# Functional Transformations and Octatonality in Selected Works by George Crumb 

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Graduate Program in Music
A thesis submitted in partial fulfillment of the requirements for the degree in Doctor of Philosophy
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# Functional Transformations and Octatonality in Selected Works by George Crumb 

 (Thesis format: Monograph)by

Peter Lea

Graduate Program in Music

A thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy

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#### Abstract

The music of George Crumb has been analyzed using a variety of analytic methods including pitch-class set theory, transformational theory, intertextual analyses, and various tonal and Schenkerian approaches. The results of these types of analyses invariably identify certain consistencies in Crumb's compositional style within a single work or volume of similar works, but often are unable to relate or compare procedures between different works.

In the present study, I propose a transformational model that accounts for characteristic gestures in numerous works by Crumb. Unlike many transformational models of twentieth-century music, the proposed model does not require significant alterations for each work, and is able to model transformations between sets with a cardinality of three through twelve. Because of the relatively stable nature of this model, it is possible to compare gestures within and between different works in a manner analogous to comparisons of functional progressions in tonal music.

In the introductory chapter, the basis for a functional model of Crumb's music is introduced, and the scope of the study is defined. In the second chapter, certain characteristics of Crumb's music identified by other scholars such as trichordal structures, referential collections, and the principles of opposition and completion are discussed. Based upon these characteristics, the model of octatonality is proposed and defined in the third chapter. The fourth and fifth chapters include analyses of Crumb's A Haunted Landscape (1984) and "Come Lovely and Soothing Death..." from Apparition (1980) as representative examples of octatonality. Crumb's sketch materials for these and other related works are


included to provide support for segmentation as well as octatonic considerations. In the concluding sixth chapter, the functional designs of these examples are compared.

The generic model, divorced from the octatonic collection, offers a method to compare different works at various functional levels where the intervallic components of the music, a limited pitch-class set vocabulary, and an underlying functionality are perceived as fundamental features of the music. The utility of the generic model in analyzing Crumb's non-octatonic works is demonstrated and an extension of the generic model for the analysis of music by other composers is suggested.

## Keywords

George Crumb, Crumb, octatonic, octatonality, function, diatonic, tonal, referential collection, transformational theory, neo-Riemannian, K-net, set-theory, intertextuality, A Haunted Landscape, Come Lovely and Soothing Death, Apparition, Gargoyles, Makrokosmos, Music of the Starry Night, Music of Shadows, Litany of the Galactic Bells, Pastoral, Primeval Sounds, Processional, Whitman, Lorca, Paisaje, Siguiriya, Cante Jondo, Deep Song, sketch

For Heather and Ruby

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## 1 Introduction

Within George Crumb's music there often seems to be a balance between tonal and post-tonal elements. In "Litany of the Galactic Bells" from Crumb's Makrokosmos II, music that does not sound tonal-despite the pronounced usage of a major-minor seventh sonority-interrupts a quotation of an excerpt from Beethoven's "Hammerklavier" Sonata, Op. 106, accompanied by the marking "abrupt, intrusive!" Crumb inscribes the performance direction "wistfully elegant" above a passage from Chopin's FantaisieImpromptu in "Dream Images (Love-Death Music)" from Makrokosmos I, which follows a passage of music that is not tonal. Richard Bass has characterized such music as "neither consistently tonal in a conventional sense, nor designedly atonal." ${ }^{11}$ In his essay "Music: Does It Have a Future?" Crumb asserts his admiration for tonality noting, "Not only is the question of tonality still unresolved but we have not yet invented anything comparable to the sure instinct for form which occurs routinely in the best traditional music. Instead, each new work seems to require a special solution, valid only in terms of itself. ${ }^{, 2}$ Later in the essay he continues:

In general, I feel that the more rationalistic approaches to pitch-organization, including specifically serial technique, have given way, largely, to a more intuitive approach. There seems to be a growing feeling that we must somehow evolve a new kind of tonality. Probably the ideal solution, anticipated, it seems to

[^0]me, by Bartók, is to combine the possibilities of our chromatic language-which is so rich and expressive in its own right-with a sense of strong tonal focus. ${ }^{3}$ Although Crumb seems to be addressing composers in general, reference to his own compositional agenda is thinly veiled. Crumb often mentions that his music contains a certain "natural acoustic" because of growing up in an Appalachian river valley in West Virginia. ${ }^{4}$ Such elusive rhetoric-couched in intuition and myth-is how Crumb routinely describes his compositional process. ${ }^{5}$ The definition of Crumb's "natural acoustic" is difficult to pinpoint, but it appears that Crumb believes that this acoustic forms the basis for his style, which appears to be influenced by both the common-practice tonal tradition as well as his geographical environment. ${ }^{6}$

The chord progression illustrated in Figure 1.1 is an example of a progression that contains both tonal and post-tonal elements. The bass line projects an ascending Eb-major

[^1]Figure 1.1. Crumb, Apparition, "Come Lovely," p. 12, s. 3-p. 13, s. 1. Copyright © 1980 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

scale; the first three chords prolong tonic harmony with the second-inversion sub-mediant chord substituting for a typical first-inversion tonic chord. The fourth chord might be analyzed as a predominant chord with an unresolved 4-3 suspension; however, it is at this point that a tonal reading becomes untenable. In the fifth and sixth chords, the tenor and alto voices deviate from the diatonic model: the tenor introduces a Gb on the fifth chord and the alto includes both an Fb and Gb on the fifth and sixth chords respectively. Within a tonal framework, it may be possible to view both chords as applied dominants; however, they do not resolve as one might expect. Alternatively, if the tenor and alto voices were considered separately, the tenor projects an incomplete pentatonic scale and the alto, except for the first pitch, consists of a whole-tone scale. As is the case in this initial interpretation of the progression, Bass suggests that studies of such music usually examine either "the relationship between one of these collections and an overarching tonal structure, treating the octatonic or whole-tone elements as chromatic inflections or
tonally deviant intrusions within a diatonic framework" or "the special properties and applications of the individual collections themselves." ${ }^{, 7}$

Passages that draw upon tonal and post-tonal elements are prevalent in Crumb's music and scholars approach it in a variety of ways. In the second chapter, I will lay the foundation for a functional octatonic model using both existing and original analyses of various works. In sections 2.1 and 2.2, I discuss the correlation between specific trichords within and between works and the ability of these trichords, as well as tetrachords, to identify specific transpositions of a referential collection. Both trichordal structures and referential collections are important aspects of the octatonic model, and lay the foundation for the transformational component of the model. Sections 2.3 and 2.4, which examine the principles of opposition and completion respectively, are pertinent for the functional aspect of octatonality. As I argue, the principle of opposition introduces tension whereas the principle of completion conveys resolution.

The model of octatonality is presented in the third chapter, which integrates the observations of the second chapter. In section 3.1, the metaphorical transfer of tonal and post-tonal concepts to the octatonic model is explored in detail; which was implicit in the discussion of the principles of opposition and completion in the second chapter. Within each of the following sections, which deal with core aspects of the model, the implications of the metaphorical transfers are made explicit. This is most noticeable in section 3.7 where hierarchical functions in octatonality are proposed, which is reliant upon the metaphorical transfer of tonal functionality to the model of octatonality.

[^2]Although the material is presented in a linear format, each aspect of the model is interrelated and predicated upon observations of a number of Crumb's works. For these reasons, as well as to mitigate the abstract presentation of the model, the reader may find it useful to preview the analyses presented in the fourth and fifth chapters before reading sections 3.2-3.8.

The fourth and fifth chapters include analyses of Crumb's A Haunted Landscape (1984) and "Come Lovely and Soothing Death" from Apparition (1980) as representative examples of octatonality. Sketch materials are used to demonstrate the interconnectedness of the two works as well as provide support for interpretation of form, segmentation, determination of referential collections, and evidence of Crumb's new tonality.

In the final chapter, the functional design of A Haunted Landscape and "Come Lovely" are compared. In addition, a non-octatonic portion of Crumb's Processional is analyzed using a similar model but Messiaen's fourth mode of limited transposition is substituted for the octatonic collection. As is demonstrated, the generic functional-yettransformational model, divorced from the octatonic collection, offers a method to compare works at various functional levels where the intervallic components of the music, a limited pitch-class set vocabulary, and an underlying functionality are perceived as fundamental features of the music.

## 2 Recurring Aural Signatures in George Crumb's Music

In this chapter, I will explore various analytic methods utilized to describe Crumb's music and demonstrate the consistent emphasis on and correlation between triadic structures, referential collections and symmetry, opposition, and completion. These attributes will provide the foundation for the proposed octatonic model and illuminate aspects of Crumb's stated desire for a "more organic and well-ordered procedure." ${ }^{1}$

### 2.1 Trichordal Structures

Crumb's music is saturated with triadic structures, many of which are recognizable as major or minor triads. ${ }^{2}$ In his essay, "Music: Does It Have a Future?" Crumb wished for a new kind of tonality that could combine the chromaticism of present resources with a strong sense of tonal focus. ${ }^{3}$ In "God-music" from Black Angels, Crumb describes a sustained B-major tonality throughout the movement (see Figure 2.1). ${ }^{4}$ In Figure 2.1, the set-class (sc) membership of the major and minor triads are identified on

[^3]Figure 2.1. Crumb, Black Angels, "God-music," reduction of the accompaniment Copyright © 1971 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

the first line and the triadic roots on the second line. ${ }^{5}$ The normal form of the trichords $\{125\}$ and $\{9 \mathrm{~A} 1\}$ are used because they are distinctly post-tonal constructs.

The sheer repetition of the B-major triads in "God-music," as compared to other sets, is one aspect that contributes to Crumb's sense of a B-major tonality (see Table 2.1). ${ }^{6}$ Moreover, the relationship between the sets also implies a functional design (see Table 2.2). An entry in Table 2.2 represents the number of times the trichord in the corresponding column-header immediately follows the trichord in the corresponding rowheader. ${ }^{7}$ For example, the numeral 3 in the B column indicates that a B-major triad happens three times after the corresponding \{9A1\} trichord. The most significant finding is that the F-major triad only occurs before a B-major triad. In addition, the B-major triad moves to the F-major triad six times and the F-major triad moves to the B-major triad seven times; a correlation suggestive of a routine procedure. ${ }^{8}$

[^4]Table 2.1. Set-class occurrences in "God-music"

| Set Class | Instance | Occurrences |
| :--- | :--- | :---: |
| (037) | B | 15 |
|  | F | 7 |
|  | d | 4 |
|  | $\mathrm{~B} b$ | 1 |
|  | Total | $\mathbf{2 7}$ |
| (014) | $\{9 \mathrm{~A} 1\}$ | 4 |
|  | $\{\mathrm{~B} 23\}$ | 3 |
|  | $\{125\}$ | 2 |
|  | $\{589\}$ | 1 |
|  | (026) | $\{9 \mathrm{~B} 3\}$ |
|  | $\{680\}$ | $\mathbf{1 0}$ |
|  | $\{8 \mathrm{~A} 2\}$ | 4 |
|  | $\{7 \mathrm{~B} 1\}$ | 2 |
|  | Total | $\mathbf{1 1}$ |
| (016) | $\{056\}$ | 2 |
|  | $\{892\}$ | 1 |
|  | $\{\mathrm{~B} 45\}$ | 1 |
|  | Total | $\mathbf{4}$ |

Table 2.2. Set correlations in "God-music"

|  |  | 037 |  |  |  | 014 |  |  |  | 026 |  |  |  | 016 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | B | F | d | Bb | 9A1 | B23 | 125 | 589 | 9B3 | 680 | 8A2 | 7B1 | 056 | 892 | B45 |
| 037 | B |  | 6 | 1 |  | 4 |  | 2 |  |  |  |  |  |  |  |  |
|  | F | 7 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | d | 1 |  |  |  |  | 3 |  |  |  |  |  |  |  |  |  |
|  | Bb |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 014 | 9A1 | 3 |  | 1 |  |  |  |  |  |  |  |  |  |  |  |  |
|  | B23 | 1 |  |  |  |  | 2 |  |  |  |  |  |  |  |  |  |
|  | 125 | 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 589 |  |  |  |  |  |  |  |  |  | 1 |  |  |  |  |  |
| 026 | 9B3 |  |  |  |  |  |  |  |  |  | 1 |  |  | 2 |  | 1 |
|  | 680 |  |  |  |  |  |  |  | 1 | 2 |  |  |  |  | 1 |  |
|  | 8A2 |  |  |  |  |  |  |  |  |  |  |  | 1 |  |  |  |
|  | 7B1 |  |  |  |  |  |  |  |  |  |  | 1 |  |  |  |  |
| 016 | 056 |  |  |  |  |  |  |  |  | 2 |  |  |  |  |  |  |
|  | 892 |  |  |  |  |  |  |  |  |  | 1 |  |  |  |  |  |
|  | B45 |  |  |  |  |  |  |  |  |  |  | 1 |  |  |  |  |

In "God-music," instances of sc (016) and sc (026) are the most common set classes of the entire B section; the only sets that are not members of these set classes are $\{589\},\{\mathrm{B} 4\}$, and the final $\mathrm{B}^{b}$-major triad. This correlation between instances of sc (016) and sc (026) is also featured in "Gargoyles" from Makrokosmos II, where a (016) ostinato accompanies linear instances of sc (026) (see Figure 2.2). ${ }^{9}$ In mm. 12-14, semi-tone or whole-tone transpositions of the vertical (016) trichordal ostinato is contrasted with instances of sc (026) in a higher register. In "God-music," instances of sc (026) and sc (016) acted as neighbour chords within an entire section but in "Gargoyles," the instances of sc (026) trichords behave differently, but are still in close proximity to instances of sc (016).

Figure 2.2. Crumb, Makrokomos II, "Gargoyles," mm. 12-14. Copyright © 1973 by C. F. Peters Corporation. All Rights Reserved. Used by permission.


[^5]In Crumb's "Pastorale" from Makrokosmos I, instances of scs (016) and (026) are again found in close proximity and include instances of sc (025) (see Figure 2.3). ${ }^{10}$ Bass highlights the importance of the rising minor second from the first pitch of m .1 (pc A) to the last pitch of m. 2 (pc B) and the falling minor second from the first pitch of m .3 (pc A) to the last pitch of $\mathrm{m} .5(\mathrm{pc} 9)$ as well as the descending major-second transposition of the (016) trichords in m. 3. ${ }^{11}$ The melodic material fuses with the harmonic material through the minor- and major-second motives and emphasizes the interconnectedness of the three different set classes.

Figure 2.3. Crumb, Makrokosmos I, "Pastorale," mm. 1-5. Copyright © 1973 by C. F. Peters Corporation. All Rights Reserved. Used by permission.


[^6]In A Haunted Landscape Crumb makes use of alternating major and minor triads (see Figure 2.4). In addition, although not notated in Figure 2.4, there is a Bb pedal in the lower register held throughout the entire piece. ${ }^{12}$ In Figure 2.4, the progression is modeled by a series of compound neo-Riemannian operations. ${ }^{13}$ The actual progression is notated in whole notes, and the imaginary transformational path of the compound operations is notated with filled-in note heads. The transformations that relate the first chord to the second chord and the third chord to the fourth chord are identical <PRP> transformations. In addition, the relationship of the final chord of the first progression (Bminor) to the first chord of the second progression ( Eb -major, not shown) would be <PLP>-the foil of <PRP>, which linked the first-two and last-two chords of each progression.

When this progression is recapitulated, there is an important difference: the piano part introduces another pitch to the triads as an insistent repeated-note motive (see Figure 2.5). In Figure 2.5, a filled-in note head represents the repeated note, which has been

[^7]Figure 2.4. Crumb, A Haunted Landscape, mm. 23-27. Copyright © 1984 by C. F. Peters Corporation. All Rights Reserved. Used by permission.


Figure 2.5. Crumb, A Haunted Landscape, mm. 141-144

confined to one register to highlight the abstract parsimonious voice leading. The repeated note supplies the seventh to the first and third chords, transforming them into major-minor seventh chords; however, the second and fourth chords do not correspond to a typical tonal chord. Neither a contextually inverted neo-Riemannian transformation nor a four-node Klumpenhouwer network (K-net) can model the transformation of one tetrachord to the next.

The relationship between the trichords and tetrachords is apparent. A common interval in each of these chords is the perfect fifth formed between the root and the fifth of the original triad. Omitting the "third" of both the major and minor tetrachords obtains sc (025) and sc (016) respectively. In "God-music," instances of sc (016) separated instances of sc (026), which is a subset of sc (0258) (cf. Figure 2.1). In "Pastorale" an instance of sc (026) transformed into an instance of sc (025); both sets are subsets of sc
(0258) (cf. Figure 2.3). The interaction between instances of the above-mentioned set classes is because of their shared membership in a specific referential collection, which will be discussed in the next section.

The appearance of the first major chord in "Come Lovely" is symbolic of the death's arrival, anticipated by the preceding text "serenely arriving, arriving" (see Figure 2.6). This is a distinctive arrival point and, as Straus suggests, "When triads occur in contexts other than the traditionally tonal one, careful critical attention must be paid." ${ }^{14}$ Tonality's association with death here is quite clear; not only does a major triad accompany the arrival of death but also the beginning of the progression appears to be tonal. The first three chords take part in a tonic prolongation moving from tonic to the submediant through a passing dominant. A functionally suspicious subdominant chord with an unresolved 4-3 suspension begins to veil the sense of tonality, which then leads to a series of whole-tone related incomplete applied dominants. It is noteworthy that the final chord of this progression is the dominant of Eb major. Within a short span of seven chords, tonality is born (initial tonic prolongation), dies (subdominant and subsequent non-functional applied dominants) and is reborn (the final dominant). ${ }^{15}$ Not only is this

[^8]Figure 2.6. Crumb, Apparition, "Come Lovely," p. 12, s. 2-p. 13, s. 1. Copyright © 1980 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

view of death consistent with Whitman's text, but also with Crumb's regret over the loss of a tonal analogue in the twentieth century. ${ }^{16}$

In "Come Lovely," tonality's arrival also signals the death of tonality. As Crumb stated, "Perhaps we have come to think of ourselves as philosophically contemporaneous

[^9]with all earlier cultures. And it could be that today there are more people who see culture evolving spirally rather than linearly. With the concentric circles of the spiral, the points of contact and the points of departure in music can be more readily found." ${ }^{17}$ The arrival of the major chord thus represents life and death-a point of contact and departure. This progression is repeated twice, transposed down a whole-step with subsequent iterations, and in this way one could imagine the cycle of life and death, or death and life, repeated ad infinitum. Tonality flows to post-tonality and back again cyclically. In "Come Lovely," neither idiom is considered contemporaneous with the other but simply part of musical culture's ebb and flow.

The close proximity of different chords in "Come Lovely" is similar to those demonstrated in earlier examples. In "God-music," instances of sc (014) acted as neighbour chords to instances of sc (037) and in "Come Lovely" three instances of sc (014) prepare the arrival of the first major chord, an instance of sc (037) (cf. Figure 2.1). In the second section of "God-music," instances of sc (016) alternated with instances of sc (026) and in "Come Lovely," an instance of sc (016) led to three instances of sc (026). An instance of sc (016) separated instances of sc (025) and sc (026) in "Pastorale," while in "Come Lovely," the $\mathrm{IV}^{4-}$ chord, an instance of sc (016), acted as a divider between the opening tonic prolongation and the final three trichords, all instances of sc (026) (cf.

Figure 2.3). Furthermore, in "Pastorale," an instance of sc (026) was transformed into an instance of sc (025) and in the tonal analysis of "Come Lovely" both of these set classes were interpreted as incomplete dominant-seventh chords. In A Haunted Landscape, the

[^10]major-minor seventh chords and alternating instances of sc (0147) tetrachords are supersets of the trichords already discussed (cf. Figure 2.5).

Whereas in A Haunted Landscape the initial triadic sequence could be modeled using neo-Riemannian operations, the same is not possible in "Come Lovely;" however, the entire progression may be graphed as K-nets (see Figure 2.7). ${ }^{18}$ In Figure 2.7, a pivot is shown because not all of the triads share a common interval—graph 4a shares a perfect fifth with the triads preceding it and graph 4 b shares a tritone with the triads following it. This network expresses some of the relationships heard but not all; the segregation of the first four chords from the last four reflects the shift to a post-tonal space but does not mimic the voice leading of the example except in the bass voice, which is represented as the lowest node of each graph. As has been demonstrated, the close proximity of select pitch-class sets in Crumb's music suggests at the minimum an instinctual gravitation to certain intervallic configurations and, at the maximum, an innovation that routinely utilizes characteristic gestures, not unlike tonality.

[^11]Figure 2.7. K-net interpretation of the "Come Lovely" trichordal progression

$\left\langle\mathrm{I}_{\mathrm{B}}\right\rangle \quad\left\langle\mathrm{T}_{9}\right\rangle \quad\left\langle\mathrm{I}_{8}\right\rangle \mathbf{4 b}$

$\left\langle\mathrm{T}_{5}\right\rangle$
$\left\langle\mathrm{T}_{4}\right.$ >
< $\mathrm{T}_{4}$ >

### 2.2 Referential Collections

Responding to a question about how committed he was to using certain referential collections, Crumb replied:
"How committed are you to whole tones?" Debussy would probably say, "Oh, it's a valuable addition to the vocabulary. I'll move in and out of whole tones. Whole tones will be followed by diatonic functional harmony. That'll be followed by modality" [laughs heartily]. I feel the same way! You can create almost a prismatic sense by going from one to the other. I'm not a purist at all. I'm very sloppy about these things. ${ }^{19}$

This supposed sloppiness can be challenging for the analyst for a number of reasons: the boundaries between collections may become blurred because of shared subsets; operative collections are difficult to distinguish if they are incomplete; interactions between foreign

[^12]collections may be difficult to model; and layered collections obfuscate the perception of a distinct collection. These difficulties will be discussed in the following analyses.
"Music of Shadows" from Makrokosmos I is an example of Crumb's prismatic technique of moving between different collections (see Figure 2.8). As Bass argues, "The importance of symmetry and transpositional projections in Crumb's works can hardly be overstated. Crumb routinely achieves clarity through the manipulation of a limited number of pitch constructions within some large-scale symmetrical progression. ${ }^{20}$ In particular, Bass's analysis of "Music of Shadows" is a prime example. Figure 2.8 replicates many aspects of Bass's analysis. The octatonic and whole-tone subsets, which Bass describes in his article, are indicated directly below the staff following the label $\subseteq .^{21}$ Below this, the pitch-class intersection, $\cap$, between each chord is indicated; an underlined pitch class indicates that the melody note in the higher stratum is required to account for the pitch-class intersection between collections. For example, the pitch-class intersection between the third and fourth pentachords in B is [06] but pc 0 is only found in the higher stratum of the final pentachord. Finally, Bass's formal divisions are indicated above each system in bold font. ${ }^{22}$ In addition to Bass's observations, a whole-step motive is

[^13]Figure 2.8. Crumb, Makrokosmos I, "Music of Shadows." Copyright © 1973 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

annotated with a bracketed $x$ or -x to differentiate between the motive's ascending or descending form respectively.

The invariant tetrachord, $\{56 \mathrm{~B} 0\}$, in the middle section is labeled as representative of $\mathrm{Oct}^{2}$. The reasons for this include: the prevalence of complete octatonic
collections in Crumb's music; the opening, middle, and closing sections each end with a subset of the same octatonic collection and thus provide some coherence; and the prevailing collection in this movement is $\mathrm{Oct}^{2}$. Additional relationships emerge if the pitch classes that are missing-those that are required to complete the parent collectionare considered. The missing pitch class of each whole-tone subset is supplied in the highest voice of the consequent hexachordal octatonic subset in both the opening and closing sections (the bracketed pitches in Figure 2.8). The relationship between the highest notes in these adjacent chords again reflects the pervasive whole-step motive. In addition, the invariant tetrachordal pedal supplies the missing pitch classes from the first hexachordal octatonic subset of the piece, $\{06\}$, and the missing pitch classes from the chord which conclude the $A$ section and begin the $A^{1}$ section, $\{5 B\}$.

The form of "Music of Shadows" resembles a variation of binary form. Analogous to tonal archetypes, the material of the digression (the $B$ and $B^{1}$ section) is less stable-less pitch class intersection and greater variability in the highest stratumand the whole-step motive unifies the digression with the $A$ and $A^{1}$ sections. ${ }^{23}$ In addition, the invariant tetrachord in the B and $\mathrm{B}^{1}$ sections might be construed as a dominant pedal. Instead of "standing on the dominant," the digression "stands on Oct ${ }^{2}$." In an interview, Crumb acknowledged his admiration of tonality:

I love tonality. I feel that what happened with Schoenberg and others was in a way an aberration and not an entirely successful one, philosophically or simply musically. My ear rejects that sort of symmetry. Melody itself is based on asymmetry, and therefore our inner ear is asymmetrically constructed. That's

[^14]another reason I admire composers like Bartók, who avail themselves of all the rich new possibilities while retaining the tonal principle as the spine of the music. ${ }^{24}$

In "Music of Shadows" the spine, or tonal principle, appears to be the form, which is emphasized by the interaction between octatonic and whole-tone collections.

Although the hexachords in "Music of Shadows" were analyzed as representative of an octatonic collection, they also constitute the well-known Petroushka collection, sc (013679). Dimitri Tymoczko takes issue with numerous scholars who, in the music of Stravinsky, have taken surface-level octatonicism and promoted it so that it informs deep level structures. ${ }^{25}$ Instead, he suggests that "many instances of purported octatonicism actually result from two other compositional techniques: modal use of non-diatonic minor scales, and superimposition of elements belonging to different scales. ${ }^{, 26}$ Consider the short segment from Crumb's "Gargoyles" illustrated in Figure 2.9. The collection to which each vertical trichord belongs is indicated below the staff and the transpositions of the linear whole-tone scale is indicated to the right of the staff. ${ }^{27}$

Although the pool of representative collections used to analyze the short segment was limited to those listed in Appendix 1, the number of potential representative collections is quite extensive. To determine the operative referential collection, I believe

[^15]Figure 2.9. Crumb, Makrokosmos II, "Gargoyles," mm. 9-10. Copyright © 1973 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

that specificity is an important criterion. ${ }^{28}$ For example, given a C major triad progressing to a G major triad, one is likely to assume a diatonic key area given no additional information. In a tonal system, this ambiguity can be dispelled by either adding more functionally related triads or through adding a minor seventh to either one of the two triads creating a dominant-seventh chord.

A tabulation of prominent collections and subsets found in Crumb's music is illustrated in Table 2.3. Instances of sc (024) can be found in four different collections; however, set $\{024\}$ can be found in only one transposition of a unique whole-tone or

[^16]Table 2.3. Identification of collection by subset

| Set Class | Octatonic | Messaien's <br> Mode 4 | Harmonic <br> Minor | Melodic <br> Minor (asc.) | Diatonic | Whole <br> Tone |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $(013)$ | U | C | C | C | C |  |
| $(014)$ | U | U | C | U |  |  |
| $(016)$ | U | C | U | U |  |  |
| $(024)$ |  |  | U | C | C | U |
| $(025)$ | U | U | C | C | C |  |
| $(026)$ | U | C | U | C | U | U |
| $(036)$ | C | C | C | U | U |  |
| $(037)$ | U | U | C | C | C |  |
| $(0167)$ | U | C |  |  |  |  |
| $(0147)$ | U | U | U |  |  |  |
| $(0158)$ |  | U | U |  | C |  |
| $(0268)$ | U | C |  | U |  | U |
| $(0258)$ | U | U | C | C | U |  |
| $(0358)$ | U |  | U | C | C |  |
| $(0369)$ | C | C | C |  |  |  |
| U 's | 11 | 6 | 6 | 4 | 3 | 3 |
| C's | 2 | 7 | 6 | 7 | 6 | 0 |
| Total | $\mathbf{1 3}$ | $\mathbf{1 3}$ | $\mathbf{1 2}$ | $\mathbf{1 1}$ | $\mathbf{9}$ | $\mathbf{3}$ |

harmonic minor collection. This property is identified with " $U$ " in the corresponding column in Table 2.3, which represents its ability to identify a Unique instance of a set from a set class. On the other hand, $\{024\}$ is a subset of three different transpositions of the diatonic collection- $D^{4}, D^{6}$, and $D^{B} —$ and is identified with " $C$ " in the corresponding column, which represents its Commonality with other transpositions. A blank cell indicates that the set class is not found in the corresponding collection.

In Table 2.3 a particular instance of sc (037), (0258), or (0358) can identify a unique transposition of an octatonic collection. If one considers the C-major progression
from $C\{047\}$ to $d^{7}\{9025\}$ to $G^{7}\{B 257\}$ as an octatonic progression, it would pass through $\mathrm{Oct}^{1}$ to $\mathrm{Oct}^{2}$ to $\mathrm{Oct}^{3}$. While the specificity of chords that point to a unique collection is important, it is also imperative that all such chords belong to as few collections as possible. The choice of referential collection should also weigh the ability of the collection to represent all of the objects that appear in the piece, but not entail many objects that do not appear. ${ }^{29}$ In the above case, all the chords belong to only one diatonic collection but to three octatonic progressions; therefore, the diatonic progression is preferred.

Returning to the excerpt from "Gargoyles" shown in Figure 2.9, one could eliminate some of the vertical interpretations of the chords. A member of sc (016) does not identify a unique instance of Messiaen's fourth mode and although the trichords can distinguish between transpositions of the other collections, the harmonic- and melodicminor interpretations cycle through half of the twelve possible unique collections from their respective parent collections. The octatonic interpretation cycles through all the possible unique octatonic collections and corresponds with the notated articulations. Although the octatonic, harmonic minor and melodic minor collections encompass all the chords of the excerpt, the octatonic collection is minimal because all the unique instances of the collection are represented in the music.

[^17]Ciro Scotto investigates the interaction between diatonic and chromatic collections and the procedures used to organize pitch content in Crumb's Processional. ${ }^{30}$ In particular, Scotto shows how the diatonic collection and various TINV collections interact through an exchange of object and process. Two interesting features of Processional are the notated key signatures of the opening and closing sections and the corresponding diatonic collections in the music (see Figure 2.10). The opening hexachord, shown as a vertical simultaneity in Figure 2.10, implies membership in either $D^{0}$ or $D^{5}$. The pitches that would confirm or deny one of the two collections are the two pitches that form the inversional axis between the right- and left-hand trichords of the hexachord, C or $\mathrm{Cb} .{ }^{31}$ As Scotto points out, the final pitch to enter, which completes the aggregate, is the only pitch that invades the center registral stream-the area between the right- and left-hand trichords of the hexachord—and the pitch that confirms the $\mathrm{D}^{0}$ collection implied by the key signature. ${ }^{32}$ Using Scotto's registral streams concept, parsing the last six notes of each aggregate (enclosed within large boxes) and either the lowest or highest note of the opening hexachord reveal two other diatonic collections, which are indicated by dashed boxes and arrows in Figure 2.10. ${ }^{33}$

[^18]Figure 2.10. Crumb, Processional, reduction of the opening section. Copyright © 1984 by C. F. Peters Corporation. All Rights Reserved. Used by permission.


On a larger scale, the diatonic collections occupying the middle register of each segment $\left(D^{0}, D^{3}, D^{6}, D^{A}\right)$, might be expressed as $\{A 036\}$, using the index number of each collection, which is a member of sc (0258). ${ }^{34}$ The (0258) structure that ties the four segments of the opening section together is particularly interesting in a piece that alternates between diatonic and chromatic passages. From the collections listed in Table 2.3, an instance of sc (0258) can uniquely identify a particular octatonic collection, Messiaen's fourth mode of limited transposition, and the diatonic collection.

Figure 2.11 is a reduction of the opening portion of the middle section of Processional. ${ }^{35}$ Although each diatonic hexachord could belong to two unique diatonic collections, the collection that corresponds to its counterpart in the opening section was

[^19]Figure 2.11. Crumb, Processional, reduction of the first segment of the middle section. Copyright © 1984 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

chosen based upon the left- and right-hand trichordal pairings and the interval-class 3 gap between the hands. Transpositional relationships between collections are indicated below the staff, many of which conform to a $\mathrm{T}_{(0369)}$ schema. ${ }^{36}$ The transpositions in Figure 2.11 are represented in a slightly different manner than the way in which Scotto presents them: the combined tetrachords that complete instances of Messiaen's fourth mode are merged into octachords and the $\mathrm{F}^{3}$ collection completes the hyperaggregate of the tetrachordal structures. Considering the last two mode- 4 collections, $\mathrm{F}^{3}$ and $\mathrm{F}^{4}$, the individual tetrachords within each collection exhibit a $\mathrm{T}_{(0167)}$ schema comprised of two overlapping $\mathrm{T}_{(016)}$ patterns. These overlapping patterns are reminiscent of the interlocking diatonic collections shown in Figure 2.10, and thus forge a connection between the different referential collections of the opening and middle sections.

[^20]"Primeval Sounds," the first movement of Crumb's Makrokosmos legacy, summarizes many of the issues discussed so far (see Figure 2.12). David Lewin describes this opening as an example of up-shift voice leading in which "fundamental-position triads, whose basses are related by step . . . move the upper three voices, in contrary motion to the bass, to the next available notes of the new harmony., ${ }^{37}$ It is clear that Lewin is considering the left- and right-hand minor triads in their own streams: the roots of the right-hand triads descend by semitone from F while the roots of the left-hand triads descend by semitone from B. Bass acknowledges these triads, and the tritone registral exchange indicated with arrows in Figure 2.12, but prefers to conceptualize the verticallyaligned triads in the upper and lower staves as hexachords. ${ }^{38}$ The grace-note connection between triads supports this reading: each pair of triads is held for three seconds and the sustain pedal is held for the passage's duration. Regardless of the interpretation chosen, triadic or hexachordal, each vertical chord can uniquely identify a specific octatonic or Messiaen's mode-4 collection. The tritone relationship between the roots of the first and last chord in each hand, as well as the tritone relationship between the left- and right-hand chords, exudes a symmetry that is common in Crumb's music. Crumb's predilection for borrowing is thinly veiled here: two tritone-related minor triads are comparable to the two tritone-related major triads that constitute the Petrushka chord (see Figure 2.13). Rewriting each of the constituent triads of the Petrushka chord so that they are in root
${ }^{37}$ David Lewin, "Some Ideas about Voice-Leading between PCSets," Journal of Music Theory 42 (1998): 72.

38 Bass, "Pitch Structure," 73-75.

Figure 2.12. Crumb, Makrokosmos I, "Primeval Sounds," opening Copyright © 1973 by C. F. Peters Corporation. All Rights Reserved. Used by permission.


Figure 2.13. Stravinsky, Petrushka, Second Tableau, R. 49


Figure 2.14. Inversional relationship between the Petrushka and Primeval chords

position and inverting around each triad's root obtains the opening two chords of "Primeval Sounds" (see Figure 2.14).

The next section of "Primeval Sounds" provides additional support for an octatonic interpretation (see Figure 2.15). Whereas the first two trichords in Figure 2.15 belong to $\mathrm{F}^{1}$, the first seven trichords all belong to $\mathrm{Oct}^{3}$. In addition, the octatonic collection is completed on the fourth trichord, the central trichord of the seven. A similar configuration occurs with the final seven trichords, indicated with a dotted arrow above the staff in Figure 2.15. The central seven chords complete the arch design alternating between subsets of $\mathrm{Oct}^{1}$, Oct ${ }^{2}$ and Oct ${ }^{1}$. After the first three trichords of the central seven, only pc 4 is required to complete the $\mathrm{Oct}^{1}$ collection, but the $\mathrm{Oct}^{2}$ collection disrupts this. The missing pc 4 of the Oct $^{1}$ collection is finally obtained in the lowest voice of the first chord in the closing $\mathrm{Oct}^{3}$ collection. The insistent repeated notes occurring above the trichords are beamed together and represented with a single note. ${ }^{39}$

Figure 2.15. Crumb, Makrokosmos I, "Primeval Sounds," p. 1, s. 2-3. Copyright © 1973 by C. F. Peters Corporation. All Rights Reserved. Used by permission.


[^21]After a reiteration of the opening four trichords and a completion of $\mathrm{Oct}^{2}$, two opposing whole-tone collections are presented (see Figure 2.16). Crumb's selfproclaimed sloppiness, or prismatic sense of moving between collections, is highlighted throughout this passage. Each vertical dyad's octatonic membership is indicated below the whole-tone interpretation, highlighting another symmetrical arch progression. While the supposition that a dyad could be representative of a collection four times larger than itself might seem unduly promiscuous, the boundary semi-tonal interval-class is reminiscent of the earlier (014) progressions in the piece (see Figure 2.15). The possible notes, which would create a vertical (014) trichord between the whole-tone scales, are notated as x-note heads. ${ }^{40}$

The end of this section features a seven-second tremolo divided between the right and left hands. The right-hand tremolo is a subset of either $\mathrm{WT}^{1}$ or $\mathrm{Oct}^{2}$ while the lefthand tremolo is a subset of either $\mathrm{WT}^{2}$ or $\mathrm{Oct}^{1}$; however, the most complete and minimal referential collection would be the $\mathrm{F}^{3}$ collection. This interpretation is justifiable since it would be virtually impossible to distinguish between the two tetrachords because of the low and similar register, rapid oscillation between the same notes, and the pitches being sustained by the sustain pedal.

A return is achieved with some slight changes (see Figure 2.17). The first four dyads in Figure 2.17 all belong to $\mathrm{F}^{1}$ while the next four dyads all belong to $\mathrm{F}^{4}$. This observation reinforces the interpretation of the previous tremolo as $\mathrm{F}^{3}$. At the same time,

[^22]Figure 2.16. Crumb, Makrokosmos I, "Primeval Sounds," p. 1, s. 3-4. Copyright © 1973 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

the odd-numbered dyads belong to $\mathrm{Oct}^{3}$ and the even-numbered dyads belong to Oct ${ }^{1}$, each completing their respective collection by the fourth dyad (indicated by a dotted arrow in Figure 2.17). Following this overlap of collections, a forceful $\mathrm{Oct}^{2}$ pedal is established, which resonates throughout the segment while chords from the other octatonic collections are played in a higher register. This is similar to the Oct ${ }^{2}$ pedal used in "Music of Shadows," which was silently depressed and held with the sostenuto pedal, which enables sympathetic resonance with the other octatonic collections that were also played in a higher register (cf. Figure 2.8). What might have been considered a segment comprised of Messiaen's fourth mode following the tremolo, becomes distinctly octatonic following the $\mathrm{Oct}^{2}$ pedal. The upper-beamed dyads and trichords, which belong to $\mathrm{Oct}^{3}$ at the beginning of the excerpt in Figure 2.17, replicate the opening melodic

Figure 2.17. Crumb, Makrokosmos I, "Primeval Sounds," p. 2, s. 1-3. Copyright © 1973 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

phrase following the introductory minor chord progression of the piece (cf. Figure 2.15). Following the $\mathrm{Oct}^{2}$ pedal, the pattern of the opening section is maintained more-or-less faithfully. The repeated-note trichord's extension across the section and inclusion as a subset in the pivotal tremolo chords underscores its significance and enables a prismatic sense of moving between collections.

The frequent recurrence of, and correlation between, certain pc sets in Crumb's music, as demonstrated in section 2.1, coincides with shared membership in a select number of referential collections. In "Come Lovely," these subsets reappear and illustrate

Crumb's prismatic sense of moving between collections, as they did in "Primeval Sounds" (see Figure 2.18). The diatonic and whole-tone elements of the progressions in "Come Lovely" are clear: the first four triads of each seven-chord progression belong to, and complete, a single diatonic collection; the last three triads of each seven-chord progression belong to $\mathrm{WT}^{1}$. The symmetry in each progression is prominent: the middle trichord of each progression completes the respective diatonic collection with the highest pitch, indicated with a dotted arrow, which is similar to the completion of the octatonic collection in the seven-chord progressions in "Primeval Sounds" (cf. Figure 2.15). The trichord that completes the diatonic aggregate is the point in which a diatonic reading might be questioned. Unlike "Primeval Sounds," "Come Lovely" switches to a three-chord-whole-tone progression and the bass note of the final trichord completes the respective whole-tone collection. Interestingly, the pitch class that completes each diatonic and whole-tone collection participates in a voice exchange, indicated with dotted arrows, which links the diatonic and whole-tone segments. In a tonal interpretation, the pitch class that completes both the diatonic and whole-tone collections retains even further significance: it is the leading-tone of the respective diatonic collection, similar to the technique used in the opening section of Processional (cf. Figure 2.10).

Because of the bass's ascending diatonic scale in each progression and the correspondence between the pitch classes required to complete each collection, an allusion to a functional relationship between the trichords is enticing. Rather than separate each progression into only diatonic and whole-tone collections, it is also feasible to consider each trichord as representative of an octatonic collection. Each trichord uniquely identifies only one octatonic collection and thus provides support for this

Figure 2.18. Crumb, Apparition, "Come Lovely," trichordal progressions. Copyright © 1980 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

$\mathrm{Oct}^{1}$
reading (see Figure 2.6 and Table 2.3). In addition, the octatonic collection is the only collection from those listed in Table 2.3, which encompasses all the trichordal set classes. While hearing a shift in octatonic collections on a trichord-by-trichord basis may seem overly gratuitous, additional justification for this interpretation will be discussed in section 2.4.

### 2.3 The Principle of Opposition

In an interview with Crumb, Paul Steenhuisen commented that he felt a duality present in Crumb's music, something he equated to "a polarity of light and dark, of lifeaffirming and deathly forces." ${ }^{, 41}$ Crumb affirmed this observation and replied, "There are opposites like that, yes. I've always liked music that had the quality of presenting opposites. ${ }^{42}$ This opposition is present on many different levels in Crumb's music. At the most superficial level, even the titles of his pieces suggest this preoccupation. In Black Angels, movements are titled "Devil-music" and "God-music," or in Makrokosmos II the self-reflexive title "Twin Suns (Doppelgänger aus der Ewigkeit)" hints at an oppositional duality. Edward Pearsall describes "several structural paradigms that engender goal-direction in Crumb's music. These paradigms are dialectical in nature:

[^23]equilibrium and disequilibrium, stasis and motion, and closure and continuity., ${ }^{, 43}$ In particular, Pearsall situates symmetrical and asymmetrical pitch collections as opposing pairs because of the dialectic in which "asymmetry resists symmetry, while symmetry seeks to overcome asymmetry." ${ }^{44}$ Resulting from this oppositional battle, Pearsall suggests, "Motion in this case is brought about through an interplay of contrasting ideas." ${ }^{45}$

Such interplay between contrasting ideas is highlighted in Night of the Four Moons (see Figure 2.19). Composed as "an artistic response to an external event," the work symbolizes Crumb's "rather ambivalent feelings vis-à-vis Apollo 11.,"46 This ambivalence, and perhaps disappointment, regarding the decreasing mystery of Earth's opposing partner is reflected in the text of the first movement: La luna está muerta, mиerta. The farewell music shown in Figure 2.19, which alludes to Mahler's "Der Abschied" from Das Lied von der Erde and Haydn's "Farewell" Symphony No. 45, "is Crumb's Das Lied von den Mond, a regretful farewell to the poetic and mystical qualities long associated with the moon, forever altered in 1969 by our arrival on the lunar

[^24]Figure 2.19. Crumb, Night of the Four Moons, IV. Copyright © 1973 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

surface. ${ }^{, 47}$ Not only does this intertextual connection highlight opposition in Crumb's music but, also the intra-opus opposition between the white-note "Musica Mundana" \{7902\} tetrachord, which is played on stage and the primarily black-note "Musica Humana" $D^{5}$ collection, which is played off stage.

Opposition is highlighted in both the objects and the transformational processes found in "Litany of the Galactic Bells" (see Figure 2.20). The piece begins with the right hand playing three hexachords belonging to $\mathrm{WT}^{2}$ while the left hand plays grace-note

[^25]Figure 2.20. Crumb, Makrokosmos II, "Litany of the Galactic Bells," opening. Copyright © 1973 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

hexachords belonging to $\mathrm{WT}^{1}$. Following a four-second pause, the registral arrangement of collections is reversed and the right-hand pitches outline $\mathrm{WT}^{1}$ and the left-hand pitches outline $\mathrm{WT}^{2}$. In addition, the transformation of one hexachord to the next within each three-note segment is reversed: the first hexachord is transposed up by six semitones and then down by two semitones while the fourth hexachord is transposed down by six semitones and then up by two semitones. ${ }^{48}$

A similar grace-note technique, which also exhibits the principle of opposition, is found in Crumb's "Music of the Starry Night" from Music for a Summer Evening (see

Figure 2.21). In the program notes Crumb states:

[^26]Figure 2.21. Crumb, Music for a Summer Evening, "Starry Night." Copyright © 1974 by C. F. Peters Corporation. All Rights Reserved. Used by permission.
a) p. 18, s. 1
b) p. 18, s. 1
c) p. 19, s. 2

d) p. 24, s. $2-$ p. 25 , s. 1
e) p. 25, s. 1


In its overall style, Summer Evening might be described as either more or less atonal, or more or less tonal. The more overtly tonal passages can be defined in terms of the basic polarity $\mathrm{F} \# / \mathrm{d} \#$ minor (or, enharmonically, $\mathrm{Gb} / \mathrm{eb}$ minor). This (most traditional) polarity ... is stated once again in "Music of the Starry Night," with the quotation of passages from Bach's $\mathrm{d} \#$ minor fugue (Well-Tempered

Clavier, book II) and a concluding "Song of Reconciliation" in Gb (overlaid by an intermittently resounding "Five-fold Galactic Bells" in F\#). ${ }^{49}$

This polarity occupies a significant portion of "Music of the Starry Night" and separates the structure of the movement into two parts: the first part contains the quotations of Bach's fugue and the second part consists of the "Song of Reconciliation." Although Crumb mentions a polarity between a major key and its relative minor key, polarity is also present within the transformational structure of the bell-like figures. ${ }^{50}$ The transformations of these figures within the first section of "Starry Night" remain unchanged, featuring a $\mathrm{T}_{-5}$ to $\mathrm{T}_{2}$ pattern (see Figure 2.21a-c). This corresponds with the D\#-minor fugue quotations in this section. ${ }^{51}$ In the opposing second section, characterized by the Gb "Song of Reconciliation" and F\# "Galactic Bells," the transformational scheme is altered for the final version of the grace-note figure (see Figure 2.21e). Instead of conforming to the established $\mathrm{T}_{-5}$ to $\mathrm{T}_{2}$ pattern, the indices of transposition are inverted to a $\mathrm{T}_{5}$ to $\mathrm{T}_{-2}$ pattern.

In "Litany," defining the opposite whole-tone collection was relatively simple since there are only two distinct whole-tone collections; however, in "Starry Night" each hexachord could belong to either an octatonic collection or Messiaen's fourth mode, which is indicated below the staff. There is always an intersection of four pitch classes between two octatonic collections: the intersection of Oct ${ }^{2}$ and Oct ${ }^{1}$ is $\{0369\}$ while the

[^27]intersection of $\mathrm{Oct}^{2}$ and $\mathrm{Oct}^{3}$ is $\{258 \mathrm{~B}\}$. Therefore, considering pc intersection, both $\mathrm{Oct}^{1}$ and $\mathrm{Oct}^{3}$ are equally opposite $\mathrm{Oct}^{2}$. This battle with opposition, and the symmetry which this opposition represents, is played out in the music. The first three progressions, shown in Figure 2.21a-c, are firmly entrenched in an $\mathrm{Oct}^{2}-\mathrm{Oct}^{1}-\mathrm{Oct}^{2}$ pattern. The first progression of the opposing section transposes the original progression, which was framed by $\mathrm{Oct}^{2}$, and is comprised of the opposing collections to $\mathrm{Oct}^{2}$ (see Figure 2.21d). The final progression of the piece is the polar version of the first progression: instead of proceeding to $\mathrm{Oct}^{1}$, $\mathrm{Oct}^{3}$ is utilized, which changes the transformational schema (see Figure 2.21e). This reversal corresponds with the key-area polarity between the sections. The interpretation of each chord as a subset of Messiaen's fourth mode does not reflect a similar oppositional pattern. ${ }^{52}$ Therefore, the preference to choose the octatonic reading over the mode-4 reading rests upon the two-fold assumption that both opposition and symmetry are essential characteristics of Crumb's music.

The principle of opposition is evident in the progressions from "Come Lovely"
(see Figure 2.6 and Figure 2.18). ${ }^{53}$ The tonal reading hinted at a tonic-dominant-tonic

[^28]opposition for the first three chords and the octatonic reading mimics this with an $\mathrm{Oct}^{1}-\mathrm{Oct}^{3}-\mathrm{Oct}^{1}, \mathrm{Oct}^{3}-\mathrm{Oct}^{2}-\mathrm{Oct}^{3}$, or $\mathrm{Oct}^{2}-\mathrm{Oct}^{1}-\mathrm{Oct}^{2}$ progression in the respective iterations. Following the fourth chord, the fifth, sixth, and seventh chords cycle through each octatonic collection. An alternate segmentation of this chord progression also highlights the opposition within the progression. The segment from the third to fifth chords, as well as the segment from the fourth to sixth chords of each progression, exhibits the same type of polarity.

### 2.4 The Principle of Completion

The completion of collections and patterns is an important aspect of Crumb's music. ${ }^{54}$ Although Crumb's admiration for tonal forms and his subsequent desire to find some sort of analogue in post-tonal music might suggest that closure is an important principle in his music, Pearsall believes that this is not always the case:

Not all of Crumb's pieces end with decisive closure. Indeed, it is a hallmark of his style that many of them do not. This inconclusiveness enhances the atmosphere of mystery that has often been associated with these works; ideas are left hanging in the air so to speak, inviting the listener to supply his or her own meaning. Understood in this way, the lack of closure constitutes a rhetorical marker in Crumb's music, one that engenders expectancy in the listener's imagination rather than frustration. ${ }^{55}$

[^29]Pearsall's definition of closure is based upon an opposition between symmetry and asymmetry and the notion that a return to a symmetrical structure constitutes a stable event. ${ }^{56}$ Stephanie Lind also negotiates aspects of closure through a network analysis of Crumb's "Portent" from Zeitgeist and considers which layout of nodes, arrows and transformations best accounts for surface-level voice leading, and suggests higher-level groupings and hierarchies. ${ }^{57}$ Lind uncovers a compelling transformational pattern within "Portent" that illustrates a repetition-with-contrast principle and demonstrates the relative completeness or incompleteness of the networks based upon previous networks. ${ }^{58}$

The problem of composing music without the guidance of tonality was not lost on Crumb, and likely prompted him to wish for "a new kind of tonality." ${ }^{59}$ For Crumb, sonata form is superior to the forms of recent compositions that are based upon what he believes are two opposite principles: "the 'non-repetitive' principle, which implies a progression along a straight line without ever referring back to itself" and the "'minimal' type, which usually consists of a repetition ad infinitum of one idea. ${ }^{60}$ Instead of conforming to one of these procedures, Crumb felt that composers of the future should "reevaluate the more traditional principle of repetition-with-contrast, which served the

[^30]earlier composers so well. ${ }^{,{ }^{61}}$ Apprehensive about strict serialism, Crumb stated that both "new chord structures [and] new modes of introducing tension, are important for the musical vocabulary." ${ }^{62}$ Joseph DeBaise comments upon the lack of tension in the "Song of Reconciliation," the second section of "Music of the Starry Night" from Music for a

## Summer Evening:

Because of the non-functional nature of the harmony, this music has a stationary character. There may be activity, but no feeling of movement or progression toward a cadential goal. There may be a dynamic buildup, yet there is a lack of forward motion. Small scale melodic arch designs return back rather than move onward, giving a feeling of in-place motion or prolongation. There is gravitation around instead of going to and from, succession without progression, elaboration rather than propulsion. In this sense, Crumb's music is vertically conceived instead of horizontal, and reflects an Eastern influence. ${ }^{63}$

Bass, however, believes that such an emphasis on the static quality of Crumb's music is over-represented in Crumb scholarship:

Crumb does occasionally use stasis as a musical device, but when he does so it is usually associated with an attempt to portray vastness, or a sense of the infinite, and this is often accomplished through some abstract numerical scheme. Such strictly mechanical procedures are the exception rather than the rule. It is my contention that, for the most part, Crumb's music is far from the kind of static assemblage some people consider it to be-that even though it employs novel timbres and performance techniques, it is usually constructed in ways that follow very closely our Western compositional traditions. ${ }^{64}$

[^31]Although DeBaise noted small changes in the "Song of Reconciliation," he did not feel that it was enough to create a sense of motion in the music. Throughout the musical examples discussed in this chapter, it is clear that strict repetition in the music of Crumb is not the norm. For example, the transformational pattern in the first section of Processional resisted the repetitive $\mathrm{T}_{3}$ relationship, which prompted Scotto to declare the fourth diatonic collection an "interloper," rather than a completion of a (0258) structure (cf. Figure 2.10). ${ }^{65}$ In Crumb's music, through such small changes in patternssometimes manifesting as a tension between opposites-it is possible that one might find a sense of closure and completion.

In his analysis of "Approach Strong Deliveress!" from Apparition, Bass uses language appropriate for describing tonal music to describe the relationship between different octatonic collections. In the second section of the piece, he describes how "the highest pitches of the piano part 'modulate' from an Oct-II to an Oct-III subset." ${ }^{\text {" }}$ ' Bass also implies a return at the end of the work after a coda where "the final two sonorities $\ldots$ are the same as the first two . . ., but in reverse order, so that the movement ends with the same chord as it began." ${ }^{, 67}$

The existence of such tonal analogues is often mentioned in analyses of Crumb's music, but rarely explored in any detail. As demonstrated earlier, a specific instance of a

[^32]set class can identify a referential collection (see Table 2.3). ${ }^{68}$ Many of these set classes are also found in a diatonic collection. A comparison of the common functional associations in a tonal setting with their respective octatonic collection membership illuminates some additional similarities. In Table 2.4, the columns represent the three functional relationships found in tonality. ${ }^{69}$ The first row under the headings indicates chords typically associated with the respective functions in such a way that they also align with a unique octatonic collection. Certain chords that can often function in various ways in a tonal setting are relegated to only one column, and therefore one functional category, to align with a unique octatonic collection. For example, bVI may have either a tonic or pre-dominant function in a tonal setting but a specific instance of a major chord only occurs in one octatonic collection. Therefore, bVI is only included in the predominant column. The Neapolitan chord is not included because its function is typically predominant; however, its octatonic membership would place it in the dominant category of this table. ${ }^{70}$ For a similar reason, vii ${ }^{\varnothing 7}$ is not included because its function is typically

[^33]Table 2.4. Correspondence of functional harmonies and octatonic collections

| Keys | Tonic (T) | Predominant (PD) | Dominant (D) |
| :---: | :---: | :---: | :---: |
|  | I, i, vi ${ }^{(7)}$, bIII | $\begin{aligned} & \text { ii }^{(7)}, \text { IV, iv }{ }^{(7)} \text {, } \\ & \text { Gr, Fr, It, bVI, } \\ & \text { V }^{(7)} / \mathrm{V}, \text { vii }{ }^{\otimes 7} / \mathrm{V} \end{aligned}$ | $\mathrm{V}^{(7)}, \mathrm{v}^{(7)}, \mathrm{iii}, \mathrm{bII}{ }^{\text {b }}$ |
| C, Eb, F\#, A (+/-) | Oct ${ }^{1}$ | Oct $^{2}$ | $\mathrm{Oct}^{3}$ |
| D, F, Ab, B (+/-) | Oct ${ }^{2}$ | $\mathrm{Oct}^{3}$ | Oct ${ }^{1}$ |
| $\mathrm{C} \#, \mathrm{E}, \mathrm{G}, \mathrm{B}{ }^{(+/-)}$ | $\mathrm{Oct}^{3}$ | Oct ${ }^{1}$ | $\mathrm{Oct}^{2}$ |

dominant, but its octatonic membership would place it in the pre-dominant category. In addition, diminished triads and fully-diminished seventh chords are not included since they do not specify a unique octatonic collection; although vii ${ }^{\circ}$ and vii ${ }^{07}$ could both be found in the dominant column, they would also require inclusion in the pre-dominant column, which is not their typical function.

