391 | 1.68 | 30.5 | 167 | 150 | 317 | 317 | 94 | 411 |
| 41 | 4 | 45 | 10.31 | 333 | 1.57 | 20 | 127 | 124 | 251 | 171 | 77 | 248 |
| 42 | 4 | 60 | 7.58 | 399 | 1.66 | 27.5 | 166 | 149 | 315 | 282 | 93 | 375 |
| 43 | 4.5 | 45 | 9.85 | 340 | 1.7 | 18 | 127 | 122 | 249 | 152 | 76 | 228 |
| 44 | 4.5 | 60 | 7.31 | 354 | 2.66 | 25.5 | 168 | 147 | 315 | 259 | 92 | 351 |
| 45 | 5 | 45 | 9.41 | 346 | 1.66 | 17 | 126 | 120 | 246 | 141 | 75 | 216 |
| 46 | 5 | 60 | 7.04 | 406 | 1.68 | 24 | 164 | 145 | 309 | 241 | 91 | 332 |
| 47 | 5.5 | 30 | 11.62 | 343 | 1.68 | 13 | 116 | 110 | 226 | 99 | 69 | 168 |
| 48 | 5.5 | 45 | 8.97 | 347 | 1.69 | 16 | 124 | 118 | 242 | 130 | 73 | 203 |
| 49 | 5.5 | 60 | 6.78 | 407 | 2.09 | 22.5 | 168 | 143 | 311 | 223 | 89 | 312 |
| 50 | 6 | 30 | 10.95 | 342 | 1.67 | 12 | 112 | 107 | 219 | 89 | 67 | 156 |
| 52 | 6 | 45 | 8.55 | 348 | 1.67 | 15 | 122 | 115 | 237 | 120 | 72 | 192 |



Figure 5-1 Cost Analysis for Tanks Capacity $500 \mathrm{~m}^{3}$


Figure 5-2 Cost Analysis for Tanks Capacity $1750 \mathrm{~m}^{3}$


Figure 5-3 Cost Analysis for Tanks Capacity $3000 \mathrm{~m}^{3}$

As a result of the analysis of variance of the first case study, small capacity tanks show significant differences in cost due the difference in construction material (i.e., concrete and steel). It is noticed that for $500 \mathrm{~m}^{3}$ and $1750 \mathrm{~m}^{3}$ capacities, where ( $\mathrm{p} \leq 0.05$ ) as presented in Table 5-3, the cost of steel conical tanks is significantly less than concrete counterparts. On the other hand, for large tanks having $3000 \mathrm{~m}^{3}$ capacity, there is no significant difference in cost. As such, it can be stated that for large capacity tanks the effect of the type of construction material (steel or concrete) on the cost is negligible.

Table 5-3 Effect of Material Type on Cost of Conical Tanks Descriptive of ANOVA

| Effect of tank material on the cost ( $\left.\alpha_{\mathrm{a}}=0.05\right)$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Capacity: $500 \mathrm{~m}^{3}$ <br> summary of analysis |  |  |  |  |  |
| Groups | Count | Sum | Average | Variance |  |
| Concrete | 14 | 4731 | 337.92857 | 1794.6868 |  |
| Steel | 14 | 2765 | 197.5 | 604.42308 |  |
| Source of Variation | SS | df | MS | F | P-value |
| Between Groups | 138041.2857 | 1 | 138041.29 | 115.07708 | 4.81715E-11 |
| Within Groups | 31188.42857 | 26 | 1199.5549 |  | Fcrit |
| Total | 169229.7143 | 27 |  |  | 4.225201273 |
| $\frac{\text { Capacity: } \mathbf{1 7 5 0} \mathbf{m}^{\mathbf{3}}}{\text { summary of analysis }}$ |  |  |  |  |  |
| Groups | Count | Sum | Average | Variance |  |
| Concrete | 22 | 6267 | 284.86364 | 1748.0281 |  |
| Steel | 22 | 5011 | 227.77273 | 5201.2316 |  |
| Source of Variation | SS | df | MS | F | P -value |
| Between Groups | 35853.09091 | 1 | 35853.091 | 10.318535 | 0.00252908 |
| Within Groups | 145934.4545 | 42 | 3474.6299 |  | Fcrit |
| Total | 181787.5455 | 43 |  |  | 4.072653759 |
| Capacity: $\mathbf{3 0 0 0} \mathrm{m}^{3}$ summary of analysis |  |  |  |  |  |
| Groups | Count | Sum | Average | Variance |  |
| Concrete | 15 | 4067 | 271.13333 | 1432.2667 |  |
| Steel | 15 | 4208 | 280.53333 | 8138.1238 |  |
| Source of Variation | SS | df | MS | F | P-value |
| Between Groups | 662.7 | 1 | 662.7 | 0.1384896 | 0.712588948 |
| Within Groups | 133985.4667 | 28 | 4785.1952 |  | Fcrit |
| Total | 134648.1667 | 29 |  |  | 4.195971819 |

