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The Construction of Edmond Halley's 1701 Map of Magnetic Declination

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A thesis submitted in partial fulfillment of the requirements for the degree in Master of Science

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I Abstract

Using the navigational instruments of his time, Edmond Halley collected data during sea voyages of the HMS Paramore. Following these voyages, in 1701 he published a map showing lines of equal magnetic declination. Magnetic declination or variation is the angular difference between magnetic north and geographical or true north for any point on the earth's surface. The map has been held up by many as an early, and good, example of statistical graphics. Halley did not reveal the data analytic techniques that he used in his map construction and they remain unknown to this day. Using some mathematical tools of his day, namely arithmetical averages and Newton's divided difference method to fit a line to data, a plausible method for the map's construction is given.

II Keywords

Halley, chart, Atlantic, map, magnetic declination, variation

III Acknowledgements

My sincere thanks to my advisor Dr. David Bellhouse for providing an opportunity to work on such an interesting project.

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1 Introduction

Edmond Halley published the world's first map, shown in Figure 1, of the Atlantic Ocean in 1701 showing lines of equal magnetic declination, known today as isogones. Halley constructed the isogones using observations he collected during two sea voyages. For reference, his observations have been marked with symbols on the map. Each triangle and circle represents a position of latitude and longitude west of London, and an associated magnetic declination from the first and second voyage respectively. Halley did not publish the analytic techniques he used to construct the map. The purpose of this thesis is to propose a plausible method as to how Halley went from the data to the finished map.

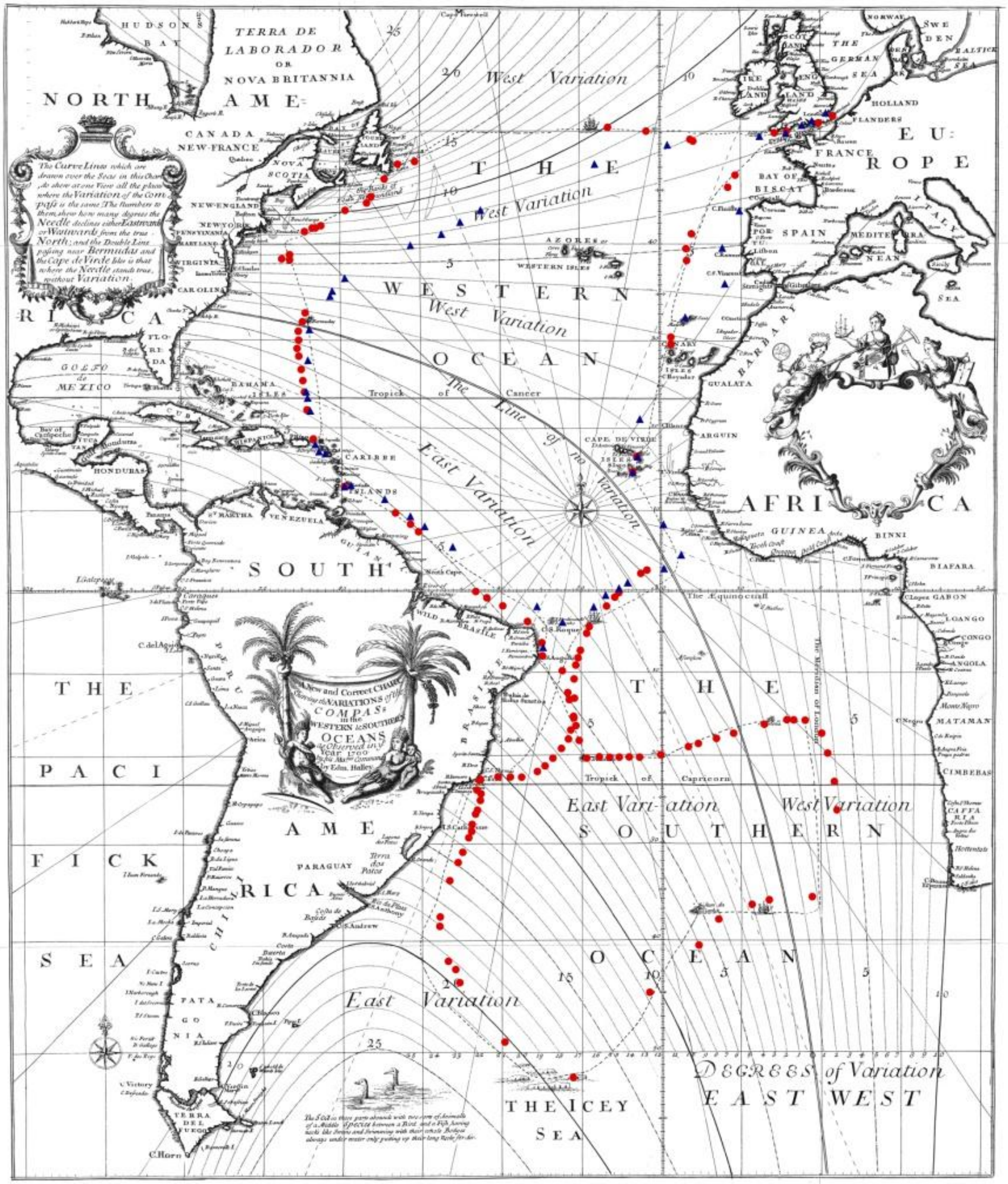

Figure 1: The 1701 Atlantic Map showing Halley's Data.

2 Background

In 1701 Edmond Halley constructed the world's first published map showing magnetic declination. Magnetic declination or variation is the angular difference between magnetic north and geographical or true north for any point on the earth's surface. If the angle is greater than true north, the variation is east, if the angle is less than true north, the variation is west, and if magnetic north and true north are in the same direction, there is no variation. At the time, sea navigators could calculate latitude wherever they were by observing the Sun, provided it was visible to them; however, calculating longitude was not as straightforward (Cook 1998, p.21-22). Determining longitude required knowing the time at some arbitrary reference point or meridian such as London. Pendulum clocks existed in the 17th century but were not accurate at sea due to changes in temperature and the motion of the ship. For the safety of oceanic navigation, solving the longitude problem was a serious problem requiring investigation (Cook 1998, p.23). Halley's interest in magnetic declination and longitude earned him the opportunity to try to solve the longitude problem.

Halley's interest in the Earth's magnetism began in his youth and continued until the end of his life. In 1683, Halley published, "A theory of the variation of the magnetical compass" describing the magnetic declinations in various parts of the world based on the observations of sea captains and explorers. Halley gives a sample of 55 observations from 47 locations and discusses the direction and the rate of change of the variation of the compass. Halley knew the magnetic declination changed with time (secular variation) and included five readings from London spanning over 100 years. He gives a general magnetic theory as to why the magnetic declination changes with time proposing that the "Earth is one great Magnet, having Four Magnetical poles, or points of attraction" (Halley, 1683). Halley extends his idea in greater detail in a paper published nearly a decade later (Halley, 1692). To account for the secular variation, Halley hypothesized that the earth contained four magnetic poles: two fixed on the earth's crust and two moving internally within the core (Halley, 1692). The North and South Poles each contained one fixed and one movable magnetic point of attraction. Halley makes the point that his hypothesis needs further investigation because the variation of the compass had not been studied long enough. A sea voyage would allow Halley to observe the most recent magnetic declination, to gain further insight into his theory of the Earth's magnetism, and to possibly find a connection to help solve the longitude problem at sea.

In 1693, Halley, together with Benjamin Middleton, elected Fellow of the Royal Society in 1687, petitioned the Royal Society for their support of a worldwide oceanic voyage to observe the magnetic declination. The Royal Society agreed to help by supplying a small vessel; Middleton would assist in the cost of the voyage, and the observations would be made by Halley. After years of delays, the Royal Navy took full responsibility for the voyage, and in 1698, King William III commissioned Halley as Royal Naval Captain of the HMS Paramore and provided him with a complete set of instructions. The Admiralty's instructions to Halley dated 15 October 1698 were (Thrower, 1981, p.268-269):

> Whereas his Maty. has been pleased to lend his Pink the Paramour for your proceeding with her on an Expedition, to improve the knowledge of the Longitude and variations of the Compasse, which Shipp is now compleatly Man'd, Stored and Victualled at his Mats. Charge for the said Expedition ...

The voyage was restricted to the Atlantic Ocean, and in addition, Halley was ordered to search for the discovery of unknown land in the southern Atlantic Ocean. In October 1698, Halley set sail on what would be the first of two Atlantic sea voyages. This was the first time a sea voyage had been planned for the sole purpose of scientific discovery (Thrower, 1981, p.15-16). Less than two months after his return to London from the second voyage, Halley presented a map to the Royal Society showing lines of equal magnetic declination. The Atlantic map was published a few months later in 1701.

Halley did not reveal the data analysis techniques he used in the construction of the map and they remain unknown to this day. For example, Friendly (2008) describes the map in the context of the development of statistical graphics but makes no mention of how the map was constructed. Using some of the mathematical tools of his day, we carried out a reconstruction of the map; the tools that Halley used are an early form of data smoothing.

3 The Atlantic Map

The Atlantic map is a nautical map on a Mercator projection that includes cartographic elements such as lines of latitude and longitude and a compass rose. The original Atlantic map was engraved and printed on a broadsheet measuring 22.5 by 19.5 inches (Thrower, p. 368).

The electronic copy of the original map used in this thesis shows where the sheet had been folded. The map features decorative cartouches including a native American family in feathered garments and headdress under palm trees and rococo-style cartouches with mythological figures representing astronomy (with armillary sphere and telescope), navigation (with ship and compass), and mathematics (with triangle and dividers). A dedication to King William III was included in later publications of the map in the blank cartouche in Northern Africa. Halley’s route is marked by a dashed line with ornamental ships superimposed. There is a notice of discovery of birds and icebergs in “The Icey Sea” now known as the Antarctic Ocean, which Halley makes note of in his journal.

The upper Atlantic Ocean is referred to as the Western Ocean while the lower Atlantic Ocean is referred to as The Southern Ocean. The spelling of some locations such as Brasile (now Brazil) has been changed and modernized since the map was published. Represented with a double line on the map, the line of no variation indicates when the reading of the compass stands true. Today, it is known as the agonic line. The map consists of 60 lines of equal magnetic declination: the agonic line, the line of 1 degree west variation, 25 lines of east variation, 24 lines of west variation in the upper Atlantic Ocean, and 9 lines of west variation in the lower Atlantic Ocean. Halley called the lines of variation ‘curve lines’, however, they became known as Halleyan or Halleian lines (Thrower, 1981, p.58). Today, the lines of equal magnetic declination are known as isogones.

4 Magnetic Data Collection

Halley recorded the latitude, longitude and magnetic declination during his voyages in two separate journals now located in the British Library (British Library, Add. MS 30,368). The following methods that Halley used are taken from his journals (British Library, Add. MS 30,368). The latitude was taken at noon and the longitude was obtained by reckoning from the previous day’s noon position, except in some cases when a celestial observation was made. Nearly all of the observations of magnetic declination were made by observing the Sun’s magnetic amplitude, the angular distance when on the horizon at sunrise or sunset. The evening amplitude was combined with the amplitude of the following morning. Then the magnetic declination was one-half the difference between the two amplitudes and applied to the geographical position at midnight. If the morning and evening amplitudes were observed on the same day, then half the difference was taken as the magnetic declination and applied to the position at noon. When cloudy or foggy weather prevented Halley from taking the Sun's amplitude, the magnetic declination was obtained by observing the azimuth of the Sun or Moon, when at a low altitude above the horizon.

Halley recorded the amplitudes; however, the magnetic declinations were not always deduced and entered into the journal. In addition, Halley made note of instances when he was off course, but he did not record his course corrections. In 1913, James P. Ault and William F. Wallis, members of the Department of Terrestrial Magnetism, Washington, D.C., performed a compilation of Halley's original data. Using Halley's methods given in the journal, Ault and Wallis (1913) corrected the geographical positions and calculated the missing magnetic declinations from the given amplitudes. The complete data set of 170 magnetic observations is used in the reconstruction of the Atlantic map. Table 6 shows the observations taken on the first voyage and Table 7 shows the observations taken from the second voyage.

5 Map Construction

Halley had a number of techniques available to him such as arithmetic mean and Newton's divided difference methods at the time he constructed the map. We conjecture that Halley used the arithmetic mean to reduce the error in magnetic declination. By the end of the seventeenth century, the calculation of the mean was thought to be a better value than a single measurement. Plackett (1958) illustrates this when he refers to Flamsteed's (Halley's predecessor as Astronomer Royal) excerpt on a discussion of errors with respect to astronomy. Using the arithmetic mean with respect to magnetic declination is found in a letter written by D. B. to the publisher of *Philosophical Transactions* (1668), a paper likely known to Halley.

Halley now had several points of latitude and longitude with associated magnetic declinations. We conjecture that he fit lines through these points using the technology available to him: Newton's divided differences. Newton's divided difference method was well known to Halley when he constructed the map. The formula is in Lemma V, Book III of the *Principia*, published in 1687 with Halley's influence and assistance (Ronan, 1969, p. 81).

5.1 Use of the Arithmetic Mean

The arithmetic mean is applied to calculate the average latitude, longitude, and the corresponding magnetic declination for a set of observations that are in close proximity to one another. The mean position C is calculated as follows where y is latitude and x is longitude:

$$C = \left[\frac{1}{n}(y_1 + y_2 + \cdots + y_n), \frac{1}{n}(x_1 + x_2 + \cdots + x_n)\right]$$

The mean magnetic declination M is calculated as follows where m is the magnetic declination for a single observation:

$$M = \frac{1}{n}(m_1 + m_2 + \cdots + m_n)$$

Group size (n) ranges from one, where a good single observation is present, to four, where a cluster of points exist. There are several possible combinations of observations that Halley may have used to calculate averages when constructing his map (Tables 4 and 5 in the appendix) and not all of the averages result in a position exactly on a line of variation. However, it is assumed that Halley used interpolation to find the points needed to construct the lines. Examples are shown in Figure 2 where averages were taken from groups of points (circled) on the East Coast of North America. The positional means (red points) were calculated from each cluster of Halley's individual data points (blue points).

The errors in magnetic declination were measured electronically. If the error in magnetic declination was greater than the predicted value (the line of variation), it was given a positive sign, and if the error in magnetic declination was less than the predicted value, it was given a negative sign. Figures 3 and 4 (upper Atlantic Ocean) and Figures 5 and 6 (lower Atlantic Ocean) show how the error in magnetic declination is reduced when using averages compared to Halley's individual observations.

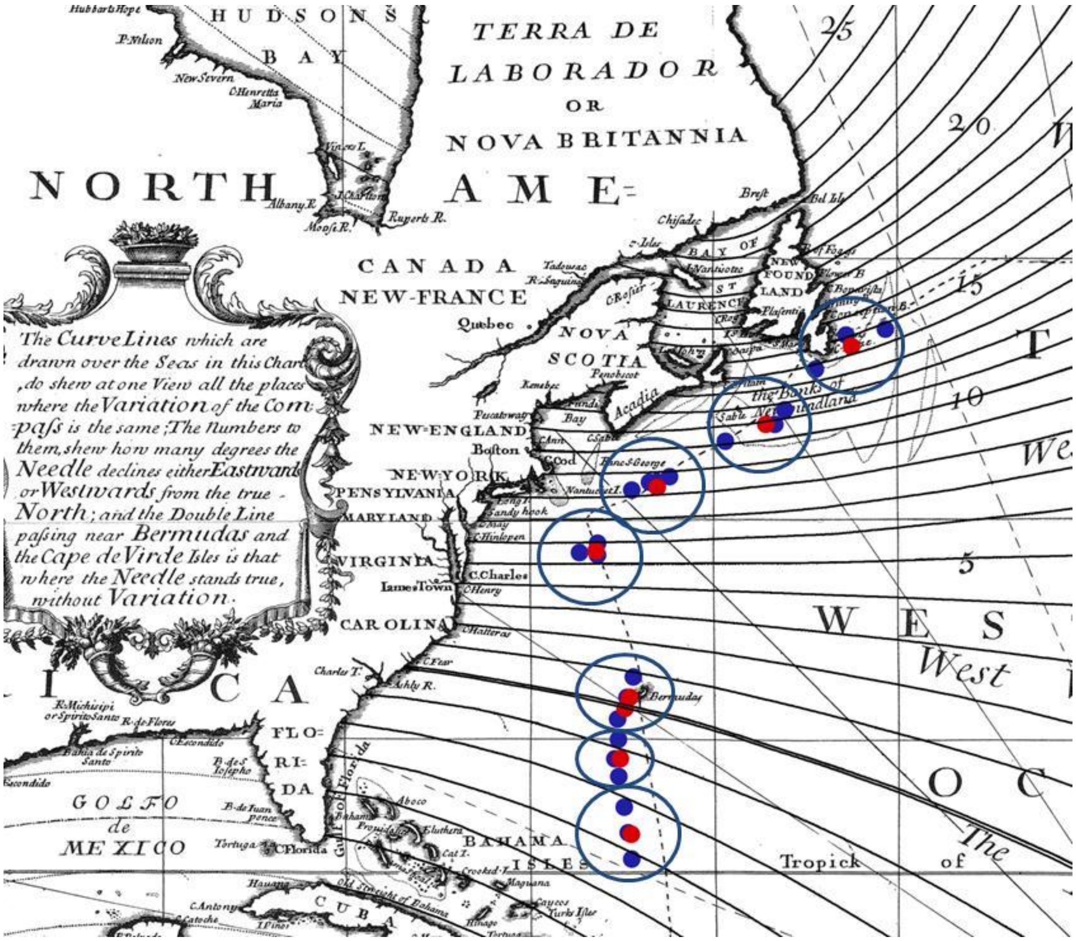

Figure 2: Averages on East Coast of North America.

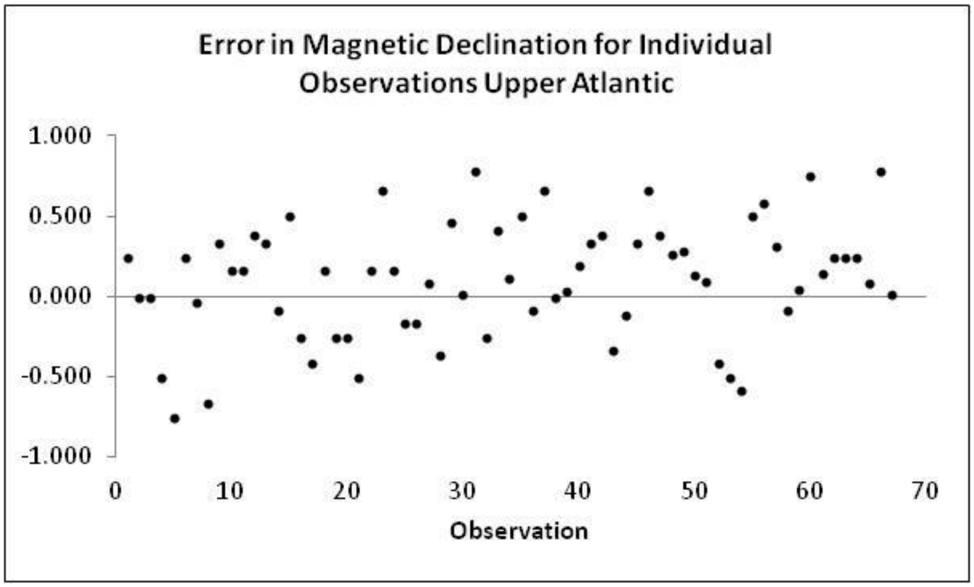

Figure 3: Error in Magnetic Declination for Individual Observations Upper Atlantic.

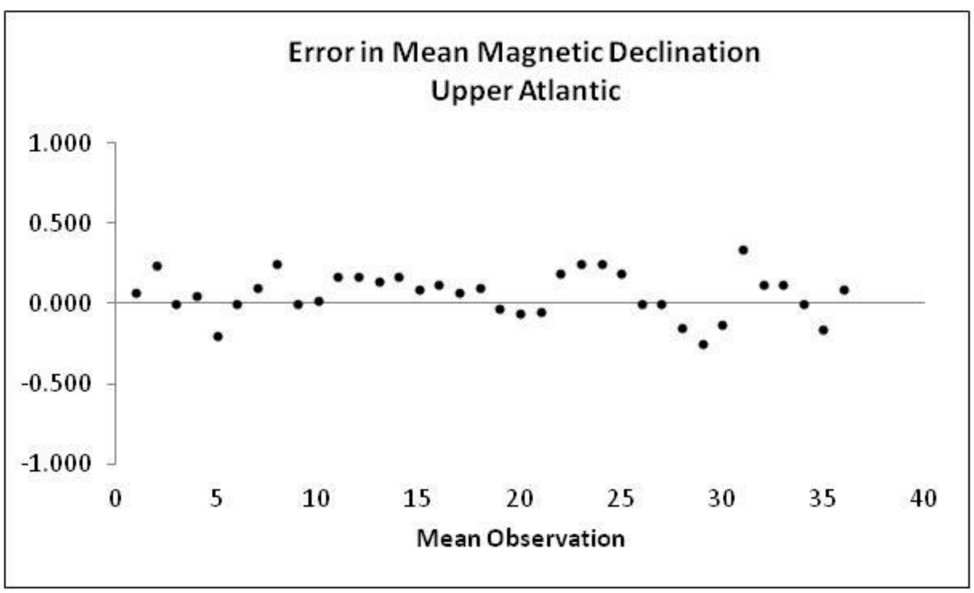

Figure 4: Error in Mean Magnetic Declination Upper Atlantic.