For the most part, typical tonal harmonies seem to separate into functional tonal categories that also define the octatonic collections. The following analysis of excerpts from Crumb's "Music of the Starry Night," which include quotations from Bach's D\#minor fugue from the second book of Das Wohltemperierte Clavier, will demonstrate how an octatonic interpretation of a tonal passage can reveal the principle of completion.

The first three entries of the subject (S) and answer (A) of the D\#-minor fugue are analyzed in Figure 2.22. The subject's head-motive repeatedly emphasizes the tonic and prolongs tonic function through neighbour motion to the leading-tone. The real answer is an ascending-fifth transposition of the subject. Although the head-motive of the fugue conforms to what one might expect of a real answer, the tail-motive is less clear. A descent from $\hat{5}$ to $\hat{1}$ at the tail of each entry is indicated with the corresponding key

Figure 2.22. Bach, Das Wohltemperierte Clavier, Vol. II, VIII, mm. 1-9

$a \#: \hat{5} \hat{4} \quad \hat{3} \hat{2} \hat{1}$

relationship. A dotted bracket at the end of each subject indicates that the final two notes of each entry may or may not be considered part of the respective subject or answer. After the first three entries, subsequent entries do not maintain the same descent of $\hat{5}$ to $\hat{1}$, instead ending on 3. In Figure 2.23, the final entry of the four-voiced exposition, which was left incomplete in Figure 2.22, is included. The final entry of the answer in the exposition maintains the descent only until $\hat{3}$ (a dotted bracket indicates a delayed descent to $\hat{1}$, passing through $b \hat{2}$ rather than $\hat{2}$ ). The initial key of the descent begins in $A \#$ minor and pivots to $\mathrm{G} \#$ minor by the end of the passage; however, the entire descent is notated in the initial key. The delayed $\hat{1}$ in $A \#$ minor (pc A) can also be thought of as $\hat{2}$ in $G \#$ minor, which then resolves to $\hat{1}$ in $G \#$ minor (pc 8) over $\mathrm{i}^{6}$.

Figure 2.23. Bach, Das Wohltemperierte Clavier, Vol. II, VIII, mm. 9-12


In a skeletal model of Bach's subject, Crumb switches from a diatonic collection to a whole-tone collection to emphasize the ambiguity of the final descent (see Figure 2.24; the corresponding notes of the skeletal structure are circled in Figure 2.23). This whole-tone version is played by the vibraphone and whistle in the second half of the final movement near the beginning of the "Song of Reconciliation." DeBaise comments on this, and subsequent appearances, noting, "The theme quoted from Bach's d\# minor fugue (W.T.C., Bk. II), is found several times in part two with alterations based on the wholetone scale. ${ }^{" 71}$ A later example of this theme shown in Figure 2.25 "is in augmentation and altered to conform to a whole-tone scale. Moreover, each note of the melody is harmonized with this same whole-tone hexachord., ${ }^{, 72}$ The seed of ambiguity present in the tail-motive, expressed as a descending whole-tone scale, has spread throughout the entire subject near the end of "Music of the Starry Night."

[^34]Figure 2.24. Crumb, Music for a Summer Evening, "Starry Night," p. 20, s. 2. Copyright © 1974 by C. F. Peters Corporation. All Rights Reserved. Used by permission.


Figure 2.25. Crumb, Music for a Summer Evening, "Starry Night," p. 24, s. 1-2. Copyright © 1974 by C. F. Peters Corporation. All Rights Reserved. Used by permission.


These appearances of the altered Bach subject in the "Song of Reconciliation" are consistent with how Crumb quotes materials in other works. In an interview, Steenhuisen asked Crumb if he was aware of the distinction between quoted and invented material to which Crumb replied:

When I think of it consciously, I'm aware of when I'm quoting. But I usually cut in and out, like a film technique I could cut right at any point or I could cut out of it before a cadence or something. I feel completely free to distort a quotation in a hundred different ways. It's never a literal quotation in my music. Something is always changing. The timbre is different, or the instrumentation, or there is an overlay of some kind that has nothing to do with the original music. All kinds of things can happen. ${ }^{73}$

[^35]Quotation marks did not enclose the previous two examples in the score, a technique that Crumb often utilizes to make a clear distinction between quoted and invented material; however, in the first part of "Starry Night" quotation marks are used three times, which correspond with three different segments of Bach's fugue. The first quotation is based upon the subject of mm. 9-12 as shown in Figure 2.26 (cf. Figure 2.23). The quotation marks encircling the quotation of the subject are deceiving: they are not exact quotations. ${ }^{74}$ X-note heads in Figure 2.26 indicate the altered pitches compared to Bach's original (subsequent figures will also use this notation). Crumb's first quotation of Bach's fugue alters the final notes in a way that suggests that the progression ends on $V^{7}$ with the following chord, $\mathrm{VI}^{6}$, functioning as a neighbour chord to V . The use of the sustain pedal throughout these passages would retain the $D \#$ in the lowest voice, which emphasizes the dominant arrival. Crumb includes an upward arrow above the quotation with a caption reading "striving," which extends to 5 and then terminates. A downward arrow with a caption reading "but falling" begins when the notes of the descent are revisited, and extends beyond the subject's terminal note. If read as a tonal progression, it would seem that the "fall" might correspond to a lack of tonal closure. ${ }^{75}$ The quotation concludes just before the point that Bach returns to $i^{6}$ in the fugue. In Bach's fugue, the motion to tonic is anything but stable-a point that Crumb highlights again in the next quotation.

[^36]Figure 2.26. Crumb, Music for a Summer Evening, "Starry Night," p. 18, s. 2. Copyright © 1974 by C. F. Peters Corporation. All Rights Reserved. Used by permission.


There is another small change just before the final dominant arrival in the alto part: instead of $D \#$ and $E \#$ sixteenth-notes the $D \#$ is deleted and the $E \#$ becomes the chord tone; as a result, the former tonic triad may be interpreted as a half-diminished seventh chord. Such an alteration can be accounted for as a modally mixed \#viø7 chord, which, although unusual in common-practice literature, might be seen to function as a tonic substitute. Because the $\mathrm{E} \#$ is now a chord tone, the subsequent Fx might be considered a passing tone to the $\mathrm{G} \#$. As a result, the chord would change from a vii ${ }^{06}$ to a $\mathrm{ii}^{7}$ that prepares the dominant arrival on the downbeat of the next measure (cf. Figure 2.23).

I propose an octatonic reading of this progression, even though such an interpretation may seem absurd given the source material. ${ }^{76}$ As shown previously in Table 2.4, many tonal chords can also uniquely identify an octatonic collection. In each

[^37]example, a chord's membership in an octatonic collection is indicated below the staff. The corresponding Roman numeral indicates the pitches of each chord; other pitches should be considered non-chord tones. In all of the examples, chords that arise from voice leading are indicated in parentheses and will not be considered as the operative harmony. Circled octatonic collections in the examples of Crumb's Bach quotations indicate the collections that open and close each progression. Crumb's first quotation of Bach's fugue begins with a subset of an $\mathrm{Oct}^{3}$ collection and ends with a subset of an $\mathrm{Oct}^{1}$ collection, which contributes to a sense of incompleteness because a return to the original octatonic collection was not achieved (see Figure 2.26). This lack of closure in the octatonic interpretation is analogous to the unresolved tension in a tonal interpretation, exemplified by a dominant arrival that does not resolve to tonic, and corresponds to Crumb's annotation of "but falling."

The second quotation is also altered in significant ways (see Figure 2.27 and Figure 2.28). Preceding this quotation in Bach's fugue, there is a relatively strong cadence in B major. Because of this strong arrival, it is unlikely that the second eighth note would be heard as a separate chord and, viewing the progression through an octatonic lens, the chord at this point would be a subset of Oct ${ }^{2}$. Crumb alters this quotation significantly by not including the cadence, deleting the bass note at the beginning of the quotation, and reinforcing the root of the iv chord by changing the alto pitch. Because of these changes, there is a strong feeling of iv being tonicized, and therefore the first chord can be understood as a subset of Oct ${ }^{1}$. The excerpt closes on a tonally tense chord: a German augmented sixth chord that resolves in Bach's fugue but subsequently evades a cadence. Crumb mimics this lack of completion in Bach's fugue

Figure 2.27. Bach, Das Wohltemperierte Clavier, Vol. II, VIII, mm. 21-24


Figure 2.28. Crumb, Music for a Summer Evening, "Starry Night," p. 19, s. 1. Copyright © 1974 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

by simply ending on a French augmented sixth chord, which is altered slightly to avoid the sense of a new subject starting, but retains the same octatonic membership as in Bach's fugue. Whereas the first quotation moved from $\mathrm{Oct}^{3}$ to $\mathrm{Oct}^{1}$, this progression moves from Oct ${ }^{1}$ back to Oct ${ }^{3}$. Neither subject exhibits a straightforward descent from $\hat{5}$ to $\hat{1}$. Like the first quotation, the second quotation begins and ends with different
octatonic subsets, again exhibiting incompleteness in both the tonal and octatonic interpretations, and again corresponds to Crumb's annotation "but falling."

The final quotation supplies the most convincing evidence for an octatonic reading because of Crumb's substantial alterations to Bach's fugue (see Figure 2.29 and Figure 2.30). In Bach's fugue, there are two chords that do not identify a specific octatonic collection: $\mathrm{ii}^{\mathrm{o6}}$ and $\mathrm{vii}^{\mathrm{o7}} / \mathrm{V}$ in m .29 . These are the only two places that Crumb alters the quotation. The $5^{\text {th }}$ of the $\mathrm{ii}^{\text {o6 }}$ in the tenor is raised, which changes it from a diminished triad to a minor triad that then passes through the original pitch to the root of the V chord, and the $7^{\text {th }}$ of the $\mathrm{vii}^{\mathrm{o} 7} / \mathrm{V}$ in the tenor is raised, which changes it from a fullydiminished seventh to a half-diminished seventh chord. These changes solidify each chords octatonic membership-the original diminished triad or diminished-seventh chord does not uniquely identify a specific octatonic collection. In Bach's fugue, a perfect authentic cadence completes the subject, which is the first strong cadence of the entire piece and is well beyond the limits of the exposition. In Crumb's version, this definitive tonal closing is excised but the notation "(attaining!)" above the staff indicates that this subject entry was successful. In the tonal interpretation, this supposed success is not apparent because the final unresolved dominant chord again thwarts closure. Unlike the first two quotations, which opened and closed in different octatonic collections and thus lacked a sense of closure, the final progression is framed by subsets of the same octatonic collection. Thus, the framing effect of the Oct ${ }^{3}$ subsets corresponds with Crumb's annotation "attaining!" While it is likely most listeners would hear the quotation of Bach's fugue as a tonal element transported into a post-tonal setting, the points of

Figure 2.29. Bach, Das Wohltemperierte Clavier, Vol. II, VIII, mm. 27-30


Figure 2.30. Crumb, Music for a Summer Evening, "Starry Night," p. 20, s. 1. Copyright © 1974 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

contact and departure between both environments are more readily found through a shift in perspective.

This sense of a shifting perspective is what characterizes the triadic progressions in "Come Lovely" (see Figure 2.6 and Figure 2.18). The major triad's appearance, symbolizing death, immediately awakens the tonal impulse. The fourth chord causes one
to question the tonal implications of the progression and the last three "dominantseventh" chords complete a whole-tone scale. The mix of diatonic and whole-tone elements is further complicated when the trichord-by-trichord octatonic element is introduced. The first three chords of each progression exhibit an almost straightforward example of a prolongation of tonic through a passing dominant. This motion from tonic to dominant and back to tonic is mirrored in the motion from one octatonic collection to another and back again. The progression of these collections corresponds to the tonal functions identified in Table 2.4. The first progression translated to its tonal function, assuming the first chord is tonic, would thus read $\mathrm{T}-\mathrm{D}-\mathrm{T}-\mathrm{PD}-\mathrm{T}-\mathrm{PD}-\mathrm{D}$. The same pattern obtains with each transposition of the progression and subsequent reinterpretation of the first chord as tonic. Because the progression is repeated two times, a representative of each octatonic collection occurs at least once at its respective position within the song. For example, the first chord of the first progression is a subset of Oct ${ }^{1}$; the first chord of the second, Oct ${ }^{3}$; the first chord of the third, Oct ${ }^{2}$. The final A-major chord of the incomplete fourth progression represents a return to the original octatonic collection that started the progression, which completes the cycle. In a tonal interpretation, the first and last triads are polar opposites, transpositionally related by a tritone; however, in the octatonic reading both chords belong to the same octatonic collection and thus represent completion.

### 2.5 Toward a New Tonality

The diversity of Crumb's music becomes apparent when examining analyses of his works where within even one work tonal and post-tonal methodologies are utilized. ${ }^{77}$ Such a mixture of analytic methods, which on the surface seem to stand in stark contrast to one another, is a reaction to Crumb's juxtaposition of diatonic and non-diatonic elements within a single work. Bass and Scotto have explored these connections to some extent: Bass investigates the special properties of the octatonic, whole-tone and diatonic collections whereas Scotto demonstrates how such structures can be related through aggregate partitions and transpositional combination with their constituent components related through various transformational networks. ${ }^{78}$ Bass's observations are very general, and point out interesting structural features when the interaction between different referential collections is considered. Scotto's analysis of Processional is highly individuated, and demonstrates a sense of the "psychological curve" of the piece but the process he uncovers is not applicable to Crumb's other works. ${ }^{79}$ In the analyses cited above, Crumb's music is either described too generally and thus fails to illuminate the processes of the individual work or is constantly reinvented, which seems to be at odds

[^38]with Crumb's preference for a more encompassing organic procedure that synthesizes the resources available to the contemporary composer.

There is evidence of routine and functional procedures in the music of Crumb. A limited variety of trichordal set-classes are often found within close proximity to one another and have proven useful in identifying specific transpositions of different referential collections. The principles of opposition and completion, as exhibited in Crumb's music, proved to be important in helping the analyst decide between competing representational choices of collections, and illustrated a possible way to understand completion in a post-tonal context. It is because of these correspondences to tonality-the prevalence of and correlation between trichords, emphasis on octatonic collections, opposition between harmonies and pattern completion-that I propose a model of octatonality for the music of Crumb.

## 3 A Model of Octatonality

The octatonic model to be developed in this chapter accounts for the features of Crumb's music discussed in the second chapter. Most importantly, the model will be applicable to many of Crumb's works, and will uncover recurring patterns at both surface and background levels of structure. At the same time, the individual character of a work will not be subsumed by the generality of the model and will be visible in diverse transformational patterns, key areas, and functional levels. The model will expand traditional metaphors used to describe music, such as "high" and "low," by introducing additional dimensions to the musical space; however, the model's representation will remain minimal and economical to enhance the significance of the transformational events. ${ }^{1}$ Aspects of tonal theory are utilized from a primarily Riemannian perspective and its subsequent revitalization in the work of Daniel Harrison. ${ }^{2}$ In addition, features of Darryl White's "Octatonic Metaphor"-although quite divorced from its original intention of explicating certain chromatic passages in tonal works-are adjusted to introduce the concept of hybrid chords within an octatonic system. ${ }^{3}$ The post-tonal

[^39]component of the octatonic model pays homage to Perle-Lansky cycles, Klumpenhouwer networks, neo-Riemannian theory, and set-class genera, although the similarity of the octatonic model to be developed here to these theories will be more conceptual than concrete. In general, I will approach the model of octatonality with a transformational attitude espoused by Lewin in Generalized Musical Intervals and Transformations. ${ }^{4}$

Nevertheless, such a transformational attitude will not overshadow various objects within the octatonic system, and the relative permanence of these objects will allow a sense of harmonic function to obtain. This balance between transformations and function in Crumb's music may be analogous to what Crumb believed was the "balance between the technical and intuitive aspects" of his music. ${ }^{5}$ The metaphor of octatonality, and its realization in a helical model of octatonic space, will be investigated for its internal consistency and its relationship to the above-mentioned theories that influenced the creation of the model. Unlike other analytic models used to analyze Crumb's music, the metaphor of octatonality can demonstrate consistent and functional motions in a large body of Crumb's works while at the same time highlighting the unique "psychological curve" of an individual piece. ${ }^{6}$

[^40]
### 3.1 Models and Metaphors

The use of metaphors to describe music is pervasive in writings about music. For example, the terms "high" or "low" are used to describe the different frequencies of pitches, or "thick" or "thin" to describe the "texture" or "density" or "spacing" of the pitches. Such terms have become so commonplace that they are what Richard Boyd would describe as "frozen metaphors." ${ }^{7}$ Boyd further explains:
[Frozen metaphors] seem to lose their insightfulness through overuse: the invitation to explore the various analogies and similarities between the primary literal subject and the metaphorical secondary subject becomes pointless or trite if repeated too often. Theory-constitutive scientific metaphors, on the other hand, become, when they are successful, the property of the entire scientific community, and variations on them are explored by hundreds of scientific authors without their interactive quality being lost. ${ }^{8}$

Schenkerian theory is an example of a theory-constitutive metaphor, which many theorists have utilized and modified. Variations on the Ursatz, expansion of prolongational techniques, and similar hierarchic notational practices used in both tonal and post-tonal analyses attest to the interactive quality of the theory. Janna Saslaw explores theory-constitutive metaphors through a study of Hugo Riemann's work and the role of body-derived image schemas in conceptualizing music through the conduit metaphor. ${ }^{9}$ Many musicians understand the metaphor key areas are containers, and, as

[^41]Saslaw points out, modulation to a new key is a bumpy path while return to the tonic is a smooth path. ${ }^{10}$ The path (modulation) between the containers (keys) and the forces acting upon the music between the containers (unique musical events) are effective metaphors for describing an otherwise abstract concept. This type of metaphor will also be utilized in the model of octatonality.

Adrian Childs extends the metaphor of key areas are containers to transformational networks are containers in his cubic transformational model for seventh chords. ${ }^{11}$ Within the larger metaphor of transformational networks are containers, seventh chords define the structural limits of the container or seventh chords are the structural foundation for the container. What is most interesting about Child's model is his use of space. Although illustrated in two dimensions, the implied third dimension adds a sense of perspective to the chords and the metaphor of transformations are a journey is enhanced by our basic understanding of geometry. The utilization of multiple dimensions is also a key feature in the octatonic model.

Transformational networks, such as Childs's cube, have become increasingly popular in analytic writing about music in part because they align with common conceptual metaphors and exemplify theory-constitutive metaphors not yet frozen
metaphor; however, I will use italics instead. The principle construction of these metaphors is targetdomain is source-domain.
${ }^{10}$ Saslaw, "Forces, Containers, and Paths," 229.
${ }^{11}$ See Adrian P. Childs, "Moving beyond Neo-Riemannian Triads: Exploring a Transformational Model for Seventh Chords," Journal of Music Theory 42 (1998): 181-93.
because of overuse. However, as Roeder states, "Much more needs to be said about the goals and principles that should guide the choice of object families and transformations, type of network, and visual layout in order to make analyses persuasive." ${ }^{12}$ Because there are no general guidelines for creating transformational networks, according to Buchler this has lead to the inability to pinpoint a good or bad transformational analysis as easily as a good or bad Schenkerian analysis. ${ }^{13}$ Roeder outlines general criteria for creating transformation networks, which informed the creation of the octatonic model. ${ }^{14}$ In general, a balance between economy of structure and the ability of the octatonic model to be relevant for many works will be maintained.

Crumb seems to have an aversion to overly detailed analyses of music; in particular, the types of analyses presented in Perspectives of New Music, at least prior to 1977. In an interview with Robert Shuffet, Crumb elaborates:

Very fussy analysis of this kind [those found in Perspectives of New Music] doesn't interest me at all. I've always felt that beyond a certain point, one has to think in musical terms entirely, and language - in fact - becomes a liability-a defective vehicle for describing what is actually occurring in the music. Music is much too subtle for analysis beyond a certain basic level. ${ }^{15}$

[^42]In 1965, Crumb published an article analyzing Peter Westergaard's Variations for Six Players in Perspectives of New Music. ${ }^{16}$ His analysis was brief—remaining true to his aversion of detailed analyses-and illuminated as much about Variations as it did about himself. ${ }^{17}$ In particular, Crumb commented on how the row used in the work-or in his words, "germ-series"-was present at different levels of structure and, according to Crumb, was "something aurally tangible rather than an abstraction whose relationships must be isolated by close analysis." ${ }^{18}$ The "germ metaphor" crops up often in Crumb's discussion of his own music. Responding to a question about his favorite composition, Crumb replied that Songs, Drones and Refrains of Death "seemed to remain closest and most faithful to the original germ of the idea." ${ }^{, 19}$ In a conversation with Shuffet, Crumb again mentions the concept of a germ:

Arriving at a form is, for me, the most difficult part of composition; once I have the basic germs-the cells-the basic inspirations (whatever those might be)-, I then elaborate these tiny elements into a larger sequence. This elaboration for me is the most difficult thing. In fact, that is composing: the real sense of the term composing is putting together little things to make something larger. ${ }^{20}$
${ }^{16}$ George Crumb, "Peter Westergaard: Variations for Six Players," Perspectives of New Music 3, no. 2 (1965): 152-59.
${ }^{17}$ Marion A. Guck explores such analytic investments and self-reflections in analysis in "Analytic Fictions," Music Theory Spectrum 16 (1994): 217-30.

18
Crumb, "Peter Westergaard," 159.

19
Strickland, "George Crumb," 173.
${ }^{20}$ Shuffet, "The Music," 409.

Crumb's use of the term "germ cell" seems to be aligned with Schenker's early idea of the motif, which "constitutes the only and unique germ cell of music as an art." ${ }^{\text {"21 }}$ Schenker describes the reciprocal relationship between the germ cell and tonality:

Basically, the two experiments [the discovery of the motif and tonality] are mutually dependent: any exploration of the function of the motif would, at the same time, advance the development of the tonal system, and, vice versa, any further development of the system would result in new openings for motivic association. ${ }^{22}$

Such a symbiotic relationship is perhaps what Crumb alluded to when he said that "I sense that it will be the task of the future to somehow synthesize the sheer diversity of our present resources into a more organic and well-ordered procedure." ${ }^{23}$ Although the following discussion of the model of octatonality may appear overly rational, the essence of the model relies upon the intuitive nature of the metaphors discussed above.

[^43]
### 3.2 Chords

The graphical representation of the proposed model resembles a helix wrapped around upon itself (see Figure 3.1). ${ }^{24}$ Each node represents a pitch class and the cylinders, which connect the nodes, represent the interval between the pitch classes. ${ }^{25}$ Each cylinder is rotated around the inner axis of the helix and is bounded. ${ }^{26}$ Connecting three or more adjacent nodes on the helix forms a chord. ${ }^{27}$ Pitch classes from five different rotations of an ordered Oct ${ }^{1}$ collection are aligned to the nodes of the five different rings on the helix. The rings, which are indicated by lines that vary in intensity from light grey to black, are labeled to the right of the model according to their relative position. The pitch-class content of these rings is included in tabular form in Table 3.1. ${ }^{28}$

[^44]

Table 3.1. Rings of the $\mathrm{Oct}^{1}$ helix

| Ring | Pitch Classes |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Upper | 0 | 4 | 9 | 1 | 6 | A | 3 | 7 |
| Outer | 4 | 9 | 1 | 6 | A | 3 | 7 | 0 |
| Lower | 9 | 1 | 6 | A | 3 | 7 | 0 | 4 |
| Lower-inner | 3 | 7 | 0 | 4 | 9 | 1 | 6 | A |
| Upper-inner | 7 | 0 | 4 | 9 | 1 | 6 | A | 3 |

In Table 3.1, the five different rotations of an ordered $\mathrm{Oct}^{1}$ collection are included in the rows and the columns represent eight pentagonal segments that partition the helix. Each ring is constructed using every fourth pitch of an octatonic scale, which alternate between interval-classes 4 and 5 in modulo-12 space. ${ }^{29}$ For example, the Oct ${ }^{1}$ scale ordered <0134679A> is rearranged into <04916A37>. ${ }^{30}$ This arrangement of pitch classes, $<04916 \mathrm{~A} 37>$, is found in the first row of Table 3.1 and to the left of the arrow indicating the upper ring in Figure 3.1, and continues clockwise around the upper ring in the figure. The next node counterclockwise around the helix's inner axis, the outside ring, begins the ordered collection <4916A370>, which is a rotation of the upper ring's

[^45]collection. All of the rings are organized in such a way that the successive interval array (SIA) between consecutive pitch classes, formed along the cylindrical route, is $<4564954659 \ldots 9>.^{31}$

The highest eight nodes that constitute the upper ring is one permutation of the ordered $\mathrm{Oct}^{1}$ collection, which, starting from pc 0 and read clockwise around the upperring, contains the pcs <04916A37>. A dotted line with an arrow indicates the correlation between the pitch classes in Table 3.1 and their orientation on the helical model shown in Figure 3.1. In Table 3.1, an array of pitches is formed by concatenating the pitches from the top cell of the first column downwards and then continuing with the top cell of the next column once the lowest row is reached. In Figure 3.1, the same pitch array is found following a counterclockwise motion along the cylinders around the helix's inner axis, which corresponds to a clockwise motion around the center of the bounded helix.

The trichords found in this model, using the above-mentioned SIA, include all the unique members of scs (037), (016), (025), (014), and (026) without duplication of a constituent set. The tetrachords include all the unique instances of scs (0258), (0146), and (0147), although some sets are duplicated. In Figure 3.1, joining the three lowest nodes in the foreground, <493>, results in an instance of (016). ${ }^{32}$ Using the same set and including the nearest counter-clockwise node, relative to the center of the bounded helix, would

[^46]result in <0493>, an instance of (0147). Extending the same trichord, <493>, the opposite direction would result in <4937>, an instance of (0146). All of the set classes contained within this model are the prominent set classes found in Crumb's music identified in the second chapter.

A skeletal view of the three-dimensional model with each node labeled generically is illustrated in Figure 3.2. The "inside view" is the perspective from within each pentagonal segment, in this case nodes 1.1 to 1.5 , which corresponds to the same nodes when viewing the helix from above. The number before the decimal place indicates the node's location within a pentagonal segment of the overall helix and the number after the decimal place indicate the node's location within each pentagonal segment. Therefore, a node labeled 2.5 is the second iteration of a pentagonal segment within the overall helix and is located in the fifth position, moving counterclockwise from 1, within the pentagonal segment. ${ }^{33}$

The labeling of nodes may be represented more abstractly: an "o" before a decimal place indicates that the digit that corresponds to the node must be an odd number; thus, node o. 1 would correspond to nodes $1.1,3.1,5.1$, and 7.1 since they all begin with odd numbers. ${ }^{34}$ An "e" before a decimal place indicates the digit must be an even number; thus, e. 1 refers to nodes $2.1,4.1,6.1$, and 8.1. An " $h$ " before a decimal place (short for helical) indicates that the digit may be either even or odd. Following a decimal place, an x will represent any possible number. The inside view shows a certain

[^47]Figure 3.2. Skeletal view of the octatonic model

perspective, looking directly through the center of one pentagonal segment in threedimensional space. Therefore, it appears that all sides are of equal length, whereas in three-dimensional space they are not. ${ }^{35}$ In addition, a break is added to the line segment between nodes $1.5-1.1$ in the inside view since the next node would actually be 2.1 .

The SIA illustrated in Figure 3.1 may be applied to each octatonic collection. One of the many possible orderings of each octatonic collection is listed in Table 3.2. The first column indicates the nodal position within the helix as illustrated in Figure 3.2. The next three columns contain the five rings of the $\mathrm{Oct}^{1}, \mathrm{Oct}^{2}$, and $\mathrm{Oct}^{3}$ collections respectively, which create an octatonic array. ${ }^{36}$ The SIA column indicates the interval formed by the directed intervals between consecutive pitch classes within each octatonic array. The interval entered in a cell in the SIA column corresponds to the directed interval of the corresponding pitch class immediately above the interval entry to the pitch class directly below the interval entry for each octatonic column. ${ }^{37}$ For example, in the $\mathrm{Oct}^{3}$ column the interval from A (node 1.3) to 7 (node 1.4) corresponds to the interval 9. A concatenation of two or more consecutive intervals would constitute a SIA. After ten successive intervals, the pattern repeats, indicated with shading in the corresponding cells.

[^48]Table 3.2. Ordered arrays of the $\mathrm{Oct}^{1}$, $\mathrm{Oct}^{2}$, and $\mathrm{Oct}^{3}$ models

| N | Oct ${ }^{1} \mathbf{R}^{1<65>}$ | Oct ${ }^{2} \mathbf{R}^{9<65>}$ | Oct ${ }^{3} \mathrm{R}^{\text {B<65> }}$ | SIA | Trichord | Tetrachord |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.1 | 1 | 9 | B | 6 | (037) | (0258) |
| 1.2 | 7 | 3 | 5 | 6 | (026) | (0258) |
| 1.3 | 0 | 8 | A | 5 | (016) | (0146) |
| 1.4 | 9 | 5 | 7 | 9 | (025) | (0146) |
| 1.5 | 1 | 9 | B | 4 | (014) | (0146) |
| 2.1 | 6 | 2 | 4 | 5 | (037) | (0147) |
| 2.2 | 0 | 8 | A | 6 | (016) | (0147) |
| 2.3 | 4 | 0 | 2 | 4 | (026) | (0146) |
| 2.4 | 1 | 9 | B | 9 | (014) | (0146) |
| 2.5 | 6 | 2 | 4 | 4 | (025) | (0146) |
| 3.1 | A | 6 | 8 | 4 | (037) | (0258) |
| 3.2 | 4 | 0 | 2 | 5 | (026) | (0258) |
| 3.3 | 9 | 5 | 7 | 5 | (016) | (0146) |
| 3.4 | 6 | 2 | 4 | 9 | (025) | (0146) |
| 3.5 | A | 6 | 8 | 4 | (014) | (0146) |
| 4.1 | 3 | B | 1 | 5 | (037) | (0147) |
| 4.2 | 9 | 5 | 7 | 4 | (016) | (0147) |
| 4.3 | 1 | 9 | B | 4 | (026) | (0146) |
| 4.4 | A | 6 | 8 | 5 | (014) | (0146) |
| 4.5 | 3 | B | 1 | 5 | (025) | (0146) |
| 5.1 | 7 | 3 | 5 | 4 | (037) | (0258) |
| 5.2 | 1 | 9 | B | 6 | (026) | (0258) |
| 5.3 | 6 | 2 | 4 | 9 | (016) | (0146) |
| 5.4 | 3 | B | 1 | 4 | (025) | (0146) |
| 5.5 | 7 | 3 | 5 | 5 | (014) | (0146) |
| 6.1 | 0 | 8 | A | 5 | (037) | (0147) |
| 6.2 | 6 | 2 | 4 | 4 | (016) | (0147) |
| 6.3 | A | 6 | 8 | 9 | (026) | (0146) |
| 6.4 | 7 | 3 | 5 | 9 | (014) | (0146) |
| 6.5 | 0 | 8 | A | 4 | (025) | (0146) |
| 7.1 | 4 | 0 | 2 | 4 | (037) | (0258) |
| 7.2 | A | 6 | 8 | 6 | (026) | (0258) |
| 7.3 | 3 | B | 1 | 9 | (016) | (0146) |
| 7.4 | 0 | 8 | A | 9 | (025) | (0146) |
| 7.5 | 4 | 0 | 2 | 4 | (014) | (0146) |
| 8.1 | 9 | 5 | 7 | 5 | (037) | (0147) |
| 8.2 | 3 | B | 1 | 6 | (016) | (0147) |
| 8.3 | 7 | 3 | 5 | 4 | (026) | (0146) |
| 8.4 | 4 | 0 | 2 | 5 | (014) | (0146) |
| 8.5 | 9 | 5 | 7 | 5 | (025) | (0146) |

In a similar manner, set-class designations for three consecutive pitches are entered in the trichord column such that (016), found in the row-header labeled 1.3, corresponds with the set $\{\mathrm{AB} 5\}$ found in nodes 1.1-1.3 in the $\mathrm{Oct}^{3}$ column, as well as the corresponding trichords in the Oct ${ }^{1}$ and Oct ${ }^{2}$ columns. The same method is used for identifying the set-class membership of the tetrachords; that is, the row of the set-class designation corresponds with the pitch class in the same row of any octatonic column and the pitch classes contained three rows above it. As the cyclical nature of the threedimensional model suggests, the nodal positions wrap around on themselves such that node 1.1 comes after node 8.5. Therefore, set-class designations for trichords in rowheaders labeled 1.1 and 1.2 correspond with the pitch classes at nodes $8.4-1.1$ and 8.51.2 respectively. For set-class designations of tetrachords, the row-headers labeled 1.1, 1.2, and 1.3 correspond with the pitch classes at nodes $8.3-1.1,8.4-1.2$, and $8.5-1.3$ respectively. Because of the alternating intervals between odd- and even-numbered pentagonal segments within the helix, a distinction can be made between chords that share an identical set class, but are not transpositionally related. Using the arrays shown in Table 3.2, major triads are found on nodes e.4-o.1 whereas minor triads are found on nodes o.4-e.1. Table 3.3 itemizes these positions based upon the orderings in Table 3.2. ${ }^{38}$ In the first column of Table 3.3, the generic positions of the trichords are listed and in the second column, the corresponding set class. In the third column, the corresponding SIA from the model is listed to distinguish between the inversionally related ordered sets. In the fourth column, the SIA is used to describe the unordered sets in normal form.

[^49]Table 3.3. Positions of trichords in the octatonic model

| N | Set Class | Model SIA | Normal Form <br> Set SIA |
| :---: | :---: | :---: | :---: |
| e.2-e. 4 | (014) | <49> | <13> |
| 0.3-0.5 |  | <94> | <31> |
| 0.5-e. 2 | (016) | <56> | <15> |
| 0.1-0.3 |  | <65> | <51> |
| e.3-e. 5 | (025) | <95> | <32> |
| 0.2-0.4 |  | <59> | <23> |
| e.5-o.2 | (026) | <46> | <24> |
| e.1-e. 3 |  | <64> | <42> |
| e.4-o. 1 | (037) | <54> | <43> |
| o.4-e. 1 |  | <45> | <34> |

Instances of sc (037) always occupy the same position in every pentagonal segment, alternating between instances exhibiting an SIA on the model of <54> and <45> respectively (minor and major triads). With the exception of instances of sc (037), instances of the other trichordal set classes are asymmetrically placed on the helix. A comparison of the other trichords at the same even- and odd-numbered locations on the helix highlights an interesting pattern. Instances of sc (014) with the model SIA <49> correspond with instances of sc (025) with the model SIA <59> on nodes h.2-h. 4 whereas instances of sc (014) with the model SIA <94> correspond with instances of sc (025) with the model SIA <95> on nodes h.3-h.5. Similarly, instances of sc (026) with the model SIA <46> correspond with instances of sc (016) with the model SIA <56> on nodes h. $5-\mathrm{h} .2$ whereas instances of sc (026) with the model SIA <64> correspond with instances of sc (016) with the model SIA <65> on nodes h.1-h.3. Thus, considering the geometrical model, instances of scs (014) and (016) share a common nodal position with instances of scs (025) and (026) respectively.

The addition of an adjacent node to any one of the trichords on the helix creates one of three different tetrachordal set classes. Unlike the trichords, which include every member of the set class, the tetrachords include only half of the possible tetrachordal sets of a given set class (see Table 3.2). The nodal positions of these tetrachords within the octatonic model are listed in Table 3.4. Whereas the trichords included both transpositionally and inversionally related sets, tetrachords utilize only one form of each set class. Thus, while an instance of sc (0146) may be found on three different nodal positions within a pentagonal segment, each set uses the <132> SIA when configured in normal form and not the inversionally-related <231> SIA.

Table 3.4. Positions of tetrachords in the octatonic model

| N | Set Class | Model SIA | Normal Form Set SIA |
| :---: | :---: | :---: | :---: |
| h.5-h. 3 | (0146) | <564> | <132> |
| h.1-h. 4 |  | <649> |  |
| h.2-h. 5 |  | <495> |  |
| o.3-e. 2 | (0147) | <945> | <331> |
| o.4-e. 3 |  | <456> |  |
| e.3-o.2 | (0258) | <954> | <332> |
| e. $4-0.3$ |  | <546> |  |

Another important difference between the trichords and tetrachords found in the model is the exclusivity of sets: a specific trichord can only be found at one location on the entire octatonic model while a specific tetrachord can be found at more than one location. For example, $\{\mathrm{AB} 24\}$ is found in the $\mathrm{Oct}^{3}$ column at nodes 1.5-2.3, 2.1-2.4 and 2.2-2.5 (cf. Table 3.2). This is because of the repetition of pitch classes surrounding the trichords. In fact, joining nodes 1.5-2.5 would still create the set $\{\mathrm{AB} 24\}$ or, accounting
for the repetition of pitches, create the multi-set $\{\mathrm{ABB} 244\}$. This overlap is present for the other two tetrachordal set classes; however, only one pitch is duplicated rather than two, which creates pentachordal, rather than hexachordal, multi-sets of set classes (0147) and (0258). It may appear that the inclusion of multiple instances of the same set within the model may be overgenerous and therefore the elegance or the economical structure of the model deteriorates; however, the usefulness of multi-sets will be demonstrated in the analyses of A Haunted Landscape and "Come Lovely" in the following chapters.

There are a number of conceptual metaphors implied in the above discussion. Foremost is the metaphor that pitch classes are objects which may be at odds with the traditional perception of a pitch class as a category which contain all representatives of pitches with similar names in mod-12 chromatic space; however, the extrapolation of the concept of pitch class into an object is necessary to show transformations of these objects. To a lesser extent, the metaphor nodes are containers is implied by the skeletal view of the model, which generalizes the octatonic model, and will be expanded upon later. Creating a chord from consecutive nodes draws upon the metaphor that chords are regions in octatonic space and chords are containers. While the latter metaphor is commonplace in theoretical discourse, the former metaphor is perhaps less familiar and will be explored in section 3.7.

The discussion of chords in the model has been primarily reserved for trichords and tetrachords for a number of purely pragmatic reasons. First, specific instances of dyads are repeated throughout the model. Second, no dyad found in the model can form a multi-set by including an adjacent node. Third, in Crumb's music, a dyad is usually better interpreted as a subset of a trichord or tetrachord. Finally, in Crumb's music, sets and
multi-sets with a cardinality of greater than four within an octatonic mode are relatively rare and can often be interpreted as trichords and tetrachords. Although discussion of these types of chords has been brief, their inclusion in the model and subsequent transformations will be discussed in sections 3.5 and 3.8.

### 3.3 Modes, Keys, and Key Areas

In his dissertation, William Richards proposes a model of generic set-class space for music of Stravinsky that evinces "idiosyncratic expressions of serial techniques intermixed with non-serial linear constructions, and the commingling of diatonic and non-diatonic pitch objects." ${ }^{, 39}$ Such a statement aptly describes the progression in "Come Lovely" and many of Crumb's works more generally. Richards's model relies heavily on aspects of Allen Forte's and Richard Parks's independently conceived theories of pitchclass set genera. ${ }^{40}$ Richards chooses three genera based upon the diatonic, chromatic, and octatonic collections, which are defined by the scs (013568A), (0123456), and ( 0134679 A ) respectively. ${ }^{41}$ He illustrates how each of the genera may be mapped onto
${ }^{39}$ William H. Richards, "Transformation and Generic Interaction in the Early Serial Music of Igor Stravinsky," (Ph.D. diss., University of Western Ontario, 2003), iii.
${ }^{40}$ Forte, "Pitch-Class Set Genera" 187-270; Richard S. Parks, "Pitch-Class Set Genera: My Theory, Forte's Theory," Music Analysis 17 (1998): 206-26.
${ }^{41}$ Richards includes only seven pitch classes as representative of the chromatic genera since it is "equal or near-equal" to the size of the diatonic and octatonic genera; Richards "Transformation and Generic Interaction," 76.
and into each other using various transformational processes. ${ }^{42}$ According to Richards, "Each genus is defined by a referential collection, a progenitor trichordal sc, a collection of primary members, and a collection of characteristic members., ${ }^{43}$ The referential collection would be either the diatonic, chromatic, or octatonic genera listed above. Although his model favors the genera model and preference rules put forward by Parks, the "progenitor trichordal sc" is related to Forte's theory in which each of the twelve trichordal set classes determines each of the twelve genera. ${ }^{44}$ The collection of primary members is comprised of those set classes that are subsets or supersets of the referential collection. The collection of primary members is quite expansive and may be shared between the different genera; however, the number of characteristic members is much more limited. Based upon Parks's eighth preference rule, and expanded by including Forte's progenitor set class, Richards lists four criteria for a characteristic member of a genus: 1) it must be a subset or superset of the referential collection and contain a welldefined progenitor set class; 2) all the characteristic members must be subsets or supersets of each other; 3) there is some uniformity of interval-class distribution within

[^50]the interval-class vectors of the characteristic sets; 4) there is some uniformity in interval patterns within the SIAs of the characteristic sets. ${ }^{45}$

Although this may only seem to be a classification system, Richards is able to explain continuities and discontinuities in Stravinsky's music using generic transformational networks based upon the three genera. Whereas the transformations and networks change to reflect the different pieces he analyzes, the presence of the three genera provides a foundation for understanding more than just one of Stravinsky's works. Richards admits, "Pitch-class set theory, however, does not predict functional relationships among pitch classes. The potential of pitch-class set theory as a means of explanation lies in its ability to draw pitch objects into abstract associations without relying on an a priori functional model. ${ }^{36}$ While functional models may be, at times, contrived independently from experience, there is no reason a functional model cannot be based upon experience, which Richards's statement seems to imply. Even Richards's generic model of three genera might initially be considered as an a priori construct, except that it was based upon his experience of the music. In the same way, the "chord of nature" and Schenker's Urlinie and Bassbrechung might be considered a universal construct, but his theories were also an explanation of his experience with the music he admired. The transformations used to relate segments within Richards's model, which are

[^51]based upon mathematics and logic, are a priori knowledge, whereas the model itself is representative of a posteriori knowledge gained through the a priori knowledge of mathematics and logic. Because knowledge, especially within theoretical discussions about music, seems to be entangled between "what is known" and "what becomes known," the argument that knowledge exists somewhere between these two categories seems most plausible. Because of this, the proposed model of octatonality will incorporate both a priori and abstract concepts.

The ordered arrays listed in Table 3.2 suggest a notion of mode. In this context, it will be used to refer to a specific ordering of an octatonic collection's array. ${ }^{47}$ As will be discussed later, order and function are important aspects of the model and thus mode is perhaps a more suitable term than other terms considered such as referential collection or genera. Each of the octatonic modes listed in Table 3.2 is aligned such that each array's SIA is equivalent; however, because the same SIA is repeated every ten nodes, indicated by shading in Table 3.2, there are four possible ways to arrange each of the octatonic modes to align the respective SIAs. ${ }^{48}$ The nomenclature used to describe the octatonic modes is as follows: $\mathbf{R}$ is appended to the octatonic collection designation to identify the specific Rotation of the array. The first superscript following $\mathbf{R}$ indicates the pitch class occupying the 1.1 node of the mode and the two superscripts in angle brackets indicate

[^52]the interval between the first and second pitch classes and second and third pitch classes of the mode respectively. The use of the first pitch class and of the first two intervals is necessary to uniquely identify each mode because of the repetition of pitch classes within the array (five in total) and the repetition of the SIA every 10 nodes (four in total). For example, Oct $^{1} \mathrm{R}^{1<65>}$ is the octatonic mode which is based upon the hyper-aggregate constructed from the Oct ${ }^{1}$ collection such that pc 1 is in nodal position 1.1 and the interval between the first and second pitch, 1 and 7 , is 6 and the interval between the second and third pitch, 7 and 0 , is 5 . Subsequent iterations will use the same SIA, which repeats every 10 nodes, of < $6594564954>$.

The governing octatonic mode, which will be contextually determined, will determine the octatonic key; thus, the octatonic key is the most structurally significant octatonic mode. ${ }^{49}$ A key is notated the same as a mode, but to distinguish between the two in analytic figures the key notation will be enclosed in a box. A key area is a merger of two or three octatonic modes. Whereas in tonality, the terms key and key area are virtually interchangeable locutions, in octatonality this is not the case. A reversal of the metaphorical terminology—applying an octatonic key area to a tonal key area-would be equivalent to the interpretation that a particular tonal sonata has the key area of C major, G major, and A minor. For example, a hypothetical octatonic piece may employ the modes Oct $^{1} \mathrm{R}^{1<65>}$ and $\mathrm{Oct}^{2} \mathrm{R}^{9<65>}$. The correlation between octatonic modes can be consolidated into one expression by the notation $\mathrm{Oct}^{1,2} \mathrm{R}^{1,9,665>}$. The first two superscripts separated by commas after "Oct" identify which octatonic modes merge; the second two

49 The definitions of key and mode will be expanded upon in sections 3.4 and 3.7.
superscripts separated by commas after " $R$ " identify which rotations of the respective modes merge; the SIA is the same for both modes and is therefore not duplicated. If, based upon a contextual analysis, it was determined that the mode $\mathrm{Oct}^{2} \mathrm{R}^{9665>}$ was the most structurally significant, it would be notated first (e.g. Oct ${ }^{2,1} \mathrm{R}^{9,1<65>}$ ). In cases where the octatonic key area includes modes from all three octatonic collections, the indices of the other collections will be appended such that it follows the cyclical order 1-2-3-1... . For example, if $\mathrm{Oct}^{2} \mathrm{R}^{9<65>}$ is the key, then a key area might be represented as Oct $^{2,3,1} \mathrm{R}^{9, \mathrm{~B}, 1<65>}$. The significance of keys and key areas will be discussed in detail in the following sections.

This theory of octatonic modes and keys departs significantly from Forte's setclass genera since a progenitor set is not utilized. The theory more closely resembles Parks's set-class genera because of the reliance on successive interval arrays, referential collections, and interval vectors. The concept of octatonic key areas aligns most closely with Richards's model, which is limited to the diatonic, chromatic, and octatonic genera. The octatonic model primarily differs from Richards's because of the importance placed upon the ordered octatonic array; however, it is similar in that it is an a priori construction based upon my experience with the music.

The conceptual metaphors presented in the above discussion rely heavily upon tonal metaphors. Key areas and modes are containers is a metaphor that is featured prominently throughout the section and is similar to how one views keys and modes in tonality. In addition, the construction of the model is based upon a repeating base SIA that implies two natural metaphors: the base SIA is a germ and the octatonic key is a reproduction of the germ. The metaphorical implication, however, that an octatonic key
is in any way organic or related to nature should be supplanted with the notion that an octatonic key is a metaphorical extension of the theory-constitutive metaphor of tonality.

### 3.4 Hybrid Chords and Families

The equivalency between the four different rotations of each octatonic mode is borne from the method in which the order of each mode was constructed: each ring is created using every fourth pitch of an octatonic scale, which alternate between intervalclasses 4 and 5. Because each octatonic mode's SIA is identical, the individual octatonic modes can be aligned in a generic multi-dimensional octatonic system. In tonality, the same type of concept applies when considering the tonic of a given key. In C major, pc 0 resides within the category of tonic and in G major, pc 7 also resides within the same category. In the model of octatonality, each node acts as a category.

A hybrid chord involves the merger of two or three octatonic modes such that one node represents the pitch classes of each octatonic mode. ${ }^{50}$ Consider the diatonic collections of C major and G major arranged in a method similar to the way rotations of the octatonic modes were arranged (see Table 3.5). Each diatonic collection is arranged such that the tonic is at position 1.1 and each subsequent pitch is a third higher. The SIA demonstrates the alternation between minor and major thirds in the same way the SIA was shown in Table 3.2. Triads and seventh chords that result from the union of nodes

[^53]Table 3.5. Two diatonic collections arranged within a heptagonal segment

| Position | C Major | G Major | SIA | Triad | Seventh Chord |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1.1 | C | G | 4 | IV | $i i^{7}$ |
| 1.2 | E | B | 3 | vi | IV ${ }^{7}$ |
| 1.3 | G | D | 3 | I | $\mathrm{vi}^{7}$ |
| 1.4 | B | F\# | 4 | iii | $\mathrm{I}^{7}$ |
| 1.5 | D | A | 3 | V | iii ${ }^{7}$ |
| 1.6 | F | C | 3 | vii | $\mathrm{V}^{7}$ |
| 1.7 | A | E | 4 | ii | vii ${ }^{\text {¢7 }}$ |

within a single mode are identified with Roman numerals, instead of pitch-set notation, and the Roman numeral corresponds to the pitches in the same manner as the set classes in Table 3.2. Joining two corresponding nodal positions from each mode would create what White might term either inherently polysemic or hybrid chords. ${ }^{51}$ Those that are inherently polysemic naturally occur in each respective collection. For example, the polysemic chord $\langle\mathrm{CEGB}\rangle$ is created through the union of nodes $1.1-1.2$ between C and G major, which is a $\mathrm{I}^{7}$ chord in the key of C major and a $\mathrm{IV}^{7}$ chord in the key of G major (hybrid chords are not shown in the table). A hybrid chord with respect to C major is formed by merging the nodes $1.3-1.4$ or $1.4-1.5$ of $C$ and $G$ major since $F \#$ is not an element of the C-major collection. In a similar manner, a hybrid chord with respect to G major is formed through the union of nodes $1.5-1.6$ or $1.6-1.7$ of C and G major since Fq is not an element of the G-major collection.

[^54]The merger of corresponding nodal locations of different octatonic modes obtains inherently polysemic and hybrid chords within the octatonic model. The four different rotations of each octatonic mode with an initial SIA of <54> are included in Appendix 3. Depending on which rotations are used, different families of polysemic and hybrid chords are obtained through the combination of two or three octatonic modes. Appendix 4 provides an overview of the shared set classes in each hybrid family and Appendix 5 lists the sets and set classes contained through the nodal union of different octatonic modes. The first column in Appendix 5 indicates the requisite number of nodes required to produce the set class in the second column. An asterisk after the set class indicates that a particular instance of the set class is duplicated more than once within the key area. The third column includes the interval-class vector for each set class. The third to sixth columns indicate the sets obtained by the union of octatonic modes in a key area. ${ }^{52}$ The notation of the key area in the column header is generalized because the same sets obtain given a certain configuration of octatonic modes. For example, the sets obtained through the union of modes $\operatorname{Oct}{ }^{1} \mathrm{R}^{0<54>}$ and $\mathrm{Oct}^{2} \mathrm{R}^{\mathrm{B}<54>}$ are the same sets obtained through the union of modes Oct ${ }^{1} \mathrm{R}^{3<54\rangle}$ and Oct $^{2} \mathrm{R}^{2<54\rangle}$. Since the rotational index of the $\mathrm{Oct}^{2}$ mode is one less than the rotational index of the $\mathrm{Oct}^{1}$ mode, the generic interpretation is notated as Oct $^{1,2} \mathrm{R}^{\mathrm{x}, \mathrm{x}-1<\mathrm{yz}\rangle}$.

The union of different octatonic modes can be divided into hybrid families.
Membership within a certain hybrid family is based upon the difference in the rotational index value of the requisite modes, which also corresponds to values within the interval

[^55]vector of the set classes found within the family. For example, the Hybrid 1 family (H1) has rotational indices that differ by $+/-1$. Thus the above example, $\mathrm{Oct}^{1,2} \mathrm{R}^{\mathrm{x}, \mathrm{x}-1<\mathrm{yz}\rangle}$, is a member of the H 1 family. The superscript notation appended to the hybrid family designation indicates the number of modes and nodes used respectively. For example, $\mathrm{H} 1^{2,3}$ indicates the union of two modes and three nodes in the H 1 family. ${ }^{53}$ The title of each table within Appendix 5 includes " $n$ " as a superscript, which changes based upon the number of nodes in the first column of each table. The resulting influence found in the interval vector is not surprising. For example, in the $\mathrm{H} 1^{2,2}$ family, ic 1 is the most common and numerous interval class. This generally obtains for all of the other hybrid families, although as the number of modes and nodes increases, the distinctness of each hybrid family decreases.

Hybrid families that utilize three octatonic modes have more than two rotational indices to compare and, in most cases, the indices are not equal. For example, the generic key area Oct $^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+2, \mathrm{x}+1<\mathrm{yz}>}$ has indices of $\mathrm{x}, \mathrm{x}+2$, and $\mathrm{x}+1$. Taking the indices as a set class, $(x, x+1, x+2)$ or (012), the interval vector which describes the relationships between the indices would be [210000]. This is reflected in the two-part hybrid name H112. The integer 1 is listed first because of its relative frequency. The interval vectors of the set classes contained within the $\mathrm{H} 1 \backslash 2^{3,2}$ family also reflect this since ic 1 is more common and numerous that ic 2 . The only three-mode hybrid family that does not require a slash notation is $\mathrm{H} 4^{3, \mathrm{n}}$ because of the symmetrical division of the octave by ic 4 . The

[^56]Hybrid $1 \backslash 4 \backslash 5$ family equally weights all of the rotation indices and corresponding interval classes; however, it is divided into the A and B families because they contain inversionally related sets. For example, the generic key areas of $\mathrm{Oct}^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-4, \mathrm{x}-5<\mathrm{yz}>}(\mathrm{A})$ and $\mathrm{Oct}^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+5, \mathrm{x}+4<\mathrm{yz}>}(\mathrm{B})$ both have indices that project sc (015); however, (A) in normal form is $\{x-5, x-4, x\}$ while (B) is $\{x, x+4, x+5)$. This difference is manifest in inversionally-related hybrid families with inversionally-related sets.

The four set classes in each two-mode-two-node hybrid family may be further divided into two groups: referential and true hybrids. For example, in $\mathrm{H} 4^{2,2},\{0347\}$ is a subset of the Oct ${ }^{1}$ collection. Using White's terminology, $\{0347\}$ is a hybrid chord with respect to Oct ${ }^{2}$. For this group of set classes, each hybrid chord has only one possible nodal position in the octatonic key area. The set classes in the second group are not found as subsets in any octatonic collection. These are hybrids regardless of the reference set. For this reason, an important distinction should be made between the types of hybrids: a set within the first group might be classified as a "referential hybrid" while a set within the second group would be a "true hybrid." ${ }^{\text {" }} 4$ While referential hybrids may be a subset of an octatonic collection, they-and true hybrids-are not present as consecutive pitches within an octatonic mode. Hybrid families that include greater than two modes or nodes all contain true hybrid chords.

Generally, referential and true hybrid chords will operate, and therefore will be notated, on a different functional level than the chords found within one octatonic mode.