For the second case study, ANOVA results as presented in Table 5-4 show that for 1750 $\mathrm{m}^{3}$ and $3000 \mathrm{~m}^{3}$ capacities, regardless the type of the construction material, the inclination angle $\theta v$ has a significant effect on the cost of the tanks. As such, increasing the inclination angle increases the cost of both concrete and steel conical tanks. It is also
noticed that there is no effect of the inclination angle in case of small capacity tanks (i.e., $\left.500 \mathrm{~m}^{3}\right)$. The reason of this negligible effect is that the minimum wall thickness governs the design of these small capacity tanks. Moreover, the results show that the base radius has a minor effect on the cost of conical tanks except in case of small capacity ( $500 \mathrm{~m}^{3}$ ) tanks.

Table 5-4 P-values of ANOVA - effect of tank dimensions on cost based on a significance level ( $\alpha_{a}=0.05$ )

| Tank capacity $\left(\mathrm{m}^{3}\right)$ | Effect of $\theta_{\mathrm{v}}$ |  | Effect of $\mathrm{R}_{\mathrm{b}}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Concrete | Steel | Concrete | Steel |
| 500 | 0.288133 | 0.326551 | 0.055180847 | 0.04254477 |
| 1750 | $1.66 \mathrm{E}-09$ | $1.12 \mathrm{E}-06$ | 0.971745071 | 0.812747932 |
| 3000 | $3.23 \mathrm{E}-10$ | 0.000494 | 0.999773457 | 0.804062029 |

### 5.6 Conclusions

The current study presents a cost analysis to compare the effectiveness of using reinforced concrete versus steel as a construction material for conical tanks. In order to conduct this comparison, 52 conical tanks having different capacities (i.e., $500 \mathrm{~m}^{3}, 1750$ $\mathrm{m}^{3}, 3000 \mathrm{~m}^{3}$ ) and different dimensions are designed first as reinforced concrete tanks and then as steel. Two simplified design approaches that were developed in previous investigations are utilized in designing the studied tanks. The cost analysis conducted in this study includes the cost of materials, formwork, labour and life-cycle. At the end of the study, statistical analyses using one way ANOVA are conducted to study the significance of type of construction material on the cost function and to investigate the
effect of dimension parameters on the cost for both reinforced concrete tanks and steel counterparts. The main conclusions of this study are listed below:

- Compared to reinforced concrete, steel is a more cost-effective construction material for conical tanks with capacities of $1750 \mathrm{~m}^{3}$ or less. Steel tanks provide a reduction in the cost up to $42 \%$, and $22 \%$ for $500 \mathrm{~m}^{3}$, and $1750 \mathrm{~m}^{3}$, respectively. This conclusion can be generally applied for conical tanks having different dimensions except those tanks with inclination angle $60^{\circ}$ and base radiuses of 3 m and 3.5 m .
- For $1750 \mathrm{~m}^{3}$ capacity conical tanks having dimensions of $60^{\circ}$ inclination angle and base radius less than 4 m , reinforced concrete is considered to be more economical construction material compared to steel.
- Cost analysis for conical tanks with $3000 \mathrm{~m}^{3}$ volume shows that concrete is more economical for tanks that have inclination angle of $60^{\circ}$ and base radiuses of ( 3 m to 3.5 m ). For all other studied cases, no general conclusion is reached.
- ANOVA technique demonstrates that the angle of wall inclination has the main effect on the cost of conical tanks as increasing the wall inclination increases the cost. Moreover, the angles of inclination $15^{\circ}$ and $30^{\circ}$ are found to be more economics than angles of $45^{\circ}$ and $60^{\circ}$ for tanks having the same capacities. On the other hand, the change in the base radius has a slight effect on the cost function. The effect of the base radius is only noticed in case of small capacity ( $500 \mathrm{~m}^{3}$ ) tanks, where the increase in base radius leads to a slight reduction in cost.


## CHAPTER 6 CONCLUSIONS

### 6.1 Summary and Conclusions

This thesis studies the behaviour of reinforced concrete conical tanks under the effect of hydrostatic pressure. The available design codes provide provisions on rectangular and cylindrical reinforced concrete tanks and there are no clear guidelines for conical shaped tanks. As such, there are two main objectives considered in this study. The first objective is to assess the adequacy of such available codes when applied to conical tanks using an equivalent cylindrical approach provided by AWWA-D100 (2005). This is achieved by comparing the internal forces in the tank walls that are predicted according to code provisions to those obtained from finite element analysis models. These numerical models are based on a sub-parametric triangular shell element.

The second objective is to make use of the conclusions reached in assessing the adequacy of available code provisions in order to provide a simple and adequate design approach. This simplified approach is based on utilizing accurate internal forces predicted by finite element analysis together with code requirements for serviceability and strength design to develop a set of design charts. Finally, the developed design charts are validated and utilized to design and estimate the cost of reinforced concrete conical tanks. The cost of these tanks is compared to steel counterparts in order to present a comparison between the two different types of construction materials for conical tanks. Based on all the above findings, the following conclusions are drawn.

- The available design codes for reinforced concrete tanks do not provide any clear provisions for analyzing the forces acting on the walls of conical shaped vessels. Therefore, there is a need for guidelines for such tanks.
- The equivalent cylindrical approach recommended by the AWWA-D100 (2005) is used to transform the geometry of conical vessels to equivalent cylindrical tanks. A large disagreement has been found between the maximum forces resulting from the finite element method and those resulting from the PCA-CCTWP approach combined with the equivalent cylinder approach. It is noticed that this disagreement is directly proportional to the wall inclination angle.
- The PCA-CCTWP leads to larger hoop tension and smaller meridional moment compared to internal forces obtained from finite element analysis. Therefore, the PCA-CCTWP leads to an inadequate design if applied to conical shaped tanks.
- The internal forces acting on tank walls obtained from a built in-house finite element model together with code requirements are successfully employed to develop a set of simplified design charts. These design charts are developed for a certain practical range of conical tank dimensions, including tank height, base radius, and angle of inclination. The accuracy of such charts was also assessed.
- The proposed simplified design approach can be generally used for any reinforced concrete conical tank within the assumed bounds of dimensions. The outputs of this approach include the wall minimum required thickness and the associated maximum internal forces.
- The developed design charts are utilized to design a number of reinforced concrete conical tanks then the cost of such tanks is estimated and compared to steel tanks
having the same capacity. It is concluded from this comparison that steel tanks are considered as a more economical choice for medium and small capacity tanks, regardless their dimensions. On the other hand, for large capacity conical tanks (more than $1750 \mathrm{~m}^{3}$ ), the tank dimensions (i.e., tank height, base radius, and angle of inclination) govern which construction material (reinforced concrete vs. steel) is more cost effective.
- A cost analysis for conical tanks having the same material and specific capacities has been conducted in order to study the effect of changing dimension parameters on the cost function. It is concluded that the cost of conical tanks is mainly affected by the wall inclination angles, which is directly proportional to the cost.