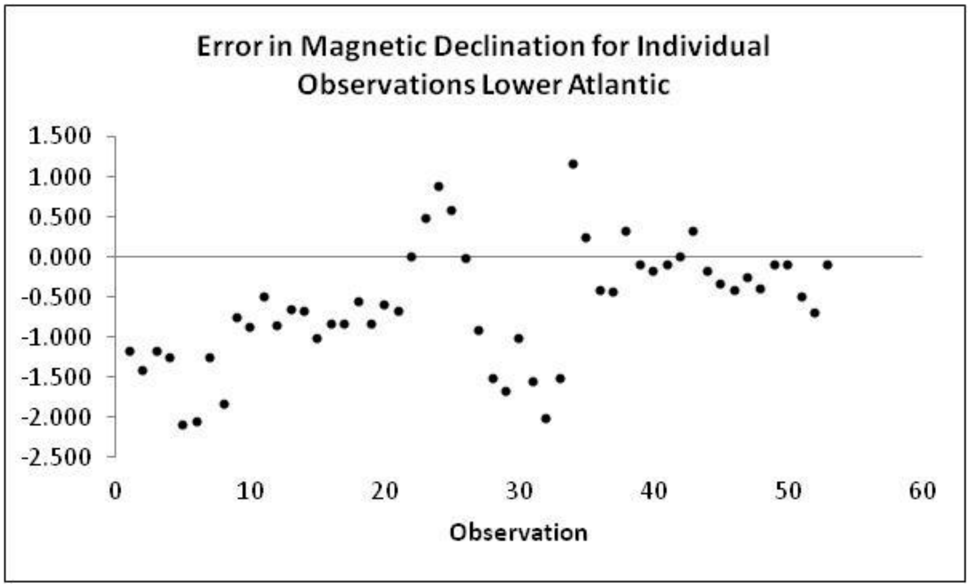

Figure 5: Error in Magnetic Declination for Individual Observations Lower Atlantic.

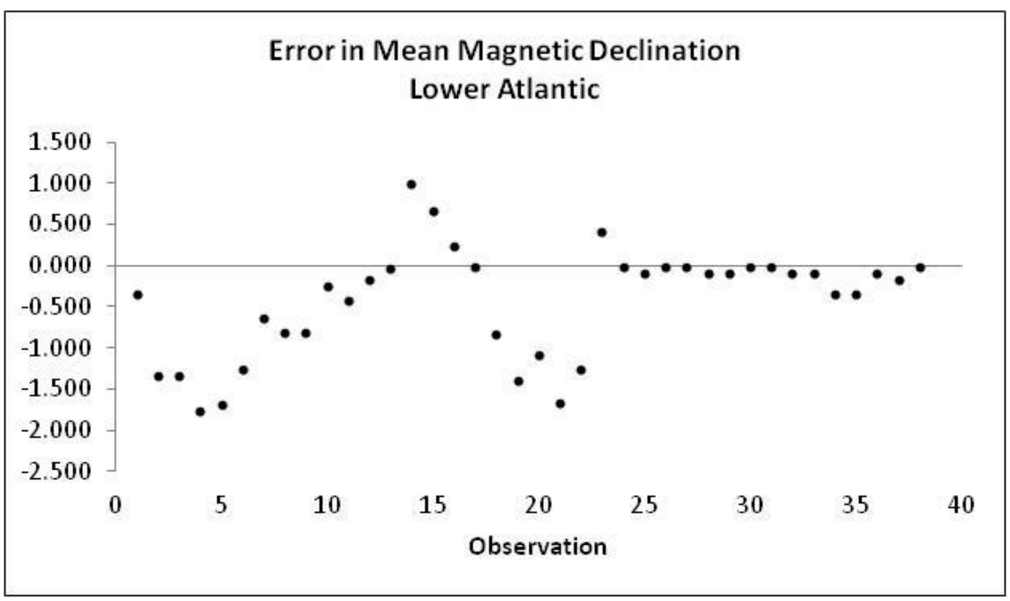

Figure 6: Error in Mean Magnetic Declination Lower Atlantic.

5.2 Use of Newton's Divided Difference Method

Newton's divided difference method is a way to fit a polynomial to the data. Given n data points, the result will be an interpolating polynomial of degree at most $n-1$ that passes through the data points. Let f be some function and let

$$(x_1, f(x_1)), \cdots, (x_n, f(x_n)),$$

be a set of data points. The following differences are real numbers,

$$f[x_k] = f(x_k)$$

$$f[x_k \;\; x_{k+1}] = \frac{f[x_{k+1}] - f[x_k]}{x_{k+1} - x_k}$$

$$f[x_k \;\; x_{k+1} \;\; x_{k+2}] = \frac{f[x_{k+1} \;\; x_{k+2}] - f[x_k \;\; x_{k+1}]}{x_{k+2} - x_k}$$

$$f[x_k \;\; x_{k+1} \;\; x_{k+2} \;\; x_{k+3}] = \frac{f[x_{k+1} \;\; x_{k+2} \;\; x_{k+3}] - f[x_k \;\; x_{k+1} \;\; x_{k+2}]}{x_{k+3} - x_k}$$

and so on. These numbers are the coefficients of the interpolating polynomial for the data points, which is given by the Newton's divided difference formula,

$$\begin{aligned} P(x) = f[x_1] + f[x_1 \;\; x_2](x - x_1) + f[x_1 \;\; x_2 \;\; x_3](x - x_1)(x - x_2) \\ + \; f[x_1 \;\; x_2 \;\; x_3 \;\; x_4](x - x_1)(x - x_2)(x - x_3) + \; \cdots \\ + f[x_1 \;\; \cdots \;\; x_n](x - x_1) \cdots (x - x_{n-1}). \end{aligned}$$

Each line of magnetic declination was digitized by recording the position for every one degree of longitude. The digitized lines were then tested for all possible combinations of third and fourth degree polynomials using Newton's divided difference method. The root mean square errors for each set of points were calculated and compared. It was found that all lines of magnetic declination could be represented by at most a fourth degree polynomial.

5.3 The Line of No Variation

Represented by a double line on the map, the line of no variation indicates when the reading of the compass stands true. Since the agonic line is the longest line on the map and divides the east variation from the west variation, it is likely the first line of variation Halley constructed.

There are four places in the Atlantic Ocean where Halley crossed the line of no variation: near Bermuda, near the Equator, west of St. Helena, and east of Tristan de Cunha. During both voyages, Halley was very close to the line of no variation when he visited the Cape Verde Islands. Halley used data from both voyages to construct the agonic line. The data are in Tables 6 and 7 in the Appendix for the first and second voyages respectively.

The first point is found near Bermuda by averaging two observations from the second voyage. Halley recorded two magnetic declination readings at several locations in this particular area of the map. The two magnetic readings for observation 103 were averaged and the result was used, along with observation 104, in the calculation to find the overall average magnetic declination. The location of the point is on the bottom of the double line and the magnetic declination is only 30 seconds east.

The second point is found near the Cape Verde Islands. Although Halley does not cross over the line in this area of the map, he travelled within one degree of it. Averaging observations 9 and 10 from the first voyage, results in a point with magnetic declination of 31 minutes west. The position of the averaged point is in the center of the Cape Verde Islands and agrees with the variation on the map. From the map, it is apparent that Halley wanted to include the entire collection of islands to have an average magnetic declination of approximately ½ degrees west and that the line of no variation must be west of the Cape Verde Islands. In his 1683 paper, he states that the needle stands true and constant in a northwest direction. As well, Halley was aware of the secular declination, the change in magnetic declination over time, in various regions of the map. In this area of the map, he knew the secular declination was slow compared to other areas of the map, and the line of no variation in 1700 remained in a northwest direction. Observations 11 and 12 from the first voyage have an average magnetic declination of ½ degrees west, which closely agrees with the map. Halley knew the line of one degree west variation is east of these two observations for reasons stated above, and may have, as a first approximation, corrected observation 11, which has a magnetic declination of one degree west, to 18 degrees north latitude. Observation 11 has a longitude of 19 ½ degrees west, and the midpoint between 19 ½ degrees west longitude and 30 ½ degrees west longitude is 25 degrees west longitude, the center of the Cape Verde Islands. Since the midpoint is the center of the Cape Verde Islands, Halley likely used 30 ½ degrees west longitude and 18 degrees north latitude as the position for the second point. When used in the calculation of Newton's divided difference formula, this point gives the best fit polynomial when used together with the other three points found along his route.

During the first voyage, Halley required significant course corrections as he crossed the agonic line near the equator although it is unknown how he corrected his course. When he crossed the equator during the second voyage, bad weather prevented him from observing the magnetic declination. Therefore, none of the observations in this area are used when reconstructing the agonic line.

The third point is found by averaging three observations near the agonic line west of St. Helena. The average of the three observations from the second voyage has a position and magnetic declination that agrees with the map.

Halley used Tristan da Cunha as a local meridian. On 17 February 1700, Halley writes in his journal "I Determine the Latitud of the most Southerly of the Isles of Tristan da cunha 37°25'." Halley also writes that there is no variation east of Tristan de Cunha. On 24 February 1700, Halley says, "No variation 11 ½ to the Eastwards of the Islands." This measurement agrees with the map. Measuring 11 ½ degrees east of the islands, a point is found on the agonic line and is used as the fourth point. Table 1 shows the position and magnetic declination for the four points.

Table 1: Position of the four points for the line of no variation.

Point	Observations	Mean Position				Mean Magnetic Declination	
		Latitude		Longitude			
		D°M'	N/S	D°M'	E/W	D°M'	E/W
1	V2-103, V2-104	31°33'	N	64°59'	W	30 sec.	E
2	W of Cape Verde Islands	18°00'	N	30°30'	W	0°00'	-
3	V2-66, V2-67, V2-68	17°22'	S	10°19'	W	0°00'	-
4	East of Tristan da Cunha	37°25'	S	4°00'	W	0°00'	-

A position was recorded for every one degree of longitude along the agonic line. Since Halley used a double line to represent the agonic line, the top line was used for the recorded points. Newton's divided difference method is sensitive to small changes; it is dependent on the spacing of the points. Therefore, the fit of the polynomial would differ slightly if Halley used the middle of the double line or the bottom line of the double line. Extrapolation was used to extend the polynomial to 50 degrees south latitude where the lines of variation end in a gridline on the map. The resulting cubic polynomial for latitude as a function of longitude for the agonic line is:

$$P_3(x) = -0.000447369x^3 - 0.0728533x^2 - 4.14404x - 52.8591$$

The following is the resulting cubic polynomial on a Mercator projection graph. Shown in Figure 7, the graph compares the fit of the polynomial, the four averaged points shaded, with the digitized line recorded from the map marked with circles.

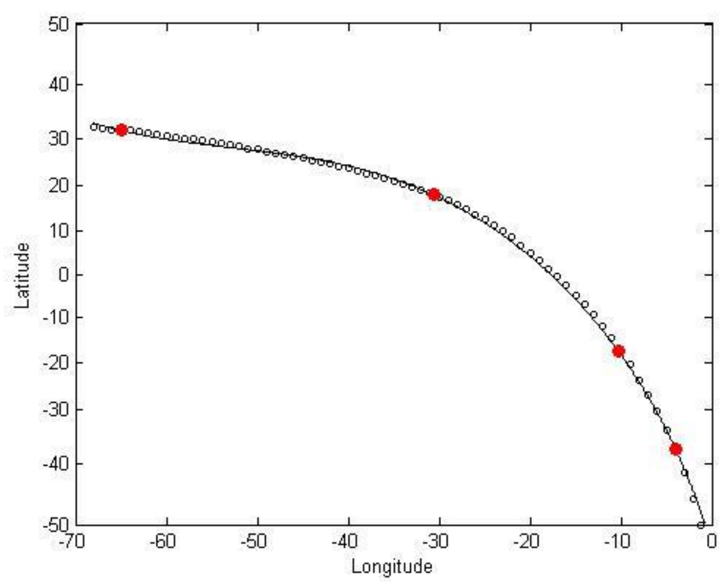

Figure 7: Averaged points on Halley's route.

As mentioned above, Newton's divided difference method is sensitive to small changes and will produce different fits to the polynomial. The method is dependent on the spacing of the points. When the points are still on the digitized line, shown in Table 2, but deviate from Halley's route, the result is a lack of fit, shown in Figure 8.

Table 2: Position of the four points that deviate from Halley's route.

Point	Position Deviated from Halley's route			
	Latitude		Longitude	
	D°M'	N/S	D°M'	E/W
1	32°25'	N	68°00'	W
2	13°30'	N	26°00'	W
3	26°53'	S	7°00'	W
4	36°00'	S	4°30'	W

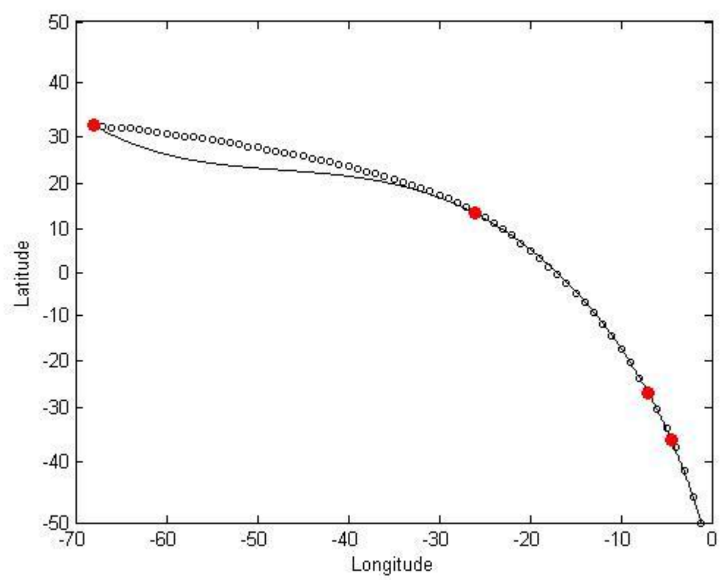

Figure 8: Points deviate from Halley's route – lack of fit.

5.4 Five Degrees East Variation

The line of 5 degrees east variation has more observations along it than does any other line of variation on the map. The shape of the line is not as curved as the agonic line, and it was found that a quadratic polynomial has the lowest order that offers the best fit.

The first point is an average of the locations at Antigua, St. Christopher's, the Road of Anguilla and a little west of Anguilla. The position of the averaged point is on the line of variation and yields an error in magnetic declination of 3 minutes. The second point is the average of two observations near the northeast coast of South America. The position of the point is on the line of variation and yields an error in magnetic declination of 4 minutes. The error in magnetic declination for both points is small. Perhaps Halley rounded the average of the magnetic declinations such that they were an even 5 degrees east variation. It was common for Halley to simplify and round his data as noted by Bellhouse (2011).

During the second voyage, Halley records that he can see the islands of Tristan da Cunha, and observes a magnetic variation of 5°48' east. Although he writes in his journal "about 3 degrees too much" in longitude, he knew his position relative to the islands. Since the line of 5 degrees east variation passes through the Islands, his recorded observation is used to help construct the line of magnetic declination. Therefore, the location of the third point is found at Tristan da Cunha, along Halley's route marked with a dashed line. Table 3 shows the three points for the line of 5 degrees east variation.

Table 3: Position of the three points for the line of 5 degrees east variation.

Point	Observations	Mean Position				Mean Magnetic Declination	
		Latitude		Longitude			
		D°M'	N/S	D°M'	E/W	D°M'	E/W
1	V1-27, V1-28, V1-29, V2-95	17°50'	N	62°48'	W	5°03'	E
2	V2-88, V2-89	0°24'	S	42°31'	W	5°04'	E
3	Tristan da Cunha	37°25'	S	14°45'	W	5°00'	E

Figure 9 shows the quadratic polynomial for the line of 5 degrees east variation and the agonic line. The averaged points are shaded, and the digitized line recorded from the map is marked with circles. The following is the quadratic polynomial.

$$P_2(x) = -0.00903596x^2 - 1.85058x - 62.7502$$

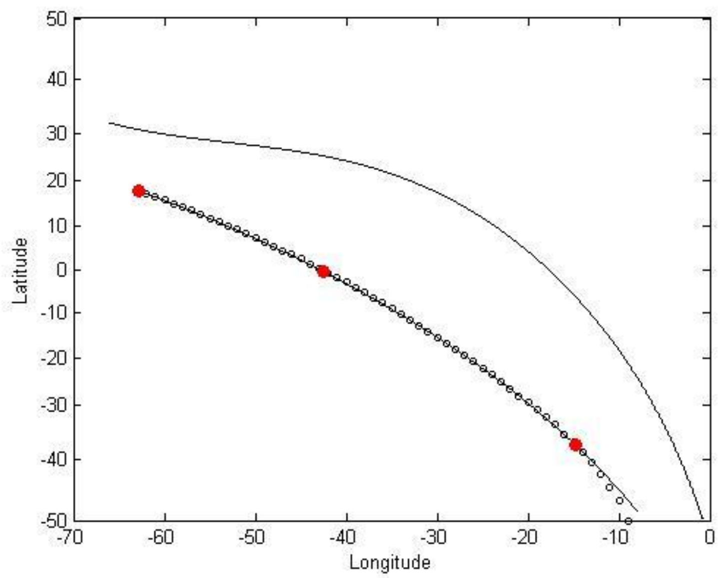

Figure 9: Quadratic polynomial for the line of 5 degrees east variation.

Extrapolation was used to extend the polynomial to 50 degrees south latitude where the lines of variation end in a gridline on the map. The graph shows how the line slightly deviates from the recorded data near the gridline. Perhaps Halley made a slight adjustment at the time he constructed the gridline at 50 degrees south latitude.

5.5 The Gridline at 50 Degrees South Latitude

Far south in the lower Atlantic Ocean, Halley crossed 50 degrees south latitude shown in Figure 10, a remote region where very few navigators had travelled. Despite having to navigate around icebergs and encountering severe weather conditions, Halley managed to make a few magnetic observations (British Library, Add. MS 30,368). Using these observations, along with spacing and his magnetic theory, Halley constructed a gridline along 50 degrees south latitude. Having the gridline would allow him to select points required in the calculation of the quadratic polynomials in the lower Atlantic Ocean.

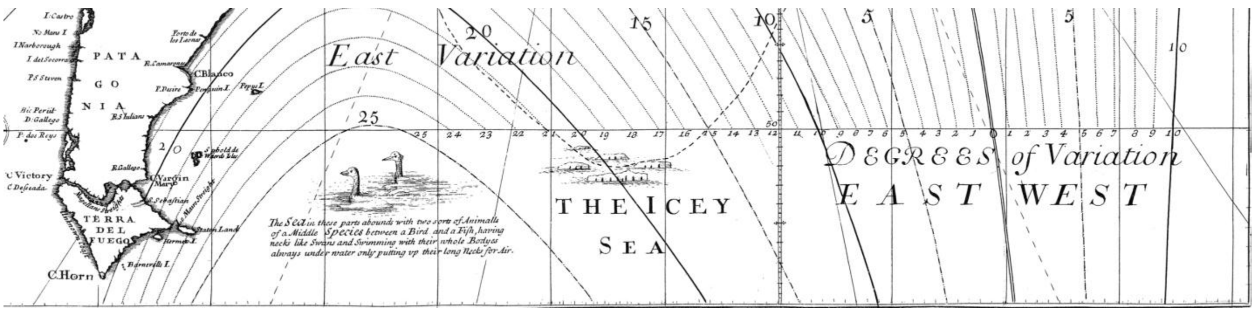

Figure 10: The gridline along 50 degrees south latitude.

Representing the value of the agonic line with a zero, the gridline includes 0 to 25 degrees east variation and 0 to 10 degrees west variation. The spacing between each line of variation from 0 to 10 degrees west variation is a mirror image of the spacing between each line of variation from 0 to 10 degrees east variation. In addition, the spacing is nearly linear as shown in the graphs below in Figures 11 and 12. From 10 and 15 degrees west variation, the spacing in between each line remains nearly linear but is slightly increased over the previous group of lines. The pattern continues for the spacing of the group of lines from 15 to 20 degrees west variation and again for the group of lines from 20 to 25, although between 24 and 25 there is a notable increase ending the pattern.

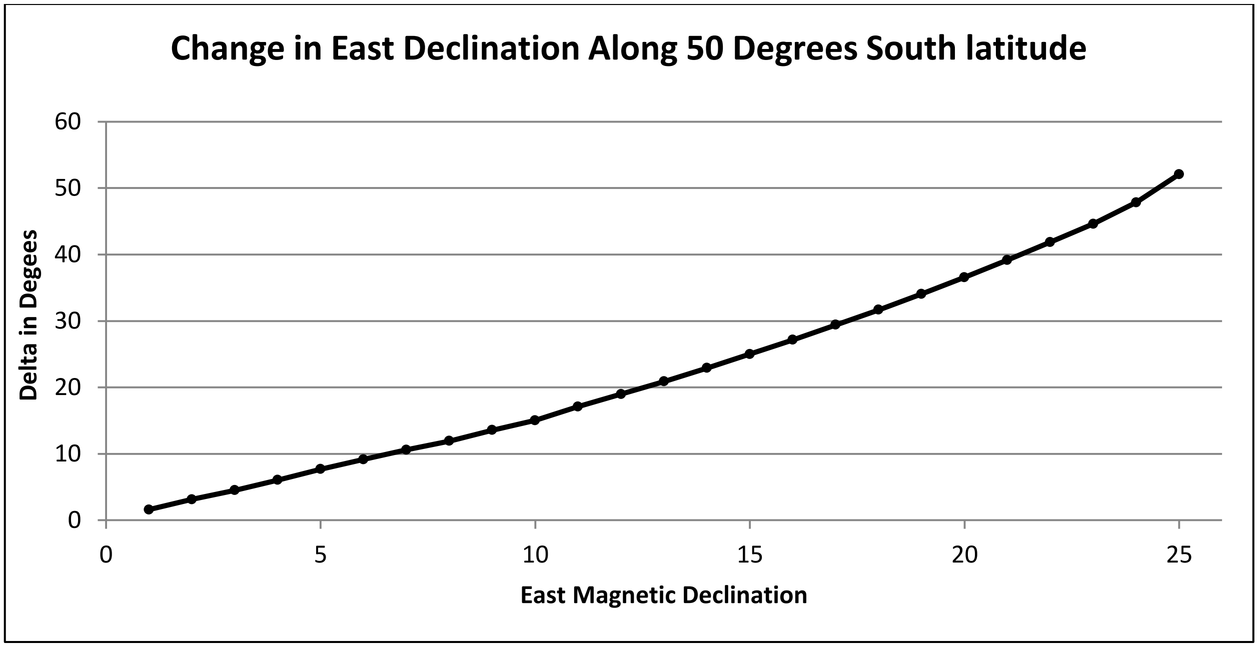

Figure 11: Change in east declination along 50 degrees south latitude.