[^57]In Crumb's music, hybrid chords either destabilize an operative key or transition between keys, which will be reflected in the notation of the key area. For example, $\{027\}$ is a true hybrid from the H 5 family occurring through the union of Oct $\mathrm{R}^{1} \mathrm{~A}\left\langle 54>\right.$ and $\mathrm{Oct}^{3} \mathrm{R}^{5<54>}$. If this hybrid was functioning as a pivot chord to modulate from $\mathrm{Oct}^{3} \mathrm{R}^{5<54>}$ to $\mathrm{Oct}^{1} \mathrm{R}^{\mathrm{A}\langle 54>}$, the respective superscripts will be reversed to reflect this using the notation $\mathrm{Oct}^{3,1} \mathrm{R}^{5, \mathrm{~A}<54>}$. A set may be a hybrid between more than one pair of octatonic modes. In this case, the hybrid that makes contextual sense should be chosen. For example, $\{0347\}$ can be found as a hybrid between $\mathrm{Oct}^{1} \mathrm{R}^{\mathrm{A}\langle 54>}$ and $\mathrm{Oct}^{2} \mathrm{R}^{6<54>}$ (nodes 4.5-5.1) or Oct ${ }^{1} \mathrm{R}^{\mathrm{A}<54>}$ and Oct $^{3} \mathrm{R}^{2<54>}$ (nodes 3.5-4.1); if the material before and after the hybrid is from Oct ${ }^{1}$ and Oct ${ }^{2}$ respectively, then the notation Oct $^{1,2} \mathrm{R}^{\mathrm{A}, 6<54>}$ would make more contextual sense. ${ }^{55}$ Thus, when notating key areas of hybrid chords the overarching octatonic key need not be notated first. In less clear situations, as is the case in most analyses, a choice will be have to be made that should reflect some larger pattern.

As was previously mentioned, some hybrid chords are represented only once in the union of two or more modes, while others are represented more than once. ${ }^{56}$ Therefore, when deciding upon which hybrid chord to partake in a transformation, choose those that are closer together on the model. ${ }^{57}$ As will be shown in the analyses of $A$ Haunted Landscape and "Come Lovely" in the following chapters, the determination of

[^58]the operative hybrid family and the octatonic key areas of the excerpt are inextricably linked.

Because hybrid chords reside in multiple dimensions, there was a heavier reliance on metaphor. In particular, the metaphor nodes are containers, which was described in section 3.2, is particularly important here. Not only does a node contain a pitch class but also contains multiple pitch classes from each octatonic mode. Therefore, the metaphor can be revised as follows: nodes are containers of pitch classes from multiple octatonic keys. Whereas chords are regions in an octatonic mode, hybrid chords are multidimensional regions in octatonic key areas. The influence of tonality is found again in the metaphor hybrid chords are transitional chords or hybrid chords are pivot chords, and evidence of these associations will be presented in later chapters.

### 3.5 T Transformations

T transformations are equivalent to the traditional operation of transposition but transferred to the octatonic model. Within the model, comprised of 40 nodes representing 5 different rotations of the octatonic collection, certain set classes always appear in the same nodal positions (see Table 3.3 and Table 3.4). For example, instances of sc (037) always appear in the outer three nodes of the helix in Figure 3.1. For this reason, the helix can be further subdivided into pentagonal segments comprised of five nodes, one node from each ring. The T-transformation $\mathrm{T}_{x} Y$ is notated in two parts: the subscript $x$ indicates the transformation of the chord from one pentagonal segment to the next and $Y$ indicates the transformation within a pentagonal segment. The preference for using normal case to
indicate motion within a pentagonal segment is based upon a preference to emphasize the transformations within a pentagonal segment over transformations between pentagonal segments, which are notated using subscript case.

Consider a trichord from one octatonic mode, found on nodes 1.1-1.3, which is transposed to nodes 2.1-2.3 (see Figure 3.3; the nodes of the two trichords have been filled in). To realize the transposition, first transpose the chord between pentagonal segments and then transpose the chord within a pentagonal segment. Since the digits before the decimal change by 1 (indicating a change between pentagonal segments) and the digits after the decimal change by 0 (indicating no relative change within each respective pentagonal segment), the corresponding transformation is notated $\mathrm{T}_{1} 0$. Additionally, the smallest integer to represent the motion within a pentagonal segment will be preferred over the smallest integer to represent the motion between pentagonal segments. For example, the transformation between nodes 1.1-1.3 to nodes 1.5-2.2 would be notated as $\mathrm{T}_{1}-1$ rather than $\mathrm{T}_{0} 4$ (see Figure 3.4). First, the trichord found on nodes $1.1-1.3$ is transposed to nodes $2.1-2.3$ via $\mathrm{T}_{1} 0$ and then transposed from nodes $2.1-$ 2.3 to nodes $1.5-2.2$ via $\mathrm{T}_{0}-1$; resulting in a net transformation of $\mathrm{T}_{1}-1$. In this case, the transformation $\mathrm{T}_{0}-1$ occurs across pentagonal segments but does not exceed the span of a generic five-node segment. The preference for using the smallest integer will also extend to the subscript notation if there is no change to the digit trailing the decimal point of the nodal designation. For example, the transformation between nodes $1.1-1.3$ to nodes $8.1-$ 8.3 would be $\mathrm{T}_{-1} 0$ rather than $\mathrm{T}_{7} 0$. In cases where the transformation would be $\mathrm{T}_{4} 0$ or $\mathrm{T}_{-4} 0$, the preference for notation will depend on the context.

Figure 3.3. $\mathrm{A}_{1} 0$ transformation


Figure 3.4. $\mathrm{A}_{1}-1$ transformation


If one set is expanded or contracted to become another set of a greater or lesser cardinality, a subscript corresponding to the increase or decrease in cardinality will be appended to the transformational notation in the form of $\mathrm{T}_{\mathrm{x}} Y_{z}$. The order of operations for these transformations are: 1) transpose the first set to the second set such that the node furthest clockwise of the first set aligns with the node furthest clockwise with the second set in relation to the overall helix as viewed from above; 2) subtract or add the required number of nodes furthest counter-clockwise on the respective set in relation to the overall helix as viewed from above. For example, a transformation from nodes 1.1-1.3 to nodes 1.5-2.3 would be notated $\mathrm{T}_{1} 0_{1}$ (see Figure 3.5). The inverse transformation from nodes $1.5-2.3$ to nodes $1.1-1.3$ would be notated $\mathrm{T}_{-1} 0_{-1}$ (see Figure 3.6). If there is no change in cardinality, no subscript is necessary.

Figure 3.5. $\mathrm{A} \mathrm{T}_{1} 0_{1}$ transformation


Figure 3.6. $\mathrm{A}_{-1} 0_{-1}$ transformation


The T transformations discussed thus far have only dealt with transformations within one octatonic key. A T transformation can take place between hybrid chords that reside in the same multi-dimensional key; however, if the hybrid chords are in different multi-dimensional key areas a hyper-T transformation will be required. Hyper-T transformations are also required between chords within an octatonic mode moving to a chord within a different octatonic mode or between chords within an octatonic mode moving to a hybrid in a multi-dimensional octatonic key area. These types of transformations will be discussed in section 3.8.

A T transformation relies primarily upon the metaphor that $T$ transformations are spiral journeys; however, the notation of the transformation in two parts breaks the spiral nature of the transformation. Instead, the first part of the transformation (the $x$ of $\mathrm{T}_{x} Y$ ),
framed as a conceptual metaphor, would be $x$ is a warp between segments in octatonic space while the second part of the transformation could be framed as $Y$ is a journey within a segment in octatonic space. These two conceptual metaphors contained within the notation of a two-part transformation should not obscure the cyclical nature of the model; the two-part transformation merely simplifies transformations in a mod-40 universe to highlight features within the octatonic model and, as will be shown in the fourth and fifth chapters, characteristic gestures in Crumb's music.

### 3.6 ROT Transformations

Although the nodal numbering system and its relationship to the octatonic modes were arbitrary (see Table 3.2), when analyzing Crumb's music certain configurations of octatonic arrays are preferable. The importance of rotations of octatonic modes is somewhat analogous to the key and mode system of tonality. For example, the $D^{B}$ collection contains all the pitch classes that comprise C major. If these pitch classes were ordered $<024579$ B $>$, and through a contextual analysis of a given piece of music it was determined that pc 0 was $\hat{1}$, then the work in question might be considered to be in C major. However, if these pitch classes were ordered <24579B0> and it was determined by the above criteria that pc 2 was $\hat{1}$, then the work in question would be in the Dorian mode of C major. These collections have identical pitch content; however, the characteristic functional gestures will change according to certain idiomatic features characteristic of each mode. The same type of process is applicable to Crumb's using the octatonic model: a correct alignment of octatonic modes illuminate certain characteristic
gestures that otherwise may remain undetectable. As discussed above with reference to hybrid chords, the alignment of octatonic modes depends on the specific rotation of each mode. If the rotation of each mode remained static throughout a work, only one set of hybrid families would be possible. Although this would produce an economical transformational structure, certain sets may not be able to participate in the transformational structure of the piece, which otherwise could.

The ROT transformation is similar to the T transformation but operates on the entire mode rather than a single chord. ROT is utilized in order to gain access to chords in different hybrid families and thus different key areas. In the analysis of Crumb's music, only modes that align the SIA of the respective modes will be employed. For this reason, the two-part notation that was used to describe T transformations, $\mathrm{T}_{x} Y$, will not be required because there will be no transformation within a pentagonal segment $(Y)$, only between pentagonal segments that contain the same SIA (x). Therefore, only four rotations are possible, based upon the initial octatonic key, and will be notated in the form $\mathrm{ROT}_{x}$. For example, if $\mathrm{Oct}^{1} \mathrm{R}^{\mathrm{A}\langle 54>}$ was rotated to become $\mathrm{Oct}^{1} \mathrm{R}^{7<54>}$ the transformation would be notated as ROT $_{-2}$ (see Appendix 3). ${ }^{58}$ For example, pc A, which was originally in position 1.1 in $\operatorname{Oct}^{1} \mathrm{R}^{\mathrm{A} 54 \gg}$, is shifted two pentagonal segments counterclockwise around the helix so that its replacement, pc 7, originally in position 3.1, now occupies node 1.1 in $\mathrm{Oct}^{1} \mathrm{R}^{7<54>}$. In a T transformation, an extra integer is included

[^59]illustrating the motion within a pentagonal segment. In this case, the motion that is implied is $\mathrm{ROT}_{-2} 0$; however, the 0 is deleted from the notation since it is implicit. ${ }^{59}$ If there is both a ROT and a T transformation required to transform one set into another, first perform the ROT transformation followed by the T transformation. For example, if $\{37 \mathrm{~A}\}$ from Oct $^{1} \mathrm{R}^{\mathrm{A}\langle 54\rangle}$ was transformed into $\{047\}$ of $\operatorname{Oct}^{1} \mathrm{R}^{7<54\rangle}$, first rotate $\operatorname{Oct}^{1} \mathrm{R}^{\mathrm{A}\langle 54>}$ by ROT $_{-2}$ to transform it into Oct $^{1} \mathrm{R}^{7<54\rangle}$ then transform $\{37 \mathrm{~A}\}$ by $\mathrm{T}_{2} 0$ into $\{047\}$ (see Appendix 3). The order of operations when combining ROT and T transformations does not matter between octatonic modes because the operations are commutative. The preference for performing the transformations in this order will be discussed later in regards to hyper-transformations between one-dimensional modes and multi-dimensional key areas.

If key areas are containers then a ROT transformation is a selective force. Such a force acts upon only one octatonic mode and rotates it in relationship to the other octatonic modes. The use of a ROT transformation is closely aligned to the creation of different hybrid families; without the ROT transformation, only one family of hybrid chords could occur in a single piece. The ROT transformation's role in allowing relationships between musical objects that might not otherwise interact could be understood as the metaphor ROT transformations are bridges.

[^60]
### 3.7 Hierarchical Functions

As discussed in the second chapter, there is a correspondence between the functional relationship of certain tonal chords and their membership in octatonic collections (see Table 2.4). Because of this correspondence, White has proposed "a model of chromatic functional chord relations based on the hyper-octatonic cycle." ${ }^{60}$ Predicated upon the interaction metaphor of Max Black, he parses various trichords and tetrachords into tonic, predominant, and dominant functional categories based upon their membership in the three different octatonic collections. This metaphor is utilized for various passages in chromatic tonal repertoire that resist easy explication using the elements of tonal theory. White's model, an example of a theory constitutive metaphor, can be modified and applied to Crumb's octatonic music.

The model of octatonality includes all three octatonic collections and their respective arrays. Each node on the model may represent a single pitch class of an octatonic mode or multiple pitch classes of an octatonic key area. Such a metaphor, which adds dimensions that are not easily fathomable, is reasonable given our familiarity with a similar phenomenon in the diatonic system (see Figure 3.7). In the key of C major, the third chord would typically be interpreted as an applied dominant. A common way of notating this is by connecting that chord with an arrow to the resulting harmony; however, even if the applied dominant does not resolve to the intended harmonyperhaps instead of $\mathrm{V}^{7}$ a iii ${ }^{6}$ chord was substituted-the applied dominant might still be

[^61]Figure 3.7. Generic interpretation of an applied dominant

written using the slash notation as shown in Figure 1.1. Generally, both of these methods are preferred over the alternative, $\mathrm{II}_{5}^{6}$, since it describes a functional relationship between the home key of C major and the tonicized key of G major. In the tonal system, this implicit transformation of a II chord into an applied V chord is consistent with Roeder's suggestion to choose transformations that occur prominently and repeatedly as well as transformations that can be applied to other families of objects. In tonality, the motion from dominant to tonic in the home key is a prominent feature. Considering the home key as a family of objects, and using a similar notation to illustrate a tonicization, is in keeping with Roeder's criterion that the same transformations may be applied to a different family of similar objects. In tonality, this notation is usually reserved for dominant function chords because they uniquely identify one diatonic collection over another. In the octatonic model, the combination of three or more nodes uniquely identifies a single octatonic key; therefore, this metaphorical transfer may happen between any given nodes given equivalent SIAs.

While it may be tempting to label a progression from Oct ${ }^{1}$ to Oct ${ }^{2}$ to Oct ${ }^{3}$ and back to Oct ${ }^{1}$ as a progression that moves through tonic (T) to predominant (PD) to dominant (D) and back to tonic (T), such a labeling system would likely confuse some of the properties exhibited in tonality with octatonality (see Figure 3.7). Instead, the same progression in the helical model would be read as progressing from the octatonic $(\mathrm{O})$ to the lower-octatonic $(\mathrm{L})$ to the upper-octatonic $(\mathrm{U})$ and back to the octatonic $(\mathrm{O}) .{ }^{61} \mathrm{~A}$ cyclical spatial metaphor of the functional space within octatonality will help to conceptualize this paradigm. In Figure 3.8 three large circles indicate three octatonic regions with large bold font in the center of each circle indicating the functional designation of each system: the circle on the right is identified as the octatonic, the circle on the lower-left side is the lower-octatonic and the circle on the upper-left side is the upper-octatonic. ${ }^{62}$ Within each larger system a similar nested structure occurs. Proceeding clockwise around each circle corresponds to a cyclic rotation through increasing index numbers identifying each octatonic collection. The larger background structure implies a hierarchical organization of these spaces in reference to Oct ${ }^{1}$. In the larger circle on the right (the octatonic), Oct ${ }^{1}$ is designated as the octatonic of the smaller system and therefore the octatonic of the entire system. This is analogous to how a tonal work may modulate between the key areas of C major to F major to G major and back to C major, which exhibits a large scale motion from tonic to predominant to dominant and

[^62]Figure 3.8. Functional relationships within octatonality

back to tonic. Within each mode, and at different levels of structure, the same type of motions may exist. Although the system shown in Figure 3.8 is based upon $\mathrm{Oct}^{1}$ as the octatonic it could be easily reconfigured to accommodate either one of the two other octatonic collections as the octatonic.

In Figure 3.9 the tonal model from Table 3.5 is arranged in a similar manner as the inside view of a pentagonal segment of the octatonic model shown in Figure 3.2. Continuous, dashed and dotted lines enclose the pitches that comprise chords that may

Figure 3.9. Functional relationships within the key of C major

participate in tonic, predominant and dominant functional relationships respectively. Any three consecutive pitch classes from nodes 1.4-1.7, which correspond to either vi, I, or iii chords in C major, may have tonic function in tonal music. At the same time, vi may be considered predominant or iii may be considered dominant; this overlap in function also obtains for the chords in the predominant and dominant function categories. Rotating
counter-clockwise through the functions of the tonal model shown in Figure 3.9 corresponds to a progressive motion while rotating clockwise corresponds to a retrogressive motion. The term progressive is used in the sense that predominant function chords typically lead to dominant function chords, which then lead to tonic function chords. Alternatively, when this motion is reversed such a motion is termed retrogressive. Motion within a pentagonal segment of the octatonic model may also exhibit the clockwise or counter-clockwise motions between octatonic collections but on a surface level. Thus a counter-clockwise transformation of nodes $1.1-1.3$ to nodes $8.5-1.2, \mathrm{~T}_{0}-1$, would be similar to rotating counter-clockwise in a higher-level octatonic space, moving from O to $\mathrm{U}, \mathrm{U}$ to L , or L to O . If the transformation were reversed, $\mathrm{T}_{0} 1$, a corresponding clockwise rotation between higher-level octatonic functions would result.

Whereas in tonality chords may reside in more than one functional category, in the helical model such a plural function is absent because of the limited arrangement of chords within a pentagonal segment compared to the arrangement of chords in a heptagonal segment of the diatonic collection. For example, if one were to assign $\mathrm{O}, \mathrm{L}$, and U functions arbitrarily to three groups of five consecutive nodes within a pentagonal segment-in a manner similar to which T, PD and D functions were assigned to the nodes of the tonal model - each trichord would belong to every functional category in the octatonic system. Rather than base surface-level function upon the object, the characteristic gesture of the transformation within the pentagonal segment will determine the function. Within each pentagonal segment, a positive or negative transformationclockwise or counter-clockwise-will correspond to a progressive or retrogressive function respectively. For example, assuming the octatonic for a piece corresponds to
nodes 1.1-1.3, moving to nodes 1.4-2.1 may be considered a clockwise motion through the transformation $\mathrm{T}_{0} 3$; however, since the preference is to show the smallest distance within a pentagonal segment compared to the smallest distance between segments, the transformation $\mathrm{T}_{1}-2$ would be used. The negative integer indicates a counter-clockwise motion and thus this transformation may be thought of as a retrogressive motion from the octatonic to the upper-octatonic. ${ }^{63}$

Determining which octatonic mode to identify as the octatonic would rest upon a contextual analysis of a given piece of music, and would consider all the levels of structure and characteristic motions contained within it. In particular, the concepts of opposition and completion that were discussed in the second chapter will be of primary importance. The persuasiveness of hierarchical functionality will only be realized in the analyses of Crumb's A Haunted Landscape and "Come Lovely" in the following chapters.

The description of function within octatonality relied heavily upon the theoryconstitutive metaphor of tonality. Because of the correspondence between subsets of octatonic collections and tonal chords, and their respective functions in tonality, the concept of progressive and retrogressive motions was introduced to explain characteristic gestures in octatonality. A few of the metaphors encapsulated within this section are as follows: motion between chords is functional, motion between modes is functional,

[^63]clockwise transformations are progressive and counter-clockwise transformations are retrogressive.

### 3.8 Hyper-T Transformations

The T transformations discussed in section 3.5 applied only to sets within one octatonic mode. As discussed above, the generic octatonic model, in which pitch classes of each octatonic mode share the same nodal position, allows for the transformation of one type of set into another through a hyper-T transformation. The adjective hyper is used in the sense that the transformation exceeds the dimensionality of a single octatonic mode. ${ }^{64}$ In order to allow transformations of all objects contained within the model I will outline three categories of hyper-T transformations: transformations between chords of different octatonic modes; transformations of chords between different two-dimensional key areas; and, transformations of a chord into another chord where the dimensionality of the respective mode or key area differs.

All of the following hyper- T transformations extend the metaphors found in T transformations. Whereas $T$ transformations are cyclical journeys, hyper-T transformations are cyclical journeys through multi-dimensional space. The exact nature

[^64]of the multi-dimensional space changes for each of the three categories described below; however, each transformation must leave one realm and enter another.

### 3.8.1 Hyper-T transformations between chords of different octatonic modes

As mentioned above, there is an equivalency between the octatonic modes because the SIAs of similarly rotated modes are identical. Angle brackets enclosing the transposition transformation indicate a hyper-transformation between different octatonic modes with identical SIAs. To clarify the modes involved in the hyper-transformations, a superscript notation is appended to the angle brackets using the index number of the affected octatonic modes connected by an arrow. For example, moving from an $\mathrm{Oct}^{1}$ mode to an Oct ${ }^{2}$ mode from nodes 1.1-1.3 to nodes 2.2-2.4 respectively, results in the transformation $\left\langle\mathrm{T}_{1} 1\right\rangle^{1 \rightarrow 2}$. The superscript notation of the hyper-transformation implicitly includes reference to the progressive or retrogressive functional motion. ${ }^{65}$ For example, the transformation of $\{701\}$, which occurs on nodes $1.1-1.3$ of Oct $^{1} \mathrm{R}^{1<65>}$, to $\{890\}$, which occurs on nodes $2.2-2.4$ of $\mathrm{Oct}^{2} \mathrm{R}^{9<65\rangle}$, is labeled $\left\langle\mathrm{T}_{1} 1\right\rangle^{1 \rightarrow 2}$ (see Table 3.2). Because the motion from $\mathrm{Oct}^{1}$ to $\mathrm{Oct}^{2}$ involves a clockwise motion around the functional circle, it would represent a progressive function (see Figure 3.8). Moving from \{890\} back to $\{701\}$, using the same rotations of the octatonic modes, is notated as $\left\langle\mathrm{T}_{-1}-1\right\rangle^{2 \rightarrow 1}$. This type of transformation between networks fulfills Lewin's requirement that nodes of

[^65]semigroups communicate in order to participate within one network. ${ }^{66}$ In addition, two different functional levels are present in this transformation: the surface-level retrogressive function indicated by the normal case -1 transformation within the angle brackets and a higher-level retrogressive function between octatonic modes.

Hyper-T transformations can also be combined with a ROT transformation. Consider again $\{701\}$ in Oct $^{1} \mathrm{R}^{1<65>}$ moving to $\{890\}$ in Oct $^{2} \mathrm{R}^{0<65>}$ instead of $\mathrm{Oct}^{2} \mathrm{R}^{9<65\rangle}$. Implicit is the notion that $\mathrm{Oct}^{2} \mathrm{R}^{9<65>}$ has been established prior to this transformation and Oct $^{2} \mathrm{R}^{0<65>}$ is operative later. To accomplish this, $\mathrm{ROT}_{-2}$ transforms $\mathrm{Oct}^{2} \mathrm{R}^{9<65>}$ into Oct $^{2} \mathrm{R}^{0<65>}$. After this rotation, $\{701\}$ is transposed by $\left\langle\mathrm{T}_{3} 1\right\rangle^{1 \rightarrow 2}$ to $\{890\}$ in $\operatorname{Oct}^{2} \mathrm{R}^{9<65\rangle}$. Unlike transformations within a single octatonic mode, the order of operations in hyper-T transformations is important. If $\{701\}$ of Oct $\mathrm{R}^{1<65\rangle}$ was first transposed by $\left\langle\mathrm{T}_{3} 1\right\rangle^{1 \rightarrow 2}$, rather than first rotated, the result would be $\{569\}$ of $\mathrm{Oct}^{2} \mathrm{R}^{0<65>}$ which would not change with a subsequent ROT $_{-2}$ transformation.

### 3.8.2 Hyper-T transformations between different two-dimensional key areas

A hyper-T transformation that only involves two-dimensional key areas will include two different hybrid chords. Because both are hybrids, and hybrids are utilized primarily for transitional passages in Crumb's compositions, only a surface-level harmonic functionality will result if there is motion within a pentagonal segment between the respective chords. For example, the transformation of $\{1379\}$ found in the key area of

[^66]Oct ${ }^{1,2} \mathrm{R}^{1,9<65>}$ at nodes $1.1-1.2$ to $\{57 \mathrm{~B} 1\}$ found in the key area of $\mathrm{Oct}^{1,3} \mathrm{R}^{1, \mathrm{~B}<65>}$ would be notated simply as $\left\langle\mathrm{T}_{0} 0\right\rangle^{1,2 \rightarrow 1,3}$ (see Table 3.2). A reversal of this transformation would be notated as $\left\langle\mathrm{T}_{0} 0\right\rangle^{1,3 \rightarrow 1,2}$.

### 3.8.3 Hyper-T transformations between a mode (or key area) and a key area with a difference in dimensionality

Because a hyper-T transformation already includes the concept of moving between two octatonic modes or different two-dimensional key areas, a suitable representation for the union or merger of two octatonic modes is $>\mathrm{T}<.{ }^{67}$ Consider the merging hyper- T transformation of $\{701\}$ in the mode $\operatorname{Oct}^{1} \mathrm{R}^{1<65>}$ to $\{789013\}$ in the key area of Oct ${ }^{1,2} \mathrm{R}^{1,9<65>}$, a $\mathrm{H} 4^{2,2}$ family (see Table 3.2). Pitch-set $\{701\}$ occurs between nodes $1.1-1.3$ in Oct ${ }^{1} \mathrm{R}^{1<65>}$ and $\{789013\}$ is found at the same union of modes in Oct ${ }^{1,2} \mathrm{R}^{1,9<65>}$. A merging transposition of these nodes would be notated as $>\mathrm{T}_{0} 0<{ }^{1 \rightarrow 1,2}$. The space that this chord now occupies is in the two-dimensional key area of Oct ${ }^{1,2} \mathrm{R}^{1,9<65>}$. Because a new dimension is created, a further transformation of $\{789013\}$ to $\{12569\}$ (nodes $1.4-2.1$ in Oct ${ }^{1,2} \mathrm{R}^{1,9<65>}$ ), would simply be a transposition transformation of $\mathrm{T}_{1}-2 .{ }^{68}$ Although the cardinalities of the set class differ, they occupy the same number of nodes in the model. A splitting transformation of $\{12569\}$ back to the original $\left\{701\right.$ ) of $\operatorname{Oct}^{1} \mathrm{R}^{1<65>}$, which is in a single dimension, would be notated as

[^67]$>\mathrm{T}_{-1} 2<^{1,2 \rightarrow 1}$. Although only an example of moving from a one-dimensional mode to a two-dimensional key area and vice-versa has been presented, the same process applies to transformations between two- and three-dimensional key areas or between a onedimensional mode and a three dimensional key area.

### 3.9 Application

The contingent relationships found within the model of octatonality allow for multiple interpretations of a given work. The most important decision often rests upon the interpretation of mode or key, which influences the subsequent transformations and functional designations. However, in some cases-particularly in transitional passagesmode or key appear to be subservient to transformations. This dynamic balance between harmonic structures and transformations will be explored in the following analyses of Crumb's A Haunted Landscape and "Come Lovely" in the fourth and fifth chapters.

## 4 A Haunted Landscape

From approximately 1972-1984, A Haunted Landscape underwent many revisions and various conceptions before the final product was sent to the publisher. ${ }^{1}$ During such an extended period, Crumb exchanged materials between different pieces, which may have been a result of his simultaneous compositional process (see Figure 4.1). Examples such as this abound throughout the Sketchbooks where Crumb lists the number of compositions he works on at one time, which may be one reason for his inter-opus consistency of style. ${ }^{2}$ In the following analysis, I will discuss the opening and closing sections of A Haunted Landscape and expose the stratified levels of octatonic harmonic function and similarities and differences in transformational patterns. This chapter will

[^68]Figure 4.1. Crumb’s simultaneous compositional process. Sketchbook "B," box 9, George Crumb Papers, Music Division, Library of Congress, Washington, D.C., 93. Courtesy of George Crumb. ${ }^{3}$

be divided into nine parts: part one will define the formal structure of the work and a justification for omitting the middle section of the work will be presented based on sketch materials; part two will provide the context for the determination of the octatonic key; the introduction, transition to the A section, A section, transition to the $\mathrm{A}^{1}$ section and the $\mathrm{A}^{1}$ section will be analyzed in parts three to seven; in part eight the overall octatonic functional design of the introduction, opening and closing sections will be discussed; and finally, the last section will discuss the significance of the findings. This will provide a foundation in which to compare the harmonic and transformational processes in $A$ Haunted Landscape with those in "Come Lovely" in the sixth chapter.

[^69]
### 4.1 Formal Design

Crumb's early formal sketches for A Haunted Landscape consistently reference the trichordal and tetrachordal progressions as important structural features (see Figure 2.4 and Figure 2.5). Figure 4.2 is an early sketch that divides the work into seven phrases and a coda (see Figure 4.3). ${ }^{4}$ The first and second phrases do not indicate the opening material; however, the trichordal progression completes each phrase. These elements appear again at the end of the fourth and seventh phrases accompanied by a sketch for a trumpet fanfare. The third and fifth phrases are noticeably blank, and only a turtledove effect along with "etc" is indicated for the coda.

What appears to be a later version of this formal plan is written on the title page for A Haunted Landscape (see Figure 4.4and Figure 4.5). This sketch also includes seven phrases and a coda; however, in this version, the trichordal progressions of each phrase include only the highest pitch, which are analyzed by Crumb as refrains. Additionally, every phrase ends with a trichordal progression and the initial portion of each phrase includes sketches of other ideas.

[^70]Figure 4.2. Formal sketch of A Haunted Landscape with triadic refrains notated. A Haunted Landscape: for Orchestra, 1984, sketches, box 3/folder 1, George Crumb Papers, Music Division, Library of Congress, Washington, D.C. Courtesy of George Crumb.


Figure 4.3. Transcription of the triadic-refrain formal sketch of A Haunted Landscape


Figure 4.4. Formal sketch of A Haunted Landscape with highest pitches of triadic refrains notated and concept sketches of penultimate material. A Haunted Landscape: for Orchestra, 1984, sketches, box 3/folder 1, George Crumb Papers, Music Division, Library of Congress, Washington, D.C. Courtesy of George Crumb.


Figure 4.5. Transcription of the formal sketch of A Haunted Landscape with the highest pitches of triadic refrains notated and concept sketches of penultimate material


Another formal plan abandons the seven-part phrase plus coda design and instead plots out a five-part symmetrical arch form (see Figure 4.6 and Figure 4.7). ${ }^{5}$ On the right side, five phrases are numbered, each ending with the trichordal progression in the strings and a letter designating the formal structure of each phrase: $\mathrm{A}-\mathrm{B}-\mathrm{C}-\mathrm{B}-\mathrm{A}$. On the left side, a different formal plan is included, which segments the work into three movements. Crumb indicates that the first movement would use "existing material (+ one more stringchords passage)," nothing for the second movement, and for the third movement, simply "tonality again." This allusion to tonality likely references the return of the alternating major and minor triads. The apparent importance of the trichordal progression in each of the above sketches and their relationship to the overall form, in both the early and final versions of the work, is clear. The tripartite structure illustrated in Figure 4.6 closely resembles the final overall form of A Haunted Landscape. An early sketch of the middle section, rather than a second movement as shown above, suggests a five part-form with the aesthetic of a development section (see Figure 4.8 and Figure 4.9). Noticeably missing is any reference to the trichordal progression, although it may be implied by his notation "[quasi development];" however, in the final published version, the trichordal progression is not found in the middle section.

[^71]Figure 4.6. Multi-movement form versus arch-form sketch of A Haunted Landscape. A Haunted Landscape: for Orchestra, 1984, sketches, box 3/folder 1, George Crumb Papers, Music Division, Library of Congress, Washington, D.C. Courtesy of George Crumb.


Figure 4.7. Transcription of the multi-movement form versus arch-form sketch of $A$ Haunted Landscape

Haunted Landscape form phrase \#1

```
Another idea: -
"Haunted Landscapes"
I. the existing material
(+ one more string-chords passage)
II.
III. tonality again
```

leading to explosive then str. chords [A]
ppp

| phrase \#2 | leading to explosive - then string chords [B] |
| :--- | :--- | :--- |
| phrase \#3 | - a $f f z$ section, |
|  | then the languido 10 note-chords |
|  | -then string chords [C] |
| [overlay xylophone] |  |

phrase \#4
then string chords [B]
[overlay
phrase \#5 - the "alla marcia" dominates
then string chords [A]
[overlay
distant tpts.?]

Figure 4.8. Formal sketch of the middle section of A Haunted Landscape. A Haunted Landscape: for Orchestra, 1984, sketches, box 3/folder 1, George Crumb Papers, Music Division, Library of Congress, Washington, D.C. Courtesy of George Crumb.


Figure 4.9. Transcription of the middle-section formal sketch of A Haunted Landscape


Figure 4.10 is an example of the most complete formal sketch that resembles the final version of A Haunted Landscape (see Figure 4.11). This is similar to the formal structure of the published work (see Table 4.1). In Table 4.1, uppercase letters indicate the larger formal divisions, which create a compound ternary form while lowercase letters indicate the internal formal components of the larger formal divisions with measure numbers listed below each segment. ${ }^{6}$ I have maintained Crumb's formal

[^72]Figure 4.10. Final formal sketch of A Haunted Landscape. A Haunted Landscape: for Orchestra, 1984, sketches, box 3/folder 1, George Crumb Papers, Music Division, Library of Congress, Washington, D.C. Courtesy of George Crumb.


Figure 4.11. Transcription of the final formal sketch of A Haunted Landscape

alla marcia

materials)
"Octatonicism in the Music of George Crumb: An Analysis of A Haunted Landscape," (Ph.D. diss., University of Chicago, 1997), 16-18 and 80.

Table 4.1. Formal structure of A Haunted Landscape

|  |  | A |  |  | TR. | B |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| intro. | tr. | a | $\mathrm{tr}^{1}$ | $a^{1}$ | n/a | b | c | d |
| 1-16 | 17-22 | 23-27 | 27-30 | 31-35 | 35 | 36-49 | 50-62 | 63-69 |
|  |  |  |  |  |  |  |  |  |
|  |  | $B$ cont. |  |  | RTR. |  | $\mathrm{A}^{1}$ (coda) |  |
| e | $\mathrm{b}^{1}$ | growth | climax | decay | $c^{1}$ | $\mathrm{a}^{2}$ | $\mathrm{d}^{1}$ | $\mathrm{a}^{3}$ |
| 70-92 | 93-101 | 102-6 | 107-12 | 113-28 | 129-41 | 142-54 | 155 | 156-63 |

designations where possible; although in some cases I diverge slightly. For example, in Crumb's sketch an introduction precedes two A sections which are comprised of the "str. chords + drum pattern + brass explosion," whereas the drum pattern and brass explosion-each time they occur-is analyzed as a transition and differentiated between Crumb's two A sections, which are annotated as a and a ${ }^{1}$ respectively. There are only two other major differences: first, in Crumb's version section A followed section C, which does not occur in the final version; second, Crumb's final C section is merged with the
transition for reasons which will be discussed later. ${ }^{7}$ As notated in Crumb's final formal sketch, the E section contains many contrasting ideas, some of which have appeared in other sections and are quite fragmentary. Similar fragmentary techniques are used in the growth, climax and decay sections. What remains consistent among all of Crumb's formal sketches is the importance of the major and minor triadic progressions, which serve as bookends of the compound ternary form.

The transition to the $B$ section and the $B$ section itself will not be analyzed.
Although this represents a significant portion of the work, its fragmentary and developmental nature is not suited to analysis using the functional octatonic model, although some sections can be modeled to a certain extent. ${ }^{8}$ For example, most of the $b$ and $b^{1}$ sections alternate between different octatonic collections and could be segmented as normal or hybrid chords, but such an analysis is not compelling. Other sections, such as the e section, include many fragments that simply cannot be explained by the model. A discussion of overall octatonic functional design can still be accomplished using only the A and $A^{1}$ sections, which are clear examples of octatonality. Crumb's tripartite formal schematic shown in Figure 4.6 implies a suspension of tonality for the middle movement because of the return of tonality for the final movement. Therefore, the same type of suspension of octatonality, or a suspension to a varying degree, might be argued for the B section in the final version.

[^73]
### 4.2 Key

The determination of an octatonic key is based upon a contextual analysis of a given work. As was demonstrated previously, the triadic progressions of A Haunted Landscape are important structural components that frame the $\mathrm{ABA}^{1}$ form. An examination of the sketch materials helps to reveal the octatonic framework and primacy of the Bb -major triad in the published version.

An early version of the trichordal progression from A Haunted Landscape, discussed in the second chapter (see Figure 2.4), first appeared in drafts for "The Night in Silence under Many a Star ..." originally conceived as the seventh movement of Apparition (see Figure 4.12). In the final version of Apparition, "The Night in Silence" frames the work as the first and last movements-switching roles with "Come Lovely" which framed the work in the earlier version-and does not include the trichordal progression. ${ }^{9}$ Such repurposing of material was common for Crumb and examples similar to that of the trichordal progression from "The Night in Silence" and A Haunted Landscape abounds in the sketch materials. A reduction of Crumb's sketch for "The Night in Silence" is included in Figure 4.13. Black whole notes indicate pitches sketched in ink and grey whole notes indicate pitches sketched in pencil. The alternating trichords, which complete each system, are illustrated as a hexachord and an x through a note corresponds to Crumb's correction of a chord. Below the staff is an analysis of each triad's membership in an octatonic collection and a box indicates the collection is

[^74]Figure 4.12. Crumb, "The Night in Silence," Apparition, sketches for a trichordal progression, circa 1972. Sketchbook " $A$ ", box 9, George Crumb Papers, Music Division, Library of Congress, Washington, D.C., 101. Courtesy of George Crumb.


Figure 4.13. Crumb, "The Night in Silence," reduction and analysis of the sketch

complete. The two triads that Crumb corrected have the corresponding octatonic membership indicated above the staff with a line through it. The fifth system, left incomplete in the sketch, is completed in the figure following Crumb's instructions: "like $1^{0 \prime \prime}$ (see Figure 4.12).

The octatonic analysis highlights important principles guiding the triadic progression. The first, second, and fifth system each start and end with the same octatonic
collection; however, the final segment is only a subset of an octatonic collection. The final octatonic segment of the first and fifth system each lack pc 2 while the final octatonic segment of the second system misses pcs 0 and 6. Either a $B b$-major or G-minor triad would have supplied the missing pc 2 in the final octatonic segment of the first and fifth systems but instead pc A is used, rather than a complete triad. Additionally, the span of each octatonic segment in the song varies widely from one chord to seven chords: the first, second, and fifth systems are quite similar whereas the third system features frequent changes between octatonic collections and conversely all of the triads in the fourth system are members of $\mathrm{Oct}^{2}$. Except for the middle system, the importance of framing the respective phrases and the song as a whole with corresponding octatonic collections reiterates the importance of the principle of completion in Crumb's music.

A revised sketch of "A Night in Silence" demonstrates a reduced fluctuation between octatonic collections and replaces the singular $B b$ at the end of the first and fifth systems with a triad. The revised sketch of the first system of Figure 4.12 is shown in Figure 4.14. Figure 4.15 is an analytic reduction of the entire revised sketch. Alterations, as compared to Figure 4.13, are indicated with an asterisk; in some cases, the octatonic collection does not change even though the pitches of the triad do. In this version, each system begins with three chords that complete the respective octatonic aggregates. The first, fourth and fifth systems share the same pattern of segmentation. Although the segmentation is different in the fourth system, there is a progressive motion between each collection, similar to the motion in the other systems, and the first and final octatonic collections frame the segment. The second system also cycles through each octatonic collection, but does not return to the initial collection, whereas the third system only

Figure 4.14. Crumb, "The Night in Silence," Apparition, revised sketch of the trichordal progression. Apparition: Elegiac Songs and Vocalises for Soprano and Amplified Piano (On Texts from Walt Whitman's "When lilacs last in the dooryard bloom 'd"), 1979, sketches, box 5/folder 9, George Crumb Papers, Music Division, Library of Congress, Washington, D.C. Courtesy of George Crumb.

cycles through two octatonic collections. The most obvious similarity between both sketches is the use of $\mathrm{Oct}^{3}$ as a frame. Interestingly, the alternating minor-triads present in the previous sections are omitted in this sketch (indicated by a question mark in Figure 4.15); however, Crumb draws a line to the location where they would have occurred and writes " 3 flutes then 3 clar." ${ }^{10}$ In another sketch, these chords are first played by one flute

[^75]Figure 4.15. Crumb, "The Night in Silence," reduction and analysis of the revised sketch

and two clarinets and then presumably the strings (see Figure 4.16). ${ }^{11}$ The alternating chords are played not once, as was the case in the previous systems, but twice. The first instance utilizes overlapping C\#- and G-minor triads, members of $\mathrm{Oct}^{3}$, whereas the
${ }^{11}$ The clefs in Figure 4.16 from the bottom system to the top are treble, bass, treble, and treble.

Figure 4.16. Crumb, "The Night in Silence," Apparition, alternate revised sketch of the final measures. Apparition: Elegiac Songs and Vocalises for Soprano and Amplified Piano (On Texts from Walt Whitman's "When lilacs last in the dooryard bloom'd"), 1979, sketches, box 5/folder 9, George Crumb Papers, Music Division, Library of Congress, Washington, D.C. Courtesy of George Crumb.

second instance, overlapping F- and B-minor triads, are members of Oct ${ }^{2}$. If the pattern of only a triad plus one hexachord was followed, as was the case in the previous systems, the return to the $\mathrm{Oct}^{3}$ collection would represent the principle of completion. This inconclusive ending, however, was not retained in the final published version.

Many aspects of the octatonic framework found in "The Night in Silence" are also found in A Haunted Landscape. The string progression of the A and A ${ }^{1}$ sections utilize the same cyclical sequence of octatonic collections as illustrated in the first and last systems of Figure 4.15 (see Figure 4.17). The A section cycles once through

Figure 4.17. Octatonic progressions of the "string chords" in the A (mm. 23-35) and $\mathrm{A}^{1}$ (mm. 142-54) sections of Crumb's A Haunted Landscape. Copyright © 1984 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

$\mathrm{Oct}^{3}-\mathrm{Oct}^{2}-\mathrm{Oct}^{1}-\mathrm{Oct}^{3}$, although none of these collections are complete. ${ }^{12}$ The $\mathrm{A}^{1}$ section cycles twice through the progression, the second time using a different sequence of chords. After the first four string chords of the $\mathrm{A}^{1}$ section, two pairs of alternating minor chords complete the Oct ${ }^{2}$ section (written as two hexachords inError! Reference source not found. Figure 4.17) in a manner similar to the sketch of "The Night in Silence" shown in Figure 4.16; however, the hexachords in A Haunted Landscape are members of the same octatonic collection. ${ }^{13}$ This alternating figure happens after every four string chords and completes the respective octatonic collection in all but one iteration of the progression: the second progression, which corresponds to the final progression of the A section, reverts to an Oct ${ }^{1}$ hexachordal subset. This lack of completion corresponds with Crumb's analytic notation "but falling" written above the final pitches of the incomplete subject from Bach's D\#-minor fugue quoted in "Starry Night" (see Figure 2.26 and Figure 2.28). The expected completion of the $\mathrm{Oct}^{3}$ collection, and subsequent denial, propels the music forward until the $\mathrm{Oct}^{3}$ collection is complete.

The reiteration of the octatonic progression in the $\mathrm{A}^{1}$ section allows not only for the $\mathrm{Oct}^{3}$ collection, but also the B --major and $\mathrm{C} \#$-minor string triads, to frame the

[^76]passage. These two triads consistently appeared in the early sketches of the piece, starting the first and last phrases of "The Night in Silence" (see Figure 4.13 and Figure 4.15). In addition, in the published version of A Haunted Landscape they are the only sequence of two or more chords that remain the same. The $\mathrm{B} b$-major triad appears to be more significant than the C\#-minor triad for two reasons: first, throughout the entire work, a low $\mathrm{B} b$-pedal is sustained; and second, some of the string instruments sustain the $\mathrm{B} b$ major triad throughout the $A^{1}$ section while the other strings play the progression shown in Figure 4.17. The ratio of the number of strings responsible for maintaining the sustained $\mathrm{B} b$-major triad compared to the octatonic sequence gradually shifts over the course of the section in favour of the sustained $\mathrm{B} b$-major triad.

The importance of the $\mathrm{Oct}^{3}$ collection and the Bb -major triad were significant factors in determining the octatonic key of A Haunted Landscape-Oct ${ }^{3} \mathrm{R}^{5<54>}$. This specific rotation positions the $\mathrm{B} b$-major triad on the first three nodes of the octatonic mode. While the nodes that the triad occupies are arbitrary, the ordering of this mode compared to other octatonic modes is important for the determination of hybrid families, which are found in transitional passages, and the determination of key areas. In the analysis that follows, the octatonic, $\mathrm{Oct}^{3} \mathrm{R}^{5<54>}$, will never be rotated, rather the other two octatonic collections and respective modes will be rotated in relationship to this octatonic mode.

### 4.3 Introduction Section

The introduction primarily utilizes instances of sc (014) played by the harp. Figure 4.18 segments the (014) progressions into three systems. This segmentation correlates with the separation of each progression in the score by moments of relative repose, which feature the low sustained $\mathrm{B} b$-pedal and a brief melody played by the flutes between the first and second as well as the second and third systems (m. 7 and m. 13 respectively). Table 4.2 lists the specific nodal positions for each chord in relationship to its mode. The octatonic key area is indicated in the top row, which is defined by the octatonic modes included in each column. Within each column, each chord is shaded. Thick lines with a border correspond to the (014) sections separated by a measure line in Figure 4.18 and medium-thick lines without a border indicate segments within the final (014) section.

The key of the introduction, $\mathrm{Oct}^{2} \mathrm{R}^{3<54>}$ (indicated by a box in Figure 4.18), was chosen for two reasons. First, the first two (014) progressions are all found within the $O c t^{2}$ collection, which is indicative of the $\mathrm{Oct}^{2}$ collection's significance. Second, the sustained $\mathrm{Oct}^{2}$ segment in A Haunted Landscape resembles the extended $\mathrm{Oct}^{2}$ segment before the "string chords" in a sketch of "The Night in Silence." The same two "string chords," which follow the extended Oct ${ }^{2}$ segment in "The Night in Silence," begin the A section in A Haunted Landscape (cf. Figure 4.12 and Figure 4.13). ${ }^{14}$ Because the third (014) progression cycles through $\mathrm{Oct}^{2}, \mathrm{Oct}^{3}$ and $\mathrm{Oct}^{1}$ collections, the key area of the

[^77]Figure 4.18. Octatonic analysis of the introduction. Copyright © 1984 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

introduction is notated as $\mathrm{Oct}^{2,3,1} \mathrm{R}^{3,5,7<54>}$. Below the staff, the functional designations ( O , L , and U ) reflect the progressive functional motion between the different modes in relationship to the key of the introduction, $\operatorname{Oct}^{2} \mathrm{R}^{3<54>}$. ${ }^{15}$

[^78]Table 4.2. Tabular representation of the chords, modes, and key areas of the introduction

| N | $\mathrm{Oct}^{2,3,1} \mathrm{R}^{3,5,7<54>}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.1 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 5 | 5 | 5 | 7 | 7 | 7 |
| 1.2 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | A | A | A | 0 | 0 | 0 |
| 1.3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 4 | 4 | 4 |
| 1.4 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 8 | 8 | 8 | A | A | A |
| 1.5 | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | 1 | 1 | 1 | 3 | 3 | 3 |
| 2.1 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | A | A | A | 0 | 0 | 0 |
| 2.2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 4 | 4 | 4 |
| 2.3 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 7 | 7 | 7 | 9 | 9 | 9 |
| 2.4 | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | 1 | 1 | 1 | 3 | 3 | 3 |
| 2.5 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 5 | 5 | 5 | 7 | 7 | 7 |
| 3.1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 4 | 4 | 4 |
| 3.2 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 7 | 7 | 7 | 9 | 9 | 9 |
| 3.3 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | B | B | B | 1 | 1 | 1 |
| 3.4 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 5 | 5 | 5 | 7 | 7 | 7 |
| 3.5 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | A | A | A | 0 | 0 | 0 |
| 4.1 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 7 | 7 | 7 | 9 | 9 | 9 |
| 4.2 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | B | B | B | 1 | 1 | 1 |
| 4.3 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 4 | 4 | 6 | 6 | 6 |
| 4.4 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | A | A | A | 0 | 0 | 0 |
| 4.5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 4 | 4 | 4 |
| 5.1 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | B | B | B | 1 | 1 | 1 |
| 5.2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 4 | 4 | 6 | 6 | 6 |
| 5.3 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 8 | 8 | 8 | A | A | A |
| 5.4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 4 | 4 | 4 |
| 5.5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 7 | 7 | 7 | 9 | 9 | 9 |
| 6.1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 4 | 4 | 6 | 6 | 6 |
| 6.2 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 8 | 8 | 8 | A | A | A |
| 6.3 | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | 1 | 1 | 1 | 3 | 3 | 3 |
| 6.4 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 7 | 7 | 7 | 9 | 9 | 9 |
| 6.5 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | B | B | B | 1 | 1 | 1 |
| 7.1 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 8 | 8 | 8 | A | A | A |
| 7.2 | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | 1 | 1 | 1 | 3 | 3 | 3 |
| 7.3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 5 | 5 | 5 | 7 | 7 | 7 |
| 7.4 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | 9 | B | B | B | 1 | 1 | 1 |
| 7.5 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 4 | 4 | 6 | 6 | 6 |
| 8.1 | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | 1 | 1 | 1 | 3 | 3 | 3 |
| 8.2 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 5 | 5 | 5 | 7 | 7 | 7 |
| 8.3 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | 8 | A | A | A | 0 | 0 | 0 |
| 8.4 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 4 | 4 | 4 | 6 | 6 | 6 |
| 8.5 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 8 | 8 | 8 | A | A | A |

The registration of and voice leading between instances of sc (014) is replicated in the octatonic model (see Table 4.2). In pitch space, the instances of sc (014) are built, from the lowest to highest pitch, with the interval order <94>. On the octatonic model, the same order of pitches is preserved: the node furthest clockwise corresponds to the lowest pitch, the central node corresponds to the central pitch, and the node furthest counter-clockwise corresponds to the highest pitch. A result of this relationship is a one-to-one transformational mapping of each pitch according to its register. ${ }^{16}$

The rotation of each octatonic mode reflects the sequential motion indicated by brackets below the third system in Figure 4.18. ${ }^{17}$ ROT transformations are indicated above the staff at the end of the introduction. A $\mathrm{T}_{-7}$ relationship between the three-chord segments in mod-12 pitch space translates to a $\left\langle\mathrm{T}_{-2} 0\right\rangle$ relationship in octatonic space. Within each of these segments, the same order of transformations obtains; $\mathrm{T}_{4} 0$ in pitch space becomes $\mathrm{T}_{2} 0$ in octatonic space. Connecting the last chord of one three-chord segment with the first chord of the next three-chord segment, results in null transpositional indices. With the exception of the hyper-transformation, there is little movement between these segments. This corresponds with the performance directions for this passage: each three-chord segment is separated by a rest and includes graduated

[^79]dynamics- $m f>, p>$, and $p p p$ respectively-which creates the sense that each threechord segment attempts to continue a pattern the previous segment was not able to complete.

The transformations of the first (014) passage exhibit a symmetrical arch form: $\mathrm{T}_{4} 0$ transformations alternate first with $\mathrm{T}_{-2} 0$ then $\mathrm{T}_{2} 0$ and back again to $\mathrm{T}_{-2} 0$. To complete the arch form, a final $\mathrm{T}_{4} 0$ transformation would be required. This final transformation is indicated by parenthesis in Figure 4.18 because the moment of repose which follows (mm. 6-9) separates the (014) passages. Although the delayed gratification of the pattern completion may be perceived, the narrative of incompleteness may be equally compelling. The second (014) progression begins with the same transformational gestures as the first, but it is truncated. The sense that each (014) progression is incomplete is highlighted in the final (014) progression. Rather than start from the beginning of the entire progression, in a manner similar to a fastidious student practicing the end of a difficult work, the final segment of the progression is attempted three times without success-in each case, the expected $\mathrm{T}_{4} 0$ transformation is denied.

The incompleteness of the transformational structure is replicated in the harmonic functional progression of the introduction. On a higher level, the first two (014) progressions all belong to the local octatonic mode of the introduction. The first segment of the last progression also belongs to the local octatonic mode but subsequent transpositions of the three-chord segments progress to the local lower- and upperoctatonic modes respectively. The lack of resolution to the local octatonic mode draws attention to the incompleteness of the harmonic structure. Except for the final hyper-T
transformation to the transition section, there are no functional surface-level motions since all of the (014) chords share the same nodal positions within a pentagonal segment.

### 4.4 Transition to Section A

The transition to the A section primarily consists of the low $\mathrm{B} b$ - pedal, gradual drum-rolls that crescendo-the opposite of the gradual decrescendos at the end of the introduction-and an explosive figure played by the horns and trumpets, which closes the transition in m. 22 (see Figure 4.19). A harmonic and rhythmic reduction of the explosive figure is illustrated in Figure 4.20 and a tabular version in Table 4.3. ${ }^{18}$ The first and fifth chords of the transition are grace notes to the second and sixth chords respectively. The third and fourth chords are performed as a trill, as are the seventh and eight chords. Therefore, the most likely segmentation to be perceived would divide the entire transition into four sets of heptachords. Using traditional set-theoretic operations, both the first and second as well as the third and fourth heptachords are related by $\mathrm{T}_{\mathrm{A}}$ while the second to third heptachords are related by $\mathrm{T}_{6}$. This transformational pattern is similar to the transpositions in octatonic space: $\mathrm{T}_{\mathrm{A}}$ becomes $\mathrm{T}_{-1}-2$ and $\mathrm{T}_{6}$ becomes $\mathrm{T}_{-1} 2$ in pitch and octatonic space respectively.

[^80]Figure 4.19. Crumb, A Haunted Landscape, mm. 20-22. Copyright © 1984 by C. F. Peters Corporation. All Rights Reserved. Used by permission.


