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Graduate Program in Business

A thesis submitted in partial fulfillment of the requirements for the degree in Doctor of  
Philosophy

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INCENTIVES FOR OPTIMAL ALLOCATION OF HIV PREVENTION  
RESOURCES

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by

Monali M. Malvankar

Graduate Program in Business Administration

A thesis submitted in partial fulfillment  
of the requirements for the degree of  
Doctor of Philosophy

The School of Graduate and Postdoctoral Studies  
The University of Western Ontario  
London, Ontario, Canada

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THE UNIVERSITY OF WESTERN ONTARIO  
School of Graduate and Postdoctoral Studies

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entitled:

**Incentives for Optimal Allocation of HIV Prevention Resources**

is accepted in partial fulfillment of the  
requirements for the degree of  
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Chair of the Thesis Examination Board

## ABSTRACT

The thesis consists of three main chapters on optimal incentives for a multi-level allocation process of HIV/AIDS prevention funds. HIV/AIDS prevention funds often traverse several levels of distribution. At each level, equity-based heuristics are often used by decision-makers that may lead to sub-optimal allocation. Mathematical programming models may help to allocate prevention funds optimally. Thus, incentives could be given to decision-makers to encourage optimal allocation.

Chapter 4 investigates the impact of incentives by developing a model in which an upper-level decision-maker (UD) allocates funds to a single lower-level decision-maker (LD) who then distributes funds to local programs. The UD makes use of an incentive scheme to encourage a LD to allocate optimally. The optimal decision at the lower-level depends on the strength of the incentive provided by the upper-level and the preferences for equity by the lower-level. The results demonstrate that under certain conditions an incentive may help the upper-level to encourage optimal allocation at the lower-level.

Chapter 5 extends the model developed in Chapter 4 to incorporate information asymmetry. The information about the total infections prevented per dollar and preferences of the LD regarding equity-based allocation is known at the lower-level, but unknown at the upper-level. We seek to answer the following questions: What is the impact of incentives under information asymmetry? We examine conditions when loss of efficiency is higher or zero at the upper-level.

Chapter 6 evaluates the impact of two types of incentives between and within the two LDs. The UD sets the level of two types of incentives and then the two LDs sets the fraction of the funds to be reserved for proportional allocation and the amounts allocated to the lower-level programs. We analyze each decision-makers' behaviour at the equilibrium when either or both incentive schemes are incorporated.

Keywords: HIV/AIDS, resource allocation, incentives, optimization

## **CO-AUTHORSHIP**

I hereby state that all of the work presented in this thesis was solely my own and was under the supervision of Greg Zaric and Xinghao (Shaun) Yan.

## **DEDICATION**

To my dearest Gurudev (master), parents and sweet sister Hemali, without whom this would not have happened.

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# TABLE OF CONTENTS

CERTIFICATE OF EXAMINATION.....	ii
ABSTRACT.....	iii
CO-AUTHORSHIP.....	iv
DEDICATION.....	v
ACKNOWLEDGEMENTS.....	vi
TABLE OF CONTENTS.....	vii
LIST OF TABLES.....	x
LIST OF FIGURES.....	xi
Chapter 1 Introduction.....	1
Chapter 2 Practical Applications of HIV/AIDS Prevention Funds Allocation Process	
2.1 United Nations Programme on HIV/AIDS (UNAIDS).....	4
2.2 The World Bank Global AIDS Program.....	5
2.3 World Health Organization (WHO).....	6
2.4 U.S. President’s Emergency Plan for AIDS Relief (PEPFAR).....	7
2.5 Global Fund to Fight AIDS, Tuberculosis and Malaria.....	9
2.6 Bill and Melinda Gates Foundation.....	10
2.7 Center for Disease Control and Prevention (CDC).....	11
2.8 The Ryan White Comprehensive AIDS Resources Emergency Act (RWCA).....	13
2.9 Federal Initiative to Address HIV/AIDS in Canada.....	14
2.10 Non-Governmental Organizations (NGOs).....	15
2.11 Faith-Based Organizations (FBOs).....	16
2.12 Community-Based Organizations (CBOs).....	17
2.13 Summary.....	17
Chapter 3 Theoretical Models of HIV/AIDS Prevention Funds Allocation Process	
3.1 Fiscal Federalism.....	41
3.2 Resource Allocation in Health Care.....	42
3.2.1 Equity-based Heuristic.....	43
3.2.2 Optimization Models.....	44
3.2.3 Operations Research Models.....	45
3.3 HIV Resource Allocation.....	46
3.3.1 Equity-based Heuristics.....	46



3.3.2	Optimization Models.....	46
3.3.3	Operations Research Models.....	46
3.4	Bi-level Optimization.....	47
3.5	Incentives in Health Care Settings.....	48
3.6	Game Theoretic Models.....	49

#### Chapter 4 Optimal Incentives for Multi-level allocation of HIV Prevention Resources

4.1	Introduction.....	56
4.2	The Model.....	58
4.2.1	The Lower-level Model.....	59
4.2.2	The Upper-level Model.....	61
4.3	Analysis.....	61
4.3.1	Linear Utility Function.....	62
4.3.2	Multiplicative Utility Function .....	64
4.4	Example.....	66
4.4.1	Linear Utility Function.....	66
4.4.2	Multiplicative Utility Function.....	68
4.5	Discussion.....	69
	References.....	79

#### Chapter 5 Optimal Incentives for HIV Prevention Funds Allocation under Asymmetric Information

5.1	Introduction.....	82
5.2	Model Formulation.....	84
5.2.1	The Lower-level Model.....	85
5.2.2	The Upper-level Model.....	88
5.3	Analysis.....	89
5.3.1	Case 1.....	89
5.3.2	Case 2.....	91
5.4	Example.....	94
5.5	Discussion.....	96
	References.....	106

#### Chapter 6 Multi-level Allocation of HIV Prevention Funds: a Multiple Incentive Model

6.1	Introduction.....	108
6.2	Model Formulation.....	109
6.3	Analysis.....	114
6.3.1	Two Incentives (“fg” Case).....	114
6.3.2	Between Region Incentives (“f” Case).....	115
6.3.3	Within Region Incentives (“g” Case) .....	116
6.4	Example.....	118
6.5	Discussion.....	119
	References.....	134

Chapter 7 Summary, Conclusions and Future Directions.....	135
Appendices.....	139
8 Proof of Chapter 4.....	139
9 Proof of Chapter 5.....	151
10 Proof of Chapter 6.....	160
VITA.....	169

## LIST OF TABLES

Table 2.1: Funds Allocation by International Funding Agencies.....	19
Table 2.2: UNAIDS Allocation in \$US by Principal Outcomes.....	20
Table 2.3: Budget Allocation by Agency and Regions (in \$US).....	21
Table 2.4: FY 2009 Approved Funding by PEPFAR and 2008 HIV/AIDS Epidemic Overview.....	22
Table 2.5: Approved Funding by the Global Fund by Region and Disease for Round 8, 2008.....	24
Table 2.6: Funding History from FY 2000 to FY 2010 for HIV/AIDS by CDC.....	25
Table 2.7: Estimated numbers of persons living with HIV infection (not AIDS) or with AIDS at the end of 2007, by area of residence—United States and dependent areas....	26
Table 2.8: Discretionary State/Formula Grants by CDC for HIV/AIDS Prevention and Surveillance Programs for State and Local Health Departments.....	28
Table 2.9: Distribution of Ryan White Program Funding by Region and Part, FY 2007. .....	31
Table 2.10: The Distribution of ACAP Allocation Funds (in CAD\$).....	33
Table 2.11: NGOs working to reduce spread of HIV/AIDS.....	34
Table 2.12: Financial Summary from September 2007-08 by FHI.....	35
Table 2.13: Financial Highlights by World Vision.....	36
Table 5.1: Summary of Notations.....	98
Table 5.2: LOE for multiplicative utility function when preferences for allocating based on equity are unknown at the upper level.....	101
Table 5.3: LOE for multiplicative utility function when infections prevented per dollar are unknown at the upper level.....	102
Table 6.1: Summary of Notations.....	121
Table 6.2: Conditions for existence of Nash equilibriums for “ $fg$ ” case.....	125
Table 6.3: Optimal $f^*$ and $IA_i^{fg}$ for “ $fg$ ” case.....	126
Table 6.4: Conditions for existence of Nash equilibriums for “ $g$ ” case.....	129
Table 6.5: Optimal $g^*$ and $IA_i^g$ for “ $g$ ” case.....	130
Table 6.6: Conditions for existence of Nash equilibriums for “ $f$ ” case.....	131
Table 6.7: Optimal $f^*$ and $IA_L^f$ for “ $f$ ” case.....	132
Table 5.4: Upper level problem for linear utility function at the lower level when $b$ is unknown.....	156
Table 5.5: Upper level problem for multiplicative utility function at the lower level when $b$ is unknown.....	157
Table 5.6: Upper level problem for linear utility function at the lower level when $h$ is unknown.....	158
Table 5.7: Upper level problem for multiplicative utility function at the lower level when $h$ is unknown.....	159

## LIST OF FIGURES

Figure 2.1: The Global Fund Proposal Process.....	37
Figure 2.2: Distribution of funding by geographic region by the Global Fund.....	38
Figure 2.3: Distribution of funding by type of Principal Recipient by the Global Fund.....	38
Figure 2.4: Funding process in Bill and Melinda Gates Foundation.....	39
Figure 3: Taxonomy Diagram.....	50
Figure 4.1: Threshold $f_t^L$ versus budget for different values of $c$ (coefficient of the total infections averted). Three lines are shown corresponding to the base case value of $c$ , $c/2$ , and $2c$ .....	72
Figure 4.2: Threshold values $b_u^L$ and $b_i^L$ for different values of $a$ , $b$ , and $c$ . ....	73
Figure 4.3: Threshold $f_t^L$ versus budget for different values of $n_1$ ( the size of the risk group with highest number of infections averted).....	74
Figure 4.4: Utility function versus $r$ .....	75
Figure 4.5: Optimal value $r^*$ versus $f$ .....	76
Figure 4.6: Total infections averted versus $f$ .....	77
Figure 4.7: Multiplicative utility function $U_M(r)$ versus $r$ .....	78
Figure 5.1:HIV Infections Averted versus budget for full information and asymmetric information for different values of $h_1$ (infections averted/\$ in program one).....	103
Figure 5.2:HIV Infections Averted versus budget for full information and asymmetric information for different values of $b$ (coefficient of the fraction of budget to be reserved for proportional allocation).....	104
Figure 5.3:Expected HIV infections averted for various values of $p$ for the two cases.....	105
Figure 6.1: HIV infections averted versus budget for both regions for all three cases.....	133

## Chapter 1

### Introduction

Human immunodeficiency virus (HIV)/ Acquired Immune Deficiency Syndrome (AIDS) prevention funds are often allocated at multiple levels. A national level decision maker may allocate funds to regional decision makers, who then distribute funds to local organizations, risk groups, or programs. For example, for the FY 2002-2006, the World Bank distributed US\$1.7 billion to national and regional AIDS programs that then dispersed funds to local programs, risk groups or organizations. In 2009, the Global Fund to Fight AIDS, Tuberculosis, and Malaria dispersed US\$ 9 billion to principal recipients (PRs) nominated by various countries through country coordinating mechanisms (CCMs) and these PRs further distributed funds to various sub-recipients to prevent and treat HIV/AIDS.

Simple allocation rules are often used by the decision makers at each level of the decision making process to help guide the allocation of HIV/AIDS prevention funds, e.g., allocating in proportion to HIV prevalence, incidence, or population size. The Global Fund's technical review panel (TRP) review proposals are based on technical merit, including the soundness of the approach, the feasibility of the proposal, its potential sustainability and the anticipated degree of impact. Reviewers make use of epidemiological information or the implementation of previous financing related to the proposal under review. TRP recommendations are made by consensus and if a consensus cannot be reached then the chair calls for a decision by majority vote of those present. However, these rules may lead to sub-optimal decisions.

Mathematical programming models have been developed to aid in the healthcare resource allocation processes at both single and multiple levels of decision making by effectively using constrained resources and significantly improving healthcare outcomes. My thesis investigates the impact of incentives to encourage the optimal allocation of prevention funds by developing a set of mathematical models. Specifically, the thesis addresses the following questions: Under what circumstances can financial

incentives help encourage the optimal allocation of prevention funds? And what is the optimal level of such incentives?

In Chapter 2, we provide an overview of major international aid organizations and their allocation processes for HIV/AIDS prevention funds. We then highlight several major issues of importance for researchers in the field of HIV/AIDS policy modeling as well as for policy makers with the aim to convey both the effectiveness, and challenges of the allocation of limited HIV/AIDS prevention funds. In Chapter 3, we review various resource allocation models and incentive-based models that are used in a variety of healthcare settings and for HIV/AIDS in particular.

In Chapter 4, we develop a dynamic programming model for a multi-level HIV/AIDS prevention funds allocation process in which a single upper-level decision maker (UD) uses incentives to promote optimal allocation by a single lower-level decision maker (LD) who then allocates funds to three programs. The UD uses an incentive scheme to encourage the LD to reduce the fraction of funds reserved for equity by making the amount received by the lower level dependent on this fraction. In particular, the upper level may withhold funds to encourage an allocation that is more efficient.

In Chapter 5, we incorporate information asymmetry in the model developed in Chapter 4. This Chapter consists of two cases. In the first case, we assume that the preferences of the LD with respect to allocating HIV/AIDS prevention funds based on equity are unknown to the UD. In the second case, the number of infections prevented per dollar in a program is known to the LD, but unknown to the UD.

In Chapter 6, we model an incentive-based multi-level resource allocation process with an UD allocating funds to two LDs who then allocate funds to three programs. The UD sets level of two types of incentives, between and within regions that maximizes the total number of infections averted and then the two LDs simultaneously set the fraction of the funds to be reserved for the proportional allocation and the amounts allocated to lower-level programs. The UD uses two types of incentives to encourage LDs to allocate optimally.

Our numerical analysis suggests that under certain conditions incentives can encourage an optimal allocation based on the possession of symmetric and asymmetric information. However, there is a loss of efficiency when we compare an asymmetric information case with a symmetric information case. Finally, in Chapter 7, we describe the lessons learned in the doctoral research.

## Chapter 2

### **Practical Applications of HIV/AIDS Prevention Funds Allocation Process**

Funding for HIV/AIDS has increased considerably in the last decade. In 2007, the estimated funding to prevent HIV/AIDS worldwide was approximately \$10 billion (Table 2.1), an almost forty-fold increase since 1996, when the funding was \$260 million [1]. This increase from a “millions” to a “billions” of dollars was largely due to a series of international funding initiatives.

Governments coordinate the majority of the international funding initiatives (Table 2.1). Examples include the joint United Nations programme on HIV/AIDS (UNAIDS); the World Bank's Global AIDS Programme; World Health Organization (WHO); the U.S. President's Emergency Plan for AIDS Relief (PEPFAR); the Global Fund for AIDS, Tuberculosis and Malaria; and the Gates Foundation. The major national funding agencies to prevent HIV/AIDS in the U.S. are: the Centers for Disease Control and Prevention (CDC), and the Ryan White Comprehensive AIDS Resources Emergency (CARE) Act (RWCA); in Canada, there is the Federal Initiative to Address HIV/AIDS.

#### **2.1 United Nations Programme on HIV/AIDS (UNAIDS)**

The Joint United Nations Programme on HIV/AIDS (UNAIDS) coordinates the HIV/AIDS related efforts of the UNAIDS Secretariat and 10 funds, programmes, and agencies of the UN system organizations. These 10 agencies are: The Office of the United Nations High Commissioner for Refugees (UNHCR), The United Nations Children's Fund (UNICEF), The World Food Programme (WFP), The United Nations Development Programme (UNDP), The United Nations Population Fund (UNFPA), The United Nations Office on Drugs and Crime (UNODC), The International Labour Organization (ILO), The United Nations Educational, Scientific and Cultural Organization (UNESCO), The World Health Organization (WHO), and The World Bank. The UNAIDS Secretariat has staff in more than 80 countries and headquarters in



Geneva, Switzerland. Its stated goals are to carry out work in the areas of leadership, mobilization, planning, financing, technical assistance, human rights, gender discrimination, and most at-risk populations, etc. (Table 2.2) [1].

UNAIDS helps various countries launch their national AIDS programmes. It also keeps track of the financial resources that are required at the global and country levels in order to generate information about the epidemic and the response to it. In 2008-09, UNAIDS spent approximately US\$484,820,000 on HIV/AIDS prevention and treatment programmes [2]. Table 2.3 provides the budget allocation at global and regional levels [3].

The UNAIDS Programme Coordinating Board (PCB) approves how the Unified Budget and Work plan (UBW) are detailed. The UBW Performance Monitoring and Evaluation Framework then make use of the qualitative and quantitative indicators to monitor the results of the programme at the country level. It covers information about the implementation of activities and expenditures and incorporates a mid-term review based on two criteria to determine the release of funds for the following year. These criteria are (i) progress against indicators, and (ii) the implementation of the allocated funds. Thus, funding decisions made by the PCB are based on performance. A UBW information system tracks expenditures and the results of investments of each Cosponsor, the Secretariat, and Interagency activities at a country level [2].

## **2.2. World Bank Global AIDS Program**

The World Bank provides financial and technical support by providing low-interest loans, interest-free credits and grants to developing countries to invest in education, health, environment, agriculture, and other areas. The Bank consists of two institutions: the International Bank for Reconstruction and Development (IBRD), and the International Development Association (IDA), which has 186 member countries. The IBRD focuses on middle income and poor countries, while IDA focuses on the poorest countries. For the poorest countries, the grants could be 100% financed, in contrast to middle income countries where they are only partially financed [4].

The Bank works with other UN agencies to prevent HIV/AIDS in various countries by implementing different activities such as providing policy advice on how to design build and monitor evaluation systems; providing technical and financial support to national AIDS authorities, private, and public sectors; and developing evidence-based AIDS strategies and action plans. The bank contributes to the knowledge base for HIV/AIDS prevention, treatment, and care through policy research. It also conducts research and reports on global surveillance of HIV/AIDS and related risk behaviours. The World Bank has committed about US\$1.7 billion through grants, loans and credits to programs to prevent HIV/AIDS since 2002 [4].

The World Bank uses its evaluation system to assess the progress of its activities and submits reports in the form of case studies from specific countries or regions or on major initiatives to the Joint Programme of UNAIDS. For 2008–2009, the report supplemented by evaluation study, called the “Evaluation of the World Bank’s Assistance to AIDS National Coordination Authorities” was submitted to the UBW Performance Monitoring and Evaluation Framework [4].

### **2.3. World Health Organization (WHO)**

WHO is a part of the United Nations System, which includes membership from 193 countries with six regional offices and headquarters in Geneva, Switzerland. It coordinates and directs various international healthcare activities in 85 countries by partnering with UN Agencies, Ministries of Health, Non-Governmental Organizations (NGOs), Community-Based Organizations (CBOs), health service providers, health care institutions, and people living with HIV/AIDS to help, plan, and implement programmes to prevent HIV/AIDS. The HIV/AIDS policies and the annual budget are set at the headquarters by the HIV/AIDS team whereas the policies at the regional level are set by the regional offices and are specific to the needs in their regions [5].

WHO spent \$3.3 billion US in 2006-07, out of which 70% came from donations from various countries, agencies, and other partners; the remaining one quarter came from regular “dues” from Member States. WHO allocates its budget to crucial health interventions such as response to epidemics such as HIV/AIDS and the reduction of

child mortality, health systems policies such as quality of medicines, determinants of health such as nutrition and tobacco usage, and Member States. In 2006-2007, WHO allocated 53% of its budget to health interventions; 21% to support Member States; 13% to health systems, policies, and products; and 11% to determinants of health [5].

In 2003, UNAIDS and WHO jointly launched the “3 by 5” initiative in which the target was to provide antiretroviral treatment (ART) to 3 million people living with HIV/AIDS in low- and middle-income countries by 2005. By 2006, 2,040,000 were receiving ART [6]. In 2005, UNAIDS and WHO set a goal of universal access to HIV/AIDS prevention, treatment, care, and support by 2010 which consisted of five objectives in which one of them was to maximize the health sector’s contribution to HIV/AIDS prevention. For its prevention efforts, WHO focuses on evidence-based interventions targeted towards at-risk populations such as men who have sex with men, injection drug users (IDUs), prisoners, etc. The organization aims to prevent HIV/AIDS transmission among vulnerable populations and to promote interventions in high-prevalence regions [5].

WHO uses a monitoring and reporting framework for the key areas such as universal access to testing, counseling, prevention in health care settings, sexual HIV transmission, and transmission through injection drug use, treatment and care, sexually transmitted infections control, and drug procurement, health financing and health information systems based on various indicators [5].

#### **2.4. U.S. President’s Emergency Plan for AIDS Relief (PEPFAR)**

President George W. Bush launched a health initiative, the U.S. President’s Emergency Plan for AIDS Relief (PEPFAR) in 2003, which was renewed on July 30, 2008 for 5 more years, authorizing up to \$48 billion to combat HIV/AIDS, tuberculosis, and malaria globally. Of this amount, \$39 billion was for PEPFAR bilateral HIV/AIDS programs and for the Global Fund to fight AIDS, Tuberculosis and Malaria [7].

To ensure a unified approach to prevent an HIV/AIDS epidemic, a new system was established at every level of the U.S. Government under the leadership of the U.S.

Global AIDS Coordinator (OGAC). Primary implementing departments and agencies are the Department of State (DoS), the U.S. Agency for International Development (USAID), the Department of Defense (DoD), the Department of Commerce (DoC), the Department of Labor (DoL), the Department of Health and Human Services (HHS), the Peace Corps [7].

Through FY 2013, PEPFAR plans to support life-saving treatments for 3 million HIV/AIDS-infected men, women and children, to seek the prevention of 12 million new infections, and to care for 12 million orphans and vulnerable children (Table 4). PEPFAR supports evidence-based prevention programs by targeting interventions based on the epidemiology of HIV/AIDS infection in each country by reducing sexual transmission, preventing mother-to-child transmission, and reducing the transmission of HIV/AIDS through unsafe blood and medical injections, and male circumcision [7]. Table 2.4 shows country-by-country approved funding by PEPFAR and provides an overview of the HIV/AIDS epidemic.

In 2003, PEPFAR excluded the purchase of generic drugs with PEPFAR funds. The funding agreement required that the drugs purchased with PEPFAR funding must be approved by the U.S. Food and Drug Administration (FDA) or a regulatory agency in Canada, Japan, or Western Europe [8]. WHO had previously approved generic drugs, which are less costly than anti-retroviral medications for HIV/AIDS treatment. However, that policy was no longer sufficient under the PEPFAR regulations. Thus, the rollout of lifesaving drugs slowed as 70 percent of antiretroviral drugs bought in Nigeria, Haiti, and Zambia are expected to be generic. In 2006, the FDA approved nearly 30 generic HIV/AIDS drugs. However, none of these could be distributed by PEPFAR because several African countries refused to trust the FDA, and insisted that the drugs be approved by WHO before importing them [9]. To solve this problem, FDA officials shared their files on the drugs with WHO so that WHO could add them to its list of approved medicines [9]. PEPFAR eventually began distributing generics by the end of 2005 and in FY 2007 some 73% of all the antiretroviral drugs delivered by PEPFAR were generic [10].

PEPFAR does not support needle and syringe exchange programmes. However, many people have objected, as needle exchange programmes have proved beneficial to protect IDUs from HIV transmission. PEPFAR focuses on abstinence until marriage programs rather than on sex education and condom distribution [11].

The economic crisis could affect the delivery of PEPFAR funding in 2010. Domestic concerns may take precedence over the global health initiative, resulting in flat funding for PEPFAR [12]. In a Joint Clinical Research Centre in Uganda, clinics have been forced to stop enrolling new patients due to the uncertainty in PEPFAR's budget [13]. At times, the controversial areas have overshadowed what has already been achieved by PEPFAR.

## **2.5. Global Fund to Fight AIDS, Tuberculosis and Malaria**

The Global Fund to Fight AIDS, Tuberculosis and Malaria, established in 2002, is collaboration between governments, the private sector, and affected communities. It attracts and allocates constrained resource to prevent and treat HIV/AIDS, tuberculosis and malaria. The Global Fund works in partnership with bilateral and multilateral organizations to support efforts related to the three diseases. Since 2002, the Global Fund, with its total budget of US\$ 15.6 billion for more than 572 programs in 140 countries, has allocated a quarter of all its budget to prevent HIV/AIDS and the remaining funds to prevent tuberculosis and malaria [14]. In November 2008, the Global Fund approved US\$1.164 billion to prevent and treat HIV/AIDS in 37 countries. Table 2.5 sets out the funding allocations by region and disease.

The funding process works as follows: The Global Fund's Board issues a call for proposals annually. Various countries then submit their proposals to the Global Fund through a country coordinating mechanism (CCM) based on priority needs at the national level. The Global Fund Secretariat screens the submitted proposals and the Technical Review Panel (TRP) reviews eligible grant proposals with respect to their technical merit, based on the soundness of the approach, feasibility and potential for sustainability and impact. Reviewers may make use of epidemiological information or the efficiency of the implementation of previous financing concerning the proposal

under review to evaluate a proposal. TRP's recommendations are based on consensus, and if a consensus cannot be reached, then decisions are based on a majority vote of those present. The TRP then provides funding recommendations to the Board. The Board then makes the appropriate funding decisions by consensus (Figure 2.1). The Global Fund signs a legal grant agreement with Principal Recipients (PRs), designated by the CCM. PRs directly receive grants from the Global Fund to prevent and treat HIV/AIDS and to pass the funds on to the sub-recipients. There can be multiple PRs in one country. Additional funding can be requested by PRs based on their demonstrated progress towards the intended results. Figures 2.2 and 2.3 show the distribution of funding by geographic region and by type of PR, respectively [14].

## **2.6. Bill and Melinda Gates Foundation**

The Bill and Melinda Gates Foundation helps people in developing countries escape their poverty and in the U.S. it supports people with the fewest resources to gain access to various opportunities to succeed in school and life. The Foundation has its headquarters in Seattle, Washington, and has four regional offices in three different countries to manage three core programs. The Global Development Program helps people in developing countries to escape hunger and poverty. The Global Health Program focuses on discovering insights to fight serious diseases, developing effective and affordable vaccines and medicines, and delivering proven health solutions in developing countries. The United States Program focuses on improving public education. The Foundation has 781 employees and an endowment of US\$30.8 billion. It has granted US\$20.1 billion since its inception. In 2008, the foundation granted US\$2.8 billion to support programs in more than 100 countries [15].

The Global Health Program focuses on diseases that cause the highest levels of illness and death in developing countries. These diseases include HIV/AIDS. The Foundation has worked in partnership with The Collaboration for AIDS Vaccine Discovery, the Consortium to Respond Effectively to the AIDS and TB Epidemic, The Global Fund, and PEPFAR to prevent HIV/AIDS. It awarded US\$338 million for the India AIDS Initiative, US\$33 million to improve TB control strategies in China, and US\$86 million

to develop a new and low-cost diagnosis for HIV/AIDS. In total, the Foundation has awarded US\$ 424 million to prevent and treat HIV/AIDS [15].

The Foundation awards the majority of its grants to U.S. tax-exempt organizations called grantees, which are identified by their staff. Grantees then work with beneficiaries in the field to manage the three core programs (Figure 2.4). A small percentage of the grant-making is done by issuing requests for proposals (RFPs). Proposals are prioritized based on measurable results, the use of preventive approaches, and the promise of significant and long-lasting change. If a proposal is accepted, then a grant is issued. Grantees are expected to measure their progress and to report their results to the Foundation [15].

## **2.7. Centers for Disease Control and Prevention (CDC)**

The Centers for Disease Control and Prevention (CDC) is a major component of the Department of Health and Human Services (HHS). CDC's top organizational component includes the Office of the Director as well as six Coordinating Centers and Offices and the National Institute for Occupational Safety and Health (NIOSH). The Office of the Director coordinates and directs all CDC activities and medical programs. The six coordinating centers include: the Coordinating Center for Environmental Health and Injury Prevention (CCEHIP), the Coordinating Center for Health Information and Service (CCHIS), the Coordinating Center for Health Promotion (CCHP), the Coordinating Center for Infectious Diseases (CCID), the Coordinating Office for Global Health (COGH), the Coordinating Office for Terrorism Preparedness and Emergency Response (COTPER) [16]. NIOSH ensures safety and health for all those in the workplace through research and prevention. CCID includes the National Center for HIV/AIDS, Viral Hepatitis, STD, and TB Prevention (NCHHSTP), which maximizes public health and safety through prevention, control of disease, and death due to HIV/AIDS, Viral Hepatitis, STDs, and TB [16]. In FY 2009, CDC's budget was \$6.3 billion. Of this, \$1,947,827,000 was for CCID, and of this \$1,006,375,000 was for HIV/AIDS, Viral Hepatitis, STD, and TB prevention programs. For FY 2010, CDC has requested US\$6,389 million including \$2,019,622,000 for CCID that again includes

\$1,060,299,000 for HIV/AIDS, Viral Hepatitis, STD, and TB prevention programs. The CDC awards nearly 85% of its budget through grants and contracts [17].