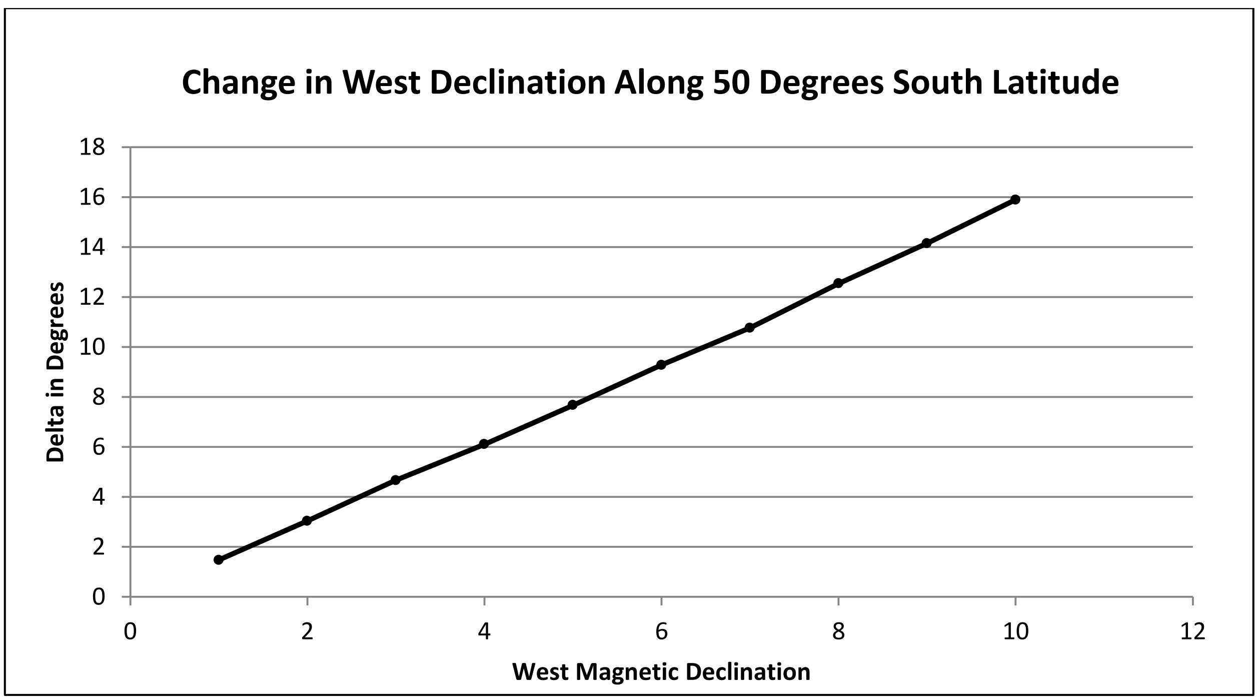

Figure 12: Change in west declination along 50 degrees south latitude.

Halley had a sufficient number of magnetic observations to construct the lines from 5 degrees east variation to 5 degrees west variation. As shown previously with the lines of the agonic and 5 degrees east variation, all of the lines from 5 degrees east to 5 degrees west can be extended to 50 degrees south latitude using extrapolation and smoothing. We conjecture that Halley then used his magnetic theory and observations to set up the spacing between each line of variation along the gridline.

To account for the secular variation, Halley hypothesized that the earth contained four magnetic poles: two fixed on the earth's crust and two moving internally within the core (1692). The North and South Poles each contained one fixed and one movable magnetic point of attraction. Since Halley was familiar with magnetism, he would have known that the lines of variation converged to the North and South Poles and that the variation was due to the interaction between the fixed and movable points of attraction at each pole. Due to the spherical shape of the earth, all of the lines of longitude converge to the poles, whereas on a two-dimensional plane such as a Mercator projection, they are vertical.

The map shows how the lines for every 5 degrees of variation from 10 west variation to 25 degrees east variation in the lower Atlantic Ocean have been extended down to 59 degrees south latitude where they tend to the vertical. Since all the lines of variation tend to vertical without crossing as the southern latitude increases, they become parallel to the lines of longitude. In addition, the lines of variation do not appear to have any disturbances, and from Halley's magnetic theory, they would converge to the South Pole. Halley, therefore, used his magnetic theory to set up the lines of variation along the gridline in a vertical pattern.

Halley used his magnetic theory to construct the gridline and then used the gridline and his observations to control the shape of the polynomials. Using three observations from the second voyage, 51, 52 and 53, Halley increased the spacing for each interval of 5 lines of variation moving west towards South America. Once the gridline was constructed, Halley would then be able to select points required in the calculation of the quadratic polynomials for each line of variation.

5.6 The Lines of East Variation

Covering thousands of kilometers of Atlantic Ocean, the set of lines from 1 to 4 degrees east variation are the longest on the map. It was found that cubic polynomials offer the best fit for these lines. Points used in the calculation for Newton's divided difference method are found along four large regions of the map: Bermuda to Anguilla, Cape Verde Islands to the coast of Brazil, west of St. Helena to the island of Trinidad, and Tristan da Cunha Islands. Starting with Bermuda to Anguilla, the first point for each line is found by using the averages from Halley's data. As stated previously, although the averaged positions and magnetic declinations agree with the map, not all of them result in a point exactly on the line of variation. Since the lines of the agonic and 5 degrees east were already constructed, it is assumed Halley used interpolation to find the first point required for the calculation. Likewise, the second and third points for each line are found along the second and third regions respectively employing the same procedure. For the purpose of reconstructing the map, a point is selected on the line of variation rather than using interpolation. For the fourth point, Halley indicates in his journal, magnetic variations at specific locations near the islands of Tristan da Cunha. For example, on 20 February 1700, Halley records the longitude from Tristan da Cunha as 6°12' and magnetic variation 2°30' east, and on 22 February 1700, Halley records the longitude from Tristan da Cunha as 12°25' and the magnetic variation ½ westerly. Holding his recorded latitude of 37°25' south constant from the agonic line to 5 degrees east, the observed positions and magnetic declinations agree with the map. The average of the two observations result in a position and magnetic declination that also agrees with the map, and therefore, is used as the fourth point in the calculation for the line of 1 degree east variation. The fourth point for the lines of 2 to 4 degrees east variation are found by holding the same latitude constant and selecting points at the intersections of the lines of variation and Halley's route.

As mentioned earlier, not all of the averages result in a position on the lines of variation, however, it is assumed that Halley would have used interpolation to locate the points needed for the calculation. Newton's divided difference method would then be used to find the interpolating polynomials. It was found that cubic polynomials offer a reasonable fit for the lines of 1 to 4 degrees east variation.

The lines of 6 to 25 degrees east variation originate at the coast of South America, starting at Brazil, move southeast, and end at Halley's gridline along 50 degrees south latitude. Halley may have used spacing as a guide when constructing the lines of variation in this area of the map. The averaged observations along the coast of South America have a reduced error in magnetic declination; it is consistently negative and at most 1 ¾ degrees. For example, the line of 6 degrees east variation goes through the island of Trinidad, however Halley observed the magnetic declination on the island as being 6 ½ degrees east variation. The line of 5 degrees east variation, the line with the most observations along it, would likely have been constructed prior to the line of 6 degrees east variation where fewer observations were made. Halley may have placed 6 degrees east variation at Trinidad rather than use the magnetic declination he observed to maintain the trend, shape and spacing between the lines of variation. This trend follows along the coast of South America except for off the coast of Rio de Janiero where Halley observes the magnetic declination as 11 ½ degrees east, which agrees with the map. Near the line of 16 degrees east variation, the magnetic declinations for observations 45, 46 and 47 of the second voyage change direction and are less than the predicted.

It was found that quadratic polynomials offer the best fit for all the lines from 6 to 25 degrees east variation. To find the quadratic polynomials, Newton's divided difference formula requires three points. For the lines of 6 to 18 degrees east, points were selected at three locations on the lines of variation: the first point on Halley's route, the second point a few degrees east of Halley's route, and the third point at the gridline along 50 degrees south latitude. Linear interpolation may have been performed by Halley to find the second points on the lines of variations a few degrees east of his route. The lines of 19 to 25 degrees east variation have a parabolic shape, and it is assumed that Halley knew the shape of these lines from the data of others given in his paper in 1683. For the purpose of reconstructing the map, two points were chosen along the gridline and one near the vertex of each line of variation. The appendix contains the table of points in Tables 8 and 9, the resulting polynomials in Table 10, and Mercator projection graphs shown in Figures 13 to 36 that are used in the reconstruction of the map. The graphs compare the interpolating polynomial with the recorded points from the map indicated with circles.

5.7 The Lines of West Variation

The line of one degree west variation is similar in length to the lines of the agonic to 5 degrees east variation. An attempt was made to reconstruct the line of one degree west variation in a similar manner, using the same four regions of the map as the lines of the agonic to 5 degrees east variation. However, it was found that to achieve a reasonable fit, a fifth point was required. In other words the line of one degree west variation requires a quartic polynomial. Four of the five points selected are on or near Halley's route and a fifth point is on the line of variation in between the Cape Verde Islands and Bermuda at 45 degrees west longitude.

Quadratic polynomials offer the best fit for all of the remaining lines of west variation located in the upper and lower Atlantic Ocean. Newton's divided difference method requires three points for each line. For the lines of 2 to 15 degrees west variation in the upper Atlantic Ocean, Halley only crosses each line twice. His route covered three large regions on the map: London to south of the Canary Islands, north of Bermuda to Newfoundland, and east of Newfoundland across 50 degrees north latitude back to London. Therefore, a third point is selected on the line of variation a few degrees west of his route along the Europe and African coastlines and as the lines of magnetic declination increase and become shorter in length, a third point is selected a few degrees east of his route along the North American coastline. Halley may have used linear interpolation to find the third points required for the calculation. Halley only collected data up to the line of 15 degrees west variation in the upper Atlantic Ocean, however, for the lines of 16 to 25 degrees west variation, he would have been able to continue in a similar manner, maintaining the shape and trend of the previous lines already constructed.

For the lines of 2 to 5 degrees west variation in the lower Atlantic Ocean, three points are obtained from three regions: near St. Helena, Tristan da Cunha, and the gridline. Extrapolating from these locations to find points help maintain the similar shape and trend for the remaining lines of 6 to 10 degrees west variation. The appendix contains the table of points in Tables 11, 12 and 14, the resulting polynomials in Tables 13 and 15, and Mercator projection graphs shown in Figures 37 to 61 (upper Atlantic Ocean) and Figures 62 to 70 (lower Atlantic Ocean) used in the reconstruction of the map. The graphs compare the interpolating polynomial with the recorded points from the map indicated with circles. The complete set of lines is shown in Figure 71.

6 The Impact of Halley's Map

Halley's 1701 map of magnetic declination was the first map printed and published with isolines, lines representing equal phenomena. According to Thrower, (1981, p.58), this makes Halley's Atlantic map one of the most important maps in the history of cartography. The nautical map helped navigators estimate their routes around the Atlantic Ocean for decades. The map proved to be so useful that Halley extended the Atlantic map to a world map published in 1702. Because of the secular variation, the change of magnetic declination over time, revisions to the Atlantic map were required after Halley's death. In 1745 and 1758, Mountaine and Dodson (1753-1754), Fellows of the Royal Society, undertook the arduous task of revising the map. The invention of the marine chronometer by John Harrison, completed in 1773, solved the longitude problem. To this day, Halley's Atlantic map is still used as a reference datum to study the change in magnetic declination.

7 Conclusions

Starting with the agonic line, each line of magnetic declination was reconstructed by choosing a set of n points used to derive the $n-1$ degree polynomial. Using Halley's original data, points were found by calculating the average latitude and longitude, and the corresponding magnetic declination. The resulting polynomial was then plotted and overlaid with the digitized data to evaluate the fit of each polynomial. Analysis reveals that Newton's divided difference can be sensitive to the points selected and small deviations can change the fit of the polynomial. The fit of the polynomial was dependent on the spacing of the points used in the calculation. It was observed that reasonable fitting polynomials were achieved when the selected points were on or near Halley's route. The fit of the polynomials were reasonable in the sense that the shape and position of the points closely match the lines of variation on the map.

It was found that a third degree polynomial has the lowest order that offers a reasonable fit for the agonic line and the lines of 1 to 4 degrees east variation. A fourth degree polynomial was required to fit the line of 1 degree west variation, and all other remaining lines were fit to quadratic polynomials. The line of 5 degrees east variation has the highest number of observations along it than any other line of magnetic declination on the map. The agonic line has the second highest number of observations along it. As well, the agonic line and the line of 5 degrees east variation have the highest number of observations on or near land, which, as previously stated, offers the smallest error in magnetic declination. This would have allowed Halley to easily construct his map using averaged observations and Newton's divided difference method.

Halley may have used spacing as a guide when constructing the lines of variation in the lower Atlantic Ocean. While the averaged observations along the coast of South America have a reduced error in magnetic declination, it is at most 1 ¾ degrees and consistently negative. In addition, Halley constructed a gridline along 50 degrees south latitude where the lines of magnetic declination end. It is possible Halley used spacing in this area of the map because once the lines from 5 degrees east variation to 5 degrees west variation were constructed, he would have been able to continue in a similar pattern to preserve the shape and trend of the lines moving upwards to the northwest and down to the southeast and southwest where little or no data existed. The lines of variation along the gridline are lined up such that they do not cross over one another and tend to run vertically to the South Pole, supporting Halley's hypothesis on the Earth's magnetism.

It has been demonstrated that Halley's 1701 map can be constructed using arithmetical means and Newton's divided difference method. The arithmetic mean and Newton's divided difference would have been well known to Halley at the time he constructed the map. The use of arithmetical means has been shown to reduce the error in magnetic declination, and Newton's divided difference method has shown that polynomials of at most fourth degree, and typically second and third degree, are required to construct the lines of variation. These calculations could have reasonably been performed by hand in 1701. The sensitivity of Newton's divided difference method shows that points on Halley's route offer reasonable polynomial fits, further supporting the reconstruction method. The map is an early and good example of statistical graphics, illustrating how a lot could be achieved with a relatively small amount of data.

8 References

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A Appendix

Table 4: Mean observations for the upper Atlantic Ocean.

Upper Atlantic Observations	Mean Position				Mean Magnetic Declination		Error	
	Latitude		Longitude					
	D°M'	N/S	D°M'	W	D°M'	E/W	D°M'	Degree
2-1, 2-124, 2-125	50°38'	N	0°55'	W	7°06'	W	0°04'	0.067
2-2, 2-3	45°48'	N	11°03'	W	6°16'	W	0°14'	0.233
1-5	39°39'	N	12°11'	W	5°00'	W	0°00'	0.000
1-4, 1-5, 1-6	39°38'	N	11°22'	W	4°57'	W	0°03'	0.050
1-5, 1-6, 1-7	36°07'	N	13°38'	W	4°27'	W	0°12'	-0.200
1-6	36°06'	N	11°52'	W	4°20'	W	0°00'	0.000
1-7, 2-8	31°09'	N	17°44'	W	2°59'	W	0°06'	0.100
1-8, 1-9	18°59'	N	22°41'	W	1°15'	W	0°15'	0.250
1-8, 2-8	25°27'	N	20°30'	W	1°59'	W	0°00'	0.000
1-9, 1-10, 2-9, 2-10	15°49'	N	23°09'	W	0°29'	W	0°01'	0.017
1-11, 1-12, 1-13, 1-14	4°19'	N	20°12'	W	0°00'		0°10'	0.167
1-13, 1-14	2°03'	N	21°52'	W	0°30'	E	0°10'	0.167
1-14, 1-15, 1-16, 1-17	0°12'	S	26°11'	W	1°32'	E	0°08'	0.133
1-17, 1-18	2°53'	S	30°23'	W	2°45'	E	0°10'	0.167
1-18, 2-86	3°48'	S	34°32'	W	3°45'	E	0°05'	0.083
2-84, 2-85, 2-86	6°04'	S	35°30'	W	4°23'	E	0°07'	0.117
2-88, 2-89	0°24'	S	42°31'	W	5°04'	E	0°04'	0.067
2-87, 2-88, 2-89	0°52'	S	41°37'	W	4°44'	E	0°06'	0.100
1-22, 1-23, 2-90, 2-91	8°22'	N	50°41'	W	4°58'	E	0°02'	-0.033
1-27, 1-28, 1-29, 2-95	17°50'	N	62°48'	W	5°03'	E	0°04'	-0.067
1-30, 1-31, 1-32, 2-96, 2-97	23°18'	N	64°10'	W	3°28'	E	0°03'	-0.050
2-97, 2-98, 2-99	25°34'	N	64°42'	W	2°24'	E	0°11'	0.183
2-98, 2-100	26°57'	N	64°58'	W	1°49'	E	0°15'	0.250
2-100, 2-101, 2-102	29°11'	N	65°17'	W	0°58'	E	0°15'	0.250
2-101,2-102	29°38'	N	65°19'	W	0°49'	E	0°11'	0.183
2-103, 2-104	31°33'	N	64°59'	W	30 sec		0°00'	0.000
1-34, 2-103, 2-104	31°26'	N	64°36'	W	0°01'	E	0°00'	0.000
2-103, 2-104, 2-105	32°02'	N	64°47'	W	0°24'	W	0°09'	-0.150
2-106, 2-107, 2-108	38°36'	N	66°39'	W	5°50'	W	0°15'	-0.250
2-109, 2-110, 2-111, 2-112	41°26'	N	63°17'	W	8°28'	W	0°08'	-0.133
2-113, 2-114, 2-115	43°46'	N	57°20'	W	10°27'	W	0°20'	0.333
2-115, 2-116	45°09'	N	55°15'	W	12°23'	W	0°07'	0.117
2-116, 2-117, 2-118	46°52'	N	52°36'	W	14°23'	W	0°07'	0.117
2-117, 2-118	47°20'	N	51°41'	W	14°50'	W	0°00'	0.000
1-38, 1-39	40°23'	N	49°14'	W	6°45'	W	0°10'	-0.167
1-42	47°16'	N	28°07'	W	8°30'	W	0°05'	0.083

Table 5: Mean observations for the lower Atlantic Ocean.

Lower Atlantic Observations (All obs from 2nd voyage)	Mean Position				Mean Magnetic Declination		Error	
	Latitude		Longitude					
	D°M'	N/S	D°M'	W	D°M'	E/W	D°M'	Degree
77, 78, 79	18°30'	S	30°07'	W	6°11'	E	0°20'	-0.333
23, 24, 25	16°57'	S	31°19'	W	7°00'	E	1°20'	-1.333
24, 25, 26	18°14'	S	31°49'	W	7°30'	E	1°20'	-1.333
27, 28, 29	21°19'	S	34°38'	W	9°46'	E	1°46'	-1.767
28, 29, 30	22°03'	S	35°57'	W	10°21'	E	1°41'	-1.683
30, 31, 32	22°42'	S	39°03'	W	10°57'	E	1°15'	-1.250
33, 34	22°53'	S	41°43'	W	11°19'	E	0°38'	-0.633
35, 36, 37	24°45'	S	42°53'	W	12°18'	E	0°48'	-0.800
37, 38, 39	26°17'	S	43°05'	W	12°58'	E	0°48'	-0.800
40, 41, 42	28°50'	S	43°58'	W	13°49'	E	0°15'	-0.250
41, 42	29°13'	S	44°03'	W	14°03'	E	0°25'	-0.417
42, 43, 44	30°59'	S	44°54'	W	14°38'	E	0°10'	-0.167
43, 44, 45	32°30'	S	45°21'	W	15°02'	E	0°02'	-0.033
45, 46	36°00'	S	47°11'	W	16°02'	E	1°00'	1.000
46, 47	38°18'	S	47°48'	W	17°09'	E	0°40'	0.667
47, 48	40°26'	S	47°19'	W	18°23'	E	0°15'	0.250
48	42°06'	S	46°49'	W	19°16'	E	0°00'	0.000
48, 49, 50	43°01'	S	46°02'	W	20°14'	E	0°50'	-0.833
49, 50, 51	45°22'	S	43°40'	W	21°08'	E	1°23'	-1.383
51, 52	50°35'	S	35°26'	W	20°30'	E	1°05'	-1.083
55, 56	37°20'	S	10°47'	W	4°54'	E	1°40'	-1.667
57, 58	35°58'	S	3°50'	W	1°00'	E	1°15'	-1.250
58, 59	31°10'	S	0°26'	W	2°00'	W	0°25'	0.417
59, 60	24°53'	S	1°51'	E	3°50'	W	0°00'	0.000
59, 60, 61	23°20'	S	1°32'	E	3°55'	W	0°05'	-0.083
62	17°33'	S	0°10'	E	3°30'	W	0°00'	0.000
62, 63	16°46'	S	0°53'	W	3°10'	W	0°00'	0.000
62, 63, 64	16°29'	S	1°42'	W	2°57'	W	0°05'	-0.083
63, 64, 65	15°56'	S	3°09'	W	2°27'	W	0°05'	-0.083
66	16°27'	S	7°02'	W	1°00'	W	0°00'	0.000
66, 67, 68	17°22'	S	10°19'	W	0°00'		0°00'	0.000
68	18°08'	S	13°03'	W	1°00'	E	0°05'	-0.083
68, 69	18°27'	S	14°11'	W	1°30'	E	0°05'	-0.083
69, 70, 71	19°20'	S	17°20'	W	2°40'	E	0°20'	-0.333
70, 71, 72	19°53'	S	19°49'	W	3°28'	E	0°20'	-0.333
71, 72, 73	20°18'	S	22°19'	W	4°08'	E	0°05'	-0.083
72, 73	20°24'	S	23°26'	W	4°27'	E	0°10'	-0.167
75	20°25'	S	26°31'	W	5°06'	E	0°00'	0.000

Table 6: Data for the First Voyage.