### 6.2 Recommendations for Future Research

This thesis studies the behaviour and design of reinforced concrete conical tanks having constant thickness under the effect of hydrostatic pressure. For future research, the following investigations are recommended:

- Study the effects of variable thickness along the tank height on the behaviour of reinforced concrete conical tanks.
- Investigate the non-linear behaviour of reinforced concrete tanks.
- Examine the response of reinforced concrete tanks under the effect of hydrodynamic pressure resulting from earthquake loading.
- Deflection of conical tanks should be investigated by using a computer program that considers cracking ad nonlinearity.
- Additional experimental study of wall specimens subjected to both axial tension and combined meridional moment and axial compression is needed.
- Assess the effect of pre-stressing on the design procedure of reinforced concrete conical tanks
- Investigate the applicability of using steel/concrete composite section for conical shaped tanks.


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## APPENDIX A - PCA-CCTWP (1993) COEFFICIENTS TO DETERMINE THE RING TENSION AND MOMENT

Table A-1 The Hoop Coefficient $C_{\mathrm{H}}$ according to PCA-CCTWP (1993)
Positive sign indicates tension

| $\boldsymbol{H}^{\mathbf{2}}$ |  |  | Coefficients at point |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{D t}$ | $\mathbf{0 . 0 H}$ | $\mathbf{0 . 1 H}$ | $\mathbf{0 . 2 H}$ | $\mathbf{0 . 3 H}$ | $\mathbf{0 . 4 H}$ | $\mathbf{0 . 5 H}$ | $\mathbf{0 . 6 H}$ | $\mathbf{0 . 7 H}$ | $\mathbf{0 . 8 H}$ | $\mathbf{0 . 9 H}$ | $\mathbf{H}$ |  |  |  |  |  |  |  |
| 0.4 | 0.474 | 0.44 | 0.395 | 0.352 | 0.308 | 0.264 | 0.215 | 0.165 | 0.111 | 0.057 | 0 |  |  |  |  |  |  |  |
| 0.8 | 0.423 | 0.402 | 0.381 | 0.358 | 0.33 | 0.297 | 0.249 | 0.202 | 0.145 | 0.076 | 0 |  |  |  |  |  |  |  |
| 1.2 | 0.35 | 0.355 | 0.361 | 0.362 | 0.358 | 0.343 | 0.309 | 0.256 | 0.186 | 0.098 | 0 |  |  |  |  |  |  |  |
| 1.6 | 0.271 | 0.303 | 0.341 | 0.369 | 0.385 | 0.385 | 0.362 | 0.314 | 0.233 | 0.124 | 0 |  |  |  |  |  |  |  |
| 2 | 0.205 | 0.26 | 0.321 | 0.373 | 0.411 | 0.434 | 0.419 | 0.369 | 0.28 | 0.151 | 0 |  |  |  |  |  |  |  |
| 3 | 0.074 | 0.179 | 0.281 | 0.375 | 0.449 | 0.506 | 0.519 | 0.479 | 0.375 | 0.21 | 0 |  |  |  |  |  |  |  |
| 4 | 0.017 | 0.137 | 0.253 | 0.367 | 0.469 | 0.545 | 0.579 | 0.553 | 0.447 | 0.256 | 0 |  |  |  |  |  |  |  |
| 5 | -0.008 | 0.114 | 0.235 | 0.356 | 0.469 | 0.562 | 0.617 | 0.606 | 0.503 | 0.294 | 0 |  |  |  |  |  |  |  |
| 6 | -0.011 | 0.103 | 0.223 | 0.343 | 0.463 | 0.566 | 0.639 | 0.643 | 0.547 | 0.327 | 0 |  |  |  |  |  |  |  |
| 8 | -0.015 | 0.096 | 0.208 | 0.324 | 0.443 | 0.564 | 0.661 | 0.697 | 0.621 | 0.386 | 0 |  |  |  |  |  |  |  |
| 10 | -0.008 | 0.095 | 0.2 | 0.311 | 0.428 | 0.552 | 0.666 | 0.73 | 0.678 | 0.433 | 0 |  |  |  |  |  |  |  |
| 12 | -0.002 | 0.097 | 0.197 | 0.302 | 0.417 | 0.541 | 0.664 | 0.75 | 0.72 | 0.477 | 0 |  |  |  |  |  |  |  |
| 14 | 0 | 0.098 | 0.197 | 0.299 | 0.408 | 0.531 | 0.659 | 0.761 | 0.752 | 0.513 | 0 |  |  |  |  |  |  |  |
| 16 | 0.002 | 0.1 | 0.198 | 0.299 | 0.403 | 0.521 | 0.65 | 0.764 | 0.776 | 0.536 | 0 |  |  |  |  |  |  |  |