Figure 4.20. Octatonic analysis of the transition to the A section. Copyright © 1984 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

n.f. seq. $(\mathbf{U} \rightarrow \mathbf{O})$

Table 4.3. Tabular representation of the hybrid chords and key areas of the transition

| N | Oct ${ }^{2,3,1} \mathrm{R}^{0,5,4<54>}$ |  | Oct ${ }^{3,1,2} \mathrm{R}^{5,1,6<54>}$ |  |
| :---: | :---: | :---: | :---: | :---: |
| 1.1 | 054 | 054 | 516 | 516 |
| 1.2 | 5A9 | 5A9 | A6B | A6B |
| 1.3 | 921 | 921 | 2A3 | 2A3 |
| 1.4 | 387 | 387 | 849 | 849 |
| 1.5 | 810 | 810 | 192 | 192 |
| 2.1 | 5A9 | 5A9 | A6B | A6B |
| 2.2 | 921 | 921 | 2A3 | 2A3 |
| 2.3 | 276 | 276 | 738 | 738 |
| 2.4 | 810 | 810 | 192 | 192 |
| 2.5 | 054 | 054 | 516 | 516 |
| 3.1 | 921 | 921 | 2A3 | 2A3 |
| 3.2 | 276 | 276 | 738 | 738 |
| 3.3 | 6BA | 6BA | B70 | B70 |
| 3.4 | 054 | 054 | 516 | 516 |
| 3.5 | 5A9 | 5A9 | A6B | A6B |
| 4.1 | 276 | 276 | 738 | 738 |
| 4.2 | 6BA | 6BA | B70 | B70 |
| 4.3 | B43 | B43 | 405 | 405 |
| 4.4 | 5A9 | 5A9 | A6B | A6B |
| 4.5 | 921 | 921 | 2A3 | 2A3 |
| 5.1 | 6BA | 6BA | B70 | B70 |
| 5.2 | B43 | B43 | 405 | 405 |
| 5.3 | 387 | 387 | 849 | 849 |
| 5.4 | 921 | 921 | 2A3 | 2A3 |
| 5.5 | 276 | 276 | 738 | 738 |
| 6.1 | B43 | B43 | 405 | 405 |
| 6.2 | 387 | 387 | 849 | 849 |
| 6.3 | 810 | 810 | 192 | 192 |
| 6.4 | 276 | 276 | 738 | 738 |
| 6.5 | 6BA | 6BA | B70 | B70 |
| 7.1 | 387 | 387 | 849 | 849 |
| 7.2 | 810 | 810 | 192 | 192 |
| 7.3 | 054 | 054 | 516 | 516 |
| 7.4 | 6BA | 6BA | B70 | B70 |
| 7.5 | B43 | B43 | 405 | 405 |
| 8.1 | 810 | 810 | 192 | 192 |
| 8.2 | 054 | 054 | 516 | 516 |
| 8.3 | 5A9 | 5A9 | A6B | A6B |
| 8.4 | B43 | B43 | 405 | 405 |
| 8.5 | 387 | 387 | 849 | 849 |

The pitch-material presented by the horns and trumpets are hybrid chords which belong to two different rotations of the $\mathrm{H} 1 \backslash 4 \backslash 5^{3,3}$ (B) family; the first two heptachords belong to $\mathrm{Oct}^{2,3,1} \mathrm{R}^{0,5,4<54>}$ while the second two heptachords belong to $\mathrm{Oct}^{3,1,2} \mathrm{R}^{5,1,6<54>}$. The relative rotations of each octatonic key area differ: $\mathrm{Oct}^{2,3,1} \mathrm{R}^{0,5,4<54\rangle}$, using the generic notation discussed in section 3.4, is representative of $\mathrm{Oct}^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-4, \mathrm{x}+1<54>}$ whereas Oct $^{3,1,2} \mathrm{R}^{5,1,6<54>}$ is representative of Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+5, \mathrm{x}+4<54>}$. This partitioning of keys corresponds with the segmentation of the passage and highlights the pivotal role of the transition in moving from the local key of the introduction, $\operatorname{Oct}^{2} \mathrm{R}^{3<54>}$, to the local key of the A section and global key of the work, $\mathrm{Oct}^{3} \mathrm{R}^{5<54>}$. The operative octatonic key is notated first in the superscript annotation for each key area. The final key area of the transition, $\mathrm{Oct}^{3,1,2} \mathrm{R}^{5,1,6<54>}$, is important as it determines the modes used in the A section and the corresponding transformational patterns. ${ }^{19}$

All four heptachords of the transition are related by opposing motions within a pentagonal segment (indicated by the normal case index of transposition in Figure 4.20). The first to second chords move two nodes counter-clockwise (upwards in Table 4.3), whereas the second to third chords move two nodes clockwise (downwards in Table 4.3). The third to fourth chords revert to the two-node counter-clockwise motion, and the fourth chord of the transition to the first chord of the A section continues this opposing motion with a two-node clockwise motion. On a surface level, this motion corresponds to an alternation between retrogressive and progressive harmonic functions; however,

[^81]because two different key areas are used, which are comprised of all three octatonic modes, a specific designation of $\mathrm{O}, \mathrm{U}$, or L for these chords is difficult, if not impossible, to determine. The lack of functionality on the surface is described in the notation "n.f. sequence $(\mathrm{U} \rightarrow \mathrm{O})$," short for "non-functional harmonic sequence moving from the upperoctatonic to the octatonic." The motion from $\mathrm{U} \rightarrow \mathrm{O}$ refers to a higher-level harmonic function between the local key of the introduction to the local key of the A section, which is also the global key of the work.

### 4.5 Section A

The A section of A Haunted Landscape is comprised of two string chord passages separated by a short transition (see Figure 4.21). Transformations using traditional settheoretic transformations uncover many interesting patterns. The inversional index decreases by 2 between the string chords in the a and the $\mathrm{a}^{1}$ sections. A comparison of the inversional indices between the first two chords of the a section and the last two chords of the $\mathrm{a}^{1}$ section highlights the interval 6-an important interval in Crumb's music. The transition contrasts with the a and $a^{1}$ sections through the inclusion of minor-major seventh chords, which are related by transposition rather than inversion. Finally, the transpositions between the first and second as well as the third and fourth heptachords are the same as the transpositions in the transition leading to the A section whereas the transpositions between the second and the third heptachords differ. Although the brief

Figure 4.21. Set-theoretic transformations of the A section (mm. 23-35), reduction. Copyright © 1984 by C. F. Peters Corporation. All Rights Reserved. Used by permission.


## trans.


analysis using traditional set-theoretic operations is able to highlight many aspects of the progression, it is not able to relate the final chord of one section with the first chord of another.

The octatonic model replicates many of the features identified in the set-theoretic analysis, is able to link each section, and uncover additional patterns (see Figure 4.22 and Table 4.4). Similar to the transition to the A section, each of the heptachords in the second transition are members of the H1\415 3,3 (B) family. The first two heptachords are both found in $\mathrm{Oct}^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-1, \mathrm{x}-5<\mathrm{yz>}}$ and the last two heptachords are both found in Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-4, \mathrm{x}+1<\mathrm{yz>}}$, which segments the passage into two key areas similar to the first transition (cf. Figure 4.20). Moreover, all four heptachords are related by opposing -2/+2

Figure 4.22. Octatonic analysis of the A section. Copyright © 1984 by C. F. Peters Corporation. All Rights Reserved. Used by permission.


## n.f. seq. $(\mathbf{U} \rightarrow \mathbf{L})$


motions within a pentagonal segment, as was the case in the first transition. The transformations within the second transition are also analogous to the set-theoretic transformations of the heptachords in pitch space: $\mathrm{T}_{\mathrm{A}}$ becomes $\mathrm{T}_{-1}-2$ and $\mathrm{T}_{3}$ becomes $\mathrm{T}_{-3} 2$ in pitch and octatonic space respectively. Thus, transformations in octatonic space are also able to model the different transformations between the second and third heptachords of the first and second transitions, as was the case with transpositions in pitch space. The second transition completes a pattern of rotating through the three

Table 4.4. Tabular representation of the chords, modes, and key areas of the A section

| N | Oct ${ }^{3,2} \mathrm{R}^{5,6<54>}$ |  |  |  | Oct ${ }^{3,1,2} \mathrm{R}^{5, \mathrm{~A}, 9<54>}$ |  | Oct ${ }^{3,1,2} \mathrm{R}^{5,4,0<54>}$ |  | Oct ${ }^{3,1} \mathrm{R}^{5,4<54>}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.1 | 5 | 5 | 6 | 6 | 5A9 | 5A9 | 540 | 540 | 4 | 4 | 5 | 5 |
| 1.2 | A | A | B | B | A32 | A32 | A95 | A95 | 9 | 9 | A | A |
| 1.3 | 2 | 2 | 3 | 3 | 276 | 276 | 219 | 219 | 1 | 1 | 2 | 2 |
| 1.4 | 8 | 8 | 9 | 9 | 810 | 810 | 873 | 873 | 7 | 7 | 8 | 8 |
| 1.5 | 1 | 1 | 2 | 2 | 165 | 165 | 108 | 108 | 0 | 0 | 1 | 1 |
| 2.1 | A | A | B | B | A32 | A32 | A95 | A95 | 9 | 9 | A | A |
| 2.2 | 2 | 2 | 3 | 3 | 276 | 276 | 219 | 219 | 1 | 1 | 2 | 2 |
| 2.3 | 7 | 7 | 8 | 8 | 70B | 70B | 762 | 762 | 6 | 6 | 7 | 7 |
| 2.4 | 1 | 1 | 2 | 2 | 165 | 165 | 108 | 108 | 0 | 0 | 1 | 1 |
| 2.5 | 5 | 5 | 6 | 6 | 5A9 | 5A9 | 540 | 540 | 4 | 4 | 5 | 5 |
| 3.1 | 2 | 2 | 3 | 3 | 276 | 276 | 219 | 219 | 1 | 1 | 2 | 2 |
| 3.2 | 7 | 7 | 8 | 8 | 70B | 70B | 762 | 762 | 6 | 6 | 7 | 7 |
| 3.3 | B | B | 0 | 0 | B43 | B43 | BA6 | BA6 | A | A | B | B |
| 3.4 | 5 | 5 | 6 | 6 | 5A9 | 5A9 | 540 | 540 | 4 | 4 | 5 | 5 |
| 3.5 | A | A | B | B | A32 | A32 | A95 | A95 | 9 | 9 | A | A |
| 4.1 | 7 | 7 | 8 | 8 | 70B | 70B | 762 | 762 | 6 | 6 | 7 | 7 |
| 4.2 | B | B | 0 | 0 | B43 | B43 | BA6 | BA6 | A | A | B | B |
| 4.3 | 4 | 4 | 5 | 5 | 498 | 498 | 43B | 43B | 3 | 3 | 4 | 4 |
| 4.4 | A | A | B | B | A32 | A32 | A95 | A95 | 9 | 9 | A | A |
| 4.5 | 2 | 2 | 3 | 3 | 276 | 276 | 219 | 219 | 1 | 1 | 2 | 2 |
| 5.1 | B | B | 0 | 0 | B43 | B43 | BA6 | BA6 | A | A | B | B |
| 5.2 | 4 | 4 | 5 | 5 | 498 | 498 | 43B | 43B | 3 | 3 | 4 | 4 |
| 5.3 | 8 | 8 | 9 | 9 | 810 | 810 | 873 | 873 | 7 | 7 | 8 | 8 |
| 5.4 | 2 | 2 | 3 | 3 | 276 | 276 | 219 | 219 | 1 | 1 | 2 | 2 |
| 5.5 | 7 | 7 | 8 | 8 | 70B | 70B | 762 | 762 | 6 | 6 | 7 | 7 |
| 6.1 | 4 | 4 | 5 | 5 | 498 | 498 | 43B | 43B | 3 | 3 | 4 | 4 |
| 6.2 | 8 | 8 | 9 | 9 | 810 | 810 | 873 | 873 | 7 | 7 | 8 | 8 |
| 6.3 | 1 | 1 | 2 | 2 | 165 | 165 | 108 | 108 | 0 | 0 | 1 | 1 |
| 6.4 | 7 | 7 | 8 | 8 | 70B | 70B | 762 | 762 | 6 | 6 | 7 | 7 |
| 6.5 | B | B | 0 | 0 | B43 | B43 | BA6 | BA6 | A | A | B | B |
| 7.1 | 8 | 8 | 9 | 9 | 810 | 810 | 873 | 873 | 7 | 7 | 8 | 8 |
| 7.2 | 1 | 1 | 2 | 2 | 165 | 165 | 108 | 108 | 0 | 0 | 1 | 1 |
| 7.3 | 5 | 5 | 6 | 6 | 5A9 | 5A9 | 540 | 540 | 4 | 4 | 5 | 5 |
| 7.4 | B | B | 0 | 0 | B43 | B43 | BA6 | BA6 | A | A | B | B |
| 7.5 | 4 | 4 | 5 | 5 | 498 | 498 | 43B | 43B | 3 | 3 | 4 | 4 |
| 8.1 | 1 | 1 | 2 | 2 | 165 | 165 | 108 | 108 | 0 | 0 | 1 | 1 |
| 8.2 | 5 | 5 | 6 | 6 | 5A9 | 5A9 | 540 | 540 | 4 | 4 | 5 | 5 |
| 8.3 | A | A | B | B | A32 | A32 | A95 | A95 | 9 | 9 | A | A |
| 8.4 | 4 | 4 | 5 | 5 | 498 | 498 | 43B | 43B | 3 | 3 | 4 | 4 |
| 8.5 | 8 | 8 | 9 | 9 | 810 | 810 | 873 | 873 | 7 | 7 | 8 | 8 |

possible generic key areas of the H1 $1415^{3,3}$ (B) family, which the first transition started. The first transition moved from Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-4, \mathrm{x}+1<54>}$ to Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+5, \mathrm{x}+4<54>}$ and the second transition moves from Oct $^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-1, \mathrm{x}-5\langle\mathrm{yz}\rangle}$ back to the original $\mathrm{Oct}^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-4, \mathrm{x}+1<\mathrm{yz}\rangle}$ key area.

Like the (014) progression of the introduction, the registration of and voice leading between the string chords is replicated in the octatonic model; however, a one-toone mapping between pitches of the string chords only happens in octatonic space (see Table 4.4). ${ }^{20}$ For example, the first $\mathrm{B} b$-major triad is arranged with the lowest pitch, 5, at node 1.1, the central pitch, A, at node 1.2, and the highest pitch, 2 , at node 1.3. The higher the pitch in the table, the further clockwise it is placed on the helical model (c.f. Figure 3.1 and Figure 3.2). The C\#-minor triad that follows also has the lowest pitch, 4, at node 6.1 , the central pitch, 8 , at node 6.2 , and the highest pitch, 1 , at node 6.3 . Therefore, 5 moves to 4 , A moves to 8 , and 2 moves to 1 . In pitch space, using the $\mathrm{T}_{6} \mathrm{I}$ operation, 5 moves to 1 , A moves to 8 , and 2 moves to 4 , which does not imitate the smooth voice leading of the string chords. The ability to reproduce the voice leading used by Crumb is one of the strengths of the octatonic model.

After the first two heptachords, the flutes, piccolo, oboes and clarinets introduce material, not included in the analysis shown in Figure 4.22, above the last two heptachord segments of the transition (see Figure 4.23). This additional layer is comprised of two

[^82]Figure 4.23. Crumb, A Haunted Landscape, mm. 29-31. Copyright © 1984 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

instances of sc (048), which combine to form $\mathrm{WT}^{1},\{02468 \mathrm{~A}\}$, followed by $\{9 \mathrm{~A} 25\}$ (see Figure 4.24). Pitch-set $\{048\}$ is a subset of the lower heptachord and its registral arrangement corresponds to the upper three pitches of the first tetrachord of the segment, which is indicated with a dashed arrow. Pitch-set $\{26 \mathrm{~A}\}$ is not a subset of either of the first two tetrachords, but comprises the top three pitches of the third tetrachord. The trichords and the heptachord combine to form the pitch-set $\{01245689 \mathrm{AB}\}$. This set can be found in the H1 $44 \backslash 5^{3,5}$ (B) family-the same family as the lower heptachords except five nodes are combined rather than three. Pitch-set \{9A25\} duplicates pitch material of the final heptachord but its registral arrangement is interesting: it is written as a majormajor seventh chord, a contrast to the minor-major seventh chords that combine to form each heptachord.

Figure 4.24. Interaction of the woodwind and brass layers in the first transition, m. 30. Copyright © 1984 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

\{9A25\}


The 10-note chord created by the union of the heptachord and the whole-tone collection forms the basis for the c section of the work (see Figure 4.25 and Figure 4.26). In the final formal sketch of the work, Crumb describes these as "the big chords [languid]" (see Figure 4.10). Figure 4.25 is a reduction of the first and second languid chords of the c section and Figure 4.26 is a reproduction of the score where the first chord changes into the second. The filled-in note heads of the second chord in Figure 4.25 represent the pitches emphasized following the onset of the 9-note chord, which combine with the 9 -note chord to form the 11 -note chord. The first 10 -note chord appears to be based upon the minor-major seventh chords that combined to form the heptachord of the transition. The lowest four pitches form a minor-major seventh chord, <7A26>, and a minor third higher the next four pitches also form a minor-major seventh chord, <9048>. The pattern continues by ascending a minor third to the next pitch; however, the last two pitches form a major third rather than a minor third. The top four pitches form a majormajor seventh chord, <48B3>, the same quality as the chord above the last heptachord of the transition. If this major-major seventh chord were stacked in the same way as were the minor-major sevenths, the next chord would be <6A15>, which are the lowest pitches of the 9-note chord that follows.

Figure 4.25. Languid chords from the c section of A Haunted Landscape, mm. 50-52. Copyright © 1984 by C. F. Peters Corporation. All Rights Reserved. Used by permission.


Figure 4.26. Crumb, A Haunted Landscape, m. 52. Copyright © 1984 by C. F. Peters Corporation. All Rights Reserved. Used by permission.


The relationship between the last two heptachords of the transition and the c section extends beyond the tetrachordal subsets. The first languid chord of the c section is a member of the same set class as the 10 -note chord of the transition and thus can be represented in the same hybrid family. The second 11 -note languid chord can also be found in the same hybrid family. Out of three possible generic key areas, the 10-note chord from the transition and the first 10-note and 11-note chords from the c section can only be found in Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-1, \mathrm{x}-5}$. This is the same generic key area of the first two heptachords of the A section's transition-the key area which completed the rotation through the other possible key areas of the hybrid family. ${ }^{21}$

The possibility of representing the third heptachord of the transition in Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-1, \mathrm{x}-5}$ is problematic since the same key area is not possible for the final heptachord. While it is possible to represent the first three heptachords as members of Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{A}, 9,5<54>}$, this contradicts the segmentation discussed above. In addition, the transpositional values would no longer reflect the pattern established between the firsttwo and last-two heptachordal segments of each transition: the transformation to the final heptachord would be $\mathrm{T}_{-1} 2$. $^{22}$ Although the motion between pentagonal segments would remain the same, -1 , the motion within a pentagonal segment would change from -2 to +2 and disrupt the alternation between retrogressive and progressive motions previously established.

[^83]Alternatively, the heptachords could be separated from the added notes according to the respective registral streams. Scotto utilized a similar approach in his analysis of Processional discussed in section 2.2. Additionally, evidence that Crumb made use of different layers of music is found throughout the sketches. For example, in his arch-form sketch of A Haunted Landscape, Crumb indicates that the string chords of certain sections include other material as an overlay (see Figure 4.6). Considering the upper stream separately, (02468) can be found in the $\mathrm{H} 244^{3,3}$ family. As discussed earlier, the rotation of modes in the $\mathrm{H} 24^{3,3}$ family replicate the key area of the introduction. As will be demonstrated below, this is also the case in the retransition to $A^{1}$ and $A^{1}$ itself. The major-major seventh chord is a subset of and contained within the registral stream of the heptachords. Although the instance of $\{02468 \mathrm{~A}\}$ is important in defining relationships between different sections of the work, its position in a different registral stream excludes it from participating in the established harmonic layer.

In the a and a ${ }^{1}$ sections, the patterns illuminated using traditional transposition and inversion operations in pitch space are not isographic with the transformations in octatonic space (see Figure 4.21 and Figure 4.22). Although the traditional operations exhibit a decreasing inversional index between chords, the indices of the octatonic transpositions are identical. Moreover, the merging hyper-T transformation between the a section and the transition, $>\mathrm{T}_{-2} 0<^{2 \rightarrow 3,1,2}$, as well as the splitting hyper-T transformation between the transition and the $\mathrm{a}^{1}$ section, $>\mathrm{T}_{0} 2<^{3,1,2 \rightarrow 1}$, is able to connect the respective sections whereas traditional set-theoretic operations could not.

The relationship between the quasi-inverted indices of the hyper- T transpositions between and within a pentagonal segment is tempting to discuss. However, the -2 motion
between pentagonal segments compared to the +2 motion within a pentagonal segment, as well as the lack of motion within a segment and between segments, is likely not something that could be perceived. Rather, the relative distance traveled on the octatonic model of the merging transposition, which leads to the transition, is greater than that of the splitting transformation, which leads back to the $\mathrm{a}^{1}$ section. This difference in distances is suggestive of the rhetorical function of the short transitional passage: the commencement of the passage after the string chords of the a section is abrupt while the transition to $\mathrm{a}^{1}$ is gradual.

The string triads of the $a$ and $a^{1}$ sections all occur at the same nodal location within a pentagonal segment of the model; this is reflected by the normal case 0 in the notation of each transformation; therefore there is no surface-level change of function between each chord. The hyper-T transformations between the second and third chords of the $a$ and $a^{1}$ sections are retrogressive: the a section moves from the octatonic to the upper-octatonic while the $\mathrm{a}^{1}$ section moves from the lower-octatonic back to the octatonic. The final chord of the a section is connected to the first chord of the $\mathrm{a}^{1}$ section by the transition, which reflects the motion from the upper- to lower-octatonic in its parenthetical notation.

### 4.6 Retransition to Section $A^{1}$

The retransition to the final $\mathrm{A}^{1}$ section begins with the languid chords of the c section; however, instead of 10- or 11-note chords these are only 8-note chords (see

Figure 4.27 and Table 4.5). ${ }^{23}$ In Figure 4.27, the third chord of each system includes the label " $1 / 4 \downarrow$," indicating the pitch is to be played a quarter-tone lower. The second chord of each system is sustained in the vibraphones for the duration of the third chord and the violins alternate between the second and third chords. I interpret the third chord as a neighbour harmony for two reasons: the second chord of each system is sustained in the vibraphones for the duration of the third chord and the violins alternate between the second and third chords. If the tuning of the third chord was not considered, this chord could partake in the transformational structure. Darker lines around the corresponding nodes indicate this chord's nodal position and an arrow connects the respective alternating chords in Table 4.5.

The section containing the languid chords corresponds with Crumb's second "C" section in the final formal sketch of the work (see Figure 4.10). This section is considered a retransition for the following reasons: the use of hybrid chords, which characterized previous transitional material; sequential transpositions of the hybrid chords; a lack of harmonic motion following the hybrid chords in mm. 134-41; and a return to the string chords of the A section. Crumb's use of a dashed box around his final "C" section also may indicate his awareness of the looser structure of the "C" section compared to the first c section.

[^84]Figure 4.27. Octatonic analysis of the transition to the $A^{1}$ section. Copyright © 1984 by C. F. Peters Corporation. All Rights Reserved. Used by permission.


Measures 134-41 most likely correspond to what Crumb considered a transition. Measure 134 begins with pitch-set $\{045\}$ in the timpani and harp, which cannot be easily accounted for in the octatonic model. The arrangement of pitches implies some relationship to the generic key of $\mathrm{Oct}^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+5, \mathrm{x}+4\langle\mathrm{yz}\rangle}$, a member of the $\mathrm{H} 1 \backslash 415^{3, \mathrm{n}}$ (B) family, which represented the key areas found in the first and second transitions. Specifically, the pitch-classes 0,4 , and 5 were the rotational indices of the key areas of the first-two and last-two heptachords of the first and second transition respectivelyOct $^{2,3,1} \mathrm{R}^{0,5,4<54>}$ and Oct ${ }^{3,1,2} \mathrm{R}^{5,4,0<54>}$. Following the $\{045\}$ trichord, drum rolls and short melodic fragments disrupt any sense of harmonic motion and thus the entire segment is

Table 4.5. Tabular representation of the hybrid chords and key areas of the retransition

| N | Oct ${ }^{3,1,2} \mathrm{R}^{5,7,9<54>}$ |  | Oct ${ }^{3,1,2} \mathrm{R}^{5,1,3<54>}$ |  | Oct ${ }^{3,1,2} \mathrm{R}^{5,7,3<54>}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.1 | 79 | 57 | 53 | 13 | 57 | 53 |
| 1.2 | 02 | A0 | A8 | 68 | A0 | A8 |
| 1.3 | 46 | 24 | 20 | A0 | 24 | 20 |
| 1.4 | A0 | 8A | 486 | 46 | 8A | 86 |
| 1.5 | 35 | 13 | 1B | 9B | 13 | 1B |
| 2.1 | 02 | A0 | A8 | 68 | A0 | A8 |
| 2.2 | 46 | 24 | 20 | A0 | 24 | 20 |
| 2.3 | 9B | 79 | 75 | 35 | 79 | 75 |
| 2.4 | 35 | 13 | 1B | 9B | 13 | 1B |
| 2.5 | 79 | 57 | 53 | 13 | 57 | 53 |
| 3.1 | 46 | 24 | 20 | A0 | 24 | 20 |
| 3.2 | 9B | 79 | 75 | 35 | 79 | 75 |
| 3.3 | 13 | B1 | B9 | 79 | B1 | B9 |
| 3.4 | 79 | 57 | 53 | 13 | 57 | 53 |
| 3.5 | 02 | A0 | A8 | 68 | A0 | A8 |
| 4.1 | 9B | 79 | 75 | 35 | 79 | 75 |
| 4.2 | 13 | B1 | B9 | 79 | B1 | B9 |
| 4.3 | 68 | 46 | 42 | 02 | 46 | 42 |
| 4.4 | 02 | A0 | A8 | 68 | A0 | A8 |
| 4.5 | 46 | 24 | 20 | A0 | 24 | 20 |
| 5.1 | 13 | B1 | B9 | 79 | B1 | B9 |
| 5.2 | 68 | 46 | 42 | 02 | 46 | 42 |
| 5.3 | A0 | 8A | 86 | 46 | 8A | 86 |
| 5.4 | 46 | 24 | 20 | A0 | 24 | 20 |
| 5.5 | 9B | 79 | 75 | 35 | 79 | 75 |
| 6.1 | 68 | 46 | 42 | 02 | 46 | 42 |
| 6.2 | A0 | 8A | 86 | 46 | 8A | 86 |
| 6.3 | 35 | 13 | 1B | 9B | 13 | 1B |
| 6.4 | 9B | 79 | 75 | 35 | 79 | 75 |
| 6.5 | 13 | B1 | B9 | 79 | B1 | B9 |
| 7.1 | A0 | 8A | 86 | 46 | 8A | 86 |
| 7.2 | 35 | 13 | 1B | 9B | 13 | 1B |
| 7.3 | 79 | 57 | 53 | 13 | 57 | 53 |
| 7.4 | 13 | B1 | B9 | 79 | B1 | 4 B 9 |
| 7.5 | 68 | 46 | 42 | 02 | 46 | 42 |
| 8.1 | 35 | 13 | 1B | 9B |  | 1B |
| 8.2 | 79 | 57 | 53 | 13 | 57 | 53 |
| 8.3 | 02 | A0 | A8 | 68 | A0. | A8 |
| 8.4 | 68 | 46 | 42 | 02 | 46 | 42 |
| 8.5 | A0 | 8A | 86 | 46 | 8A | 86 |

not included in the octatonic analysis of the retransition. Instead, the languid chords from mm. 129-33 are shown to connect with the following $\mathrm{A}^{1}$ section, which begins at m. 142 .

The hybrid chords of the retransition can only be found in the three different key areas of the $\mathrm{H} 2^{2,5}$ family. These hybrid chords use the same number of nodes as the vertical interpretation of the third heptachord plus upper registral stream of the transition between the a and a ${ }^{1}$ sections and the 10 -note and 11 -note chords of the c section (cf. Figure 4.24 and Figure 4.25). Additionally, when combining the adjacent two-mode key areas, the $\mathrm{H} 2 \backslash 4^{3,5}$ family is revealed. For example, the first chord is in the key area of Oct ${ }^{1,2} \mathrm{R}^{7,9<54>}$ while the second chord is in the key area of $\mathrm{Oct}^{3,1} \mathrm{R}^{5,7<54>}$ and thus the relationship between all the modes is $\mathrm{Oct}^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+2, \mathrm{x}-2<54>}$. This reveals an important change with respect to the earlier languid chords: the earlier languid chords all resided within a key representative of the $\mathrm{H} 1 \backslash 4 \backslash 5^{3, \mathrm{n}}$ family in the c section and allusions were made to this key area in the transition between the a and $a^{1}$ sections. This fundamental shift in the key area reflects a return to the key organization of the introduction, which utilized a key area representative of the $\mathrm{H} 2 \backslash 4^{3,3}$ family, and the key area of the entire $\mathrm{A}^{1}$ section, which will be discussed in the following section.

Concerning the transformations between heptachords, there are no shifts within each pentagonal segment, uncovering a symmetrical transpositional scheme:
$\mathrm{T}_{-2} 0-\mathrm{T}_{0} 0-\mathrm{T}_{-2} 0 .{ }^{24}$ The $\left.\left\langle\mathrm{T}_{0} 0\right\rangle\right\rangle^{3,1 \rightarrow 1,2}$ transformation is similar to the $\left\langle\mathrm{T}_{0} 0\right\rangle$ transformation

[^85]between the three different octatonic modes at the end of the introduction (cf. Figure 4.18 and Table 4.2). In the introduction, this transformation produced a "one more time" effect, by way of attempting to complete the established pattern. The $\left\langle\mathrm{T}_{0} 0_{-1}\right\rangle$ transformation, which links the retransition to the $\mathrm{A}^{1}$ section, also follows this pattern, although with a change in nodal and pitch cardinality. The overall harmonic function of the progression has not been indicated because of the lack of a distinct key area in the decay section, which precedes the retransition.

### 4.7 Section $A^{1}$ (Coda)

The hybrid chords of the retransition lead seamlessly to the string chords that begin the $\mathrm{A}^{1}$ section. As mentioned previously, the piano part articulates an additional repeated pitch, which transforms the triads into tetrachords; in Figure 4.28, this is indicated with a filled-in note head both above and below the string chords. The positioning of this repeated note on the octatonic model is found both above and below each major and minor string triad (see Table 4.6). Although the repeated notes in earlier figures were notated in one register to draw attention to the parsimonious voice leading, Crumb's actual registral placement of these pitches is significant: every major string triad includes the repeated note above the highest pitch of the triad while every minor string triad includes the repeated note below the lowest pitch of the triad. A circle indicates the actual registral placement of the repeated notes in Figure 4.28.

Figure 4.28. Octatonic analysis of the $\mathrm{a}^{2}$ section in $\mathrm{A}^{1}$. Copyright © 1984 by C. F. Peters Corporation. All Rights Reserved. Used by permission.


Table 4.6. Tabular representation of the chords of the $\mathrm{a}^{2}$ section in $\mathrm{A}^{1}$

| N | Oct ${ }^{3,1,2} \mathrm{R}^{5,7,3<5}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.1 | 5 | 5 | 3 | 3 | 3 | 3 | 7 | 7 | 5 | 5 | 5 | 77 | 5 | 5 | 3 | 3 | 3 | 3 | 7 | 7 | 5 | 5 | 5 | 5 |
| 1.2 | A | A | 8 | 8 | 8 | 8 | 8 | 0 | A | A | 0 | 0 | A | A | 8 | 8 | 8 | 8 | 0 | 0 | A | A | A | A |
| 1.3 | 2 | 2 | 0 | 0 | 0 | 0 | 04 | 44 | 2 | 2 | 4 | 4 | 2 | 2 | 0 | 0 | 0 | 0 | 4 | 4 | 2 | 2 | 2 | 2 |
| 1.4 | 8 | 8 | 6 | 6 | 6 | 6 | 6 A | A | 8 | 8 |  | A A | 8 | 8 | 6 | 6 | 6 | 6 | A | A | 8 | 8 | 8 | 8 |
| 1.5 | 1 | 1 | B | B | B | B | B | 33 | 1 | 1 | 3 | 3 3 | 1 | 1 | B | B | B | B | 3 | 3 | 1 | 1 | 1 | 1 |
| 2.1 | A | A | 8 | 8 | 8 | 8 | 0 | 0 | A | A | 0 | 0 | A | A | 8 | 8 | 8 | 8 | 0 | 0 | A | A | A | A |
| 2.2 | 2 | 2 | 0 | 0 | 0 | 0 | 4 | 44 | 2 | 2 | 4 | 4 | 2 | 2 | 0 | 0 | 0 | 0 | 4 | 4 | 2 | 2 | 2 | 2 |
| 2.3 | 7 | 7 | 5 | 5 | 5 | 5 | 5 | 99 | 7 | 7 | 9 | 9 | 7 | 7 | 5 | 5 | 5 | 5 | 9 | 9 | 7 | 7 | 7 | 7 |
|  | 1 | 1 | B | B | B | B | 3 | 33 | 1 |  | 3 | 33 | 1 | 1 | B | B |  | B | 3 | 3 | 1 | 1 |  | 1 |
| 2.5 | 5 | 5 | 3 | 3 | 3 | 3 | 7 | 77 | 5 | 5 | 57 | 77 | 5 | 5 | 3 | 3 | 3 | 3 | 7 | 7 | 5 | 5 | 5 | 5 |
|  | 2 | 2 | 0 | 0 | 0 | 0 | 4 | 44 | 2 | 2 |  | 44 | 2 | 2 | 0 | 0 | 0 | 0 | 4 | 4 | 2 | 2 | 2 | 2 |
| 3.2 | 7 | 7 | 5 | 5 | 5 | 5 | 9 | 9 | 7 | 7 | 9 | 99 | 7 | 7 | 5 | 5 | 5 | 5 | 9 | 9 | 7 | 7 | 7 | 7 |
| 3.3 | B | B | 9 | 9 | 9 | 9 | 1 | 11 | B | B B | B 1 | 11 | B | B | 9 | 9 | 9 | 9 |  |  | B | B | B | B |
|  | 5 | 5 | 3 | 3 | 3 | 3 | 7 | 77 | 5 | 5 | 7 | 77 | 5 | 5 | 3 | 3 | 3 | 3 | 7 | 7 | 5 | 5 | 5 | 5 |
| 3.5 | A | A | 8 | 8 | 8 | 8 | 8 | 0 | A | A A | 0 | 0 | A | A | 8 | 8 | 8 | 8 | 0 | 0 | A | A | A | A |
| 4.1 | 7 | 7 | 5 | 5 | 5 | 5 | 9 | 9 | 7 |  | 7 | 99 | 7 | 7 | 5 | 5 | 5 | 5 | 9 | 9 | 7 | 7 | 7 | 7 |
| 4.2 | B | B | 9 | 9 | 9 | 9 | 1 | 1 | B | B | 1 | 1 | B | B | 9 | 9 | 9 | 9 | 1 | 1 | B | B | B | B |
| 4.3 | 4 | 4 | 2 | 2 | 2 | 2 | 6 | 66 | 4 |  | 6 | 66 | 4 | 4 | 2 | 2 | 2 | 2 | 6 | 6 | 4 | 4 | 4 | 4 |
|  | A | A | 8 | 8 | 8 | 8 | 0 | 0 | A | A | O | 90 | A | A | 8 | 8 | 8 | 8 | 0 | 0 | A | A | A | - ${ }^{\text {A }}$ |
| 4.5 | 2 | 2 | 0 | 0 | 0 | 0 | 4 | 44 | 2 | 2 | 4 | 44 | 2 | 2 | 0 | 0 | 0 | 0 | 4 | 4 | 2 | 2 | 2 | 2 |
| 5.1 | B | B | 9 | 9 | 9 | 9 | 1 | 1 | B | B | B 1 | 11 | B | B | 9 | 9 | 9 | 9 | 1 | 1 | B | B | B | B |
|  | 4 | 4 | 2 | 2 | 2 | 2 | 6 | 6 | 4 | 4 |  | 6 | 4 | 4 | 2 | 2 | 2 | 2 | 6 | 6 | 4 | 4 | 4 | 4 |
|  | 8 | 8 | 6 | 6 | 6 | 6 | A | A | 8 | 8 |  | A A | 8 | 8 | 6 | 6 | 6 | 6 | A | A | 8 | 8 | 8 | 8 |
|  | 2 | 2 | 0 | 0 | 0 | 0 | 4 | 4 | 2 | 2 | 4 | 4 | 2 | 2 | 0 | 0 | 0 | 0 | 4 | 4 | 2 | 2 | 2 | 2 |
|  | 7 | 7 | 5 | 5 | 5 | 5 | 9 | 9 | 7 | 7 | 9 | 9 | 7 | 7 | 5 | 5 | 5 | 5 | 9 | 9 | 7 | 7 | 7 | 7 |
|  | 4 | 4 | 2 | 2 | 2 | 2 | 6 | 6 | 4 |  |  | 66 | 4 | 4 | 2 | 2 | 2 | 2 | 6 | 6 | 4 | 4 | 4 | 4 |
|  | 8 | 8 | 6 | 6 | 6 | 6 | A | A | 8 | 8 |  | A A | 8 | 8 | 6 | 6 | 6 | 6 | A | A | 8 | 8 | 8 | 8 |
|  | 1 | 1 | B | B | B | B | 3 | 3 | 1 |  | 3 | 3 | 1 | 1 | B | B | B | B | 3 | 3 | 1 | 1 |  |  |
|  | 7 | 7 | 5 | 5 | 5 | 5 | 9 | 9 | 7 | 7 |  | 99 | 7 | 7 | 5 | 5 | 5 | 5 | 9 | 9 | 7 | 7 |  |  |
|  | B | B | 9 | 9 | 9 | 9 | 1 | 1 | B | B B | B 1 | 11 | B | B | 9 | 9 | 9 | 9 |  |  | B | B |  | B |
|  | 8 | 8 | 6 | 6 | 6 | 6 | A | A | 8 | 8 |  | A A | 8 | 8 | 6 | 6 | 6 | 6 | A | A | 8 | 8 |  | 8 |
| 7.2 | 1 | 1 | B | B | B | B | 3 | 3 | 1 | 1 | 3 | 3 | 1 | 1 | B | B | B | B | 3 |  | 1 | 1 |  |  |
| 7.3 | 5 | 5 | 3 | 3 |  | 3 | 7 | 7 | 5 | 5 | 7 | 7 | 5 | 5 | 3 | 3 | 3 | 3 | 7 | 7 | 5 | 5 |  | 5 |
| 7.4 | B | B | 9 | 9 | ¢ | 9 | 1 | 1 | B | B |  | 1 | B | B | 9 | 9 | 9 | 9 | 1 | 1 | B | B | B | B |
| 7.5 | 4 | 4 | 2 | 2 | 1 | 2 | 6 | 6 | 4 | 4 |  | 66 | 4 | 4 | 2 | 2 | 2 | 2 | 6 | 6 | 4 | 4 |  | 4 |
| 8.1 | 1 | 1 | B | B | B | B | 3 | 3 | 1 |  |  | 3 | 1 | 1 | B | B | B | B | 3 | 3 | 1 | 1 | 1 | 1 |
| 8.2 | 5 | 5 | 3 | 3 | 3 | 3 | 7 | 7 | 5 | 5 |  | 7 | 5 | 5 | 3 | 3 | 3 | 3 | 7 | 7 | 5 | 5 | 5 | 5 |
| 8.3 | A | A | 8 | 8 | 8 | 8 | 0 | 0 | A | A | 0 | 00 | A | A | 8 | 8 | 8 | 8 | 0 | 0 | A | A | A | A |
| 8.4 | 4 | 4 | 2 | 2 | 2 | 2 | 6 | 6 | 4 |  |  | 6 | 4 |  | 2 | 2 | 2 |  | 6 | 6 | 4 |  | 4 | 4 |
| 8.5 | 8 | 8 | 6 | 6 | 6 | 6 | A | A A | 8 |  |  | A A | 8 | 8 | 6 | 6 | 6 | 6 | A |  | 8 | 8 | 8 | 8 |

Alternatively, if the tetrachord were interpreted as a pentachordal multi-set, which includes the repeated pitches on either side of the string triad in the octatonic model, the retransition would fuse with the $\mathrm{A}^{1}$ without a decrease in cardinality of nodes. This equality of nodal cardinality was present in both previous transitions and thus one might expect the same retention of nodal cardinality leading to the $A^{1}$ section. As was previously mentioned, both the (014) chords of the introduction and the string chords of the A section were similar in their registral realization of the chords in pitch space and nodal positions in octatonic space. In addition, T and hyper-T transformations in octatonic space enabled a one-to-one mapping of individual voices in the above passages. If the chords of the $\mathrm{A}^{1}$ section were interpreted as tetrachords, the equivalence between the pitch's register and nodal placement would remain but the transformations would not produce the one-to-one mapping. Conversely, if the chords of the $\mathrm{A}^{1}$ section were considered pentachordal multi-sets, which include a duplicate of the repeated-note motive's pitch, then it would disturb the relationship between register in pitch space and nodal placement on the model but preserve the one-to-one transformational mapping. ${ }^{25}$ Each interpretation is equally appealing, but neither option alone equally accounts for the relationships found within this passage. For this reason, both transformational interpretations are included below the staff in Figure 4.28.

In the A section, a transition connected the two string chord progressions; however, similar transitional material is not included in the $\mathrm{A}^{1}$ section. The alternating

[^86]minor chords notated on the upper staff of each system might allude to a transitional passage but, as was shown earlier in sketches of the work, they appear to either confirm an octatonic collection by completing the respective octatonic aggregate or deny completion by moving to a different octatonic mode. Additionally, these chords are not hybrid chords, which were typical in earlier transitional passages. Because these chords appear to have a subordinate function to the tetrachords and pentachords, the transformations linking the tetrachord to the trichord and vice versa are listed between the staves and transformations between the trichords are listed above the upper staff. This segmentation is also supported by the orchestration whereby the strings, crotales, piano, and harps are assigned to the tetrachords, and the wind instruments, vibraphone, and tubular bells are assigned to the alternating minor triads.

The final arrangement of modes in earlier transitions determined the key of the respective sections that followed. The choice of modes was based upon the ability to illustrate repetitive transformational patterns. The key of the retransition is particularly interesting since it establishes keys representative of the $\mathrm{H} 2 \backslash 4$ family for the $\mathrm{A}^{1}$ section rather than of the H1\4\5 family found in the previous A section. As was shown, the chords in the retransition are similar in many respects to the languid chords of the c section except for their hybrid family membership: in the c section, the languid chords belonged to the H1 1415 family while the chords of the retransition belong to a H 214 family. The consequence of this change is that now similar transformations relate all of the chords of the $A^{1}$ section. If hybrid chords from the H1\4\5 family had been used, this would not have been the case. Additionally, because of the rotations of the modes, no subsequent rotations are required to maintain equivalent transformations between
chords-previously, these rotations happened only within the transitional sections. Thus, the key of the coda does not require transitions to maintain equivalent transformations and shares a similar hybrid key area with the introduction section, providing a framing effect consistent with the principle of completion.

The octatonic key area for the $\mathrm{A}^{1}$ section affects the directionality of the transformations between the respective string chords as compared to the A section. In the A section, the string chords of the a and $\mathrm{a}^{1}$ sections each utilized a $\mathrm{T}_{-3} 0-<\mathrm{T}_{-3} 0>-\mathrm{T}_{-3} 0$ structure-all of the chords moved counter-clockwise around the octatonic model, or upwards in Table 4.4. The pentachordal progressions of the $\mathrm{A}^{1}$ section also move through three pentagonal segments, but the directionality oscillates with a $\mathrm{T}_{-3} 0-\mathrm{T}_{3} 0-\mathrm{T}_{-3} 0$ pattern (the same base pattern happens within the tetrachordal interpretation with a similar oscillation within a pentagonal segment). This type of oscillation has been found throughout the work: the $\mathrm{T}_{4} 0$ and $\mathrm{T}_{-4} 0$ transformations between the tritone related minor chords at the end of each progression in the $\mathrm{A}^{1}$ section; the $\mathrm{T}_{-2} 0$ and $\mathrm{T}_{2} 0$ transformations, which were arranged symmetrically around the $\mathrm{T}_{4} 0$ transformation in the introduction; and the directionality-changes between heptachords in the first and second transitions. All of these alternating transformations underscore the importance of opposition in Crumb's music.

The functional relationship between tetrachords, due to the oscillation back and forth within a pentagonal segment, also draws attention to this opposition. This surfacelevel functionality is indicated as the upper row of functional descriptions and is not in bold font. Each chord oscillates between the local octatonic, the first chord of each pair, and the lower-octatonic, the second chord of each pair. On a higher functional level, the
chord progression maintains a retrogressive functional motion as was the case in the A section. The second iteration of the pattern allows both a completion of the $\mathrm{Oct}^{3}$ aggregate, which had been avoided until this point, and a return to the original string chord pair, which initiated the $A$ and $A^{1}$ sections of the work.

Measures 155-63 may be considered a post-cadential extension. The $\mathrm{d}^{1}$ section is a single measure devoted to fanfare figures in the trumpets, which outline concatenated major then minor triads which all can be modeled in $\mathrm{Oct}^{3} \mathrm{R}^{5<54>}$. Following this, the first B b-major and $\mathrm{C} \#$-minor string chords are repeated, followed by percussion and a turtledove effect, and then are repeated again until the music fades. ${ }^{26}$ Because all of the music is in the key of $\mathrm{Oct}^{3} \mathrm{R}^{5<54>}$, the octatonic established at the end of the $\mathrm{a}^{2}$ section extends through the $\mathrm{d}^{1}$ and $\mathrm{a}^{3}$ sections.

### 4.8 Octatonic Functional Design

A Haunted Landscape includes many different levels of harmonic function. Table 4.7 itemizes the harmonic functions of each section within the piece. For most of $A$ Haunted Landscape, there is very little motion at the first functional level because most of the chords within a given section occupy the same nodal locations within a pentagonal segment. For this reason, the first level of harmonic function is not included. The second level of harmonic function indicates the functional harmonic changes initiated by hyper-T

[^87]Table 4.7. Hierarchical octatonic functions in A Haunted Landscape

|  | A |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | intro | tr. | a | tr. | $\mathrm{a}^{\text {I }}$ |
| 2nd | $\mathrm{O} \rightarrow \mathrm{L} \rightarrow \mathrm{U}$ |  | $\mathrm{O} \rightarrow \mathrm{U}$ | $(\mathrm{U} \rightarrow \mathrm{L})$ | $\mathrm{L} \rightarrow \mathrm{O}$ |
| 3rd | O |  | $\mathrm{O} \rightarrow \mathrm{U} \rightarrow \mathrm{L} \rightarrow \mathrm{O}$ |  |  |
| 4th | U | $(\mathrm{U} \rightarrow \mathrm{O})$ | O |  |  |


transformations between octatonic modes. The third functional level indicates the local function of each section, while the fourth function level contextualizes the local key areas in relation to the global key of $\mathrm{Oct}^{3} \mathrm{R}^{5<54>}$.

The introduction features a progressive motion through its local octatonic, loweroctatonic and upper-octatonic modes in relation to the local key of $\mathrm{Oct}^{2} \mathrm{R}^{3<54\rangle}$. A sense of incompleteness is felt upon reaching the upper-octatonic, which reinforces $\mathrm{Oct}^{2} \mathrm{R}^{3<54>}$ as the local octatonic. Because of this, on the third functional level the entire introduction extends the local octatonic. On a global level, Oct $^{2} \mathrm{R}^{3<54>}$ functions as the upper-octatonic to the global octatonic $\mathrm{Oct}^{3} \mathrm{R}^{5<54>}$, which is indicated on the fourth functional level.

On the second functional level, the second transition connects the upper-octatonic of the a section with the lower-octatonic of the $\mathrm{a}^{1}$ section. At a higher level, both the a and the $\mathrm{a}^{1}$ sections complete a retrogressive motion starting and ending on the octatonic. Because the global octatonic frames these sections, this function extends across the entire A section on the fourth functional level.

The first and second transitions contain similar progressions, but function at different levels. The first transition connects the introduction to the A section while the second transition connects the a and the $\mathrm{a}^{1}$ sections. Even though the octatonic key for the B section was unable to be determined, the retransition is notated as moving to the global octatonic at the fourth functional level. Although not discussed in the analysis of the retransition, the two-mode hybrid chords may instill a sense of motion even if a retrogressive or progressive motion is difficult to establish. Consider the union of an octatonic and upper-octatonic mode moving to an upper-octatonic and lower-octatonic mode. Referring to Figure 3.8, either a progressive or a retrogressive transformation relates the respective merged modes; moving clockwise or counter-clockwise on the figure would map the position of one two-mode hybrid onto another.

The $\mathrm{a}^{2}$ section consolidates the alternating octatonic to lower-octatonic motion at the first functional level into the retrogressive motion to and from the octatonic on the second level. On the third functional level, the octatonic of the $\mathrm{a}^{2}$ section joins with the $\mathrm{d}^{1}$ and $\mathrm{a}^{3}$ sections to extend the octatonic through the entire $\mathrm{A}^{1}$ section.

At the deepest level of structure, not shown in Table 4.7, the introduction and first transition function as a large upper-octatonic upbeat to the compound ternary form, framed by the octatonic key of the A and $A^{1}$ sections. Although the A section utilized key areas representative of the $\mathrm{H} 1 \backslash 4 \backslash 5$ family, the eventual shift to key areas representative of the H2 4 family in the retransition and $A^{1}$ section represents a return to the key area of the introduction. This motion away from and back to key areas representative of the $\mathrm{H} 2 \backslash 4$ family may correspond to the endpoints of Crumb's "psychological curve."

### 4.9 Significance

As was demonstrated in the preceding analysis, the octatonic model is able to relate, through various transformations, the many disparate sets contained within $A$ Haunted Landscape ranging from trichords within a single octatonic mode to 10- and 11note hybrid chords. Although the difference in cardinality between such chords is large in pitch space, the difference in nodal cardinality is much smaller and, as was shown, increased or decreased gradually in octatonic space. Not only were these chords able to relate to one another on a surface level, but a number of transformational patterns emerged across sections. At higher functional levels, patterns emerged that related the A and $A^{1}$ sections of $A$ Haunted Landscape. In particular, the octatonic model was able to highlight the principles of opposition and completion, discussed in sections 2.3 and 2.4 respectively, and demonstrates the pervasiveness of octatonality. In the following chapter, the octatonic model's effectiveness beyond the scope of one work will be demonstrated through an analysis of Crumb's "Come Lovely," and will uncover comparable patterns at various levels of structure.

## 5 Come Lovely and Soothing Death

The text for Apparition is excerpted from the bird song verses of Walt Whitman's poem "When Lilacs Last in the Dooryard Bloom'd," Whitman's elegy for Abraham Lincoln and the soldiers that died in the Civil War. "Come Lovely" is the fifth movement of Crumb's Apparition; however, it was originally conceived as both the first and final movements (see Figure 5.1 and Figure 5.2). The verses in the early version of Apparition, shown in Figure 5.1, follow the same temporal order of verses as the bird song section of Whitman's "Lilacs." In the final version of Apparition, "Night in Silence" frames the work rather than "Come Lovely," which becomes the penultimate movement. Moreover, the first verse of Whitman's poem-not a section of the bird song-is included in the second movement of the final version, adding to the discontinuity between Whitman's text and Crumb's setting. This discontinuity is heightened within "Come Lovely" by Crumb's repetition of certain lines from Whitman's verse (see Table 5.1). Crumb repeats the first two lines of Whitman's verse and appends a hummed "mm." The third and fourth lines of Whitman's verse are not repeated and the hum is noticeably absent. In Crumb's setting, the return of the first line of Whitman's verse plus the appended hum, concludes the song.

[^88]Figure 5.1. Early movement structure of Apparition. Sketchbook " $A$," box 9, George Crumb Papers, Music Division, Library of Congress, Washington, D.C., 100. Courtesy of George Crumb.


Figure 5.2. Transcription of an early movement structure of Apparition


Table 5.1. Comparison of Whitman's 28th verse of "Lilacs" and Crumb's setting in "Come Lovely"

| Whitman | Crumb |
| :--- | :--- |
| Come, lovely and soothing Death, | Come, lovely and soothing death, mm <br> Come, lovely and soothing death, mm <br> Undulate round the world, mm <br> serenely arriving, arriving, mm <br> Undulate, undulate round the world, <br> Undulate round the world, serenely arriving, arriving, |
| serely arriving, arriving |  |
| In the day, in the night, to all, to each, | In thy, in the night, to all, to each, <br> Sooner or later delicate death. <br> Come, lovely and soothing death, mm |

Mutlu Blasing argues that various poetic conventions and devices, as well as grammatical constructions, "make for different temporal orders and create different fictions of time" in Whitman's "Lilacs," which include "natural-cyclical time, historical narrative time, and lyric time with its eternal present." ${ }^{2}$ Blasing notes that "while the [bird song] offers an acceptance of death, the form—a highly controlled, well-modulated temporal composition-escapes the kind of time that kills." ${ }^{3}$ Within the first verse of the bird's song-the verse which constitutes Crumb's "Come Lovely"-the first line represents natural-cyclical time through the willful invocation of death. The knowledge that death will arrive, rather than the possibility that death might arrive, attests to the

[^89]bird's and Poet's knowledge about the natural cycle of life and death. The second line represents lyric time and the eternal present signified by the suffix "-ing," which forms the present participle "arriving." The ever-present arrival of death at any time is represented by the repetition of the word "arriving." The final two lines utilize historical narrative time exemplified by the use of the words "day" and "night" in the third verse and the parallel words "sooner" and "later" in the final line.

These different aspects of time are present in Crumb's adaptation of Whitman's text. The repetition of the second line reinforces the never-ending phase of life and death as represented in the first line. The last two lines are the only two lines that do not repeat, remaining true to Whitman's temporal arrangement exemplifying historical narrative time. ${ }^{4}$ As will be shown, the musical setting of the text further emphasizes the distinction between the different temporal fictions of the text.

Jeffrey Steele argues that Whitman's bird song in "Lilacs" mediates between unexpressed grief and conscious mourning, and suggests "the thrush presents itself as an 'other' which the Poet recognizes as his own reflection." ${ }^{5}$ The song represents the Poet's attempt to regain artistic control and the Poet realizes "that his voice must rise from death and darkness, as well as from life and light, he starts to liberate the creative energy which had been swallowed." ${ }^{6}$ This sentiment is echoed by Patricia Young, who proposes, "On

[^90]the purely historical level . . . a man's unnatural death at the hands of another man-a violent 'crime'-becomes on more symbolic levels a natural, necessary creative event." ${ }^{7}$ Mark Edmundson expands this argument stating that Lincoln's death presented a moment of recreation because of the artist's realization that the established societal order was contingent and thus forms could be reinvented. ${ }^{8}$

The musical setting of the opening section of "Come Lovely" was originally set with the text from Lorca's "Paisaje" [Landscape] from the set of poems titled Poema de la Siguiriya gitana [Poem of the Gypsy Siguiriya] found in Poema del Cante Jondo [Poem of the Deep Song] (see Figure 5.3). ${ }^{9}$ The influence of Lorca's text remains after its replacement with Whitman's text, as will be demonstrated in the subsequent sections. In a lecture, Lorca described what he conceived as deep song:

Like the primitive Indian musical systems, deep song is a stammer, a wavering emission of the voice, a marvelous buccal undulation that smashes the resonant cells of our tempered scale, eludes the cold, rigid staves of modern music, and makes the tightly closed flowers of the semitones blossom into a thousand petals. . . . Deep song is akin to the trilling of birds, the song of the rooster, and the natural music of forest and fountain. ${ }^{10}$

[^91]Figure 5.3. The musical setting of "Come Lovely" with words from Lorca's "Paisaje."Sketches, box 5/folder 9, George Crumb Papers, Music Division, Library of Congress, Washington, D.C. Courtesy of George Crumb.


Many aspects of deep song align with Crumb's compositional predilections: a penchant for non-western music; a wistfulness for tonality brought about by a realization of the limitations of modern music; and an emphasis on a natural acoustic. ${ }^{11}$ More specifically, deep song relates in many ways to Whitman's poem, which utilizes a bird song to examine the undulating relationship between life, death, and artistic creation. ${ }^{12}$

For Lorca, the artist's relationship with death was important in order to convey duende, an essential characteristic of poetry, dance, bull fighting, music, and, most relevant for the analysis of "Come Lovely," deep song. In Lorca's lecture "Play and Theory of the Duende," the exact definition of duende is difficult to pinpoint. ${ }^{13}$ A couple of Lorca's many descriptions of duende appear in the following metaphorical forms:

Angel and muse escape with violin and compass; the duende wounds. In the healing of that wound, which never closes, lies the invented, strange qualities of a man's work. ${ }^{14}$
or
But the torero who is bitten by duende gives a lesson in Pythagorean music and makes us forget he is always tossing his heart over the bull's horns. ${ }^{15}$

[^92]death's importance in each description of duende is paramount and is personified as something visceral yet calculating. Christopher Maurer argues that Lorca's "Play and Theory of the Duende" is an "attempt to come to grips with an eternal artistic problem that had troubled him from early youth: a simultaneous longing for form and respect for chaos." ${ }^{16}$ Both Whitman and Lorca engage with this artistic problem in their poetry through the spectacle of death and, as will be demonstrated below, Crumb, in his setting of "Come Lovely." ${ }^{17}$

### 5.1 Formal Design

The formal analysis and segmentation of "Come Lovely" highlights aspects of the aforementioned temporal fictions contained within Whitman's "Lilacs" as well as Lorca's notion of deep song and duende. These topics, combined with supporting evidence found in Crumb's sketch materials, will converge to create a multi-dimensional framework in which to explore octatonality.