To help control the HIV/AIDS epidemic, CDC works with community, state, national, and international partners in a variety of surveillance, research, prevention, and evaluation activities. Most of CDC's HIV/AIDS prevention efforts are the responsibility of the Coordinating Center for Infectious Diseases (CCID), and the National Center for HIV/AIDS, Viral Hepatitis, STD, and TB Prevention (NCHHSTP). Within this Center, there are the two Divisions of HIV/AIDS Prevention (DHAP) that seek to prevent HIV/AIDS infection and reduce the incidence of HIV/AIDS-related illness and death. The Division of HIV/AIDS Prevention–Intervention Research and Support (DHAP-IRS) provides support for HIV/AIDS prevention research and the development, implementation, and evaluation of evidence-based HIV/AIDS prevention programs. The Division of HIV/AIDS Prevention–Surveillance and Epidemiology (DHAP–SE) conduct HIV/AIDS prevention research, and surveillance as well as the development, and testing of effective biomedical interventions to reduce HIV transmission and disease progression [16]. In FY 2009, CDC's budget for HIV/AIDS domestic and research was \$691,860,000 and for the FY 2010, CDC has requested \$744,914,000. Table 2.6 shows the funding history of CDC from FY 2000 to FY 2010 for HIV/AIDS [17].

CDC allocates funds to 65 states and local governments called “grantees” who then distribute funds to risk groups, local programs, and organizations. All grantees have one or more community planning groups (CPGs) that identify a list of priority populations based on risk behaviour, gender, and race/ethnicity categories. CPGs provides a comprehensive plan for HIV/AIDS prevention consisting of priority populations ranked from lowest to highest priority by the health departments. Based on CPG's plan, the health department prepares a proposal to be submitted to CDC. Following the award from CDC, grantees issue requests for proposals (RFPs). NGOs/CBOs/FBOs or local organizations then submit proposals and further distribution of funds takes place [16]. Table 2.7 shows a list of living HIV/AIDS cases by areas of residence and Table 2.8 shows CDC's funding for HIV/AIDS prevention and surveillance programs by state and local health departments for FY 2008.



## **2.8. The Ryan White Comprehensive AIDS Resources Emergency Act (RWCA)**

The Health Resources and Services Administration (HRSA) is a federal agency of the U.S. Department of Health and Human Services established in 1982. It consists of 6 bureaus and 13 offices. It helps to improve access to health care services for uninsured, underserved, and special needs populations as well as people living with HIV/AIDS, pregnant women, mothers and children by providing financial support to health care providers in every state and U.S. territory. It also supports programs that protect civilians against bioterrorism, and that compensate individuals harmed by a vaccination; it also maintains databases that protect against health care malpractice and health care waste, fraud and abuse. In FY 2008, HRSA provided health care to 23 million people with a budget of \$7 billion [18].

The agency's Ryan White Comprehensive AIDS Resources Emergency (CARE) Act (RWCA) allocates funds to various cities, states, public, and private entities to support underserved people suffering from HIV/AIDS. In FY 2008, RWCA allocated \$2.1 billion to five major parts (A, B, C, D, and E) described below [19].

Part A provides grants to Eligible Metropolitan Areas (EMAs), an area with a population of 50,000 or more and over 2000 reported AIDS cases in the last five years, and to Transitional Grant Areas (TGAs), defined as an area with 1000-2000 new AIDS cases in last 5 years. In FY 2008, \$627.148 million was awarded to 22 EMAs and 34 TGAs. Part A funding includes formula grants, supplemental grants, and Minority AIDS Initiative Funds for minority populations. Formula grants are based on the number of reported HIV/AIDS cases, Supplemental grants are based on need and other criteria, and Minority AIDS Initiative Funds are based on need as well as the distribution of minority populations living with HIV/AIDS [19].

Part B provides grants to all 50 states, the District of Columbia, Puerto Rico, Guam, the U.S. Virgin Islands and 5 U.S. Pacific Territories and Associated Jurisdictions. Part B awarded \$1.195 million in FY 2008 including a base grant to States and Territories using a formula based on the number of people living with HIV/AIDS, \$808.5 million to the AIDS Drug Assistance Program (ADAP), \$5 million to supplemental grants for

States with “Emerging Communities” having 500-1000 reported AIDS cases, and \$7 million to the Minority AIDS Initiative. ADAP provides medication to people suffering from HIV/AIDS [19].

In FY 2008, Part C granted \$198.754 million for planning, capacity and development, and early intervention services and Part D granted \$73.69 million for family-centered care, support, logistics, and coordination services. Part E granted \$34.09 million to AIDS Educational Training Centers, \$12.85 million to Dental Reimbursement Program, and \$25 million was set aside for Special Projects of National Significance (SPNS) [19]. Table 2.9 shows distribution of Ryan White Program Funding by Region and Part for FY 2007.

The Ryan White HIV/AIDS Treatment Modernization Act of 2006 directs 75% of the total funds to core services such as medical, dental and prescription assistance and also allows greater flexibility to the U.S. Secretary of Health and Human Services to reallocate funds to respond to changing epidemic requirements.

## **2.9. The Federal Initiative to Address HIV/AIDS in Canada and Abroad**

The Federal Initiative was launched to: address HIV/AIDS in Canada and abroad--including the prevention of a number of new infections--reduce the social and economic impact of HIV/AIDS, slow the progression of the disease to improve quality of life, and contribute to the global relief effort. The Federal Initiative is a partnership between the Public Health Agency of Canada (PHAC), Health Canada, the Canadian Institutes of Health Research (CIHR), and the Correctional Services Canada. It provides funding for prevention, support programs, research, surveillance, public awareness, and evaluation for HIV/AIDS.

PHAC is also responsible for the coordination of the Federal Initiative and has a budget of \$13,900,000 for preventing and treating HIV/AIDS. In addition, it is responsible for HIV/AIDS communications, social marketing, national and regional programs, policy development, surveillance, laboratory science and global engagement focusing on technical assistance, and policy advice. PHAC funds national and regional level

programs. Funds are distributed to five national level programs to support a voluntary-sector response; help engage people living with HIV/AIDS; encourage an integrated approach to disease prevention; enhance the capacity of individuals, organizations to respond to the epidemic; enable the development of effective interventions, and enhance a broader response to the HIV/AIDS epidemic.

PHAC funds regional level programs through six regional offices and the Northern Secretariat responsible for administering the AIDS Community Action Program (ACAP), which provides funding to different regions. For FY 2005-06, ACAP grants were allocated based on weighted criteria that consisted of 40% allocation based on population, 25% based on base amount for each province and territory, 25% based on provincial/territorial rates of AIDS cases per million, 10% based on the extent to which funding is available from provincial/territorial governments for ACAP-type activities. For FY 2006-07, 2007-08, 2008-09, grants were based on the new framework using three principles: burden, vulnerability, and equity with the weightings that result in these allocations being directed towards those geographic areas that have the highest proportions of vulnerable populations to HIV/AIDS. Table 2.10 shows the distribution of ACAP funds.

Health Canada is responsible for community-based HIV/AIDS education, prevention, and related services for First Nations and Inuit communities. It provides funding to support global engagement to non-profit organizations and institutions through the HIV/AIDS Global Engagement Grants Programme. CIHR is responsible for setting priorities and administering the research program. It provides funding to support research and helps build research capacity. Correctional Service Canada is responsible for providing services related to the prevention, care, treatment of HIV/AIDS to prisoners.

### **2.10. Non-Governmental Organizations (NGOs)**

Non-Governmental Organizations (NGOs) are legally constituted with no participation of any government and they maintain their status by excluding government representatives from membership in the organization. NGOs emphasize humanitarian

issues, socio-economic and sustainable development. For example, the International Federation of Red Cross and the Red Crescent Societies work to eliminate stigma and discrimination for people living with HIV/AIDS. There are many other NGOs working to reduce the spread of HIV/AIDS such as the Family Health International (FHI), the Global Network of People Living with HIV, the International HIV/AIDS Alliance, AVERT, and the International Council for AIDS Service Organizations. Table 2.11 lists the major NGOs and their budgets.

Governments collaborate with NGOs to prevent HIV/AIDS. For example, FHI implemented the AIDS Control and Prevention (AIDSCAP) project funded by the United States Agency for International Development (USAID) in 54 countries including Latin America and the Caribbean. AIDSCAP worked with over 500 different NGOs, community groups, and universities, reached 19 million people and distributed more than 254 million condoms. FHI gets revenue support from the U.S. government, and other governments as well as from a variety of foundations, individuals, multilaterals, and corporations (Table 2.12). In 2008, FHI spent 85% of its operational funds, \$322.290 million US, in preventing and treating HIV/AIDS.

### **2.11. Faith Based Organizations (FBOs)**

Faith Based Organizations (FBOs), are defined by USAID, as groups of individuals volunteering for a stated spiritual or belief system. FBOs often have a good understanding of the local culture and are able to reach isolated areas due to their organizational network, and thus, can effectively work in rural parts of poor countries. FBOs are major providers of care and support for people living with HIV/AIDS in developing countries. FBOs collaborate with major agencies to provide counseling and testing services to people suffering from HIV/AIDS. For example, World Vision collaborated with USAID to implement innovative HIV/AIDS prevention strategies in Asia among high-risk groups. Revenue sources and operating expenses of World Vision are given in Table 2.13. Faith Summit, 2010 organized by the Art of Living Foundation, an FBO in India brought over 500 spiritual leaders together against HIV/AIDS. They delved into specific action plan to address HIV/AIDS across all states in India. In Uganda, religious leaders of Roman Catholic, Anglican, and Muslim faiths, worked with

the Ministry of Health to employ funds from the World Health Organization Global Program on AIDS (WHO/GPA) to prevent HIV/AIDS infections.

### **2.12. Community Based Organizations (CBOs)**

Community Based Organizations (CBOs) are non-profit organizations run primarily by volunteers that provide social services at the local level. CBOs are valued for their vital contributions to the health and well-being of the society. Many CBOs receive funding from a variety of sources including grants, donations, fees, and fundraising. But government is the primary source of funding for most of these agencies. For example, USAID granted \$2 million to CBOs to care and support people living with HIV/AIDS under the new Community REACH (Rapid and Effective Action Combating HIV/AIDS) program. The “Community REACH” program is designed to promote community-based programs. These programs issue requests for applications and NGOs and CBOs working on either local or worldwide bases in selected countries are eligible to apply for grants. Many CBOs in different countries have received grants from the Community REACH program, e.g., the Dawn of Hope Ethiopia Association, CARE-Rwanda, Cambodian HIV/AIDS Education and Care.

### **2.13 Summary**

We have described many international funding initiatives that are distributing up to billions of dollars to prevent and treat HIV/AIDS. These organizations use various approaches to make funding decisions. For example, they use qualitative and quantitative indicators, epidemiological information, and the efficiency of the implementation of previous financing as well as consensus or majority vote decision making. While these approaches may be useful for allocating treatment funds, for allocating prevention funds they may lead to sub-optimal allocations since they may be based on either consensus or vote [20]. Further, there is a lack of coordination between the different agencies sharing the responsibility of HIV/AIDS prevention since these agencies are funded through different sources, serve different constituents, have different responsibilities other than the HIV/AIDS issue, and report to different committees. These agencies also compete for funding and public attention. Hence,

federal leadership is needed to coordinate these agencies and their activities as well as other organizations like NGOs, CBOs, and FBOs in order to prevent, cure, and treat the maximum number of people suffering from HIV/AIDS [20].

Further, there is lack of tools to translate research findings into action at the community level, and the way in which activities are prioritized, conducted, monitored, and assessed needs a more pragmatic and reasonable base[20].

Table 2.1: Funds Allocation by International Funding Agencies

Funding Initiative	(in US\$)
UNAIDS	484,820,000
World Bank`s Global AIDS Program	1,700,000,000
WHO	93,300,000
PEPFAR	3,733,100,000
Global Fund to Fight AIDS, Tuberculosis, and Malaria	1,164,000,000
Gates Foundation	457,000,000
Federal Initiative to Address HIV/AIDS in Canada	13,900,000

Table 2.2: UNAIDS Allocation in \$US by Principal Outcomes<sup>1</sup>.

Principal Outcomes	Revised Budget for 2008–2009 (\$US)
1. Leadership and Resource Mobilization	218,009,374
2. Planning, financing, technical assistance and coordination	107,411,487
3. Strengthened evidence base and accountability	30,520,600
4. Human resources and systems capacities	45,615,495
5. Human rights, gender, stigma and discrimination	29,855,935
6. Most at-risk populations	16,090,000
7. Women and girls, young people, children and populations of humanitarian concern	32,317,109
Contingency	5,000,000
Total	484,820,000

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<sup>1</sup>Source: UNAIDS,  
[http://www.unaids.org/en/KnowledgeCentre/HIVData/GlobalReport/2008/2008\\_Global\\_report.asp](http://www.unaids.org/en/KnowledgeCentre/HIVData/GlobalReport/2008/2008_Global_report.asp)



Table 2.3: Budget Allocation by Agency and Regions (in \$US)<sup>2</sup>.

Agency	Sub-Saharan Africa	Middle East and N. Africa	Asia and Pacific	Europe and Central Asia	Americas	Global
UNHCR	2,830,000	1,356,042	1,650,833	766,458	353,750	8,607,917
UNICEF	18,999,576	3,194,984	13,637,783	5,061,392	5,611,611	46,195,654
WFP	3,679,250	1,132,330	3,113,634	566,714	566,714	13,592,358
UNDP	12,000,000	5,460,000	8,500,000	6,000,000	5,000,000	10,500,000
UNFPA	35,817,440	2,463,800	20,981,440	4,714,910	4,947,210	16,665,200
UNODC	3,533,433	3,533,433	15,435,522	14,877,612	3,533,433	21,386,567
ILO	8,000,000	2,250,000	4,800,000	3,400,000	3,400,000	8,050,000
UNESCO	10,864,400	865,000	7,647,400	3,251,600	5,639,800	6,331,800
WHO	50,703,984	3,413,895	18,012,118	7,108,035	10,802,175	39,259,793
World Bank	9,360,000	1,086,000	5,496,000	1,550,000	1,850,000	27,660,000
Secretariat	38,814,040	11,415,894	28,539,735	19,026,490	17,123,842	67,480,000
Inter-agency	63,883,265	10,560,929	30,233,247	20,811,242	18,326,318	2,935,000
Total	258,485,388	46,732,306	158,047,713	87,134,453	77,154,852	268,664,289

<sup>2</sup>Source: UNAIDS,  
[http://www.unaids.org/en/KnowledgeCentre/HIVData/GlobalReport/2008/2008\\_Global\\_report.asp](http://www.unaids.org/en/KnowledgeCentre/HIVData/GlobalReport/2008/2008_Global_report.asp)

Table 2.4: FY 2009 Approved Funding by PEPFAR and 2008 HIV/AIDS Epidemic Overview<sup>3</sup>.

Region/Country	Sum of Approved Funding (in US\$ millions) for FY 2009	Adult HIV Prevalence Rate (%)	Number of People Living with HIV	Number of Orphans Due to AIDS
<b>Africa</b>	<b>\$ 3,430.6</b>			
Angola	\$ 7.0	2.1	190,000	50,000
Botswana	\$ 91.2	23.9	300,000	95,000
Cote d'Ivoire	\$ 116.0	3.9	480,000	420,000
Democratic Republic of Congo	\$ 16.1	1.2-1.5	400,000-500,000	
Ethiopia	\$ 310.9	2.1	980,000	650,000
Ghana	\$ 5.3	1.9	260,000	160,000
Kenya	\$ 528.9	7.8	1,100,000	
Lesotho	\$ 12.1	23.2	270,000	110,000
Malawi	\$ 25.2	11.9	930,000	550,000
Mozambique	\$ 202.2	12.5	1,500,000	400,000
Namibia	\$ 106.8	15.3	200,000	
Nigeria	\$ 440.6	3.1	2,600,000	1,200,000
Rwanda	\$ 122.6	2.8	150,000	220,000
South Africa	\$ 546.3	18.1	5,700,000	1,400,000
Sudan	\$ 8.8	1.4	320,000	
Swaziland	\$ 14.3	26.1	190,000	56,000
Tanzania	\$ 301.1	6.2	1,400,000	970,000
Uganda	\$ 282.4	5.4	940,000	1,200,000
Zambia	\$ 266.3	15.2	1,100,000	600,000
Zimbabwe	\$ 26.5	15.3	1,300,000	1,000,000
<b>East Asia and Pacific</b>	<b>\$ 126.3</b>			
Cambodia	\$ 18.0	0.8	75,000	
China	\$ 10.3	0.1	700,000	
Indonesia	\$ 7.8	0.2	270,000	
Thailand	\$ 5.5	1.4	610,000	
Vietnam	\$ 84.7	0.5	290,000	
<b>Europe and Eurasia</b>	<b>\$ 14.7</b>			
Russia	\$ 8.0	1.1	940,000	
Ukraine	\$ 6.7	1.6	440,000	
<b>South and Central Asia</b>	<b>\$ 29.3</b>			
India	\$ 29.3	0.3	2,400,000	
<b>Western Hemisphere</b>	<b>\$ 132.3</b>			

<sup>3</sup>Source: PEPFAR, <http://www.pepfar.gov/documents/organization/124050.pdf>

Table 2.4:Continued.

Region/Country	Sum of Approved Funding (in US\$ millions) for FY 2009	Adult HIV Prevalence Rate (%)	Number of People Living with HIV	Number of Orphans Due to AIDS
Antigua and Barbuda**				
Bahamas**		3.0	6,200	
Barbados**		1.2	2,200	
Belize**		2.1	3,600	
Dominica**				
Dominican Republic	\$ 8.3	1.1	62,000	
Grenada**				
Guyana	\$ 20.5	2.5	13,000	
Haiti	\$ 100.5	2.2	120,000	
Jamaica**		1.6	27,000	
St. Kitts and Nevis**				
Saint Luda**				
St. Vincent and the Grenadines**				
Suriname**		2.4	6,800	
Trinidad and Tobago **		1.5	14,000	
<b>Total</b>	<b>\$3,733.1</b>			

This table provides estimates of HIV/AIDS epidemic as of December 2007. The adult prevalence rate provides proportion of adults (15-49 years) living with HIV in 2007. The number of people living with HIV provides an estimate of adult and children with HIV infection in 2007. The number of orphans due to AIDS represents the estimated number of children (0-17 years) in 2007 who have lost one or both parents to AIDS. For countries where no recent data were available, country specific estimates have not been listed in the table.

\*Adult ages 15-49, Kenya AIDS Indicator Survey 2007

\*\*Countries that comprise the Caribbean Region Platform were awarded US\$3.0 million

Table 2.4 explanation: Provides an overview of the HIV/AIDS epidemic in PEPFAR countries/regional platforms, organized by region of PEPFAR investment. Only countries/regional platforms preparing PEPFAR operational plans, reflecting most of the PEPFAR country investments, are included in the table above.

Table 2.5: Approved Funding by the Global Fund by Region and Disease for Round 8, 2008<sup>4</sup>.

Region	Approved Funding (in US\$ millions)
East Asia and the Pacific	387.4 for 17 programs in 9 countries
Eastern Europe & Central Asia	141.9 for 16 programs in 9 countries
Latin America & the Caribbean	161.9 for 11 programs in 10 countries
North Africa & the Middle East	147.4 for 10 programs in 7 countries
South Asia	98.3 for 8 programs in 5 countries
Sub-Saharan Africa: East Africa	796.1 for 15 programs in 10 countries
Sub-Saharan Africa: Southern Africa	414.0 for 13 programs in 5 countries
Sub-Saharan Africa: West & Central Africa	912.3 for 20 programs in 13 countries
Disease	
HIV/AIDS	1.164 for programs in 37 countries
Malaria	1.568 for programs in 28 countries
Tuberculosis	327 for programs in 29 countries

<sup>4</sup>Source: The Global Fund, <http://www.theglobalfund.org/en/trp/reports/>

Table 2.6: Funding History from FY 2000 to FY 2010 for HIV/AIDS by CDC<sup>5</sup>.

Fiscal Year	Research and Domestic HIV Prevention (Infectious Disease)	Other Domestic HIV Prevention	Global AIDS Program <sup>6</sup>	CDC-Wide HIV Total <sup>7</sup>
2000	\$564,458,000	\$87,706,000	\$35,000,000	\$687,164,000
2001	\$653,462,000	\$96,199,000	\$104,527,000	\$854,188,000
2002	\$689,169,000	\$96,038,000	\$168,720,000	\$953,927,000
2003 <sup>1</sup>	\$699,620,000	\$93,977,000	\$182,569,000	\$976,166,000
2004 <sup>8</sup>	\$667,940,000	\$70,032,000	\$266,864,000	\$1,004,836,000
2005	\$662,267,000	\$69,438,000	\$123,830,000	\$855,535,000
2006 <sup>9</sup>	\$651,657,000	\$64,008,000	\$122,560,000	\$838,225,000
2007	\$695,454,000	\$62,802,000	\$120,985,000	\$879,241,000
2008	\$691,860,000	\$40,223,000	\$118,863,000	\$850,946,000
2009	\$691,860,000	\$40,223,000	\$118,863,000	\$850,946,000
2010	\$744,914,000	\$40,223,000	\$118,979,000	\$904,116,000

Note: Global AIDS amounts include funding for the Prevention of Mother to Child HIV Transmission initiative, which was transferred to the Department of State Office of the Global AIDS Coordinator in 2005.

<sup>5</sup>Source: Department of Human and Health Services, FY 2010, CDC  
[http://www.cdc.gov/fmo/topic/Budget%20Information/appropriations\\_budget\\_form\\_pdf/FY2010\\_CDC\\_CJ\\_Final.pdf](http://www.cdc.gov/fmo/topic/Budget%20Information/appropriations_budget_form_pdf/FY2010_CDC_CJ_Final.pdf)

<sup>6</sup>Amount for Global AIDS Program does not include PEPFAR funding.

<sup>7</sup>From 2000 to 2003 CDC-wide HIV/AIDS funding is comprised of specific activities within the National Center for HIV/AIDS, Viral Hepatitis, STD, and TB Prevention (NCHHSTP), the National Center for Chronic Disease Prevention and Health Promotion (NCCDPHP), and the National Center for Infectious Diseases (NCID). CDC-wide HIV/AIDS amounts shown for 2004 to 2007 are comprised of activities conducted by NCHHSTP, other parts of the Coordinating Center for Infectious Diseases, NCCDPHP, and the National Center for Birth Defects and Developmental Disabilities (NCBDDD). For the 2010 budget submission, funds supporting hemophilia/HIV activities in NCBDDDP and for oral health/HIV, BRFS/HIV, and Safe Motherhood/HIV activities in NCCDPHP have been removed from the HIV-wide table. FY 2008 and FY 2009 figures have been adjusted to become comparable to FY 2010 figures.

<sup>8</sup>In FY 2004, CDC's budget was restructured to separate actual program costs from the administration and management of those programs. Funding levels are not comparable to those of previous years. Also in that year, funding for the HIV lab activities was moved from the Infectious Disease budget activity to the Research and Domestic HIV Prevention sub-line in the HIV, STD and TB prevention budget activity.

<sup>9</sup>In 2006, HIV/AIDS Basic Research was moved from the Infectious Disease budget activity to the CDC Research and Domestic HIV Prevention sub-line under HIV/AIDS, Viral Hepatitis, STD, and TB Prevention.

Table 2.7: Estimated numbers of persons living with HIV infection (not AIDS) or with AIDS at the end of 2007, by area of residence—United States and dependent areas<sup>10</sup>.

Areas of Residence	Adults or adolescents and children (<13 years)	
	Living with HIV infection (not AIDS) <sup>11</sup>	Living with AIDS
Alabama	5,740	4,046
Alaska	289	343
Arizona	6,226	5,110
Arkansas	2,425	2,286
California	-	65,582
Colorado	6,067	4,286
Connecticut	-	6,930
Delaware	-	1,844
District of Columbia	-	8,895
Florida <sup>12</sup>	39,686	48,059
Georgia	13,873	18,011
Hawaii	-	1,136
Idaho	409	318
Illinois	-	17,075
Indiana	3,939	4,019
Iowa	644	917
Kansas	1,370	1,390
Kentucky	-	2,286
Louisiana	7,738	8,491
Maine	-	537
Maryland	-	15,682
Massachusetts	-	9,181
Michigan	6,501	7,088
Minnesota	3,380	2,439
Mississippi	4,376	3,341
Missouri	5,139	5,725
Montana	-	205
Nebraska	708	835
Nevada	3,564	2,997

<sup>10</sup>Source : The Centers for Disease Control and Prevention (CDC), <http://www.cdc.gov/hiv/topics/surveillance/resources/reports/2007report/table14.htm>

<sup>11</sup>Total number of persons living with HIV infection (not AIDS) includes persons reported from areas with confidential name-based HIV infection reporting who were residents of other states or whose area of residence is unknown. Total number of persons living with AIDS includes persons whose area of residence is unknown.

<sup>12</sup>Florida has confidential name-based HIV infection reporting on and after July 1997.

Table 2.7:Continued.

Areas of Residence	Adults or adolescents and children (<13 years)	
	Living with HIV infection (not AIDS) <sup>13</sup>	Living with AIDS
New Hampshire	-	588
New Jersey	17,612	17,671
New Mexico	962	1,339
New York	46,390	75,253
North Carolina	13,122	9,129
North Dakota	87	80
Ohio	8,557	7,426
Oklahoma	2,237	2,274
Oregon	-	2,951
Pennsylvania	-	19,236
Rhode Island	-	1,350
South Carolina	6,626	7,510
South Dakota	207	147
Tennessee	7,154	6,834
Texas	26,605	34,940
Utah	954	1,207
Vermont	-	239
Virginia	10,577	8,872
Washington	-	5,629
West Virginia	670	785
Wisconsin	2,432	2,296
Wyoming	98	106
Sub-total	256,363	455,636

Table 2.7 explanation: the numbers represent point estimates after adjusting reported case counts for reporting delays, but not for incomplete reporting. Dashes indicate data not shown because the state did not have laws requiring confidential name-based HIV infection reporting since at least 2003.

<sup>13</sup>Total number of persons living with HIV infection (not AIDS) includes persons reported from areas with confidential name-based HIV infection reporting who were residents of other states or whose area of residence is unknown. Total number of persons living with AIDS includes persons whose area of residence is unknown.

Table 2.8: Discretionary State/Formula Grants by CDC for HIV/AIDS Prevention and Surveillance Programs for State and Local Health Departments<sup>14</sup>.

State/Territory/ Grantee	FY 2008 Prevention Projects	FY 2008 Case Surveillance	Total <sup>15</sup>
Alabama	\$2,129,587	\$855,835	\$2,985,422
Alaska	\$1,417,619	\$120,010	\$1,537,629
Arizona	\$3,028,369	\$630,733	\$3,659,102
Arkansas	\$1,582,922	\$215,333	\$1,798,255
California	\$13,618,189	\$2,503,358	\$16,121,547
Colorado	\$4,387,622	\$1,483,874	\$5,871,496
Connecticut	\$6,260,601	\$992,965	\$7,253,566
Delaware	\$1,888,920	\$218,628	\$2,107,548
District of Columbia	\$5,736,854	\$1,757,516	\$7,494,370
Florida	\$19,255,996	\$3,278,335	\$22,534,331
Georgia	\$8,090,047	\$1,235,185	\$9,325,232
Hawaii	\$2,041,255	\$175,975	\$2,217,230
Idaho	\$883,103	\$69,747	\$952,850
Illinois	\$4,068,878	\$729,058	\$4,797,936
Indiana	\$2,508,313	\$758,488	\$3,266,801
Iowa	\$1,649,372	\$176,112	\$1,825,484
Kansas	\$1,617,269	\$143,735	\$1,761,004
Kentucky	\$1,921,570	\$133,063	\$2,054,633
Louisiana	\$5,227,602	\$1,479,984	\$6,707,586
Maine	\$1,613,073	\$105,487	\$1,718,560
Maryland	\$9,737,986	\$1,749,181	\$11,487,167
Massachusetts	\$8,655,094	\$1,096,037	\$9,751,131
Michigan	\$6,386,659	\$1,701,840	\$8,088,499
Minnesota	\$3,171,739	\$257,870	\$3,429,609
Mississippi	\$1,835,920	\$334,518	\$2,170,438
Missouri	\$3,737,842	\$1,161,182	\$4,899,024
Montana	\$1,263,843	\$66,893	\$1,330,736
Nebraska	\$1,205,605	\$142,515	\$1,348,120
Nevada	\$2,756,285	\$785,703	\$3,541,988
New Hampshire	\$1,598,713	\$93,099	\$1,691,812
New Jersey	\$13,192,984	\$3,372,243	\$16,565,227
New Mexico	\$2,270,963	\$234,483	\$2,505,446

<sup>14</sup>Source: Department of Human and Health Services, FY 2010, CDC

[http://www.cdc.gov/fmo/topic/Budget%20Information/appropriations\\_budget\\_form\\_pdf/FY2010\\_CDC\\_CJ\\_Final.pdf](http://www.cdc.gov/fmo/topic/Budget%20Information/appropriations_budget_form_pdf/FY2010_CDC_CJ_Final.pdf)

<sup>15</sup>Amounts reflect new funding only. Approximately \$3 million in unobligated funds was also awarded to support the new funds.



Table 2.8: Continued.