Data - First Voyage								
Ref. No	Place	Date	Latitude		Longitude from London		Magnetic Declination	
			D°M'	N/S	D°M'	E/W	D°M'	E/W
1-1	River Thames	Oct 27 (1698)	51°30'	N	0°45'	E	7°00'	W
1-2	Portland Road	Nov 1	50°30'	N	2°15'	W	6°30'	W
1-3	Portsmouth Harbor	Nov 10	50°50'	N	1°00'	W	7°00'	W
1-4	At sea	Dec 5	43°10'	N	10°02'	W	5°30'	W
1-5	" "	Dec 8	39°39'	N	12°11'	W	5°00'	W
1-6	" "	Dec 10	36°06'	N	11°52'	W	4°20'	W
1-7	Town of Funchal	Dec 20	32°37'	N	16°50'	W	4°00'	W
1-8	At sea	Dec 28	21°12'	N	22°31'	W	2°00'	W
1-9	Island of Sal	Jan 1 (1699)	16°45'	N	22°50'	W	0°30'	W
1-10	Bay of Praya	Jan 6	14°53'	N	23°25'	W	0°32'	W
1-11	At sea	Jan 9	8°28'	N	19°39'	W	1°00'	W
1-12	" "	Jan 14	4°40'	N	17°25'	W	0°00'	
1-13	" "	Jan 23	2°57'	N	19°35'	W	0°00'	
1-14	" "	Feb 1	1°09'	N	24°09'	W	1°00'	E
1-15	" "	Feb 3	0°26'	N	25°14'	W	1°07'	E
1-16	" "	Feb 8	0°26'	S	26°55'	W	1°30'	E
1-17	" "	Feb 12	1°55'	S	28°25'	W	2°30'	E
1-18	Fernando do Noronha	Feb 19	3°50'	S	32°20'	W	3°00'	E
1-19	Coast of Brazil	Mar 5	7°00'	S	34°45'	W	2°44'	E
1-20	At sea	Mar 18	1°51'	S	35°31'	W	3°00'	E
1-21	" "	Mar 25	5°37'	N	46°05'	W	4°00'	E
1-22	" "	Mar 27	8°10'	N	49°36'	W	5°00'	E
1-23	" "	Mar 29	9°25'	N	51°24'	W	5°15'	E
1-24	" "	Mar 30	11°32'	N	54°29'	W	5°30'	E
1-25	" "	Apr 1	13°14'	N	58°43'	W	4°45'	E
1-26	Bridgetown, Barbados	Apr 2	13°04'	N	59°32'	W	5°00'	E
1-27	Antigua	Apr 23	17°04'	N	61°55'	W	5°00'	E
1-28	St Christopher's	Apr 30	17°18'	N	62°42'	W	5°30'	E
1-29	Road of Anguilla	May 7	18°10'	N	63°10'	W	5°15'	E
1-30	At sea	May 11	22°07'	N	63°46'	W	4°15'	E
1-31	" "	May 12	23°36'	N	64°06'	W	3°15'	E
1-32	" "	May 13	24°20'	N	64°16'	W	2°30'	E
1-33	" "	May 16	27°52'	N	64°04'	W	1°00'	E
1-34	" "	May 18/19	31°13'	N	63°51'	W	0°00'	
1-35	" "	May 23/24	34°44'	N	61°16'	W	3°00'	W
1-36	" "	May 24/25	35°16'	N	60°59'	W	3°30'	W
1-37	" "	May 26	36°36'	N	59°25'	W	4°30'	W
1-38	" "	May 31	39°50'	N	51°13'	W	6°30'	W
1-39	" "	June 2	40°56'	N	47°15'	W	7°00'	W
1-40	" "	June 4	42°04'	N	44°56'	W	9°20'	W
1-41	" "	June 6	43°11'	N	42°22'	W	10°20'	W
1-42	" "	June 11	47°16'	N	28°07'	W	8°30'	W
1-43	" "	June 13/14	48°54'	N	19°02'	W	8°30'	W
1-44	" "	June 18	49°36'	N	7°32'	W	6°25'	W
1-45	" "	June 20	49°51'	N	4°14'	W	5°40'	W

Table 7: Data for the Second Voyage.

Data - Second Voyage								
Ref. No	Place	Date	Latitude		Longitude from London		Magnetic Declination	
			D°M'	N/S	D°M'	E/W	D°M'	E/W
2-1	The Downs	Sept 27 (1699)	51°15'	N	1°35'	E	7°32'	W
2-2	At sea	Oct 1	46°20'	N	10°35'	W	6°07'	W
2-3	" "	Oct 2	45°16'	N	11°30'	W	6°24'	W
2-4	" "	Oct 6	41°00'	N	16°02'	W	4°42'	W
2-5	" "	Oct 7	39°34'	N	16°44'	W	3°28'	W
2-6	" "	Oct 8	38°25'	N	16°40'	W	3°45'	W
2-7	" "	Oct 14	30°28'	N	18°37'	W	2°00'	W
2-8	" "	Oct 15	29°41'	N	18°38'	W	1°58'	W
2-9	Island of Sal	Oct 23/24	16°44'	N	22°55'	W	0°55'	W
2-10	Bay of Praya	Oct 26	14°54'	N	23°25'	W	0°00'	
2-11	At sea	Nov 11	2°42'	N	21°44'	W	1°20'	E
2-12	" "	Nov 12	2°17'	N	22°30'	W	1°45'	E
2-13	" "	Nov 16/17	0°09'	S	25°16'	W	2°00'	E
2-14	" "	Nov 17	0°43'	S	25°47'	W	2°00'	E
2-15	" "	Nov 18/19	1°50'	S	26°49'	W	2°30'	E
2-16	" "	Nov 21/22	4°24'	S	28°59'	W	3°00'	E
2-17	" "	Nov 22	5°15'	S	29°18'	W	3°20'	E
2-18	" "	Nov 24	7°22'	S	30°05'	W	4°12'	E
2-19	" "	Nov 25	8°18'	S	30°24'	W	4°26'	E
2-20	" "	Nov 26	9°12'	S	30°40'	W	5°00'	E
2-21	" "	Nov 27	10°55'	S	30°55'	W	5°35'	E
2-22	" "	Nov 28	13°17'	S	30°52'	W	5°30'	E
2-23	" "	Nov 29/30	15°29'	S	30°59'	W	6°30'	E
2-24	" "	Nov 30/Dec 1	17°08'	S	31°20'	W	7°10'	E
2-25	" "	Dec 1	18°13'	S	31°37'	W	7°20'	E
2-26	" "	Dec 2	19°20'	S	32°30'	W	8°00'	E
2-27	" "	Dec 3/4	20°30'	S	33°39'	W	9°30'	E
2-28	" "	Dec 4/5	21°24'	S	34°39'	W	10°03'	E
2-29	" "	Dec 5	22°03'	S	35°36'	W	9°45'	E
2-30	" "	Dec 6	22°43'	S	37°35'	W	11°15'	E
2-31	" "	Dec 7	22°42'	S	39°00'	W	10°30'	E
2-32	" "	Dec 9	22°41'	S	40°33'	W	11°07'	E
2-33	Off Rio de janeiro	Dec 13/14	23°05'	S	42°53'	W	11°30'	E
2-34	" " " "	Dec 29	23°00'	S	42°45'	W	11°46'	E
2-35	At sea	Jan 1 (1700)	24°12'	S	43°13'	W	12°04'	E
2-36	" "	Jan 2	24°38'	S	42°41'	W	12°10'	E
2-37	" "	Jan 3	25°24'	S	42°46'	W	12°40'	E
2-38	" "	Jan 4/5	26°30'	S	43°08'	W	13°00'	E
2-39	" "	Jan 5	26°57'	S	43°22'	W	13°15'	E
2-40	" "	Jan 7	28°05'	S	43°48'	W	13°23'	E
2-41	" "	Jan 8/9	28°52'	S	43°51'	W	14°00'	E
2-42	" "	Jan 9	29°34'	S	44°14'	W	14°05'	E
2-43	" "	Jan 10/11	31°08'	S	44°57'	W	15°00'	E
2-44	" "	Jan 11	32°14'	S	45°32'	W	14°49'	E
2-45	" "	Jan 12	34°09'	S	46°35'	W	15°16'	E
2-46	" "	Jan 15	37°50'	S	47°46'	W	16°47'	E
2-47	" "	Jan 16/17	38°46'	S	47°49'	W	17°30'	E
2-48	" "	Jan 18/19	42°06'	S	46°49'	W	19°16'	E
2-49	" "	Jan 19/20	42°52'	S	45°53'	W	20°15'	E
2-50	" "	Jan 21	44°04'	S	45°23'	W	21°10'	E
2-51	" "	Jan 25/26	49°10'	S	39°43'	W	22°00'	E

2-52	" "	Jan 30/31	51°59'	S	31°08'	W	19°00'	E
2-53	" "	Feb 9	44°57'	S	21°32'	W	12°53'	E
2-54	" "	Feb 14	40°35'	S	15°21'	W	9°00'	E
2-55	" "	Feb 16	38°03'	S	12°52'	W	5°48'	E
2-56	" "	Feb 18	36°36'	S	8°41'	W	4°00'	E
2-57	" "	Feb 19/20	36°07'	S	6°32'	W	2°30'	E
2-58	" "	Feb 23	35°49'	S	1°08'	W	0°30'	W
2-59	" "	Mar 1	26°30'	S	2°00'	E	3°30'	W
2-60	" "	Mar 3	23°15'	S	1°41'	E	4°10'	W
2-61	" "	Mar 5	19°55'	S	0°55'	E	4°06'	W
2-62	" "	Mar 6/7	17°33'	S	0°10'	E	3°30'	W
2-63	" "	Mar 8/9	15°58'	S	1°55'	W	2°50'	W
2-64	" "	Mar 9/10	15°55'	S	3°20'	W	2°30'	W
2-65	" "	Mar 10	15°54'	S	4°12'	W	2°00'	W
2-66	" "	Mar 31	16°27'	S	7°02'	W	1°00'	W
2-67	" "	Apr 2	17°31'	S	10°53'	W	0°00'	
2-68	" "	Apr 3	18°08'	S	13°03'	W	1°00'	E
2-69	" "	Apr 4	18°45'	S	15°19'	W	2°00'	E
2-70	" "	Apr 5	19°07'	S	16°34'	W	2°30'	E
2-71	" "	Apr 7	20°07'	S	20°06'	W	3°30'	E
2-72	" "	Apr 9	20°24'	S	22°46'	W	4°24'	E
2-73	" "	Apr 10	20°23'	S	24°05'	W	4°30'	E
2-74	" "	Apr 10/11	20°24'	S	25°19'	W	5°00'	E
2-75	" "	Apr 11	20°25'	S	26°31'	W	5°06'	E
2-76	" "	Apr 13	20°22'	S	27°50'	W	6°25'	E
2-77	Trinidad	Apr 15-19	20°30'	S	29°25'	W	6°30'	E
2-78	At sea	Apr 21	18°23'	S	30°16'	W	6°27'	E
2-79	" "	Apr 22	16°37'	S	30°41'	W	5°36'	E
2-80	" "	Apr 23	15°42'	S	30°56'	W	5°51'	E
2-81	" "	Apr 24	13°28'	S	31°31'	W	5°04'	E
2-82	" "	Apr 25	12°36'	S	31°42'	W	5°31'	E
2-83	" "	Apr 26	9°54'	S	32°21'	W	5°15'	E
2-84	Pernambuco	May 1	8°03'	S	34°50'	W	4°38'	E
2-85	At sea	May 5	6°25'	S	34°56'	W	4°00'	E
2-86	" "	May 6/7	3°45'	S	36°44'	W	4°30'	E
2-87	" "	May 8	1°49'	S	39°50'	W	4°05'	E
2-88	" "	May 9	0°48'	S	41°49'	W	5°00'	E
2-89	" "	May 10	0°00'		43°13'	W	5°08'	E
2-90	" "	May 15	7°29'	N	50°22'	W	4°53'	E
2-91	" "	May 16	8°25'	N	51°23'	W	4°45'	E
2-92	" "	May 17	9°49'	N	53°16'	W	4°48'	E
2-93	Bridgetown, Barbados	May 22	13°04'	N	59°32'	W	5°25'	E
2-94	" "	May 23	13°04'	N	59°32'	W	5°21'	E
2-95	At sea	June 10	18°48'	N	63°26'	W	4°27'	E
2-96	" "	June 12	22°13'	N	64°11'	W	4°05'	E
2-97	" "	June 13	24°16'	N	64°30'	W	3°17'	E
2-98	" "	June 14	25°35'	N	64°42'	W	2°20'	E
2-99	" "	June 15	26°50'	N	64°55'	W	1°35'	E
2-100	" "	June 16	28°19'	N	65°13'	W	1°17'	E
2-101	" "	June 17	29°10'	N	65°26'	W	1°04'	E
2-102	" "	June 18	30°05'	N	65°12'	W	0°33'	E
2-103	" "	June 19	31°03'	N	65°14'	W	0°12'	E
2-104	" "	June 20	32°03'	N	64°43'	W	0°09'	W
2-105	" "	July 12	32°59'	N	64°23'	W	1°15'	W
2-106	" "	July 17	38°30'	N	67°18'	W	6°00'	W
2-107	" "	July 18	38°24'	N	66°19'	W	6°00'	W
2-108	" "	July 19	38°54'	N	66°19'	W	5°30'	W

2-109	" "	July 21	41°07'	N	64°22'	W	7°45'	W
2-110	" "	July 22	41°27'	N	63°25'	W	8°26'	W
2-111	" "	July 23	41°28'	N	63°01'	W	8°50'	W
2-112	" "	July 24	41°40'	N	62°20'	W	8°52'	W
2-113	" "	July 26	43°07'	N	59°18'	W	9°30'	W
2-114	" "	July 28	43°48'	N	56°37'	W	10°36'	W
2-115	" "	July 29	44°22'	N	56°05'	W	11°15'	W
2-116	" "	July 30	45°56'	N	54°25'	W	13°30'	W
2-117	Toad's Cove	Aug 5	47°13'	N	52°45'	W	15°00'	W
2-118	At sea	Aug 7/8	47°26'	N	50°37'	W	14°40'	W
2-119	" "	Aug 17	50°18'	N	26°31'	W	9°10'	W
2-120	" "	Aug 19	50°00'	N	21°42'	W	8°15'	W
2-121	" "	Aug 21	49°21'	N	16°14'	W	7°32'	W
2-122	" "	Aug 22	49°12'	N	15°51'	W	6°17'	W
2-123	" "	Aug 26	49°52'	N	5°40'	W	7°13'	W
2-124	Off the Eddystone	Aug 27	50°00'	N	4°10'	W	6°33'	W
2-125	Off Beachy	Aug 31	50°40'	N	0°10'	W	7°14'	W

Table 8: Points for the lines of 1 to 4 degrees east variation.

Line of Variation	Point 1		Point 2		Point 3		Point 4	
	Latitude	Longitude	Latitude	Longitude	Latitude	Longitude	Latitude	Longitude
	D°M’	D°M’	D°M’	D°M’	D°M’	D°M’	D°M’	D°M’
1 degrees east	29°30’ N	65°00’ W	9°75’ N	30°00’ W	18°45’ S	13°00’ W	37°45’ S	6°00’ W
2 degrees east	26°57’ N	64°58’ W	6°35’ N	33°30’ W	18°56’ S	15°57’ W	37°25’S	8°15’ W
3 degrees east	24°30’ N	64°30’ W	3°45’ N	36°30’ W	20°00’ S	19°30’ W	37°25’ S	10°30’ W
4 degrees east	20°45’ N	62°30’ W	1°00’ N	39°15’ W	20°18’ S	22°19’ W	37°25’ S	12°30’ W

Table 9: Points for the lines of 6 to 25 degrees east variation.

All of the following positions of latitudes are south and longitudes are west of London.

Line of Variation	Point 1		Point 2		Point 3	
	Latitude	Longitude	Latitude	Longitude	Latitude	Longitude
	D°M’	D°M’	D°M’	D°M’	D°M’	D°M’
6 degrees east	15°00’	34°00’	20°15’	29°35’	50°00’	10°15’
7 degrees east	17°30’	35°15’	20°15’	32°35’	50°00’	11°35’
8 degrees east	19°30’	36°15’	20°15’	35°30’	50°00’	13°00’
9 degrees east	21°15’	38°00’	23°30’	35°45’	50°00’	14°38’
10 degrees east	22°20’	40°00’	23°30’	38°45’	50°00’	16°20’
11 degrees east	23°00’	42°55’	30°00’	35°00’	50°00’	18°00’
12 degrees east	25°10’	44°15’	26°45’	42°00’	50°00’	20°15’
13 degrees east	27°30’	45°10’	30°00’	41°20’	50°00’	21°50’
14 degrees east	29°40’	46°00’	32°00’	42°30’	50°00’	23°50’
15 degrees east	31°45’	46°30’	34°00’	43°15’	50°00’	26°00’
16 degrees east	34°00’	47°20’	37°00’	42°45’	50°00’	28°15’
17 degrees east	36°15’	48°00’	38°30’	44°20’	50°00’	30°30’
18 degrees east	49°00’	38°00’	45°20’	40°00’	50°00’	32°45’
19 degrees east	50°00’	74°30’	38°75’	52°00’	50°00’	35°15’
20 degrees east	50°00’	72°30’	40°21’	55°00’	50°00’	37°30’
21 degrees east	50°00’	70°30’	42°45’	54°00’	50°00’	40°15’
22 degrees east	50°00’	68°00’	44°48’	54°00’	50°00’	43°00’
23 degrees east	50°00’	65°15’	46°27’	55°00’	50°00’	45°35’
24 degrees east	50°00’	62°15’	46°27’	55°00’	50°00’	48°45’
25 degrees east	50°00’	57°52’	49°45’	55°00’	50°00’	53°00’

Table 10: Polynomials for the lines of 1 to 25 degrees east variation.

Line of Variation	Polynomials
1 degrees east	$P_3(x) = -0.000370409x^3 - 0.0613923x^2 - 3.77591x - 58.2754$
2 degrees east	$P_3(x) = -0.000293905x^3 - 0.0506761x^2 - 3.42723x - 62.4106$
3 degrees east	$P_3(x) = -0.000113591x^3 - 0.0282652x^2 - 2.70454x - 62.8329$
4 degrees east	$P_3(x) = -0.000159396x^3 - 0.029947x^2 - 2.63735x - 66.019$
5 degrees east	$P_2(x) = -0.00903596x^2 - 1.85058x - 62.7502$
6 degrees east	$P_2(x) = -0.0147906x^2 - 2.12817x - 70.2598$
7 degrees east	$P_2(x) = -0.0164972x^2 - 2.14897x - 72.7523$
8 degrees east	$P_2(x) = -0.013859x^2 - 1.99438x - 73.5848$
9 degrees east	$P_2(x) = -0.0109001x^2 - 1.80388x - 74.0578$
10 degrees east	$P_2(x) = -0.0103921x^2 - 1.75438x - 75.8777$
11 degrees east	$P_2(x) = -0.0117429x^2 - 1.79884x - 78.5745$
12 degrees east	$P_2(x) = -0.0150958x^2 - 2.00868x - 84.4855$
13 degrees east	$P_2(x) = -0.0159837x^2 - 2.03517x - 86.8108$
14 degrees east	$P_2(x) = -0.0133307x^2 - 1.84834x - 86.4758$
15 degrees east	$P_2(x) = -0.0114746x^2 - 1.72215x - 87.0191$
16 degrees east	$P_2(x) = -0.0126588x^2 - 1.79533x - 90.6155$
17 degrees east	$P_2(x) = -0.0124827x^2 - 1.7656x - 92.2389$
18 degrees east	$P_2(x) = -0.0153818x^2 - 1.99592x - 98.8685$
19 degrees east	$P_2(x) = -0.0298507x^2 - 3.27612x - 128.392$
20 degrees east	$P_2(x) = -0.0314286x^2 - 3.45714x - 135.446$
21 degrees east	$P_2(x) = -0.0319559x^2 - 3.53912x - 140.679$
22 degrees east	$P_2(x) = -0.0337662x^2 - 3.74805x - 148.732$
23 degrees east	$P_2(x) = -0.0367666x^2 - 4.07484x - 159.347$
24 degrees east	$P_2(x) = -0.0413793x^2 - 4.5931x - 175.573$
25 degrees east	$P_2(x) = -0.0434783x^2 - 4.82065x - 183.364$

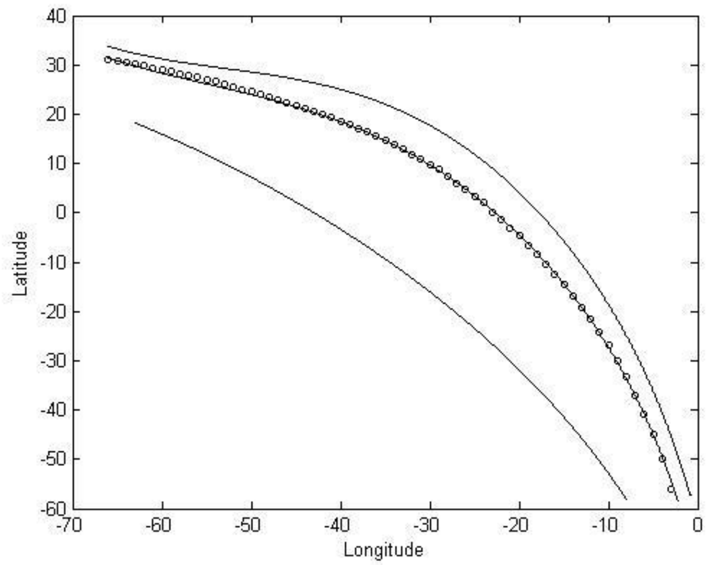

Figure 13: Addition of 1 degree east variation.