Supplemental Coefficients

| $\boldsymbol{H}^{\mathbf{2}}$ | Coefficients at point |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{D t}$ | $\mathbf{0 . 7 5 H}$ | $\mathbf{0 . 8 H}$ | $\mathbf{0 . 8 5 H}$ | $\mathbf{0 . 9 H}$ | $\mathbf{0 . 9 5 H}$ | $\mathbf{H}$ |
| 20 | 0.812 | 0.817 | 0.756 | 0.603 | 0.344 | 0 |
| 24 | 0.816 | 0.839 | 0.793 | 0.647 | 0.377 | 0 |
| 32 | 0.814 | 0.861 | 0.847 | 0.721 | 0.436 | 0 |$|$| $\boldsymbol{D} \boldsymbol{t}$ | $\mathbf{0 . 7 5 H}$ | $\mathbf{0 . 8 H}$ | $\mathbf{0 . 8 5 H}$ | $\mathbf{0 . 9 H}$ | $\mathbf{0 . 9 5 H}$ | $\mathbf{H}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 40 | 0.802 | 0.866 | 0.88 | 0.778 | 0.483 | 0 |
| 56 | 0.781 | 0.864 | 0.9 | 0.82 | 0.527 | 0 |

Table A-2 The Moment Coefficient $C_{\mathrm{M}}$ according to PCA-CCTWP (1993)
Positive sign indicates tension