State/Territory/ Grantee	FY 2008 Prevention Projects	FY 2008 Case Surveillance	Total <sup>16</sup>
New York	\$26,785,716	\$2,733,243	\$29,518,959
North Carolina	\$4,208,066	\$792,412	\$5,000,478
North Dakota	\$672,678	\$63,329	\$736,007
Ohio	\$5,206,904	\$911,402	\$6,118,306
Oklahoma	\$2,434,358	\$484,092	\$2,918,450
Oregon	\$3,018,171	\$291,031	\$3,309,202
Pennsylvania	\$4,377,928	\$616,209	\$4,994,137
Rhode Island	\$1,642,131	\$224,293	\$1,866,424
South Carolina	\$4,460,943	\$809,337	\$5,270,280
South Dakota	\$642,291	\$61,003	\$703,294
Tennessee	\$3,913,051	\$942,399	\$4,855,450
Texas	\$12,936,907	\$2,229,005	\$15,165,912
Utah	\$1,071,870	\$177,801	\$1,249,671
Vermont	\$1,460,681	\$84,325	\$1,545,006
Virginia	\$4,938,495	\$827,536	\$5,766,031
Washington	\$3,337,579	\$1,704,245	\$5,041,824
West Virginia	\$1,684,759	\$208,934	\$1,893,693
Wisconsin	\$2,788,528	\$399,453	\$3,187,981
Wyoming	\$787,249	\$61,819	\$849,068
Chicago	\$5,443,889	\$1,433,107	\$6,876,996
Houston	\$5,092,037	\$1,705,603	\$6,797,640
Los Angeles	\$12,888,698	\$2,369,850	\$15,258,548
New York City	\$21,281,593	\$3,968,220	\$25,249,813
Philadelphia	\$6,327,782	\$1,212,151	\$7,539,933
San Francisco	\$9,005,739	\$1,849,740	\$10,855,479
American Samoa	\$174,435	\$6,719	\$181,154
Guam	\$499,622	\$22,975	\$522,597
Marshall Islands	\$122,518	\$17,672	\$140,190
Micronesia	\$212,866	\$17,273	\$230,139
Northern Mariana Islands	\$192,386	\$22,712	\$215,098
Palau	\$235,697	\$22,091	\$257,788

<sup>16</sup>Amounts reflect new funding only. Approximately \$3 million in unobligated funds was also awarded to support the new funds.

Table 2.8: Continued.

State/Territory/ Grantee	FY 2008 Prevention Projects	FY 2008 Case Surveillance	Total <sup>17</sup>
Puerto Rico	\$4,051,694	\$1,136,524	\$5,188,218
Virgin Islands	\$407,698	\$120,495	\$528,193
Total States/Cities/Te rritories	\$297,045,753	\$55,585,985	\$352,631,738

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<sup>17</sup>Amounts reflect new funding only. Approximately \$3 million in unobligated funds was also awarded to support the new funds.

Table 2.9: Distribution of Ryan White Program Funding by Region and Part, FY 2007<sup>18</sup>.

	Part A Total	Part B (Total)	Part C	Part D	AETC	SPNS	Part F Dental Reimbu- rsement Program	Commu- nity- Based Dental Partner- ship Program
United States	28.2%	56.1%	8.9%	3.3%	1.8%	1.1%	0.4%	0.2%
Alabama	0.0%	74.3%	21.2%	4.4%	0.0%	0.0%	0.1%	0.0%
Alaska	0.0%	56.6%	43.4%	0.0%	0.0%	0.0%	0.0%	0.0%
Arizona	30.6%	59.4%	5.9%	2.7%	0.0%	1.3%	0.0%	0.0%
Arkansas	0.0%	82.5%	11.8%	5.7%	0.0%	0.0%	0.0%	0.0%
California	35.3%	49.2%	7.8%	2.6%	3.2%	1.3%	0.5%	0.1%
Colorado	27.8%	52.8%	6.7%	3.5%	8.1%	0.0%	0.0%	1.2%
Connecticut	28.1%	51.2%	13.2%	3.7%	0.0%	3.8%	0.0%	0.0%
Delaware	0.0%	83.6%	11.5%	4.9%	0.0%	0.0%	0.0%	0.0%
District of	52.1%	35.5%	4.5%	2.5%	4.0%	1.1%	0.3%	0.0%
Florida	33.2%	55.9%	5.1%	3.4%	1.4%	0.3%	0.5%	0.1%
Georgia	24.0%	56.5%	13.0%	1.5%	3.3%	1.5%	0.2%	0.0%
Hawaii	0.0%	93.0%	7.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Idaho	0.0%	59.8%	40.2%	0.0%	0.0%	0.0%	0.0%	0.0%
Illinois	33.2%	48.0%	9.8%	3.2%	3.6%	1.4%	0.4%	0.4%
Indiana	18.6%	74.9%	6.5%	0.0%	0.0%	0.0%	0.0%	0.0%
Iowa	0.0%	64.8%	34.8%	0.0%	0.0%	0.0%	0.4%	0.0%
Kansas	0.0%	80.8%	19.2%	0.0%	0.0%	0.0%	0.0%	0.0%
Kentucky	0.0%	69.2%	20.7%	2.4%	0.0%	0.0%	4.4%	3.3%
Louisiana	25.9%	53.0%	9.7%	4.8%	3.8%	2.0%	0.0%	0.9%
Maine	0.0%	57.4%	42.6%	0.0%	0.0%	0.0%	0.0%	0.0%
Maryland	31.2%	61.8%	3.9%	2.5%	0.0%	0.6%	0.0%	0.0%
Massachusetts	32.3%	38.5%	15.2%	5.3%	3.9%	2.8%	1.5%	0.6%
Michigan	28.1%	56.9%	9.4%	4.1%	0.0%	1.0%	0.5%	0.0%
Minnesota	34.9%	55.3%	7.0%	2.2%	0.0%	0.0%	0.6%	0.0%
Mississippi	0.0%	76.3%	16.5%	5.6%	0.0%	0.0%	0.0%	1.5%
Missouri	33.5%	51.3%	8.6%	6.6%	0.0%	0.0%	0.0%	0.0%
Montana	0.0%	60.1%	39.9%	0.0%	0.0%	0.0%	0.0%	0.0%
Nebraska	0.0%	77.4%	22.3%	0.0%	0.0%	0.0%	0.3%	0.0%
Nevada	32.2%	55.3%	9.7%	2.8%	0.0%	0.0%	0.0%	0.0%
New	0.0%	68.7%	15.6%	15.7%	0.0%	0.0%	0.0%	0.0%
New Jersey	31.5%	56.7%	6.8%	2.8%	1.0%	0.5%	0.2%	0.5%
New Mexico	0.0%	69.5%	24.8%	5.7%	0.0%	0.0%	0.0%	0.0%

<sup>18</sup>Source: Kaiser Family Foundation, <http://www.statehealthfacts.org/comparebar.jsp?ind=535&cat=11>

Table 2.9: Continued.

	Part A Total	Part B (Total)	Part C	Part D	AETC	SPNS	Part F Dental Reimbu- rsement Program	Commu- nity- Based Dental Partner- ship Program
New York	34.8%	50.7%	7.2%	3.5%	1.2%	1.2%	1.2%	0.2%
North Carolina	9.0%	72.5%	10.7%	5.3%	0.0%	2.6%	0.0%	0.0%
North Dakota	0.0%	100.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Ohio	12.4%	72.5%	10.4%	3.4%	0.0%	1.2%	0.0%	0.0%
Oklahoma	0.0%	82.0%	14.0%	3.4%	0.0%	0.7%	0.0%	0.0%
Oregon	25.9%	55.1%	7.0%	0.0%	0.0%	8.5%	1.4%	2.1%
Pennsylvania	27.3%	48.8%	13.4%	3.5%	5.1%	1.6%	0.4%	0.0%
Rhode Island	0.0%	59.0%	18.0%	10.2%	0.0%	12.8%	0.0%	0.0%
South Carolina	0.0%	81.8%	14.2%	1.6%	0.0%	2.3%	0.0%	0.0%
South Dakota	0.0%	67.4%	32.6%	0.0%	0.0%	0.0%	0.0%	0.0%
Tennessee	30.0%	59.5%	6.1%	4.3%	0.0%	0.0%	0.0%	0.0%
Texas	29.6%	60.4%	5.1%	3.3%	1.1%	0.5%	0.1%	0.0%
Utah	0.0%	83.5%	16.3%	0.0%	0.0%	0.0%	0.2%	0.0%
Vermont	0.0%	64.2%	35.8%	0.0%	0.0%	0.0%	0.0%	0.0%
Virginia	13.7%	78.1%	5.9%	2.2%	0.0%	0.1%	0.0%	0.0%
Washington	25.8%	50.9%	9.3%	3.9%	10.1%	0.0%	0.0%	0.0%
West Virginia	0.0%	75.9%	24.1%	0.0%	0.0%	0.0%	0.0%	0.0%
Wisconsin	0.0%	73.5%	16.8%	6.6%	0.0%	3.1%	0.0%	0.0%
Wyoming	0.0%	73.6%	26.4%	0.0%	0.0%	0.0%	0.0%	0.0%
Guam	0.0%	100.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Puerto Rico	27.5%	57.9%	12.6%	2.0%	0.0%	0.0%	0.0%	0.0%
Virgin Islands	0.0%	72.2%	13.8%	14.0%	0.0%	0.0%	0.0%	0.0%

Table 2.10: The Distribution of ACAP Allocation Funds (in CAD\$)<sup>19</sup>.

Region	2005/06	2006/07	2007/08	2008/09
British Columbia	1,339,956	1,515,709	1,761,891	2,329,827(17%)
Alberta	906,129	906,129	906,129	984,575 (7%)
Manitoba/Saskatchewan	941,949	941,949	941,949	941,949(7%)
Ontario	2,702,466	3,017,212	3,473,671	4,556,482(33%)
Quebec	2,072,765	2,229,137	2,493,496	3,189,937(23%)
New Brunswick, Newfoundland and Labrador, Nova Scotia, Prince Edward Island	1,456,184	1,456,184	1,456,184	1,456,184(10%)
Yukon, Northwest Territories, Nunavut	396,680	396,680	396,680	441,046(3%)
Total	\$9,816,129	\$10,463,000	\$11,430,000	\$13,900,000(100%)

<sup>19</sup>Source: Public Health Agency of Canada,  
<http://www.phac-aspc.gc.ca/aids-sida/publication/reports/acap-pacs/acap-pacs-eng.php>

Table 2.11: NGOs working to reduce spread of HIV/AIDS

NGOs	Budget Available (US\$ millions)
International Federation of Red Cross and Red Crescent Societies <sup>7</sup>	-
Family Health International	369.876 <sup>20</sup>
Global Network of People Living with HIV	-
International HIV/AIDS Alliance	71.1 <sup>21</sup>
International Council for AIDS Service Organizations	-
AVERT	0.175 <sup>22</sup>
Elton John AIDS Foundation	2.99 <sup>23</sup>
NAM	1.19 <sup>24</sup>

<sup>20</sup>Source: Family Health International.

<http://www.fhi.org/NR/rdonlyres/edhbtg4dyeob2k7wod26lf3qu45sbalmy6ezh5dds5mwgni4auhd4h4yq7nufuwwqyz4tqakn5gcoj/FHIAnnualReport2009.pdf>.

<sup>21</sup>Source: International HIV/AIDS Alliance, <http://www.aidsalliance.org/homepagedetails.aspx?id=1>.

<sup>22</sup>Source: AVERT. 2007 [cited 2008 June 1]; <http://www.avert.org/worldstats.htm>.

<sup>23</sup>Source: Elton John AIDS Foundation; <http://www.ejaf.org/>.

<sup>24</sup>Source: NAM, <http://www.aidsmap.com/cms1038153.aspx>.

Table 2.12: Financial Summary from September 2007-08 by FHI<sup>25</sup>.

<b>Revenue and Support</b>	<b>2008 (in US\$ million)</b>
US Government	302.126
Other Governments	22.787
Foundations and Individuals	19.687
Corporations	12.555
Multilaterals	11.278
Interest, investment and lab services income	1.443
Total revenues, gains and support	369.876
<b>Expenses</b>	
Operational use of funds	322.290
General and Administrative	44.080
Fundraising	0.186
Total expenses	366.556
Net Assets, Beginning of Year	12.368
Change in Net Assets	3.320
Net Assets, End of Year	15.688
<b>Work by Health Area</b>	
HIV/AIDS	85%
Reproductive Health	11%
Other Public Health and Development	4%
<b>Work by Practice Area</b>	
Health and Development Programs	76%
Research	24%

<sup>25</sup>Source: Family Health International, <http://www.fhi.org/NR/rdonlyres/edhbtg4dyeob2k7wod26lf3qu45sbalmy6ezh5dds5mwgni4auhd4h4yq7nufwqwyz4tqakn5gcoj/FHIAAnnualReport2009.pdf>

Table 2.13: Financial Highlights by World Vision<sup>26</sup>.

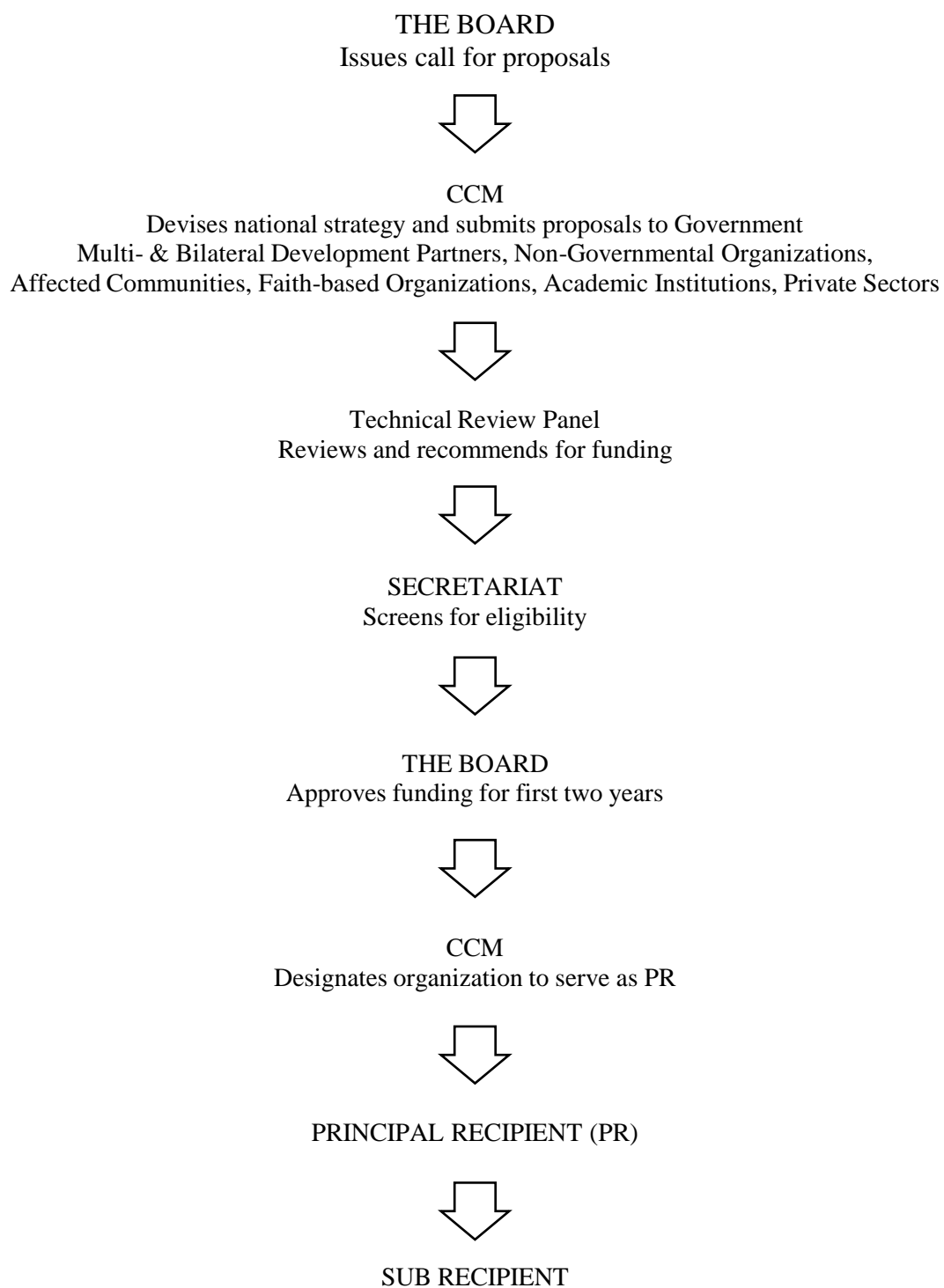
<b>Revenue Sources (in US\$ millions)</b>	<b>2008</b>
Private cash contributions	468
Government grants (food and cash)	281
Gifts-in-kind	366
Other income, net	6
Total revenue	1,109
<b>Operating Expenses (in US\$ millions)</b>	
Programs that benefit children, families, and communities in need	979
Fundraising	97
Management & general	52
Total operating expenses	1,128

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<sup>26</sup>Source: World Vision, <http://www.worldvision.org/content.nsf/about/ar-financials>.



Figure 2.1: The Global Fund Proposal Process<sup>27</sup>.



<sup>27</sup> Source : The Global Fund to Fight AIDS, Tuberculosis and Malaria,  
<http://www.theglobalfund.org/en/structures/?lang=en>

Figure 2.2: Distribution of funding by geographic region by the Global Fund<sup>28</sup>.

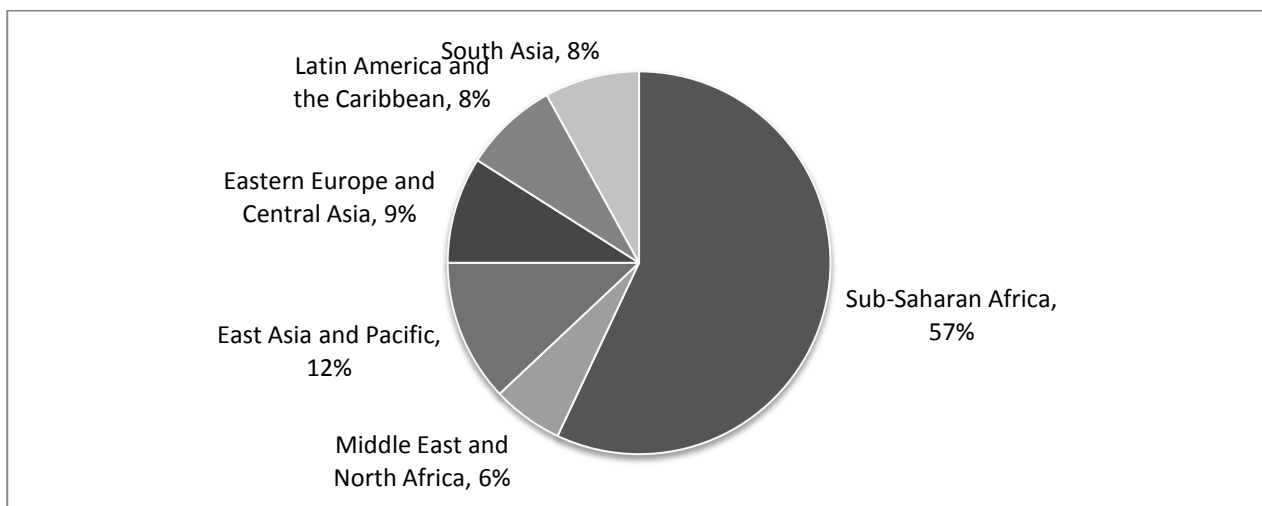
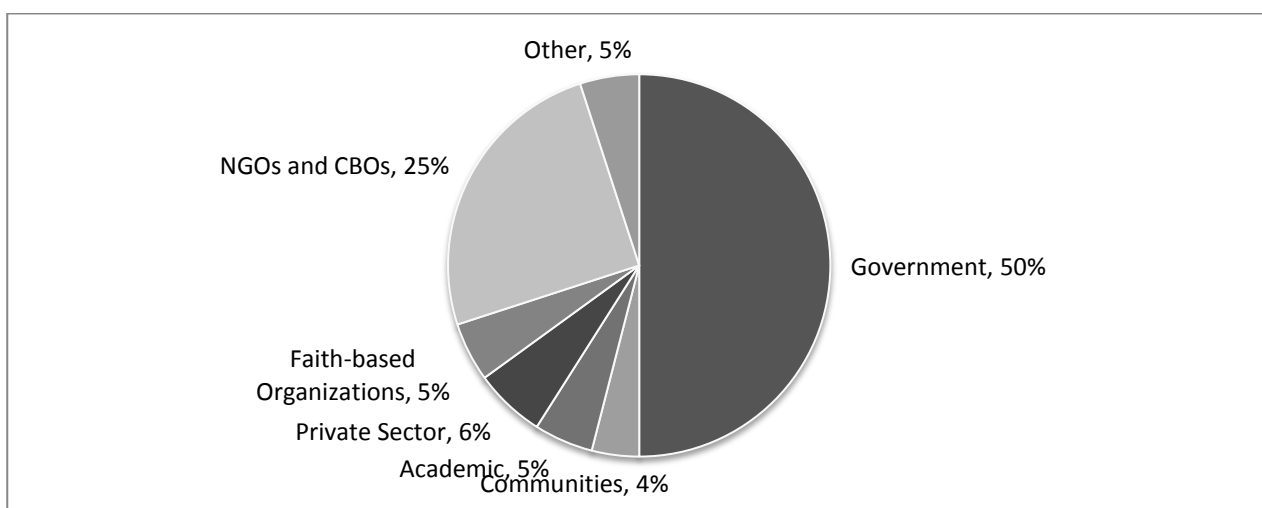


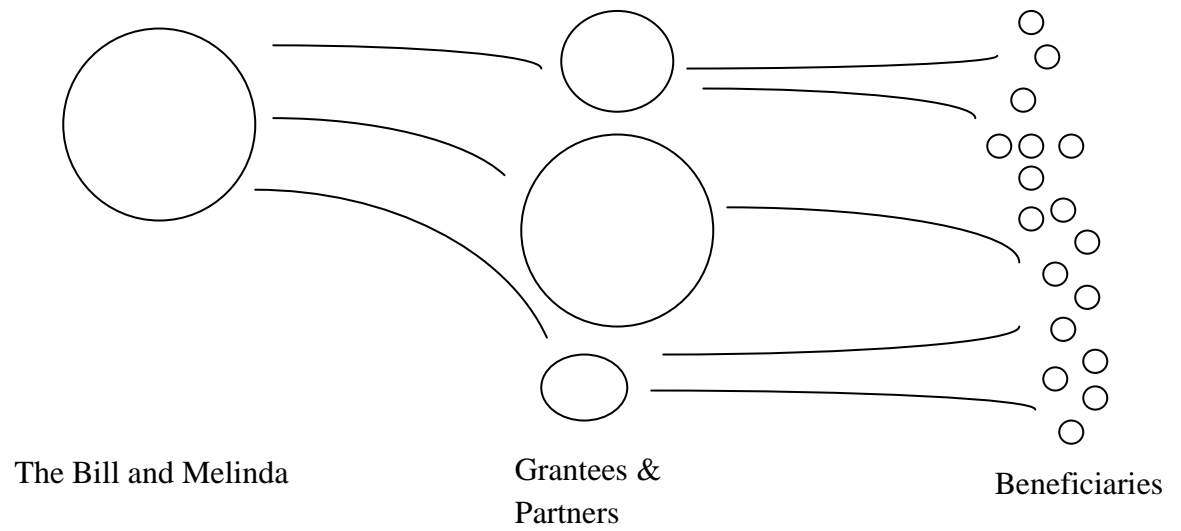
Figure 2.3: Distribution of funding by type of Principal Recipient by the Global Fund<sup>29</sup>.



<sup>28</sup> Source : The Global Fund to Fight AIDS, Tuberculosis and Malaria, [http://www.theglobalfund.org/en/distributionfunding/?lang=en#geographic\\_region](http://www.theglobalfund.org/en/distributionfunding/?lang=en#geographic_region).

<sup>29</sup> Source : The Global Fund to Fight AIDS, Tuberculosis and Malaria, <http://www.theglobalfund.org/en/recipients/?lang=en>

Figure 2.4: Funding process in Bill and Melinda Gates Foundation<sup>30</sup>.



<sup>30</sup>Source: The Bill and Melinda Gates Foundation,  
<http://www.gatesfoundation.org/grantseeker/Pages/overview.aspx>

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## Chapter 3

### Theoretical Models of HIV/AIDS Prevention Funds Allocation Process

#### 3.1. Fiscal Federalism

Fiscal federalism means decentralization of decision making to sub-national levels of government regarding allocation of financial resources. Rondinelli pioneered the formal analysis regarding decentralization [1] which Mills applied to health care sector [2].

Fiscal federalism in health care sector is often adopted in many countries. In Denmark, the federal government allocates funds to 5 main regions which are responsible for all the health care activities in their respective regions. A region has an incentive to provide service that will substitute for hospital care. For instance, a region will pay a certain amount for a hospitalized citizen. If via prevention hospitalization is reduced, then savings will accrue to the region [3].

In Norway, the federal government allocates funds to the Ministry of Health and Care Services, which then distributes funds to 5 regional health authorities, who then distribute funds to 32 health enterprises consisting of hospitals and clinics. Norwegian decentralized health care system is believed to increase welfare since local authorities are free to act according to local preferences, availability of data, and local cost structure [4]. In Canada, the federal and provincial relationship in the health care sector has been viewed as cooperative and collaborative federalism wherein the federal government cooperates and collaborates with provincial governments to make health care policy decisions [5]. In other countries, for example, India and China [6], Uganda, Ghana, Zambia, and the Philippines [7] all tiers of government share responsibility for all health care activities.

There are various advantages and disadvantages associated with fiscal federalism or the decentralization of decision making in health sector. Fiscal federalism provides autonomy to local government to decide on how much to spend and where according to local preferences and local needs under symmetric information [8, 9]. Brueckner uses a growth model to show that decentralization leads sub-national governments to provide

public services tailored to the needs of local consumers which increases the incentive to save. This incentive directs higher investment in human capital which in turn leads to faster economic growth [10]. Magnussen et al. showed that decentralization has been viewed as a means to improve the outcomes of the health care sector in European countries. However, in case of information asymmetry about local preferences at the national level, this approach leads to ambiguity in health outcomes [4]. Breuille and Gary-Bobo developed a principal multi-agent model to characterize optimal inter-governmental funds transfer under information asymmetry [11].

One major disadvantage of decentralization is that the sub-national levels of government have varying capacities or some of them may possess a weak capacity to distribute resources optimally. Further, resources are distributed based on local priorities which may lead to sub-optimal health outcomes [9]. Thus, conditionality or performance criteria are often used by federal government to dictate the terms of how to utilize limited resources. Sometimes incentives are also provided by the federal government to effectively guide local governments' behaviour towards distributing resources optimally [7].

HIV prevention resources, too, are often allocated at multiple levels. At the sub-national levels, resources are distributed based on local priorities which may lead to sub-optimal health outcomes. We consider optimal incentives in a decentralized HIV prevention resource allocation process under symmetric and asymmetric information.

### **3.2 Resource Allocation in Health Care**

Resource allocation in health care is defined as the dispersion of limited resources, for example, the distribution of funds to various regions, populations, risk groups, or programs. Various methods have been proposed to disperse these limited resources in an optimal way. The following paragraphs summarize some of the methods used in the allocation of health care resources.

Economic evaluation is used to identify the most efficient way of allocating scarce healthcare resources to alternative activities where the costs and consequences of these

decisions are compared to provide evidence to help policy makers and healthcare planners make effective choices. Cost-effectiveness analysis (CEA) is one basic type of economic evaluation.

### **3.2.1 Equity-based Heuristics**

#### *Cost-effectiveness Analysis*

Cost-effectiveness Analysis (CEA) provides valid criteria to decision makers for choosing health care interventions by comparing the costs versus the effectiveness of various interventions in cases where effectiveness can be measured by disease-specific outcomes [12]. An incremental cost-effectiveness ratio (ICER), is defined as the total incremental cost divided by the total incremental benefit of an intervention, and is calculated for each intervention. ICER's of different health care interventions are then compared to allocate funds. Cost is expressed in dollars and health benefits in units of health such as the reduction in the length of hospital stays, life years saved or additional years of life gained adjusted for quality of life defined as quality adjusted life years (QALYs) gained [12].

Hoel showed the significance of CEA in allocating health care resources by maximizing the expected utility of the policy maker subject to net health expenditures [13]. Cohen et al. demonstrated the value of CEA by developing an optimization model that lead to the allocation of funds among interventions in descending order of cost-effectiveness [14]. Weinstein showed that the optimal solution was comprised of allocating resources in increasing order of their effectiveness ratio until the budget is exhausted [15]. Most CEA analyses assume that all competing programs exhibit mutual exclusivity, perfect divisibility, and constant returns to scale. However, these assumptions may not be satisfied in practice [14]. Because the CEA needs realistic estimates of the cost and effectiveness of all interventions, the practicality of this option is decreased [16]. Many statistical issues arising from the comparison of different types of intervention [17], the ethical issues [18], and the theoretical issues [19, 20] in the CEA have been discussed by several researchers. Though, the CEA can offer valid criteria to the decision maker on which to base the choice of various health care interventions, there may be issues related

to the cost-effectiveness ratios and translating them into realistic criteria for resource allocation [15].