There are three main categories of musical motives within "Come Lovely": fragments of bird song, vestiges of tonality, and symbols of narrative-time. The fragments of bird song are clearly identifiable in the vocalise preceding "Come Lovely," which include a warble, trill, and a repeated-note staccato alarm (see Figure 5.4).

[^93]Figure 5.4. Bird-song components in Crumb's "Vocalise 3: Death Carol ('Song of the Nightbird')." Copyright © 1980 by C. F. Peters Corporation. All Rights Reserved. Used by permission.


Whitman describes the bird in "Lilacs" as follows:
Solitary, the thrush,
The hermit, withdrawn to himself, avoiding the settlements,
Sings by himself a song. ${ }^{18}$

While this description seems to align with the characterization of a Hermit Thrush,
Crumb's setting of the bird's song seems to imply the song of a Wood Thrush. Donald Kroodsma describes the Wood Thrush's song in two parts: "the loud, whistled prelude and the softer, fluty flourish that follows." ${ }^{, 19}$ The whistled prelude alternates between a wider range of pitches than the flourish and thus I have labeled the respective components of the song Warble and Trill..$^{20}$ The staccato Alarm is not a part of the Wood

[^94]Thrush's song proper; it is sung at a lower pitch when the Wood Thrush first awakes in the morning and before it sleeps at a night. ${ }^{21}$ If another Wood Thrush encroaches on the Wood Thrush's territory, the Alarm is used to ward off the invader using increasingly higher pitches. ${ }^{22}$ Whereas the Hermit Thrush also uses a Warble and Trill as components of its song, the Hermit Thrush uses a high-pitched screech for an alarm rather than the repeated notes of the Wood Thrush. ${ }^{23}$

Each of these bird-song components has a counterpart in "Come Lovely;"
representative examples of these components are provided in Figure 5.5. ${ }^{24}$ There are two distinct Warble motives; the first, labeled "Bird," is introduced in the piano and the second, labeled "Poet," in the voice. ${ }^{25}$ Similarly, there are two Trill motives; the first, labeled "Call," utilizes a lower-neighbour-note figure while the second, labeled "Undulate," utilizes an upper-neighbour-note figure. The Alarm motive is found almost exclusively in the repeated notes of the piano's quasi-tonal progression; Figure 5.5

[^95]Figure 5.5. Bird-song components in Crumb’s "Come Lovely." Copyright © 1980 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

## Warble



Trill
c) Call

d) Undulate

includes only the first chord of the first progression labeled "Elegy" (cf. Figure 2.6 for the complete progression). ${ }^{26}$

A vestige of tonality usually accompanies Warble (Bird) (see Figure 5.6). While the pianist's right-hand performs Warble (Bird) on the piano keys, the pianist's left-hand plucks a linear F-major triad on the piano strings using the fingertip (f.t.) and fingernail (f.n.), which is labeled "Vestige." Because of the difference in timbre and register between the two lines, these motives are understood as comprising two separate streams of music. ${ }^{27}$

The final motivic category includes motives that evince historical narrative characteristics. Two of these motives have a negligible impact on an octatonic analysis of the piece: the chord clusters achieved by striking the palms on the piano strings in the lower register, hereafter referred to as "Cluster," and the knocks on either the piano's soundboard or crossbeams, hereafter referred to as "Knock". As will be shown, Cluster often corresponds with hybrid chords or a change of octatonic key(s). Knock is used only

[^96]Figure 5.6. Tonal vestiges in Crumb's "Come Lovely." Copyright © 1980 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

three times: once at the beginning with Cluster, following the first Warble, and preceding the final Trill (Call). The primary symbol of historical narrative time is comprised of solid and arpeggiated chords based upon the interval of a perfect fourth, hereafter referred to as "Fourths" (see Figure 5.7). Kristina Szutor describes Fourths as both referential, relating to transformations within the earlier movement "Dark Mother," and "like a 'curtain', used to close off one section and set the stage for another., ${ }^{28}$ A number of other metaphors may be equally appropriate: the chimes of a children's book indicating when to turn a page, the downward and upward motion of sunset and sunrise, the cyclical representation of death and life, or the Pythagorean music of the torero as he tosses his

[^97]Figure 5.7. Historical narrative time symbols in Crumb's "Come Lovely." Copyright © 1980 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

## Fourths


heart over the bull's horns. ${ }^{29}$ Regardless of the metaphor ascribed to Fourths, they function as markers between temporal sections of "Come Lovely."

As discussed in the introduction to this chapter, the text portrays natural-cyclical, lyric, and historical narrative time. Table 5.2 correlates the motivic components of the bird song, tonal vestiges and historical narrative symbols with the three categories of fictional time. Motives related to the bird song are categorized in either natural-cyclic or lyric time. As mentioned above, the text "Come Lovely" is an example of natural-cyclic time, and thus Trill (Call) belongs to this category. The first half of the bird song, which precedes Trill (Call), includes Warble (Bird); to conform to the temporal progression of the Wood Thrush's song, Warble (Bird) is included in the same category as Trill (Call). For a similar reason Vestige, which represents a separate stream of music accompanying Warble (Bird), is included in this category. Although Elegy is not part of the bird song

[^98]Table 5.2. Correspondence of fictional times and motives in Crumb's "Come Lovely"

| Natural-Cyclic Time | Lyric Time | Historical Narrative Time |
| :--- | :--- | :--- |
| Warble (Bird) | Warble (Poet) | Cluster |
| Trill (Call) | Trill (Undulate) | Knock |
| Vestige |  | Fourths (Solid) |
| Alarm (Elegy) |  | Fourths (Arpeggio) |

proper (Warble + Trill), the low register of the progression, the tonal allusion the progression shares with Vestige, its identification as a component of the Wood Thrush's vocabulary, and the sequential nature of the progression is suggestive of natural-cyclic time. Both Warble (Poet) and Trill (Undulate) are examples of lyric time because of the accompanying text, which conveys a sense of the eternal present. Cluster, Knock, and Fourths are not related to parts of the bird song and, as discussed above, serve to reinforce various structural elements within the piece; thus, their function as markers of time situate them in the category of historical narrative time. ${ }^{30}$

The importance of Elegy is highlighted in Crumb's formal sketches of the work (see Figure 5.8). For each B section of the sketch, Crumb includes the key he associates with each statement of Elegy. An early sketch of this progression written with a $\mathrm{D} b$-major

[^99]Figure 5.8. Formal sketch of "Come Lovely." Sketchbook "B," box 9, George Crumb Papers, Music Division, Library of Congress, Washington, D.C., 48. Courtesy of George Crumb.

key signature includes the annotation "a germinal idea-but develop" (see Figure 5.9 and
Figure 5.10). ${ }^{31}$ The primacy of this progression is also highlighted by the "overlay" or "interposition" of additional streams of music over the $\mathrm{D} b$ and B 亿 Elegy themes (see

Figure 5.8). Crumb refers to this overlay as "falling tonality," similar to his annotation of

[^100]Figure 5.9. Elegy theme as a germinal idea. Sketchbook " $A$," box 9, George Crumb Papers, Music Division, Library of Congress, Washington, D.C., 42. Courtesy of George Crumb.


Figure 5.10. Transcription of the sketch of the elegy theme as a germinal idea

the "but falling" subject in Bach's D\#-minor fugue discussed in section 2.4 (see Figure 5.11). The additional dimensions created through different streams of music may be what Crumb was referring to in his penciled annotation to the right of "Come Lovely" in a near-final formal sketch of Apparition, which reads "multi dimension write climax?" (see

Figure 5.12 and Figure 5.13). A formal sketch of the different Elegy concatenations includes a drawing of a tesseract - a four-dimensional hypercube - to the right side of

Figure 5.11. Overlay of "falling tonality." Sketches, box 5/folder 9, George Crumb Papers, Music Division, Library of Congress, Washington, D.C. Courtesy of George Crumb.


Figure 5.12. Formal sketch of Apparition with reference to multiple dimensions in "Come Lovely." Sketchbook "B," box 9, George Crumb Papers, Music Division, Library of Congress, Washington, D.C., 46. Courtesy of George Crumb.


Figure 5.13. Transcription of formal sketch referencing multiple dimensions

another formal diagram (see Figure 5.14). ${ }^{32}$ Sidney Sheldon describes a tesseract as a non-Euclidian concept in his The Other Side of Midnight: "For Catherine time had lost its circadian rhythm; she had fallen into a tesseract of time, and day and night blended into one.,"33 This quote exemplifies all three of the fictional times discussed above: the narrator describes, through historical narrative time, the natural-cyclic time of Catherine's circadian rhythms, which meld into the lyric time of the eternal present where day and night become one. As will be shown in "Come Lovely," the different fictional times also merge, which demonstrates the Poet's gradual realization of death's inevitability.

[^101]Figure 5.14. Formal sketch of Elegy theme and four-dimensional tesseract. Sketches, box 5/folder 9, George Crumb Papers, Music Division, Library of Congress, Washington, D.C. Courtesy of George Crumb.


Table 5.3 correlates the motives and fictional times discussed above as they pertain to the formal structure of "Come Lovely." Abbreviations used in the table are defined in the legend below. ${ }^{34}$ Although "Come Lovely" alternates between unmeasured and measured time, I have included measure numbers that correspond to the bar lines placed in the score. A repetition of a motive is not indicated unless another motive interrupts it. For example, in the first measure there are multiple repetitions of Warble (Bird) which are interrupted by Knock before Warble (Bird) resumes. These segments within a single measure are indicated with dotted bar lines. Dashed horizontal lines separate the two streams of music contained within natural-cyclic time. At times, some

[^102]Table 5.3. Formal structure of "Come Lovely"

| Section | A |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Measure(s) | 1 |  |  |  | 2-3 | 4 | 5-6 | 7 |  |
| Lyric |  |  |  |  |  |  |  | T(U) | W(P) |
| Natural-cyclic |  | W(B) |  | W(B) | T(C) | W(B) | T(C) |  |  |
|  |  | V |  | V |  | V |  |  |  |
| Historical narrative | C, K |  | K | C | C | C | C |  |  |


| Section | B |  | $\mathrm{A}^{1}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Measure(s) | 8-14 | 14 | 15 |  |  |  |
| Lyric | > |  |  |  | T(U) | W(P) |
| Natural-cyclic |  |  |  |  |  |  |
| Natural-cyctic | E |  | (E) | > |  |  |
| Historical narrative |  | F(S) |  | $\mathrm{F}(\mathrm{A} \downarrow \uparrow)$ |  |  |


| Section | B ${ }^{1}$ |  | $\mathrm{A}^{2}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Measure(s) | 16-22 | 22 | 23 |  |  |  |  |
| Lyric | > |  |  |  |  |  | T(U) |
| Natural-cyclic |  |  |  |  |  | W(B) |  |
|  | E |  | (E) | > | L. 4 | V |  |
| Historical narrative | L. 3 | $\mathrm{F}(\mathrm{S})$ |  | $\mathrm{F}(\mathrm{A} \downarrow \uparrow)$ |  |  |  |


| Section | $\mathrm{B}^{2}$ |  |  | Coda |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Measure(s) | 24-30 | 27 | 30 | 31 |  |  | 32-33 |
| Lyric |  |  |  |  |  |  |  |
| Natural-cyclic | W(B), T(C), A |  |  |  |  |  | T(C) |
|  | E |  |  | (E) | > |  |  |
| Historical narrative |  | $\mathrm{F}(\mathrm{A} \downarrow)$ | F(S) |  | $\mathrm{F}(\mathrm{A} \uparrow)$ | K | C |

## Legend

| W(B) | Warble (Bird) | L. 3 | $3^{\text {rd }}$ line "Lilacs," Voice |
| :--- | :--- | :--- | :--- |
| W(P) | Warble (Poet) | L. 4 | $4^{\text {th }}$ line "Lilacs," Voice |
| T (C) | Trill (Call) | F(S) | Fourths (Solid) |
| T (U) | Trill (Undulate) | F (A $\downarrow \uparrow)$ | Fourths (Arpeggio) Down \& Up |
| E | Elegy | F (A $\downarrow$ or $\uparrow)$ | Fourths (Arpeggio) Down or Up |
| (E) | $1^{\text {st chord of next Elegy }}$ | K | Knock |
| A | Alarm | C | Cluster |
| V | Vestige | $>$ | Fade |

cells are merged in one time stream while cells are separated in another. For example, Elegy occurs in mm. 8-14 and Fourths (Solid) only happen in m. 14. ${ }^{35}$
"Come Lovely" is an example of modified strophic form; the A sections contain considerably more variety than the B sections, which are almost entirely comprised of Elegy. There are many similarities between the form of "Come Lovely" and the siguiriya gitana, which is generally considered "the deepest and saddest flamenco style." ${ }^{36}$ The siguiriya prominently features two devices that mimic weeping: queijíos ("ay-ay-ay" vocalizations) and jipío (extended wailing). ${ }^{37}$ The shorter Trill (Call) motives and the longer Trill (Undulate) motive in "Come Lovely" may be analogues to queijíos and jipío respectively. Lorca describes the siguiriya in his lecture on Deep Song:

The Gypsy siguiriya begins with a terrible scream that divides the landscape into two ideal hemispheres. It is the scream of dead generations, a poignant elegy for lost centuries, the pathetic evocation of love under other moons and other winds. Then the melodic phrase beings to pry open the mystery of the tones and remove the previous stone of the sob, a resonant tear on the river of the voice. ${ }^{38}$

[^103]The hemispheric divisions in "Come Lovely" correspond with the A and B sections of the work; the A sections are comprised of Warble and Trill while the B sections contain Elegy. ${ }^{39}$ The vocalise, which precedes "Come Lovely," may then represent the beginning terrible scream that divides the landscapes. As discussed previously, the first and second lines of Whitman's "Lilacs" are repeated, each ending with a hum (see Table 5.1). Rugoff argues that the hum breaks down the boundary between words and pure sound and that this breakdown is extended in the pitch domain for the final Trill (Call), which utilizes indistinct pitches. ${ }^{40}$ Candelas Newton describes a similar breakdown or lack of distinction as a characteristic of the siguiriya: "the siguiriya begins traditionally with an anguish-filled scream and continues with a series of vocal inflections with no distinct beginnings or ends." ${ }^{, 41}$ These indistinct endings found in "Come Lovely" are indicated by the fade symbol ">" in Table 5.3. The blending of the A and B sections points to another shared characteristic between the siguiriya and "Come Lovely." The rhythm of a siguiriya is generally looser when the vocalist sings with the guitar, similar to a recitative, (equivalent to the A sections in "Come Lovely") versus when the instrumentalist plays alone (equivalent to the B sections in "Come Lovely").

In the following analysis, transformations will be analyzed within and between natural-cyclic and lyric time; however, if two streams of natural-cyclic time are operative

[^104]at the same time they will be analyzed separately. Historical narrative time will not participate in the transformational networks of natural-cyclic or lyric time and only transformations between Fourths will be analyzed. Although Fourths will be represented using the octatonic model, it only serves to accentuate the underlying progressions of natural-cyclic and lyric time.

### 5.2 Section A

An octatonic analysis of the A section is illustrated in Figure 5.15. The three types of fictional times are indicated to the left of each stave: $\mathrm{H} / \mathrm{N}$ stands for historical narrative time; N-C stands for natural-cyclic time and U or L on either side of N-C identifies the upper or lower streams respectively; L, by itself, represents lyric time. A diamond and an X in the historical narrative time staff identify Cluster and Knock respectively. The first pitch-specific motives of "Come Lovely" include Vestige in the lower natural-cyclic stream and Warble (Bird) in the upper natural-cyclic stream. Notes without stems indicate a rhythmic reduction of pitch material. For example, the first Warble (Bird) tetrachord in the upper natural-cyclic stream is repeated twice as thirty-second note quintuplets but is represented only once in the reduction. A box around linear pitches indicates the object of transformation; if there is no box, the preceding vertical simultaneity is the object of transformation. As discussed earlier, the two streams of natural-cyclic time often occur in different transformational networks therefore the transformations between chords in the upper and lower natural-cyclic streams are notated

Figure 5.15. Octatonic analysis of the A section. Copyright © 1980 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

(Figure 5.15 continued)

(5)

above and below the respective staves. Transformations enclosed by parentheses at the beginning of each system remind the reader of the transformation leading to the chord from the same time stream. ${ }^{42}$ At times, either the upper or the lower streams of naturalcyclic time may transform into lyric time, or vice versa, which is indicated with a curved dashed arrow. Indicators of octatonic functional design are only included below the lower natural-cyclic staff for reasons that will be discussed later. The global key of "Come Lovely" is indicated inside a box at the top of the figure, enclosed with solid lines, while the local key of a specific passage is indicated with a box enclosed with small dashed lines. The key area for historical narrative time is illustrated using a box enclosed in larger dashed lines and is placed under the historical narrative staves (not shown in Figure 5.15 but appears in subsequent figures). A ROT transformation of a mode will apply to either natural-cyclic and lyric time (written above these staves) or historical narrative time (written below this stave). The notational practices described above will also apply for the subsequent sections of "Come Lovely." Additionally, although all transformations are included in the analytic figures, only the pertinent transformations will be discussed.

The first measure of upper natural-cyclic time alternates between different chords found in $\mathrm{Oct}^{3} \mathrm{R}^{\mathrm{B}<54>}$ and various hybrid chords found in $\mathrm{Oct}^{2,3} \mathrm{R}^{0, \mathrm{~B}<54>}$. Warble (Bird) could be found by concatenating six nodes duplicating two pitch classes to create a multiset; however, only four nodes are utilized to minimize the cardinality change to the

[^105]following hybrid chords, which are created using only two nodes. As a result, the cardinalities of the respective pc sets are equal while the cardinality of the nodes changes. The transposition values of all the chords in the first measure of natural-cyclic time do not change within a pentagonal segment. Although there is motion between the first four chords within a pentagonal segment, the last four chords maintain the same furthestclockwise nodal positions resulting in no change of the transposition values (except for an increase or decrease of nodal cardinality).

The second and third measures of upper natural-cyclic time feature a shift from $\mathrm{Oct}^{2,3} \mathrm{R}^{0, \mathrm{~B}<54>}$ to $\mathrm{Oct}^{1,2} \mathrm{R}^{\mathrm{A}, 0<54>}$ with Trill (Call). This reference to the global octatonic of the piece, $\mathrm{Oct}^{1} \mathrm{R}^{\mathrm{A}}$, materializes in the fourth measure, which alternates between chords found in $\mathrm{Oct}^{2} \mathrm{R}^{0<54>}$ and $\mathrm{Oct}^{1} \mathrm{R}^{\mathrm{A}\langle 54>}$. Although there is still no motion within a pentagonal segment, the motion between pentagonal segments increases. A rotation occurs, corresponding to Cluster in historical narrative time, and the two hybrid chords leading to m. 5 shift from Oct ${ }^{1,2} \mathrm{R}^{\mathrm{A}, 9<54>}$ to $\mathrm{Oct}^{2,3} \mathrm{R}^{9, \mathrm{~B}<54>}$. This pivotal moment, which is emphasized by Cluster, transitions to the second Trill (Call) in mm. 5-6.

The first and second Trill (Call) are found in octatonic keys representative of a H112 family, although the union of the two specific octatonic modes for each Trill (Call) imply a two-mode H 2 and H 1 family respectively. Trill (Undulate), which follows the second Trill (Call) in m. 7, utilizes all three octatonic modes to complete the aggregate in pitch-class space. This represents a convergence of modes as well as natural-cyclic and lyric time. The piano part's right- and left-hand complete the $\mathrm{Oct}^{2}$ and $\mathrm{Oct}^{3}$ collections respectively, and the voice projects a subset of $\mathrm{Oct}^{3}$ (see Figure 5.16). Although such a layering of octatonic collections might suggest a two-mode hybrid, this interpretation

Figure 5.16. Comparison of the three iterations of Undulate. Copyright © 1980 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

would require a concatenation of a significant number of nodes. As shown in Figure 5.16, later examples of Trill (Undulate) in m. 15 and m. 23 utilize all three octatonic collections. Three different three-mode hybrid families allow all twelve pitch classes to be represented using only five nodes: H112, H4, and H5\2. H112 was chosen because of its prevalence thus far, the prominence of semitones in the music, and its ability to model consistent transformations in subsequent iterations of Elegy. In H1 $12^{3,5}$, the aggregate can be found eight times through the union of nodes $h .1-\mathrm{h} .5$. The specific choice of nodes 2.1-2.5 to represent Trill (Undulate) in section A is based upon minimizing the transformational distance to and from Trill (Undulate) and maintaining similar transformations between motives later in the song.

The back-to-back setting of Trill (Call) and Trill (Undulate) reverses the temporal order of the Wood Thrush's song and highlights the temporal discontinuity of the text. Although both are classified as a motive relating to the Wood Thrush's song, Trill (Call) and Trill (Undulate) may be analogous to the weeping and extended wailing of the siguiriya respectively. Whereas the weeping of Trill (Call) is relegated to the voice part and two octatonic modes, the extended wailing of Trill (Undulate) is present in all parts and octatonic modes. Warble (Poet) follows Trill (Undulate), which leads to Elegy of the $B$ section. The $T_{2} 0$ transformations between trichords of Warble (Poet) are similar to the transformations between the first two Warble (Bird)'s in m. 4, which was the first time a subset of the global octatonic key was utilized, and the $\mathrm{T}_{2} 0$ transformation is the inverse of the $\mathrm{T}_{-2} 0$ transpositions found throughout the section. Although the transformational gestures are similar, it is the first time two $\mathrm{T}_{2} 0$ transpositions occur in a row, leading to the E-flat major chord of the B section.

In the A section, Vestige occurs only three times and is always set against Warble (Bird). The first instance of Vestige projects an F-major triad which occupies nodes 1.11.3 in $\mathrm{Oct}^{2} \mathrm{R}^{0<54>}$ and is transformed into an incomplete D -major triad via $\mathrm{T}_{2} 0$. The root of the D-major triad is implied in Figure 5.15, and is enclosed in parentheses, which is based upon the linear intervallic pattern established by the preceding F-major triad. In an earlier sketch of "Come Lovely," which used the text from Lorca's "Paisaje," a similar pattern was used to begin the song (see Figure 5.3). Vestige and Warble (Bird) begin the song, and the repetition of Vestige includes a root a minor seventh to form a D major-minor seventh chord. This provides additional evidence for the implied root of the D-major triad shown in Figure 5.15. In "Come Lovely," the third instance of Vestige is a C\#-minor triad found in $\mathrm{Oct}^{3} \mathrm{R}^{\mathrm{B}<54>}, \mathrm{a}\left\langle\mathrm{T}_{-1} 0\right\rangle^{2 \rightarrow 3}$ transformation of the previous Vestige, which places the chord midway between the first and second chords on the helical model. This transformational trajectory is maintained and $\left\langle\mathrm{T}_{-1} 0\right\rangle^{3 \rightarrow 1}$ connects the third Vestige chord with the first chord of Elegy of the B section, returning to the same nodal position in which Vestige started.

Harmonic function is notated only for the lower natural-cyclic stream. Although there is some surface-level functionality in the upper natural-cyclic and lyric streams, the prevalence of hybrid chords interspersed throughout, as well as Crumb's notation in the sketches that these streams are overlays, suggests that their harmonic significance is secondary to the lower natural-cyclic stream. The $\mathrm{Oct}^{2}$ collection, and its various rotations, is the local octatonic for the A section. It is the governing collection for the first two Vestige chords and is utilized in every hybrid chord of the upper natural-cyclic and lyric streams. On a local functional level, Vestige moves progressively from the octatonic
to the lower-octatonic; however, on a global functional level-in the key area of Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{A}, 9,8<54>}$-this translates to a progressive motion from lower-octatonic to upperoctatonic, which resolves to the global octatonic in the following B section.

### 5.3 Section B

The $B, B^{1}$ and $B^{2}$ sections are primarily comprised of Elegy, which was discussed in the first and second chapter (cf. Figure 1.1, Figure 2.6, Figure 2.7, and Figure 2.18). Warble (Poet) from the A section signifies the impending arrival of death, symbolized by the E-flat major triad, by anticipating the $\mathrm{Oct}^{1}$ collection (see Figure 5.17). The $\mathrm{T}_{3} 1$ transformation, which elides lyric and natural-cyclic time, also merges the A and B sections. ${ }^{43}$ The repetition of the final $\{901\}$ Warble (Poet) set to the text "mm" gradually fades from lyric to natural-cyclic time. Additional pitches notated on the lower staff of lyric time, which alternate between descending intervals of 10 and 11 , relate to the ascending interval of 11 of Warble (Poet) and are inversionally related to the whole-tone alternations of Trill (Call) and the semitone alternations of Trill (Undulate). Each of the three dyads in the lower staff of lyric time can be found in the Oct ${ }^{3}$, Oct ${ }^{1}$ and Oct ${ }^{3}$ collections respectively, which provides a foil to the underlying $\mathrm{Oct}^{1}, \mathrm{Oct}^{3}$ and Oct ${ }^{1}$ chords of Elegy. This opposition, as well as the intervallic content, is not maintained in the $B^{1}$ and $B^{2}$ sections and, as the sketch materials imply, the exact intervallic content

[^106] Figure 5.15 with arrow brackets-creating the trichord $\{901\}$.

Figure 5.17. Octatonic analysis of the B section. Copyright © 1980 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

does not seem to be of primary importance (see Figure 5.18 and Figure 5.19). In the sketch, Elegy opens with an E-major triad accompanied by \{57\}, both members of Oct ${ }^{3}$. Subsequent repetitions of the interval in the right-hand of the piano part include only the rhythm of the motive rather than any distinct pitches. For these reasons, the lower staff of lyric time can be understood as an overlay that participates in the gradual weakening of lyric time in favour of natural-cyclic time. As discussed above, Rugoff argues that this marks the boundary between words and pure sound. This lack of distinction for phrase endings is also characteristic of the siguiriya; as Lorca would say, this is the point that "the melodic phrase beings to pry open the mystery of the tones and remove the previous stone of the sob, a resonant tear on the river of the voice., ${ }^{44}$

Figure 5.18. Sketch of the Elegy theme with indistinct pitch overlay. Sketches, box 5/folder 9, George Crumb Papers, Music Division, Library of Congress, Washington, D.C. Courtesy of George Crumb.


[^107]Figure 5.19. Transcription of Elegy theme sketch with indistinct pitch overlay

Episode \#4 [soft intimation elegaic music]

[PIII. sempre]

Elegy, found in the lower natural-cyclic time stream, pries open the mystery of the tones and is entirely comprised of chords that each reside in a single octatonic mode-no hybrid chords are utilized. On a surface level, not shown in Figure 5.17, the motion within pentagonal segments aligns with the tonal interpretation of the passage in Figure 1.1. A retrogressive motion from the first to second chord and a progressive motion from the second to third, models the tonal interpretation moving from tonic to dominant and back to tonic. The motion to the fourth chord is progressive and coincides with the motion from tonic to predominant in the tonal interpretation. One step removed from the surface, the motion between the octatonic modes for each chord is analogous to the functions of the tonal interpretation for the first four chords and is notated below each
chord in the lower natural-cyclic time stream. The final three chords that conclude Elegy do not exhibit surface-level functionality since there is no motion within a pentagonal segment; however, progressive motion between octatonic modes of the final three chords moves from the octatonic to the lower-octatonic to the upper-octatonic. The last three Elegy chords are related, in turn, by the base transformation of $\left\langle\mathrm{T}_{-2} 0\right\rangle$, the same transformation that was prominent in the A section as well as the inverse of Warble (Poet) that led to the B section. The final chord of Elegy is related to the first chord in the lower natural-cyclic stream of the $\mathrm{A}^{1}$ section by $\mathrm{T}_{-2}-1$, which is a surface-level retrogressive function and remains in the same octatonic mode.

As a whole, Elegy functions on a global level as the octatonic since the local key is based upon Oct ${ }^{1} \mathrm{R}^{\text {A<54> }}$. This mode's importance, as compared to the other modes, is reinforced by its position at the beginning of the section and alternation with the other two opposing collections in $\mathrm{mm} .8-12$. The final two measures suggest that a return to the octatonic on the local and global levels is imminent, but does not materialize.

On a separate level, Fourths (Solid) interject while the final chord of Elegy is still ringing. The key is written below the historical narrative staff and is representative of a H5\2 family. While this chord, and subsequent Fourths, could also be found in a key defined by the rotations of the $\mathrm{H} 2 \backslash 4$ family, the alignment of modes found in a $\mathrm{H} 5 \backslash 2$ family corresponds with the left- and right-hand (027) set class in the piano part. In addition, if keys from a H2\4 family were utilized, ROT transformations would have to occur between Fourths (Solid) and Fourths (Arpeggio), which occur at the beginning of the next section; however, keys representative of the $\mathrm{H} 5 \backslash 2$ family do not require a ROT transformation between the consecutive Fourths. Fourths, both Solid and Arpeggio, are
interpreted as three-mode-three-node hybrid chord multi-sets even though they could be interpreted as three-mode-two-node hybrid chords. The representation of Fourths as a multi set maintains the same cardinality of nodes with Fourths in the $\mathrm{B}^{2}$ section and coda, which can only be constructed from the union of three nodes. Although the $\mathrm{T}_{-1} 1$ transformation to Fourths (Arpeggio) of the $\mathrm{A}^{1}$ section indicates a surface-level progressive functionality, both Fourths are not ascribed any higher-level octatonic function since they are hybrid chords. As discussed previously, the motives in historical narrative time simply accentuate the formal boundaries within the song.

### 5.4 Section $A^{1}$

The $A^{1}$ section begins with a $D b$-major triad, which anticipates the triad that begins Elegy in the following section (see Figure 5.20). The Pythagorean Fourths (Arpeggio) follow the Db-major triad-throwing the cloak over the bull's horns-and switch the focus from natural-cyclic to lyric time. ${ }^{45}$ A rotation of $\mathrm{Oct}^{3} \mathrm{R}^{8<54>}$ into Oct $^{3} \mathrm{R}^{\mathrm{B}\langle 54>}$, via $\mathrm{ROT}_{2}{ }^{3}$, shifts the key of both lyric and natural-cyclic time to Oct $^{3,2,1} \mathrm{R}^{\mathrm{B}, \mathrm{A}, 9<54>}$ for the remainder of the $\mathrm{A}^{1}$ and $\mathrm{B}^{1}$ sections. Unlike the first A section, $A^{1}$ only includes the temporally reversed sequence of Trill (Undulate) and Warble (Poet) in lyric time. The word "undulate" is repeated twice and Trill (Undulate) completes two aggregates in mod-12 pitch-class space as well as completing the aggregate of each

[^108]Figure 5.20. Octatonic analysis of the $\mathrm{A}^{1}$ section. Copyright © 1980 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

octatonic collection (see Figure 5.16). The global importance of the Oct ${ }^{1}$ collection is highlighted by the repetition of the $\mathrm{Oct}^{1}$ collection in the first two and last two dyads of the right- and left-hand piano parts respectively. The motion from Oct ${ }^{2}$ to $\mathrm{Oct}^{3}$ in the leftand right-hand piano parts respectively-note the reversal of the hands-is the same global motion of Vestige in the A section. The second Trill (Undulate) is transformed to Warble (Poet) by $>\mathrm{T}_{-1} 2_{-2} \ll^{1,2,3 \rightarrow 1,2}$, which is the same base transformation that connected the corresponding motives in the A section (cf. Figure 5.15, m. 7). ${ }^{46}$

Warble (Poet) occurs in both the vocal and piano parts: the trichords of the vocal part are all found in $\mathrm{Oct}^{1}$ while the trichords echoed by the piano are found in $\mathrm{Oct}^{2}$. Within the key area of $\mathrm{Oct}^{3,1,2} \mathrm{R}^{\mathrm{B}, \mathrm{A}, 9<54>}$, each trichordal pair occurs at the same nodal positions and thus could be modeled either as $\left\langle\mathrm{T}_{0} 0\right\rangle$ transformations between modes or joined as hybrid chords; the latter option is chosen for this analysis for reasons which will be made clear below. The $T_{2} 0$ transformation between the first two hybrid chords is the same transformation as was found in the corresponding passage of the A section; however, the piano does not accompany the final Warble (Poet) in the voice and thus a splitting hyper-transformation is notated which still retains the base $\mathrm{T}_{2} 0$ transposition value. Until this point, only Warble (Bird) was played by the piano. The piano's union with the voice in Warble (Poet) may reflect Jeffrey Steele's observation that the bird song in Whitman's "Lilacs" represents the point in which the Poet recognizes his own reflection in the Bird and attempts to regain artistic control. ${ }^{47}$ Thus, the merger of two

[^109]octatonic modes into a hybrid chord—a type of Doppelgänger-is fitting. Lorca's concept of duende is present here: the doubly-extended wailing of Trill (Undulate) emphasizes the pain and suffering that is necessary to allow the invented and strange qualities of the Poet's work to surface.

The harmonic function of the $\mathrm{D} b$-major triad changes as the $\mathrm{A}^{1}$ section progresses. Although the triad was the upper-octatonic of the B section's key, the function of the triad gradually transforms into the octatonic of the $B^{1}$ section's key, which is notated below the staff as U/O. The pivotal function of the $\mathrm{ROT}_{2}{ }^{3}$ transformation and subsequent transitional hybrids of lyric time gradually shift the focus from Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{A}, 9,8<54>}$ to Oct $^{3,1,2} \mathrm{R}^{\mathrm{B}, \mathrm{A}, 9<54>}$; at a global level this is notated as " $\rightarrow \mathrm{U}$, " read as "becoming the global upper-octatonic."

### 5.5 Section B ${ }^{1}$

The Elegy theme in the $B^{1}$ section contains equivalent transformations and surface-level functionality as the previous B section; however, they differ in many ways (see Figure 5.21). The local octatonic key has shifted to $\mathrm{Oct}^{3} \mathrm{R}^{\mathrm{B}<54>}$, rather than Oct ${ }^{1} \mathrm{R}^{\mathrm{A}\langle 54>}$ and the Elegy progression functions on a global level as the upper-octatonic rather than the octatonic. In the B section, the final Warble (Poet) was transformed into the first chord of Elegy by $\mathrm{T}_{3} 1$ whereas in the $\mathrm{B}^{1}$ section, a change of mode necessitates a hyper-transformation of $\left\langle\mathrm{T}_{3} 1\right\rangle^{1 \rightarrow 3}$ (cf. Figure 5.17). In the $\mathrm{B}^{1}$ section, the right-hand of the piano fades, like in the $B$ section, but with different pitches and interval sizes, which

Figure 5.21. Octatonic analysis of the $\mathrm{B}^{1}$ section. Copyright © 1980 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

echo the contour and rhythm of last two pitches of Warble (Poet). Most importantly, the repetition of Warble (Poet) with the indistinct text " mm " is absent; instead, the vocal part shifts to historical narrative time.

The Poet's attempt to regain artistic control, implied by the hybrid chords of Warble (Poet) of the preceding section, proceeds with a relatively strict and uncharacteristic vocal phrase in historical narrative time. Alternations between quasiscalar whole-tone and octatonic passages lead to disjunct motion accompanying the text "to all, to each." The dual interpretation of the last segment as a subset of WT ${ }^{2}$ or Oct $^{2}$ alludes to the merger of the Bird and Poet in the previous section. The segments of the historical narrative vocal line identified in Figure 5.21 are not suited to an octatonic analysis, although hybrid chords could account for all but the $\mathrm{Oct}^{1}$ segment. ${ }^{48}$ Instead, the vocal line is interpreted as the Poet's disembodied attempt to regain artistic control through an exploration of alternative musical artifacts. With this section's final Elegy chord, Fourths (Solid) marks the end of the section returning to the same nodal position as Fourths (Solid) of the B section.

### 5.6 Section $A^{2}$

Like the previous $A^{1}$ section, $A^{2}$ begins with a major triad, transformed from the previous Elegy's final chord by $\mathrm{T}_{-2}-1$, which anticipates the first Elegy chord of the next

[^110]section (see Figure 5.22). Fourths (Arpeggio) follows but is extended an additional fourth. The voice joins the piano's Fourths (Arpeggio) with the text "sooner or later," continuing in the previously established historical narrative time and emphasizing the lowest fourths of the preceding piano part. The vocal part continues with the words "delicate death," which is set to a descending arpeggiation of an A-major triad. ${ }^{49}$ This triad no longer conforms to the Fourths motive nor is similar to the scalar or disjunct motion found in the preceding line of the $B^{1}$ section. The triad shares the same register as Vestige and, as shown in Figure 5.22, is connected to Vestige in lower natural-cycle time forming a major-minor seventh chord. ${ }^{50}$ The text, which is representative of historical narrative time, is accompanied by music representative of natural-cyclic time. This marks the point that the Poet realizes death's necessity and shifts from historical narrative to natural-cyclic time.

Trill (Undulate) is an abbreviation of the previous statement found in the $A^{1}$ section: the first two and last two dyads in the right and left hands found in Trill (Undulate) in $\mathrm{A}^{1}$ comprise Trill (Undulate) of $\mathrm{A}^{2}$ (cf. Figure 5.16). Trill (Undulate) serves as a nexus of transformational paths: both streams of natural-cyclic time merge to lyric time and then diverge to their respective time streams for the $\mathrm{B}^{2}$ section. Warble (Poet) does not follow Trill (Undulate) as it did in the A and $\mathrm{A}^{1}$ sections; however, the components of the bird's song remain intact: Warble (Bird) proceeds directly to Trill

[^111]Figure 5.22. Octatonic analysis of the $\mathrm{A}^{2}$ section. Copyright © 1980 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

(Undulate). The union of natural-cyclic and lyric time mirrors the Poet's realization that both life and death are required to liberate creative energy.

### 5.7 Section $B^{2}$

The Elegy progression of $B^{2}$ follows a similar transformational route charted in the $B$ and $B^{1}$ sections (see Figure 5.23). In this section, $\operatorname{Oct}^{2} \mathrm{R}^{0<54>}$ is the local octatonic and functions as the lower-octatonic on the global level. Unlike the previous B sections, an extensive bird song is overlaid, which previously only occurred in the A sections. The majority of the bird song is quite different from the bird motives discussed earlier:

Warble (Bird) only happens once at the end of m . 25. While the bird song is different, it still evinces the characteristics of the Wood Thrush's song. For example, the upper natural-cyclic stream in m. 24 includes Warble, Trill and Alarm in a linear succession. The wide grace-note arpeggios and reversal of direction is characteristic of Warble, the neighbour-note motion from C to D is similar to the Thrush's Trill, and the repeated shrill A's, which finish the measure, are the closest approximation of the Wood Thrush's Alarm presented thus far. The improvisatory nature of the bird song may allude to the Poet's understanding and acceptance of natural-cyclic time rather than simply representing it with constructed motives. Although the Poet may have found duende, a moment of doubt appears. In m. 25 Warble (Bird) disrupts the natural cycle resulting in the highest and lengthiest Alarm of the song in mm. 26-27. Following this Alarm, the cloak of the torero prematurely falls—Fourths (Arpeggio $\downarrow$ )—and the Oct ${ }^{1}$ stab of historical narrative time in m .28 seals the torero's fate.

Figure 5.23. Octatonic analysis of the $\mathrm{B}^{2}$ section. Copyright © 1980 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

(Figure 5.23 continued)


### 5.8 Coda

The Coda is similar to the other A sections in that the major triad of natural-cyclic time is transformed via $\mathrm{T}_{-2}-1$ from the final Elegy chord of the previous section (see Figure 5.24). If the song had continued the cyclic pattern, the octatonic would have been reestablished in the following $\mathrm{B}^{3}$ section. Fourths (Arpeggio $\uparrow$ ) also implies a continuation of the life-death cycle as the torero rises from the dust. The return of the wailing Trill (Bird) includes the indistinct hum from the A section and a glissando of indistinct pitches. ${ }^{51}$ The narrative presented thus far has established a trend of increasing self-awareness for the Poet but the blurring of both pitch and text imply a regression and loss of artistic creativity. This indistinctness, however, could represent the cyclical nature of the artistic process, which mimics the cycle of life and death and requires constant rejuvenation.

[^112]Figure 5.24. Octatonic analysis of the coda. Copyright © 1980 by C. F. Peters Corporation. All Rights Reserved. Used by permission.

(31)


### 5.9 Octatonic Functional Design

The different levels of octatonic function of the lower natural-cyclic time stream are illustrated in Table 5.4. As was the case in the analysis of A Haunted Landscape, the surface-level functionality is not included in the table. At the second functional level, the $B$ and $B^{1}$ sections feature isographic hyper-transformations between different octatonic modes because the local key area and local octatonic modes change for each section. As discussed previously, the second-level functionality of each B section can be understood

Table 5.4. Hierarchical octatonic functions in the lower natural-cyclic time stream of "Come Lovely"

|  | A | B | $\mathrm{A}^{\mathrm{L}}$ | $\mathrm{B}^{1}$ |  |
| :---: | :--- | :--- | :---: | :--- | :---: |
| 2nd | $\mathrm{O} \rightarrow \mathrm{L}$ | $\mathrm{O} \rightarrow \mathrm{U} \rightarrow \mathrm{O} \rightarrow \mathrm{L} \rightarrow \mathrm{O} \rightarrow \mathrm{L} \rightarrow \mathrm{U}$ | $\mathrm{U} / \mathrm{O}$ | $\mathrm{O} \rightarrow \mathrm{U} \rightarrow \mathrm{O} \rightarrow \mathrm{L} \rightarrow \mathrm{O} \rightarrow \mathrm{L} \rightarrow \mathrm{U}$ |  |
| 3rd | $\mathrm{L} \rightarrow \mathrm{U}$ | O | $\rightarrow \mathrm{U}$ | U |  |
| 4th | $\mathrm{L} \rightarrow \mathrm{U}$ | O | U |  |  |
| 5th | $\mathrm{L} \rightarrow \mathrm{U}$ | O |  |  |  |


|  | $\mathrm{A}^{2}$ | $\mathrm{~B}^{2}$ | Coda |
| :--- | :--- | :--- | :--- |
| 2nd | $\mathrm{U} \rightarrow \mathrm{L} / \mathrm{U}$ | $\mathrm{O} \rightarrow \mathrm{U} \rightarrow \mathrm{O} \rightarrow \mathrm{L} \rightarrow \mathrm{O} \rightarrow \mathrm{L} \rightarrow \mathrm{U}$ | $\mathrm{U} / \mathrm{O}$ |
| 3rd | $\rightarrow \mathrm{L}$ | L | $\rightarrow \mathrm{O}$ |
| 4th | L | O |  |
| 5th | (O cont. ) |  |  |

as representing a single octatonic function at the third and fourth functional levels, which moves retrogressively from the octatonic to the upper-octatonic to the lower-octatonic.

The intervening $\mathrm{A}^{1}$ and $\mathrm{A}^{2}$ sections facilitate this retrogressive motion by anticipating the first chord of the subsequent Elegy progression and utilizing hybrid chords that enable the transition from one octatonic key to another and maintain the transformational and functional structure of the B sections. Therefore, the "becoming" function of the $A^{1}$ and $A^{2}$ sections on the third hierarchical level merges with the global function of the $B^{1}$ and $B^{2}$ sections respectively on the fourth level. The right side of cells in the third to fifth functional levels are left open to represent the implied continuation of the cycle and potential return of the octatonic in the fourth Elegy progression.

### 5.10 Significance

The above analysis of "Come Lovely" integrated narrative aspects of both Whitman's "Lilacs" and Lorca's "Paisaje" to paint an intertextual interpretation of the creative necessity of the natural cycle of life and death. The segmentation of the music into different streams-based upon a textual analysis, register, pitch-class content, and timbre-correlated with various octatonic structures. The upper natural-cyclic time stream primarily alternated between chords from singular octatonic modes and hybrid chords while the lower natural-cyclic time stream consisted entirely of chords from singular octatonic modes. The lyric time stream was similar to the upper natural-cyclic time stream, but emphasized hybrid chords over single-mode chords. Together, the natural-cyclic and lyric time key areas were representative of a H112 family, which corresponds with the chromatic nature of the song in pitch space. The narrative time stream was represented by key areas of a $\mathrm{H} 5 \backslash 2$ family, which did not participate in the transformational structures of natural-cyclic and lyric time streams. Pragmatically, approaching the analysis of "Come Lovely" in this way allowed a distinct functional progression to emerge, and allowed the diversity of the overlaid musical objects to be recognized.

## 6 Conclusions

In this closing chapter, I will compare the analyses of A Haunted Landscape and "Come Lovely" to demonstrate how the model of octatonality can illuminate similarities and differences between the two works. I will then discuss the possible extension of the model to other works by Crumb, and will discuss suggestions for the application of the theory. Finally, I will propose suggestions for future research in octatonality as well as generic functional-yet-transformational (FYT) models.

### 6.1 Comparison of A Haunted Landscape and "Come Lovely"

The overall functional schemas of A Haunted Landscape and "Come Lovely" are similar in many respects (see Table 6.1). In A Haunted Landscape the introduction served as a global upper-octatonic upbeat that moved progressively to the octatonic of the A section. In "Come Lovely," the A section also features an off-octatonic opening, moving progressively from the global lower- to upper-octatonic and finally resolving to the octatonic of the B section at the third, fourth, and fifth functional levels. In addition, the retrogressive motions in both the A and $\mathrm{A}^{1}$ sections of A Haunted Landscape, which begin and end on the octatonic at the third and second functional levels respectively, are analogous to the retrogressive motion at the fourth functional level in "Come Lovely" between the B sections and coda. At the fourth level, the octatonic of the B section and

Table 6.1. Comparison of A Haunted Landscape and "Come Lovely"
a) A Haunted Landscape

|  | A |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | intro | tr. | a | tr. | $\mathrm{a}^{1}$ |
| 2nd | $\mathrm{O} \rightarrow \mathrm{L} \rightarrow \mathrm{U}$ |  | $\mathrm{O} \rightarrow \mathrm{U}$ | $(\mathrm{U} \rightarrow \mathrm{L})$ | $\mathrm{L} \rightarrow \mathrm{O}$ |
| 3rd | O |  | $\mathrm{O} \rightarrow \mathrm{U} \rightarrow \mathrm{L} \rightarrow \mathrm{O}$ |  |  |
| 4th | U | $(\mathrm{U} \rightarrow \mathrm{O})$ | O |  |  |


|  | TR. | B | RTR. | $\mathrm{A}^{1}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | $\mathrm{a}^{2}$ |  | $\mathrm{d}^{1}$ | $\mathrm{a}^{3}$ |
| 2nd | N/A |  |  | $\mathrm{O} \rightarrow \mathrm{U} \rightarrow \mathrm{L} \rightarrow \mathrm{O}$ | $\mathrm{O} \rightarrow \mathrm{U} \rightarrow \mathrm{L} \rightarrow \mathrm{O}$ | O | O |
| 3rd |  |  |  | O | O |  |  |
| 4th |  |  | $\rightarrow \mathrm{O}$ |  | O |  |  |

b) "Come Lovely"

|  | A | B | $\mathrm{A}^{1}$ | $\mathrm{~B}^{1}$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| 2nd | $\mathrm{O} \rightarrow \mathrm{L}$ | $\mathrm{O} \rightarrow \mathrm{U} \rightarrow \mathrm{O} \rightarrow \mathrm{L} \rightarrow \mathrm{O} \rightarrow \mathrm{L} \rightarrow \mathrm{U}$ | $\mathrm{U} / \mathrm{O}$ | $\mathrm{O} \rightarrow \mathrm{U} \rightarrow \mathrm{O} \rightarrow \mathrm{L} \rightarrow \mathrm{O} \rightarrow \mathrm{L} \rightarrow \mathrm{U}$ |  |
| 3rd | $\mathrm{L} \rightarrow \mathrm{U}$ | O | $\rightarrow \mathrm{U}$ | U |  |
| 4th | $\mathrm{L} \rightarrow \mathrm{U}$ | O | O |  |  |
| 5th | $\mathrm{L} \rightarrow \mathrm{U}$ |  |  |  |  |


|  | $\mathrm{A}^{2}$ | $\mathrm{~B}^{2}$ | Coda |
| :--- | :---: | :---: | :---: |
| 2nd | $\mathrm{U} \rightarrow \mathrm{L} / \mathrm{U}$ | $\mathrm{O} \rightarrow \mathrm{U} \rightarrow \mathrm{O} \rightarrow \mathrm{L} \rightarrow \mathrm{O} \rightarrow \mathrm{L} \rightarrow \mathrm{U}$ | $\mathrm{U} / \mathrm{O}$ |
| 3rd | $\rightarrow \mathrm{L}$ | L | $\rightarrow \mathrm{O}$ |
| 4th | L | O |  |
| 5th | (O cont. ) |  |  |

the implied octatonic of the coda frame "Come Lovely" in a manner similar to the way in which the octatonic of the A and $\mathrm{A}^{1}$ section frame $A$ Haunted Landscape.

In both works, the refrain sections-the $\mathrm{a}, \mathrm{a}^{1}$ and $\mathrm{a}^{2}$ sections in A Haunted Landscape and the B, B' ${ }^{1}$, and $\mathrm{B}^{2}$ sections in "Come Lovely"-primarily used trichordal structures found within a single octatonic mode. Whereas in A Haunted Landscape there was no surface-level functionality within the refrains, "Come Lovely" alternated between
surface-level functional motion and stasis between the first-four and last-four chords in Elegy respectively. ${ }^{1}$ The intervening sections of both works-transitions in A Haunted Landscape and the $\mathrm{A}^{1}$ and $\mathrm{A}^{2}$ sections in "Come Lovely"-primarily utilized hybrid chords which functioned to pivot to the key areas of the respective refrains.

The key areas of both works differed dramatically: A Haunted Landscape utilized key areas representative of a $\mathrm{H} 2 \backslash 4$ family for the introduction and refrains and a $\mathrm{H} 1 \backslash 4 \backslash 5$ family for the transitions, whereas in "Come Lovely," key areas representative of a H1\2 family were primarily used for the entire song. Although major and minor triads were central to each work, the hybrid chords utilized in the sections between refrains helped define the key of each work. The octatonic model's strength is its ability to highlight major differences between works, such as key areas, while relating them functionally.

### 6.2 Applicability to Other Works by Crumb

The similarities between A Haunted Landscape and "Come Lovely" are not surprising given that they were composed during the same period and Crumb's predilection for a repurposing of musical material between different works. ${ }^{2}$ For the octatonic model to be applicable to other works, the work should feature trichords or

[^113]tetrachords of a single octatonic mode, which include instances of scs (037), (016), (025), (014), (026), (0258), (0146), and (0147). As discussed above, sets representative of these set classes usually appear in refrain-like passages, and form the basis for functional octatonality. The number of such set classes is limited and, as shown in the first chapter, occur repeatedly throughout Crumb's catalogue. A non-exhaustive list of potential pieces that could be interpreted using the octatonic model include: "God-music" from Black Angels (see Figure 2.1); "Primeval Sounds" (see Figure 2.12, Figure 2.15, Figure 2.16, and Figure 2.17), "Pastorale" (see Figure 2.3), and "Music of Shadows" (see Figure 2.8) from Makrokosmos I; and "Music of the Starry Night" from Music for a Summer Evening (see Figure 2.26, Figure 2.28, and Figure 2.30). In general, it appears that most of the works suitable for interpretation using the model of octatonality were composed in the early 1970s to the mid-1980s.

Although hybrid chords are not essential for a functional octatonic analysis, their presence often indicates various levels of functionality. As demonstrated in the analysis of A Haunted Landscape in the fourth chapter, hybrid chords were utilized to modulate between keys representative of different hybrid families. In the analysis of "Come Lovely" in the fifth chapter, hybrid chords modulated between different modal orderings of key areas representative of the same hybrid family. It is reasonable to presume that the structural significance of hybrid chords will be maintained in other octatonic works by Crumb. To maintain the significance of hybrid chords, it is important that they serve to accentuate chords found within a single octatonic mode. Because eighty-three different set classes are found within the different hybrid families, versus eight set classes of cardinality three or four within a single octatonic mode in this model, it may be tempting
to elevate hybrid chords over chords found in a single octatonic mode; however, doing so would obfuscate octatonic functionality. ${ }^{3}$

Other works by Crumb do not seem to fit this model; in some of these cases, such as in Processional, I sense a similar process at work. The opening of this piece has the introductory character found in both A Haunted Landscape and "Come Lovely" while the middle section includes verticalities characteristic of the refrains in the latter two pieces (see Figure 2.10 and Figure 2.11). ${ }^{4}$ The six-note diatonic subset, (024579), which begins each sub-section of the opening section can be found in either the $\mathrm{H} 2 \backslash 4^{3,2}$ or $\mathrm{H} 5 \backslash 2^{3,2}$ or 3 families. Instances of the (0158) tetrachords of the middle section, which combine to form instances of Messiaen's fourth mode, can be found in the $\mathrm{H} 4^{2,2}$ or $\mathrm{H} 5^{2,2}$ families. The similar intervallic structure of the three- or two-mode hybrid families of the hybrid chords from the opening and middle sections suggests a good fit with the model of octatonality; however, without chords contained within a single octatonic mode, higherlevel functional motions cannot be ascertained. This suggests that the motion between two-mode hybrid chords in works such as Processional may be functional and requires further investigation or an alternate version of the model should be considered.

The model of octatonality presented in this dissertation utilized a specific SIA for each octatonic mode, which accounted for functional motions in A Haunted Landscape

[^114]and "Come Lovely" and could be useful for analyzing the works suggested in this section. Using the octatonic collection as a base, there are numerous other possible configurations, which could model a different collection of set classes. One prevalent set class in Crumb's music that is missing from the octatonic model, either as a chord from a single octatonic mode or as a hybrid chord, is sc (012). Although only the octatonic collection was utilized for the model, many other collections could also be used to create analogous functional models. In particular, the diatonic collection, whole-tone collection and Messiaen's fourth mode may be particularity suited for other works by Crumb. For example, if the sc (0158) tetrachords from Processional were to be integrated into a functional model, an instance of that set class should be a subset and unique identifier of the originating collection. For example, the first pitch set of the middle section, $\{9 \mathrm{~A} 25\}$, is found in two diatonic collections, $\mathrm{D}^{4}$ and $\mathrm{D}^{9}$, the harmonic minor collection, $\mathrm{Hm}^{4}$, and Messiaen's fourth mode, $\mathrm{F}^{2}$ (see Figure 2.11). Because the set cannot distinguish between transpositions of the diatonic collections, the model should incorporate either the melodic minor collection or Messiaen's fourth mode. Given the context of the (0158) tetrachords in Processional, Messiaen's fourth mode is preferable because adjacent tetrachords combine to complete the collection.

In Table 6.2, an instance of Messiaen's fourth mode, the $\mathrm{F}^{5}$ collection, is ordered in a similar way to the Oct ${ }^{1}$ collection shown in Table 3.1. As was the case with the octatonic collection, each ring is constructed using every fourth pitch of Messiaen's fourth mode. Unlike octatonic space, which alternated between interval-classes 4 and 5, the intervals of Messiaen's mode-4 space alternate between two groups of

Table 6.2. Rings of an $\mathrm{F}^{5}$ helix

ic $\left\langle 3,5,5,5>.^{5}\right.$ The concatenation of the pitch classes of all transpositions of Messiaen's fourth mode on the helix is illustrated in Table 6.3.

The intervallic arrangement of pitch classes accommodates instances of scs (0158) and (012) as chords within a single mode and an instance of the diatonic subset (024579) as a two-mode hybrid chord, which are all prominent set classes in Processional. A transformational analysis of the opening segment of the middle section of Processional utilizing the same transformational apparatus of the octatonic model is shown in Figure 6.1 (cf. Figure 2.11). Mode-4 key areas and ROT transformations are listed above each system while T and hyper-T transformations are listed below. ${ }^{6} \mathrm{~A}$ noticeable difference, as compared to the octatonic model, is the number of modes included in the definition of each key area. This is because of the number of distinct transpositions of the mode that can be utilized in the model.