| $\boldsymbol{H}^{\boldsymbol{2}}$ |  |  |  | Coefficients at point |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\overline{\boldsymbol{D t}}$ | $\mathbf{0 . 0 H}$ | $\mathbf{0 . 1 H}$ | $\mathbf{0 . 2 H}$ | $\mathbf{0 . 3 H}$ | $\mathbf{0 . 4 H}$ | $\mathbf{0 . 5 H}$ | $\mathbf{0 . 6 H}$ | $\mathbf{0 . 7 H}$ | $\mathbf{0 . 8 H}$ | $\mathbf{0 . 9 H}$ | $\mathbf{H}$ |
| 0.4 | 0 | 0.002 | 0.0072 | 0.0151 | 0.023 | 0.0301 | 0.0348 | 0.0357 | 0.0312 | 0.0197 | 0 |
| 0.8 | 0 | 0.0019 | 0.0064 | 0.0133 | 0.0207 | 0.0271 | 0.0319 | 0.0329 | 0.0292 | 0.0187 | 0 |
| 1.2 | 0 | 0.0016 | 0.0058 | 0.0111 | 0.0177 | 0.0237 | 0.028 | 0.0296 | 0.0263 | 0.0171 | 0 |
| 1.6 | 0 | 0.0012 | 0.0044 | 0.0091 | 0.0145 | 0.0195 | 0.0236 | 0.0255 | 0.0232 | 0.0155 | 0 |
| 2 | 0 | 0.0009 | 0.0033 | 0.0073 | 0.0114 | 0.0158 | 0.0199 | 0.0219 | 0.0205 | 0.0145 | 0 |
| 3 | 0 | 0.0004 | 0.0018 | 0.004 | 0.0063 | 0.0092 | 0.0127 | 0.0152 | 0.0153 | 0.0111 | 0 |
| 4 | 0 | 0.0001 | 0.0007 | 0.0016 | 0.0033 | 0.0057 | 0.0083 | 0.0109 | 0.0118 | 0.0092 | 0 |
| 5 | 0 | 0 | 0.0001 | 0.0006 | 0.0016 | 0.0034 | 0.0057 | 0.008 | 0.0094 | 0.0078 | 0 |
| 6 | 0 | 0 | 0 | 0.0002 | 0.0008 | 0.0019 | 0.0039 | 0.0062 | 0.0078 | 0.0068 | 0 |
| 8 | 0 | 0 | 0 | -0.0002 | 0 | 0.0007 | 0.002 | 0.0038 | 0.0057 | 0.0054 | 0 |
| 10 | 0 | 0 | 0 | -0.0002 | -0.0001 | 0.0002 | 0.0011 | 0.0025 | 0.0043 | 0.0045 | 0 |
| 12 | 0 | 0 | 0 | -0.0001 | -0.0002 | 0 | 0.0005 | 0.0017 | 0.0032 | 0.0039 | 0 |
| 14 | 0 | 0 | 0 | -0.0001 | -0.0001 | -0.0001 | 0 | 0.0012 | 0.0026 | 0.0033 | 0 |
| 16 | 0 | 0 | 0 | 0 | -0.0001 | 0.0002 | -0.0004 | 0.0008 | 0.0022 | 0.0029 | 0 |

## Supplemental Coefficients

| $\frac{H^{2}}{D t}$ | Coefficients at point |  |  |  |  |  | $H^{2}$ | Coefficients at point |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0.75H | 0.8 H | 0.85H | 0.9H | 0.95H | H | $\overline{\text { Dt }}$ | 0.75H | 0.8 H | 0.85H | 0.9 H | 0.95H | H |
| 20 | 0.0008 | 0.0014 | 0.002 | 0.0024 | 0.002 | 0 | 40 | 0 | 0.0003 | 0.0006 | 0.0011 | 0.0011 | 0 |
| 24 | 0.0005 | 0.001 | 0.0015 | 0.002 | 0.0017 | 0 | 48 | 0 | 0.0001 | 0.0004 | 0.0008 | 0.001 | 0 |
| 32 | 0 | 0.0005 | 0.0009 | 0.0014 | 0.0013 | 0 | 56 | 0 | 0 | 0.0003 | 0.0007 | 0.0008 | 0 |

APPENDIX B - Shape Function of Consistent Shell Element (Koziey and Mirza, 1997)
Global displacement $(u, v, w)$
$\left\{\begin{array}{c}u \\ v \\ W\end{array}\right\}=\sum_{i=1}^{13} \bar{N}_{l}\left\{\begin{array}{c}u_{i} \\ v_{i} \\ w_{i}\end{array}\right\}+\sum_{i=1}^{13} N_{i} M_{1}\left[\bar{V}_{l}\right]\left\{\begin{array}{l}\alpha_{i} \\ \beta_{i}\end{array}\right\}+\sum_{i=1}^{13} N_{i} M_{2}\left[\bar{V}_{l}\right]\left\{\begin{array}{c}\emptyset_{i} \\ \varphi_{i}\end{array}\right\}$

## Quadratic interpolation functions $\left(N_{i}\right)$

$$
\begin{array}{llll}
N_{1}=L_{1}\left(2 L_{1}-1\right), & N_{2} & N_{3} & N_{4}=0, \quad N_{5}=4 L_{1} L_{2}, \\
& =L_{2}\left(2 L_{2}-1\right), & =L_{3}\left(2 L_{3}-1\right), & \\
N_{6}=0, & N_{7}=0, & N_{8}=4 L_{2} L_{3}, & N_{9}=0, \quad N_{10}=0, \\
N_{8}=4 L_{1} L_{3}, & N_{12}=0, & N_{13}=0 &
\end{array}
$$