### *League Tables*

League tables rank alternative interventions based on their ICER's and are valuable tools to inform decision makers about the allocation of scarce healthcare resources. Mason et al. presented guidelines to help decision makers interpret league tables and made recommendations for future league tables' development [21]. Mauskopf et al. proposed a reference case that expanded league tables, making league tables a more effective tool for decision making [22]. However, construction of such tables is difficult because it requires standardization across studies, with respect to both method and underlying assumptions [23].

### *Program Budgeting and Marginal Analysis (PBMA)*

PBMA helps maximize the impact of limited healthcare resources on the health needs of a population by assisting decision makers in effectively allocating resources. This approach is based on two key economic principles: opportunity cost and marginal analysis. The objective is to either minimize the opportunity costs or maximize the benefits [24]. PBMA involves assessing the costs and benefits of proposed changes in healthcare delivery and focusing on the marginal benefits to assess the impact of proposed changes [25]. Various studies have given a fuller description of PBMA [26], in the South Australian context [27], and in the U.K. and Canadian contexts [26]. Dionne et al. conducted an empirical study which showed PMBA to be a useful priority setting framework [28]. However, PBMA is affected by organizational barriers, inadequate resourcing, and a culture dedicated to supporting proactive change [29].

### **3.2.2 Optimization Models**

Several studies have proposed linear programming (LP) models where the assumptions of program divisibility and constant returns to scale are not required. Kuo et al. developed an LP model to allocate operating room time among surgeons to maximize



total weekly revenue [30]. Similarly, Saaty et al. proposed an LP model to solve human resource allocation problems [31].

Earnshaw et al. presented various formulations of integer programming (IP) problems to allocate interventions to specific populations with diabetes mellitus [32]. Earnshaw and Dennett also provided an overview of the potential use of LP/IP models in various healthcare resource allocation settings [33]. Epstein et al. and others proposed an IP model with the objective of determining the optimal value of the available health care treatments subject to budgetary constraints and evaluated various budgetary policies showing the opportunity loss, in terms of health benefits forgone, for each policy [34]. Numerous researchers have developed IP models to efficiently distribute surgical cases to various multifunctional operating rooms [35, 36].

Zaric and Brandeau formulated a dynamic programming (DP) model in which the budget is allocated over multiple time periods to different populations where the objective is to maximize the quality of adjusted life years (QALYs) gained or minimize the total number of new infections subject to the applicable budget constraints [37]. It was shown that optimal allocation consists of investing as much as possible in some populations and nothing in other populations and that the reallocation of funds may lead to more benefits. Bala and Mauskopf proposed a DP model to optimally assign treatments to different health states [38].

### **3.2.3 Operations Research Models**

Zaric and Brandeau combined epidemic modeling with optimization techniques with the objective to maximize the quality of adjusted life years (QALYs) gained or minimize the total number of new infections for interacting populations and assuming a non-linear production function that relates the amount invested in an intervention to change in risky behaviour [39]. Obtained solutions were then compared with simple allocation strategies, which often led to suboptimal allocations. Brandeau et al. developed a similar formulation for a set of non-interacting populations presented by a simple susceptible/infected (S/I) epidemic model and assumed a general cost function that related cost with the reduction in the sufficient contact rate in each population and

showed that the optimal allocation of limited prevention funds depended on numerous factors [40].

### **3.3 HIV Resource Allocation**

#### **3.3.1 Equity-based Heuristics**

Bautista-Arredondo et al. suggested a general framework focusing on CEA for the allocation of HIV/AIDS prevention funds for developing countries [41]. Pinkerton et al. developed a resource allocation model based on league tables suggesting the allocation of funds to interventions in increasing order of their cost-effectiveness ratio until the budget is exhausted [42].

#### **3.3.2 Optimization Models**

LP models with the objective to maximize the weighted number of HIV infections averted subject to constraints on budget, funds distributed to at-risk individuals, and equity have been proposed and studied under several scenarios using information from Florida showing an improvement of 73% over simple allocation strategies [43, 44]. Stinnett and Paltiel developed a mixed integer programming model incorporating partial indivisibilities, non-constant returns to scale, and global mutual exclusivity of various prevention programs [45]. Kaplan and Pollack reviewed various budget allocation rules employed by HIV community planning groups and devised a dynamic programming model to maximize the number of HIV infections prevented subject to budget constraints [46]. Richter et al. proposed a dynamic programming model for HIV transmission in injection drug users and non-users with the objective to minimize the number of new HIV infections over a fixed time period [47].

#### **3.3.3 Operations Research Models**

Various simulation models have been proposed to evaluate the impact of HIV interventions. Zaric et al. performed computer simulations to analyze the effects of interventions on an HIV epidemic [48]. Korenromp et al. studied the effects of intervention by stochastic simulation in a rural African population [49]. Nagelkerke et

al. developed a dynamic compartmental simulation model to identify the best policy to inhibit the spread of HIV in Botswana and India [50].

Zaric and Brandeau developed an optimal investment portfolio that maximizes the number of HIV infections averted based on an allocation method that considers non-linear epidemic dynamics [51]. They showed that simple allocation methods may lead to sub-optimal allocation. Lasry et al. combined epidemic modeling with an optimization technique for a two-level decision-making process which consisted of allocating funds to four sub-populations at the lowest level which were modeled using a susceptible-infected epidemic model. They showed that if optimization modeling is to be applied at one level of decision-making process then it is beneficial to apply it at both the lower and upper level. Hence, the upper-level decision maker should develop incentives to encourage optimal allocations at the lower-levels [52].

### **3.4 Bi-level Optimization**

Bi-level programming problem (BLP) is a special case of multi-level programming problem (MLP) solving. MLP is a set of nested optimization problems where the control over the decision variables is partitioned among the various levels, but a decision variable at one level may affect the objective function of other levels. Thus, the decision maker may influence the policies utilized at other levels and thereby improve his own objective function. Two-levels in a hierarchy define a BLP.

The formal formulation of the linear BLP has been provided by Fortuny-Amat and McCarl [53] and by Candler and Townsley [54]. Numerous versions of the BLP are provided by various authors [55-57]. Fortuny-Amat and McCarl formulated a linear BLP with no upper-level constraints and a unique lower-level solution [53]. Bialas and Karwan formulated the BLP as a non-convex programming problem and demonstrated its tractability [55]. Several researchers have studied the fundamental concept of BLP. Simultaneously, several algorithms have also been proposed to solve the BLP. Shimizu and Aiyoshi developed a computational method based on a series of non-linear programming problems approximating the original problem and obtaining the sequence

of approximating solutions converging on the true solution [58]. Bard presented an algorithm to solve BLP using sensitivity analysis [56].

In this thesis, we formulate a BLP in which a decision at one level affects the objective function at other level and the decision maker at one level can influence the decisions made at other level, thereby improving the objective function.

### **3.5 Incentives in Health Care Settings**

The concept of using incentives to encourage optimal behaviour in health care has been investigated in several settings. In principal-agent problem, the principal has a primary stake in the performance of the system, but delegates operational control of that system to an agent. Two situations arise, in the first-best situation, the agent's actions can be observed by the principal, whereas, in the second-best situation, the agent's actions are hidden from the principal. However, in both situations, the principal can observe the outcome of the action taken by an agent. The principal provides incentives or compensation contracts based on the observed outcome to induce the agent to operate in the principal's best interests [59].

Mas-Colell et al. provided an overview of principal-agent models [60]. Fuloria and Zenios considered an incentive-based contractual agreement between a purchaser and a provider of health care in which the purchaser reimburses the provider according to a pre-specified payment system in the context of Medicare's End Stage Renal Disease Program. They formulated a principal-agent model where the purchaser maximizes the amount of social welfare and the provider maximizes its expected utility from the level of consumption and derived an outcome-adjusted payment system that motivates the provider to adopt actions promoting the purchaser's welfare [61]. Su and Zenios considered a mechanism-design problem in the context of the kidney transplant waiting system. The authors maximized the objective function, which is the sum of expected utilities of all patients and minimum improvement in the expected utility of different patients' types. They showed that the allocation mechanism does induce patients to declare the type of kidney he/she would be willing to accept at the time they join the kidney waiting-list by ensuring that patients who wait longer receive better kidneys [62].

However, we are not aware of any research on incentives to promote the optimal allocation of HIV prevention resources. My thesis investigates the impact of using incentives in a multi-level allocation process of HIV prevention funds.

### **3.6 Game Theoretic Models**

Some work incorporating game theory has been done in health care resource distribution settings. Nagel modeled group decision making as a one-shot zero sum game and developed strategies that bring into consideration the expectation of individual players by influencing their perceptions [63]. Cohen and Burg proposed a zero-sum game that chooses between efficiency and fairness in the distribution of health care resources in the United States. In the U.S., there is a wide gap between more-favoured and less-advantaged groups in terms of access to health care resources. A dynamic setting focusing on the redistribution of the health care resources was proposed in such a way that the inequality gap was reduced by distributing a larger share of health care resources to less-advantaged groups without making the more-favoured group worse off [64].

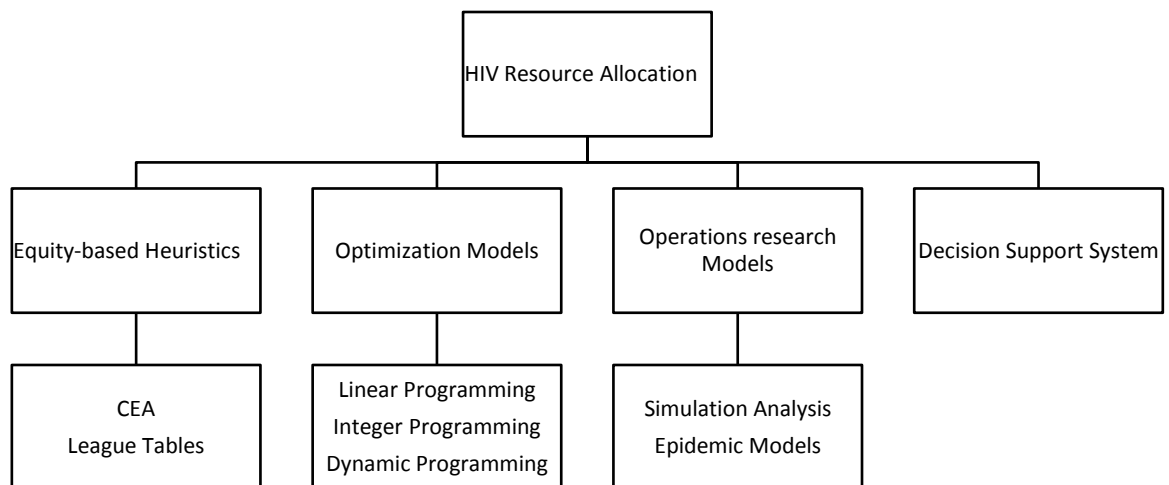
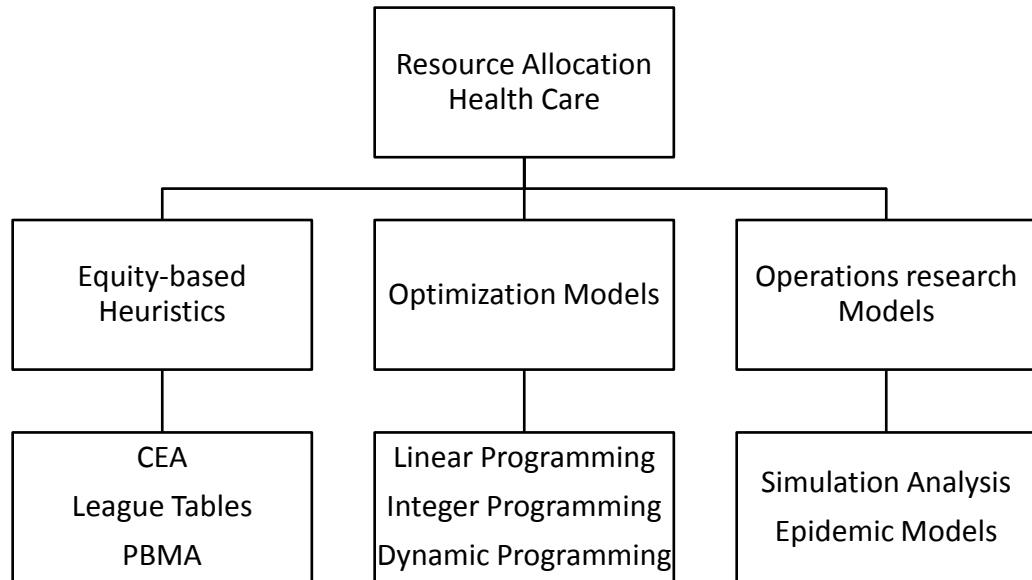
Jan formulated “short-termism”, a decision making tool used in the public sector that favours choices that yield short-term gains, as prisoner’s dilemma and coordination games are used to address the incentives that occurs in such games. The allocation of resources across programs is based on a marginal cost-effectiveness/cost-benefit analysis, which was modeled as a prisoner’s dilemma game and the adoption of information systems across hospitals as a coordination game. They showed the role of credible commitment in facilitating long-term decision making in a health care setting [65]. Jan et al. analyzed the group decision-making process as a one-shot zero sum game in the context of Divisions of General Practice in Queensland, Australia and demonstrated a consultative process in which the relevant stakeholders (players) were encouraged to take into consideration the global allocation issue and to move beyond their localized interests [66].

McPake et al. modeled two-tier charging, the practice in which hospitals offer two separate qualities of service, basic and premium, at different prices, as a Stackelberg

game in which the Ministry of Health (MOH) is the leader and the hospital is a follower. MOH sets the prices that maximize its utility function subject to its budget constraint set by Ministry of Finance and then the hospital follows by setting its quality levels by maximizing the use of any surplus. The case in which MOH sets prices but provides only lump-sum subsidies to the hospital was compared to the case in which MOH sets prices and also provides an activity-based subsidy for the provision of a basic service that reflects the volume of the service provided. McPake et al. showed that switching to activity-based payment doubles the quality level of the basic and premium service [67]. Sun et al. modeled various countries as players in a game during an outbreak of an influenza epidemic by making optimal decisions about allocating their own drug stockpiles to protect their populations. They developed a two-period multivariate model to represent the epidemic within and across countries by capturing three types of uncertainties: the number of initial infections, the spread of the disease, and drug efficacy. Their analysis showed that Nash equilibrium exists for between-country infections suggesting that countries should agree on an allocation scheme that would benefit everyone [68]. However, we are not aware of any modeling of HIV prevention funds allocation process as a Stackelberg game.

In Chapter 6, we model an incentive-based multi-level resource allocation process with an upper-level decision maker (UD) by allocating funds to two lower-level decision makers (LDs) who then allocate funds to three programs. The UD sets the level of incentive that maximizes the total infections averted and then the two LDs simultaneously set the fraction of the funds to be reserved for the proportional allocation maximizing their utility functions. The UD uses an incentive scheme to encourage a LD to reduce the fraction of funds reserved for equity by making the amount received by each LD dependent on this fraction and the decision of other LD.

Figure 3: Taxonomy Diagram.



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## Chapter 4

### Optimal Incentives for Multi-level Allocation of HIV Prevention

#### Resources

##### 4.1. Introduction

HIV prevention funds are often allocated at multiple levels. For example, a national level decision maker may allocate funds to regional decision makers who then distribute funds to local organizations, risk groups, or programs. In the U.S., the Centers for Disease Control and Prevention (CDC) disperses funds to several community planning groups (CPGs) who then distribute the funds to local programs or risk groups. An international organization such as The Global Fund to Fight AIDS, Tuberculosis and Malaria distributes funds to different countries. Funds received by countries may then be allocated to sub-recipients such as regional decision makers or to specific programs [1].

In many countries health care funds are often allocated by central government to regional governments' who then distribute funds to hospitals and clinics in their respective regions. This is called fiscal federalism or decentralization or transfer of financial power from a central to less central authority [2, 3]. In fiscal federalism, local authorities make decisions in accordance with local preferences which may not lead to optimal outcome [4, 5].

Similarly, many regional-level decision makers allocate HIV prevention funds often using equity-based heuristics including various forms of proportional allocation which may not lead to optimal health care outcome [6-8]. Some examples of proportionality include dividing the budget equally among competing programs; dividing resources equally among programs without considering the effectiveness of targeted programs; or allocating in proportion to HIV prevalence, incidence, or population size.

There are numerous mathematical programming models of health care resource allocation that are applicable when there is a single decision maker. These include linear programming models [9-12], mixed-linear integer programming models [13], dynamic programming models [14-16], and stochastic programming models [13, 17, 18].

Resource allocation models that incorporate epidemic dynamics have also been developed for the specific case of HIV [19-23].

The issue of equity versus efficiency tradeoffs in HIV resource allocation has been examined, both at a single-level [20, 24] and at multiple-levels of decision-making [25, 26]. Lasry et al. modeled a two-level decision-making process combining epidemic modeling with optimization technique in the context of Sub-Saharan Africa. They built a model in which an upper level decision maker (UD) allocates funds to lower-level decision makers (LD), who further distribute the funds to two sub-populations, each funding two different prevention programs. The four sub-populations are modelled using susceptible/infected epidemic model. The authors showed an improvement of 7% over the number of new HIV infections if optimal allocation takes place at the lower level [25].

Zaric and Brandeau modeled a two-level decision-making process with a single UD, multiple LDs, and three sub-populations in each region. The model was based on 40 U.S. states with 3 risk groups per state [26]. As in Lasry et al, the authors found that optimal allocation at the lower level often yields greater gains than optimal allocation at the higher level, and in some cases differences can be substantial. They concluded that the UDs such as donor organizations, should develop incentives to promote optimal allocation at the lower level [26]. We expand on the work of Lasry et al. and Zaric and Brandeau by investigating the impact of an incentive program to encourage optimal allocation at the lower level.

The concept of using incentives to encourage optimal behaviour in health care has been investigated in several settings. Fuloria and Zenios considered an incentive-based contractual agreement between a purchaser and a provider of health care in which the purchaser reimburses the provider according to a pre-specified payment system in the context of Medicare's End Stage Renal Disease Program. They formulated a principal – agent model where the purchaser maximizes social welfare and the provider maximizes its expected utility from consumption and derived an outcome- adjusted payment system that motivates the provider to adopt actions promoting the purchaser's welfare [27]. Su and Zenios considered a mechanism-design problem in the context of the kidney

transplant waiting system by maximizing the objective function, which is the sum of expected utilities of all patients and minimum improvement in the expected utility of different patients' types. They showed that the allocation mechanism does induce patients to declare the type of kidney he/she would be willing to accept at the time they join kidney waiting-list by ensuring that patients who wait longer receive better kidneys [28]. There are many other examples of research on the use of incentives in health care [29-34]. However, we are not aware of any research on incentives to promote optimal allocation of HIV prevention resources.

In this paper, we model a two-level resource allocation problem in which the UD uses incentives to promote optimal allocation by the LD. Our study attempts to answer the following questions. Under what situations does giving incentives to the LD help encourage optimal allocation at the lower level? and What is the optimal level of incentives? The rest of the paper is organized as follows: Section 2 provides the mathematical description of the model. Section 3 introduces mathematical analysis of the model and illustrates with a numerical example. Concluding comments are provided in Section 4.

#### **4.2. The Model**

We developed a single period model of a two-level decision-making process. There is a single decision-maker at each level and a fixed time horizon of length  $T > 0$ . The LD has two decisions: 1. What proportion of the funds received to allocate based on equity? And 2. How should the remaining funds be allocated to programs? The UD has one decision, which is the amount to allocate to the lower level. The UD uses an incentive scheme to encourage the LD reduce the fraction of funds reserved for equity by making the amount received by the lower-level dependent on this fraction. In particular, the upper-level may withhold funds to encourage allocation that is more efficient.

The LD chooses  $r$  ( $0 \leq r \leq 1$ ), which is the fraction of the funds to be reserved for proportional allocation, and then distributes amount  $y_j, j = 1, 2, \dots, m$  to program  $j$ . As in Zaric and Brandeau, we assume that one program is available for each risk group, that

the programs do not interact, and that the costs and benefits scale linearly [26]. Let  $B$  be the budget of the upper-level and  $Z$  be the amount allocated to the lower-level. The UD has a total budget  $B$ , chooses a fraction  $f$ ,  $0 \leq f \leq 1$  and allocates an amount  $Z$  to the lower-level using the following equation:

$$Z(r) = B(1 - rf). \quad (4.1)$$

We refer to  $f$  as the “strength of the incentive” in that a higher value of  $f$  corresponds to a stronger penalty for equity at the lower-level. To illustrate, consider the following examples. The LD receives the total budget  $B$  when  $f = 0$  and receives  $(1 - r)$  times the budget  $B$  when  $f = 1$ . When  $r = 0$ , all funds are reserved for an optimal allocation and the lower-level receives  $B$  regardless of  $f$ . When  $r = 1$ , all the funds are reserved for a proportional allocation and the lower-level receives  $B(1 - f)$ .

We formulate this problem as a dynamic program in which the time sequence is as follows: the upper-level chooses  $f$ ; then the lower-level chooses  $r$  for the given value of  $f$ ; then the lower-level determines  $y_j$  for the given values of  $f$  and  $r$ . We solve this problem using backward induction and present the details in the reverse time sequence.

#### 4.2.1 The Lower-level Model

**Stage 3:** We develop a model at this stage similar to the lower-level models of Zaric and Brandeau [26] and Kaplan and Merson [24]. Let  $h_j$  be the number of HIV infections prevented per dollar invested in a program  $j$  over time  $T$ . Let  $n_j$  be the size of the risk group  $j$  and  $N = \sum_j n_j$ . We assume that the programs have been indexed so that

$h_1 > h_2 > \dots > h_m$ . The total number of HIV infections averted,  $IA$ , is given by the following equation:

$$IA = h_1 y_1 + h_2 y_2 + \dots + h_m y_m. \quad (4.2)$$

In the last step of the dynamic program, given  $r$  and  $f$ , the total number of infections averted is found by solving the following linear programming problem:

$$\max_{y_1, y_2, \dots, y_m} IA = h_1 y_1 + h_2 y_2 + \dots + h_m y_m \quad (4.3)$$

$$\text{S.t. } \sum_{j=1}^m y_j \leq Z = B(1 - rf) \quad (4.4)$$

$$y_j \geq rZ \frac{n_j}{N} \quad (4.5)$$

This is similar to the “Knapsack LP” formulation at the lower-level in Zaric and Brandeau [26] and the resulting optimal solution is of the following form:

$$y_1 = Z - rZ \frac{n_2}{N} - \dots - rZ \frac{n_m}{N},$$

$$y_j = rZ \frac{n_j}{N}, \quad j = 2, \dots, m.$$

$$IA(r) = Z(h_1 - rk) = B(1 - rf)(h_1 - rk), \quad (4.6)$$

$$\text{where, } k = (h_1 - h_2) \frac{n_2}{N} + \dots + (h_1 - h_m) \frac{n_m}{N}.$$

Note that  $h_1 \geq k$  and the result of stage 3 is the function  $IA(r)$ .

**Stage 2:** In the second step, the LD chooses  $r$  to maximize his utility function. We assume that the LD’s utility function considers equity (as captured through  $r$ ), efficiency (as captured through  $IA(r)$ ), and funds received (as captured through  $Z(r)$ ). We investigate two different forms for the utility function, linear and multiplicative,  $U_L(r)$  and  $U_M(r)$ , given by

$$U_L(r) = aZ(r) + br + cIA(r) \quad (4.7)$$



and

$$U_M(r) = Z(r)^a r^b IA(r)^c. \quad (4.8)$$

In the case of  $U_L(r)$ , the parameters  $a, b, c > 0$  represent the relative weights applied to funds received, equity, and infections averted. In the case of  $U_M(r)$ , the parameters  $a, b, c > 0$  are exponents of budget, equity, and infections averted. We assume that values of  $a$ ,  $b$ , and  $c$  are known by both the LD and the UD. Depending on which utility function is used, the lower-level optimization problem is written as:

$$L_L : \max_r U_L(r) = aZ(r) + br + cIA(r) \quad (4.9)$$

or

$$L_M : \max_r U_M(r) = Z(r)^a r^b IA(r)^c \quad (4.10)$$

$$\text{s.t. } 0 \leq r \leq 1 \quad (4.11)$$

The LD solves  $L_L$  or  $L_M$  to obtain  $r^*(f)$ .

#### 4.2.2 The Upper-level Model

**Stage 1:** As in other models [9, 10, 26], we assume that the objective at the upper-level is to maximize the number of infections averted. Thus, upper-level resource allocation problem is:

$$IA_i : \max_f IA(r^*(f)), \text{ where } i = L, M \quad (4.12)$$

$$\text{s.t. } 0 \leq f \leq 1 \quad (4.13)$$

$$r^*(f) = \arg \max (L_i(r)) \quad (4.14)$$

#### 4.3. Analysis

In this section, we analyze the optimization problem for both utility functions. All proofs are shown in the appendix. We focus on the case of three sub-populations (i.e.  $m=3$ ) and  $k = (h_1 - h_2) \frac{n_2}{N} + (h_1 - h_3) \frac{n_3}{N}$ , although the results are easily generalizable beyond this.

### 4.3.1 Linear Utility Function

Before stating the solution, we define three threshold values:

$$f_t^L = \frac{b - cBk}{B(a + c(h_1 - k))}, \quad b_{ut}^L = B(a + ch_1), \quad \text{and} \quad b_{it}^L = cBk.$$

**Proposition 4.1:** For problem  $L_L$  with three sub-populations:

- (i)  $U_L$  is a convex function of  $r$  and therefore the optimal solution is either  $r^* = 0$  or  $r^* = 1$ .
- (ii) If  $f \leq f_t^L$  then  $r^* = 1$ .
- (iii) If  $f > f_t^L$  then  $r^* = 0$ . ■

**Corollary 4.1:** When  $b > b_{ut}^L$  then  $r^* = 1$  and when  $b < b_{it}^L$ , then  $r^* = 0$ . ■

Part (i) of Proposition 4.1 says that an extreme point solution of allocating all or none based on equity is always optimal. Part (ii) says that all of the funds received from the upper-level are reserved for proportional allocation if the strength of the incentive is less than a threshold value  $f_t^L$ . Alternatively, part (iii) says that that all funds will be allocated optimally if the strength of the given incentive is greater than  $f_t^L$ . Corollary 4.1 relates the optimal lower-level decision to various problem parameters. It says that all funds are allocated optimally if the LD has a lower preference for equity ( $b < b_{it}^L$ ) and all funds are allocated proportionally if it has a higher preference for equity ( $b > b_{ut}^L$ ).

We next present results that characterize the optimal solution to the upper-level resource allocation problem.

**Proposition 4.2:** For problem  $IA_L$  with three sub-populations:

(i) If  $b \leq b_{it}^L$  then  $r^* = 0$  at the lower level regardless of  $f$ .  $IA_L$  is independent of  $f$  and any  $f$  is optimal. Thus,  $IA_L^*(r^*(f)) = Bh_1$ .

(ii) If  $b > b_{ut}^L$  then  $r^* = 1$  at the lower level regardless of  $f$ .  $IA_L = B(1-f)(h_1 - k)$  and  $f^* = 0$  is optimal. Thus,  $IA_L^*(r^*(f)) = B(h_1 - k)$ .

(iii) If  $b_{it}^L < b \leq b_{ut}^L$  then any  $f^* > f_t^L$  is optimal, resulting in  $r^* = 0$  and  $IA_L^*(r^*(f)) = Bh_1$ .

(iv)  $Z = B$ . ■

Part (i) of Proposition 4.2 says that any level of incentive is optimal if the coefficient of  $r$  in the lower-level linear utility function is less than  $b_{it}^L$ . In this case, the LD will always choose to allocate the entire budget optimally even without incentives from the upper-level and will therefore receive maximum budget. Part (ii) says that if the coefficient of  $r$  is greater than  $b_{ut}^L$ , then the LD will always allocate all funds proportionally, regardless of any incentives. Since  $IA$  is decreasing in  $f$ , it is optimal to set  $f^* = 0$ . This results in the lower-level receiving the maximum possible budget. Part (iii) says that the level of incentive provided should be greater than  $f_t^L$  if the coefficient of  $r$  is in between  $b_{ut}^L$  and  $b_{it}^L$ . For  $b$  between these levels, a choice of incentive above  $f_t^L$  will ensure optimal allocation and the LD will receive entire budget  $B$ .

This may have implications for LD about revealing their preferences for proportional allocation. A feasible amount of incentive may help the UD to encourage the LD for doing optimal allocation. However, if the LD strongly prefers proportional or optimal

allocation then would always choose to allocate proportionally or optimally even with or without incentives.