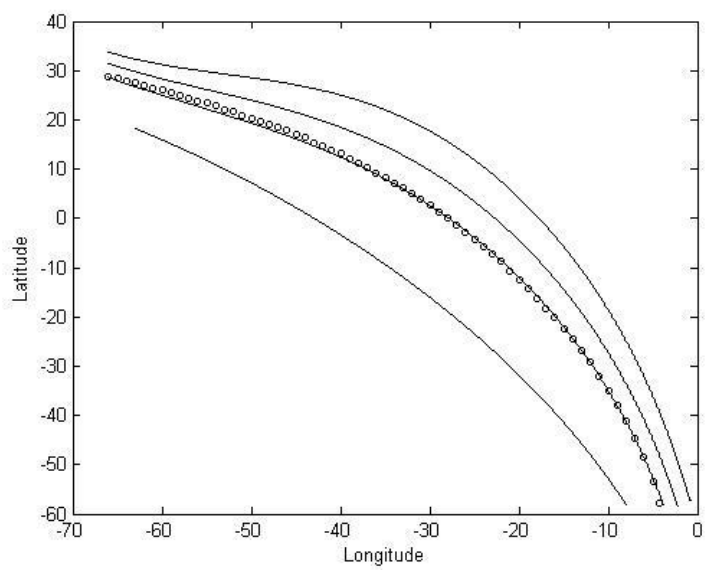

Figure 14: Addition of 2 degrees east variation.

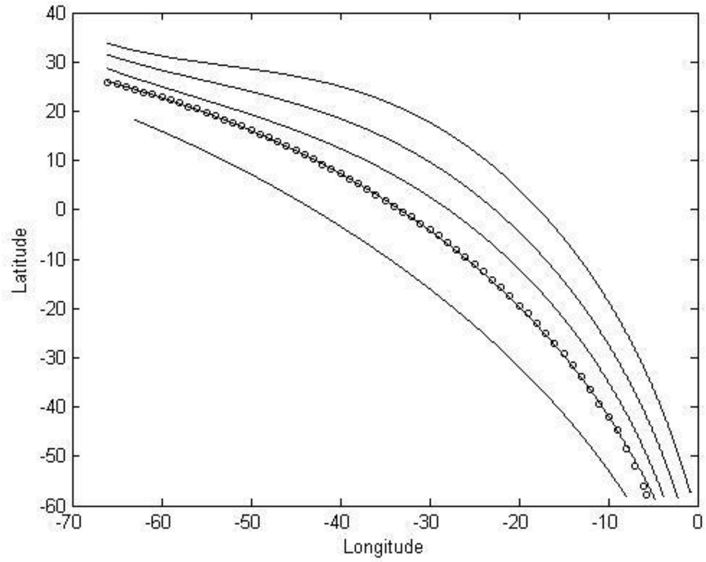

Figure 15: Addition of 3 degrees east variation.

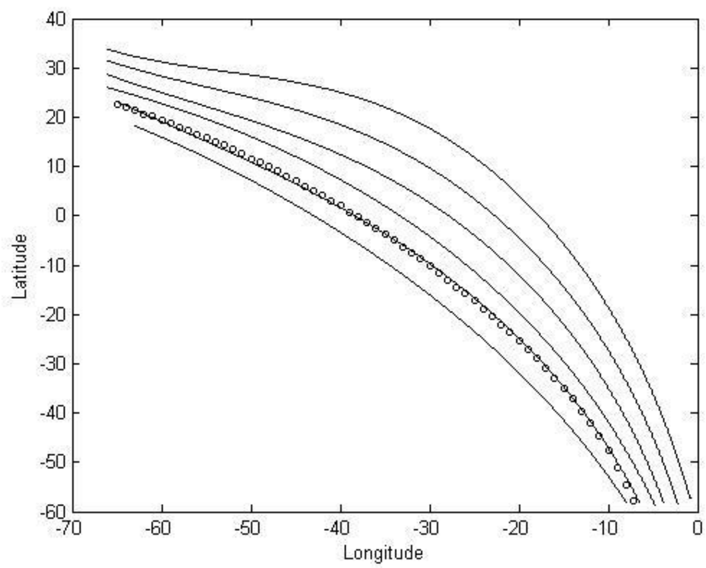

Figure 16: Addition of 4 degrees east variation.

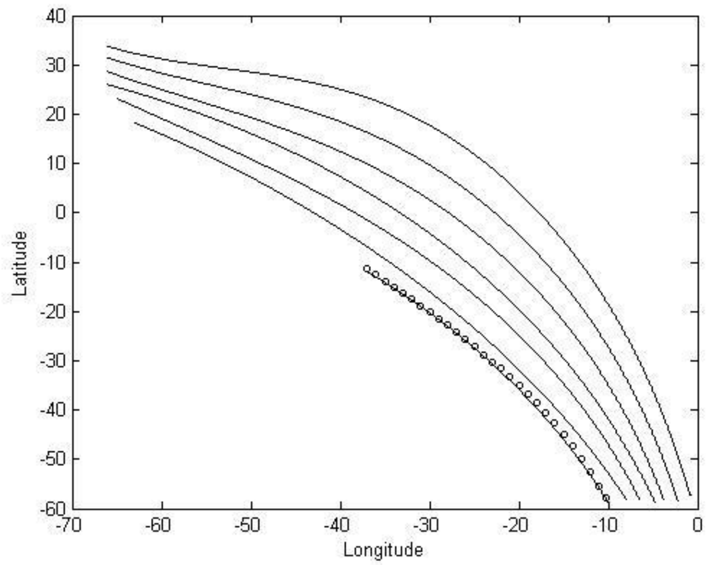

Figure 17: Addition of 6 degrees east variation.

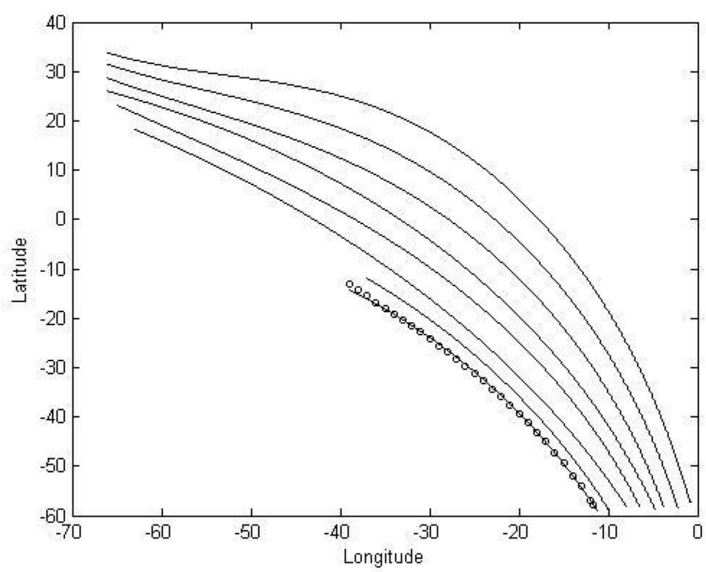

Figure 18: Addition of 7 degrees east variation.

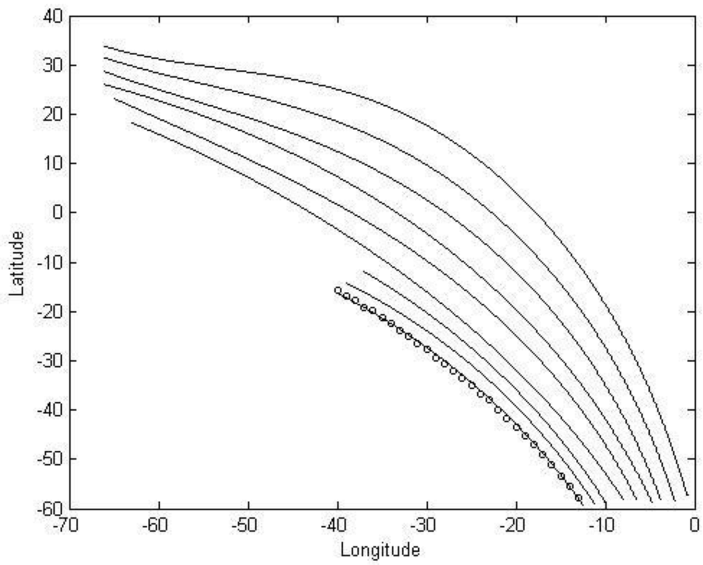

Figure 19: Addition of 8 degrees east variation.

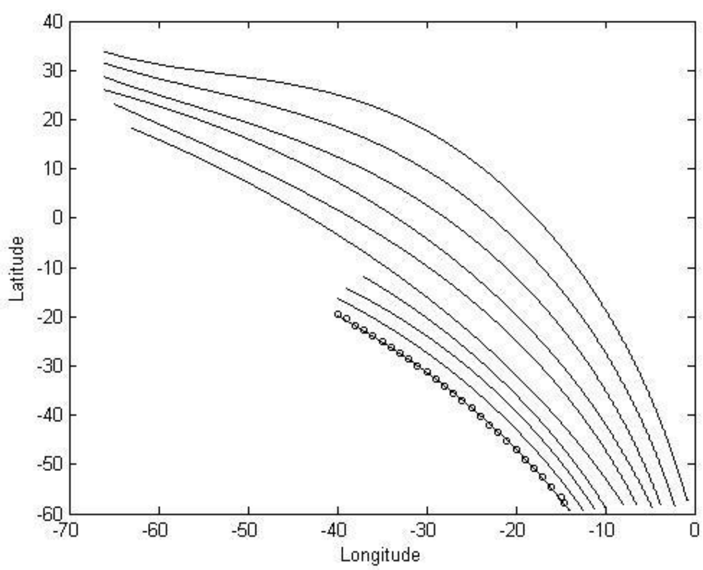

Figure 20: Addition of 9 degrees east variation.

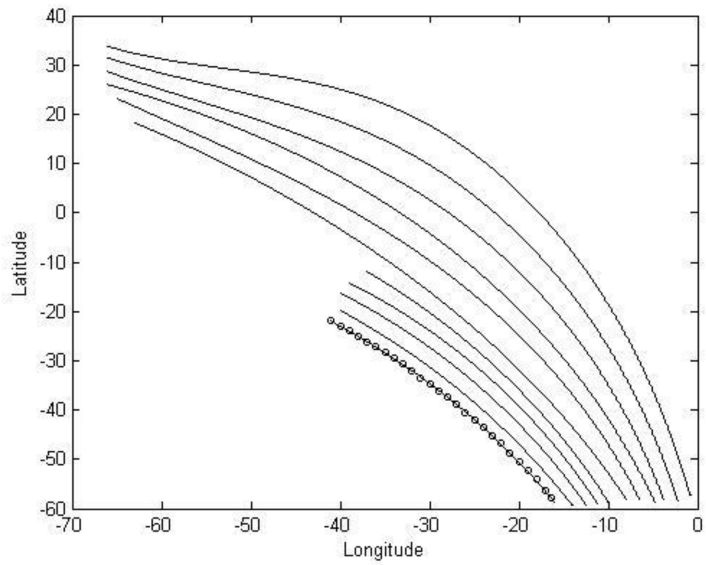

Figure 21: Addition of 10 degrees east variation.

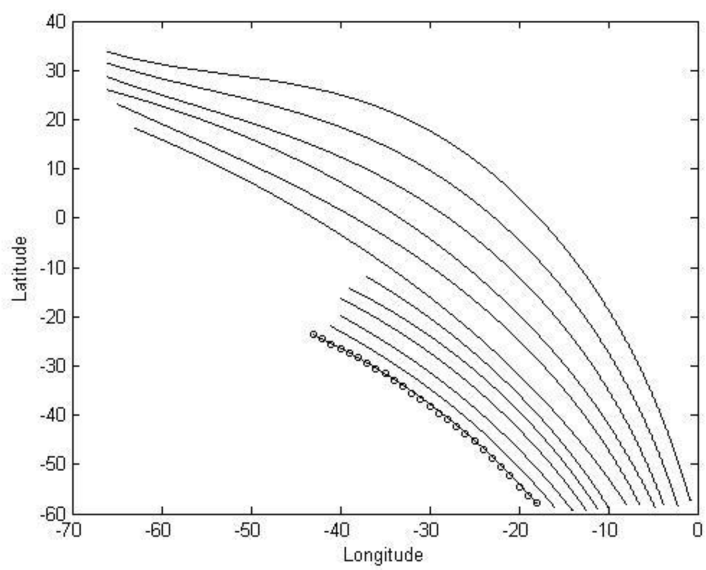

Figure 22: Addition of 11 degrees east variation.

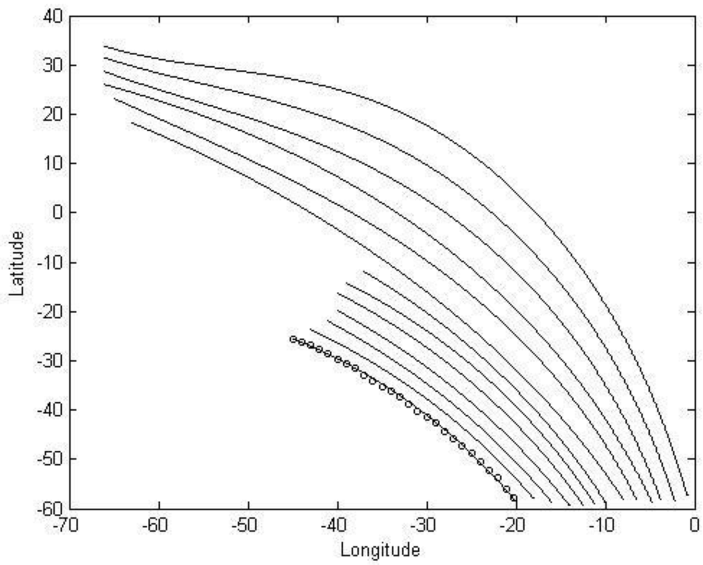

Figure 23: Addition of 12 degrees east variation.

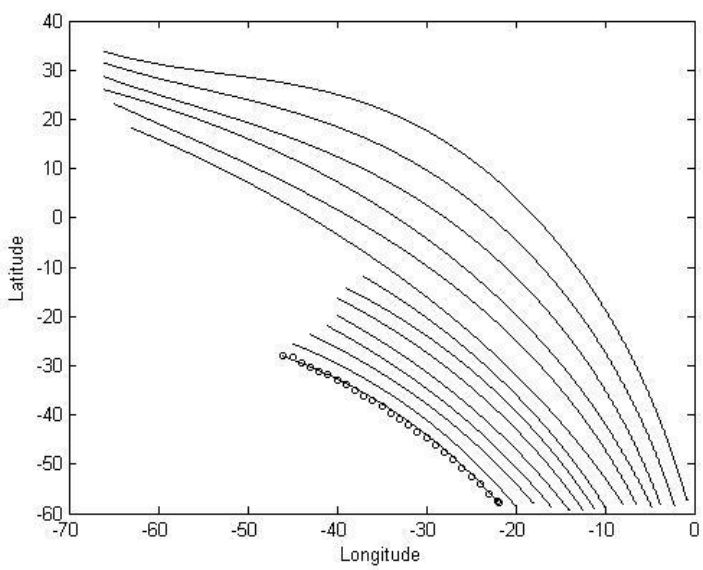

Figure 24: Addition of 13 degrees east variation.

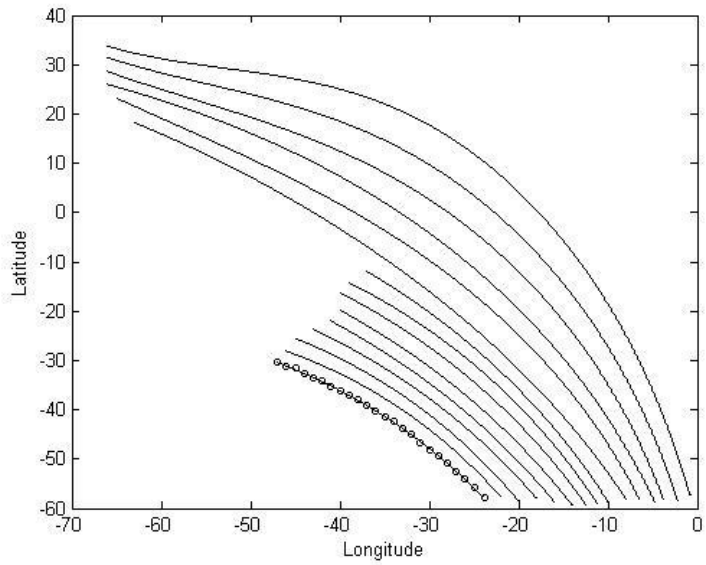

Figure 25: Addition of 14 degrees east variation.

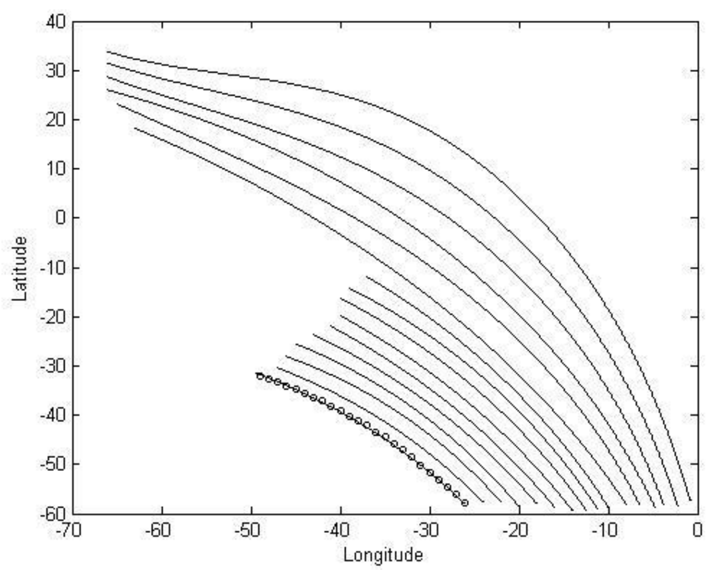

Figure 26: Addition of 15 degrees east variation.

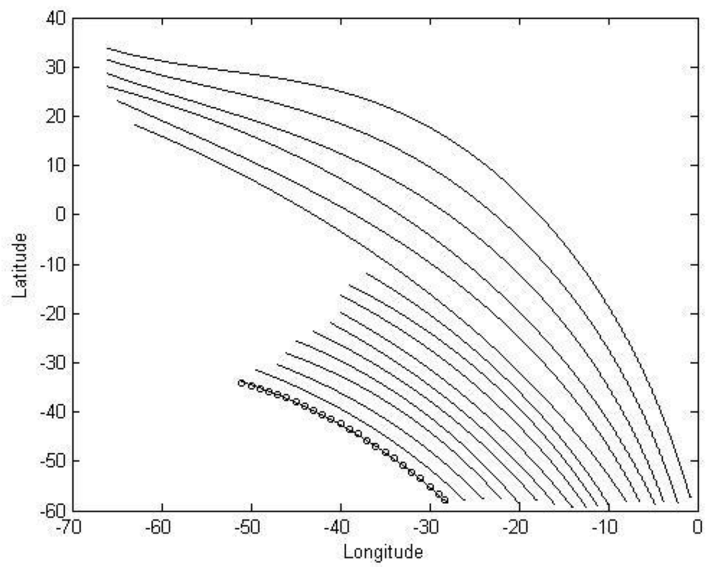

Figure 27: Addition of 16 degrees east variation.

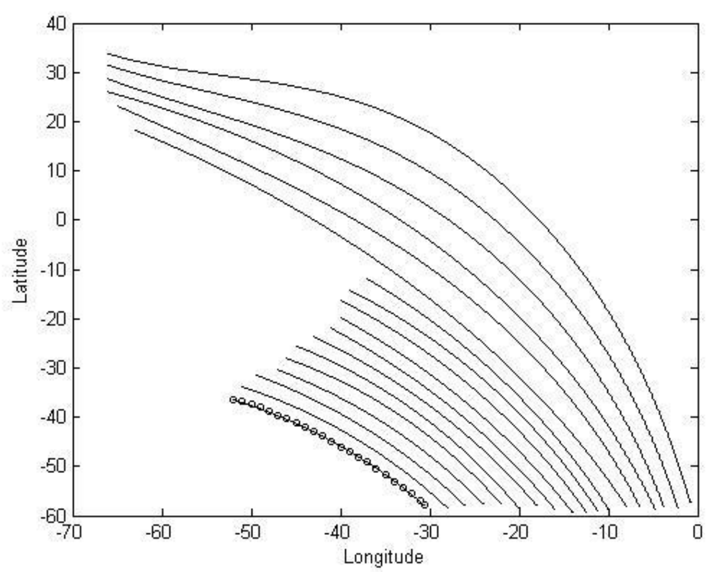

Figure 28: Addition of 17 degrees east variation.

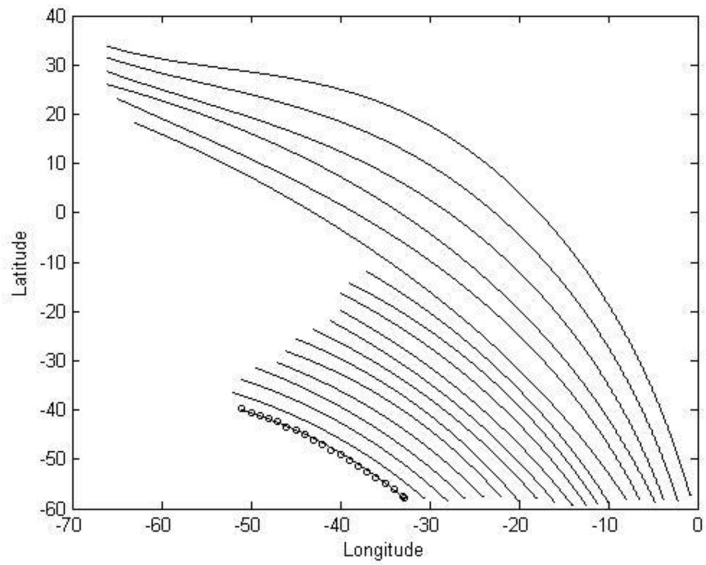

Figure 29: Addition of 18 degrees east variation.