## Cubic interpolation Functions ( $\bar{N}_{L}$ )

$$
\begin{array}{ll}
\overline{N_{1}}=\frac{1}{2} L_{1}\left(3 L_{1}-1\right)\left(3 L_{1}-2\right), & \overline{N_{2}}=\frac{1}{2} L_{2}\left(3 L_{2}-1\right)\left(3 L_{2}-2\right), \\
\overline{N_{3}}=\frac{1}{2} L_{3}\left(3 L_{3}-1\right)\left(3 L_{3}-2\right), & \overline{N_{4}}=\frac{9}{2} L_{1} L_{2}\left(3 L_{1}-1\right), \\
\overline{N_{5}}=0, & \overline{N_{6}}=\frac{9}{2} L_{1} L_{2}\left(3 L_{2}-1\right), \\
\overline{N_{7}}=\frac{9}{2} L_{2} L_{3}\left(3 L_{2}-1\right), & \overline{N_{8}}=0 \\
\overline{N_{9}}=\frac{9}{2} L_{2} L_{3}\left(3 L_{3}-1\right), & \overline{N_{10}}=\frac{9}{2} L_{3} L_{1}\left(3 L_{3}-1\right), \\
\overline{N_{11}}=0, & \overline{N_{12}}=\frac{9}{2} L_{3} L_{1}\left(3 L_{1}-1\right), \\
\overline{N_{4}}=27 L_{1} L_{2} L_{3}, &
\end{array}
$$

Where $L_{1}, L_{2}$ and $L_{3}$ designate the area coordinates of the triangular parent element.

## Matrix $\left[\bar{V}_{1}\right]$

$\left[\bar{V}_{i}\right]=\left[\bar{V}_{1 i}-\bar{V}_{2 i}\right]$
Where $\bar{V}_{1 i}$ and $\bar{V}_{2 i}$ are directed along the $\dot{x}$ and $\dot{y}$ axes, respectively.
Shape function ( $M_{1}$ and $M_{2}$ )
$M_{1}=\frac{h_{i} t}{2}$
$M_{2}=\frac{h_{i} t}{2}\left(1-t^{2}\right)$
Where $h_{i}$ is the shell thickness at the $i$ th- node.

## The Jacobian matrix

$J=\left\{\begin{array}{l}R \\ S \\ T\end{array}\right\}$
Where vectors $R$ and $S$ are tangent to the surface defined by $t=$ constant. A vector $V_{3}$ normal to this surface is found as:
$V_{3}=R \times S$
The remaining vectors $V_{2}$ and $V_{1}$ of the orthogonal basis are given by $V_{2}=V_{3} \times R, \quad V_{1}=V_{2} \times V_{3}$

Normalizing $V_{1}, V_{2}$, and $V_{3}$ gives the set of unit vectors $\bar{V}_{1}, \bar{V}_{2}$ and $\bar{V}_{3}$ from which the transformation matrix of direction cosines is constructed as


## APPENDIX C - MAGNIFICATION FUNCTION

Table C-1 Regression Coefficients (Sweedan, and El Damatty 2009)