### 4.3.2 Multiplicative Utility Function

We next present results that characterize the optimal solution to the lower-level resource allocation problem when a multiplicative utility function is used. First, we define three terms.

$$f_{r1}^M = \frac{bh_1 - (b+c)k}{(a+b+c)h_1 - (a+b+2c)k}, b_i^M = \frac{ck}{h_1 - k}$$

$$r^l = \frac{(a+b+c)h_1f + (b+c)k - \sqrt{((a+b+c)h_1f + (b+c)k)^2 - 4(a+b+2c)kfbh_1}}{2(a+b+2c)kf}$$

**Proposition 4.3:** For problem  $L_M$  with three populations,  $r^* = \min\{r^l, 1\}$ . ■

#### Corollary 4.2:

i) If  $b > b_i^M$  then

- a. If  $0 \leq f \leq f_{r1}^M$  then  $r^* = 1$ .
- b. If  $f_{r1}^M < f \leq 1$  then  $r^* = r^l < 1$ .

ii) If  $b \leq b_i^M$  then  $r^* = r^l < 1$ . ■

Proposition 4.3 says that either all or a fraction ( $0 < r^l < 1$ ) of the funds are reserved for proportional allocation. Corollary 4.2 gives threshold conditions under which each result is optimal.

The multiplicative utility function has similarities with the linear utility function. In both cases, a UD can use incentives to encourage the LD allocate to the budget optimally. In addition, in both cases there are situations where the lower-level will always choose to

allocate proportionally ( $r^* = 1$ ). We next present results for the upper-level when the LD has a multiplicative utility function.

**Proposition 4.4:** For problem  $IA_M$  with three populations:

(i) If  $b > b_t^M$  then

a)  $f^* = 0$  is optimal if  $h_1 < h_t$ , resulting in  $r^* = 1$  at the lower-level and

$$IA_M^*(r^*) = B(h_1 - k).$$

b)  $f^* = 1$  is optimal if  $h_1 \geq h_t$ , resulting in  $r^* = r^l < 1$  at the lower-level and

$$IA_M^*(r^*) = B(1 - r^l)(h_1 - r^l k).$$

(ii) If  $b \leq b_t^M$  then  $f^* = 1$  is optimal, resulting in  $r^* = r^l < 1$  at the lower-level and

$$IA_M^*(r^*) = B(1 - r^l)(h_1 - r^l k). \quad \blacksquare$$

Part i-a of Proposition 4.4 says that no incentive is given if the exponent of  $r$  in the lower-level multiplicative utility function is greater than  $b_t^M$  and infections averted/\$ in program 1 are lower than a threshold  $h_t$ . In this case, the UD may not provide any incentive if the LD have higher preference for proportional allocation and less infections are averted/\$ in the region. The LD will choose to allocate the entire budget proportionally without incentives and will receive the budget  $B$ . Part i-b says that maximum incentive is given if the exponent of  $r$  in the lower-level multiplicative utility function is greater than  $b_t^M$  and infections averted/\$ in program 1 are higher than a threshold  $h_t$ . In this case, the UD may provide full incentive if the LD have higher preference for proportional allocation and higher infections are averted/\$ in the region. The LD will choose to allocate optimally with incentives and will receive  $B(1 - r^l)$ . Part (ii) says that maximum incentive is optimal if the coefficient of  $r$  in the lower-level multiplicative utility function is less than  $b_t^M$ . The LD will choose to allocate optimally

with incentives. Since  $IA$  is increasing in  $f$  it is optimal to set  $f^* = 1$ . This results in the LD receiving a fraction of the budget,  $B(1 - r')$ .

Implications for the LD with multiplicative utility function are similar to that with the linear utility function. Incentives may help to maximize the total infections averted/\$ invested. Maximum incentives could be provided to the LD with higher preference for proportional allocation and higher infections averted/\$ in the region which may result in optimal allocation. However, no incentive could be provided to the LD with higher preference for proportional allocation and less infections averted/\$ in the region in order to maximize the total infections averted/\$. This may result in proportional allocation of the entire budget. Maximum incentives could be given to the LD with less preference for proportional allocation since it may switch from proportional to optimal allocation with incentives.

#### 4.4. Example

We illustrate with an example. We used data for California from Zaric and Brandeau [26] and the California Department of Public Health [35]. We assumed three risk groups ( $m=3$ ): injection drug users (IDUs),  $i = 1$ , heterosexuals (HET),  $i = 2$ , and men who have sex with men (MSM),  $i = 3$ . Risk group 1 consists of 17,759 IDUs, risk group 2 consists of 12,167 HET, and risk group 3 consists of 121,128 MSM. We estimated the potential cost and effectiveness of interventions in each population by calculating the number of infections averted per dollar invested using a formula published elsewhere [9] as 0.00012, 0.000046, and 0.0000088, in risk groups 1, 2, and 3, respectively. We estimate that approximately \$35,512,626 of the CDC's \$297,049,344 budget is allocated to California [36].

##### 4.4.1 Linear Utility Function

To estimate  $a$ ,  $b$ , and  $c$ , in the lower-level linear utility function we set  $b = 1$  and assumed  $U_L(r = 0) = \gamma$  and  $U_L(r = 1) = 1$ ,  $0 \leq \gamma \leq 1$  (i.e., we assumed that, given a choice between no equity and all equity, the decision-maker prefers all equity). In the

base case we assumed that  $\gamma = 0.5$ . This resulted in estimates of  $a = 0.00000004$ ,  $b = 1$ , and  $c = 0.0004$ .

Figure 4.1 shows the value of  $f_i^L$  for budget ranging from \$10 million to \$50 million and three lines represent three different values of  $c$  relative to the base case estimates. Since  $f_i^L$  is decreasing in  $B$ , as the budget increases the LD will choose optimal allocation for a smaller incentive. When the budget is large enough,  $f_i^L = 0$ , suggesting that no incentive is needed. When the budget is small,  $f_i^L > 1$ , suggesting that proportional allocation is always chosen. The threshold  $f_i^L$  is decreasing in  $c$ , implying that the lower-level will choose optimal allocation for a larger budget and higher number of HIV infections averted/\$ in a program.

Figure 4.2 contains two graphs showing  $b_u^L$  and  $b_{it}^L$  for different values of  $a$  and  $c$ . The two lines divide each graph into three regions defined by the optimal lower-level decision. The topmost region in each graph corresponds to a region where the LD will always choose to allocate the entire budget proportionally regardless of the incentive provided. The lowermost region in both graphs shows the region where the LD will always choose to allocate entire budget optimally regardless of the incentive provided. The middle region in both the graphs shows that the LD may choose to allocate optimally depending on the incentive and total budget. The threshold  $b_u^L$  is increasing in  $c$  and  $a$ . That is, the topmost region is decreasing in  $c$  and  $a$  suggesting that the LD is less likely to choose proportional allocation as its preferences for the budget and the number of infections averted increases. In Figure 4.2a, the threshold  $b_{it}^L$  is increasing in  $c$  and in the second graph, is constant in  $a$ . That is, the lowermost region is increasing in  $c$  and constant in  $a$  suggesting that the LD is most likely to choose optimal allocation as its preferences for the number of infections averted increases and the choice of allocating optimally does not depend on the budget received.

Figure 4.3 shows the value of  $f_t^L$  for budget ranging from \$10 million to \$50 million. The graph has three lines for three different values of  $n_1$  relative to the base case estimate. The threshold value  $f_t^L$  is decreasing in  $B$  and  $n_1$ , meaning that the lower-level will choose optimal allocation even for a small incentive if the size of the risk group with highest number of infections averted per dollar invested is small and the budget received from the upper-level is high. The threshold  $f_t^L$  is less than one for  $n_1$  at half the base case and reaches zero for  $B$  equal to \$20 million. Thus, for a large budget the LD may choose to allocate optimally. The threshold  $f_t^L$  is less than one for  $B$  less than \$14 million and base case  $n_1$  and zero for  $B$  greater than \$30 million. This suggests that for larger budget and larger risk group with highest number of infections averted per dollar no incentive is needed to encourage optimality.

Figure 4.4 contains two graphs showing linear utility function for different values of  $r$  and the three lines in each graph represents three different values of  $f$ . In the first graph, if the coefficient of  $r$  in the linear utility function is less than  $b_t^L$  then the utility function gets maximized at  $r = 0$ , suggesting that the LD with less preferences for equity may choose optimal allocation. In the second graph, if the lower-level has high preference for equity then it may choose proportional allocation even with incentives.

#### 4.4.2 Multiplicative Utility Function

To estimate  $a$ ,  $b$ , and  $c$ , in the lower-level multiplicative utility function we set  $a = b = 1$  and assumed  $U_L(r = 0) = \gamma$  and  $U_L(r = 1) = 1$ ,  $0 \leq \gamma \leq 1$  (i.e., we assumed that, given a choice between no equity and all equity, the decision-maker prefers all equity). This resulted in estimates of  $a = 1$ ,  $b = 1$ , and  $c = 2.584$ .

Figure 4.5 shows that  $r^*$  is non-increasing in  $f$ . As the incentive provided increases, a non-increasing fraction of the funds are reserved for proportional allocation. As long as,  $f_{t1}^M < 1$ , incentives from the upper-level can be used to improve the outcomes.



Figure 4.6 contains three graphs showing total infection averted for different values of  $f$  for the case i-a)  $b > b_i^M, h_1 < h_i$ , i-b)  $b > b_i^M, h_1 > h_i$ , and ii)  $b \leq b_i^M$  of Proposition 4.4. In the first and the second graph, total infected averted decreases in  $f$  for  $f \in [0, f_{i1}^M]$  and increases in  $f$  for  $f \in (f_{i1}^M, 1]$ . However, the total infections averted in the first graph are maximized at  $f^* = 0$  and in the second graph at  $f^* = 1$ . This suggests that if the LD prefers proportional allocation above a threshold then no incentive is given if the infections averted/\$ in the program is less than a threshold, however, full incentive is given if the infections averted/\$ in the program is above a threshold in order to maximize total infected averted. In the second graph, total infected averted increases in  $f$  for all  $f$ . Thus, if the LD prefers proportional allocation below a threshold then maximum incentives should be provided.

Figure 4.7 shows multiplicative utility function for different values of  $r$  and the three lines in the graph represents three different values of  $f$ . As the incentives increase, the LD's utility function is maximized at an interior point suggesting that the lower-level chooses to allocate optimally. This implies that as the incentives increase, the LD chooses to allocate optimally, however, switches to proportional allocation if no incentives are provided.

#### 4.5. Discussion

We considered a two-level resource allocation problem where the objective at the upper-level is to maximize the total number of HIV infections averted and the objective at the lower-level is to maximize a utility function that contains terms for infections averted, budget, and equity. We considered two general forms for the utility function at the lower-level. The linear objective function is convex in the fraction of the funds reserved for proportional allocation and therefore the optimal allocation has either all or none of the funds reserved for equity. The choice of all or none depends on several factors including the level of the incentive provided by the upper-level and the coefficient of the proportional allocation term in the lower-level utility function.

The possible way to implement this model in practice is that the UD chooses  $f$  and the

LD chooses  $r$  and  $y_j$ , then the upper-level directly allocates the funds to program  $j$  so there is no opportunity for the LD to renege on its plan.

Our analysis suggests that incentives may not be needed if the LD strongly prefers optimal allocation as it would always choose to allocate optimally even without incentives. For a larger budget and higher number of HIV infections averted/\$ in a program, the LD may always prefer optimal allocation. Further, for the larger budget and bigger size of the risk group with highest number of infections averted/\$ no incentive is needed to encourage optimality. However, incentives might be very beneficial if the LD has less preference for optimal allocation since it may switch from optimal to proportional allocation without incentives.

The function  $Z = B(1 - rf)$  incorporating the LD's concerns about equity and upper-levels choice about the strength of incentive is proposed to help demonstrate the significance of incentives in encouraging optimal allocation of HIV prevention funds. However, different incentive schemes could be developed. Our analysis can be generalized for different incentive schemes by adjusting for the upper and lower-level utility functions. Further, the paper looks at a type of incentive. Other types of incentives based on quality or performance could be considered. Recognition of the LDs who are making optimal allocation decisions by the UD or introducing rebate which is paid by the LD to the UD if he is unable to prevent targeted number of new infections can also be considered.

We are not aware of any information specifying functional forms for the utility functions of the regional-level decision maker in practice. However, our analysis has demonstrated the value of such information to the UD. This research is first step in generation of such utility functions that includes significant components that are trivial at the lower level. The results of this evaluation will be useful for policy makers in order to put forward a useful incentive scheme to promote optimal decisions at the lower-level.

We considered a single region at the lower-level. However, it could be extended to multiple regions. A short time horizon model was considered which does not incorporate epidemic dynamics. Models for longer time horizons could be developed. These would need to consider epidemic dynamics and may lead to different allocation decisions. We assumed that benefits of intervention scale linearly with respect to amounts invested. This assumption is common in cost effectiveness analysis but may not always be valid. We assumed that there is no upper limit on amount invested in the programs. However, in the presence of comprehensive information regarding the upper limit on the amount to be invested in the program, decision-makers can invest the amount equal to the upper limit to the program preventing highest number of infections, then to the program preventing second highest number of infections and so on. The remaining budget can then be withheld for future use. The information regarding the upper limit on the amount to be invested in the program can be incorporated into the model and decisions can be made accordingly.

Figure 4.1: Threshold  $f_i^L$  versus budget for different values of  $c$  (coefficient of the total infections averted). Three lines are shown corresponding to the base case value of  $c$ ,  $c/2$ , and  $2c$ .

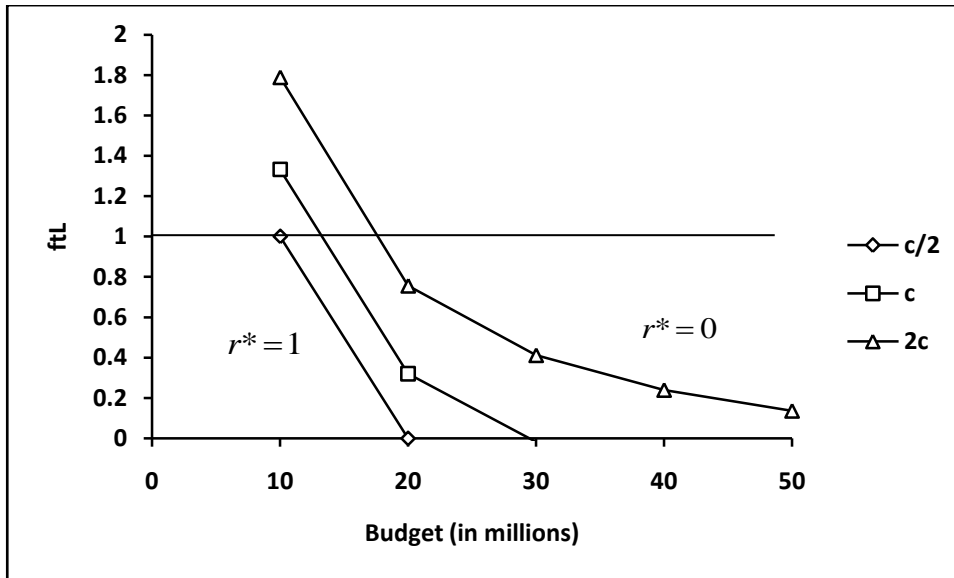


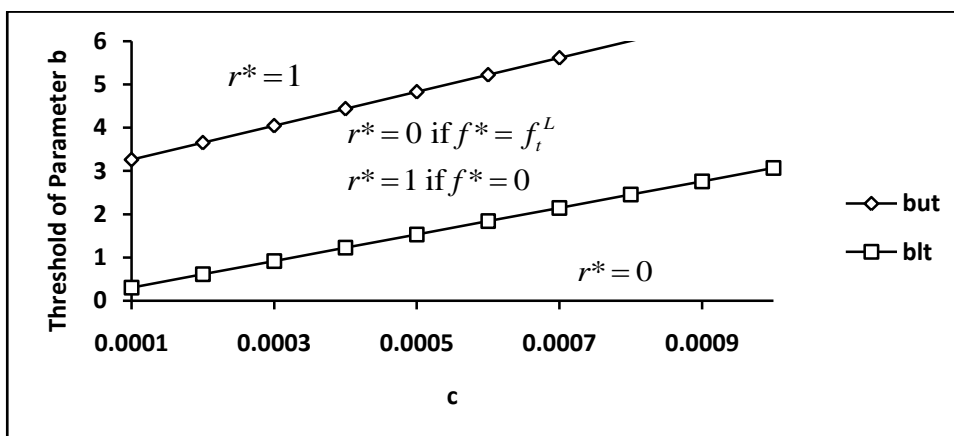
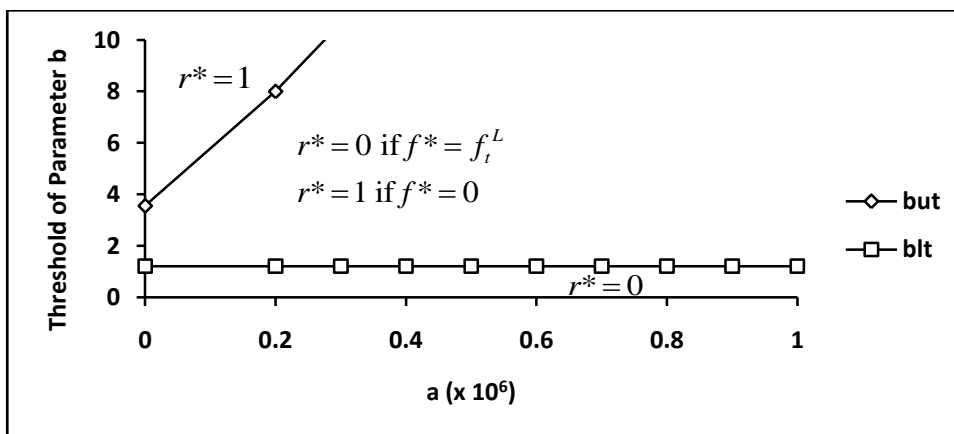
Figure 4.2: Threshold values  $b_{ut}^L$  and  $b_{lt}^L$  for different values of  $a$ ,  $b$ , and  $c$ .Figure 4.2a: Threshold values  $b_{ut}^L$  and  $b_{lt}^L$  for different values of  $b$  and  $c$ .Figure 4.2b: Threshold values  $b_{ut}^L$  and  $b_{lt}^L$  for different values of  $a$  and  $b$ .

Figure 4.3: Threshold  $f_i^L$  versus budget for different values of  $n_1$  (the size of the risk group with highest number of infections averted).

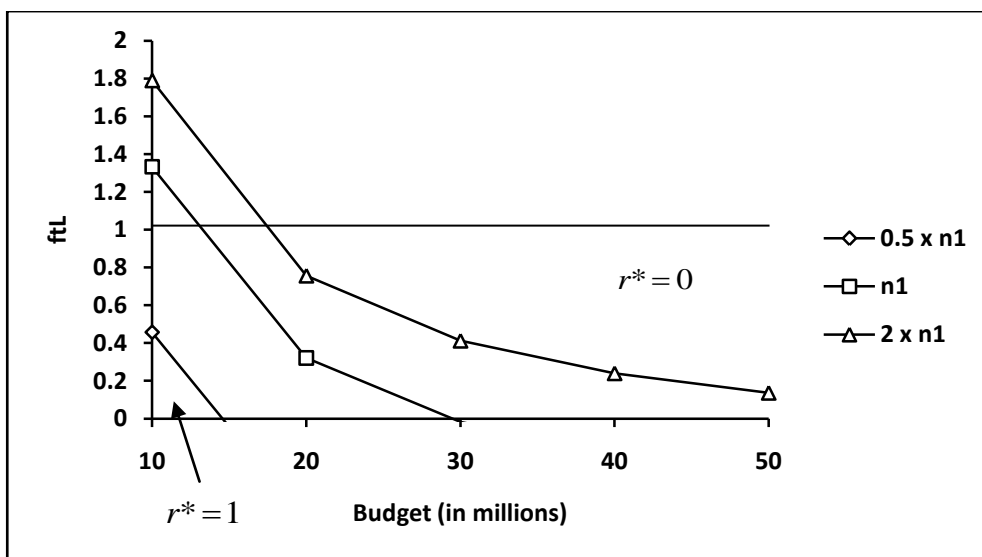


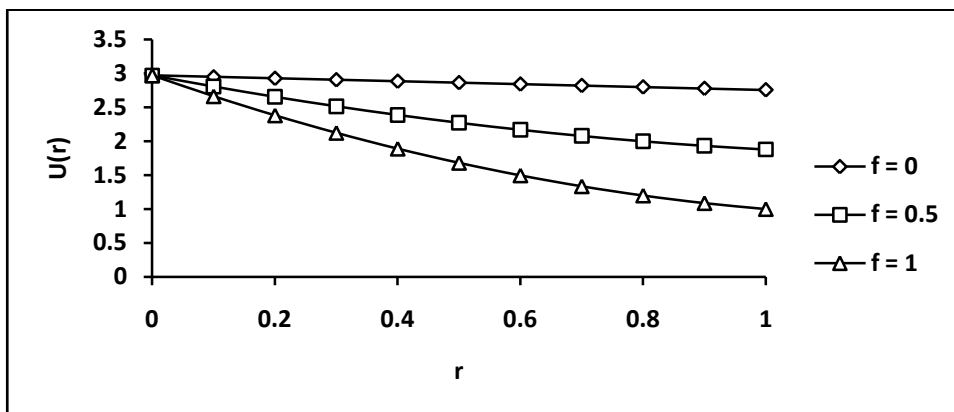
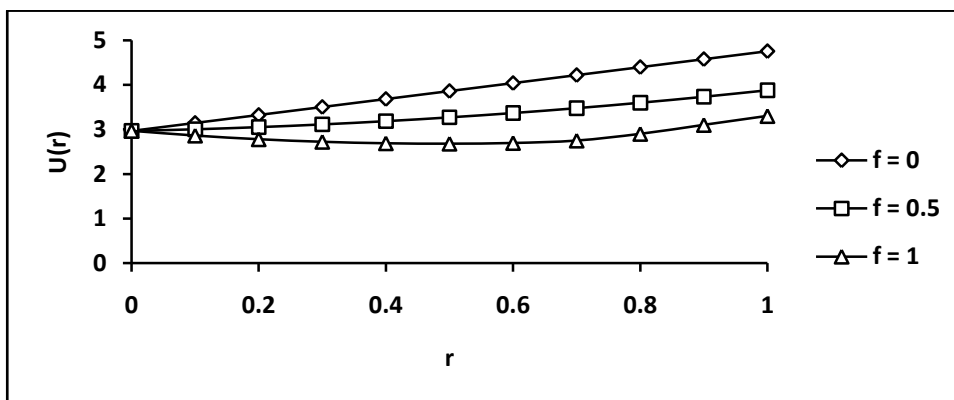
Figure 4.4: Utility function versus  $r$ .Figure 4.4a: Linear utility function  $U_L(r)$  versus  $r$  for  $b < b_{ut}^L$ .Figure 4.4b: Linear utility function  $U_L(r)$  versus  $r$  for  $b > b_{ut}^L$ .

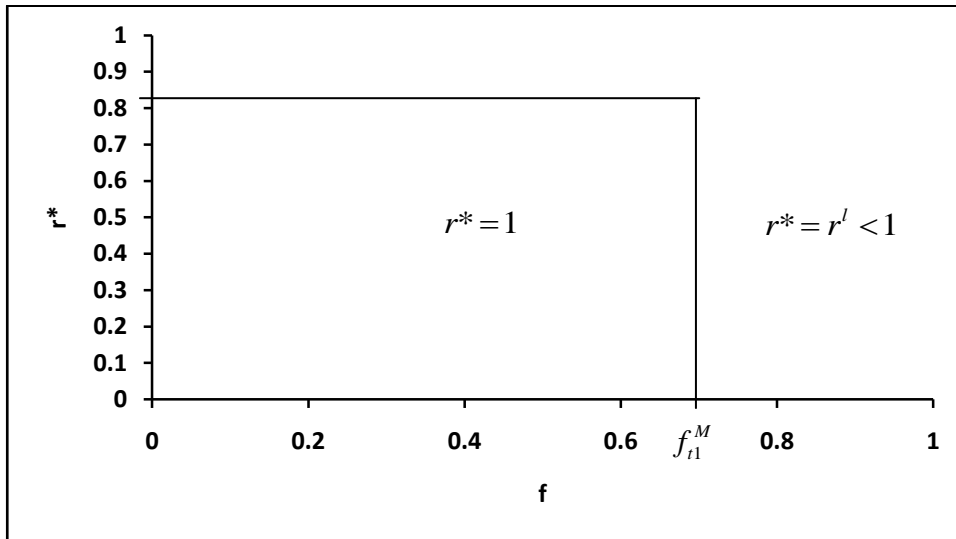
Figure 4.5: Optimal value  $r^*$  versus  $f$ .



Figure 4.6: Total infections averted versus  $f$ .

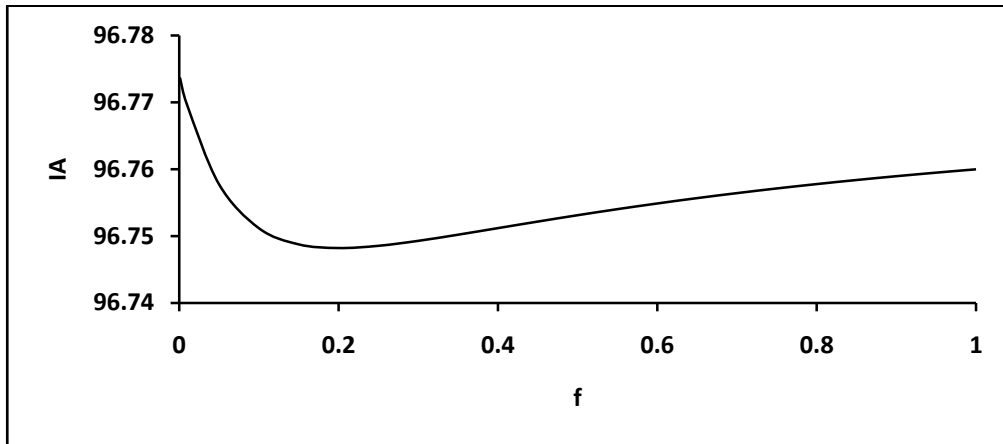


Figure 4.6a: Total infections averted versus  $f$  for the case  $b > b_i^M$  and  $h_1 < h_i$ .

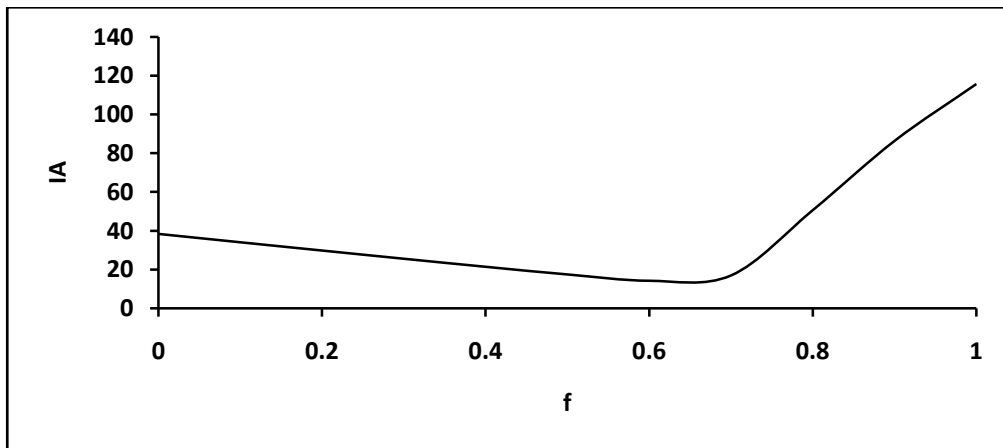


Figure 4.6b: Total infections averted versus  $f$  for the case  $b > b_i^M$  and  $h_1 > h_i$ .

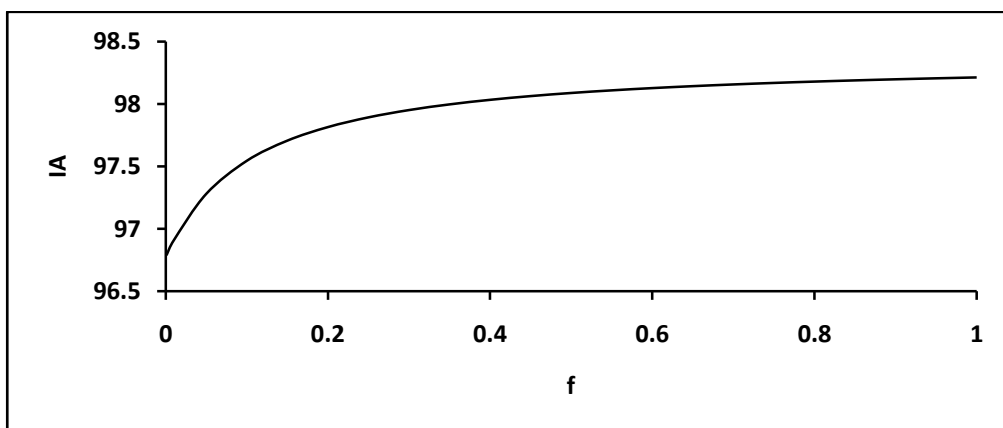
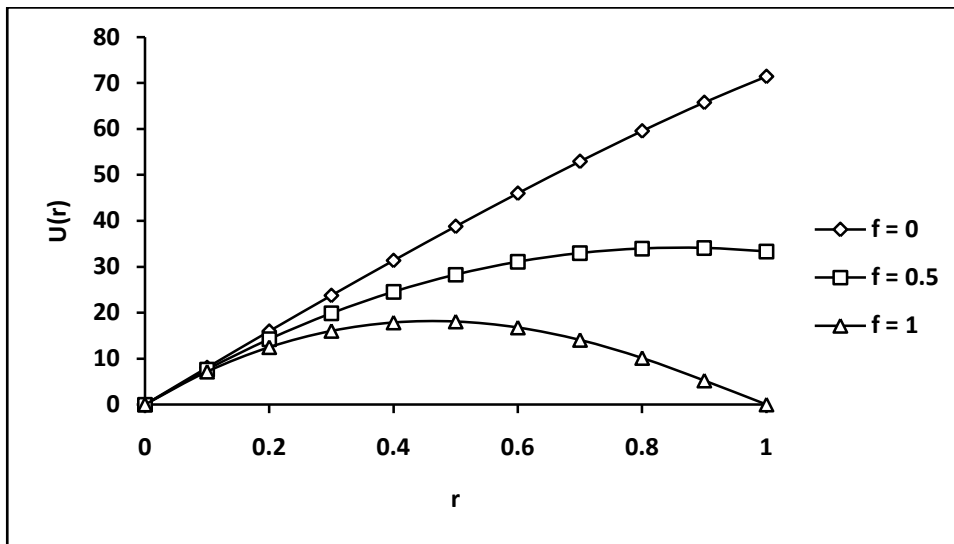


Figure 4.6c: Total infections averted versus  $f$  for the case  $b \leq b_i^M$ .