[^115]Table 6.3. <114> rotational arrays of Messiaen's fourth mode

| N | $\mathbf{F}^{0}$ | $\mathrm{F}^{1}$ | $\mathrm{F}^{2}$ | $\mathbf{F}^{3}$ | $\mathbf{F}^{4}$ | $\mathrm{F}^{5}$ | SIA | Trichord | Tetrachord |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.1 | 0 | 1 | 2 | 3 | 4 | 5 | 1 | (016) | (0147) |
| 1.2 | 1 | 2 | 3 | 4 | 5 | 6 | 1 | (016) | (016) |
| 1.3 | 2 | 3 | 4 | 5 | 6 | 7 | 4 | (012) | (0126) |
| 1.4 | 6 | 7 | 8 | 9 | A | B | 3 | (015) | (0126) |
| 1.5 | 9 | A | B | 0 | 1 | 2 | 6 | (037) | (0158) |
| 2.1 | 3 | 4 | 5 | 6 | 7 | 8 | 6 | (036) | (0147) |
| 2.2 | 6 | 7 | 8 | 9 | A | B | 1 | (036) | (036) |
| 2.3 | 7 | 8 | 9 | A | B | 0 | 1 | (014) | (0236) |
| 2.4 | 9 | A | B | 0 | 1 | 2 | 5 | (013) | (0236) |
| 2.5 | 2 | 3 | 4 | 5 | 6 | 7 | 6 | (027) | (0237) |
| 3.1 | 8 | 9 | A | B | 0 | 1 | 1 | (016) | (0127) |
| 3.2 | 9 | A | B | 0 | 1 | 2 | 3 | (016) | (016) |
| 3.3 | 0 | 1 | 2 | 3 | 4 | 5 | 2 | (014) | (0146) |
| 3.4 | 2 | 3 | 4 | 5 | 6 | 7 | 2 | (025) | (0146) |
| 3.5 | 7 | 8 | 9 | A | B | 0 | 6 | (027) | (0257) |
| 4.1 | 1 | 2 | 3 | 4 | 5 | 6 | 6 | (016) | (0127) |
| 4.2 | 2 | 3 | 4 | 5 | 6 | 7 | 1 | (016) | (016) |
| 4.3 | 3 | 4 | 5 | 6 | 7 | 8 | 4 | (012) | (0126) |
| 4.4 | 7 | 8 | 9 | A | B | 0 | 5 | (015) | (0126) |
| 4.5 | 0 | 1 | 2 | 3 | 4 | 5 | 5 | (037) | (0237) |
| 5.1 | 6 | 7 | 8 | 9 | A | B | 6 | (016) | (0147) |
| 5.2 | 7 | 8 | 9 | A | B | 0 | 1 | (016) | (016) |
| 5.3 | 8 | 9 | A | B | 0 | 1 | 4 | (012) | (0126) |
| 5.4 | 0 | 1 | 2 | 3 | 4 | 5 | 3 | (015) | (0126) |
| 5.5 | 3 | 4 | 5 | 6 | 7 | 8 | 6 | (037) | (0158) |
| 6.1 | 9 | A | B | 0 | 1 | 2 | 6 | (036) | (0147) |
| 6.2 | 0 | 1 | 2 | 3 | 4 | 5 | 3 | (036) | (036) |
| 6.3 | 1 | 2 | 3 | 4 | 5 | 6 | 2 | (014) | (0236) |
| 6.4 | 3 | 4 | 5 | 6 | 7 | 8 | 2 | (013) | (0236) |
| 6.5 | 8 | 9 | A | B | 0 | 1 | 6 | (027) | (0237) |
| 7.1 | 2 | 3 | 4 | 5 | 6 | 7 | 6 | (016) | (0127) |
| 7.2 | 3 | 4 | 5 | 6 | 7 | 8 | 1 | (016) | (016) |
| 7.3 | 6 | 7 | 8 | 9 | A | B | 3 | (014) | (0146) |
| 7.4 | 8 | 9 | A | B | 0 | 1 | 5 | (025) | (0146) |
| 7.5 | 1 | 2 | 3 | 4 | 5 | 6 | 5 | (027) | (0257) |
| 8.1 | 7 | 8 | 9 | A | B | 0 | 6 | (016) | (0127) |
| 8.2 | 8 | 9 | A | B | 0 | 1 | 1 | (016) | (016) |
| 8.3 | 9 | A | B | 0 | 1 | 2 | 1 | (012) | (0126) |
| 8.4 | 1 | 2 | 3 | 4 | 5 | 6 | 4 | (015) | (0126) |
| 8.5 | 6 | 7 | 8 | 9 | A | B | 5 | (037) | (0237) |

Figure 6.1. Alternate analysis of the first segment of the middle section of Processional. Copyright © 1984 by C. F. Peters Corporation. All Rights Reserved. Used by permission.


The instances of (0158) tetrachords are transformed into each other by either $\mathrm{T}_{4} 0$ or $\left\langle\mathrm{T}_{4} 0\right\rangle^{x \rightarrow y}$ depending if the mode changes. The grouping of, and motion between, tetrachords is similar to the trichords in the a and $\mathrm{a}^{1}$ sections of A Haunted Landscape (cf. Figure 4.22). In both cases, the first two chords are members of one mode, which then move to two chords within another mode. Additionally, the chords occupy the same nodal locations within a pentagonal segment in relation to the respective works. The similarity between the two pieces continues as transitional hybrid chords emerge, which then connect with a restatement of the opening four chords in different modes. The
$\mathrm{ROT}_{4}{ }^{4}$ transformation of $\mathrm{F}^{4}$ in Processional modulates to ensure that the final two tetrachordal occurrences of the $\mathrm{F}^{4}$ collection retain the same $\mathrm{T}_{4} 0$ pattern for the final six tetrachords. This rotation is followed by an extended transitional passage that utilizes the same diatonic subset as the previous transition, but features a different transformational structure. An interesting feature of the modal arrays used in the analysis of the passage is that they allow common $\mathrm{T}_{4} 0$ transformation between the tetrachordal and hexachordal pairs, which corresponds to the $\mathrm{T}_{6}$ transformation that would relate them in pitch space.

This brief analysis of indicates that the general features of the octatonic model may be extended to other collections, although considerably more research would be required to arrive at a theory of harmonic functionality. In the case of Messiaen's fourth mode, six unique transpositions of the collection may not provide a comfortable metaphorical fit with the three functional categories of tonality or octatonality. Moreover, a detailed analysis of a number of works utilizing a specific collection would be required to uncover characteristic gestures that constitute a functional model. Selections from Crumb's seven volumes of American Songbooks (2003-2010) may also be suited to a similar type of analysis-octatonic or otherwise-because of its tonal roots in folk song. Additionally, the deep-song inspired The Ghosts of Alhambra (2008) may also warrant further study because of its relationship to deep song-an important component of "Come Lovely"-and the use of Lorca's poem "Paisaje" for the fourth movement. As was discussed in the fourth and fifth chapters, "Paisaje" played an important role in the conception of both A Haunted Landscape and "Come Lovely."

### 6.3 Considerations for Future Research

While this study is relatively limited in scope, there are a number of possible directions for future research. As suggested in the previous section, numerous other works by Crumb may fit a similar functional model. The most common characteristic of music that fits such a model seems to be a perceived rivalry between tonal and post-tonal elements. The presence of tonal artifacts has exerted a great deal of influence on the present study in both the selection of works and creation of models; however, the lack of a major or minor triad need not exclude a work from further study nor serve as the basis of a functional model.

The advantage a functional-yet-transformational (FYT) model is its potential to uncover functional and transformational relationships between many works. An FYT model is also able to differentiate between types of fully chromatic spaces whether they are octatonic, diatonic, whole-tone, or other types of spaces. Because of this, FYT models may be useful in exploring other types of music where the intervallic components of the music, a limited pitch-class set vocabulary, and an underlying functionality is perceived. Various works composed by Bartók, Debussy or Stravinsky, to name a few, might be reexamined through such a model. Interpreting Bartók's music with such a model would likely yield fruitful results and potentially corroborate further reasons for Crumb's predilection with his music. In an interview with Edward Strickland, Crumb commented: "That's another reason I admire composers like Bartók, who avail themselves of all the
rich new possibilities while retaining the tonal principle as the spine of the music. ${ }^{\circ 7}$ An FYT model might also clarify perceived functionality in music by more recent composers, such as György Ligeti or Toru Takemitsu, who demonstrate a type of posttonal diatonicism that embraces various aspects of tonality. As has been demonstrated in this study, the spine of Crumb's octatonic music retains many metaphorical principles of tonality and simultaneously utilizes the rich new possibilities of a post-tonal landscape.

[^116]
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## Appendices

Appendix 1: Naming Conventions of Pitch-Class Collections

| Collection Name | Forte Label | Prime Form | Ordered PCs | Convention |
| :---: | :---: | :---: | :---: | :---: |
| Diatonic | 7-35 | 013568A | 013568A | $\mathrm{D}^{0}$ |
|  |  |  | 124679B | $\mathrm{D}^{1}$ |
|  |  |  | 23578A0 | $\mathrm{D}^{2}$ |
|  |  |  | 34689B1 | $\mathrm{D}^{3}$ |
|  |  |  | 4579A02 | $\mathrm{D}^{4}$ |
|  |  |  | 568AB13 | $\mathrm{D}^{5}$ |
|  |  |  | 679B024 | $\mathrm{D}^{6}$ |
|  |  |  | 78A0135 | $\mathrm{D}^{7}$ |
|  |  |  | 89B1246 | $\mathrm{D}^{8}$ |
|  |  |  | 9A02357 | $\mathrm{D}^{9}$ |
|  |  |  | AB13468 | $\mathrm{D}^{\text {A }}$ |
|  |  |  | B024579 | $\mathrm{D}^{\text {B }}$ |
| Harmonic Minor | 7-32 | 0134689 | 0134689 | $\mathrm{Hm}^{0}$ |
|  |  |  | 124579A | $\mathrm{Hm}^{1}$ |
|  |  |  | 23568AB | $\mathrm{Hm}^{2}$ |
|  |  |  | 34679B0 | $\mathrm{Hm}^{3}$ |
|  |  |  | 4578A01 | $\mathrm{Hm}^{4}$ |
|  |  |  | 5689B12 | $\mathrm{Hm}^{5}$ |
|  |  |  | 679A023 | $\mathrm{Hm}^{6}$ |
|  |  |  | 78 AB 134 | $\mathrm{Hm}^{7}$ |
|  |  |  | 89B0245 | $\mathrm{Hm}^{8}$ |
|  |  |  | 9A01356 | $\mathrm{Hm}^{9}$ |
|  |  |  | AB12467 | $\mathrm{Hm}^{\text {A }}$ |
|  |  |  | B023578 | $\mathrm{Hm}^{\text {B }}$ |
| Melodic Minor | 7-34 | 013468A | 013468A | $\mathrm{Mm}^{0}$ |
|  |  |  | 124579B | $\mathrm{Mm}^{1}$ |
|  |  |  | 23568A0 | $\mathrm{Mm}^{2}$ |
|  |  |  | 34679B1 | $\mathrm{Mm}^{3}$ |
|  |  |  | 4578A02 | $\mathrm{Mm}^{4}$ |
|  |  |  | 5689B13 | $\mathrm{Mm}^{5}$ |
|  |  |  | 679A024 | $\mathrm{Mm}^{6}$ |
|  |  |  | 78AB135 | $\mathrm{Mm}^{7}$ |
|  |  |  | 89B0246 | $\mathrm{Mm}^{8}$ |
|  |  |  | 9A01357 | $\mathrm{Mm}^{9}$ |
|  |  |  | AB12468 | $\mathrm{Mm}^{\text {A }}$ |
|  |  |  | B023579 | $\mathrm{Mm}^{\text {B }}$ |

(Appendix 1 continued)

| Collection Name | Forte Label | Prime Form | Ordered PCs | Convention |
| :---: | :---: | :---: | :---: | :---: |
| Mode 4 | 8-9 | 01236789 | 01236789 | $\mathrm{F}^{0}$ |
|  |  |  | 1234789A | $\mathrm{F}^{1}$ |
|  |  |  | 234589AB | $\mathrm{F}^{2}$ |
|  |  |  | 34569AB0 | $\mathrm{F}^{3}$ |
|  |  |  | 4567AB01 | $\mathrm{F}^{4}$ |
|  |  |  | 5678B012 | $\mathrm{F}^{5}$ |
| Octatonic | 8-28 | 0134679A | 0134679A | Oct ${ }^{1}$ |
|  |  |  | 0235689B | $\mathrm{Oct}^{2}$ |
|  |  |  | 124578AB | $\mathrm{Oct}^{3}$ |
| Whole-Tone | 6-35 | 02468A | 02468A | $\mathrm{WT}^{1}$ |
|  |  |  | 13579B | $\mathrm{WT}^{2}$ |

## Appendix 2: Definitions

Definition 1. Suppose Oct ${ }^{y}$ is an ordered set of pitch classes, $b$, which comprises an instance of the octatonic collection (0134679A). Let $O c t=\left\langle b_{0}, b_{1}, \ldots, b_{7}\right\rangle$ where $b_{0}<b_{1}, b_{1}<b_{2}, \ldots, b_{6}<b_{7}$ and $O c t=(0134679 \mathrm{~A})$. Let $y=b_{0}+b_{1}$ which identifies the specific instance of (0134679A) such that where $y=$ 1, $O c t^{y}=\{0134679 \mathrm{~A}\}$; where $y=2, O c t^{y}=\{0235689 \mathrm{~B}\}$; where $y=3$, $O c t^{y}=\{124578 \mathrm{AB}\}$.

Definition 2. Let $m \neq 0$, the divisor in modulo (mod) $\boldsymbol{m}$ space, where $a=$ a pitch or network node and $b=$ a pitch class or network-node class equivalent to $a$ $\bmod m ; b \equiv a \bmod m$.

Definition 3. Suppose $J$ is an ordered network of nodes of cardinality $n$; let $J=\left\langle j_{1}, j_{2}\right.$, $\ldots, j_{\mathrm{n}}>$ where $j_{1}=1, j_{2}=2, \ldots, j_{\mathrm{n}}=\mathrm{n}$. If $J=$ the helical model of octatonic space, let $n=40$.

Definition 4. Suppose $h . x$ is the nodal position of $j$ in $J$. For all $j$ in $J$ let $x=j \bmod m$ and let $h=\frac{j-\mathrm{x}}{m}$. Where $J=$ the helical model of octatonic space, let $m=5$. Where $\frac{h}{2}=0$, then optionally $h \equiv e($ even $)$ else $h \equiv o($ odd $)$.

Definition 5. Suppose $B$ is an ordered set of pitch classes of cardinality $n$; let $B=<b_{1}$, $b_{2}, \ldots, b_{\mathrm{n}}>$. Suppose $S$ is an ordered set of intervals of cardinality $n-1$ representing the successive interval array of the ordered set of pitch classes; let $S=\left\langle s_{1}, s_{2}, \ldots, s_{\mathrm{n}-1}\right\rangle$ where $s_{1}=\left(b_{2}-b_{1}\right) \bmod m, s_{2}=\left(b_{3}-b_{2}\right)$ $\bmod m, \ldots, s_{\mathrm{n}-1}=\left(b_{\mathrm{n}}-b_{\mathrm{n}-1}\right) \bmod m$.
(Appendix 2 continued)

Definition 6. Suppose $J$ is the helical model of octatonic space; let $B \rightarrow J$ where $S=<5$, $4,6,5,9, \ldots, 9>\|<4,6,5,9,5, \ldots, 5>\|<6,5,9,5,4, \ldots, 4>\|<5,9,5,4$, $6, \ldots, 6>\|<9,5,4,6,5, \ldots, 5>$. These are the possible successive interval arrays of the different rotations in the helical model.

Definition 7. Suppose $\operatorname{Oct}^{y} \mathrm{R}^{e\langle s 1, s 2\rangle}$ is an ordered set of pitch classes from Oct ${ }^{e}$ in a helical model of octatonic space $(J)$ which is rotated, $R$; let $e=$ the pitch class that occupies node $1.1\left(j_{1}\right)$ in $J$ where $m=5$ and $n=40$. For any successive interval array $(S)$ that describes the ordered set of pitch classes (B) in $J$, let the first two elements of $S, s_{1}$ and $s_{2}$, distinguish between the other rotations of $B$ which may occur on J (cf. definition 6).

Definition 8. Consider the distance, $D$, between any two nodes, $\mathfrak{j}$, of the helical model $\operatorname{Oct}^{y} \mathrm{R}^{e<s 1, s 2\rangle}$. Let $P=\left\langle p_{1}, p_{2}, \ldots, p_{\mathrm{n}}\right\rangle$ where $P \in \operatorname{Oct}^{e} \mathrm{R}^{c<11, s 2\rangle}$ and let $Q=$ $\left\langle q_{1}, q_{2}, \ldots, q_{\mathrm{n}}>\right.$ where $Q \in \operatorname{Oct}^{y} \mathrm{R}^{e<s 1, s 2\rangle}$. Let $p_{1} \equiv j_{\mathrm{p}}$ where $B \rightarrow J$; let $q_{1} \equiv$ $j_{\mathrm{q}}$ where $B \rightarrow J(\mathrm{cf} . \operatorname{def} .6)$. Let $D=\left(j_{\mathrm{q}}-j_{\mathrm{p}}\right) \bmod 40$ where $\left(j_{\mathrm{q}}-j_{\mathrm{p}}\right) \bmod 40$ $=\left\lceil\left(j_{\mathrm{q}}-j_{\mathrm{p}}\right) \bmod 40,\left(j_{\mathrm{p}}-j_{\mathrm{q}}\right) \bmod 40\right\rceil$ else let $D=-\left(j_{\mathrm{p}}-j_{\mathrm{q}}\right) \bmod 40$ where $\left(j_{\mathrm{p}}-j_{\mathrm{q}}\right) \bmod 40=\left\lceil\left(j_{\mathrm{q}}-j_{\mathrm{p}}\right) \bmod 40,\left(j_{\mathrm{p}}-j_{\mathrm{q}}\right) \bmod 40\right\rceil$.

Definition 9. Consider the union of two networks to form a hybrid network, $K$. Let $\operatorname{Oct}^{y, z} \mathrm{R}^{e, f\langle s 1, s 2\rangle}=\operatorname{Oct}^{y} \mathrm{R}^{e\langle s 1, s 2\rangle} \cup \operatorname{Oct}^{z} \mathrm{R}^{f\langle s 1, s 2\rangle}$ where $\operatorname{Oct}^{y}=(0134679 \mathrm{~A})$, Oct $^{z}=(0134679 \mathrm{~A})$ and $y \neq z$. Let $e=$ the pitch class that occupies $j_{1}$ in Oct $^{y} \mathrm{R}^{e\langle s 1, s 2\rangle}$ and let $f=$ the pitch class that occupies $j_{1}$ in $\operatorname{Oct}^{z} \mathrm{R}^{f\langle s 1, s 2\rangle}$. Let $K=\left\langle k_{1}, k_{2}, \ldots, k_{40}\right\rangle$ where $k_{1}=j_{1}$ in $\operatorname{Oct}^{y} \mathrm{R}^{e\langle s 1, s 2\rangle} \cup j_{1}$ in $\operatorname{Oct}^{z} \mathrm{R}^{f\langle s 1, s 2\rangle}, k_{2}=j_{2}$ in $\operatorname{Oct}^{y} \mathrm{R}^{e\langle s 1, s 2\rangle} \cup j_{2}$ in $\operatorname{Oct}^{z} \mathrm{R}^{f\langle s 1, s 2\rangle}, \ldots, k_{40}=j_{40}$ in $\operatorname{Oct}^{y} \mathrm{R}^{e\langle s 1, s 2\rangle} \cup j_{40}$ in $\operatorname{Oct}^{z} \mathrm{R}^{f\langle s 1, s 2\rangle}$.

Definition 10. $\mathrm{T}_{x} Y_{z}$ is a $\mathbf{T}$ transformation that transposes a pitch or pitches a certain distance, $D$, between nodes, $j$, of the helical model Oct $^{y} \mathrm{R}^{c<s 1, s 2>}$. Let $Y=\left(j_{\mathrm{q}}\right.$ $\left.-j_{\mathrm{p}}\right) \bmod 5$ where $\left(j_{\mathrm{q}}-j_{\mathrm{p}}\right) \bmod 5=\left\lceil\left(j_{\mathrm{q}}-j_{\mathrm{p}}\right) \bmod 5,\left(j_{\mathrm{p}}-j_{\mathrm{q}}\right) \bmod 5\right\rceil$ else let $Y=-\left(j_{\mathrm{p}}-j_{\mathrm{q}}\right) \bmod 5$ where $\left(j_{\mathrm{p}}-j_{\mathrm{q}}\right) \bmod 5=\left\lceil\left(j_{\mathrm{q}}-j_{\mathrm{p}}\right) \bmod 5,\left(j_{\mathrm{p}}-j_{\mathrm{q}}\right) \bmod \right.$ 57. Let $x=\frac{D-Y}{5}$. Let $z=\# Q-\# P$.

Definition 11. $\operatorname{ROT}_{x} Y$ is a ROT transformation that rotates Oct ${ }^{y} \mathrm{R}^{c<s 1, s 2\rangle}$ a certain distance, $D$, around the helical model of octatonic space, $J$. Let $Y=\left(j_{1+D}-\right.$ $\left.j_{1}\right) \bmod 5$ where $\left(j_{1+D}-j_{1}\right) \bmod 5=\left\lceil\left(j_{1+D}-j_{1}\right) \bmod 5,\left(j_{1}-j_{1+D}\right) \bmod 5\right\rceil$ else let $Y=-\left(j_{1}-j_{1+D}\right) \bmod 5$ where $\left(j_{1}-j_{1+D}\right) \bmod 5=\left[\left(j_{1+D}-j_{1}\right) \bmod 5\right.$, $\left(j_{1}-j_{1+D}\right) \bmod 57$. Let $x=\frac{D-Y}{5}$.
(Appendix 2 continued)

Definition 12. $\left\langle\mathrm{T}_{t} U_{v}\right\rangle_{w}$ is a hyper-T transformation that transposes a pitch or pitches a certain distance, $D$, between nodes, $j$, of $\mathrm{Oct}^{y} \mathrm{R}^{c\langle s 1, s 2\rangle}$ and $\mathrm{Oct}^{z} \mathrm{R}^{f<s 1, s 2\rangle}$. Let $w=1$ where $(y+z) \bmod 3=2$ else let $w=-1$ where $(y+z) \bmod 3=1$.

Following this, the rest of the transformation is as demonstrated in definition 9 .

Appendix 3: < $54>$ Rotations of the Octatonic Modes

| Position | Oct ${ }^{1}$ |  |  |  | Oct ${ }^{2}$ |  |  |  | $\mathrm{Oct}^{3}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.1 | A | 7 | 4 | 1 | 6 | 3 | 0 | 9 | 8 | 5 | 2 | B |
| 1.2 | 3 | 0 | 9 | 6 | B | 8 | 5 | 2 | 1 | A | 7 | 4 |
| 1.3 | 7 | 4 | 1 | A | 3 | 0 | 9 | 6 | 5 | 2 | B | 8 |
| 1.4 | 1 | A | 7 | 4 | 9 | 6 | 3 | 0 | B | 8 | 5 | 2 |
| 1.5 | 6 | 3 | 0 | 9 | 2 | B | 8 | 5 | 4 | 1 | A | 7 |
| 2.1 | 3 | 0 | 9 | 6 | B | 8 | 5 | 2 | 1 | A | 7 | 4 |
| 2.2 | 7 | 4 | 1 | A | 3 | 0 | 9 | 6 | 5 | 2 | B | 8 |
| 2.3 | 0 | 9 | 6 | 3 | 8 | 5 | 2 | B | A | 7 | 4 | 1 |
| 2.4 | 6 | 3 | 0 | 9 | 2 | B | 8 | 5 | 4 | 1 | A | 7 |
| 2.5 | A | 7 | 4 | 1 | 6 | 3 | 0 | 9 | 8 | 5 | 2 | B |
| 3.1 | 7 | 4 | 1 | A | 3 | 0 | 9 | 6 | 5 | 2 | B | 8 |
| 3.2 | 0 | 9 | 6 | 3 | 8 | 5 | 2 | B | A | 7 | 4 | 1 |
| 3.3 | 4 | 1 | A | 7 | 0 | 9 | 6 | 3 | 2 | B | 8 | 5 |
| 3.4 | A | 7 | 4 | 1 | 6 | 3 | 0 | 9 | 8 | 5 | 2 | B |
| 3.5 | 3 | 0 | 9 | 6 | B | 8 | 5 | 2 | 1 | A | 7 | 4 |
| 4.1 | 0 | 9 | 6 | 3 | 8 | 5 | 2 | B | A | 7 | 4 | 1 |
| 4.2 | 4 | 1 | A | 7 | 0 | 9 | 6 | 3 | 2 | B | 8 | 5 |
| 4.3 | 9 | 6 | 3 | 0 | 5 | 2 | B | 8 | 7 | 4 | 1 | A |
| 4.4 | 3 | 0 | 9 | 6 | B | 8 | 5 | 2 | 1 | A | 7 | 4 |
| 4.5 | 7 | 4 | 1 | A | 3 | 0 | 9 | 6 | 5 | 2 | B | 8 |
| 5.1 | 4 | 1 | A | 7 | 0 | 9 | 6 | 3 | 2 | B | 8 | 5 |
| 5.2 | 9 | 6 | 3 | 0 | 5 | 2 | B | 8 | 7 | 4 | 1 | A |
| 5.3 | 1 | A | 7 | 4 | 9 | 6 | 3 | 0 | B | 8 | 5 | 2 |
| 5.4 | 7 | 4 | 1 | A | 3 | 0 | 9 | 6 | 5 | 2 | B | 8 |
| 5.5 | 0 | 9 | 6 | 3 | 8 | 5 | 2 | B | A | 7 | 4 | 1 |
| 6.1 | 9 | 6 | 3 | 0 | 5 | 2 | B | 8 | 7 | 4 | 1 | A |
| 6.2 | 1 | A | 7 | 4 | 9 | 6 | 3 | 0 | B | 8 | 5 | 2 |
| 6.3 | 6 | 3 | 0 | 9 | 2 | B | 8 | 5 | 4 | 1 | A | 7 |
| 6.4 | 0 | 9 | 6 | 3 | 8 | 5 | 2 | B | A | 7 | 4 | 1 |
| 6.5 | 4 | 1 | A | 7 | 0 | 9 | 6 | 3 | 2 | B | 8 | 5 |
| 7.1 | 1 | A | 7 | 4 | 9 | 6 | 3 | 0 | B | 8 | 5 | 2 |
| 7.2 | 6 | 3 | 0 | 9 | 2 | B | 8 | 5 | 4 | 1 | A | 7 |
| 7.3 | A | 7 | 4 | 1 | 6 | 3 | 0 | 9 | 8 | 5 | 2 | B |
| 7.4 | 4 | 1 | A | 7 | 0 | 9 | 6 | 3 | 2 | B | 8 | 5 |
| 7.5 | 9 | 6 | 3 | 0 | 5 | 2 | B | 8 | 7 | 4 | 1 | A |
| 8.1 | 6 | 3 | 0 | 9 | 2 | B | 8 | 5 | 4 | 1 | A | 7 |
| 8.2 | A | 7 | 4 | 1 | 6 | 3 | 0 | 9 | 8 | 5 | 2 | B |
| 8.3 | 3 | 0 | 9 | 6 | B | 8 | 5 | 2 | 1 | A | 7 | 4 |
| 8.4 | 9 | 6 | 3 | 0 | 5 | 2 | B | 8 | 7 | 4 | 1 | A |
| 8.5 | 1 | A | 7 | 4 | 9 | 6 | 3 | 0 | B | 8 | 5 | 2 |

## Appendix 4: Correlation between Hybrid Families and Set-class Inclusion

Entries in the following table indicate the number of nodes required to form the set class listed in the left-most column in the hybrid family listed in the top row.

| SC | $\mathrm{H} 1^{2, n}$ | $\mathrm{H} 2^{2, n}$ | $\mathrm{H} 4^{2, n}$ | $\mathrm{H} 5^{2, \mathrm{n}}$ | $\mathrm{H} 112{ }^{3, \mathrm{n}}$ | $\mathrm{H} 2 \backslash 4{ }^{3, \mathrm{n}}$ | $\mathrm{H} 4^{3, \mathrm{n}}$ | $\mathrm{H} 5 \backslash 2^{3, \mathrm{n}}$ | H1\4\5 ${ }^{3, \mathrm{n}}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  | A | B |
| (027) |  |  |  | 2 |  |  |  |  |  |  |
| (048) |  |  | 2 |  |  |  | 2 |  |  |  |
| (0134) | 2 |  |  |  |  |  |  |  |  |  |
| (0145) | 2 |  |  |  |  |  |  |  |  |  |
| (0156) | 2 |  |  |  |  |  |  |  |  |  |
| (0158) |  |  | 2 | 2 |  |  |  |  |  |  |
| (0167) | 2 |  |  | 2 |  |  |  |  |  |  |
| (0235) |  | 2 |  |  |  |  |  |  |  |  |
| (0246) |  | 2 |  |  |  |  |  |  |  |  |
| (0257) |  | 2 |  |  |  |  |  | 2 |  |  |
| (0268) |  | 2 | 2 |  |  |  |  |  |  |  |
| (0347) |  |  | 2 |  |  |  |  |  |  |  |
| (0358) |  |  |  | 2 |  |  |  |  |  |  |
| (01245) | 3 |  |  |  |  |  |  |  |  |  |
| (01267) | 3 |  |  | 3 |  |  |  |  |  |  |
| (01358) |  |  |  | 3 |  |  |  |  |  |  |
| (01458) |  |  | 3 |  |  |  |  |  |  |  |
| (01568) |  |  |  |  |  |  |  |  |  |  |
| (02357) |  | 3 |  |  |  |  |  |  |  |  |
| (02468) |  | 3 | 3 |  |  | 2 |  |  |  |  |
| (02479) |  |  |  | 3 |  |  |  |  |  |  |
| (012345) |  |  |  |  | 2 |  |  |  |  |  |
| (012346) |  | 3 |  |  |  |  |  |  |  |  |
| (012356) | 3 |  |  |  |  |  |  |  |  |  |
| (012367) | 3 |  |  |  |  |  |  |  |  |  |
| (012368) |  | 3 |  |  |  |  |  |  |  |  |
| (012456) |  |  |  |  | 2 |  |  |  |  |  |
| (012469) |  |  | 3 |  |  |  |  |  |  |  |
| (012567) |  |  |  |  | 2 |  |  |  |  |  |
| (012568) |  |  | 3 |  |  |  |  |  |  |  |
| (012569) |  |  |  | 3 |  |  |  |  |  |  |

(Appendix 4 continued)

| Family |  |  |  |  |  |  | H4 ${ }^{3, n}$ | 4512,n |  | $15^{3, n}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SC |  |  |  |  |  |  |  |  | A | B |
| (012578) |  |  |  | 3 |  |  |  |  |  |  |
| (012678) |  |  |  |  | 2 |  |  | 2 |  |  |
| (013458) |  |  |  |  |  |  |  |  |  |  |
| (013478) | 3 |  |  |  |  |  |  |  |  |  |
| (013578) |  |  |  |  |  |  |  | 2 |  |  |
| (014589) |  |  |  |  |  |  | 2, 3 |  |  |  |
| (023457) |  |  |  |  |  | 2 |  |  |  |  |
| (023579) |  | 3 |  |  |  |  |  |  |  |  |
| (024579) |  |  |  |  |  | 2 |  | 2, 3 |  |  |
| (02468A) |  |  |  |  |  | 2, 3 | 2, 3 |  |  |  |
| (0123456) |  |  |  |  | 3 |  |  |  |  |  |
| (0123468) |  | 4 |  |  |  |  |  |  |  |  |
| (0123567) | 4, 5 |  |  |  |  |  |  |  |  |  |
| (0123589) |  |  |  | 4, 5 |  |  |  |  |  |  |
| (0123678) |  |  |  |  | 3 |  |  | 3 |  |  |
| (0124569) |  |  |  |  |  |  |  |  |  |  |
| (0124578) | 4, 5 |  |  |  |  |  |  |  |  |  |
| (0124589) |  |  |  |  |  |  |  |  |  |  |
| (0124679) |  |  |  | 4, 5 |  |  |  |  |  |  |
| (0124689) |  |  | 4, 5 |  |  |  |  |  |  |  |
| (0125679) |  |  |  | 4, 5 |  |  |  |  |  |  |
| (0125689) |  |  | 4, 5 |  |  |  |  |  |  |  |
| (0134578) |  |  |  |  |  |  |  |  |  |  |
| (0134579) |  |  | 4, 5 |  |  |  |  |  |  |  |
| (013468A) |  | 4, 5 |  |  |  |  |  |  |  |  |
| (013568A) |  |  |  |  |  |  |  | 3 |  |  |
| (0234579) |  |  |  |  |  | 3 |  |  |  |  |
| (01234567) |  |  |  |  | 3 |  |  |  |  |  |
| (01234568) |  |  |  |  |  | 3 |  |  |  |  |
| (01234678) | 5 |  |  |  | 3 |  |  |  |  |  |
| (01234679) |  | 4, 5 |  |  |  |  |  |  |  |  |
| (0123468A) |  | 5 |  |  |  |  |  |  |  |  |
| (01235679) | 4 |  |  |  |  |  |  |  |  |  |
| (01235689) | 5 |  |  |  |  |  |  |  |  |  |
| (0123568A) |  |  |  |  |  | 3 |  |  |  |  |
| (01235789) |  |  |  | 5 |  |  |  | 3 |  |  |
| (01245679) |  |  |  | 5 |  |  |  |  |  |  |
| (01245689) |  |  | 5 |  |  |  |  |  |  |  |

(Appendix 4 continued)

| SC | $\mathrm{H}^{2, \mathrm{n}}$ | $\mathrm{H} 2^{2, \mathrm{n}}$ | $\mathrm{H} 4^{2, \mathrm{n}}$ | $\mathrm{H}^{2, \mathrm{n}}$ | $\mathrm{H} 12^{3, \mathrm{n}}$ | $\mathrm{H} 2 \backslash 4^{3, \mathrm{n}}$ | $\mathrm{H} 4^{3, \mathrm{n}}$ | $\mathrm{H} 512^{3, \mathrm{n}}$ | $\mathrm{H} 1 \backslash 45^{3, \mathrm{n}}$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $(0124568 \mathrm{~A})$ |  |  | 5 |  |  |  |  |  |  |  |
| $(012345678)$ |  |  |  |  | 4,5 |  |  |  |  |  |
| $(01234568 \mathrm{~A})$ |  |  |  |  |  | $3,4,5$ |  |  |  |  |
| $(012345789)$ |  |  |  |  | 3 |  |  | $3,4,5$ | 4,5 |  |
| $(01234578 \mathrm{~A})$ |  | 5 |  |  |  |  |  |  |  |  |
| $(012346789)$ | 5 | 5 | 5 | 5 |  |  |  |  |  |  |
| $(01235678 \mathrm{~A})$ |  |  |  |  |  |  |  | 4,5 | 4,5 |  |
| $(01235679 \mathrm{~A})$ | 5 |  |  |  |  |  |  |  |  |  |
| $(01245689 \mathrm{~A})$ |  |  |  |  |  |  | $3,4,5$ |  |  |  |
| $(0123456789)$ |  |  |  |  | 4,5 |  |  |  | 5 |  |
| $(012345689 \mathrm{~A})$ |  |  |  |  |  |  |  |  | 4,5 |  |
| $(012345789 \mathrm{~A})$ |  |  |  |  |  |  |  | 4,5 | 5 |  |
| $(0123456789 \mathrm{~A})$ |  |  |  |  | 4,5 | 4,5 |  | 5 | 5 |  |
| $(0123456789 \mathrm{AB})$ |  |  |  |  | 5 | 5 | 5 |  |  |  |

## Appendix 5: Hybrid Families

In the following tables, an asterisk following a set class indicates that sets within the respective set class are replicated more than once within the hybrid family. Sets are listed in ascending numerical order rather than normal form.

Hybrid $1^{2, n}$ Family

| Nodes | SC | IV | Oct ${ }^{1,2} \mathrm{R}^{\mathrm{x}, \mathrm{x}-1<\mathrm{yz}\rangle}$ | Oct ${ }^{1,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+1<\mathrm{yz}>}$ | Oct ${ }^{2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-1<\mathrm{yz}\rangle}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | (0134) referential | [212100] | 0134 | 0134 | 023B |
|  |  |  | 019A | 019A | 089B |
|  |  |  | 023B | 1245 | 1245 |
|  |  |  | 089B | 12 AB | 12 AB |
|  |  |  | 2356 | 3467 | 2356 |
|  |  |  | 3467 | 4578 | 4578 |
|  |  |  | 5689 | 679A | 5689 |
|  |  |  | 679A | 78AB | 78AB |
|  | (0145)* | [201210] | 0189 | 0145 | 078B |
|  |  |  | 034B | 129A | 1256 |
|  |  |  | 2367 | 3478 | 23 AB |
|  |  |  | 569A | 67AB | 4589 |
|  | (0156) * | [200121] | 0156 | 0178 | 045B |
|  |  |  | 067B | 1267 | 1289 |
|  |  |  | 239A | 34 AB | 2378 |
|  |  |  | 3489 | 459A | 56AB |
|  | $(0167)^{*}$ <br> referential | [200022] | 0167 | 0167 | 056B |
|  |  |  | 056B | 1278 | 1278 |
|  |  |  | 2389 | 349A | 2389 |
|  |  |  | 349A | 45AB | 45AB |
| 3 | (01245) | [322210] | 0134B | 01245 | 023AB |
|  |  |  | 0189A | 0129A | 0789B |
|  |  |  | 0189B | 01345 | 078AB |
|  |  |  | 0234B | 129AB | 12356 |
|  |  |  | 23467 | 34578 | 123 AB |
|  |  |  | 23567 | 34678 | 12456 |
|  |  |  | 5679A | 678AB | 45689 |
|  |  |  | 5689A | 679AB | 45789 |
|  | (01267) | [310132] | 01567 | 01267 | 0456B |
|  |  |  | 0156B | 01278 | 045AB |
|  |  |  | 0167B | 01678 | 056AB |
|  |  |  | 0567B | 12678 | 12378 |
|  |  |  | 23489 | 3459A | 12389 |
|  |  |  | 2349A | 345AB | 12789 |
|  |  |  | 2389A | 349AB | 23789 |
|  |  |  | 3489A | 459AB | 456AB |

$\mathrm{HI}^{2, n}$ continued

| Nodes | SC | IV | Oct ${ }^{1,2} \mathrm{R}^{\mathrm{x}, \mathrm{x}-1<\mathrm{yz}>}$ | Oct ${ }^{1,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+1<\mathrm{yz}>}$ | Oct ${ }^{2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-1<\mathrm{yz}\rangle}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 3 \\ \text { cont. } \end{gathered}$ | (012356) | [433221] | 012356 | 0134AB | 01245B |
|  |  |  | 01239A | 01789A | 01289B |
|  |  |  | 013456 | 0178AB | 02345B |
|  |  |  | 0239AB | 123467 | 1289 AB |
|  |  |  | 06789B | 1234 AB | 234578 |
|  |  |  | 0679AB | 124567 | 235678 |
|  |  |  | 345689 | 45679A | 5678AB |
|  |  |  | 346789 | 45789A | 5689AB |
|  | (012367) | [422232] | 012367 | 014567 | 01256B |
|  |  |  | 012389 | 0145AB | 01278B |
|  |  |  | 016789 | 0167AB | 05678B |
|  |  |  | 03456B | 123478 | 125678 |
|  |  |  | 0349AB | 12349A | 234589 |
|  |  |  | 0569AB | 12789A | 2345AB |
|  |  |  | 236789 | 34789A | 2389AB |
|  |  |  | 34569A | 4567AB | 4589AB |
|  | (013478) | [313431] | 013489 | 013478 | 02378B |
|  |  |  | 015689 | 014578 | 04578B |
|  |  |  | 01569A | 01459A | 04589B |
|  |  |  | 02367B | 12459A | 124589 |
|  |  |  | 03467B | 12679A | 125689 |
|  |  |  | 03489B | 1267 AB | 1256AB |
|  |  |  | 23569A | 3467AB | 2356AB |
|  |  |  | 23679A | 3478AB | 2378AB |
| 4 | (0123567)* | [543342] | 0123567 | 0124567 | 012456B |
|  |  |  | 012389A | 012789A | 012789B |
|  |  |  | 013456B | 01345AB | 02345AB |
|  |  |  | 016789B | 01678AB | 05678AB |
|  |  |  | 02349AB | 1234678 | 1235678 |
|  |  |  | 05679AB | 12349AB | 12389 AB |
|  |  |  | 2346789 | 345789A | 2345789 |
|  |  |  | 345689A | 45679AB | 45689AB |
|  | (0124578)* | [434442] | 015689B | 012679A | 04578AB |
|  |  |  | 023489B | 013459A | 12378AB |
|  |  |  | 023567B | 0134678 | 12456AB |
|  |  |  | 235689A | 34679AB | 1245789 |
|  | (01235679)* | [555553] | 0123679A | 014578AB | 0125689B |
|  |  |  | 0134569A | 123478 AB | 0234589B |
|  |  |  | 01346789 | 124567 AB | 0235678B |
|  |  |  | 034679AB | 1245789A | 235689AB |
| 5 | (0123567)* | [543342] | 0123567 | 0124567 | 012456B |
|  |  |  | 012389A | 012789A | 012789B |
|  |  |  | 013456B | 01345AB | 02345AB |
|  |  |  | 016789B | 01678AB | 05678AB |
|  |  |  | 02349AB | 1234678 | 1235678 |
|  |  |  | 05679AB | 12349 AB | 12389 AB |
|  |  |  | 2346789 | 345789A | 2345789 |
|  |  |  | 345689A | 45679AB | 45689 AB |
|  | (0124578) | [434442] | 015689B | 012679A | 04578AB |
|  |  |  | 023489B | 013459A | 12378AB |
|  |  |  | 023567B | 0134678 | 12456 AB |
|  |  |  | 235689A | 34679AB | 1245789 |

$\mathrm{H1}^{2, n}$ continued

| Nodes | SC | IV | Oct ${ }^{1,2} \mathrm{R}^{\mathrm{x}, \mathrm{x}-1<\mathrm{yz}}$, | Oct ${ }^{1,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+1<\mathrm{yz}}$ | Oct ${ }^{2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-1<\mathrm{yz}}$, |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 5 \\ \text { cont. } \end{gathered}$ | (01234678) | [654553] | 0123567B | 01234678 | 012456AB |
|  |  |  | 0156789B | 0126789A | 045678AB |
|  |  |  | 023489AB | 013459AB | 12345789 |
|  |  |  | 2345689A | 345679 AB | 123789AB |
|  | (01235689) | [546553] | 0123679A | 014578AB | 0125689B |
|  |  |  | 0134569A | 123478AB | 0234589B |
|  |  |  | 01346789 | 124567 AB | 0235678B |
|  |  |  | 034679AB | 1245789A | 235689AB |
|  | (012346789) | [766674] | 012346789 | 0124567 AB | 01235678B |
|  |  |  | 01236789A | 0145678AB | 01256789B |
|  |  |  | 0134569AB | 12345789A | 0234589AB |
|  |  |  | 0345679AB | 1234789 AB | 2345689AB |
|  | (01235679A) | [667773] | 01235679A | 01245679A | 01245689B |
|  |  |  | 01235689A | 01245789A | 01245789B |
|  |  |  | 01345689A | 01345789A | 02345789B |
|  |  |  | 01345689B | 0134578AB | 0234578AB |
|  |  |  | 01346789B | 0134678AB | 0235678AB |
|  |  |  | 02346789B | 1234678AB | 1235678AB |
|  |  |  | 0234679AB | 1234679 AB | 1235689 AB |
|  |  |  | 0235679AB | 1245679AB | 1245689AB |

Hybrid $2^{2, n}$ Family

| Nodes | SC | IV | Oct ${ }^{1,2} \mathrm{R}^{\mathrm{x}, \mathrm{x}+2<\mathrm{yz}>}$ | Oct ${ }^{1,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-2<\mathrm{yz}>}$ | Oct ${ }^{2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+2<\mathrm{yz>}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | $(0235)$ <br> referential | [122010] | 013A | 013A | 0235 |
|  |  |  | 0235 | 079A | 029B |
|  |  |  | 029B | 124B | 124B |
|  |  |  | 079A | 1346 | 18AB |
|  |  |  | 1346 | 18AB | 2457 |
|  |  |  | 3568 | 2457 | 3568 |
|  |  |  | 4679 | 4679 | 578A |
|  |  |  | 689B | 578A | 689B |
|  | (0246)* | [030201] | 0246 | 024A | 028A |
|  |  |  | 068A | 1357 | 135B |
|  |  |  | 139B | 179B | 2468 |
|  |  |  | 3579 | 468A | 579B |
|  | (0257)* | [021030] | 0279 | 057A | 0257 |
|  |  |  | 035A | 138A | 168B |
|  |  |  | 1368 | 146B | 249B |
|  |  |  | 469B | 2479 | 358A |
|  | $\begin{aligned} & (0268)^{*} \\ & \text { referential } \end{aligned}$ | [020202] | 0268 | 046A | 0268 |
|  |  |  | 046A | 1379 | 157B |
|  |  |  | 1379 | 157B | 248A |
|  |  |  | 359B | 248A | 359B |
| 3 | (02357) | [132130] | 0135A | 0138A | 02357 |
|  |  |  | 0235A | 0578A | 02457 |
|  |  |  | 0279A | 0579A | 0249B |
|  |  |  | 0279B | 1246B | 1249B |
|  |  |  | 13468 | 1346B | 1689B |
|  |  |  | 13568 | 138 AB | 168AB |
|  |  |  | 4679B | 24579 | 3568A |
|  |  |  | 4689B | 24679 | 3578A |
|  | (02468) | [040402] | 02468 | 0246A | 02468 |
|  |  |  | 0246A | 0248A | 0248A |
|  |  |  | 0268A | 0468A | 0268A |
|  |  |  | 0468A | 13579 | 1357B |
|  |  |  | 13579 | 1357B | 1359B |
|  |  |  | 1359B | 1379B | 1579B |
|  |  |  | 1379B | 1579B | 2468A |
|  |  |  | 3579B | 2468A | 3579B |
|  | (012346) | [443211] | 012346 | 01234A | 01235B |
|  |  |  | 01239B | 0124AB | 0128AB |
|  |  |  | 0139AB | 0179AB | 0289AB |
|  |  |  | 023456 | 123457 | 12345B |
|  |  |  | 06789A | 134567 | 234568 |
|  |  |  | 0689AB | 1789AB | 245678 |
|  |  |  | 345679 | 45678A | 56789B |
|  |  |  | 356789 | 46789A | 5789AB |

$H 2^{2, n}$ continued

| Nodes | SC | IV |  | Oct ${ }^{1,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-2\langle\mathrm{yz} \mathrm{\rangle}}$ | $\mathrm{Oct}^{2,3 \mathrm{~B}^{\mathrm{x}, \mathrm{x}+2\langle\mathrm{yz}\rangle} \mathrm{l}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 3 \\ \text { cont. } \end{gathered}$ | (012368) | [332232] | 012368 012379 026789 03456 A 0359 AB 0469 AB 136789 34569 B | 0146AB <br> 0157 AB <br> 04567A <br> 123479 <br> 12348A <br> 13789A <br> 14567B <br> 24789A <br> 189 | 01257 B 01268 B 025678 15678 B 23458 A 23459 B 2489 AB 3589 AB |
|  | (023579) | [143241] | 01368A 023579 024679 02469B 03568A 03579A 13469B 13689B | 01357A 02457 A 02479A 12479B 13468A 13578A 14679 B 1468 AB | 02358A 02578 A 02579 B 12468B 13568B 1358AB 24579 B 24689B |
| 4 | (0123468)* | [453432] | 0123468 012379 B 01359AB 023456A 026789A 04689AB 1356789 345679 B |  | 012357B 01268 AB 0245678 02489 AB 123459 B 156789B 234568 A 35789AB |
|  | (013468A)* | [254442] | $\begin{aligned} & \hline \text { 013468A } \\ & \text { 013579A } \\ & \text { 024679A } \\ & \text { 134679B } \\ & \hline \end{aligned}$ | $\begin{aligned} & 024578 \mathrm{~A} \\ & 124579 \mathrm{~B} \\ & 12468 \mathrm{AB} \\ & 13578 \mathrm{AB} \end{aligned}$ | 023568A 023579B 024689B 135689B |
|  | (01234679)* | [556453] | $\begin{aligned} & \hline 0123689 \mathrm{~B} \\ & 0234569 \mathrm{~B} \\ & 02356789 \\ & 035689 \mathrm{AB} \end{aligned}$ | 0123479A 0134567 A 014679 AB 1346789 A | 012578 AB 123458 AB 1245678 B 245789 AB |
| 5 | (0123468)* | [453432] |  |  | 012357B 01268 AB 0245678 02489 AB 123459B 156789B 234568 A 35789AB |
|  | (013468A) | [254442] |  |  |  |
|  | (01234679) | [556453] | $\begin{array}{\|l\|} \hline 0123689 \mathrm{~B} \\ 0234569 \mathrm{~B} \\ 02356789 \\ 035689 \mathrm{AB} \\ \hline \end{array}$ | $\begin{aligned} & \hline 0123479 \mathrm{~A} \\ & 0134567 \mathrm{~A} \\ & 014679 \mathrm{AB} \\ & 1346789 \mathrm{~A} \\ & \hline \end{aligned}$ | $\begin{aligned} & 012578 \mathrm{AB} \\ & 123458 \mathrm{AB} \\ & 1245678 \mathrm{~B} \\ & 245789 \mathrm{AB} \\ & \hline \end{aligned}$ |
|  | (0123468A) | [474643] | $\begin{aligned} & \hline 0123468 \mathrm{~A} \\ & 013579 \mathrm{AB} \\ & 0246789 \mathrm{~A} \\ & 1345679 \mathrm{~B} \end{aligned}$ | $\begin{aligned} & \hline 012468 \mathrm{AB} \\ & 0245678 \mathrm{~A} \\ & 1234579 \mathrm{~B} \\ & 135789 \mathrm{AB} \end{aligned}$ | 0123579B 0234568A 024689AB 1356789B |

$H 2^{2, n}$ continued

| Nodes | SC | IV | Oct ${ }^{1,2} \mathrm{R}^{\mathrm{x}, \mathrm{x}+2\langle\mathrm{yz}\rangle}$ | Oct ${ }^{1,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-2<\mathrm{yz}}$ | Oct ${ }^{2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+2<\mathrm{yz}>}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 5 \\ \text { cont. } \end{gathered}$ | (01234578A) | [677673] | 01234679B | 01234578A | 0123568 AB |
|  |  |  | 01234689B | 01234579A | 0123578AB |
|  |  |  | 0134689AB | 0124579AB | 0235789AB |
|  |  |  | 01356789A | 0124679AB | 02456789B |
|  |  |  | 0135689AB | 01345678A | 0245789AB |
|  |  |  | 02345679A | 1246789 AB | 12345689B |
|  |  |  | 02345679B | 1345678AB | 1234568 AB |
|  |  |  | 02356789A | 1346789AB | 12456789B |
|  | (012346789) | [766674] | 012356789 | 01234789A | 01245678B |
|  |  |  | 01236789B | 0134567 AB | 0125678AB |
|  |  |  | 0234569AB | 0145679AB | 1234589AB |
|  |  |  | 0345689AB | 12346789A | 2345789AB |

Hybrid $4^{2, n}$ Family

| Nodes | SC | IV | Oct ${ }^{1,2} \mathrm{R}^{\mathrm{x}, \mathrm{x}-4<\mathrm{yz}>}$ | Oct ${ }^{1,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+4 \mathrm{yz}>}$ | Oct ${ }^{2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-4\langle\mathrm{yz}\rangle}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | (048)* | [000300] | 048 | 048 | 048 |
|  |  |  | 159 | 159 | 159 |
|  |  |  | 26A | 26A | 26A |
|  |  |  | 37B | 37B | 37B |
|  | (0158)* | [101220] | 0378 | 047B | 0158 |
|  |  |  | 0459 | 1489 | 259A |
|  |  |  | 1269 | 156A | 267B |
|  |  |  | 36AB | 237A | 348B |
|  | (0268)* referential | [020202] | 0268 | 046A | 0268 |
|  |  |  | 046A | 1379 | 157B |
|  |  |  | 1379 | 157B | 248A |
|  |  |  | 359B | 248A | 359B |
|  | $\begin{aligned} & (0347) \\ & \text { referential } \end{aligned}$ | [102210] | 0149 | 0149 | 038B |
|  |  |  | 0347 | 0347 | 0589 |
|  |  |  | 038B | 125A | 125A |
|  |  |  | 0589 | 1458 | 1458 |
|  |  |  | 169A | 169A | 236B |
|  |  |  | 236B | 27AB | 2569 |
|  |  |  | 2569 | 367A | 27AB |
|  |  |  | 367A | 478B | 478B |
| 3 | (01458) | [202420] | 01459 | 01458 | 01458 |
|  |  |  | 01489 | 01459 | 01589 |
|  |  |  | 01589 | 01489 | 0348B |
|  |  |  | 03478 | 03478 | 0378B |
|  |  |  | 0347B | 0347B | 04589 |
|  |  |  | 0348B | 0478B | 0478B |
|  |  |  | 0378B | 1256A | 12569 |
|  |  |  | 04589 | 1259A | 1256A |
|  |  |  | 12569 | 1269A | 1259A |
|  |  |  | 1269A | 14589 | 14589 |
|  |  |  | 1569A | 1569A | 2367B |
|  |  |  | 2367A | 2367A | 236AB |
|  |  |  | 2367B | 237AB | 237AB |
|  |  |  | 236AB | 267AB | 2569A |
|  |  |  | 2569A | 3478B | 267AB |
|  |  |  | 367 AB | 367 AB | 3478B |
|  | (02468) | [040402] | 02468 | 0246A | 02468 |
|  |  |  | 0246A | 0248A | 0248A |
|  |  |  | 0268A | 0468A | 0268A |
|  |  |  | 0468A | 13579 | 1357B |
|  |  |  | 13579 | 1357B | 1359B |
|  |  |  | 1359B | 1379B | 1579B |
|  |  |  | 1379B | 1579B | 2468A |
|  |  |  | 3579B | 2468A | 3579B |