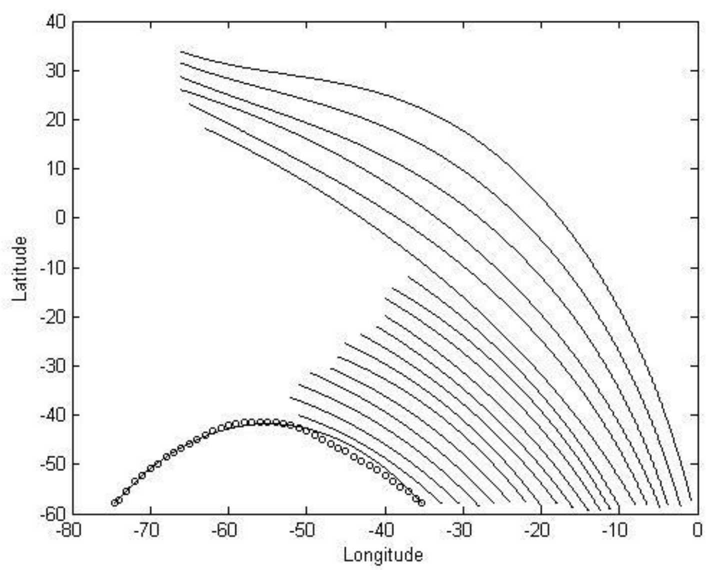

Figure 30: Addition of 19 degrees east variation.

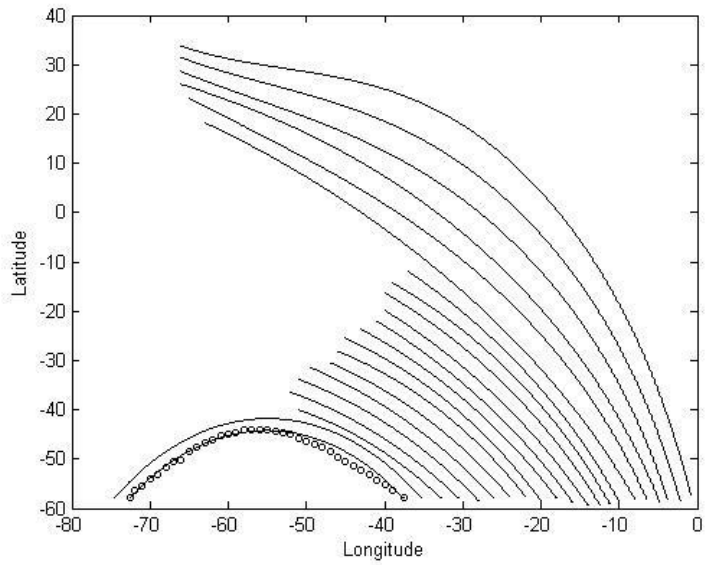

Figure 31: Addition of 20 degrees east variation.

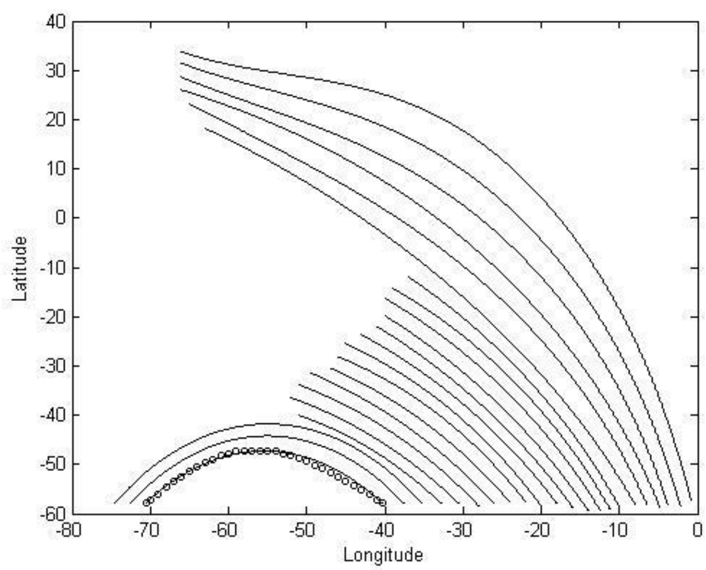

Figure 32: Addition of 21 degrees east variation.

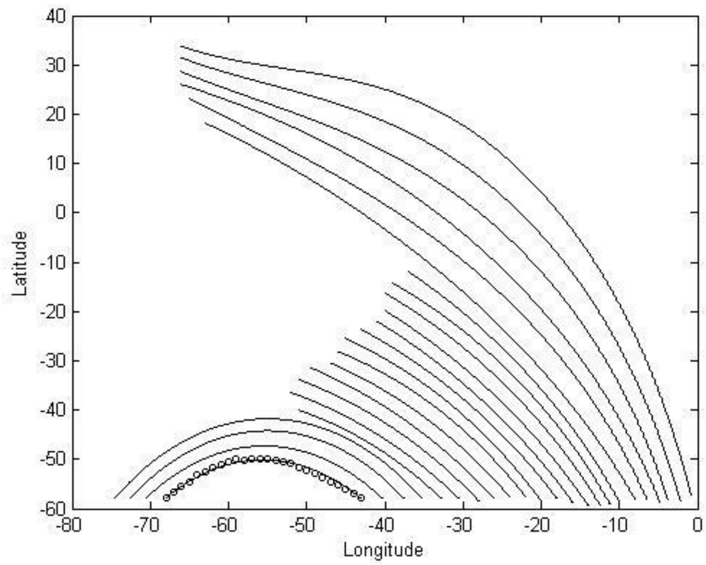

Figure 33: Addition of 22 degrees east variation.

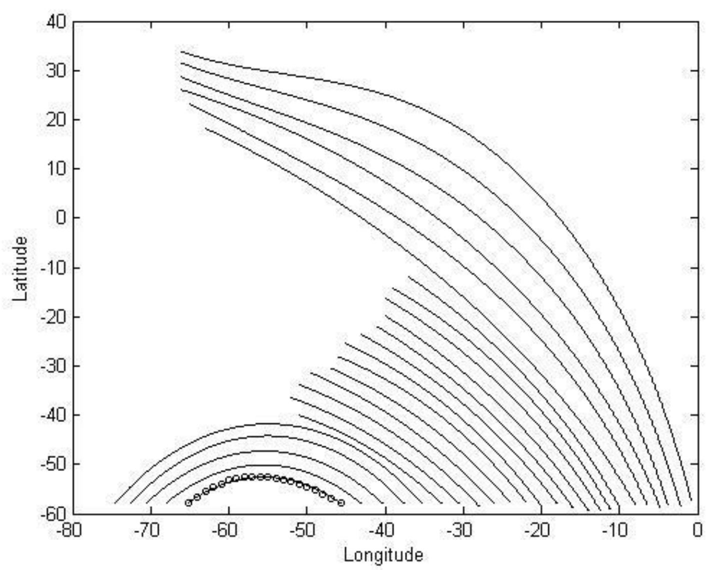

Figure 34: Addition of 23 degrees east variation.

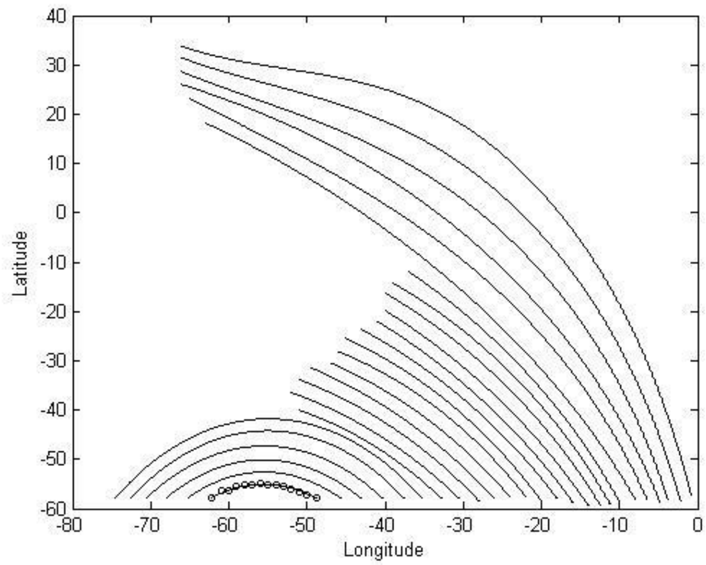

Figure 35: Addition of 24 degrees east variation.

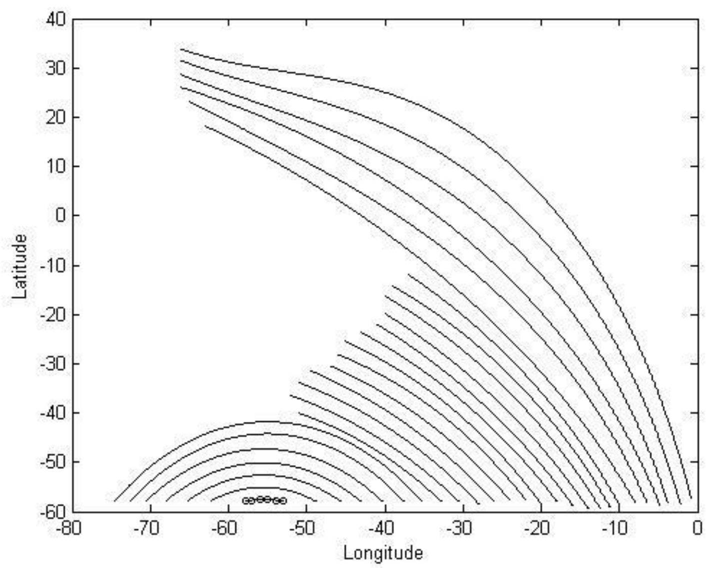

Figure 36: Addition of 25 degrees east variation.

Table 11: Points for the line 1 degrees west variation.

All of the following positions of latitudes are north and longitudes are west of London.

Line of Variation	Point 1		Point 2		Point 3		Point 4		Point 5	
	Latitude	Longitude	Latitude	Longitude	Latitude	Longitude	Latitude	Longitude	Latitude	Longitude
	D°M’	D°M’	D°M’	D°M’	D°M’	D°M’	D°M’	D°M’	D°M’	D°M’
1 degrees west	33°23’	65°00’	32°30’	60°00’	29°09’	45°00’	21°00’	25°00’	2°00’	13°00’

Table 12: Points for the lines of 2 to 25 degrees west variation – upper Atlantic Ocean.

Line of Variation Upper Atlantic	Point 1		Point 2		Point 3	
	Latitude	Longitude	Latitude	Longitude	Latitude	Longitude
	D°M’	D°M’	D°M’	D°M’	D°M’	D°M’
2 degrees west	34°20’	62°13’	25°27’	20°35’	26°45’	25°00’
3 degrees west	35°55’	65°00’	30°30’	18°00’	32°30’	30°00’
4 degrees west	37°05’	65°45’	35°50’	30°00’	35°15’	17°50’
5 degrees west	38°00’	66°00’	38°15’	50°00’	39°39’	12°11’
6 degrees west	39°00’	64°00’	42°45’	20°00’	43°33’	14°00’
7 degrees west	40°00’	64°00’	45°45’	20°00’	48°15’	8°00’
8 degrees west	40°57’	64°00’	48°15’	20°00’	50°00’	14°00’
9 degrees west	42°00’	62°00’	50°53’	20°00’	53°00’	14°00’
10 degrees west	43°00’	60°00’	50°53’	28°00’	55°53’	14°00’
11 degrees west	43°53’	58°00’	50°45’	30°00’	55°15’	20°00’
12 degrees west	44°30’	58°00’	52°15’	30°00’	57°00’	20°00’
13 degrees west	45°12’	58°00’	53°42’	30°00’	58°45’	20°00’
14 degrees west	46°00’	56°00’	49°45’	42°00’	55°08’	30°00’
15 degrees west	47°00’	54°00’	47°57’	50°00’	56°45’	30°00’
16 degrees west	47°45’	54°00’	48°45’	50°00’	58°30’	30°00’
17 degrees west	48°27’	54°00’	49°45’	50°00’	57°33’	34°00’
18 degrees west	49°12’	54°00’	50°38’	50°00’	56°38’	38°00’
19 degrees west	49°45’	54°00’	51°30’	50°00’	56°53’	40°00’
20 degrees west	50°23’	54°00’	52°30’	50°00’	57°20’	42°00’
21 degrees west	51°38’	54°00’	53°38’	50°00’	57°53’	44°00’
22 degrees west	52°45’	54°00’	55°15’	50°00’	58°15’	46°00’
23 degrees west	54°00’	54°00’	56°45’	50°00’	58°30’	48°00’
24 degrees west	55°30’	54°00’	56°53’	52°00’	58°45’	58°00’
25 degrees west	57°08’	54°00’	58°45’	52°00’	59°00’	51°45’

Table 13: Polynomials for the lines of 1 to 25 degrees west variation – upper Atlantic Ocean.

Line of Variation Upper Atlantic	Polynomials
1 degrees west	$P_4(x) = -1.1644e - 005x^4 - 0.00233755x^3 - 0.17428x^2 - 5.94159x - 50.5903$
2 degrees west	$P_2(x) = -0.00217252x^2 - 0.393141x + 18.1793$
3 degrees west	$P_2(x) = -0.000264882x^2 - 0.237086x + 26.7078$
4 degrees west	$P_2(x) = -0.000264882x^2 - 0.0603275x + 34.2586$
5 degrees west	$P_2(x) = 0.000397481x^2 + 0.0617328x + 40.3429$
6 degrees west	$P_2(x) = 0.000962121x^2 + 0.166045x + 45.6861$
7 degrees west	$P_2(x) = 0.00138663 + 0.247159x + 50.1385$
8 degrees west	$P_2(x) = 0.00251515x^2 + 0.377182x + 54.7876$
9 degrees west	$P_2(x) = 0.00297619x^2 + 0.455357x + 58.7917$
10 degrees west	$P_2(x) = 0.00436724x^2 + 0.603067x + 63.462$
11 degrees west	$P_2(x) = 0.00538064x^2 + 0.719032x + 67.4784$
12 degrees west	$P_2(x) = 0.00521617x^2 + 0.735808x + 69.6297$
13 degrees west	$P_2(x) = 0.00530075 + 0.770038x + 72.0304$
14 degrees west	$P_2(x) = 0.00692537x^2 + 0.946543x + 77.2885$
15 degrees west	$P_2(x) = 0.0090625x^2 + 1.1675x + 83.6188$
16 degrees west	$P_2(x) = 0.00989583x^2 + 1.27917x + 87.9687$
17 degrees west	$P_2(x) = 0.008125x^2 + 1.17x + 87.9375$
18 degrees west	$P_2(x) = 0.00898437x^2 + 1.29062x + 92.6953$
19 degrees west	$P_2(x) = 0.00714286x^2 + 1.18036x + 92.6607$
20 degrees west	$P_2(x) = 0.00604167x^2 + 1.15958x + 95.375$
21 degrees west	$P_2(x) = 0.0208333x^2 + 2.66667x + 134.875$
22 degrees west	$P_2(x) = 0.015625x^2 + 2.25x + 128.688$
23 degrees west	$P_2(x) = 0.03125x^2 + 3.9375x + 175.5$
24 degrees west	$P_2(x) = 0.0625x^2 + 7.3125x + 268.125$
25 degrees west	$P_2(x) = 0.0833333x^2 + 9.64583x + 335$

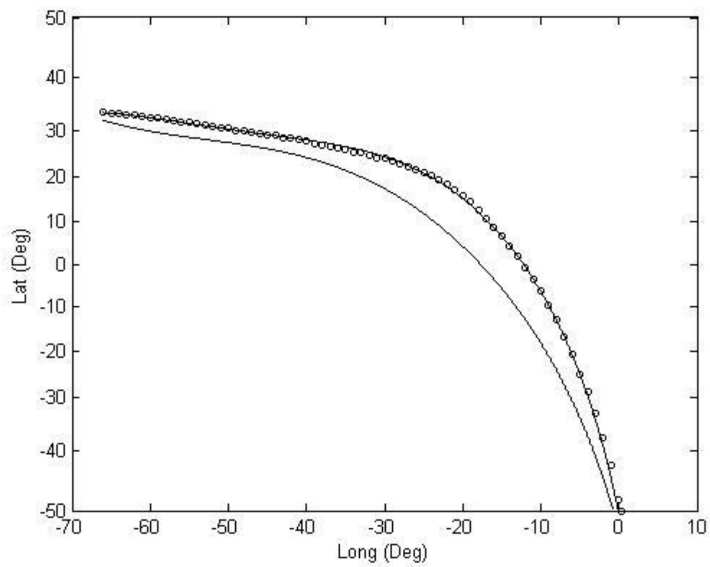

Figure 37: Addition of 1 degree west variation.

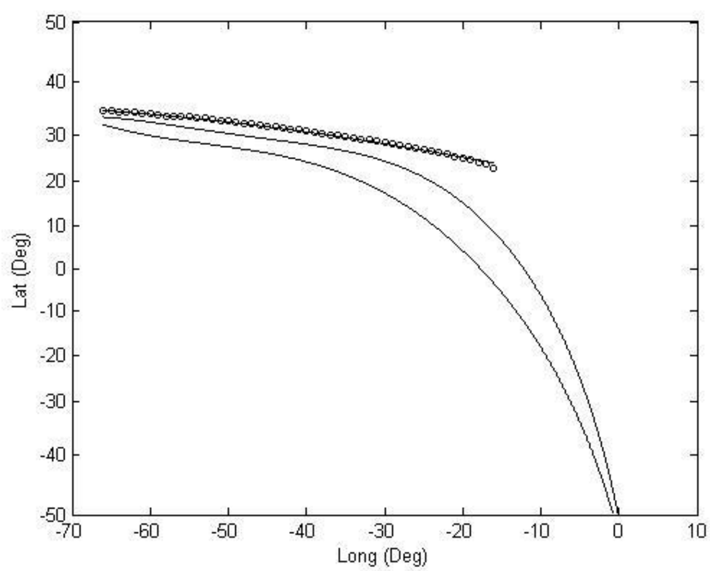

Figure 38: Addition of 2 degrees west variation.

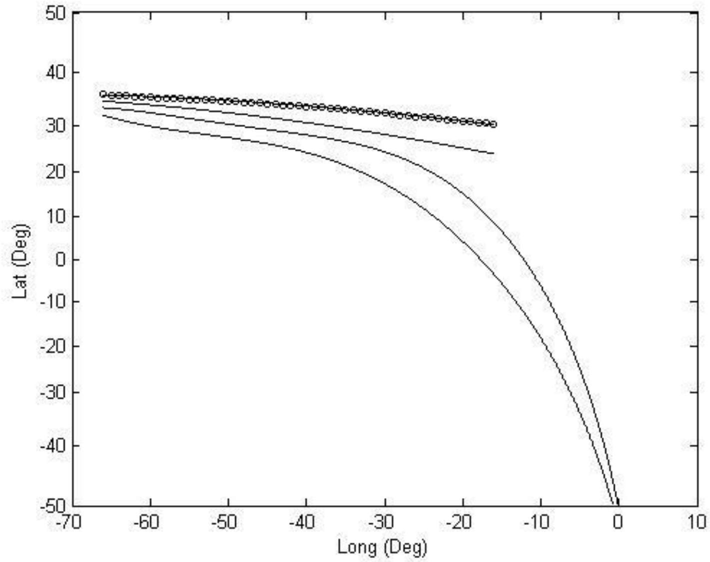

Figure 39: Addition of 3 degrees west variation.

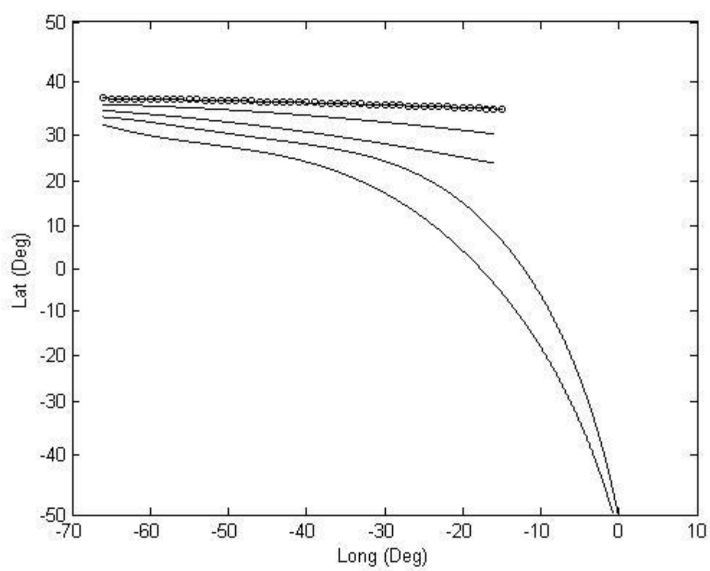

Figure 40: Addition of 4 degrees west variation.

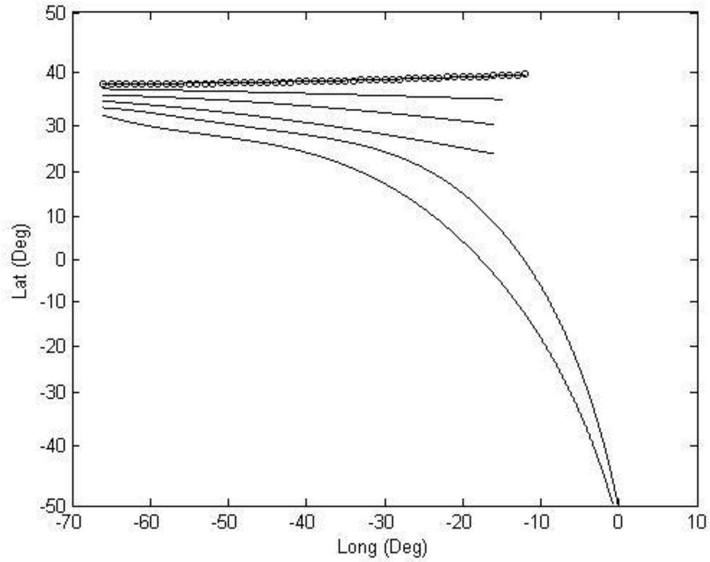

Figure 41: Addition of 5 degrees west variation.