Figure 4.7: Multiplicative utility function  $U_M(r)$  versus  $r$ .

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## Chapter 5

### **Optimal Incentives for HIV Prevention Funds Allocation under Asymmetric Information**

#### **5.1. Introduction**

Resource allocation models often require cost and effectiveness data on the results of an intervention or the number of infections prevented by an intervention. However, these data may not be available in practice due to several reasons including context-specific data requirements of a model, missing data, and lack of tools to collect the necessary information [1]. The Compendium of HIV/AIDS Prevention Interventions published by the Center for Disease Control and Prevention (CDC) lists effectiveness of various interventions. However, cost estimates are not listed [2]. Various parameters such as the preferences for equity-based allocation, resources expended at the regional level, infections prevented per dollar, and utilization of resources from various other sources at the regional level may be known at the regional level, but unknown at the national level [3].

Donors, advocacy groups, or regional-level decision-makers may have preferences for distributing the prevention funds based on equity. However, these preferences are often unknown to the national-level decision-maker. For example, Bautista-Arredondo et al. reviewed data from developing countries in Africa, Asia, and Latin America on the allocation of HIV/AIDS prevention funds and found that the regional-level decision-makers often prefer equity-based heuristics [4].

If incentives are provided by the national-level decision-maker to the regional-level decision-makers to reveal information, then this may lead to an optimal resource allocation. Lasry et al. modeled a two-level decision-making process and evaluated the impact of optimal versus simple allocation techniques by comparing four allocation strategies. They showed that if optimization modeling is to be applied at only one level of the decision-making process, then it is more beneficial to apply it at the lower level than

at the upper level, and they concluded that the upper-level decision-maker should develop incentives to encourage an optimal allocation at the lower-levels [5]. Zaric and Brandeau extended this work to include multiple regions and various sub-populations and compared optimal and proportional (in proportion to HIV incidences) allocation strategies using data from 40 U.S. states and three risk groups and obtained similar conclusions [6]. Malvankar and Zaric extended this work by developing an incentive scheme for a multi-level decision-making process if complete information is available at both the national and the regional levels [7]. We expand on the work of Malvankar and Zaric by modeling information asymmetry in the HIV/AIDS prevention funds allocation process.

Information asymmetry in a resource allocation process can occur if regional-level decision-makers possess superior information compared to the national-level decision-maker. In the fiscal federalism literature, decision-makers at the regional level often allocate resources based on regional preferences which may not be known to the decision-makers at the national level [8, 9]. Therefore, fiscal federalism under asymmetric information often leads to ambiguity in outcomes [10, 11].

Bossert and Beauvais used a principal-agent framework in which the Ministry of Health acts as the principal and the municipal and regional governments act as agents [12]. The local agents often have their own preferences and respond to local donors and advocacy groups which may have different preferences than those of the principal. The authors showed that diverse mechanisms are employed by the principal such as providing incentives, monitoring, reporting, and performance reviews to achieve the objective regarding the optimal allocation of resources.

Some studies examine incentives in cases of asymmetric information in health care resource allocation. Peterson et al. reviewed studies assessing the impact of financial incentives on improving health care quality under symmetric and asymmetric information systems [13]. Gurnani et al. investigated the impact of incentives in a two-echelon healthcare supply chain model [14]. They found that the cost structure and uncertainty in market demand govern the nature of incentives for sharing information among members of the supply chain. McKenna et al. developed a two-stage stochastic programming model for a health care resource allocation problem to evaluate different budgets and budgetary

policies [15]. There are many other examples of research on incentives under asymmetric information in health care resource allocation [16-19]. However, we are not aware of any incentive-based models developed by modeling information asymmetry in an HIV/AIDS resource allocation process.

In this paper, we model information asymmetry in a multi-level HIV/AIDS resource allocation process in which an upper-level decision-maker (UD) allocates funds to a single lower-level decision-maker (LD) who then allocates funds to two programs. Our study attempts to answer the following questions. What is the impact of incentives if the preferences of the LD about equity-based (proportional) allocation are unknown to the UD? Or What is the impact of incentives if the infections prevented per dollar at the lower level are unknown to the UD? The rest of the paper is organized as follows: Section 2 introduces the model. Section 3 provides a mathematical analysis of the proposed model and Section 4 provides a numerical example. Conclusions and extensions are discussed in Section 5.

## 5.2. Model Formulation

We develop a single period model of a two-level decision-making process similar to the model of Malvankar and Zaric [7]. There is an upper-level decision-maker (UD) using an incentive scheme to encourage a single lower-level decision maker (LD) to allocate limited resources optimally under asymmetric information.

We consider two sources of information asymmetry. In case 1, the preferences of the LD about allocating HIV/AIDS prevention funds based on equity are unknown to the UD. In case 2, the number of infections prevented per dollar in a program is known to the LD, but unknown to the UD.

We assume that with probability  $p$ , the LD is of a high type and with probability  $q$ , where  $q = 1 - p$ , the LD is of a low type. In case 1, a high type LD has higher preferences, represented as  $b^H$  for allocating HIV/AIDS prevention funds based on equity and a low type LD has lower preferences, represented as  $b^L$ ,  $b^H > b^L$ . Similarly in case 2, a high type LD has higher number of infections prevented per dollar, represented as  $h_2^H$ , in a



program and a low type LD has a lower number of infections prevented per dollar, represented as  $h_2^L$ , in a program,  $h_2^H > h_2^L$ .

The LD chooses  $r_{ij}$  ( $0 \leq r_{ij} \leq 1$ ),  $i = 1, 2$ , where  $i$  represents the case, and  $j = H, L$ , where,  $H$  represents high type and  $L$  represents low type LD in a case. The LD distributes amount  $y_m, m = 1, 2$  to program  $m$ . As in Zaric and Brandeau [6], we assume that one program is available for each risk group, that the programs do not interact, and that the costs and benefits scale linearly. Similar to Malvankar and Zaric [7], we assume that the UD has a total budget  $B$ , chooses a fraction  $f, 0 \leq f \leq 1$  and allocates an amount  $Z$  to the LD using the following equation:

$$Z(r_{ij}) = B(1 - r_{ij}f). \quad (5.1)$$

According to equation (1), the amount received by LD decreases with the decrease with  $f$  and decrease with  $r_{ij}$  is an incentive scheme developed in Malvankar and Zaric [7].

This is a dynamic programming problem with the following time sequence: the UD chooses  $f$ ; then the LD chooses  $r_{ij}$  for the given value of  $f$ ; then the LD determines  $y_m$  for the given values of  $f$  and  $r_{ij}$ . We solve this problem using backward induction and present the details in a reverse time sequence.

### 5.2.1 The Lower-level Model

**Stage 3:** The model developed in this stage is similar to that of Zaric and Brandeau [6] and Malvankar and Zaric [7]. Let  $h_m, m = 1, 2$  be the number of HIV infections prevented per dollar invested in a program  $m$ . Let  $n_m$  be the size of the risk group  $m$  and  $N = n_1 + n_2$ . The total number of HIV infections averted,  $IA$ , is given by the following equation:

$$IA = h_1 y_1 + h_2 y_2, \quad (5.2)$$

In the last step of the dynamic program, given  $r_{ij}$  and  $f$ , the total number of infections averted is found by solving the following linear programming problem:

$$\max_{y_1, y_2} IA = h_1 y_1 + h_2 y_2 \quad (5.3)$$

$$\text{S.t. } y_1 + y_2 \leq Z = B(1 - r_{ij} f) \quad (5.4)$$

$$y_j \geq r_{ij} Z \frac{n_j}{N}, j = 1, 2 \quad (5.5)$$

This is a Knapsack LP and easily solved. The solution depends on the case.

### Case 1

$$y_1 = Z - r_{ij} Z \frac{n_2}{N},$$

$$y_2 = r_{ij} Z \frac{n_2}{N}. \text{ Thus,}$$

$$IA(r_{1j}) = Z(h_1 - r_{1j} k) = B(1 - r_{1j} f)(h_1 - r_{1j} k), \quad (5.6)$$

where,  $j = H, L$ ,  $k = (h_1 - h_2) \frac{n_2}{N}$  and  $h_1 \geq k$ .

### Case 2

In case 2, the solution depends on the type at the lower level. For the high type LD,

$$y_1 = r_{ij} Z \frac{n_1}{N},$$

$$y_2 = Z - r_{ij} Z \frac{n_1}{N}. \text{ Thus,}$$

$$IA(r_{2H}) = Z(h_2^H - r_{2H} k_2) = B(1 - r_{2H} f)(h_2^H - r_{2H} k_2) \text{ with probability } p \quad (5.7)$$

where,  $k_2 = (h_2^H - h_1) \frac{n_1}{N}$  and  $h_2^H \geq k_2$  or

For the low type LD,

$$y_1 = Z - r_{ij}Z \frac{n_2}{N},$$

$$y_2 = r_{ij}Z \frac{n_2}{N}. \text{ Thus,}$$

$$IA(r_{2L}) = Z(h_1 - r_{2L}k_1) = B(1 - r_{2L}f)(h_1 - r_{2L}k_1) \text{ with probability } q \quad (5.8)$$

where,  $k_1 = (h_1 - h_2^L) \frac{n_2}{N}$  and  $h_1 \geq k_1$ .

**Stage 2:** In the second step, the LD chooses  $r_{ij}$  to maximize his utility function. We use the model developed in Malvankar and Zaric [7] at the lower-level in which equity is captured through  $r_{ij}$ , efficiency is captured through  $IA(r_{ij})$ , and funds received as  $Z(r_{ij})$ .

We investigate two different forms for the utility function, linear and multiplicative,

$U_L(r_{ij})$  and  $U_M(r_{ij})$ , given for each case below,

### Case 1

$$U_L(r_{1j}) = aZ(r_{1j}) + b^j r_{1j} + cIA(r_{1j}) \quad (5.9)$$

or

$$U_M(r_{1j}) = (Z(r_{1j}))^a (r_{1j})^{b^j} (IA(r_{1j}))^c \quad (5.10)$$

### Case 2

$$U_L(r_{2j}) = aZ(r_{2j}) + b r_{2j} + cIA(r_{2j}) \quad (5.11)$$

or

$$U_M(r_{2j}) = (Z(r_{2j}))^a (r_{2j})^b (IA(r_{2j}))^c \quad (5.12)$$

In the case of  $U_L(r_{ij})$ , the parameters  $a, b, b^H, b^L, c > 0$  represent the relative weights applied to funds received, equity, and infections averted. In the case of  $U_M(r_{ij})$ , the parameters  $a, b, b^H, b^L, c > 0$  are exponents of budget, equity, and infections averted. Depending on which utility function is used, the lower-level optimization problem is written as:

$$L_{iL} : \max_{r_{ij}} U_L(r_{ij}) \quad (5.13)$$

or

$$L_{iM} : \max_{r_{ij}} U_M(r_{ij}) \quad (5.14)$$

$$\text{s.t. } 0 \leq r_{ij} \leq 1 \quad (5.15)$$

The LD solves  $L_{iL}$  or  $L_{iM}$  to obtain  $r_{ij}^*(f)$ .

### 5.2.2 The Upper-level Model

**Stage 1:** Similar to Zaric and Brandeau and Malvankar and Zaric, we assume that the objective at the upper level is to maximize the number of infections averted [6, 20]. Thus, the upper-level resource allocation problem is:

$$IA_{iL} : \text{Max}_f E[IA(r_{ij}^*(f))] \quad (5.16)$$

or

$$IA_{iM} : \text{Max}_f E[IA(r_{ij}^*(f))] \quad (5.17)$$

$$\text{S.t. } 0 \leq f \leq 1 \quad (5.18)$$

$$r_{ij}^*(f) = \text{argmax}_{r_{ij}} U_i(r_{ij}) \quad (5.19)$$

where,  $E\left[IA\left(r_{ij}^*(f)\right)\right]$  in each case is given below,

### Case 1

$$E\left[IA\left(r_{ij}^*(f)\right)\right] = B(1-r_{1H}f)(h_1-r_{1H}k)p + B(1-r_{1L}f)(h_1-r_{1L}k)q \quad (5.20)$$

### Case 2

$$E\left[IA\left(r_{ij}^*(f)\right)\right] = B(1-r_{2H}f)(h_{2H}-r_{2H}k_2)p + B(1-r_{2L}f)(h_1-r_{2L}k_1)q \quad (5.21)$$

## 5.3. Analysis

In this section, we analyze the optimization problem for both utility functions for the two cases of asymmetric information.

### 5.3.1 Case 1

We next present the results that characterize the optimal solution to the upper-level resource allocation problem when the LD has a linear utility function and the preferences of the lower level to allocate based on equity are unknown to the UD.

**Proposition 5.1:** For problem  $IA_{1L}$  with two sub-populations:

(a) If  $b^H > b_{ut}^{bL}, b_{it}^{bL} < b^L \leq b_{ut}^{bL}, p > p_t^{bL}$  then  $f^* = 0$  is optimal, resulting in  $r_{1j}^* = 1, j = H, L$ ,  $E[IA] = B(h_1 - k)$ , and  $LOE = (1 - p)Bk$ .

(b) If  $b^H > b_{ut}^{bL}, b_{it}^{bL} < b^L \leq b_{ut}^{bL}, p \leq p_t^{bL}$  then  $f^* = f_{t2}^{bL}$  is optimal, resulting in  $r_{1H}^* = 1, r_{1L}^* = 0$ ,  $E[IA] = B(1 - f_{t2}^{bL})(h_1 - k)p + Bh_1q$ , and  $LOE = pBf_{t2}^{bL}(h_1 - k)$ .

(c) Otherwise, LOE is zero. ■

Part (a) of Proposition 5.1 states that no incentive is provided if a high type LD has preferences to allocate the budget based on equity are higher than  $b_{ut}^{bL}$ , a low type LD has

preferences between  $b_{it}^{bL}$  and  $b_{ut}^{bL}$ , and the probability that a LD is of a high type is higher than  $p_i^{bL}$  then the loss of efficiency (LOE) is  $(1-p)Bk$  compared to the full information case. This suggests that if there is a high probability that a LD is of a high type and that the LD has preferences for allocating the budget based on equity then no incentive is given since the LD will choose to allocate proportionally even with incentives. There is a LOE which increases with the decrease in the probability that a LD is of high type since if the LD is of low type then an optimal allocation can be encourage by giving incentives.

Part (b) states that an  $f_{i2}^{bL}$  level of incentive is optimal if a high type LD has preferences to allocate the budget based on equity higher than  $b_{ut}^{bL}$ , a low type LD has preferences between  $b_{it}^{bL}$  and  $b_{ut}^{bL}$ , and the probability that a LD is of high type is lower than  $p_i^{bL}$  then the LOE is  $pBf_{i2}^{bL}(h_1-k)$  compared to the full information case. In this case, if the LD is of high type then this person will always choose to allocate the entire budget based on equity even with incentives and, if the LD is of low type, then that person can be encouraged by providing incentives to allocate the entire budget optimally. If the probability that a LD will be of low type is high, then incentives should be given to encourage optimal allocation. On comparing this case with a full information case, there is a LOE incurred at the upper level. The LOE increases with the increase in the budget, level of the incentive provided, the probability that the LD is of high type, and the number of infections prevented per dollar in a program at the lower-level. This implies that if the UD provides more and more incentives to the LD and if the LD is of low type then incentives will encourage an optimal allocation. However, if the LD is of high type then that person will always choose to allocate the resources based on equity even with high incentives and thus, providing more financial incentives will result in a loss at the upper level. Part (c) states that in the other cases specified in Table 5.4 in Appendix 9, the loss of efficiency is zero.

Table 5.4 shows that for a high or a low type LD, if the preferences to allocate the budget based on equity are lower than a set threshold, then regardless of incentives, the LD would choose to allocate optimally. However, if the preferences are higher than a set threshold, then the LD would choose to allocate proportionally and no incentive is given.

If the LD has moderate preferences for allocating the budget based on equity, then an incentive is given to encourage an optimal allocation. Some cases cannot exist since  $b^H > b^L$ .

We next present the results that characterize the optimal solution to the upper-level resource allocation problem when the LD has a multiplicative utility function and the preferences of the lower level to allocate based on equity are unknown to the UD.

**Proposition 5.2:** For problem  $IA_{1M}$  with two sub-populations, under various conditions the LOE is specified in Table 5.2, otherwise, LOE is zero. ■

Table 5.2 shows that if there is a high probability that the LD is of a high type and a higher number of infections are prevented per dollar, then full incentives are given so that the LD will choose to allocate optimally even with higher preferences for allocating the budget based on equity. In this case, the LOE decreases with the increase in the probability that the LD is of high type since higher infections are prevented. Further, no incentive is given if fewer infections are prevented per dollar since both types of LDs will choose to allocate based on equity even with incentives. In this case there will be a loss of efficiency which will depend on the budget, the probability that the LD is of high type, and the number of infections prevented per dollar. In other cases specified in Tables 5.5 in Appendix 9, the loss of efficiency is zero.

Table 5.5 states that if both the LDs have fewer preferences to allocate the budget based on equity then full incentives are provided so that entire budget is allocated optimally. Further, if both the LDs have higher preferences to allocate the budget based on equity and there is a high probability that the LD is of high type then also full incentives are provided to encourage optimal allocation. However, in this case there is a loss of efficiency.

### 5.3.2 Case 2

We next present the results that characterize the optimal solution to the upper-level resource allocation problem when the LD has a linear utility function and the infections prevented per dollar at the lower level are unknown to the UD.

**Proposition 5.3:** For problem  $IA_{2L}$  with two sub-populations:

(a) If  $h_{i2}^{hL} < h_2^H \leq h_{i1}^{hL}$ ,  $h_1 \leq h_{i2}^{hL}$ ,  $p > p_t^{hL}$  then  $f^* = f_{i1}^{hL}$  is optimal, resulting in  $r_{2H}^* = 0$ ,  $r_{2L}^* = 1$ ,

$$E[IA] = Bh_2^H p + B(1 - f_{i1}^{hL})(h_1 - k_1)q, \text{ and LOE} = (1 - p)Bf_{i1}^{hL}(h_1 - k_1).$$

(b) If  $h_{i2}^{hL} < h_2^H \leq h_{i1}^{hL}$ ,  $h_1 \leq h_{i2}^{hL}$ ,  $p \leq p_t^{hL}$  then  $f^* = 0$  is optimal, resulting in  $r_{2j}^* = 1$ ,  $j = H, L$ ,

$$E[IA] = B(h_2^H - k_2)p + B(h_1 - k_1)q, \text{ and LOE} = pBk_2.$$

(c) Otherwise, LOE is zero. ■

Part (a) of Proposition 5.3 states that the  $f_{i1}^{hL}$  level of incentive is optimal if a high type LD has the number of infections prevented per dollar between  $h_{i2}^{hL}$  and  $h_{i1}^{hL}$ , a low type LD has the infections prevented per dollar below  $h_{i2}^{hL}$ , and the probability that a LD is of high type is higher than  $p_t^{hL}$  then the loss of efficiency is  $(1 - p)Bf_{i1}^{hL}(h_1 - k_1)$ . In this case, the LD with a moderate number of infections prevented per dollar can be encouraged to allocate the budget optimally by providing a suitable level of incentives. Further, with a high probability that the LD is of high type, the LOE decreases. If the LD has fewer infections prevented/dollar, then even with incentives the LD would choose to allocate based on equity. The LOE increases with the increase in the budget, infections prevented per dollar, and the level of incentives, and decreases with the increase in the probability that the LD is of high type. In this case, providing more financial incentives will result in a loss of efficiency at the upper level.

Part (b) says that no incentive is given if a high type LD has infections prevented per dollar between  $h_{i2}^{hL}$  and  $h_{i1}^{hL}$ , a low type LD has infections prevented per dollar below  $h_{i2}^{hL}$ , and the probability that a LD is of high type is lower than  $p_t^{hL}$  then the loss of efficiency is  $pBk_2$ . In this case, no incentive is given because both the LD, whether of a high or low



type, would always choose to allocate the entire budget based on equity even with incentives. Thus, where there is a lower probability that a LD will be of high type, no incentive is given to prevent a maximum number of infections. There is a loss of efficiency (LOE) incurred at the upper level. This loss is reduced to zero when the probability that the LD is of a high type is zero. The LOE increases with an increase in the budget and infections prevented per dollar are at the lower-level. Part (c) states that in other cases specified in Tables 5.6 in Appendix 9, the loss of efficiency is zero.

Table 5.6 shows that for both types of LDs, if higher number of infections is prevented per dollar, then both the LDs will choose to allocate optimal regardless of the incentive given. Further, if moderate infections are prevented per dollar in both regions then incentive will encourage optimal allocation and both the LDs will allocate optimally. However, no incentives are given if 1 of the LD has higher number of infections prevented per dollar and the other LD has fewer infections prevented per dollar since the LD with higher infections prevented will always choose to allocate optimally and the LD with fewer infections prevented will always choose to allocate proportionally.

We next present the results that characterize the optimal solution to the upper-level resource allocation problem when the LD has a linear utility function and the infections prevented per dollar at the lower level are unknown to the UD.

**Proposition 5.4:** For problem  $IA_{2M}$  with two sub-populations, under various conditions the LOE is specified in Table 5.3, otherwise, LOE is zero. ■

Table 5.3 states that full incentives are provided if there is a high probability that the LD is of a high type and a higher number of infections are prevented per dollar. In this case, entire budget will be allocated optimally preventing maximum infections. If there is a low probability that the LD is of high type and a moderate number of infections are prevented per dollar then full incentives are given to prevent maximum infections. However, no incentive is given if there is a low probability that the LD is of high type and higher number of infections is prevented per dollar. In all the cases described above, there will be a loss of efficiency when we compare the results in those achieved in the full information case. This loss will depend on the budget, the probability that the LD is of a

high type, and the number of infections prevented per dollar. In other cases specified in Tables 5.7 in Appendix 9, the loss of efficiency is zero.

Table 5.7 shows that if fewer infections are prevented per dollar in both the regions then full incentive is given to encourage both the LDs to choose optimal allocation. In this case, there is no loss of efficiency. However, if higher infections are prevented per dollar in both the regions then no incentive is given if there is a low probability that the LD is of high type. Since, it will result in a loss of efficiency.

#### 5.4. Example

We illustrate with an example from Malvankar and Zaric (2008). We assumed two risk groups: injection drug users (IDUs),  $i = 1$  and heterosexuals (HET),  $i = 2$ , where risk group 1 consists of 17,759 IDUs and risk group 2 consists of 12,167 HET. We use the estimated number of infections averted per dollar published in Malvankar and Zaric (2008) as 0.00012 and 0.000046 in risk groups 1 and 2, respectively, and approximately \$35,512,626 of the CDC's \$297,049,344 budget is allocated to California [20, 21].

To estimate  $a$ ,  $b$ , and  $c$ , we assume that  $a = 1 - b - c$  in the lower-level utility functions and the preferences for infections averted per dollar ( $c$ ) is 0.1. In case 1, we assume that preferences of the LD for allocating the budget based on equity can be as high as 0.8 with a probability of  $p = 0.7$  (i.e. we assumed that, the LD often prefers equity) and as low as 0.2 with a probability  $q = 0.3$ . In case 2, we set the number of infections averted per dollar in risk group 2 as high as twice that of 0.000046 with probability  $p = 0.4$  (i.e. we assumed that since the LD often prefers equity, it is rare that a high number of infections is averted), as low as half of 0.000046 with probability  $q = 0.6$ , and  $b = \frac{0.8+0.2}{2} = 0.5$ . In case 3, we assume  $b_H = 0.8, b_L = 0.2, h_{2H} = 2(0.000046)$ , and  $h_{2L} = 0.000046/2, p_1 = 0.7, p_2 = 0.4$ .

Figure 5.1 consists of 3 graphs showing the total number of HIV infections averted for a budget ranging from \$20 million to \$80 million and the two lines in each graph represent two cases, full and asymmetric information when the preferences of the LD for allocating

the budget based on equity are unknown to the UD and LD has a linear utility function. The 3 graphs correspond to three different values of  $h_1$  relative to the base case estimates. In all, the 3 graphs show the difference between the number of HIV infections averted in full and asymmetric information case. The difference increases as the budget increases. This shows that when the budget is higher, fewer infections are averted under asymmetric information compared to full information. In the first graph, when the budget is small enough,  $B = \$20$  million, and  $h_1$  is half the base case value, then the difference between the number of infections averted in full and asymmetric information cases is negligible. On the other hand, in the third graph, the difference between the number of infections averted in full and asymmetric information cases is highest when the budget is highest,  $B = \$80$  million, and  $h_1$  is twice the base case value. This implies that when infections prevented per dollar are higher LOE is higher.

Figure 5.2 consists of 3 graphs showing the total number of HIV infections averted for a budget ranging from \$20 million to \$80 million and the two lines in each graph represents two cases, full and asymmetric information under a linear lower-level utility function when the infections averted per dollar are unknown to the UD. The 3 graphs correspond to three different values of  $b$  relative to the base case estimates. In the case of information asymmetry, fewer infections are averted compared to the full information case when preferences for allocating the budget based on equity are lower, as shown in the first graph. If complete information is available and the preferences for allocating the budget based on equity are higher, then even with incentives the LD will choose to allocate the entire budget based on equity then the number of infections averted in full and asymmetric information cases is almost similar as shown in the third graph. However, infections averted in the third graph are lower compared to the first graph under asymmetric information. This suggests that, when preferences for allocating the budget based on equity are higher, then a lower number of infections are averted and the LOE is less.

Figure 5.3 contains two graphs. The first graph shows the number of expected infections averted for the different values of  $p$  in cases 1 and 2, where  $p$  is the probability that LD is of a high type having higher preferences for allocating the budget based on equity in case

1 and a higher number of infections averted per dollar in case 2. The expected number of infections prevented is decreasing in  $p$  for case 1 and is increasing in  $p$  for case 2. This suggests that the number of expected infections prevented increase with the increase in the probability.

Figure 5.4 shows the expected number of infections averted for the budget ranging from \$20 million to \$80 million and the 3 lines represent different values of  $N$  relative to the base case estimates. The difference between Since  $E[IA]$  is increasing in  $B$  and  $N$ , as the budget and total size of the risk groups increases, the number of infections prevented per dollar in each of the 3 cases increases.

## 5.5. Discussion

We considered asymmetric information in a two-level HIV prevention funds allocation process in which the first case of information asymmetry is about preferences for allocating the budget based on equity which are unknown to the UD. The second case is about the number of infections prevented per dollar in a program which are known to the LD, but unknown to the UD. Similar to Malvankar and Zaric (2008) we assume that the objective at the upper level is to maximize the total number of HIV/AIDS infections averted and that the objective at the lower level is to maximize a utility function. We considered a linear and a multiplicative form of the utility function for each type of information asymmetry.

Our numerical analysis suggests that incentives can encourage an optimal allocation under asymmetric information. However, there is a loss of efficiency when we compare an asymmetric information case with a full information case. Under various conditions in two different cases, the LOE increase with the increase in the budget and the size of the risk group. As the budget increases, fewer infections are averted under asymmetric information compared to full information. LOE also depends on the difference between the number of infections prevented per dollar in both risk groups. If preferences about allocating the budget based on equity are higher regardless of the type of LD, then the LOE is lower. As the preferences of the LD to prevent infections increase, the difference

between the numbers of infections averted in full and asymmetric information cases decreases.

If full information is available about the effectiveness of various interventions, the number of infections prevented per dollar invested in a region, and the total size of the infected population, etc. is available to the UD, then the information can be used to formulate a feasible incentive scheme to encourage optimal allocation. However, in practice, there is usually missing data or incomplete or no information available to the UD. We consider 2 cases of asymmetric information here. Further, our model is generalizable when various other parameters of the utility function, for example, preferences regarding the budget received and the number of infections prevented is unknown to the UD.