| Level of imperfection | Cap ratio ( $\mathrm{CP}_{\mathrm{r}}$ ) (\%) | Mean value of regression coefficient |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $a$ | $b$ | c | $d$ | $e$ | $f$ | $g$ | $h$ |
| (a) $\sigma_{y}=250 \mathrm{MPa}$ |  |  |  |  |  |  |  |  |  |
| Good shells | 0 | 1.3737 | -0.1281 | 0.5199 | -1.4468 | 4.0353 | 0.1853 | -1.2604 | 3.7744 |
|  | 15 | 1.3994 | -0.1309 | 0.4971 | -1.6456 | 3.7186 | 0.2103 | -1.2479 | 3.7425 |
|  | 30 | 1.3874 | -0.1324 | 0.4253 | -1.5879 | 3.5000 | 0.2015 | -1.2512 | 3.7463 |
|  | 45 | 1.3640 | -0.1320 | 0.3491 | -1.5680 | 3.0878 | 0.1693 | -1.2408 | 3.7476 |
| Poor shells | 0 | 1.5316 | -0.2280 | 1.0213 | -1.4156 | 6.0260 | 0.0034 | -0.9545 | 3.5209 |
|  | 15 | 1.5523 | -0.2190 | 1.0006 | -1.5432 | 5.6011 | 0.0170 | -0.9422 | 3.5007 |
|  | 30 | 1.5099 | -0.2157 | 0.8712 | -1.4236 | 5.1581 | -0.0106 | -0.9251 | 3.4775 |
|  | 45 | 1.4571 | -0.2106 | 0.7349 | -1.3386 | 4.5153 | -0.0574 | -0.9098 | 3.5046 |
| (b) $\sigma_{\mathrm{y}}=\mathbf{3 0 0} \mathrm{MPa}$ |  |  |  |  |  |  |  |  |  |
| Good shells | 0 | 1.4003 | -0.1252 | 0.5235 | -1.6221 | 4.9353 | 0.2325 | -1.4210 | 3.9860 |
|  | 15 | 1.4242 | -0.1293 | 0.5032 | -1.8295 | 4.6128 | 0.2589 | -1.4204 | 3.9936 |
|  | 30 | 1.4116 | -0.1310 | 0.4261 | -1.8200 | 4.2536 | 0.2455 | -1.4093 | 3.9455 |
|  | 45 | 1.3870 | -0.1326 | 0.3393 | -1.7355 | 3.7588 | 0.2226 | -1.4093 | 3.9975 |
| Poor shells | 0 | 1.5477 | -0.2222 | 1.0432 | -1.4616 | 6.7132 | 0.0326 | -1.0314 | 3.6333 |
|  | 15 | 1.5587 | -0.2158 | 1.0099 | -1.5437 | 6.1035 | 0.0411 | -1.0036 | 3.5881 |
|  | 30 | 1.5345 | -0.2128 | 0.8895 | -1.5246 | 5.6955 | 0.0156 | -0.9980 | 3.5889 |
|  | 45 | 1.4848 | -0.2090 | 0.7246 | -1.4561 | 5.0363 | -0.0219 | -0.9953 | 3.5862 |
| (c) $\sigma_{y}=350 \mathrm{MPa}$ |  |  |  |  |  |  |  |  |  |
| Good shells | 0 | 1.4269 | -0.1235 | 0.5281 | -1.7940 | 6.0760 | 0.2728 | -1.5913 | 4.2570 |
|  | 15 | 1.4433 | -0.1277 | 0.4982 | -2.0318 | 5.4473 | 0.2909 | -1.5554 | 4.1278 |
|  | 30 | 1.4376 | -0.1310 | 0.4248 | -2.0455 | 5.2392 | 0.2886 | -1.5878 | 4.2346 |
|  | 45 | 1.4082 | -0.1330 | 0.3300 | -1.9606 | 4.4655 | 0.2629 | -1.5615 | 4.2108 |
| Poor shells | 0 | 1.5711 | -0.2172 | 1.0703 | -1.5442 | 7.4099 | 0.0501 | -1.1022 | 3.7549 |
|  | 15 | 1.5900 | -0.2123 | 1.0192 | -1.6724 | 6.8602 | 0.0634 | -1.0910 | 3.7013 |
|  | 30 | 1.5634 | -0.2100 | 0.8773 | -1.6121 | 6.4559 | 0.0479 | -1.0974 | 3.7121 |
|  | 45 | 1.5147 | -0.2090 | 0.7166 | -1.5322 | 5.7202 | 0.0157 | -1.0982 | 3.7612 |

## CURRICULUM VITAE

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