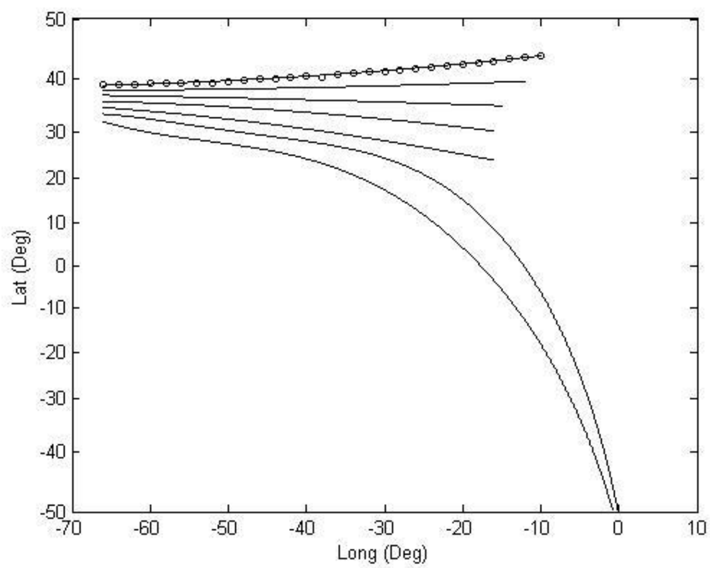

Figure 42: Addition of 6 degrees west variation.

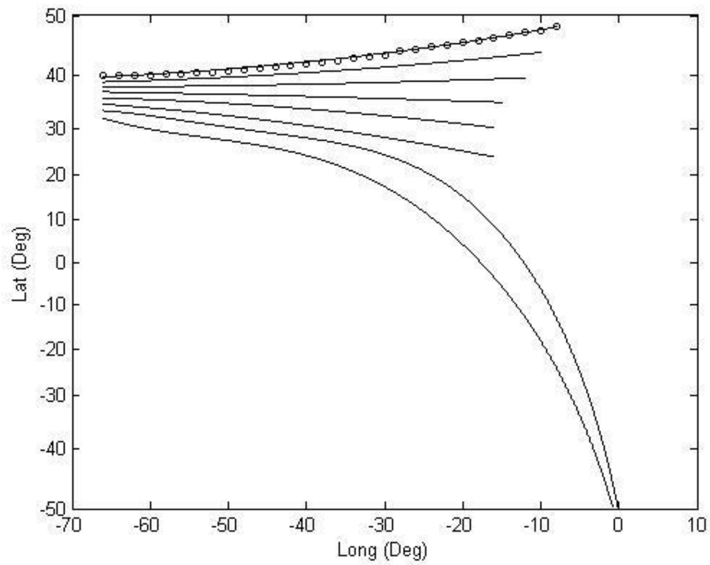

Figure 43: Addition of 7 degrees west variation.

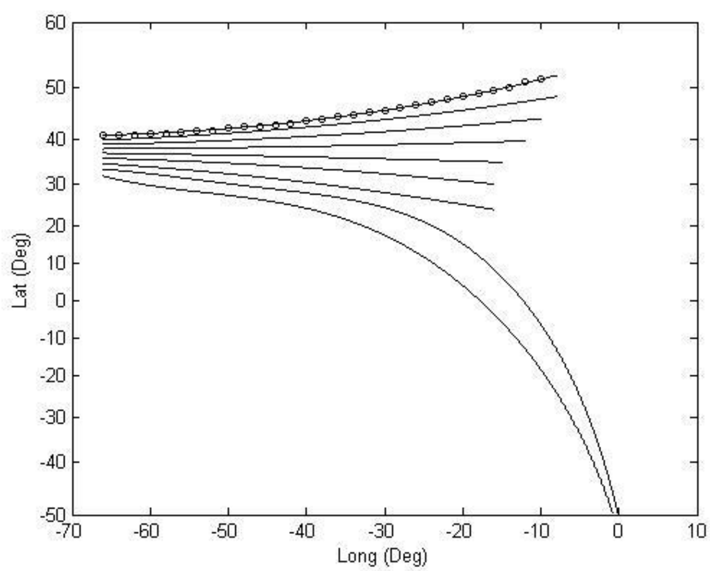

Figure 44: Addition of 8 degrees west variation.

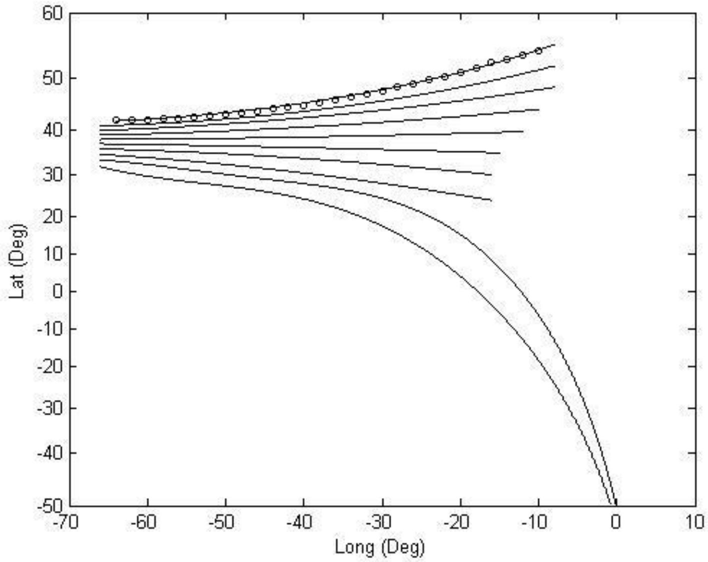

Figure 45: Addition of 9 degrees west variation.

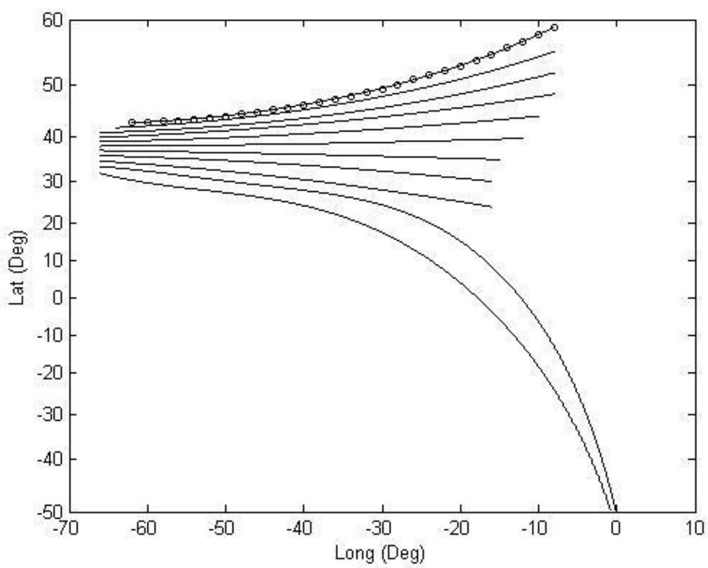

Figure 46: Addition of 10 degrees west variation.

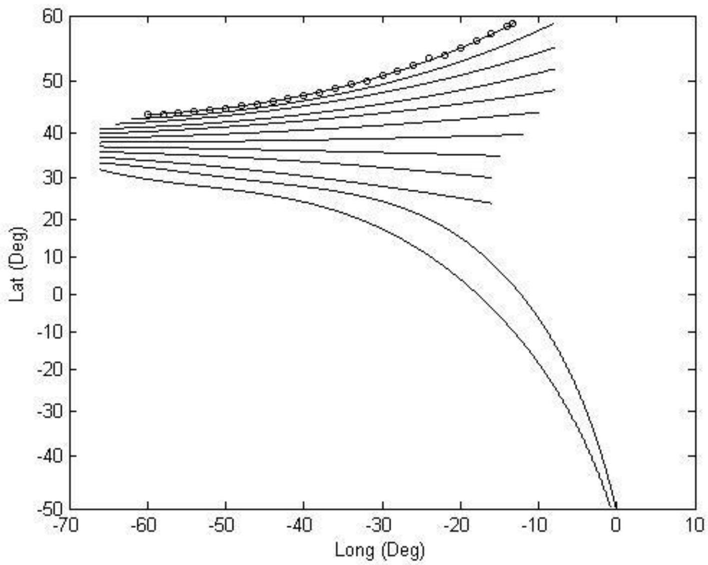

Figure 47: Addition of 11 degrees west variation.

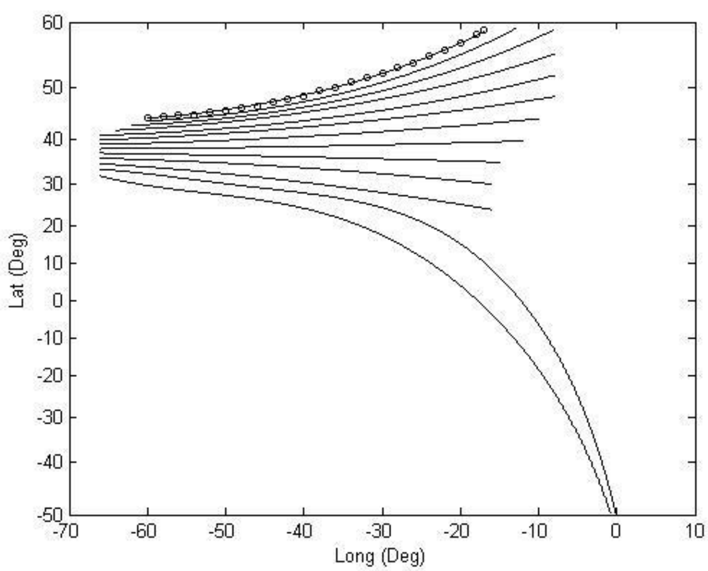

Figure 48: Addition of 12 degrees west variation.

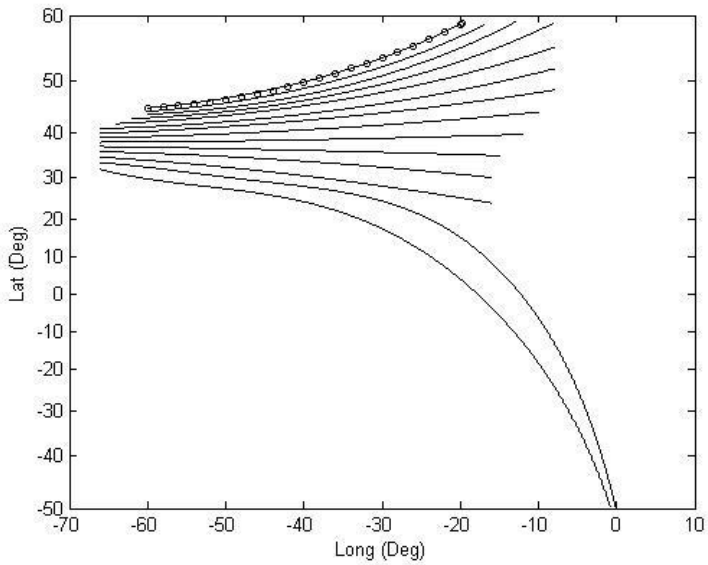

Figure 49: Addition of 13 degrees west variation.

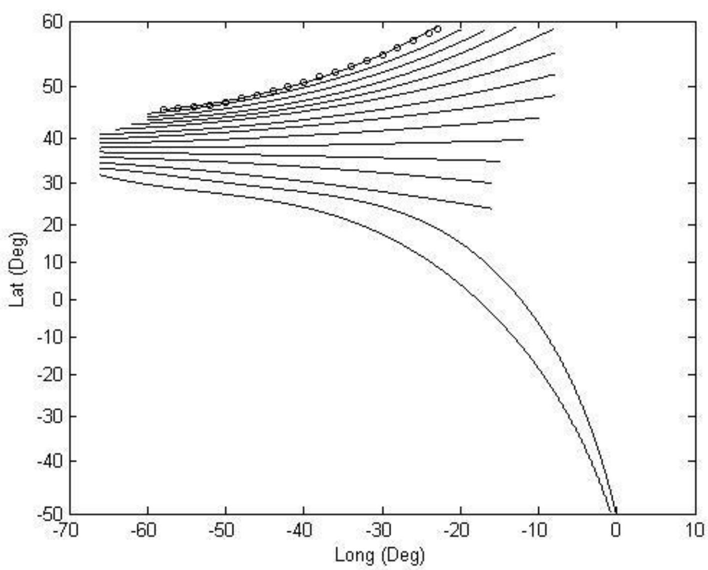

Figure 50: Addition of 14 degrees west variation.

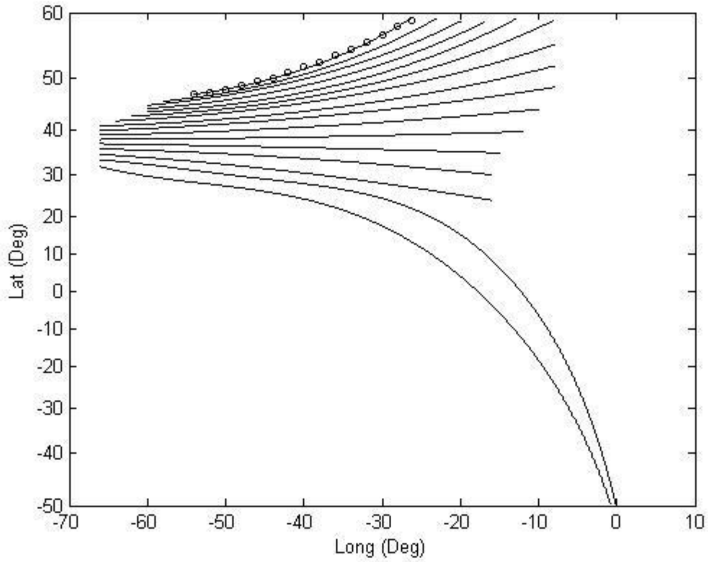

Figure 51: Addition of 15 degrees west variation.

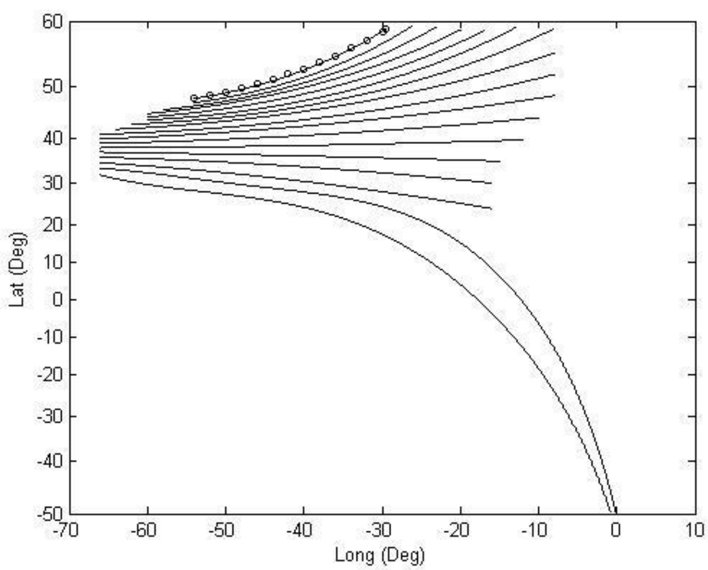

Figure 52: Addition of 16 degrees west variation.

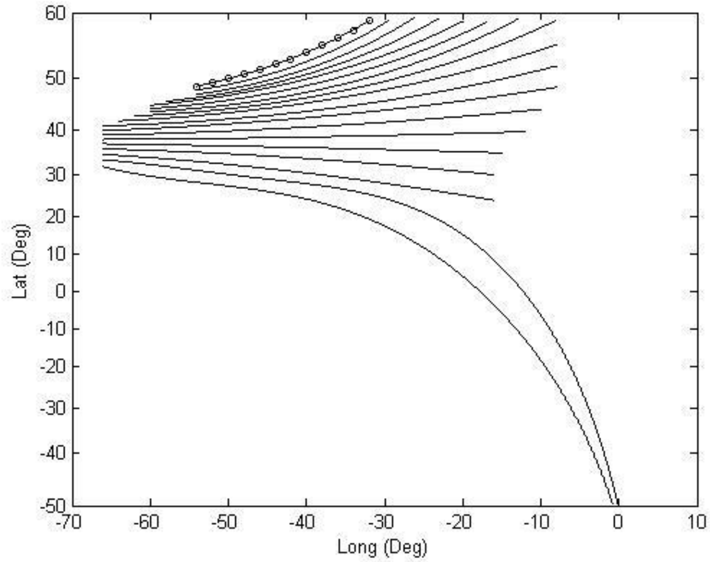

Figure 53: Addition of 17 degrees west variation.

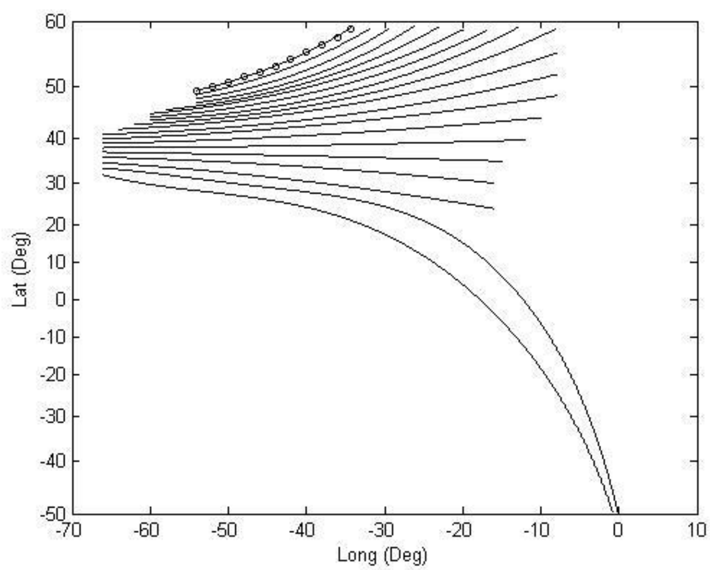

Figure 54: Addition of 18 degrees west variation.

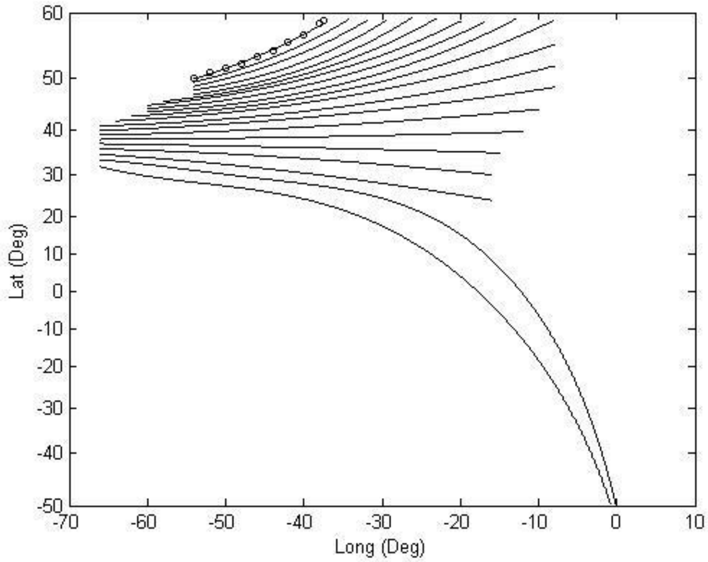

Figure 55: Addition of 19 degrees west variation.

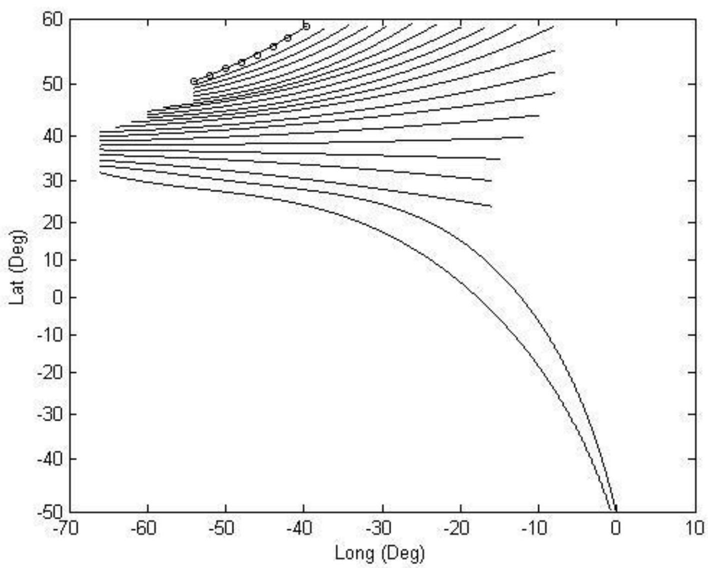

Figure 56: Addition of 20 degrees west variation.

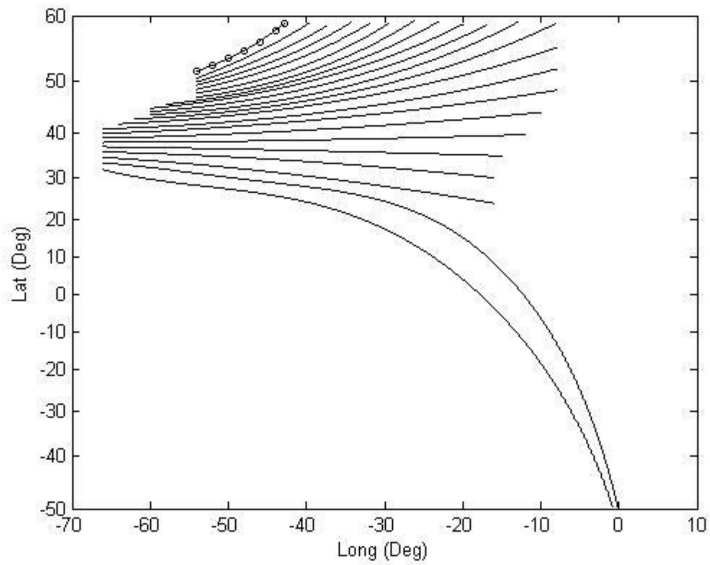

Figure 57: Addition of 21 degrees west variation.