There are multiple stakeholders including donors, advocacy groups, local governments, non-profit organizations, and community-based organizations taking part in the decision-making process at the lower level. Each stakeholder has its own priorities, objectives, and limitations (Lasry et al., 2009). We assume that there is a single decision-maker at the lower level. However, we also consider asymmetric information at the upper level about the preferences of the lower level decision-maker to allocate based on equity which can capture the impact that various stakeholders have in making a resource allocation decision about equity at the lower level that is unknown to the upper level.

Our analysis has certain limitations. We consider only 2 cases of asymmetric information. In practice, there could be a number of factors impacting the decision which are unknown to the upper level decision-maker such as local politics, and social considerations including religion, human rights and values, cultural values, etc. A model incorporating asymmetric information about these various factors could be developed. The model thus developed can be complex and may lead to different allocation decisions. Here we use an incentive scheme developed elsewhere. However, our goal was to show the impact of incentives under conditions of asymmetric information. Our model can be generalized for other incentive schemes. We also consider a short time horizon model. In the future, we would like to extend our model to a more distant time horizon under information asymmetry.

Table 5.1: Summary of Notations.

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**Thresholds** ( $i = 1, 2$ )

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$$f_{i1}^{bL} = \frac{b^H - cBk}{B(a + c(h_1 - k))},$$

$$f_{i2}^{bL} = \frac{b^L - cBk}{B(a + c(h_1 - k))},$$

$$P_t^{bL} = \frac{k}{k + f_{i2}^{bL}(h_1 - k)},$$

$$b_{ut}^{bL} = B(a + ch_1),$$

$$b_{it}^{bL} = cB(h_1 - h_2) \frac{n_2}{N},$$

$$f_{i1}^{bM} = \frac{b^H h_1 - (b^H + c)k}{(a + b^H + c)h_1 - (a + b^H + 2c)k},$$

$$f_{i2}^{bM} = \frac{b^L h_1 - (b^L + c)k}{(a + b^L + c)h_1 - (a + b^L + 2c)k},$$

$$h_{ij}^{bM} = \frac{(a + b^j + 2c) \sqrt{(h_1 + k)^2 - 4k^2} - \sqrt{((a + b^j + c)h_1 + (b^j + c)k)^2 - 4(a + b^j + 2c)kb^j h_1 - k(a + c)}}{c},$$

$$j = H, L,$$

$$b_{i1}^{bM} = \frac{ck}{h_1 - k},$$

$$P_{i1}^{bM} = \frac{(h_1 - k) - (1 - r_{iL}^l)(h_1 - r_{iL}^l k)}{(1 - r_{iH}^l)(h_1 - r_{iH}^l k) - (1 - r_{iL}^l)(h_1 - r_{iL}^l k)},$$

$$P_{i2}^{bM} = \frac{(1 - r_{iL}^l)(h_1 - r_{iL}^l k) - \frac{h_1 c}{b^L + c}}{(1 - r_{iL}^l)(h_1 - r_{iL}^l k) + (h_1 - k) - (1 - r_{iH}^l)(h_1 - r_{iH}^l k) - \frac{h_1 c}{b^L + c}},$$

$$r_{ij}^l = \frac{(a + b^j + c)h_1 f + (b^j + c)k - \sqrt{((a + b^j + c)h_1 f + (b^j + c)k)^2 - 4(a + b^j + 2c)kfb^j h_1}}{2(a + b^j + 2c)kf}.$$

$$f_{i1}^{hL} = \frac{b - cBk_2}{B(a + c(h_2^H - k_2))},$$

$$f_{i2}^{hL} = \frac{b - cBk_1}{B(a + c(h_1 - k_1))},$$

Table 5.1: Continued.

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**Thresholds** ( $i = 1, 2$ )

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$$h_{i1}^{hL} = h_1 + \frac{Nb}{cBn_1},$$

$$h_{i2}^{hL} = \frac{1}{c} \left( \frac{b}{B} - a \right),$$

$$h_{i3}^{hL} = h_1 - \frac{Nb}{cBn_2},$$

$$P_i^{hL} = \frac{f_{i1}^{hL} \frac{(h_1 - k_1)}{k_2}}{1 + f_{i1}^{hL} \frac{(h_1 - k_1)}{k_2}},$$

$$k_1 = (h_1 - h_2^L) \frac{n_2}{N},$$

$$k_2 = (h_2^H - h_1) \frac{n_1}{N},$$

$$f_{i1}^{hM} = \frac{bh_2^H - (b+c)k_2}{(a+b+c)h_2^H - (a+b+2c)k_2},$$

$$f_{i2}^{hM} = \frac{bh_1 - (b+c)k_1}{(a+b+c)h_1 - (a+b+2c)k_1},$$

$$h_{i1}^{hM} = \frac{(b+c)}{b} k_2,$$

$$h_{i2}^{hM} = \frac{h_1(cn_2 - bn_1)}{(b+c)n_2},$$

$$h_{i3}^{hM} = \frac{(a+b+2c) \sqrt{(h_2^H + k_2)^2 - 4k_2^2} - \sqrt{((a+b+c)h_2^H + (b+c)k_2)^2 - 4(a+b+2c)k_2bh_2^H - k_2(a+c)}}{c},$$

$$h_{i4}^{hM} = \frac{(a+b+2c) \sqrt{(h_1 + k_1)^2 - 4k_1^2} - \sqrt{((a+b+c)h_1 + (b+c)k_1)^2 - 4(a+b+2c)k_1bh_1 - k_1(a+c)}}{c},$$

$$P_{i1}^{hM} = \frac{(1 - r_{2L}^l)(h_1 - r_{2L}^l k_1) - (h_1 - k_1)}{(h_2^H - k_2) - (1 - r_{2H}^l)(h_2^H - r_{2H}^l k_2) + (1 - r_{2L}^l)(h_1 - r_{2L}^l k_1) - (h_1 - k_1)},$$

Table 5.1: Continued.

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**Thresholds** ( $i = 1, 2$ )

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$$p_{i2}^{hM} = \frac{(1-r_{2L}^l)(h_1 - r_{2L}^l k_1) - \frac{h_1 c}{b^L + c}}{(h_2^H - k_2) - (1-r_{2H}^l)(h_2^H - r_{2H}^l k_2) + (1-r_{2L}^l)(h_1 - r_{2L}^l k_1) - \frac{h_1 c}{b^L + c}},$$

$$p_{i3}^{hM} = \frac{(h_1 - k_1) - (1-r_{2L}^l)(h_1 - r_{2L}^l k_1)}{(1-r_{2H}^l)(h_2^H - r_{2H}^l k_2) - (1-r_{2L}^l)(h_1 - r_{2L}^l k_1) + (h_1 - k_1) - \frac{h_2^H c}{b + c}},$$

$$r_{2H}^l = \frac{(a+b+c)h_2^H f + (b+c)k_2 - \sqrt{((a+b+c)h_2^H f + (b+c)k_2)^2 - 4(a+b+2c)k_2 f b h_2^H}}{2(a+b+2c)k_2 f},$$

$$r_{2L}^l = \frac{(a+b+c)h_1 f + (b+c)k_1 - \sqrt{((a+b+c)h_1 f + (b+c)k_1)^2 - 4(a+b+2c)k_1 f b h_1}}{2(a+b+2c)k_1 f}.$$



Table 5.2: LOE for multiplicative utility function when preferences for allocating based on equity are unknown at the upper level.

Conditions		Optimal Asymmetric			LOE = $E[IA^F] - E[IA^A]$
From Table 5.4	Additional Conditions	$f^*$	$r_{1j}^*$	$E[IA^A]$	
$b^H > b_{i1}^{bM}$ , $b^L > b_{i1}^{bM}$ ,	$p > p_{i1}^{bM}$ , $h_{iH}^{bM} \leq h_1 < h_{iL}^{bM}$	1	$r_{1H}^* = r_{1H}^l$ , $r_{2H}^* = r_{2H}^l$	$B(1 - r_{1H}^l)(h_1 - r_{1H}^l k)p +$ $B(1 - r_{1L}^l)(h_1 - r_{1L}^l k)q$	$(1 - p)B((h_1 - k) - (1 - r_{1L}^l)(h_1 - r_{1L}^l k))$
	$p \leq p_{i1}^{bM}$ , $h_{iL}^{bM} \leq h_1 < h_{iH}^{bM}$				$pB((h_1 - k) - (1 - r_{1H}^l)(h_1 - r_{1H}^l k))$
$h_1 < h_{iH}^{bM}$ , $p \leq p_{i2}^{bM}$	$pB((h_1 - k) - (1 - r_{1H}^l)(h_1 - r_{1H}^l k))$				
$b^H > b_{i1}^{bM}$ , $b^L > b_{i1}^{bM}$ ,	$p \leq p_{i1}^{bM}$ , $h_{iH}^{bM} \leq h_1 < h_{iL}^{bM}$	0	$r_{1H}^* = 1$ , $r_{1L}^* = 1$	$B(h_1 - k)$	$pB((1 - r_{1H}^l)(h_1 - r_{1H}^l k) - (h_1 - k))$
	$p > p_{i1}^{bM}$ , $h_{iL}^{bM} \leq h_1 < h_{iH}^{bM}$				$(1 - p)B((1 - r_{1L}^l)(h_1 - r_{1L}^l k) - (h_1 - k))$
$h_1 < h_{iH}^{bM}$ , $p > p_{i2}^{bM}$	$(1 - p)B\left((1 - r_{1L}^l)(h_1 - r_{1L}^l k) - \frac{h_1 c}{b^L + c}\right)$				
$b^H > b_{i1}^{bM}$ , $b^L \leq b_{i1}^{bM}$ ,			$r_{1H}^* = 1$ , $r_{1L}^* = r_{1L}^l$	$B(h_1 - k)p + B\frac{h_1 c}{b^L + c}q$	

Table 5.3: LOE for multiplicative utility function when infections prevented per dollar are unknown at the upper level.

Conditions from Table 5.2	Additional Conditions	Optimal Asymmetric			LOE = $E[IA^F] - E[IA^A]$
		$f^*$	$r_{2j}^*$ , $j = H, L$	$E[IA^A]$	
$h_2^H > h_{t1}^{hM}, h_2^L > h_{t2}^{hM}$ ,	$h_2^H < h_{t3}^{hM}$ , $p \leq p_{t1}^{hM}, h_1 \geq h_{t4}^{hM}$	1	$r_{2H}^* = r_{2H}^l$ , $r_{2L}^* = r_{2L}^l$ .	$B(1 - r_{2H}^l) p \times$ $(h_2^H - r_{2H}^l k_2) +$	$pB((h_2^H - k_2) - (1 - r_{2H}^l)(h_2^H - r_{2H}^l k_2))$
	$h_2^H > h_{t3}^{hM}$ , $p > p_{t1}^{hM}, h_1 < h_{t4}^{hM}$			$B(1 - r_{2L}^l) \times$ $(h_1 - r_{2L}^l k_1) q$	$(1 - p)B((h_1 - k_1) - (1 - r_{2L}^l)(h_1 - r_{2L}^l k_1))$
$h_2^H > h_{t1}^{hM}, h_2^L \leq h_{t2}^{hM}$ ,	$h_2^H < h_{t3}^{hM}, p \leq p_{t2}^{hM}$			$pB((h_2^H - k_2) - (1 - r_{2H}^l)(h_2^H - r_{2H}^l k_2))$	
$h_2^H \leq h_{t1}^{hM}, h_2^L > h_{t2}^{hM}$ ,	$h_1 < h_{t4}^{hM}, p > p_{t3}^{hM}$			$(1 - p)B((h_1 - k_1) - (1 - r_{2L}^l)(h_1 - r_{2L}^l k_1))$	
$h_2^H > h_{t1}^{hM}, h_2^L > h_{t2}^{hM}$ ,	$h_2^H > h_{t3}^{hM}$ , $p \leq p_{t1}^{hM}, h_1 < h_{t4}^{hM}$	0	$r_{2H}^* = 1$ , $r_{2L}^* = 1$ .	$B(h_2^H - k_2) p +$ $B(h_1 - k_1) q$	$pB((1 - r_{2H}^l)(h_2^H - r_{2H}^l k_2) - (h_2^H - k_2))$
	$h_2^H < h_{t3}^{hM}$ , $p > p_{t1}^{hM}, h_1 \geq h_{t4}^{hM}$			$(1 - p)B((1 - r_{2L}^l)(h_1 - r_{2L}^l k_1) - (h_1 - k_1))$	
$h_{t1}^{hM} < h_2^H < h_{t3}^{hM}, h_2^L \leq h_{t2}^{hM}$ ,	$h_2^H < h_{t3}^{hM}, p > p_{t2}^{hM}$	1	$r_{2H}^* = 1$ , $r_{2L}^* = r_{2L}^l$	$B(h_2^H - k_2) p +$ $B \frac{ch_1}{(b+c)} q$	$(1 - p)B\left((1 - r_{2L}^l)(h_1 - r_{2L}^l k_1) - \frac{h_1 c}{b+c}\right)$
$h_2^H \leq h_{t1}^{hM}, h_2^L > h_{t2}^{hM}$ ,	$h_1 < h_{t4}^{hM}, p \leq p_{t3}^{hM}$	0	$r_{2H}^* = r_{2H}^l$ , $r_{2L}^* = 1$	$B \frac{ch_2^H}{(b+c)} p +$ $B(h_1 - k_1) q$	$pB\left((1 - r_{2H}^l)(h_2^H - r_{2H}^l k_2) - \frac{h_2^H c}{b+c}\right)$

Figure 5.1: HIV Infections Averted versus budget for full information and asymmetric information for different values of  $h_1$  (infections averted/\$ in program one).

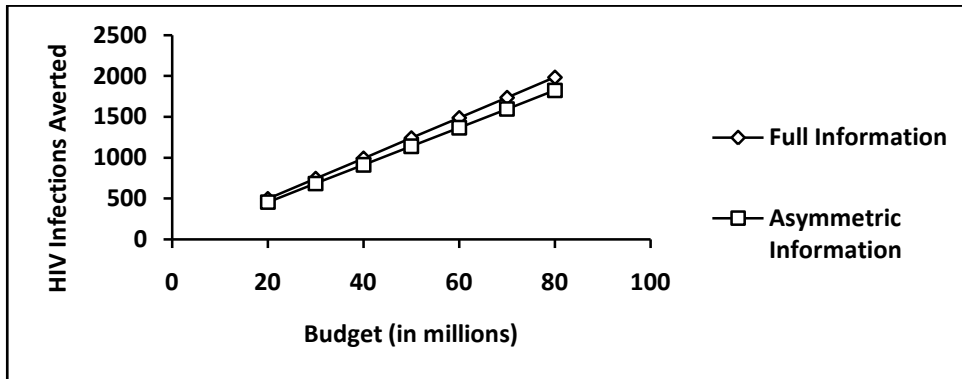


Figure 5.1a: HIV Infections Averted versus budget for the base case value of  $h_1 / 2$ .

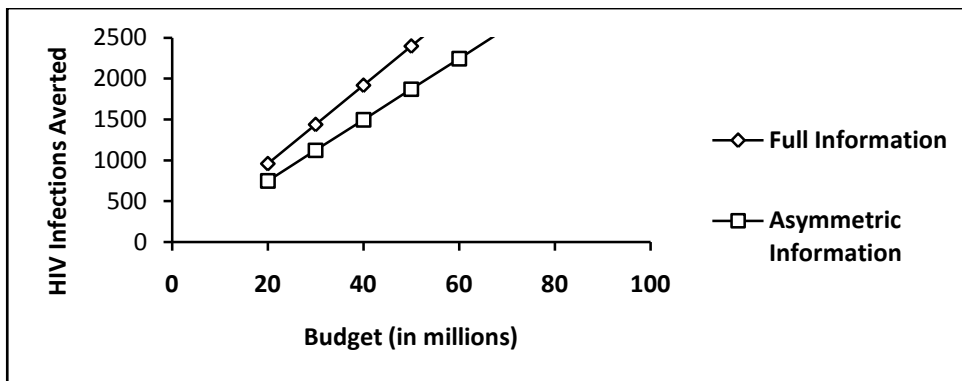


Figure 5.1b: HIV Infections Averted versus budget for the base case value of  $h_1$ .

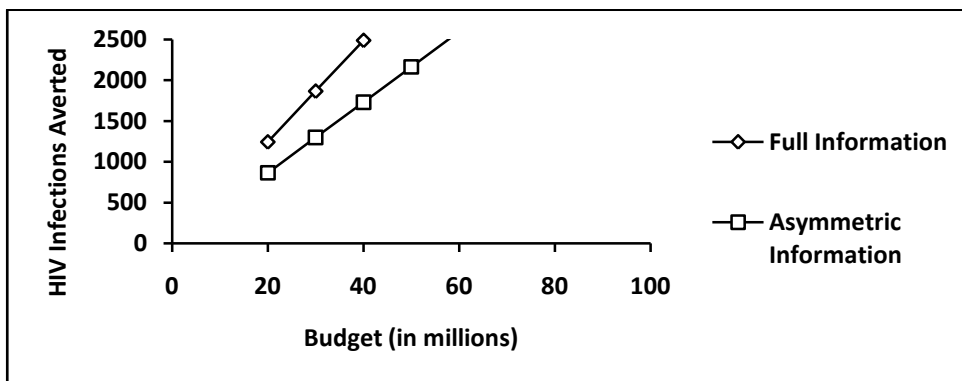


Figure 5.1c: HIV Infections Averted versus budget for the base case value of  $2h_1$ .

Figure 5.2: HIV Infections Averted versus budget for full information and asymmetric information for different values of  $b$  (coefficient of the fraction of budget to be reserved for proportional allocation).

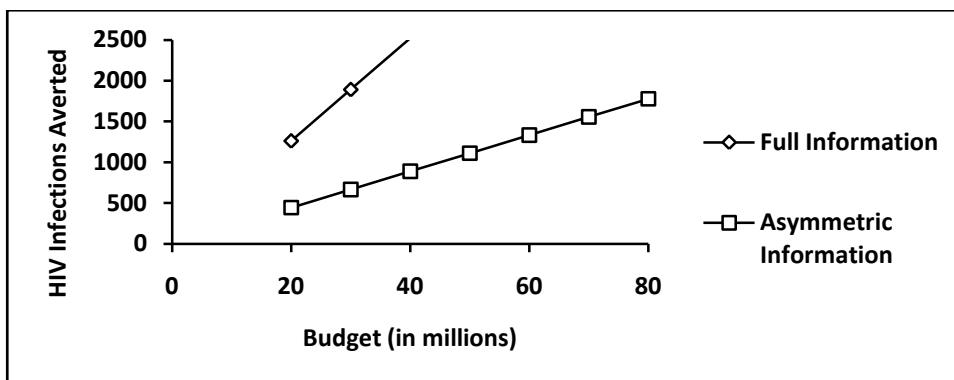


Figure 5.2a: HIV Infections Averted versus budget for the base case value of  $b/2$ .

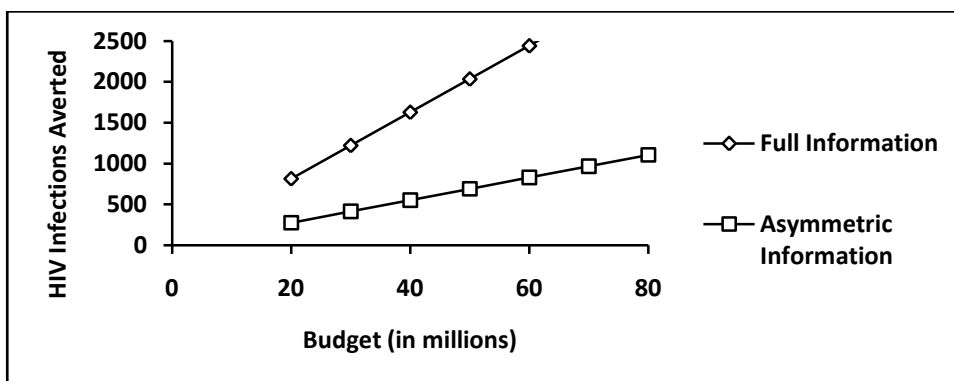


Figure 5.2b: HIV Infections Averted versus budget for the base case value of  $b$ .

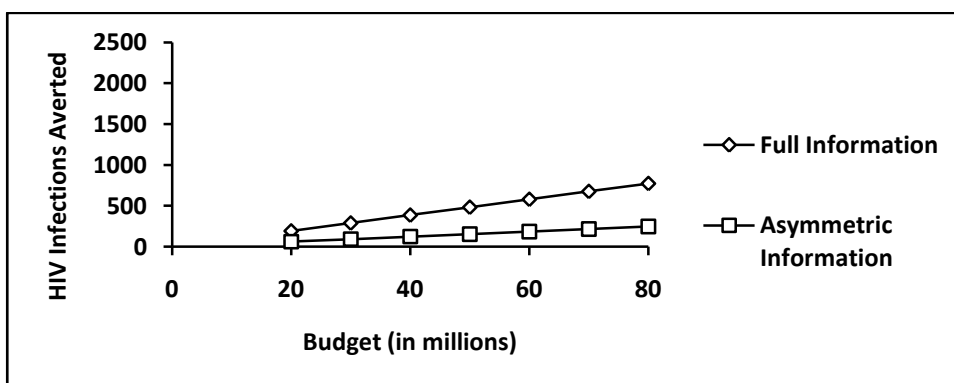


Figure 5.2c: HIV Infections Averted versus budget for the base case value of  $2b$ .

Figure 5.3: Expected HIV infections averted for various values of  $p$  for the two cases.

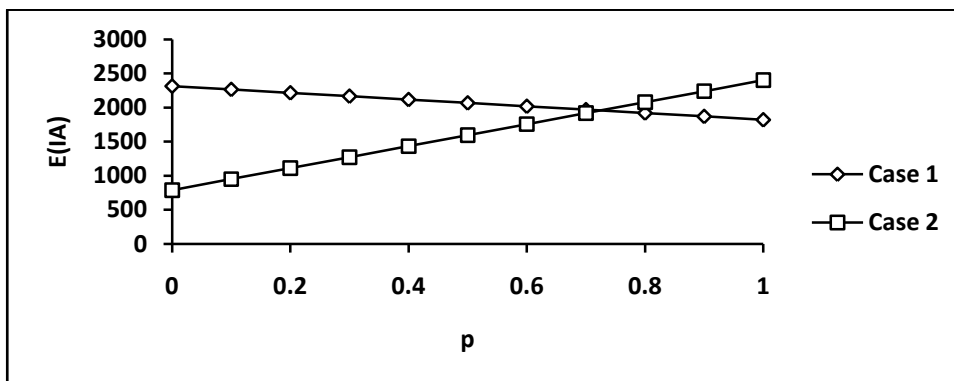
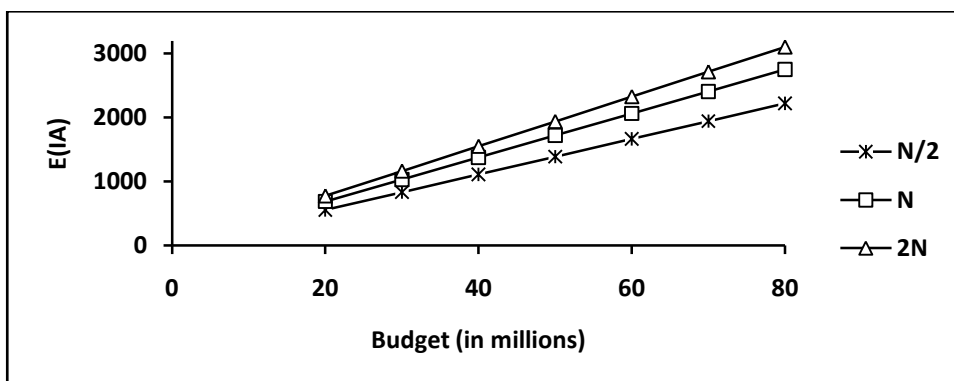


Figure 5.4: Expected HIV infections averted versus budget for both the cases for different values of  $N$  (total size of the risk groups).



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## Chapter 6

### Multi-level Allocation of HIV Prevention Funds: A Multiple Incentive Model

#### 6.1. Introduction

As described in Chapter 2, main sources of funding to prevent HIV/AIDS infections worldwide are the United Nations Program on HIV/AIDS (UNAIDS), the World Bank's Global AIDS Program, the World Health Organization, the US President's Emergency Plan for AIDS Relief, and the Global Fund for AIDS, Tuberculosis and Malaria. Funds are often allocated by these agencies at multiple levels. In Chapter 4, we consider a model in which an upper-level decision maker (UD) allocates fund to a single lower-level decision maker (LD) who then distributes funds to 3 local programs. In this chapter, we extend this model to multiple lower levels competing for the budget at the upper level.

Mathematical programming (MP) models assuming a single decision maker [1-3] and multiple decision makers [4, 5] have been developed to aid this resource allocation process. However, simple allocation techniques are often preferred [6]. In order to encourage optimal allocation of resources, incentives could be provided to decision makers [7-9].

Some work addressing incentives in health care resource allocation proposing optimization models [10, 11] has been done. Chick et al. formulated a cost-sharing contract between government and the vaccine manufacturer that provides incentives to both parties so that the vaccine supply chain gets optimized by improving the supply of vaccine [10]. Zhang and Zenios proposed a multi-period principal-agent model in which the physician is an agent and the medical insurer is a principal to design optimal contracts for the principal [11].

Some work incorporating game theory has been done in health care resource distribution settings. McPake et al. modeled two-tier charging, the practice in which hospitals offer



two separate qualities of service, basic and premium service, at different prices, as a Stackelberg game in which the Ministry of Health (MOH) is a leader and the hospital is a follower. MOH sets the prices that maximize its utility function subject to its budget constraint set by Ministry of Finance and then the hospital follows by setting its quality levels maximizing its surplus. The case in which MOH sets prices but provides only lump-sum subsidies to the hospital was compared to the case in which MOH sets prices and also provides an activity-based subsidy for the provision of a basic service that reflects the volume of the service provided. They showed that switching to activity-based payment doubles the quality level of the basic and premium services [12]. Sun et al. modeled various countries as players in a game during an outbreak of an epidemic making optimal decisions about allocating their own drug stockpiles to protect their populations. However, we not aware of any modeling of an HIV prevention funds allocation process in which multiple lower levels are competing for the budget at the upper level [13].

In this chapter, an incentive-based resource allocation process with a UD allocating funds to 2 LDs who then allocates funds to 3 programs is considered. We seek to answer the following questions: What is the impact of incentives on the optimal solution of the HIV/AIDS resource allocation process if multiple regions are competing for the budget? What is the impact of incentives provided within regions and/or between regions? The rest of the paper is organized as follows: Section 6.2 formulates the model. Section 6.3 provides mathematical analysis and Section 6.4 solves a numerical example. The conclusion and extensions are discussed in Section 6.5.

## **6.2. Model Formulation**

We modeled the allocation process for prevention funds in which a single UD allocates funds to 2 independent LDs, representing regions in a single period. The 2 LDs then allocate funds to three programs between their regions. In the first stage, the UD decides the level of incentive within regions ( $f$ ) and between regions ( $g$ ) that maximizes its utility function consisting of the total infections averted. In the second stage, responding to the UD's decision, the 2 LDs simultaneously choose the fraction of the funds received

to be reserved for proportional allocation  $(r_i^m), i = 1, 2, m = fg, f, g$ , where  $i$  represents region  $i$  and  $m$  represents the type of incentive, maximizing their own utility functions and then allocate the funds to the three programs. We consider three cases: the “ $fg$ ” case in which, the UD considers 2 types of incentives within regions and between regions, “ $f$ ” case in which, incentives within regions are considered, and “ $g$ ” case in which, incentives between regions are considered.

Let  $B$  be the budget of the UD. We assume that the budget  $B$  is divided into a fixed portion  $fB$  and an incentive portion  $(1-f)B$ , where the fraction  $f, 0 \leq f \leq 1$ , is chosen by the UD. Further, the upper level allocates an amount  $Z_i, i = 1, 2$  to the two lower levels, which is assumed to be again divided into a fixed portion  $Z_i^F$  coming from fixed portion of the budget  $Z_1^F + Z_2^F = fB$  and an incentive portion  $Z_i^I$  coming from the incentive portion of the budget  $Z_1^I + Z_2^I \leq (1-f)B$  (we assume that the UD can hold back certain amount of funds for future use). The total amount received by each region is  $Z_i = Z_i^F + Z_i^I$ . We assume that  $Z_i^F$  is proportional to the size of the infected population in region  $i$  given by,  $Z_i^F = \frac{N_i}{N_1 + N_2} fB$ , where,  $N_i$  is the size of the infected population in region  $i$ .

There are many ways in which incentives could be implemented in a multi-level HIV prevention funds allocation process. As an illustrative example, we consider an incentive scheme with the following functional form:

$$Z_i^I = (1 - r_i^m g) \frac{(1 - r_i^m + r_{-i}^m)}{2} (1 - f) B, i, -i = 1, 2, i \neq -i, \text{ where, } f \text{ and } g, (0 \leq f, g \leq 1), \text{ and}$$

$$Z_1^I + Z_2^I \leq B(1 - f) \text{ as shown below:}$$

$$\text{Clearly, if } g = 0 \Rightarrow \frac{1}{2} \left( (1 - r_1^m + r_2^m) + (1 - r_2^m + r_1^m) \right) = 1, \text{ otherwise,}$$

$$\frac{1}{2} \left( (1 - r_1^m g) (1 - r_1^m + r_2^m) + (1 - r_2^m g) (1 - r_2^m + r_1^m) \right) < 1$$

Thus,  $Z_1^l + Z_2^l = B(1-f) \frac{1}{2} \left( (1-r_1^m g)(1-r_1^m + r_2^m) + (1-r_2^m g)(1-r_2^m + r_1^m) \right) \leq B(1-f)$ .