H4 ${ }^{2, n}$ continued

| Nodes | SC | IV | Oct ${ }^{1,2} \mathrm{R}^{\mathrm{x}, \mathrm{x}-4 \mathrm{yyz}}$ | Oct ${ }^{1,3,3} \mathrm{R}^{\mathrm{x}, \times+4 \mathrm{yz}>}$ | Oct ${ }^{2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-4\langle 4 \mathrm{y}\rangle}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 3 \\ \text { cont. } \end{gathered}$ | (012469) | [233331] | $\begin{aligned} & 012469 \\ & 024569 \\ & 034579 \\ & 035789 \\ & 03678 \mathrm{~A} \\ & 0368 \mathrm{AB} \\ & 12369 \mathrm{~B} \\ & \text { 1369AB } \end{aligned}$ | 01479B 02347A 0247AB 12357A 13567A 14568 A 14689A 14789B | 01258A 02589A <br> 13458B <br> 23468B <br> 24678B <br> 25679B <br> 2579AB |
|  | (012568) | [322332] | $\begin{aligned} & 012689 \\ & 013799 \\ & 023678 \\ & 03459 \mathrm{~B} \\ & 0346 \mathrm{AB} \\ & 04569 \mathrm{~A} \\ & 123699 \\ & 3569 \mathrm{AB} \\ & \hline \end{aligned}$ | 01456A 01457 B 0467AB 12379 A 12489 A 134789 1567 AB 23478 A | 012568 01578 B 02678 B 12567B 2348 AB 2359 AB 24589 A 34589B |
| 4 | (0124689)* | [343542] | 0124689 0135789 023678A 024569A 034579B 03468AB 123679B 13569AB | 014568A 014579 B 023478 A 02467 AB 123579A 124689A 134789B 13567AB | 012568A 013578 B 024589A 024678B 125679B 134589B 23468AB 23579 AB |
|  | (0125689)* | [424542] | 0125689 023678 B 034589 B 23569 AB | $\begin{aligned} & 0134789 \\ & 014569 \mathrm{~A} \\ & 03467 \mathrm{AB} \\ & 123679 \mathrm{~A} \\ & \hline \end{aligned}$ | $\begin{aligned} & 014578 \mathrm{~B} \\ & 124589 \mathrm{~A} \\ & 12567 \mathrm{AB} \\ & 23478 \mathrm{AB} \end{aligned}$ |
|  | (0134579)* | [344532] |  |  |  |
| 5 | (0124689)* | [343542] | 0124689 0135789 023678A 024569A 034579B 03468AB 123679 B 13569 AB | 014568A 014579 B 023478 A 02467AB 123579A 124689A 134789B 13567AB | 012568A 013578 B 024589A 024678B 125679B 134589B 23468AB 23579 AB |
|  | (0125689) | [424542] | 0125689 023678 B 034589 B 23569 AB | 0134789 014569 A 03467 AB 123679 A | $\begin{aligned} & \hline 014578 \mathrm{~B} \\ & 124589 \mathrm{~A} \\ & 12567 \mathrm{AB} \\ & 23478 \mathrm{AB} \end{aligned}$ |
|  | (0134579) | [344532] | $\begin{aligned} & \hline 012469 \mathrm{~A} \\ & 0134579 \\ & \text { 034678A } \\ & \text { 13679AB } \end{aligned}$ | $\begin{aligned} & \hline 02478 \mathrm{AB} \\ & 12357 \mathrm{AB} \\ & 124568 \mathrm{~A} \\ & 145789 \mathrm{~B} \end{aligned}$ | 013589B 023468B 025689A 235679B |

H4 $4^{2, n}$ continued

| Nodes | SC | IV | Oct ${ }^{1,2} \mathrm{R}^{\mathrm{x}, \mathrm{x}-4<\mathrm{yz}>}$ | Oct ${ }^{1,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+4 \mathrm{yz} \text { > }}$ | Oct ${ }^{2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-4<\mathrm{yz}>}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 5 \\ \text { cont. } \end{gathered}$ | (01245689) | [545752] | 01245689 | 0134789B | 0124589A |
|  |  |  | 0124569A | 0145689A | 0125689A |
|  |  |  | 01345789 | 0145789B | 0134578B |
|  |  |  | 023678AB | 023467AB | 0134589B |
|  |  |  | 0345789B | 023478AB | 0234678B |
|  |  |  | 034678AB | 1235679A | 125679 AB |
|  |  |  | 123569AB | 123567 AB | 234678AB |
|  |  |  | 123679AB | 1245689A | 235679 AB |
|  | (0124568A) | [464743] | 0124689A | 0124568A | 0135789B |
|  |  |  | 0134579B | 024678AB | 023468AB |
|  |  |  | 0234678A | 123579AB | 0245689A |
|  |  |  | 135679AB | 1345789B | 1235679B |
|  | (012346789) | [766674] | 012356789 | 01234789A | 01245678B |
|  |  |  | 01236789B | 0134567AB | 0125678AB |
|  |  |  | 0234569AB | 0145679AB | 1234589 AB |
|  |  |  | 0345689AB | 12346789A | 2345789AB |

Hybrid $5^{2, n}$ Family

| Nodes | SC | IV | Oct ${ }^{1,2} \mathrm{R}^{\mathrm{x}, \mathrm{x}+5 \mathrm{y} z>}$ | Oct ${ }^{1,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-5 \mathrm{yyz}}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | (027)* | [010020] | 057 | 027 | 05A |
|  |  |  | 16B | 168 | 138 |
|  |  |  | 249 | 35A | 279 |
|  |  |  | 38A | 49B | 46B |
|  | (0158)* | [101220] | 0378 | 047B | 0158 |
|  |  |  | 0459 | 1489 | 259A |
|  |  |  | 1269 | 156A | 267B |
|  |  |  | 36AB | 237A | 348B |
|  | $\begin{array}{\|l\|} (0167)^{*} \\ \text { referential } \end{array}$ | [200022] | 0167 | 0167 | 056B |
|  |  |  | 056B | 1278 | 1278 |
|  |  |  | 2389 | 349A | 2389 |
|  |  |  | 349A | 45AB | 45AB |
|  | $\begin{aligned} & \text { (0358) } \\ & \text { referential } \end{aligned}$ | [012120] | 0259 | 037A | 0259 |
|  |  |  | 0358 | 0479 | 0358 |
|  |  |  | 037A | 136A | 148B |
|  |  |  | 0479 | 1469 | 158A |
|  |  |  | 136A | 148B | 247B |
|  |  |  | 1469 | 158A | 257A |
|  |  |  | 269B | 247B | 269B |
|  |  |  | 368B | 257A | 368B |
| 3 | (01267) | [310132] | 01567 | 01267 | 0456B |
|  |  |  | 0156B | 01278 | 045AB |
|  |  |  | 0167B | 01678 | 056AB |
|  |  |  | 0567B | 12678 | 12378 |
|  |  |  | 23489 | 3459A | 12389 |
|  |  |  | 2349A | 345 AB | 12789 |
|  |  |  | 2389A | 349 AB | 23789 |
|  |  |  | 3489A | 459 AB | 456AB |
|  | (01358) | [122230] | 02459 | 0237A | 01358 |
|  |  |  | 03578 | 0247B | 0158A |
|  |  |  | 0378A | 0479B | 0259A |
|  |  |  | 04579 | 1356A | 1348B |
|  |  |  | 12469 | 14689 | 2467B |
|  |  |  | 1269B | 1489B | 2579A |
|  |  |  | 136AB | 1568A | 2679B |
|  |  |  | 368 AB | 2357A | 3468B |
|  | (02479) | [032140] | 02479 | 02479 | 02579 |
|  |  |  | 02579 | 0257A | 0257A |
|  |  |  | 0357A | 0357A | 0358A |
|  |  |  | 0358A | 1358A | 1358A |
|  |  |  | 1368A | 1368A | 1368B |
|  |  |  | 1368B | 1468B | 1468B |
|  |  |  | 1469B | 1469B | 2469B |
|  |  |  | 2469B | 2479B | 2479B |

H5 ${ }^{2, n}$ continued

| Nodes | SC | IV | Oct ${ }^{1,2} \mathrm{R}^{\mathrm{x}, x+5 \times \mathrm{yz7}}$ | Oct ${ }^{1,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-5\langle\mathrm{yz7}}$ | $\mathrm{Oct}^{2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+5 \times \mathrm{yz7}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 3 \\ \text { cont. } \end{gathered}$ | (012569) | [313431] | 012569 | 014789 | 012589 |
|  |  |  | 014569 | 01478B | 01458B |
|  |  |  | 034589 | 0347AB | 03458B |
|  |  |  | 034789 | 12367A | 12589A |
|  |  |  | 03678B | 12567A | 23478B |
|  |  |  | 0367AB | 14569A | 23678B |
|  |  |  | 12369A | 14589A | 2567 AB |
|  |  |  | 2369AB | 2347 AB | 2569AB |
|  | (012578) | [322242] | 012679 | 01467B | 012578 |
|  |  |  | 013678 | 01567A | 01568B |
|  |  |  | 023789 | 0457AB | 02567B |
|  |  |  | 03459A | 12378A | 12678B |
|  |  |  | 0356AB | 124789 | 23489B |
|  |  |  | 04569B | 13489A | 23589A |
|  |  |  | 123689 | 1456AB | 2459 AB |
|  |  |  | 3469 AB | 23479A | 3458 AB |
| 4 | (0123589)* | [434442] | 012569B | 012367A | 01458AB |
|  |  |  | 0234589 | 0146789 | 123478B |
|  |  |  | 035678B | 03479AB | 125789A |
|  |  |  | 23689 AB | 134569A | 24567AB |
|  | (0124679)* | [344352] | 0124679 | 02457AB | 013568B |
|  |  |  | 013678A | 123578A | 023589A |
|  |  |  | 034579A | 124789B | 025679B |
|  |  |  | 13469AB | 14568AB | 234689B |
|  | (0125679)* | [433452] | 0125679 | 0124789 | 0125789 |
|  |  |  | 013678B | 012567A | 014568B |
|  |  |  | 014569B | 014678B | 02567AB |
|  |  |  | 0234789 | 03457AB | 03458AB |
|  |  |  | 034589A | 123678A | 123589A |
|  |  |  | 03567AB | 134589A | 123678B |
|  |  |  | 123689A | 14569AB | 234789B |
|  |  |  | 23469AB | 23479AB | 24569 AB |
| 5 | (0123589) | [434442] | 012569B | 012367A | 01458AB |
|  |  |  | 0234589 | 0146789 | 123478B |
|  |  |  | 035678B | 03479AB | 125789A |
|  |  |  | 23689AB | 134569A | 24567 AB |
|  | (0124679) | [344352] | 0124679 | 02457AB | 013568B |
|  |  |  | 013678A | 123578A | 023589A |
|  |  |  | 034579A | 124789B | 025679B |
|  |  |  | 13469AB | 14568AB | 234689B |
|  | (0125679)* | [433452] | 0125679 | 0124789 | 0125789 |
|  |  |  | 013678B | 012567A | 014568B |
|  |  |  | 014569B | 014678B | 02567AB |
|  |  |  | 0234789 | 03457AB | 03458AB |
|  |  |  | 034589A | 123678A | 123589A |
|  |  |  | 03567AB | 134589A | 123678B |
|  |  |  | 123689A | 14569AB | 234789B |
|  |  |  | 23469 AB | 23479 AB | 24569 AB |
|  | (01235789) | [554563] | 0125679B | 0123678A | 014568 AB |
|  |  |  | 0135678B | 01246789 | 024567 AB |
|  |  |  | 0234589A | 034579AB | 1234789B |
|  |  |  | 234689 AB | 134569 AB | 1235789A |

H5 ${ }^{2, n}$ continued

| Nodes | SC | IV | Oct ${ }^{1,2} \mathrm{R}^{\mathrm{x}, \mathrm{x}+5<\mathrm{yz}>}$ | Oct ${ }^{1,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-5<\mathrm{yz}\rangle}$ | Oct ${ }^{2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+5\langle\mathrm{yz}\rangle}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 5 \\ \text { cont. } \end{gathered}$ | (01245679) | [555562] | 01245679 | 0123567A | 0123589A |
|  |  |  | 0124569B | 0124789B | 0125789A |
|  |  |  | 013678AB | 0146789B | 0134568B |
|  |  |  | 02345789 | 023457AB | 013458AB |
|  |  |  | 0345789A | 023479AB | 025679AB |
|  |  |  | 035678AB | 1235678A | 1234678B |
|  |  |  | 123469 AB | 1345689A | 2346789B |
|  |  |  | 123689 AB | 145689AB | 245679AB |
|  | (012346789) | [766674] | 012346789 | 0124567AB | 01235678B |
|  |  |  | 01236789A | 0145678AB | 01256789B |
|  |  |  | 0134569AB | 12345789A | 0234589AB |
|  |  |  | 0345679AB | 1234789AB | 2345689AB |

Hybrid $112^{3, n}$ Family

| Nodes | SC | IV | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+2, x+1\langle y z\rangle}$ | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-1, \mathrm{x}-2\langle\mathrm{yz}\rangle}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  <br>  <br>  <br>  <br>  <br> 2 | (012345) | [543210] | $\begin{aligned} & \hline 012345 \\ & 0123 \mathrm{AB} \\ & 0129 \mathrm{AB} \\ & 0789 \mathrm{AB} \\ & 123456 \\ & 345678 \\ & 456789 \\ & 6789 \mathrm{AB} \end{aligned}$ | 01234 B 0123 AB 0189 AB 0789 AB 123456 234567 456789 56789 A | 012345 01234 B 0129 AB 0189 AB 234567 345678 56789 A 6789 AB |
|  | (012456) | [432321] | 012456 0678AB 1239AB 345789 | $\begin{aligned} & 01789 \mathrm{~B} \\ & 0234 \mathrm{AB} \\ & 123567 \\ & 45689 \mathrm{~A} \end{aligned}$ | $\begin{aligned} & \hline 01289 \mathrm{~A} \\ & 01345 \mathrm{~B} \\ & 234678 \\ & 5679 \mathrm{AB} \end{aligned}$ |
|  | (012567) | [421242] |  | $\begin{aligned} & \text { 01456B } \\ & 0567 \mathrm{AB} \\ & 12389 \mathrm{~A} \\ & 234789 \\ & \hline \end{aligned}$ | 012567 01678 B <br> 2349AB <br> 34589A |
|  | (012678) | [420243] |  | $\begin{aligned} & \hline 01567 \mathrm{~B} \\ & 0456 \mathrm{AB} \\ & 123789 \\ & 23489 \mathrm{~A} \\ & \hline \end{aligned}$ | 012678 01567B 23489A 3459AB |
| 3 | (0123456) | [654321] | $\begin{aligned} & 0123456 \\ & 01239 \mathrm{AB} \\ & 06789 \mathrm{AB} \\ & 3456789 \\ & \hline \end{aligned}$ | $\begin{aligned} & 01234 \mathrm{AB} \\ & 01789 \mathrm{AB} \\ & 1234567 \\ & 456789 \mathrm{~A} \\ & \hline \end{aligned}$ | $\begin{aligned} & 012345 \mathrm{~B} \\ & 01289 \mathrm{AB} \\ & 2345678 \\ & 56789 \mathrm{AB} \\ & \hline \end{aligned}$ |
|  | (0123678) | [532353] | 0123678 0123789 0126789 03456 AB 03459 AB 04569 AB 1236789 34569 AB | 014567B 01456 AB 01567 AB 04567 AB 1234789 123489 A 123789 A 234789A | 0125678 012567 B 012678 B 015678 B 234589 A 23459 AB 23489 AB 34589AB |
|  | (01234567) | [765442] | 012345 AB 012789 AB 12345678 456789 AB | 0123456B 012389 AB 056789 AB 23456789 | 01234567 012349 AB 016789 AB 3456789 A |
|  | (01234678) | [654553] | 01245678 012456 AB 012678 AB 045678 AB 12345789 12345 AB 12379 AB 345789 AB | 0123567B 0123789 B 0156789 B 023456 AB 023489 AB 04568 AB 12356789 2345689 A | 01234678 0123489 A 0126789 A 0134567 B 013459 AB 01567 AB 2346899 345679AB |

$H M 2^{3, n}$ continued

| Nodes | SC | IV |  | Oct ${ }^{1,2,3,} \mathrm{R}^{\mathrm{x}, \mathrm{x}-1, \mathrm{x}-2\langle\mathrm{yz}\rangle}$ | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-1, x+1\langle y \mathrm{y}\rangle}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 3 \\ \text { cont. } \end{gathered}$ | (012345789) | [766773] | 012345789 0123678 AB 012456789 0124569 AB 0345678 AB 0345789 AB 1234569 AB 1236789 AB | 01234789 B 0123567 AB 01456789 B 0145689 AB 0234567 AB 0234789 AB 12345689 A 12356789 A | 01234589 A 01234678 B 01256789 A 0125679 AB 01345678 B 0134589 AB 2345679 AB 2346789 AB |
| 4 | (012345678)* | [876663] | 012345678 0123456 AB 0123459 AB 0123789 AB 0126789 AB 0456789 AB 123456799 3456789 AB | 01234567 B 0123456 AB 0123489 AB 0123789 AB 0156789 AB 0456789 AB 123456789 23456789 A | 012345678 01234567 B 0123459 AB 0123489 AB 0126789 AB 0156789 AB 23456789 A 3456789 AB |
|  | (0123456789)* | [988884] | 0123456789 01234569 AB 01236789 AB 03456789 AB | 01234567 AB 01234789 AB 01456789 AB 123456789 A | 012345678 B 01234589 AB 01256789 AB 23456789 AB |
|  | (0123456789A)* | [AAAAA5] | 012345678 AB 012345789 AB 012456789 AB 123456789 AB | $\begin{aligned} & 0123456789 \mathrm{~B} \\ & 012345689 \mathrm{AB} \\ & 012356789 \mathrm{AB} \\ & 023456789 \mathrm{AB} \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 0123456789 \mathrm{~A} \\ & 012345679 \mathrm{AB} \\ & 012346789 \mathrm{AB} \\ & 013456789 \mathrm{AB} \\ & \hline \end{aligned}$ |
| 5 | (012345678)* | [876663] | 012345678 0123456 AB 0123459 AB 0123789 AB 0126789 AB 0456789 AB 123456789 3456789 AB | 01234567 B 0123456 AB 0123489 AB 0123789 AB 0156789 AB 0456789 AB 123456799 23456789 A | 012345678 01234567 B 0123459 AB 0123489 AB 0126789 AB 0156789 AB 23456789 A 3456789 AB |
|  | (0123456789)* | [988884] | 0123456789 01234569 AB 01236789 AB 03456789 AB | 01234567AB 01234789 AB 01456789 AB 123456789 A | 012345678B 01234589 AB 01256789 AB 23456789 AB |
|  | (0123456789A)* | [AAAAA5] | 012345678 AB 012345789 AB 012456789 AB 123456789 AB | 0123456789 B 012345689 AB 012356789 AB 023456789 AB | 0123456789 A 012345679 AB 012346789 AB 013456789 AB |
|  | (0123456789AB)* | $\begin{gathered} {[12,12,12,} \\ 12,12,6] \\ \hline \end{gathered}$ | 0123456789AB |  |  |

Hybrid $2 \backslash 4^{3, n}$ Family

| Nodes | SC | IV | Oct ${ }^{1,2,3} \mathrm{R}^{\text {x,x-4,x+2}\langle\times y z}$ | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \times+2, \mathrm{x}-2\langle\mathrm{yz}\rangle}$ | Oct ${ }^{12,3,3} \mathrm{R}^{\mathrm{x}, \times+2, \times+4 \times y z>}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | (02468)* | [040402] | $\begin{aligned} & \hline 2248 \mathrm{~A} \\ & 1357 \mathrm{~B} \\ & 1579 \mathrm{~B} \\ & 2468 \mathrm{~A} \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 0246 \mathrm{~A} \\ & 0468 \mathrm{~A} \\ & 13579 \\ & 1379 \mathrm{~B} \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 02468 \\ & 0268 \mathrm{~A} \\ & \text { 1359B } \\ & 3579 \mathrm{~B} \\ & \hline \end{aligned}$ |
|  | (023457) | [343230] | 01249B 0138 AB 023457 05789 A 12346 B 1689 AB 245679 35678 A | $\begin{aligned} & 01235 \mathrm{~A} \\ & 0138 \mathrm{AB} \\ & 0279 \mathrm{AB} \\ & 05789 \mathrm{~A} \\ & 12346 \mathrm{~B} \\ & 134568 \\ & 245679 \\ & 46789 \mathrm{~B} \end{aligned}$ | 01235A <br> 01249 B <br> 023457 <br> 0279 AB <br> 134568 <br> 1689 AB <br> 35678A <br> 46789B <br> 1 |
|  | (024579)* | [143250] | 024579 03578 A 12469 B 1368 AB | $\begin{aligned} & 01358 \mathrm{~A} \\ & 02579 \mathrm{~A} \\ & 13468 \mathrm{~B} \\ & 24679 \mathrm{~B} \\ & \hline \end{aligned}$ | $\begin{array}{\|l} \hline 02357 \mathrm{~A} \\ \text { 02479B } \\ 13568 \mathrm{~A} \\ 14689 \mathrm{~B} \\ \hline \end{array}$ |
|  | (02468A)* | [060603] | $\begin{aligned} & \text { 02468A } \\ & 13579 \mathrm{~B} \end{aligned}$ | $\begin{aligned} & \text { 02468A } \\ & 13579 \mathrm{~B} \end{aligned}$ | $\begin{aligned} & \hline 02468 \mathrm{~A} \\ & 13579 \mathrm{~B} \\ & \hline \end{aligned}$ |
| 3 | (02468A)* | [060603] | $\begin{aligned} & \hline 02468 \mathrm{~A} \\ & 13579 \mathrm{~B} \\ & \hline \end{aligned}$ | $\begin{aligned} & \text { 02468A } \\ & \text { 13579B } \end{aligned}$ | $\begin{array}{\|l} \hline 02468 \mathrm{~A} \\ \text { 13579B } \\ \hline \end{array}$ |
|  | (0234579) | [354351] | 012469B 01368 AB 0234579 0245679 035678 A 035789 A 123469 B 13689AB | 012358A 01358 AB 025789A 02579AB 123468B 134568B 245679B | 012357 A 012479 B 023457 A 02479 AB 134568A 135678 A 146789 B 14689 AB |
|  | (01234568) | [665542] | 0123457 B 012348 AB 012489 AB 015789 AB 1234567 B 156789 AB 2345678 A 2456789 A | 0123456A 012346 AB 012379 AB 013789 AB 0456789 A 046789 AB 1234579 13456789 | 01234568 0123599 B 012359 AB 012689 AB 02345678 026789 AB 3456799 B 356789 AB |
|  | (0123568A) | [465562] | 0124579B 013578 AB 0234578A 0245789 A 123468 AB 1245679 B 12468 AB 135678 AB | 0123579A 0134568 A 013468 AB 0135789 A 0245679 A 024679 AB 123469 B 1346789 B | $\begin{aligned} & \text { 0123568A } \\ & 0124689 \mathrm{~B} \\ & 0234579 \mathrm{~B} \\ & 0235678 \mathrm{~A} \\ & 023579 \mathrm{AB} \\ & 0246789 \mathrm{~B} \\ & 1345698 \mathrm{~B} \\ & 135689 \mathrm{AB} \end{aligned}$ |

H2\4 $4^{3, n}$ continued

| Nodes | SC | IV | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-4, \mathrm{x}+2<\mathrm{yz}>}$ | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+2, \mathrm{x}-2<\mathrm{yz}>}$ | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+2, \mathrm{x}+4<\mathrm{yz}>}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 3 \\ \text { cont. } \end{gathered}$ | (01234568A) | [474643] | 01234579B | 01234568A | 01234568A |
|  |  |  | 0123468AB | 0123468AB | 01234579B |
|  |  |  | 0124689AB | 0123579AB | 0123579AB |
|  |  |  | 0135789AB | 0135789AB | 0124689AB |
|  |  |  | 02345678A | 02456789A | 02345678A |
|  |  |  | 02456789A | 0246789AB | 0246789AB |
|  |  |  | 12345679B | 12345679B | 13456789B |
|  |  |  | 1356789AB | 13456789B | 1356789AB |
| 4 | (01234568A)* | [686763] | 01234579B | 01234568A | 01234568A |
|  |  |  | 0123468AB | 0123468AB | 01234579B |
|  |  |  | 0124689AB | 0123579AB | 0123579AB |
|  |  |  | 0135789AB | 0135789AB | 0124689AB |
|  |  |  | 02345678A | 02456789A | 02345678A |
|  |  |  | 02456789A | 0246789AB | 0246789AB |
|  |  |  | 12345679B | 12345679B | 13456789B |
|  |  |  | 1356789AB | 13456789B | 1356789AB |
|  | (0123456789A)* | [AAAAA5] | 012345678AB | 0123456789A | 0123456789B |
|  |  |  | 012345789AB | 012345679AB | 012345689AB |
|  |  |  | 012456789AB | 012346789AB | 012356789AB |
|  |  |  | 123456789 AB | 013456789AB | 023456789AB |
| 5 | (01234568A)* | [686763] | 01234579B | 01234568A | 01234568A |
|  |  |  | 0123468AB | 0123468AB | 01234579B |
|  |  |  | 0124689AB | 0123579AB | 0123579AB |
|  |  |  | 0135789AB | 0135789AB | 0124689AB |
|  |  |  | 02345678A | 02456789A | 02345678A |
|  |  |  | 02456789A | 0246789AB | 0246789AB |
|  |  |  | 12345679B | 12345679B | 13456789B |
|  |  |  | 1356789 AB | 13456789B | 1356789 AB |
|  | (0123456789A)* | [AAAAA5] | 012345678AB | 0123456789A | 0123456789B |
|  |  |  | 012345789AB | 012345679AB | 012345689AB |
|  |  |  | 012456789AB | 012346789AB | 012356789 AB |
|  |  |  | 123456789AB | 013456789AB | 023456789AB |
|  | (0123456789AB)* | $\begin{gathered} {[12,12,12,} \\ 12,12,6] \end{gathered}$ | 0123456789 AB |  |  |

Hybrid $4^{3, n}$ Family

| Nodes | SC | IV | $\begin{aligned} & \text { Oct }^{1,2,3} \mathbf{R}^{\mathrm{x}, \mathrm{x}-4, \mathrm{x}+4<\mathrm{yz>}>} \\ & \text { Oct }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-2, \mathrm{x}-6 \times \mathrm{yz>}} \end{aligned}$ |
| :---: | :---: | :---: | :---: |
| 2 | (048)* | [000300] | $\begin{aligned} & \hline 048 \\ & 159 \\ & 26 \mathrm{~A} \\ & 37 \mathrm{~B} \end{aligned}$ |
|  | (014589)* | [303630] | $\begin{aligned} & \hline 014589 \\ & 03478 \mathrm{~B} \\ & 12569 \mathrm{~A} \\ & 2367 \mathrm{AB} \end{aligned}$ |
|  | (02468A)* | [060603] | $\begin{aligned} & \hline 02468 \mathrm{~A} \\ & 13579 \mathrm{~B} \\ & \hline \end{aligned}$ |
| 3 | (014589)* | [303630] | $\begin{aligned} & \hline 014589 \\ & 03478 \mathrm{~B} \\ & 12569 \mathrm{~A} \\ & 2367 \mathrm{AB} \\ & \hline \end{aligned}$ |
|  | (02468A)* | [060603] | $\begin{aligned} & \hline 02468 \mathrm{~A} \\ & 13579 \mathrm{~B} \end{aligned}$ |
|  | (01245689A)* | [666963] | $\begin{aligned} & \hline 01245689 \mathrm{~A} \\ & 01345789 \mathrm{~B} \\ & 0234678 \mathrm{AB} \\ & 1235679 \mathrm{AB} \end{aligned}$ |
| 4 | (01245689A)* | [666963] | $\begin{aligned} & \hline 01245689 \mathrm{~A} \\ & 01345789 \mathrm{~B} \\ & 0234678 \mathrm{AB} \\ & 1235679 \mathrm{AB} \end{aligned}$ |
| 5 | (01245689A)* | [666963] | $\begin{aligned} & \hline 01245689 \mathrm{~A} \\ & 01345789 \mathrm{~B} \\ & 0234678 \mathrm{AB} \\ & 1235679 \mathrm{AB} \end{aligned}$ |
|  | (0123456789AB)* | [12,12,12,12,12,6] | 0123456789AB |

## Hybrid $512^{3, n}$ Family

| Nodes | SC | IV | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+5, \mathrm{x}-2<\mathrm{yz}>}$ | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+5, \mathrm{x}-5<\mathrm{yz}>}$ | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+2, \mathrm{x}-5<\mathrm{yz>}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | (0257)* | [021030] | 057A | 0257 | 0279 |
|  |  |  | 138A | 168B | 035A |
|  |  |  | 146B | 249B | 1368 |
|  |  |  | 2479 | 358A | 469B |
|  | (012678)* | [420243] | 01567B | 012678 | 012678 |
|  |  |  | 0456AB | 01567B | 0456AB |
|  |  |  | 123789 | 23489A | 123789 |
|  |  |  | 23489A | 3459AB | 3459AB |
|  | (013578)* | [232341] | 013578 | 02378A | 01568A |
|  |  |  | 02459A | 04579B | 02467B |
|  |  |  | 12679B | 124689 | 13489B |
|  |  |  | 3468AB | 1356AB | 23579A |
|  | (024579) | [143250] | 01358A | 02357A | 01358A |
|  |  |  | 024579 | 024579 | 02357A |
|  |  |  | 02579A | 02479B | 02479B |
|  |  |  | 03578A | 03578A | 02579A |
|  |  |  | 12469B | 12469B | 13468B |
|  |  |  | 13468B | 13568A | 13568A |
|  |  |  | 1368 AB | 1368AB | 14689B |
|  |  |  | 24679B | 14689B | 24679B |
| 3 | (024579) | [143250] | 01358A | 02357A | 01358A |
|  |  |  | 024579 | 024579 | 02357A |
|  |  |  | 02579A | 02479B | 02479B |
|  |  |  | 03578A | 03578A | 02579A |
|  |  |  | 12469B | 12469B | 13468B |
|  |  |  | 13468B | 13568A | 13568A |
|  |  |  | 1368 AB | 1368AB | 14689B |
|  |  |  | 24679B | 14689B | 24679B |
|  | (0123678) | [532353] | 014567B | 0125678 | 0123678 |
|  |  |  | 01456AB | 012567B | 0123789 |
|  |  |  | 01567AB | 012678B | 0126789 |
|  |  |  | 04567AB | 015678B | 03456AB |
|  |  |  | 1234789 | 234589A | 03459AB |
|  |  |  | 123489A | 23459AB | 04569AB |
|  |  |  | 123789A | 23489AB | 1236789 |
|  |  |  | 234789A | 34589AB | 34569AB |
|  | (013568A)* | [254361] | 013578A | 023578A | 013568A |
|  |  |  | 024579A | 024579B | 023579A |
|  |  |  | 124679B | 124689B | 024679B |
|  |  |  | 13468AB | 13568AB | 134689B |
|  | (01235789) | [554563] | 01235789 | 0123678A | 0124678B |
|  |  |  | 0125679B | 01246789 | 0125678A |
|  |  |  | 0135678B | 013567AB | 014568AB |
|  |  |  | 0234589A | 0145679B | 024567AB |
|  |  |  | 024569AB | 0234789A | 1234789B |
|  |  |  | 034568AB | 034579AB | 1235789A |
|  |  |  | 1236789B | 1234689A | 134589AB |
|  |  |  | 234689AB | 134569AB | 234579AB |

H5 $22^{3, n}$ continued

| Nodes | SC | IV | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+5, \mathrm{x}-2\langle\mathrm{yz}\rangle}$ | Oct ${ }^{1,2,3,} \mathrm{R}^{\mathrm{x}, \mathrm{x}+5, \mathrm{x}-5\langle\mathrm{yz}\rangle}$ | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+2, \mathrm{x}-5\langle\mathrm{yz}\rangle}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 3 \\ \text { cont. } \end{gathered}$ | (012345789) | [766773] | 012345789 | 0123567 AB | 01234678B |
|  |  |  | 01234589A | 0123678 AB | 01234789B |
|  |  |  | 0124569AB | 012456789 | 01256789A |
|  |  |  | 0125679 AB | 01456789B | 0134589AB |
|  |  |  | 01345678B | 0234789AB | 0145689AB |
|  |  |  | 0345678AB | 0345789AB | 0234567 AB |
|  |  |  | 1236789AB | 12345689A | 12356789A |
|  |  |  | 2346789AB | 1234569 AB | 2345679AB |
| 4 | (012345789)* | [766773] | 012345789 | 0123567 AB | 01234678B |
|  |  |  | 01234589A | 0123678 AB | 01234789B |
|  |  |  | 0124569AB | 012456789 | 01256789A |
|  |  |  | 0125679 AB | 01456789B | 0134589 AB |
|  |  |  | 01345678B | 0234789AB | 0145689AB |
|  |  |  | 0345678AB | 0345789AB | 0234567 AB |
|  |  |  | 1236789 AB | 12345689A | 12356789A |
|  |  |  | 2346789AB | 1234569AB | 2345679AB |
|  | (01235678A)* | [576683] | 01245679B | 01235678A | 01235789A |
|  |  |  | 0135678 AB | 01246789B | 0134568AB |
|  |  |  | 02345789A | 0234579AB | 0245679AB |
|  |  |  | 1234689AB | 1345689AB | 12346789B |
|  | (012345789A)* | [888894] | 012345789A | 01235678AB | 012346789B |
|  |  |  | 01245679 AB | 012456789B | 012356789A |
|  |  |  | 01345678 AB | 02345789AB | 01345689AB |
|  |  |  | 12346789AB | 12345689 AB | 02345679 AB |
| 5 | (012345789) | [766773] | 012345789 | 0123567 AB | 01234678B |
|  |  |  | 01234589A | 0123678 AB | 01234789B |
|  |  |  | 0124569AB | 012456789 | 01256789A |
|  |  |  | 0125679 AB | 01456789B | 0134589 AB |
|  |  |  | 01345678B | 0234789AB | 0145689AB |
|  |  |  | 0345678 AB | 0345789AB | 0234567 AB |
|  |  |  | 1236789AB | 12345689A | 12356789A |
|  |  |  | 2346789AB | 1234569AB | 2345679AB |
|  | (01235678A) | [576683] | 01245679B | 01235678A | 01235789A |
|  |  |  | 0135678 AB | 01246789B | 0134568 AB |
|  |  |  | 02345789A | 0234579AB | 0245679AB |
|  |  |  | 1234689 AB | 1345689AB | 12346789B |
|  | (012345789A) | [888894] | 012345789A | 01235678 AB | 012346789B |
|  |  |  | 01245679AB | 012456789B | 012356789A |
|  |  |  | 01345678 AB | 02345789 AB | 01345689 AB |
|  |  |  | 12346789 AB | 12345689 AB | 02345679 AB |
|  | (0123456789A) | [AAAAA5] | 0123456789B | 0123456789A | 012345678 AB |
|  |  |  | 012345689 AB | 012345679 AB | 012345789 AB |
|  |  |  | 012356789 AB | 012346789 AB | 012456789 AB |
|  |  |  | 023456789 AB | 013456789 AB | 123456789 AB |

Hybrid $1 \backslash 4 \backslash 5^{3, n}$ Family (A)

| Nodes | SC | IV | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-4, \mathrm{x}-5<\mathrm{yz}>}$ | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+5, \mathrm{x}+1<\mathrm{yz}>}$ | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-1, \mathrm{x}+4<\mathrm{yz>}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | (01458)* | [202420] | 0478B | 01459 | 01589 |
|  |  |  | 1256A | 03478 | 0348B |
|  |  |  | 14589 | 1269A | 2367B |
|  |  |  | 237 AB | 367 AB | 2569A |
|  | (01568)* | [211231] | 02378 | 01578 | 0156A |
|  |  |  | 0459B | 1267B | 0467B |
|  |  |  | 12689 | 2459A | 13489 |
|  |  |  | 356AB | 348AB | 2379A |
|  | (012678)* | [420243] | 012678 | 012678 | 01567B |
|  |  |  | 0456AB | 01567B | 0456AB |
|  |  |  | 123789 | 23489A | 123789 |
|  |  |  | 3459AB | 3459AB | 23489A |
|  | (013458) | [323430] | 01489B | 01259A | 01259A |
|  |  |  | 02347B | 013458 | 013458 |
|  |  |  | 0378AB | 0378AB | 01489B |
|  |  |  | 045789 | 045789 | 02347B |
|  |  |  | 1236AB | 1236AB | 15689A |
|  |  |  | 124569 | 124569 | 23567A |
|  |  |  | 15689A | 2679AB | 2679AB |
|  |  |  | 23567A | 34678B | 34678B |
| 3 | (0123678) | [532353] | 0123678 | 0125678 | 014567B |
|  |  |  | 0123789 | 012567B | 01456AB |
|  |  |  | 0126789 | 012678B | 01567AB |
|  |  |  | 03456AB | 015678B | 04567AB |
|  |  |  | 03459AB | 234589A | 1234789 |
|  |  |  | 04569AB | 23459AB | 123489A |
|  |  |  | 1236789 | 23489AB | 123789A |
|  |  |  | 34569AB | 34589AB | 234789A |
|  | (0124569) | [434541] | 014789B | 0124569 | 012589A |
|  |  |  | 02347AB | 0345789 | 013458B |
|  |  |  | 123567A | 03678AB | 234678B |
|  |  |  | 145689A | 12369AB | 25679AB |
|  | (0124589) | [424641] | 0145789 | 012569A | 012569A |
|  |  |  | 014589B | 0134589 | 0134589 |
|  |  |  | 023478B | 0145789 | 014589B |
|  |  |  | 03478AB | 034678B | 023478B |
|  |  |  | 12367AB | 03478AB | 034678B |
|  |  |  | 124569A | 12367AB | 125689A |
|  |  |  | 125689A | 124569A | 23567 AB |
|  |  |  | 23567AB | 23679AB | 23679AB |
|  | (0134578) | [434541] | 02378AB | 012459A | 013489B |
|  |  |  | 045789B | 0134578 | 015689A |
|  |  |  | 12356AB | 12679AB | 023467B |
|  |  |  | 1245689 | 34678AB | 235679A |
|  | (01234678) | [654553] | 012456AB | 01234678 | 0123567B |
|  |  |  | 045678AB | 0126789A | 0156789B |
|  |  |  | 12345789 | 013459AB | 023489AB |
|  |  |  | 123789AB | 345679AB | 2345689A |

$H M \backslash 45^{3, n}(A)$ continued

| Nodes | SC | IV | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-4, \mathrm{x}-5\langle\mathrm{yz}\rangle}$ | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+5, \mathrm{x}+1\langle y z\rangle}$ | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-1, x+4 \times y z>}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 3 \\ \text { cont. } \end{gathered}$ | (0123568A) | [465562] | 0124689B 0234579 B 0235678 A 135689AB | $\begin{aligned} & \hline 013578 \mathrm{AB} \\ & 0245789 \mathrm{~A} \\ & 123468 \mathrm{AB} \\ & 1245679 \mathrm{~B} \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|} \hline 0123579 \mathrm{~A} \\ 0134568 \mathrm{~A} \\ 024679 \mathrm{AB} \\ 1346789 \mathrm{~B} \\ \hline \end{array}$ |
|  | (01235789) | [554563] | 0124678 B 0125678 A 134589 AB 234579 AB | $\begin{aligned} & \hline 013567 \mathrm{AB} \\ & 0145679 \mathrm{~B} \\ & 0234789 \mathrm{~A} \\ & 1234689 \mathrm{~A} \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|} \hline 01235789 \\ \text { 024569AB } \\ \text { 034568AB } \\ \text { 1236789B } \\ \hline \end{array}$ |
|  | (01245679) | [555562] | 0124569 B 02345789 035678 AB 123689 AB | $\begin{aligned} & \hline 0125789 \mathrm{~A} \\ & 013458 \mathrm{AB} \\ & 1234678 \mathrm{~B} \\ & 245679 \mathrm{AB} \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|} \hline 0123567 \mathrm{~A} \\ 0146789 \mathrm{~B} \\ 023479 \mathrm{AB} \\ 1345689 \mathrm{~A} \\ \hline \end{array}$ |
| 4 | (012345789)* | [766773] | 0123678AB 012456789 0345789 AB 1234569 AB | $\begin{aligned} & \hline 01234589 \mathrm{~A} \\ & 0125679 \mathrm{AB} \\ & 01345678 \mathrm{~B} \\ & 2346789 \mathrm{AB} \\ & \hline \end{aligned}$ | $\begin{array}{\|l\|l\|} \hline 01234789 \mathrm{~B} \\ 0145689 \mathrm{AB} \\ 0234567 \mathrm{AB} \\ 12356789 \mathrm{~A} \\ \hline \end{array}$ |
|  | (01235678A)* | [576683] | 012345789 01235678 A 0124569 AB 01246789 B 0234579 AB 0345678 AB 1236789 AB 1345689 AB | 01234678 B 01245679 B 01256789 A 0134589 AB 0135678 AB 02335789 A 1234689 AB 2345679 AB | $\begin{array}{\|l} \hline 0123567 \mathrm{AB} \\ 01235789 \mathrm{~A} \\ 0134568 \mathrm{AB} \\ 01456789 \mathrm{~B} \\ \text { 0234789AB } \\ \text { 0245679AB } \\ \text { 12345689A } \\ \text { 12346789B } \\ \hline \end{array}$ |
|  | (012345689A)* | [888984] | $\begin{aligned} & 012345789 \mathrm{~B} \\ & 01245689 \mathrm{AB} \\ & 02345678 \mathrm{AB} \\ & 12356789 \mathrm{AB} \end{aligned}$ | 01234678 AB <br> 012456789 A <br> 01345789 AB <br> 12345679 AB | $\begin{aligned} & 012345689 \mathrm{~A} \\ & 01235679 \mathrm{AB} \\ & 013456789 \mathrm{~B} \\ & 02346789 \mathrm{AB} \end{aligned}$ |
| 5 | (012345789) | [766773] | 012345789 0123678 AB 012456789 0124569 AB 0345678 AB 034789 AB 1234569 AB 1236789 AB | 01234589 A 01234678 B 01256789 A 0125679 AB 01345678 B 0134589 AB 2346679 AB 2346789 AB | $\begin{array}{\|l\|} \hline 01233789 \mathrm{~B} \\ 0123567 \mathrm{AB} \\ 01456789 \mathrm{~B} \\ 0145689 \mathrm{AB} \\ 0234567 \mathrm{AB} \\ \text { 0234789AB } \\ 12345689 \mathrm{~A} \\ \text { 12356789A } \\ \hline \end{array}$ |
|  | (01235678A) | [576683] | $\begin{aligned} & \hline 01235678 \mathrm{~A} \\ & 01246789 \mathrm{~B} \\ & 0234579 \mathrm{AB} \\ & 1345689 \mathrm{AB} \\ & \hline \end{aligned}$ | 01245679 B 0135678 AB 02345789 A 1234689 AB | $\begin{array}{\|l\|l\|} \hline 01235789 \mathrm{~A} \\ 0134568 \mathrm{AB} \\ 0245679 \mathrm{AB} \\ 12346789 \mathrm{~B} \\ \hline \end{array}$ |
|  | (0123456789) | [988884] | 0123456789 01234569 AB 01236789 AB 03456789 AB | 012345678 B 01234589 AB 01256789 AB 23456789 AB | $\begin{aligned} & \hline 01234567 \mathrm{AB} \\ & 01234789 \mathrm{AB} \\ & 01456789 \mathrm{AB} \\ & 123456789 \mathrm{~A} \\ & \hline \end{aligned}$ |
|  | (012345689A) | [888984] | 012345789 B 01245689 AB 02345678 AB 12356789 AB | 01234678 AB 012456789 A 01345789 AB 12345679 AB | $\begin{array}{\|l\|} \hline 012345689 \mathrm{~A} \\ 01235679 \mathrm{AB} \\ 013456789 \mathrm{~B} \\ 02346789 \mathrm{AB} \\ \hline \end{array}$ |
|  | (012345789A) | [888894] | 01235678 AB 012456789 B 02345789 AB 12345689 AB | $\begin{aligned} & \hline 012345789 \mathrm{~A} \\ & 01245679 \mathrm{AB} \\ & 01345678 \mathrm{AB} \\ & 12346789 \mathrm{AB} \end{aligned}$ | $\begin{aligned} & 012346789 \mathrm{~B} \\ & 012356789 \mathrm{~A} \\ & 01345689 \mathrm{AB} \\ & 02345679 \mathrm{AB} \end{aligned}$ |

$H M \backslash 45^{3, n}(A)$ continued

| Nodes | SC | IV | Oct $^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-4, \mathrm{x}-5<\mathrm{yz}\rangle}$ | Oct $^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+5, \mathrm{x}+1<\mathrm{yz}\rangle}$ | Oct $^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-1, \mathrm{x}+4<\mathrm{yz}\rangle}$ |
| :---: | :---: | :--- | :--- | :--- | :--- |
|  |  |  | 012345678 AB | 0123456789 A | 0123456789 B |
| 5 |  | $(0123456789 \mathrm{~A})$ | [AAAAA5] | 012345789 AB | 012345679 AB |
| cont. |  |  | 012456789 AB | 012346789 AB | 01235689 AB |
|  |  |  | 123456789 AB | 013456789 AB | 023456789 AB |

## Hybrid $1 \backslash 4 \backslash 5^{3, \mathrm{n}}$ Family (B)

| Nodes | SC | IV | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+5, \mathrm{x}+4 \times \mathrm{yz}>}$ | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-1, \mathrm{x}-5\langle\mathrm{yz}\rangle}$ | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-4, \mathrm{x}+1<\mathrm{yz}>}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | (01458) * | [202420] | 0378B | 01489 | 01458 |
|  |  |  | 04589 | 0347B | 1259A |
|  |  |  | 12569 | 1569A | 267AB |
|  |  |  | 236AB | 2367A | 3478B |
|  | (01568)* | [211231] | 0457B | 01568 | 01378 |
|  |  |  | 12489 | 0267B | 0459A |
|  |  |  | 156AB | 2359A | 12679 |
|  |  |  | 2378A | 3489B | 346AB |
|  | (012678)* | [420243] | 01567B | 012678 | 012678 |
|  |  |  | 0456AB | 01567B | 0456AB |
|  |  |  | 123789 | 23489A | 123789 |
|  |  |  | 23489A | 3459AB | 3459 AB |
|  | (013458) | [323430] | 012459 | 01348B | 012459 |
|  |  |  | 0237 AB | 01589A | 01348B |
|  |  |  | 034578 | 0237AB | 01589A |
|  |  |  | 04789B | 04789B | 034578 |
|  |  |  | 12356A | 12356A | 1269AB |
|  |  |  | 1269AB | 145689 | 23467B |
|  |  |  | 145689 | 23467B | 25679A |
|  |  |  | 3678AB | 25679A | 3678AB |
| 3 | (0123678) | [532353] | 014567B | 0125678 | 0123678 |
|  |  |  | 01456AB | 012567B | 0123789 |
|  |  |  | 01567AB | 012678B | 0126789 |
|  |  |  | 04567AB | 015678B | 03456AB |
|  |  |  | 1234789 | 234589A | 03459AB |
|  |  |  | 123489A | 23459AB | 04569AB |
|  |  |  | 123789A | 23489AB | 1236789 |
|  |  |  | 234789A | 34589AB | 34569AB |
|  | (0124569) | [434541] | 0124569 | 014789B | 012589A |
|  |  |  | 0345789 | 02347AB | 013458B |
|  |  |  | 03678AB | 123567A | 234678B |
|  |  |  | 12369 AB | 145689A | 25679 AB |
|  | (0124589) | [424641] | 0124589 | 013478B | 0124589 |
|  |  |  | 0145689 | 0145689 | 013478B |
|  |  |  | 02367AB | 014589A | 014589A |
|  |  |  | 034578B | 02367AB | 034578B |
|  |  |  | 034789B | 034789B | 125679A |
|  |  |  | 123569A | 123569A | 12569 AB |
|  |  |  | 12569 AB | 125679A | 23467 AB |
|  |  |  | 23678AB | 23467AB | 23678AB |
|  | (0134578) | [434541] | 02378AB | 013489B | 012459A |
|  |  |  | 045789B | 015689A | 0134578 |
|  |  |  | 12356AB | 023467B | 12679AB |
|  |  |  | 1245689 | 235679A | 34678AB |
|  | (01234678) | [654553] | 0123789B | 0123489A | 01245678 |
|  |  |  | 023456AB | 0134567B | 012678AB |
|  |  |  | 045689AB | 015679AB | 123459AB |
|  |  |  | 12356789 | 2346789A | 345789 AB |

$H M \backslash 45^{3, n}(B)$ continued

| Nodes | SC | IV | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{R}, \times+5, \mathrm{x}+4 \times \mathrm{y}\rangle}$ | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-1, \mathrm{x}-5\langle\mathrm{yz}\rangle}$ | Oct ${ }^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-4,4+1\langle y \mathrm{y}\rangle}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 3 \\ \text { cont. } \end{gathered}$ | (0123568A) | [465562] | 0124579 B 0234578 A 124689 AB 135678 AB | 0123568A 023579AB 0246789B 1345689B | $\begin{aligned} & \hline \text { 013468AB } \\ & \text { 0135789A } \\ & \text { 0245679A } \\ & \text { 1234679B } \end{aligned}$ |
|  | (01235789) | [554563] | $\begin{aligned} & \hline 0125679 \mathrm{~B} \\ & 0135678 \mathrm{~B} \\ & 0234589 \mathrm{~A} \\ & 234689 \mathrm{AB} \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 0123678 \mathrm{~A} \\ & 01246789 \\ & 034579 \mathrm{AB} \\ & 134569 \mathrm{AB} \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 014568 \mathrm{AB} \\ & 024567 \mathrm{AB} \\ & 1234789 \mathrm{~B} \\ & 1235789 \mathrm{~A} \\ & \hline \end{aligned}$ |
|  | (01245679) | [555562] | 0124789B 023457 AB 1235678 A 145689 AB | 0123589 A 0134568 B 025679AB 2346789B | 01245679 013678 AB 0345789 A 123469 AB |
| 4 | (012345789)* | [766773] | 01234789 B 0145689 AB 0234567 AB 12356789 A | 01234589A 0125679 AB 01345678 B 2346789AB | $\begin{array}{\|l} \hline 0123678 \mathrm{AB} \\ 012456789 \\ 0345789 \mathrm{AB} \\ 1234569 \mathrm{AB} \\ \hline \end{array}$ |
|  | (01235678A)* | [576683] | 012345789 01245679 B 0124569 AB 0135678 AB 02345789 A 0345678 AB 1234689 AB 1236789 AB | 01235678 A 0123567 AB 01246789 B 01456789 B 0234579 AB 0234789 AB 12345689 A 1345689 AB | 01234678 B 01235789 A 01256789 A 0134568 AB 0134589 AB 0245679 AB 12346789 B 2345679 AB |
|  | (012345689A)* | [888984] | $\begin{aligned} & 01235678 \mathrm{AB} \\ & 012456789 \mathrm{~B} \\ & 02345789 \mathrm{AB} \\ & 12345689 \mathrm{AB} \end{aligned}$ | $\begin{aligned} & \text { 012346789B } \\ & 012356789 \mathrm{~A} \\ & 01345689 \mathrm{AB} \\ & 02345679 \mathrm{AB} \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline 012345789 \mathrm{~A} \\ & 01245679 \mathrm{AB} \\ & 01345678 \mathrm{AB} \\ & 12346789 \mathrm{AB} \\ & \hline \end{aligned}$ |
| 5 | (012345789) | [766773] | 012345789 01334789 B 0124569 AB 0145689 AB 0234567 AB 0345678 AB 12356789 A 1236789 AB | 01234589 A 0123567 AB 0125679 AB 01345678 B 01456789 B 0234789 AB 12345689 A 2346789 AB | 01234678 B 0123678 AB 012456789 01256789 A 0134589 AB 0345789 AB 1234569 AB 2345679 AB |
|  | (01235678A) | [576683] | 01245679 B 0135678 AB 02345789 A 1234689 AB | 01235678 A 01246789 B 0234579 AB 1345689 AB | 01235789 A 0134568 AB 0245679 AB 12346789 B |
|  | (012345689A) | [888984] | 012345789 B 01245689 AB 02345678 AB 12356789 AB | 012345689 A 01235679 AB 013456789 B 02346789 AB | 01234678AB 012456789 A 01345789 AB 12345679 AB |
|  | (012345789A) | [888894] | 012345789 A 0123578 AB 012456789 B 01245679 AB 01345678 AB 02345789 AB 12345899 AB 12346789 AB | 012346789 B 012356789 A 01235678 AB 012456789 B 01345689 AB 02345679 AB 02345899 AB 12345689 AB | 012345789A 012347699 B 012356789 A 01245679 AB 01345678 AB 01345689 AB 0234569 AB 12346789 AB |

$H / \backslash A 5^{3, n}(B)$ continued

| Nodes | SC | IV | Oct $^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}+5, \mathrm{x}+4<\mathrm{yz}\rangle}$ | Oct $^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-1, \mathrm{x}-5\langle\mathrm{yz}\rangle}$ | Oct $^{1,2,3} \mathrm{R}^{\mathrm{x}, \mathrm{x}-4, \mathrm{x}+1<\mathrm{yz}\rangle}$ |
| :---: | :---: | :--- | :--- | :--- | :--- |
|  |  |  | 0123456789 B | 0123456789 A | 012345678 AB |
| 5 |  | $(0123456789 \mathrm{~A})$ | [AAAAA5] | 012345689 AB | 012345679 AB |
| cont. |  |  | 012356789 AB | 012346789 AB | 01245789 AB |
|  |  |  | 023456789 AB | 013456789 AB | 123456789 AB |

## Appendix 6: Key Areas in "Come Lovely"

|  | Natural-cyclic and Lyric Time |  |  |  |  |  |  |  |  |  |  |  | Historical Narrative Time |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pos. | $\mathrm{Oct}^{2,3,1}$ |  |  | Oct ${ }^{2,3,1}$ |  |  | Oct ${ }^{1,2,3}$ |  |  | Oct ${ }^{3,1,2}$ |  |  | Oct ${ }^{1,2,3}$ |  |  | $\mathrm{Oct}^{1,2,3}$ |  |  | Oct ${ }^{1,2,3}$ |  |  |
| 1.1 | 0 | B | A | 9 | B | A | A | 3 | 8 | B | A | 9 | A | 0 | 5 | A | 3 | 8 | A | 3 | 5 |
| 1.2 | 5 | 4 | 3 | 2 | 4 | 3 | 3 | 2 | 1 | 4 | 3 | 2 | 3 | 5 | A | 3 | 2 | 1 | 3 | 3 | A |
| 1.3 | 9 | 8 | 7 | 6 | 8 | 7 | 7 | 6 | 5 | 8 | 7 | 6 | 7 | 9 | 2 | 7 | 6 | 5 | 7 | 2 | 2 |
| 1.4 | 3 | 2 | 1 | 0 | 2 | 1 | 1 | 0 | B | 2 | 1 | 0 | 1 | 3 | 8 | 1 | 0 | B | 1 | 6 | 8 |
| 1.5 | 8 | 7 | 6 | 5 | 7 | 6 | 6 | 5 | 4 | 7 | 6 | 5 | 6 | 8 | 1 | 6 | 5 | 4 | 6 | 0 | 1 |
| 2.1 | 5 | 4 | 3 | 2 | 4 | 3 | 3 | 2 | 1 | 4 | 3 | 2 | 3 | 5 | A | 3 | 2 | 1 | 3 | 5 | A |
| 2.2 | 9 | 8 | 7 | 6 | 8 | 7 | 7 | 6 | 5 | 8 | 7 | 6 | 7 | 9 | 2 | 7 | 6 | 5 | 7 | 2 | 2 |
| 2.3 | 2 | 1 | 0 | B | 1 | 0 | 0 | B | A | 1 | 0 | B | 0 | 2 | 7 | 0 | B | A | 0 | 6 | 7 |
| 2.4 | 8 | 7 | 6 | 5 | 7 | 6 | 6 | 5 | 4 | 7 | 6 | 5 | 6 | 8 | 1 | 6 | 5 | 4 | 6 | B | 1 |
| 2.5 | 0 | B | A | 9 | B | A | A | 9 | 8 | B | A | 9 | A | 0 | 5 | A | 9 | 8 | A | 5 | 5 |
| 3.1 | 9 | 8 | 7 | 6 | 8 | 7 | 7 | 6 | 5 | 8 | 7 | 6 | 7 | 9 | 2 | 7 | 6 | 5 | 7 | 9 | 2 |
| 3.2 | 2 | 1 | 0 | B | 1 | 0 | 0 | B | A | 1 | 0 | B | 0 | 2 | 7 | 0 | B | A | 0 | 6 | 7 |
| 3.3 | 6 | 5 | 4 | 3 | 5 | 4 | 4 | 3 | 2 | 5 | 4 | 3 | 4 | 6 | B | 4 | 3 | 2 | 4 | B | B |
| 3.4 | 0 | B | A | 9 | B | A | A | 9 | 8 | B | A | 9 | A | 0 | 5 | A | 9 | 8 | A | 3 | 5 |
| 3.5 | 5 | 4 | 3 | 2 | 4 | 3 | 3 | 2 | 1 | 4 | 3 | 2 | 3 | 5 | A | 3 | 2 | 1 | 3 | 9 | A |
| 4.1 | 2 | 1 | 0 | B | 1 | 0 | 0 | B | A | 1 | 0 | B | 0 | 2 | 7 | 0 | B | A | 0 | 2 | 7 |
| 4.2 | 6 | 5 | 4 | 3 | 5 | 4 | 4 | 3 | 2 | 5 | 4 | 3 | 4 | 6 | B | 4 | 3 | 2 | 4 | B | B |
| 4.3 | B | A | 9 | 8 | A | 9 | 9 | 8 | 7 | A | 9 | 8 | 9 | B | 4 | 9 | 8 | 7 | 9 | 3 | 4 |
| 4.4 | 5 | 4 | 3 | 2 | 4 | 3 | 3 | 2 | 1 | 4 | 3 | 2 | 3 | 5 | A | 3 | 2 | 1 | 3 | 8 | A |
| 4.5 | 9 | 8 | 7 | 6 | 8 | 7 | 7 | 6 | 5 | 8 | 7 | 6 | 7 | 9 | 2 | 7 | 6 | 5 | 7 | 2 | 2 |
| 5.1 | 6 | 5 | 4 | 3 | 5 | 4 | 4 | 3 | 2 | 5 | 4 | 3 | 4 | 6 | B | 4 | 3 | 2 | 4 | 6 | B |
| 5.2 | B | A | 9 | 8 | A | 9 | 9 | 8 | 7 | A | 9 | 8 | 9 | B | 4 | 9 | 8 | 7 | 9 | 3 | 4 |
| 5.3 | 3 | 2 | 1 | 0 | 2 | 1 | 1 | 0 | B | 2 | 1 | 0 | 1 | 3 | 8 | 1 | 0 | B | 1 | 8 | 8 |
| 5.4 | 9 | 8 | 7 | 6 | 8 | 7 | 7 | 6 | 5 | 8 | 7 | 6 | 7 | 9 | 2 | 7 | 6 | 5 | 7 | 0 | 2 |
| 5. | 2 | 1 | 0 | B | 1 | 0 | 0 | B | A | 1 | 0 | B | 0 | 2 | 7 | 0 | B | A | 0 | 6 | 7 |
| 6. | B | A | 9 | 8 | A | 9 | 9 | 8 | 7 | A | 9 | 8 | 9 | B | 4 | 9 | 8 | 7 | 9 | B | 4 |
| 6.2 | 3 | 2 | 1 | 0 | 2 | 1 | 1 | 0 | B | 2 | 1 | 0 | 1 | 3 | 8 | 1 | 0 | B | 1 | 8 | 8 |
| 6.3 | 8 | 7 | 6 | 5 | 7 | 6 | 6 | 5 | 4 | 7 | 6 | 5 | 6 | 8 | 1 | 6 | 5 | 4 | 6 | 0 | 1 |
| 6.4 | 2 | 1 | 0 | B | 1 | 0 | 0 | B | A | 1 | 0 | B | 0 | 2 | 7 | 0 | B | A | 0 | 5 | 7 |
| 6.5 | 6 | 5 | 4 | 3 | 5 | 4 | 4 | 3 | 2 | 5 | 4 | 3 | 4 | 6 | B | 4 | 3 | 2 | 4 | B | B |
| 7.1 | 3 | 2 | 1 | 0 | 2 | 1 | 1 | 0 | B | 2 | 1 | 0 | 1 | 3 | 8 | 1 | 0 | B | 1 | 3 | 8 |
| 7.2 | 8 | 7 | 6 | 5 | 7 | 6 | 6 | 5 | 4 | 7 | 6 | 5 | 6 | 8 | 1 | 6 | 5 | 4 | 6 | 0 | 1 |
| 7.3 | 0 | B | A | 9 | B | A | A | 9 | 8 | B | A | 9 | A | 0 | 5 | A | 9 | 8 | A | 5 | 5 |
| 7.4 | 6 | 5 | 4 | 3 | 5 | 4 | 4 | 3 | 2 | 5 | 4 | 3 | 4 | 6 | B | 4 | 3 | 2 | 4 | 9 | B |
| 7.5 | B | A | 9 | 8 | A | 9 | 9 | 8 | 7 | A | 9 | 8 | 9 | B | 4 | 9 | 8 | 7 | 9 | 3 | 4 |
| 8.1 | 8 | 7 | 6 | 5 | 7 | 6 | 6 | 5 | 4 | 7 | 6 | 5 | 6 | 8 | 1 | 6 | 5 | 4 | 6 | 8 | 1 |
| 8.2 | 0 | B | A | 9 | B | A | A | 9 | 8 | B | A | 9 | A | 0 | 5 | A | 9 | 8 | A | 5 | 5 |
| 8.3 | 5 | 4 | 3 | 2 | 4 | 3 | 3 | 2 | 1 | 4 | 3 | 2 | 3 | 5 | A | 3 | 2 | 1 | 3 | 9 | A |
| 8.4 | B | A | 9 | 8 | A | 9 | 9 | 8 | 7 | A | 9 | 8 | 9 | B | 4 | 9 | 8 | 7 | 9 | 2 | 4 |
| 8.5 | 3 | 2 | 1 | 0 | 2 | 1 | 1 | 0 | B | 2 | 1 | 0 | 1 | 3 | 8 | 1 | 0 | B | 1 | 8 | 8 |

# Curriculum Vitae 

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[^0]:    ${ }^{1}$ Richard W. Bass, "Models of Octatonic and Whole-Tone Interaction: George Crumb and His Predecessors," Journal of Music Theory 38 (1994): 156.
    ${ }^{2}$ George Crumb, "Music: Does It Have a Future?" The Kenyon Review 2, no. 3 (1980): 117.