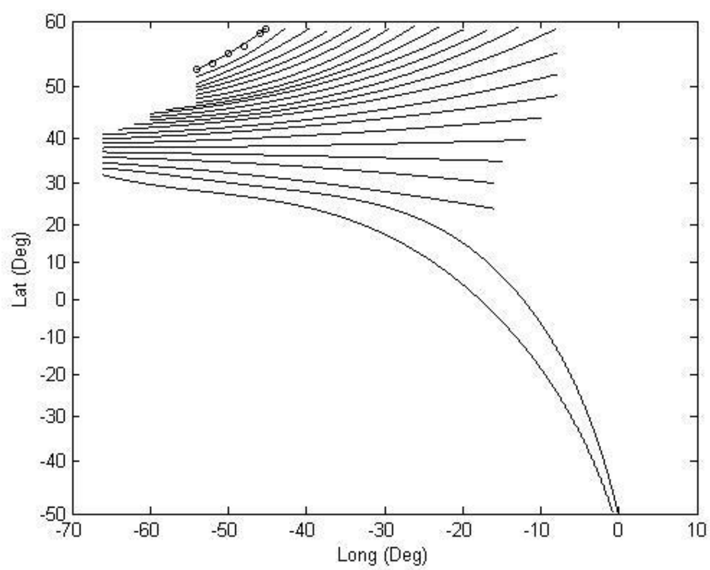

Figure 58: Addition of 22 degrees west variation.

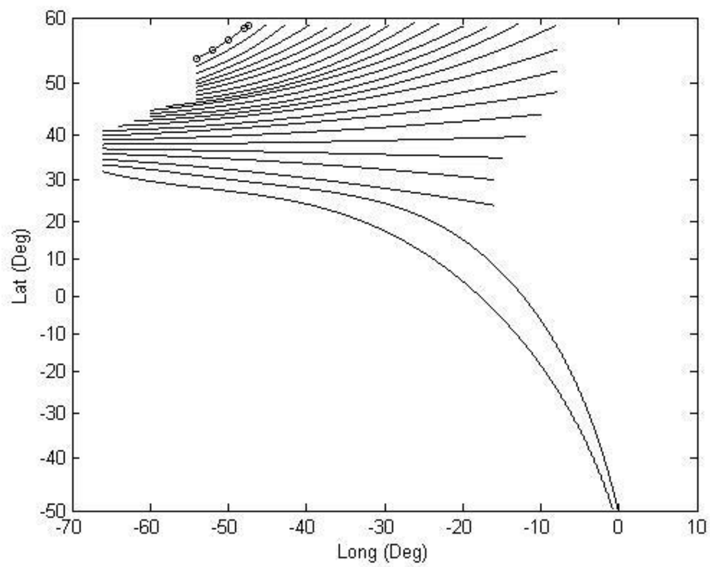

Figure 59: Addition of 23 degrees west variation.

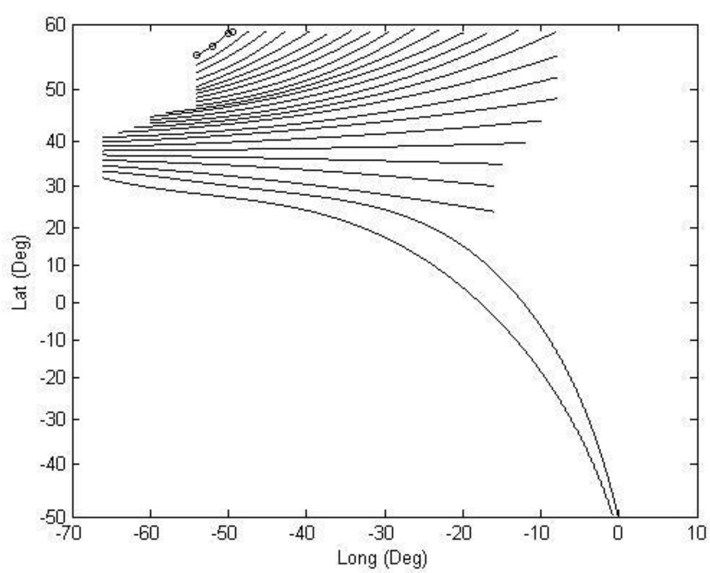

Figure 60: Addition of 24 degrees west variation.

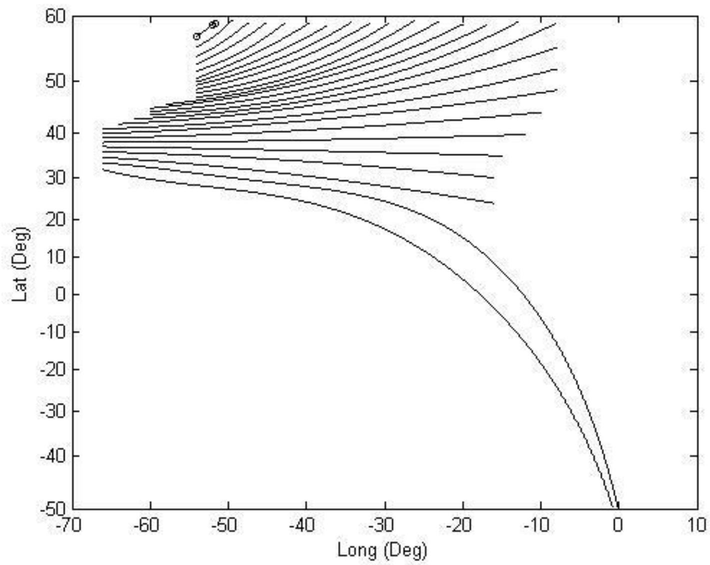

Figure 61: Addition of 25 degrees west variation.

Table 14: Points for the lines of 5 to 10 degrees east variation – lower Atlantic Ocean.

All positions of latitudes are south and longitudes are west of London.

Line of Variation Lower Atlantic	Point 1		Point 2		Point 3	
	Latitude	Longitude	Latitude	Longitude	Latitude	Longitude
	D°M’	D°M’	D°M’	D°M’	D°M’	D°M’
5 degrees west	16°00’	3°45’	25°00’	4°30’	37°25’	5°30’
6 degrees west	16°00’	6°15’	25°00’	6°45’	37°25’	7°30’
7 degrees west	10°00’	9°15’	20°00’	9°45’	30°00’	10°00’
8 degrees west	10°00’	12°00’	20°00’	11°48’	50°00’	11°23’
9 degrees west	20°00’	14°15’	30°00’	13°53’	50°00’	13°00’
10 degrees west	30°00’	16°18’	40°00’	15°30’	50°00’	14°45’

Table 15: Polynomials for the lines of 5 to 10 degrees east variation – lower Atlantic Ocean.

Line of Variation Lower Atlantic	Polynomials
5 degrees west	$P_2(x) = -0.24x^2 - 10.02x + 24.95$
6 degrees west	$P_2(x) = 1.152x^2 - 32.976x + 145.1$
7 degrees west	$P_2(x) = -26.6667x^2 + 486.667x - 2230$
8 degrees west	$P_2(x) = -32.9412x^2 + 834x - 5274.47$
9 degrees west	$P_2(x) = 3.04762x^2 - 59.0476x + 202.571$
10 degrees west	$P_2(x) = -0.537634x^2 + 29.5968x - 369.583$

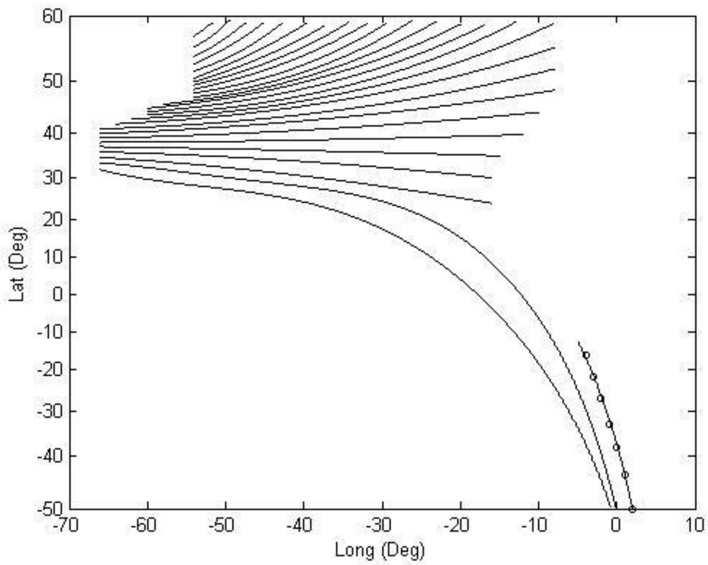

Figure 62: Addition of 2 degrees west variation, lower Atlantic.

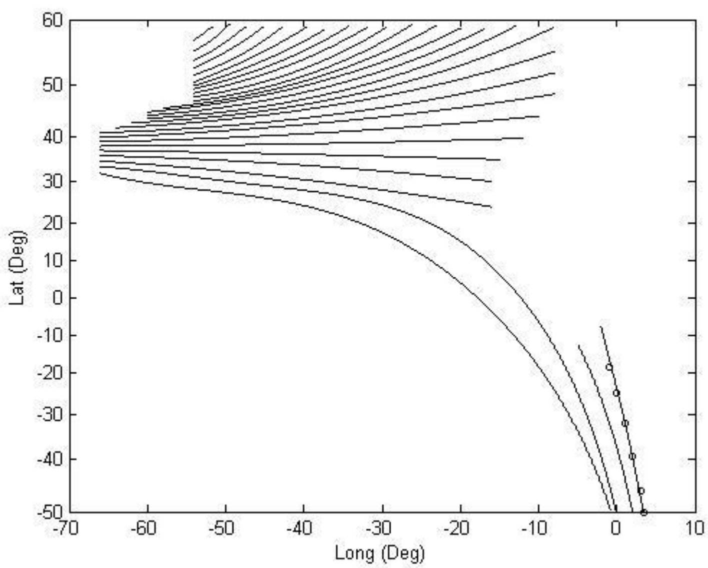

Figure 63: Addition of 3 degrees west variation, lower Atlantic.

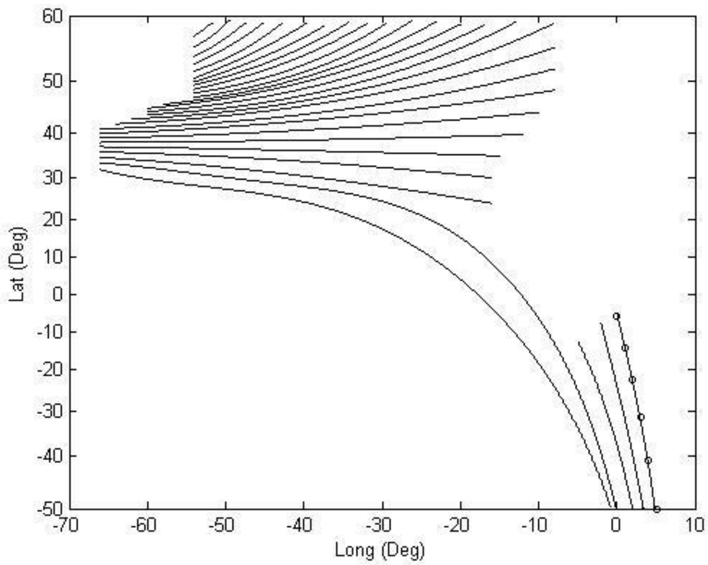

Figure 64: Addition of 4 degrees west variation, lower Atlantic.

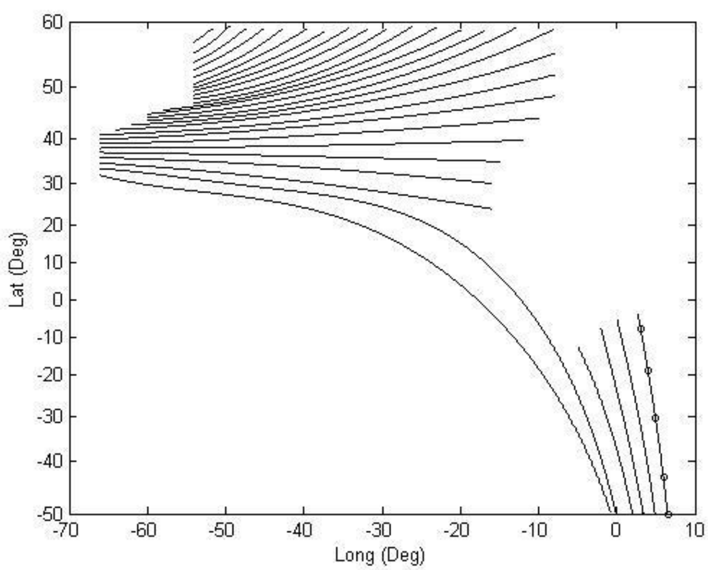

Figure 65: Addition of 5 degrees west variation, lower Atlantic.

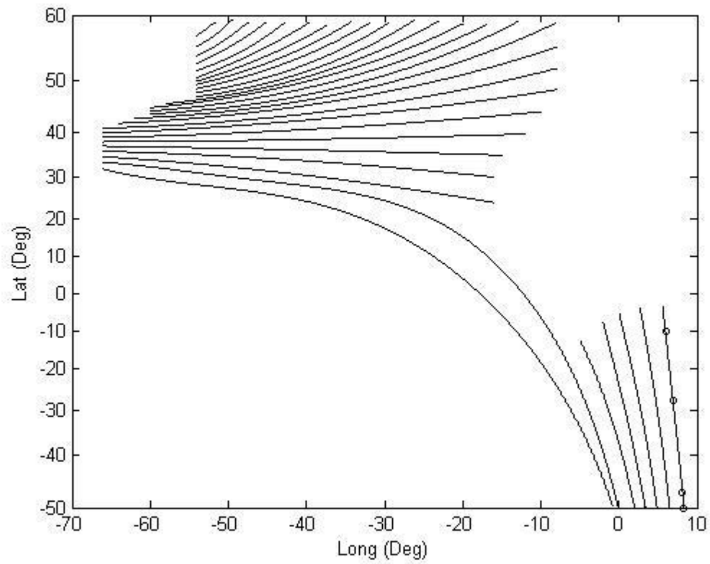

Figure 66: Addition of 6 degrees west variation, lower Atlantic.

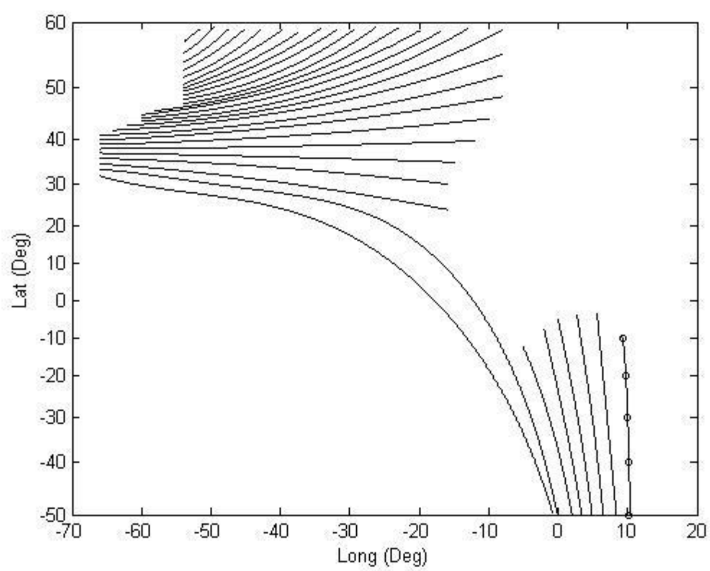

Figure 67: Addition of 7 degrees west variation, lower Atlantic.

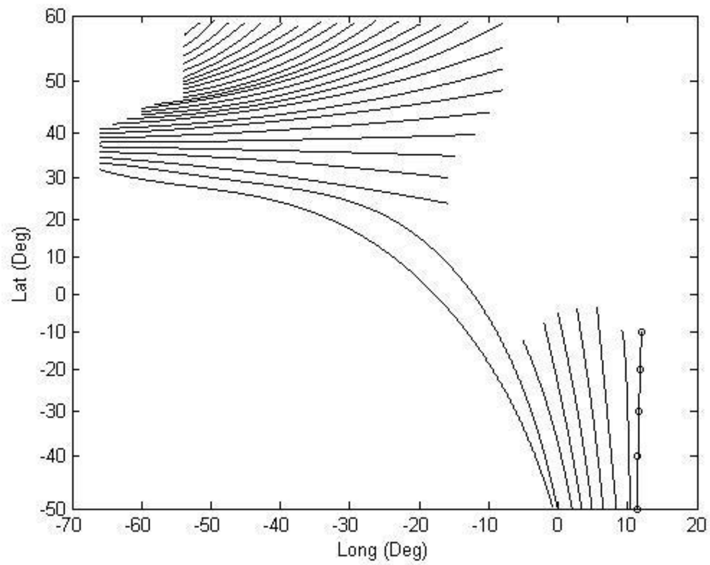

Figure 68: Addition of 8 degrees west variation, lower Atlantic.

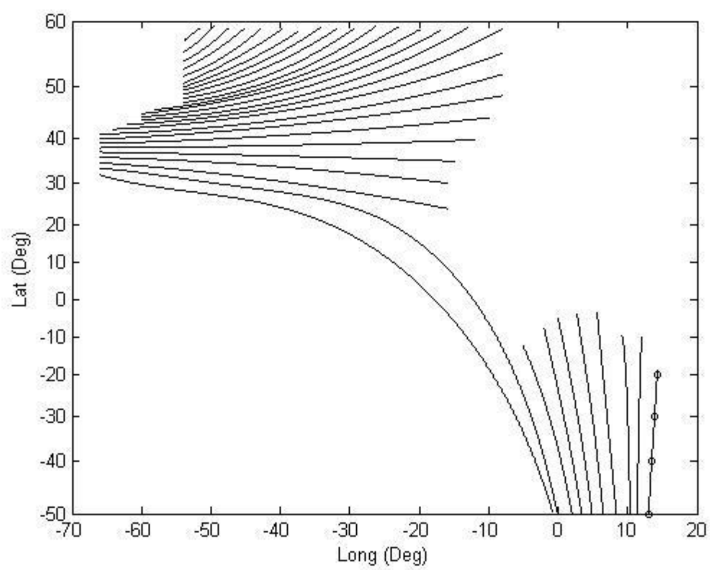

Figure 69: Addition of 9 degrees west variation, lower Atlantic.

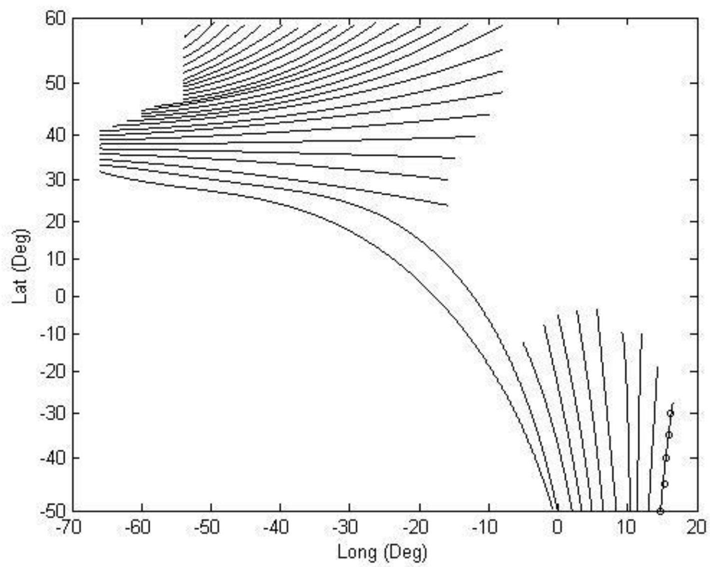

Figure 70: Addition of 10 degrees west variation, lower Atlantic.

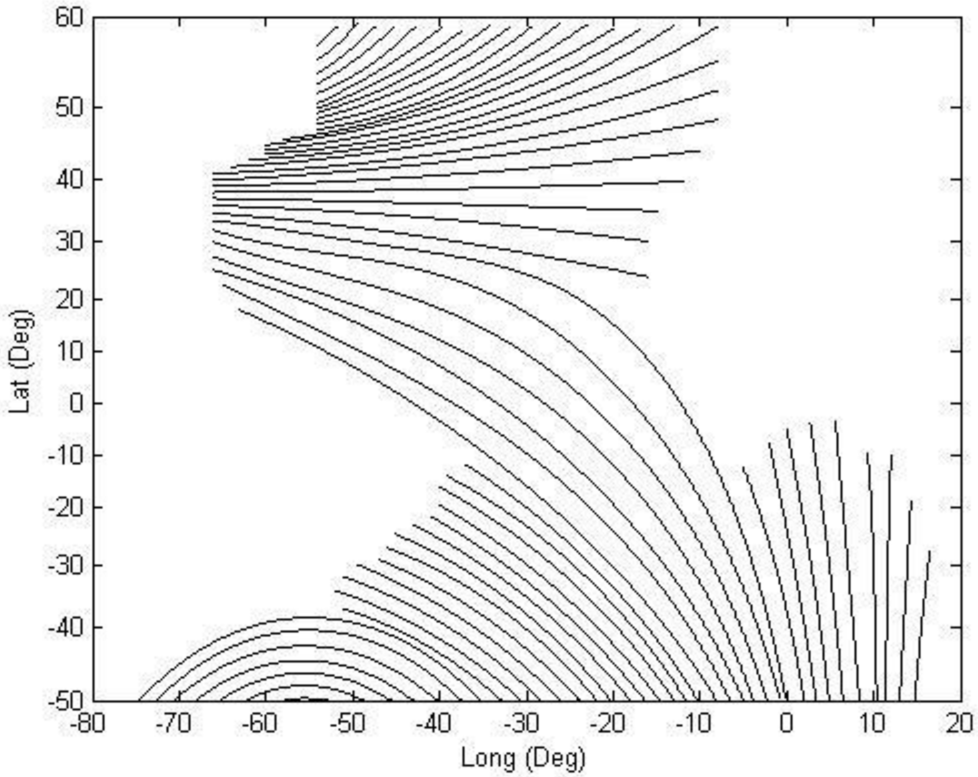

Figure 71: The complete set of lines of variation.

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Honors and Awards

Dean's Honor List, UWO, 2006 - 2010

Faculty of Science Graduate Teaching Award, 2011

University of Western Ontario

Research Poster Award, 2012

Statistical Society of Canada

Teaching Introductory Statistics, Winter 2012

University of Western Ontario

Papers Presented SSC Poster Session, 2012

Award for Best Poster Presentation