[^1]:    3
    Ibid.
    ${ }^{4}$ See Robert B. Shuffet, "The Music, 1971-1975, of George Crumb: A Style Analysis," (D.M.A. thesis, Peabody Institute, 1979), 432; Crumb, "Music: Does It Have a Future?" 121; Cole Gagne and Tracy Caras, "George Crumb," in Soundpieces: Interviews with American Composers (Metuchen: Scarecrow Press, 1982), 121; Edward Strickland, "George Crumb," in American Composers: Dialogues on Contemporary Music (Bloomington: Indiana University Press, 1991), 163; West Virginia Public Broadcasting, "A Conversation with George Crumb," YouTube (Dec 21, 2007): 4:07, http://www.youtube.com/watch?v=5xo8SHjTxpc (accessed February 10, 2014); Paul Steenhuisen, "George Crumb," in Sonic Mosaics: Conversations with Composers (Edmonton: The University of Alberta Press, 2009), 108; Bálint András Varga, Three Questions for Sixty-Five Composers (Rochester: University of Rochester Press, 2011), 50.
    ${ }^{5}$ See Shuffet, "The Music," 445; Geoff Smith and Nicola W. Smith, "George Crumb," American Originals: Interviews with 25 Contemporary Composers (London: Faber and Faber, 1994), 97; Young Gee Shim, "A Study on A Little Suite for Christmas, A.D. 1979 by George Crumb," (Ph.D. diss., Temple University, 1997), 134.
    ${ }^{6}$ For a detailed description of sounds of nature in Crumb's music see Robert C. Cook's "The Vocal Ecology of Crumb's Crickets," Journal of the Society for American Music 7 (2013): 123-45.

[^2]:    ${ }^{7}$ Bass, "Models of Octatonic and Whole-Tone Interaction," 156.

[^3]:    1
    Crumb, "Music: Does It Have a Future?" 117.
    2 Numerous scholars have looked at the intertextual relationships of the new merging with old. For a brief survey see Joseph N. Straus, Remaking the Past: Musical Modernism and the Influence of the Tonal Tradition (Cambridge: Harvard University Press, 1990); Martha M. Hyde, "Neoclassic and Anachronistic Impulses in Twentieth-Century Music," Music Theory Spectrum 18 (1996): 200-35; and Michael L. Klein, Intertextuality in Western Art Music (Bloomington: Indiana University Press, 2005).
    ${ }^{3}$ Crumb, "Music: Does It Have a Future?" 120.
    4
    ${ }^{4}$ George Crumb, program notes to Black Angles (Images I): Thirteen Images from the Dark Land (New York: CF. Peters Corporation, 1971).

[^4]:    5 In this dissertation, pitches enclosed within parentheses will denote a set-class, pitches enclosed within braces will denote a particular instance of a set-class in normal form, and pitches enclosed within angle brackets will denote a specific order of pitch classes. Pitch-class (pc) 0 will represent pc C and rise incrementally up the chromatic scale. Pitch-classes A and B will represent the pc numerals 10 and 11 respectively.
    ${ }^{6}$ Some pitches have been enharmonically respelled and shifted to a different register in Figure 2.1 to highlight the abstract voice-leading relationships. Voice-leading and metric placement are important factors contributing to the sense of a B-major tonality but are beyond the scope of the current discussion. These elements are discussed in Blair Johnston's "Between Romanticism and Modernism and Postmodernism: George Crumb's Black Angels," Music Theory Online 18, no. 2 (2012), para. 16-18, 31.
    ${ }^{7}$ The correlation between a chord that ends a section and the chord that begins a new section has not been included.
    ${ }^{8}$ The tritone relationship between the F- and B-major triads may be thought of as an ironic reflection or misreading of tonic and dominant relationships, and therefore a misreading of a functional procedure. See Straus, Remaking the Past, 74.

[^5]:    ${ }^{9}$ In his analysis of this piece, Bass notes, "(016) and (0167) pc sets are the dominant constructional units in most of the individual figures and lend a distinctive consistency to the music of 'Gargoyles.' The T-levels of simultaneities often produce (025) and (026) trichords as linear formations and whole-tone organization is prominently featured throughout much of the piece." See Richard W. Bass, "Pitch Structure in George Crumb's Makrokosmos, Volumes I and II," (Ph.D. diss., University of Texas at Austin, 1987), 283.

[^6]:    10
    As the title of the volume suggests, homage is paid to Bartók: more explicitly, the scotch-snap rhythm of the opening (025) and (016) trichords in "Pastorale," as well as the presence of a perfect fourth or fifth in these trichords, suggests a relationship to Bartók's Concerto for Orchestra (1943). See the movement Eligia, mm. 30-33, mm. 34-61, and 84-100.
    ${ }^{11}$ Bass, "Pitch Structure," 107-10.

[^7]:    12 This pedal is one reason Charles Benesh argues for tonal centrism within the work. See Charles W. Benesh, "Tonal Centrism and Symmetrical Pitch Relationships in George Crumb's A Haunted Landscape," (Ph.D. diss., University of California, Los Angeles, 2005), 11-12. Benesh's definition of tonal centrism is largely based upon Wallace Berry's interpretation of tonality as defined in Structural Functions in Music, (Englewood Cliffs: Prentice-Hall, 1976). In particular, Benesh cites Berry's classification of a "primitive ('pedal') tonality" and "tonal flux within broad, prevailing tonal unity" (Benesh, 12 n ; Berry, 172).
    ${ }^{13}$ There are three standard operations: the Parallel (P) transformation inverts the triad around the shared perfect fifth; the Leading-tone exchange (L) transformation inverts the triad around the shared minor third; the Relative (R) transformation inverts the triad around the shared major third. See Brian Hyer, "Tonal Intuitions in Tristan und Isolde," (Ph.D. diss., Yale University, 1989), 162-210. I have chosen not to include Lewin's SLIDE transformation because the operations of P, L, and R are enough to account for the progression in question. See David Lewin, Generalized Musical Intervals and Transformations (New Haven: Yale University Press, 1987; repr., Oxford: Oxford University Press, 2007): 178.

[^8]:    14 Joseph N. Straus, Introduction to Post-Tonal Theory, 3rd ed. (Upper Saddle River: Pearson Prentice Hall, 2005): 74.

    15 In my unpublished essay, "Cycles and Circles: Transformations in George Crumb's 'Come Lovely and Soothing Death,"" presented at the Graduate Music Conference at the University of Ontario (May 9, 2009), I demonstrated how many aspects of "Come Lovely" relate to the song of the Wood Thrush, the same bird discussed in Whitman's poem. In Apparition, "Come Lovely" is preceded by "Vocalise 3: Death Carol ('Song of the Nightbird')."

[^9]:    ${ }^{16}$ Kathy Rugoff observes that "Whitman reveals the song of the bird to be at the heart of his selfconcept as a poet; his awareness of the unity of life and death is associated with the song of the bird mourning the loss of his mate." See Kathy Rugoff, "Three American Requiems: Contemplating 'When Lilacs Last in the Dooryard Bloom'd'," in Walt Whitman and Modern Music: War, Desire, and the Trials of Nationhood, ed. Lawrence Kramer (New York: Garland Publishing, 2000), 134.

[^10]:    ${ }^{17}$ Crumb, "Music: Does It Have a Future?" 116.

[^11]:    18
    Each graph is labeled in bold font according to the order in which the triad appears in the music.

[^12]:    ${ }^{19}$ Strickland, "George Crumb," 172.

[^13]:    ${ }^{20}$ Bass, "Models of Octatonic and Whole-Tone Interaction," 176.
    ${ }^{21}$ I have included a complete list of all labeling conventions for various referential collections in Appendix 1. The alphabetic abbreviation refers to the collection ( $\mathrm{D}=$ diatonic, $\mathrm{Hm}=$ harmonic minor, Mm $=$ melodic minor, $\mathrm{F}=$ Messiaen's fourth mode of limited transposition, Oct $=$ octatonic, $\mathrm{WT}=$ whole-tone $)$ and the superscript differentiates between instances of the collection.
    ${ }^{22}$ For a detailed analysis of "Music of Shadows" see Bass, "Models of Octatonic and WholeTone Interaction," 176-82 and Bass, "Pitch Structure," 146-64.

[^14]:    23
    Additional evidence supporting the use of a tonal form to describe a post-tonal work by Crumb will be presented throughout this dissertation.

[^15]:    ${ }^{24}$ Strickland, "George Crumb," 171.

    25
    Dmitri Tymoczko, "Stravinsky and the Octatonic: A Reconsideration," Music Theory Spectrum 24 (2002): 68.
    ${ }^{26}$ Ibid.
    ${ }^{27}$ The label for each whole-tone scale is enclosed in a box, indicating that the collection is complete. This notational practice will be used for all complete collections; representative subsets, or incomplete collections, will not include a box.

[^16]:    ${ }^{28}$ This is similar in many respects to John Doerksen's exclusivity index, which in conjunction with Forte's theory of set-class genera, "models a set-class's power of generic association: the fewer a setclass's genera memberships, the greater its power to engage a particular genus; the greater its number of genera memberships, the lesser its power to engage one specific genus." See John Doerksen, "A Theory of Set-Class Salience for Post-tonal Music, with Analyses of Selected Lieder by Anton Webern," (Ph.D. diss., The University of Western Ontario, 1994): 39; Allen Forte, "Pitch-Class Set Genera and the Origin of Modern Harmonic Species." Journal of Music Theory 32 (1988): 187-270.

[^17]:    29
    This follows one of John Roeder's suggestions for transformational analysis: "Choose an object family that is complete (including all objects that appear in the piece) but minimal (not entailing many objects that do not appear)." See "Constructing Transformational Signification: Gesture and Agency in Bartók's Scherzo, Op. 14, No. 2, measures 1-32," Music Theory Online 15, no. 1 (2009), para. 12.1.

[^18]:    ${ }^{30}$ Ciro G. Scotto, "Transformational Networks, Transpositional Combination, and Aggregate Partitions in Processional by George Crumb," Music Theory Online 8, no. 3 (2002).
    ${ }^{31}$ Ibid., para. 8.
    32 Ibid., para. 10.
    ${ }^{33}$ An explanation of Scotto's registral streams is found in his "Transformational Networks," para. 10.

[^19]:    ${ }^{34}$ Using the same procedure, the collections in the first segment $\left(D^{6}, D^{B}, D^{0}\right)$ might be expressed as $\{6 \mathrm{~B} 0\}$, a member of $\mathrm{sc}(016)$ and an important trichord discussed in section 2.1.
    ${ }^{35}$ In the score the diatonic segments alternate between a wide and a narrow registral spacing but in the reduction are represented within a narrow registral space to underscore the relationship of these segments to the first hexachord of the opening section.

[^20]:    36
    Scotto points out that considering the tetrachords alone, all twelve transpositions are included thus completing the hyperaggregate. See "Transformational Networks," para. 27.

[^21]:    ${ }^{39}$ The repeated-note motive is a member of sc (026), which is a subset of all six collections listed in Table 2.3. It can uniquely identify transpositions of four of those collections-no other trichord or tetrachord is as versatile.

[^22]:    ${ }^{40}$ Admittedly, the whole-tone interpretation is more convincing but the sense that it is connected to the octatonic collection is still present.

[^23]:    ${ }^{41}$ Steenhuisen, "George Crumb," 112.
    ${ }^{42}$ Ibid.

[^24]:    43
    Edward Pearsall, "Dialectical Relations Among Pitch Structures in the Music of George Crumb," in George Crumb \& The Alchemy of Sound: Essays on His Music, ed. Steven Bruns, Ofer BenAmots, and Michael D. Grace (Colorado Springs: Colorado College Music Press, 2005), 58.

    44 Ibid., 63.
    45 Ibid., 65.
    46
    Crumb, program notes to Night of the Four Moons, in George Crumb \& The Alchemy of Sound, 299.

[^25]:    47
    Steven Bruns, "Les Adieux: Haydn, Mahler, and George Crumb's Night of the Four Moons," in George Crumb \& The Alchemy of Sound, 125.

[^26]:    ${ }^{48}$ As Bass argues, this figure references the chordal progression which opens the coronation scene from Mussorgsky's Boris Godunov (see "Pitch Structure," 354-57). The relationship of these whole-tone hexachords to the major-minor seventh tetrachords of Boris Godunov may seem tenuous; however, the style of a left-hand grace-note chord leading to a right-hand sustained chord is a familiar technique found in Crumb's music. In "Primeval Sounds" Crumb opened with minor chord alternations which comprised members either of an octatonic or a mode-4 collection (see Figure 2.12). The resultant hexachord was the inversion of the Petrushka chord (see Figure 2.14). This stylistic grace-note figure also appears in "Music of the Starry Night" from Music for a Summer Evening (1974) (see Figure 2.21).

[^27]:    49
    Crumb, program notes to Music for a Summer Evening (Makrokosmos III), in George Crumb \& The Alchemy of Sound, 311.
    ${ }^{50}$ These bell-like figures are reminiscent of the first three chords from "Litany of the Galactic Bells" (cf. Figure 2.20).
    ${ }^{51}$ These quotations are discussed in detail in the next section (c.f. Figure 2.22 to Figure 2.30).

[^28]:    52 The number of intersecting pitch classes between mode-4 collections depends on how far a collection is from another. For example, $\mathrm{F}^{0}$ shares six pitch classes with its immediate neighbours $\mathrm{F}^{1}$ and $F^{5}$, while sharing only four pitch classes with $F^{2}, F^{3}$ and $F^{4}$. Following the same criteria of determining an opposite as with the octatonic collection above, it would appear then that $\mathrm{F}^{2}, \mathrm{~F}^{3}$, and $\mathrm{F}^{4}$ are all equally opposite. The reading of the first three progressions $\left(\mathrm{F}^{0}-\mathrm{F}^{1}-\mathrm{F}^{3}\right)$ would result in a less symmetrical interpretation than the octatonic reading. Instead of "object-opposite-object," the progression would be read as "object-neighbour-opposite."
    ${ }^{53}$ Robert Cook has pointed out that opposition is an important principle in other sections of "Come Lovely" in "Emerson's Compensation and a Whitman Song by Crumb," $6^{\text {th }}$ European Music Analysis Conference, Freiburg, Germany, 2007 and in an adaptation of this paper in "Crumb's Apparition and Emerson's Compensation," Music Theory Spectrum 34 (2012): 1-25.

[^29]:    54 A discussion of completion in music would not be complete with a reference, at least in passing, to the work of Leonard Meyer and Eugene Narmour. In 1975, Crumb was appointed as professor at the University of Pennsylvania when both Meyer and Narmour were on faculty. The extent of Narmour's influence on Crumb, or vice versa, has received little attention. Aspects of Narmour's, and by extension Meyer's, theories may seem applicable to the topic of completion in the music of Crumb but are beyond the scope of the current study.

    55 Pearsall, "Dialectical Relations," 71-74.

[^30]:    56
    Edward Pearsall, "Symmetry and Goal-Directed Motion in Music by Béla Bartók and George Crumb," Tempo 58, no. 228 (2004): 34.
    ${ }^{57}$ Stephanie K. Lind, "Replicative Network Structures: Theoretical Definitions and Analytical Applications," (Ph.D. diss., University of British Columbia, Vancouver, 2008), 152-70.

    58
    Ibid., 170.
    ${ }^{59}$ Crumb, "Music: Does It Have a Future?" 120.
    ${ }^{60}$ Ibid., 121.

[^31]:    ${ }^{61}$ Ibid.
    ${ }^{62}$ Strickland, "George Crumb," 172.
    63 Joseph R. DeBaise, "George Crumb's 'Music for a Summer Evening': A Comprehensive Analysis," (Ph.D. diss., Eastman School of Music, University of Rochester, 1983), 22-23.
    ${ }^{64}$ Richard W. Bass, ""Approach Strong Deliveress!' from George Crumb's Apparition: A Case Study in Analysis and Performance of Post-Tonal Music," Journal of Music Theory Pedagogy 16 (2002): 59.

[^32]:    ${ }^{65}$ Scotto, "Transformational Networks," para. 20.
    ${ }^{66}$ Bass, ""Approach Strong Deliveress!'" 70. Oct-II and Oct-III are Bass's abbreviation for Oct ${ }^{2}$ and $\mathrm{Oct}^{3}$ respectively.
    ${ }^{67}$ Ibid., 73.

[^33]:    68
    The significance of the following discussion, concerning the correspondence between the function of tonal harmonies and specific octatonic collections, will be fully realized in section 3.7, which outlines hierarchical functions within the model of octatonality.

    69
    The relationship between partitions of the octatonic and functional tonal harmonies has been discussed by many. An early discussion of these relationships is found in Prince Edmond de Polignac's 1879 treatise Étude sur les Successions Alternantes de Tons et Demi-tons (Et sur la Gamme dite MajeureMineure (Translated as: A Study on the Sequences of Alternating Whole Steps and Half Steps, (and on the Scale Known as Major-Minor) by Sylvia Kahan in In Search of New Scales: Prince Edmond de Polignac, Octatonic Explorer (Rochester: University of Rochester Press, 2009): 145-300. See also: Arthur Berger, "Problems of Pitch Organization in Stravinsky," Perspectives of New Music 2 (1963): 11-18; Pieter C. van den Toorn, The Music of Igor Stravinsky, (New Haven: Yale University Press, 1983), 321-29; and Darryl L. White, "A Proposed Theoretical Model for Chromatic Functional Harmony: The Octatonic Metaphor," (M.A. thesis, Arizona State University, 2006), 20-24.
    ${ }^{70}$ I have included $b I I^{b 7}$ in the dominant category since it is the tritone substitute for the dominant.

[^34]:    ${ }^{71}$ DeBaise, "George Crumb's 'Music for a Summer Evening,"" 142-43.
    ${ }^{72}$ Ibid., 210.

[^35]:    ${ }^{73}$ Steenhuisen, "George Crumb," 111.

[^36]:    ${ }^{74}$ Quotation marks are not reproduced in this and following figures, but are found in the published score.
    ${ }^{75}$ Crumb's use of the phrase "falling tonality" is found in a sketch of the trichordal progression from "Come Lovely," reproduced in Figure 5.11, and is discussed in section 5.1.

[^37]:    ${ }^{76}$ This perspective, however, seems to be addressed directly by Crumb and his spiral view of culture and time. See Crumb, "Music: Does It Have a Future?" 116.

[^38]:    ${ }^{77}$ For example see DeBaise, "George Crumb's 'Music for a Summer Evening,'" or Benesh, "Tonal Centrism."
    ${ }^{78}$ Bass, "Models of Octatonic and Whole-Tone Interaction," 155-86; Scotto, "Transformational Networks."
    ${ }^{79}$ The term "psychological curve" is a term often used by Crumb which he defines as "the flow of emotions. The form of a composition could be expressed in terms of the spinning out or alternation of these psychological states. You could speak of them in physical terms as well; for example in such terms as inertia, points of arrival, climax, and the like." Quoted in Shuffet, "The Music," 536-37.

[^39]:    1 "Minimal" and "economical" are two of the eight criteria that Roeder suggests for creating transformational models in "Constructing Transformational Signification," para. 12.1.
    ${ }^{2}$ Daniel Harrison, Harmonic Function in Chromatic Music: A Renewed Dualist Theory and an Account of Its Precedents (Chicago: University of Chicago Press, 1994).
    ${ }^{3}$ White, "A Proposed Theoretical Model for Chromatic Functional Harmony." Other models of octatonic-based theoretical approaches were considered such as the following: Polignac's treatise translated in In Search of New Scales, 145-300; Berger's "Problems of Pitch Organization in Stravinsky," 11-42; and van den Toorn's The Music of Igor Stravinsky; however, White's model shares the greatest similarity with the model I will propose.

[^40]:    ${ }^{4}$ Lewin, Generalized Musical Intervals and Transformations, 159. Henry Klumpenhouwer succinctly describes five theses that define a transformational attitude, or "intervallic thinking," in Lewin's work. See Henry Klumpenhouwer's "In Order to Stay Asleep as Observers: The Nature and Origins of Anti-Cartesianism in Lewin's Generalized Musical Intervals and Transformations," Music Theory Spectrum 28 (2006): 279-80.
    ${ }^{5}$ Smith and Smith, "George Crumb," 97.
    6
    ${ }^{6}$ This claim will be substantiated, in part, through the analyses presented in Chapters 4 and 5, and additional possibilities will be offered in Chapter 6.

[^41]:    ${ }^{7}$ Richard Boyd, "Metaphor and Theory Change: What is a 'Metaphor' a Metaphor For?" in Metaphor and Thought, $2^{\text {nd }}$ ed., ed. Anthony Ortony (Cambridge: Cambridge University Press, 1993), 487.
    ${ }^{8}$ Ibid., 487.
    ${ }^{9}$ Janna Saslaw, "Forces, Containers, and Paths: The Role of Body-Derived Image Schemas in the Conceptualization of Music," Journal of Music Theory 40 (1996): 221. The conduit metaphor she refers to is discussed in George Lakoff and Mark Johnson, Metaphors We Live by (Chicago: The University of Chicago Press, 1980). Lakoff and Johnson use small capitals to designate an instance of the conduit

[^42]:    12 Roeder, "Constructing Transformational Signification," para. 1.9.
    13 Michael Buchler, "Are There Any Bad (or Good) Transformational Analyses?" Paper presented at the Society for Music Theory, Indianapolis, Indiana, 2010.
    ${ }^{14}$ See Roeder, "Constructing Transformational Signification," para. 12.1.
    ${ }^{15}$ Shuffet, "The Music," 453.

[^43]:    21 Heinrich Schenker, Harmonielehre.Volume I of Neue musikalische Theorien und Phantasien (Stuttgart: J.G. Cotta'sche, 1906), trans. Elizabeth Mann Borgese as Harmony, ed. Oswald Jonas (1954; repr., Chicago: The University of Chicago Press, 1980), 20.
    ${ }^{22}$ Ibid.
    ${ }^{23}$ Crumb, "Music: Does It Have a Future?" 117. Crumb's interest and awareness of Schenker's theories, although he is quite clear about not being interested in theory in the abstract, is apparent when he acknowledged that he made a translation of Schenker's Der freie Satz for a student. See Shuffet "The Music," 406.

[^44]:    24
    Although the following sections are presented in a linear format, each aspect of the model is interrelated and predicated upon observations of a number of Crumb's works. For these reasons, and to mitigate the abstract presentation of the model, the reader may find it useful to preview the analyses presented in the fourth and fifth chapters before reading sections 3.2-3.8.
    ${ }^{25}$ The use of a straight cylinder to represent a segment of a helix is not typical; instead, a curved spiral is often favoured. A similar use of straight segments is found in Jacob Reed's and Matthew Bain's article "A Tetrahelix Animates Bach: Revisualization of David Lewin's Analysis of the Opening of the F\# Minor Fugue from WTC I." Music Theory Online 13, no. 4 (2007).
    ${ }^{26}$ The bounded helical model, rather than an infinite helix, is similar to the representation of Roger N. Shepard's double helix wound around a torus; see his "Structural Representations of Musical Pitch," in The Psychology of Music, ed. Diana Deutsch (New York: Academic Press, 1982), 363.
    ${ }^{27}$ As will be shown in section 3.4, hybrid chords may consist of only two nodes, which may contain up to three pitch classes per node.
    ${ }^{28}$ The octatonic model was conceived through trial and error based upon an evolving study of Crumb's music. First, existing theoretical models were used to analyze the triadic progression in Crumb's "Come Lovely." Through the analytic process of using these models, I recognized recurring intervallic patterns. Initially, a helical model that incorporated only four ordered octatonic collections, which worked for the triadic progressions in "Come Lovely," was later expanded to five ordered octatonic collections, represented by the five rings in Figure 3.1, to include all instances of each set class.

[^45]:    ${ }^{29}$ Unless otherwise stated, modulo-12 space is operative for discussion of pitches/pitch classes, intervals/interval classes, and sets/set-classes.
    ${ }^{30}$ Formal definitions for elements of the model are included in Appendix 2; however, it should be possible to ascertain the definition less formally through the prose descriptions.

[^46]:    31
    "Successive interval array" is an ordered intervallic pattern following Richard Chrisman, "Identification and Correlation of Pitch-Sets," Journal of Music Theory 15 (1971): 60-62.
    ${ }^{32}$ Figure 3.1 shows only one possible order of the Oct ${ }^{1}$ collection. It is possible to understand this model as a frame for a mod- 8 octatonic space; however, generic octatonic space will be discussed in detail in sections 3.3-3.8.

[^47]:    ${ }^{33}$ This numbering system is arbitrary and could have started on any node of the helix without any loss of meaning.
    ${ }^{34}$ This labeling system is used later in footnote 35 and in Table 3.3.

[^48]:    ${ }^{35}$ The distance between nodes in three-dimensional space are: h. 1 to h. $2=.357$, h. 2 to h. $3=$ 0.318 , h. 3 to h. $4=0.309$, h. 4 to h. $5=0.352$, h. 5 to $h^{+1} .1=0.313$. The distance between nodes was calculated using the generic unit of measurement used in the 3D rendering program named Blender. The distance between nodes varies because the helix was constructed by wrapping it around a five-sided torus comprised of eight segments. The torus's outer circumference is greater than the inner circumference, and thus the distance between nodes varies.
    ${ }^{36}$ The headings for these columns include additional notation following the designation of each octatonic collection, which indicate a certain octatonic mode, and will be clarified in the next section.
    ${ }^{37}$ Cells in the SIA column are offset to clarify the correspondence between the intervals and pitch classes.

[^49]:    38 The information included in Table 3.3 can be verified in Table 3.2.

[^50]:    42 Ibid., 83-85.
    43
    Ibid., 86.
    ${ }^{44}$ Richards describes in detail his use of portions of Parks's and Forte's theory of pitch-class set genera in "Transformation and Generic Interaction," 80 n 100 . For additional similarities and differences between the theories refer to Parks, "Pitch-Class Set Genera," 206-26 and Richard S. Parks, "Afterword," Music Analysis 17 (1998): 237-40; Allen Forte, "Round Table: Response and Discussion," Music Analysis 17 (1998): 227-36 and Allen Forte "Afterword," Music Analysis 17 (1998): 241-44.

[^51]:    45
    Richards, "Transformation and Generic Interaction," 86. See Parks's eighth preference rule in "Pitch-Class Set Genera," 209 and Forte's progenitor in "Pitch-Class Set Genera," 190.
    ${ }^{46}$ Richards, "Transformation and Generic Interaction," 97. Doerksen's salience theory addresses, to a certain extent, the inability of set theory to determine the relative significance of events that Richards describes. See Doerksen, "A Theory of Set-Class Salience."

[^52]:    ${ }^{47}$ The concept of mode within this dissertation lies somewhere between the concept of mode in Western music theory and the Russian concept, as espoused by Boleslav Yavorsky, Sergei Protopopov and Yuri Kholopov, discussed by Phillip Ewell in "Rethinking Octatonicism: Views from Stravinsky's Homeland," Music Theory Online 18, no. 4 (2012).
    ${ }^{48}$ The importance of these alternative orderings of the arrays will be explored in the sections on transformations, hybrid chords, and hierarchical functions.

[^53]:    ${ }^{50}$ This draws upon White's concept of hybrid chords from his "A Proposed Theoretical Model for Chromatic Functional Harmony," 62-67.

[^54]:    ${ }^{51}$ Ibid.

[^55]:    52 The sets are notated using ascending values, not normal form, for easier reference.

[^56]:    53 An easy way to remember this is that " $m$ " comes before " $n$ " in the alphabet, and in the notation of the modes and nodes in the hybrid family.

[^57]:    ${ }^{54}$ Referential hybrids are indicated below the set-classes listed in Appendix 5. All other hybrids are true hybrids.

[^58]:    ${ }^{55}$ The rotations of the octatonic modes used to construct these hybrid chords are listed in Appendix 3.

    56 Asterisks following the set-class in the hybrid tables found in Appendix 5 indicate this.
    57 The emphasis on shorter distance for analytical decisions is expanded upon in the following section.

[^59]:    58
    When the mode or key area a ROT transformation is acting upon is unclear, the octatonic index for the mode will be added as a superscript following the transformation. For example, the motion between the key areas Oct ${ }^{1,2} \mathrm{R}^{\mathrm{A}, 9<54>}$ and $\mathrm{Oct}^{1,2} \mathrm{R}^{7,9<54>}$-a motion from a $\mathrm{H}^{2, \mathrm{n}}$ to a $\mathrm{H}^{2, \mathrm{n}}$ family-can be notated as ROT $_{-2}{ }^{1}$.

[^60]:    59
    If there is reason to warrant a more drastic change of the chords occupying certain nodal positions such that motion within a pentagonal segment is required then follow the format $\mathrm{ROT}_{x} Y$.

[^61]:    ${ }^{60}$ White, "A Proposed Theoretical Model for Chromatic Functional Harmony," 23-24.

[^62]:    ${ }^{61}$ Throughout this document, octatonic will refer to either the governing octatonic mode or key or a set located on specific nodes. As is the case in tonality, where the term tonic may refer to a pitch, chord, or key, in octatonality the meaning of octatonic is also contextual.
    ${ }^{62}$ White proposes a similar understanding of the relationship between functional octatonic systems. See White, "A Proposed Theoretical Model for Chromatic Functional Harmony," 21-24.

[^63]:    ${ }^{63}$ This definition of progressive and retrogressive functions as aligned to different octatonic collections is similar to White's definition; see "A Proposed Theoretical Model for Chromatic Functional Harmony," 22-23.

[^64]:    ${ }^{64}$ The use of the adjective "hyper" in relation to modes is similar to Richard Cohn's use in his "Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Triadic Progressions," Music Analysis 15 (1996): 9-40. The dimensionality of a node corresponds to the number of modes included in it. For example, a one-dimensional node contains only one octatonic mode while a threedimensional node contains three octatonic modes, each of which are representative of the three different octatonic collections.

[^65]:    65 Additional information regarding the rotation and SIA of the respective octatonic key areas will be included in the prose or analytic diagrams in the following chapters.

[^66]:    ${ }^{66}$ Lewin, Generalized Musical Intervals and Transformations, 193-95.

[^67]:    ${ }^{67}$ The inversion of the angle brackets-and their directionality towards the transformation, rather than away from the transformation-represents an inward motion I associate with merging.

    68 This would also apply to transformations between similar three-dimensional key areas.

[^68]:    ${ }^{1}$ As evidenced by Crumb's sketch materials archived at the Library of Congress, various titles for A Haunted Landscape included: "Paisage [Landscape]" the seventh movement from a planned, but not completed 10-movement work entitled Monologue of the Moon [Monólogo de la Luna] for soprano and orchestra based on Federico García Lorca's play Bodas de Sangre (found in Sketchbook " $A$," box 9, George Crumb Papers) and as the first movement of Apparitions (found in Apparition: Elegiac Songs and Vocalises for Soprano and Amplified Piano (On Texts from Walt Whitman's "When lilacs last in the dooryard bloom'd"), 1979, sketches, box 5/folder 9, George Crumb Papers); various references to Idyll or An Appalachian Idyll (found in A Haunted Landscape: for Orchestra, 1984, sketches, box 3/folder 1 and folder 3, George Crumb Papers and throughout Sketchbook " $A$," Sketchbook "B," and Sketchbook "C," box 9, George Crumb Papers) [the title of Idyll was used later in Crumb's An Idyll for the Misbegotten (Images III) (1985)]; Open Air Music also titled Orchestra Suite which were conceived as multi-movement works ranging from 7-10 movements (found in A Haunted Landscape: for Orchestra, 1984, sketches, box 3/folder 2, George Crumb Papers). Variations on the final published title include: "Haunted Landscapes" which was either the fourth or fifth movement of a multi-movement work (found in A Haunted Landscape: for Orchestra, 1984, sketches, box 3/folder 1, George Crumb Papers); Haunted Landscape with an alternative title [apparition] "a Haunted Landscape"; A Haunted Landscape with the "A" penciled in compared to the rest of the title which was in ink, and "The Haunted Landscape" a movement of the previously mentioned work (all variations found in A Haunted Landscape: for Orchestra, 1984, sketches, box 3/folder 3, George Crumb Papers).
    ${ }^{2}$ See Sketchbook " $A$," Sketchbook " $B$," and Sketchbook " $C$," box 9, George Crumb Papers.

[^69]:    ${ }^{3}$ The author photographed Crumb's sketch materials found in this dissertation on February 19-23, 2013 at the Library of Congress. I accept sole responsibility for any problems with their clarity.

[^70]:    ${ }^{4}$ The electronic version of this dissertation includes a high-quality colour version of Figure 4.2, which can be enlarged. Readers consulting a printed version of this dissertation can refer to the transcription of this sketch provided in Figure 4.3. Transcriptions of sketch materials throughout this dissertation will be included if they are difficult to read in printed form.

[^71]:    5 Only the relevant items from Figure 4.6 are included in the transcription in Figure 4.7.

[^72]:    6
    This formal structure differs significantly from Louise Mitchell's analysis that segments the work into a rondo-like structure: A (mm. 1-34), B (mm. 35-49), C (mm. 50-62), Trans. (m. 63), A1 (mm. 64-92), B1 (mm. 93-115), Trans. (116-20), A2 (mm. 121-27), C1 (mm. 128-33), A3 (mm. 134-53), Trans. (m. 154), and Closing Material (mm. 155-62). This formal design clearly does not consider the string triads as indicators of formal structure since the string chords do not occur in A1 and A2. See her

[^73]:    ${ }^{7}$ I have labeled Crumb's "(low drums beat cresc. to climax)" as "growth" in my chart and included a final $a^{3}$ section following $\mathrm{d}^{1}$, which is not a significant difference in formal structure as compared to Crumb's formal sketch shown in Figure 4.10.
    ${ }^{8}$ An octatonic treatment of these materials, while it differs considerably from the method utilized in this document, can be found in Mitchell's "Octatonicism in the Music of George Crumb," 31-71.

[^74]:    ${ }^{9}$ The varied formal designs of "Come Lovely" are discussed in the fifth chapter.

[^75]:    ${ }^{10}$ Crumb, Apparition: Elegiac Songs and Vocalises for Soprano and Amplified Piano (On Texts from Walt Whitman's "When lilacs last in the dooryard bloom'd"), 1979, sketches, box 5/folder 9, George Crumb Papers, Music Division, Library of Congress, Washington, D.C.

[^76]:    12
    Mitchell also analyzes these progressions from A Haunted Landscape using an octatonic framework and notes the relative completeness or incompleteness of the progressions. See her "Octatonicism in the Music of George Crumb," 25-27 and 73-76.

    13 Earlier sketches of Crumb's A Haunted Landscape include the alternating minor-chord figure after every two string triads comprised of all four minor triads found in one octatonic collection. This completes each octatonic segment and, because of the repetition of pitches, lends credence to the analysis of a pentachordal subset of an octatonic collection as representative of the octatonic collection. See $A$ Haunted Landscape: for Orchestra, 1984, sketches, box 3/folder 2, George Crumb Papers, Music Division, Library of Congress, Washington, D.C.

[^77]:    14 A similar procedure of starting "off-octatonic" is used for the introduction of "Come Lovely," discussed in the next chapter.

[^78]:    15 Although the key of the entire work is $\mathrm{Oct}^{3} \mathrm{R}^{5<54\rangle}$, the higher-level harmonic function of Oct $^{2} \mathrm{R}^{3<54>}$ in relation to the piece as a whole will be discussed in section 4.8.

[^79]:    ${ }^{16}$ While this mapping would also occur using the traditional transposition operation in pitch space, as will be shown in section 4.5, a T transformation in octatonic space is able to produce a one-to-one mapping between pitches of sets that are inversionally related in pitch space. In pitch space, however, a traditional $\mathrm{T}_{\mathrm{n}} \mathrm{I}$ operation will not produce a one-to-one mapping of its constituent parts.

    17 Although no hybrid chord is used in the introduction, this key area implies a $\mathrm{H} 2 \backslash 4^{3, n}$ family. The same relative rotation of modes of a $\mathrm{H} 2 \backslash 4^{3, \mathrm{n}}$ family is utilized for the transition to the coda and the coda. This correspondence is discussed in the respective sections.

[^80]:    ${ }^{18}$ In Table 4.3, the cells contain three pitch classes rather than the one because the transition uses hybrid chords which contain three pitch classes per node. This is reflected in the key area notated in the column headers.

[^81]:    ${ }^{19}$ There are numerous possibilities to represent the key areas of the transition; the significance of the particular key areas used is discussed in detail in the next section.

[^82]:    20
    Although each of the major and minor triads are realized as second- and first- inversion chords respectively, if the $\mathrm{B} b$-pedal is disregarded each of the chords are actually in root position; however, the sketch materials shown in Figure 4.2 and Figure 4.4 indicate Crumb's conception of the triads as secondand first inversion chords rather than root position chords.

[^83]:    ${ }^{21}$ Interestingly, the two pitch classes not used in the transition's 10 -note chord, $\{37\}$, are the highest and lowest pitches of the first 10 -note chord in the c section respectively.
    ${ }^{22}$ Based upon a key area of $\mathrm{Oct}^{1,2,3} \mathrm{R}^{\mathrm{A}, 9,5<54>}$ for the first three heptachords, if the key area of the final heptachord changed to Oct ${ }^{1,2,3} \mathrm{R}^{1,6,5<54>}$, the transformation to the final chord would be $\mathrm{T}_{3} 0$.

[^84]:    ${ }^{23}$ The pitch of the crotales is not included in the reduction because of the difference in timbre and a delayed onset of the pitch. Additionally, a melodic figure in the piano is not included since it occurs on a melodic rather than harmonic layer.

[^85]:    24
    If the tuning of the third chord was ignored, the transformational scheme would be $\mathrm{T}_{-2} 0-\mathrm{T}_{-2} 0-$ $\mathrm{T}_{4} 0-\mathrm{T}_{-2} 0-\mathrm{T}_{-2} 0$, also a symmetrical pattern. The double ROT transformations notated above the third chord of the first system would be divided such that $\mathrm{ROT}_{4}{ }^{2}$ would happen between the second and third chords while $\mathrm{ROT}_{4}{ }^{1}$ would happen between the third and fourth chords.

[^86]:    25
    If a pentachordal multi-set had been orchestrated with the repeated note both above and below the string triads, parallel octaves would have resulted.

[^87]:    ${ }^{26}$ The turtle-dove effect, played by the flute, is a trilled glissando between a whole tone that approximates the cooing of a turtle dove.

[^88]:    ${ }^{1}$ The bird song is found in section 16 of "Lilacs." Verse 28, which begins the section, serves as the text for "Come Lovely"; Verse 30: "Dark Mother"; Verse 31: "Approach Strong Deliveress! ..."; Verse 33: "The Night in Silence." The Vocalises between movements in Apparition do not contain text from Whitman's poem. See Walt Whitman, "When Lilac's Last in the Door-yard Bloom'd," in The Walt Whitman Archive, ed. Ed Folsom and Kenneth M. Price,
    http://www.whitmanarchive.org/published/LG/1867/poems/212 (accessed February 10, 2014).

[^89]:    ${ }^{2}$ Mutlu Konuk Blasing, "Whitman's 'Lilacs' and the Grammars of Time," PMLA 97 (1982): 32.
    ${ }^{3}$ Ibid., 36.

[^90]:    ${ }^{4}$ The last two lines also do not include the hum, which are found at the end of all the other lines; this will be discussed later.

    5 Jeffrey Steele, "Poetic Grief-Work in Whitman's 'Lilacs,"" Walt Whitman Quarterly Review 2 (1984): 13.
    ${ }^{6}$ Ibid., 14.

[^91]:    ${ }^{7}$ Patricia Lee Young, "Violence in Whitman's 'When Lilacs Last in the Dooryard Bloom'd,"" Walt Whitman Quarterly Review 4, no. 4 (1984): 12.
    ${ }^{8}$ Mark Edmundson, "'Lilacs': Walt Whitman's American Elegy." Nineteenth-Century Literature 44 (1990): 477.
    ${ }^{9}$ Translation of the Spanish text found in Lorca's Collected Poems, rev. ed., introduction and notes by Christopher Maurer, translated by Catherine Brown et al. (New York: Farrar Straus and Giroux, 1991): 94, 99. Crumb includes the full text of "Paisaje" and an English translation by J. L. Gili in Sketchbook "A," box 9, George Crumb Papers, Music Division, Library of Congress, Washington, D.C., 111.
    ${ }^{10}$ Lorca, "Deep Song" in Deep Song and Other Prose, ed. and trans. Christopher Maurer (New York: New Directions Publishing Corporation, 1980): 25.

[^92]:    ${ }^{11}$ Crumb's The Ghosts of Alhambra (2008) for baritone, guitar and percussion sets seven poems from Lorca's Poema del Cante Jondo, which includes a fourth movement titled "Paisaje."

    12 Lorca's "Paisaje" concludes with a reference to birds: "The olive trees/ are laden/ with cries./ A flock/ of captive birds/ moving their long long/ tails in the gloom." See Lorca, Collected Poems, 98-99.

    13
    Gareth Walters argues that Lorca's belief in the importance of duende is misplaced and that the Andalusian people generally do not conceive of duende in the same way as Lorca. See his Canciones and the Early Poetry of Lorca (Cardiff: University of Wales Press, 2002): 130-31.
    ${ }^{14}$ Lorca, "Play and Theory of the Duende," 50.
    15 Ibid., 51.

[^93]:    ${ }^{16}$ Maurer, Introduction to Deep Song and Other Prose, xi.
    ${ }^{17}$ Crumb also wrestled with the eternal artistic problem with regards to the balance between the technical and intuitive aspects of his music. See Smith and Smith, "George Crumb," 97.

[^94]:    ${ }^{18}$ Walt Whitman, "When Lilac's," verse 6.
    ${ }^{19}$ Donald Kroodsma, The Singing Life of Birds: The Art and Science of Listening to Birdsong, (New York: Houghton Mifflin Company, 2005): 237.
    ${ }^{20}$ For audio excerpts of the Wood Thrush's song see Cornell Lab of Ornithology, "Wood Thrush," in All About Birds (2009), ed. Hugh Powell, http://www.allaboutbirds.org/guide/Wood_Thrush/sounds (accessed February 10, 2014).

[^95]:    ${ }^{21}$ Kroodsma, The Singing Life of Birds, 253.
    ${ }^{22}$ Ibid.
    ${ }^{23}$ For an audio excerpt of the Hermit Thrush's screech (also known as the chuck or screy call) see Cornell Lab of Ornithology, "Hermit Thrush," in All About Birds (2009), http://www.allaboutbirds.org/guide/Hermit_Thrush/sounds.
    ${ }^{24}$ Robert Cook also uses the descriptive terms "Warble" and "Call". See "Crumb's Apparition and Emerson's Compensation," 3-5. I will follow Cook's style of motive identification although the actual names will differ. Thus, descriptions such as "Warble (Bird)" indicate the motive shown in Figure 5.5a.
    ${ }^{25}$ The justification for these descriptive terms will be discussed later.

[^96]:    26
    As suggested by Robert C. Cook in an email message to author, February 11, 2014, whereas the repeated notes in Elegy are similar to the Alarm of the Wood Thrush, the slow rhythmic realization and low register is suggestive of a human-voice realization of the Wood Thrush's song. An example of Alarm, realized in the Wood Thrush's voice, is found in section $\mathrm{B}^{2}$ but is not a recurring motive like the other motives listed.
    ${ }^{27}$ Kroodsma notes that the Wood Thrush sings two streams of music at once during its Warble/prelude, which are sung in lower and higher registers. This is possible since the Wood Thrush has two voice boxes; Kroodsma states that this is similar to brown thrashers, northern cardinals, mockingbirds, gray catbirds, canaries, brown-headed cowbirds, and zebra-finches (the Hermit Thrush is noticeably absent from his list). Moreover, he notes that the left voice box is responsible for the lower sounds while the right voice box is responsible for the higher sounds. This is analogous to the placement and register of Vestige in the pianist's left hand and Warble in the right hand. Additionally, like the Wood Thrush's prelude, Vestige only occurs during Warble. See Kroodsma's The Singing Life of Birds, 246-50.

[^97]:    ${ }^{28}$ Kristina F. Szutor, "Musical Coherence and Poetic Meaning in George Crumb's Apparition" (D.M.A. thesis, University of British Columbia, 1994): 61.

[^98]:    ${ }^{29}$ The metaphor of a torero is related to Lorca's description of duende discussed in the introduction to this chapter.

[^99]:    ${ }^{30}$ Lines three and four of the bird song in "Lilacs" are also examples of historical narrative time; however, the accompanying music is not as motivically interspersed as the other motives and therefore not included in Table 5.2. These lines will be addressed in the following analysis.

[^100]:    ${ }^{31}$ The length of this excerpt provides additional context to Crumb's notion of a "germ cell" and its relationship between tonality and motive, described by Schenker, which is discussed in section 3.1.

[^101]:    32 OED Online, s.v. "tesseract" (Oxford University Press, December 2013) http://www.oed.com/view/Entry/199669 (accessed February 10, 2014).
    ${ }^{33}$ Sidney Sheldon, The Other Side of Midnight (New York: William Morrow and Company, 1973); reprint, The Dark Side of Midnight (New York: HarperCollins, 2007): 305.

[^102]:    ${ }^{34}$ Most of the items included in the table and legend have already been discussed; the other items will be discussed in the respective sections of this chapter.

[^103]:    ${ }^{35}$ The fourth line, "Sooner or later delicate death," is merged with historical narrative time and the lower stream of natural-cyclic time. This will be discussed in section 5.6.
    ${ }^{36}$ Timothy Mitchell, Flamenco Deep Song (New Haven: Yale University Press, 1994): 127. The relationship of "Come Lovely" to the siguiriya is based upon the title of Lorca's set of poems Poema de la Siguiriya gitana, which includes "Pasaje," the original text of "Come Lovely."
    ${ }^{37}$ Mitchell, Flamenco Deep Song, 127. The rhythmic and harmonic elements of the siguiriya are not immediately evident in "Come Lovely." For a discussion of these features in the siguiriya see Steven K. Mullins, "Flamenco Gestures: Musical Meaning in Motion" (Ph.D. diss., University of Colorado, 2010): 21-36.
    ${ }^{38}$ Lorca, "Deep Song," 25.

[^104]:    39
    On the surface, these divisions appear to be demarcated by tonal and post-tonal elements; however, as I will show these sections can be linked transformationally and functionally.

    40 Rugoff, "Three American Requiems," 141.
    ${ }^{41}$ Candelas Newton, Understanding Federico García Lorca (Columbia, South Carolina: University of South Carolina Press, 1995): 21.

[^105]:    42 Sometimes the parenthetical notation will refer to a transformation of one chord to another separated by more than one system; such chords may reside in different sections.

[^106]:    ${ }^{43}$ Arrow brackets around the $\{90\}$ dyad in the voice (m. 8) include pc 1 (m. 7)—also shown in

[^107]:    ${ }^{44}$ Lorca, "Deep Song," 25.

[^108]:    ${ }^{45}$ Two ROT transformations and a key change, notated in Figure 5.20 in the historical narrative stream, are not realized until the end of the $\mathrm{B}^{1}$ section.

[^109]:    ${ }^{46}$ The nodal position of the first Trill (Undulate) and the preceding $>\mathrm{T}_{0} 2_{2}<^{3 \rightarrow 1,2,3}$ transformation from natural-cyclic time is based upon a similar pattern in the $\mathrm{A}^{2}$ section and will be discussed later.
    ${ }^{47}$ Steele, "Poetic Grief-Work," 13.

[^110]:    ${ }^{48}$ In addition, numerous ROT transformations would be required that would disrupt the isography of Elegy with other iterations of Elegy.

[^111]:    ${ }^{49}$ The A-major triad foreshadows the final Elegy chord of the piece found in m. 31.
    ${ }^{50}$ This is the same chord quality found in the early sketch of "Paisaje," which informed the interpretation of the incomplete D-major triad in the A section (cf. Figure 5.3 and Figure 5.15).

[^112]:    ${ }^{51}$ The final Warble (Bird) is not included as a hybrid chord because of the indistinct pitches.

[^113]:    ${ }^{1}$ The lack of surface-level functional motion in the $\mathrm{A}^{1}$ section of A Haunted Landscape is based upon the pentachordal interpretation shown in Figure 5.24.
    ${ }^{2}$ The repurposing of material is discussed in both Chapters 4 and 5, although additional evidence relating to other works is found throughout his sketches. In particular see Sketchbook "A,"Sketchbook " $B$," and Sketchbook " $C$," box 9, George Crumb Papers.

[^114]:    ${ }^{3}$ Only surface-level functionality would result as it is difficult, if not impossible, to determine the functionality of two- or three-mode hybrids as discussed in section 3.7.
    ${ }^{4}$ An initial sketch of Processional, dated 29 July 1977, is conceived of as the first movement, titled "Apparition," of a three movement collection of ballad-length pieces for Gilbert Kalish. See George Crumb, Processional (piano), 1983, holograph manuscript score paste-up and sketches, box 4/folder 8, George Crumb Papers, Music Division, Library of Congress, Washington, D.C.

[^115]:    ${ }^{5}$ The reader may wish to refer to the discussion of the octatonic model in section 3.2.
    ${ }^{6}$ The arrangement of modal index numbers in the key area designations does not indicate any hierarchical functionality as a determination of functionality is beyond the scope of the current discussion. Additionally, three SIA numbers are required to identify a unique mode-four mode while only two were required for an octatonic mode.

[^116]:    Strickland, "George Crumb," 171.