$f$  and  $g$  are chosen by the UD, represents incentives within and between a region, respectively. The functional form of the incentive scheme will allow us to separately consider the impact of each incentive.  $r_i^m$  ( $0 \leq r_i^m \leq 1$ ), fraction of the funds to be reserved for proportional allocation, is chosen by a LD. A higher value of  $r_1^m$  corresponds to a stronger penalty for equity for the lower level decision maker 1 (LD1) and a bigger reward for lower-level decision maker 2 (LD2). For example, if the LD1 chooses to allocate entire budget proportionally,  $r_1^m = 1$  and if the LD2 chooses to allocate entire budget optimally,  $r_2^m = 0$  then the LD1 will receive zero incentivized budget, whereas the LD2 will receive  $(1-f)B$  or the budget  $B$  when  $f = 0$ . When  $r_1^m = r_2^m = 0$ , entire budget is allocated optimally and incentivized budget is divided equally between the two LDs. When  $r_1^m = r_2^m = 1$ , entire budget is allocated proportionally and the LD  $i$  receives  $(1-g) \frac{1}{2} B(1-f) + \frac{N_i}{N_1 + N_2} fB$ . “ $g$ ” is similar to the incentive used in chapter 4.

The lower level then distributes amount  $y_{ij}$ ,  $j = 1, 2, 3$  to program  $j$  in region  $i$ . We assume that 1 program is available for each of 3 different risk groups, that the programs do not interact, and that the costs and benefits scale linearly, as in Zaric and Brandeau [8]. We solve this problem using backward induction and present the details in the given time sequence.

### **Stage 1: Upper-level Choice of $f$ and $g$**

Similar to chapter 4 and 5, we assume that the objective at the upper level is to maximize the number of infections averted. The upper-level resource allocation problem in stage 1 is written as follows:

$$IA_i^m : \max_{f,g} \sum_i IA_i(r_i^m * (f, g)), i=1,2, m = fg, f, g \quad (6.1)$$

$$\text{s.t. } 0 \leq f \leq 1 \quad (6.2)$$

$$0 \leq g \leq 1 \quad (6.3)$$

$$r_i^m * (f, g) = \arg \max(L_i(r_i^m)) \quad (6.4)$$

where,  $IA_i(r_i^m * (f, g))$  represents the total infections averted and is calculated by solving an optimization problem in stage 3,  $L_i(r_i^m)$  represents the lower-level optimization problem solved in stage 2, subscript  $i$  represents the utility function of LD  $i$ .

### Stage 2: Lower-level Choice of $r_i^m$

In the second step, each LD has a single decision of what proportion of the funds received to allocate based on equity and it chooses  $r_i^m$  to maximize its own utility function. We assume that the utility function is linear function of equity ( $r_i^m$ ), efficiency, defined by the number of infections averted ( $IA_i$ ), and funds received ( $Z_i$ ). This is defined by,

$$U_i = a_i Z_i(r_i^m, r_{-i}^m) + b_i r_i^m + c_i IA_i(r_i^m, r_{-i}^m) \quad (6.5)$$

where,  $U_i, a_i, b_i, c_i > 0$  represents the relative weights applied to funds received, equity, and infections averted. We assume that values of  $a_i, b_i$ , and  $c_i$  are known to both the LDs as well as the UD. The lower-level optimization problem for each LD is written as:

$$L_i^m : \max_{r_i^m} U_i = a_i Z_i(r_i^m, r_{-i}^m) + b_i r_i^m + c_i IA_i(r_i^m, r_{-i}^m) \quad (6.6)$$

$$\text{s.t. } 0 \leq r_i^m \leq 1 \quad (6.7)$$

$$r_{-i}^m = \arg \max \left( L_{-i} \left( r_{-i}^m \right) \right) \quad (6.8)$$

LD1 solves  $L_1^m$  to obtain  $r_1^m \ast (f, g)$  given the value of  $f, g$  and  $r_2^m \ast (f, g)$ . Similarly,

LD2 solves  $L_2^m$  to obtain  $r_2^m \ast (f, g)$  given the value of  $f, g$  and  $r_1^m \ast (f, g)$ .

### Stage 3: Lower-level Allocation to Programs

We develop a model at this stage similar to the lower-level model of Zaric and Brandeau [8] and Malvankar and Zaric [9]. Let  $h_{ij}$  be the number of HIV infections prevented per dollar invested in a program  $j$  in region  $i$  over time  $T$ . Let  $n_{ij}$  be the size of the risk group  $j$  in region  $i$  and  $N_i = \sum_j n_{ij}$ . We assume that the programs have been indexed so that

$h_{i1} > h_{i2} > h_{i3}$ . The total number of HIV infections averted,  $IA_i$ , is given by the following equation:

$$IA_i = h_{i1}y_{i1} + h_{i2}y_{i2} + h_{i3}y_{i3} \quad (6.9)$$

In this stage, given  $r_i^m, f$ , and  $g$ , the total number of infections averted is obtained by solving the following LP:

$$\max_{y_{i1}, y_{i2}, y_{i3}} IA_i = h_{i1}y_{i1} + h_{i2}y_{i2} + h_{i3}y_{i3} \quad (6.10)$$

$$\text{s.t. } \sum_{j=1}^3 y_{ij} \leq Z_i \quad (6.11)$$

$$y_{ij} \geq r_i^m Z_i \frac{n_{ij}}{N_i}, j = 1, 2, 3 \quad (6.12)$$

The optimal solution of this ‘‘Knapsack LP’’ is of the following form:

$$y_{i1} = Z_i - r_i Z_i \frac{n_{i2}}{N_2} - r_i Z_i \frac{n_{i3}}{N_3}$$

$$y_{ij} = r_i Z_i \frac{n_{ij}}{N_i}, \quad j = 2, 3$$

The result of stage 3 is the function  $IA_i(r_i^m)$ ,

$$IA_i(r_i^m) = Z_i \left( h_{i1} - r_i^m (h_{i1} - h_{i2}) \frac{n_{i2}}{N_2} - r_i^m (h_{i1} - h_{i3}) \frac{n_{i3}}{N_3} \right) \quad (6.13)$$

$$= \left( \frac{(1 - r_i + r_i')}{2} (1 - f) B + \left( \frac{N_i}{N_1 + N_2} \right) fB \right) (h_{i1} - k_i), \text{ where}$$

$$k_i = (h_{i1} - h_{i2}) \frac{n_{i2}}{N_2} + (h_{i1} - h_{i3}) \frac{n_{i3}}{N_3} < h_{i1}.$$

### 6.3. Analysis

In this section, we present the analysis of the problem. All notations are specified in table 6.1 and proofs are given in the appendix.

#### 6.3.1 Two Incentives (“fg” case)

We next present results that characterize the optimal solution to the lower-level problem when incentives are provided within and between regions.

**Proposition 6.1:** For problem  $L_i^{fg}$  with three sub-populations,

(i)  $U_i$  is a cubic function of  $r_i^{fg}$ . If the coefficient of  $(r_i^{fg})^3$  is greater than zero in  $U_i$  then  $r_i^{fg*} = \min\{r_i^{fgu}, 1\}$ , otherwise  $r_i^{fg*} = r_i^{fgu}$  or 0 or 1.

(ii) The conditions for existence of Nash equilibriums  $(0,0)$ ,  $(0,1)$ ,  $(1,0)$ ,  $(1,1)$ ,  $(r_1^{fgu}, 0)$ ,  $(0, r_2^{fgu})$ , and  $(r_1^{fgu}, r_2^{fgu})$  are specified in Table 6.2. ■

Part (i) of Proposition 6.1 says that either a fraction ( $0 < r_i^{fgu} < 1$ ) or none or all of the funds are reserved for proportional allocation. Part (ii) says all the conditions for existence of Nash equilibriums are specified in Table 6.2. Table 6.2 says that if the LD has bigger size of the total infected population and lower preferences for infections prevented per dollar then fewer incentives are given between regions and higher incentives are given within regions to encourage optimality if both the LDs have moderate preferences for proportional allocation. Higher incentives are given within regions and lower incentives are given between regions to encourage the LD with higher preferences for infections prevented per dollar to allocate the entire budget optimally even if both the LDs have moderate preferences for proportional allocation. However, if both the LDs have higher preferences for proportional allocation then fewer incentives are given within regions and higher incentives are given between regions to encourage the LD with higher preferences for infections prevented per dollar to allocate the entire budget optimally.

We next present results that characterize the optimal solution to the upper-level problem when incentives are provided within and between regions.

**Proposition 6.2:** For problem  $IA_i^{fg}$ , the optimal  $f^*$ ,  $g^*$ , and number of infections prevented for various conditions are specified in Table 6.3. ■

Proposition 6.2 says that optimal  $f^*$ ,  $g^*$  and  $IA_i^{fg}$  for all conditions listed in Table 6.2 are specified in Table 6.3. Table 6.3 says that if 1 of the LD has higher infections prevented/\$, moderate preferences for allocating the budget based on equity, and higher preferences for infections prevented/dollar then full incentives are given within and between so that the entire budget is allocated to that LD who will choose to allocate optimally. If 1 of the LD has higher infections prevented/dollar and other LD has higher preferences for infections prevented/dollar then moderate incentives are given within regions and full incentives are given between regions so that entire budget gets allocated optimally by the LD with higher preferences for infections prevented/dollar.

### 6.3.2 Between Region Incentives (“g” case)

We next present the results that characterize the optimal solution to the upper-level problem when incentives are provided between regions.

**Proposition 6.3:** For problem  $L_i^g$  with three sub-populations,

(i)  $U_i$  is a cubic function of  $r_i^g$ . If the coefficient of  $(r_i^g)^3$  is greater than zero in  $U_i$  then  $r_i^{f*} = \min\{r_i^{fu}, 1\}$ , otherwise  $r_i^{g*} = r_i^{gu}$  or 0 or 1.

(ii) The conditions for existence of Nash equilibriums  $(0,0)$ ,  $(0,1)$ ,  $(1,0)$ ,  $(1,1)$ ,  $(r_1^{gu}, r_2^{gu})$ ,  $(r_1^{gu}, 0)$ , and  $(0, r_2^{gu})$  are specified in Table 6.4. ■

Part (i) of Proposition 6.3 says that either a fraction  $(0 < r_i^{gu} < 1)$  or all or none of the funds are reserved for proportional allocation. Part (ii) says all the conditions for existence of Nash equilibriums are specified in Table 6.4. Table 6.4 says that if both the LDs moderately prefer proportional allocation, 1 of the LD has larger risk group, and other LD has lower preference for infections prevented/dollar then lower incentives are given between regions since the LDs will allocate the budget optimally. If one of the LDs has a higher preference for infections prevented per dollar and both the LDs have a higher preference for proportional allocation, then the LD with higher preference for infections prevented per dollar will choose to allocate optimally with higher incentives, whereas, the other LD will continue to allocate proportionally even with higher incentives.

We next present the results that characterize the optimal solution to the upper-level problem when incentives are provided between regions.

**Proposition 6.4:** For problem  $IA_i^g$ , optimal  $g^*$  and number of infections prevented for various conditions are specified in Table 6.5. ■

Proposition 6.4 says that optimal  $g^*$  and  $IA_i^{g*}$  for all conditions listed in Table 6.4 are specified in Table 6.5. Table 6.5 indicates that if 1 of the LDs has higher preference for



infections prevented per dollar then full incentives are given to allocate the entire budget to that LD if both the LDs strongly prefer a proportional allocation. In this case, the entire budget is allocated optimally. If 1 of the LD moderately prefers proportional allocation and has lower preference for infections prevented/dollar and other LD highly prefers proportional allocation then both the LDs will choose to allocate proportionally even with moderate incentives.

### 6.3.3 Within Region Incentives (“f” case)

We next present results that characterize the optimal solution to the lower-level problem when incentives are provided within regions.

**Proposition 6.5:** For problem  $L_i^f$  with three sub-populations,

- (i)  $U_i$  is a convex function of  $r_i^f$  and therefore the optimal solution is either  $r_i^{f*} = 0$  or 1.
- (ii) The conditions for existence of 4 Nash equilibriums (0,0), (0,1), (1,0), and (1,1) are specified in Table 6.6. ■

Part (i) of Proposition 6.5 states that an extreme point solution of allocating all or none of the funds is always optimal. Part (ii) states that all the conditions for existence of 4 Nash equilibriums are specified in Table 6.6. Table 6.6 shows that if the both LDs have a lower preference for proportional allocation and a higher preference for the budget received then they will allocate the budget optimally with higher incentives within their regions. If LD1 has a moderate preference for a proportional allocation and a larger infected population, then LD1 will choose to allocate optimally with lower incentives; however, higher incentives are provided to LD2 with a moderate preference for a proportional allocation and a higher preference for the budget received. If LD1 has higher preference for proportional allocation and LD2 has moderate preference for proportional allocation then full incentives within regions are given to encourage LD2 to allocate optimally if LD2 has bigger infected population.

We next present results that characterize the optimal solution to the upper-level problem when incentives are provided within regions. For problem  $IA_i^f$ , we first identified all sub-cases of the 5 cases listed in Table 6.6. Secondly, we combined some similar sub-cases and then all the sub-cases were solved to get the optimal solutions at the upper level.

**Proposition 6.6:** For problem  $IA_i^f$ , optimal  $f^*$  and number of infections prevented for various conditions are specified in Table 6.7. ■

Proposition 6.6 states that optimal  $f^*$  and  $IA_i^f$  for all conditions listed in Table 6.6 are specified in Table 6.7. Table 6.7 presents 3 different sub-cases. Sub-case 1 includes the conditions under which  $(0, 0)$  and  $(1, 1)$  exist together as multiple equilibria and unique equilibrium. Incentives are provided to encourage  $(0, 0)$  equilibrium if both the LDs have moderate preference for proportional allocation and moderate infections are prevented per dollar. Sub-case 2 includes the conditions under which  $(0, 1)$  and  $(1, 0)$  exist together as multiple equilibria. A moderate incentive is given to encourage  $(0, 1)$  if the higher infections/dollar are prevented in risk group 1 than in risk group 2 and both the LDs have moderate preferences for proportional allocation. Sub-case 3 includes the conditions under which no equilibrium and unique equilibrium exist. A full incentive is given to allocate the entire budget to a region if infections prevented per dollar in other region are lower than a threshold.

#### 6.4. Example

We illustrate with an example using data for California from Zaric and Brandeau [8], Office of AIDS [14] and for New York from Bureau of HIV/AIDS Epidemiology [15] for 3 risk groups ( $m_{i,j} = 3, i = 1, 2, j = 1, 2, 3$ ), HET, MSM, and IDUs in each state. Risk group 1 consists of 18,383 and 25,109 IDUs, risk group 2 consists of 14,701 and 18,801 HET, and risk group 3 consists of 125,351 and 32,109 MSM for California and New York, respectively. We obtained estimates of the potential cost and effectiveness of interventions in each population from elsewhere [8]. We calculated the number of infections averted per dollar invested in each intervention using a formula published

elsewhere [8] as 0.00012, 0.000046, and 0.0000088, in risk groups 1, 2, and 3, respectively. We estimate that approximately \$35,512,626 and \$48,067,309 of the CDC's \$297,049,344 budget is allocated to California and New York, respectively.

To estimate  $a_i, b_i,$  and  $c_i$  in the lower-level utility function, we set  $a_i = 1 - b_i - c_i$  and preferences for infections averted per dollar ( $c_i$ ) as 0.4. We assume that both the LDs have moderate preferences for allocating the budget proportionally  $b_1 = 0.5, b_2 = 0.55$ .

Figure 6.1 shows total HIV infections averted for budget ranging from \$20 million to \$80 million and three lines in the graph represents the three cases, “ $fg$ ” case in which both type of incentives between and within regions are given, “ $g$ ” case in which incentives are given between regions and “ $f$ ” case in which incentives are given within regions. In the 3 graphs the difference between HIV infections averted in all the 3 cases increase as the budget increases. This shows that when the budget is higher, fewer infections are averted in “ $f$ ” case compared to “ $g$ ” case. As the budget increases, this difference becomes prominent.

## 6.5. Discussion

We formulated the strategic interactions between the decision makers at multi-level in the resource allocation process for HIV prevention funds. The UD decides the level of incentive that maximizes the total infections averted and then the 2 LDs decides the fraction of the funds to be reserved for proportional allocation maximizing their utility functions. We considered 2 types of incentives, within and between regions. We analyzed each type of incentives separately as well as jointly in 3 different cases. In the “ $fg$ ” case, we consider incentives within and between regions. In the “ $g$ ” case, incentives are considered between regions and in the “ $f$ ” case, incentives are considered within regions.

We described the problem that is posed to the donor or the UD to award money among the proposed HIV prevention activities. We showed how incentives, based on the LD's concerns about equity, help encourage effective utilization of constrained prevention resources and significantly improve the health outcomes. We further compared 2

different types of incentives and its impact at the lower and the upper level. Both types of incentives encourage optimal allocation of the entire budget by the LD with higher infections prevented/dollar, higher preferences for infections prevented/dollar if both the LDs have moderate preferences for proportional allocation. However, if both the LDs have higher preferences for proportional allocation then incentives are given only between regions to encourage optimal allocation. Further, no incentives are given within regions if both the LDs have moderate preferences for proportional allocation and infections prevented/dollar in a region with larger infected population is higher than the other region.

We identified several possible extensions to the paper. First, we considered a model in which single upper levels allocate funds to 2 lower levels who then distribute funds to 3 programs. However, it could be extended to multiple UD, regions and programs by characterizing an incentive scheme that incorporates preferences and redefining the utility functions. Another important extension could be a sequential game with imperfect information in which UD leads by setting the level of incentive that maximizes its utility function and then the 2 LDs follow simultaneously by setting the fraction of the funds to be reserved for proportional allocation maximizing their utility functions where information at the lower level is unknown at the upper level. In this game, bayes rule about the lower level decisions could be incorporated to make decision at the upper level. Further, a dynamic game proceeding over time through a sequence of moves can be considered and it may lead to different allocation decisions.

Table 6.1: Summary of Notations

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**Thresholds** ( $i = 1, 2$ )

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$$r_{t-i1}^{fg} = \frac{\frac{2b_i}{(1-f)B} - (a_i + c_i h_{i1})(1-g) - \frac{c_i k_i f N_i}{(1-f)(N_i + N_{-i})} - c_i k_i}{c_i k_i + (a_i + c_i h_{i1})g},$$

$$r_{t-i2}^{fg} = \frac{\frac{b_i}{(1-f)B} - \frac{(a_i + c_i (h_{i1} - k_i))(1-g)}{2} - \frac{c_i k_i f N_i}{(1-f)(N_i + N_{-i})}}{\frac{(a_i + c_i (h_{i1} - k_i))}{2}g + \frac{c_i k_i}{2}(1-g)},$$

$$f_{ii1}^{fg} = \left( \frac{N_i + N_{-i}}{N_i} \right) \left( 1 - \frac{2b_i}{B c_i k_i} \right),$$

$$f_{ii2}^{fg} = \frac{\frac{b_i}{B}}{\frac{c_i k_i}{N_i + N_{-i}} + \frac{b_i}{B}},$$

$$f_{ii3}^{fg} = \frac{a_i + c_i (h_{i1} + 2k_i) - \frac{2b_i}{B}}{a_i + c_i (h_{i1} + 2k_i) - \frac{c_i k_i}{N_i + N_{-i}}},$$

$$f_{ii4}^{fg} = \frac{a_i + c_i h_{i1} - \frac{2b_i}{B}}{a_i + c_i h_{i1} - \frac{2N_i c_i k_i}{N_i + N_{-i}}},$$

$$g_{ii1}^{fg} = 1 + \frac{c_i k_i}{a_i + c_i h_{i1}} + \frac{1}{(1-f)(a_i + c_i h_{i1})} \left( \frac{c_i k_i f N_i}{N_i + N_{-i}} - \frac{2b_i}{B} \right),$$

$$g_{ii2}^{fg} = 1 + \frac{2}{(1-f)(a_i + c_i (h_{i1} - k_i))} \left( \frac{c_i k_i f N_i}{N_i + N_{-i}} - \frac{b_i}{B} \right),$$

$$g_{ii3}^{fg} = \frac{a_i + c_i h_{i1}}{c_i k_i} + \frac{1}{(1-f)} \left( \frac{2f N_i}{N_i + N_{-i}} - \frac{2b_i}{B c_i k_i} \right),$$

$$b_{ii1}^{fg} = \frac{(1-f)}{2} B \left[ \left( 1 + g(1 + r_{-i}^{fgu}) \right) (a_i + c_i h_{i1}) + c_i k_i (1 + r_{-i}^{fgu}) - \frac{\left( c_i k_i (1 + g(1 + r_{-i}^{fgu})) + g(a_i + c_i h_{i1}) \right)^2}{3g c_i k_i} \right] +$$

$$\frac{f B c_i k_i N_i}{(N_i + N_{-i})},$$

Table 6.1: Continued.

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**Thresholds** ( $i = 1, 2$ )

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$$c_{ii1}^{fg} = \frac{b_i - \frac{(1-f)}{2} B a_i (1 + g (1 + r_{-i}^{fgu} - r_i^{fgu}))}{\left( \frac{(1-f)}{2} B ((h_{i1} - r_i^{fgu} k_i) (1 + g (1 + r_{-i}^{fgu} - r_i^{fgu})) + k_i (1 + r_{-i}^{fgu}) (1 - r_i^{fgu} g)) + \frac{f B k_i N_i}{(N_i + N_{-i})} \right)},$$

$$c_{ii2}^{fg} = \frac{b_i - \frac{(1-f)}{2} B a_i (1 + g)}{\left( \frac{(1-f)}{2} B ((h_{i1} + k_i) + g (h_{i1} - k_i)) + \frac{f B k_i N_i}{(N_i + N_{-i})} \right)},$$

$$a_{ii1}^{fg} = \frac{2 N_i c_i k_i}{N_i + N_{-i}} - c_i h_{i1},$$

$$a_{ii2}^{fg} = \frac{2 b_i}{B (1-f) (1 + g (1 + r_{-i}^{fgu} - r_i^{fgu}))} + c_i h_{i1},$$

$$a_{ii3}^{fg} = \frac{2 b_i}{B (1-f) (1 + g (1 + r_{-i}^{fgu} - r_i^{fgu}))} + c_i h_{i1},$$

$$N_{t-i1}^{fg} = N_i^{fg} \left( \frac{1}{\frac{h_{i1}}{2 k_i} + \frac{b_i}{B (1-f) (1 + g (1 + r_{-i}^{fgu} - r_i^{fgu}))}} - 1 \right),$$

$$h_{i1}^{fg} = h_{21} + k_1 - k_2$$

$$r_{t-i1}^g = \frac{\frac{2 b_i}{B} - (a_i + c_i h_{i1}) (1-g) - c_i k_i}{c_i k_i + (a_i + c_i h_{i1}) g},$$

$$r_{t-i2}^g = \frac{\frac{b_i}{B} - \frac{(a_i + c_i (h_{i1} - k_i)) (1-g)}{2}}{\frac{(a_i + c_i (h_{i1} - k_i))}{2} g + \frac{c_i k_i}{2} (1-g)},$$

$$g_{ii1}^g = 1 + \frac{c_i k_i}{a_i + c_i h_{i1}} - \frac{2 b_i}{B (a_i + c_i h_{i1})},$$

$$g_{ii2}^g = 1 - \frac{2 b_i}{B (a_i + c_i (h_{i1} - k_i))},$$

Table 6.1: Continued.

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**Thresholds** ( $i = 1, 2$ )

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$$g_{ii3}^g = \frac{a_i + c_i h_{i1}}{c_i k_i} - \frac{2b_i}{B c_i k_i},$$

$$b_{ii1}^g = \frac{B}{2} \left( (1 + g(1 + r_{-i}^{gu})) (a_i + c_i h_{i1}) + c_i k_i (1 + r_{-i}^{gu}) - \frac{(c_i k_i (1 + g(1 + r_{-i}^{gu})) + g(a_i + c_i h_{i1}))^2}{3g c_i k_i} \right),$$

$$c_{ii1}^g = \frac{b_i - \frac{B a_i}{2} (1 + g(1 + r_{-i}^{gu} - r_i^{gu}))}{\left( \frac{B}{2} \left( (h_{i1} - r_i^{gu} k_i) (1 + g(1 + r_{-i}^{gu} - r_i^{gu})) + k_i (1 + r_{-i}^{gu}) (1 - r_i^{gu} g) \right) \right)},$$

$$c_{ii2}^g = \frac{b_i - \frac{B}{2} a_i (1 + g)}{\frac{B}{2} \left( (h_{i1} + k_i) + g(h_{i1} - k_i) \right)},$$

$$a_{ii1}^g = \frac{2b_i}{B(1 + g(1 + r_{-i}^{gu} - r_i^{gu}))} + c_i h_{i1},$$

$$a_{ii2}^g = \frac{2b_i}{B(1 + g(1 + r_{-i}^{gu} - r_i^{gu}))} + c_i h_{i1},$$

$$N_{i-1}^g = N_i \left( \frac{1}{\frac{h_{i1}}{2k_i} + \frac{b_i}{B(1 + g(1 + r_{-i}^{gu} - r_i^{gu}))}} - 1 \right)$$

$$f_{ii1}^f = \frac{B \frac{(a_i + c_i h_{i1})}{2} - b_i}{B \left[ \frac{(a_i + c_i h_{i1})}{2} - \frac{c_i k_i N_i}{N_1 + N_2} \right]},$$

$$f_{ii2}^f = \frac{B \frac{(a_i + c_i (h_{i1} + k_i))}{2} - b_i}{B \left[ \frac{(a_i + c_i (h_{i1} + k_i))}{2} - \frac{c_i k_i N_i}{N_1 + N_2} \right]},$$

$$a_{ii1}^f = \frac{2c_i k_i N_i}{N_1 + N_2} - c_i h_{i1},$$

Table 6.1: Continued.

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**Thresholds** ( $i = 1, 2$ )

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$$a_{ii2}^f > \frac{2c_i k_i N_i}{N_1 + N_2} - c_i (h_{i1} + k_i),$$

$$n_{ii}^f = \frac{h_{i1} n_{i1} (n_{i2} + n_{i3} - N_{-i})}{2(h_{i2} n_{i2} + h_{i3} n_{i3}) + h_{i1} n_{i1}},$$

$$b_{i11}^f = \frac{\frac{c_1 k_1 N_1}{N_1 + N_2} \left[ B \frac{(a_2 + c_2 h_{21})}{2} - b_2 \right] + \frac{(a_1 + c_1 h_{11})}{2} \left[ b_2 - \frac{B c_2 k_2 N_2}{N_1 + N_2} \right]}{\frac{(a_2 + c_2 h_{21})}{2} - \frac{c_2 k_2 N_2}{N_1 + N_2}},$$

$$b_{i12}^f = \frac{\frac{c_i k_i N_i}{N_1 + N_2} \left[ B \frac{(a_{-i} + c_{-i} (h_{-i1} + k_{-i}))}{2} - b_{-i} \right] + \frac{(a_i + c_i h_{i1})}{2} \left[ b_{-i} - \frac{B c_{-i} k_{-i} N_{-i}}{N_1 + N_2} \right]}{\frac{(a_{-i} + c_{-i} (h_{-i1} + k_{-i}))}{2} - \frac{c_{-i} k_{-i} N_{-i}}{N_1 + N_2}},$$

$$-i = 1, 2, -i \neq i,$$

$$b_{i13}^f = \frac{\frac{c_1 k_1 N_1}{N_1 + N_2} \left[ B \frac{(a_2 + c_2 (h_{21} + k_2))}{2} - b_2 \right] + \frac{(a_1 + c_1 (h_{11} + k_1))}{2} \left[ b_2 - \frac{B c_2 k_2 N_2}{N_1 + N_2} \right]}{\frac{(a_2 + c_2 (h_{21} + k_2))}{2} - \frac{c_2 k_2 N_2}{N_1 + N_2}}.$$



Table 6.2: Conditions for existence of Nash equilibriums for “fg” case.

Conditions	Equilibrium
$f_{ii1}^{fg} \leq f \leq f_{ii2}^{fg}, g > \max \{g_{ii1}^{fg}, g_{ii2}^{fg}\}, b_i > \max \left\{ \frac{Bc_i k_i N_i}{2(N_i + N_{-i})}, \frac{Bc_i k_i}{3} \right\}$	$r_i^{fg*} = 1$
$f < \min \{f_{ii3}^{fg}, f_{ii4}^{fg}, f_{t-i3}^{fg}, f_{t-i4}^{fg}\}, g < \min \{g_{ii3}^{fg}, g_{t-i3}^{fg}\}, a_{ii1}^{fg} < a_i < a_{ii2}^{fg}, a_{-i} > a_{t-i1}^{fg}, N_{-i}^{fg} > N_{t-i1}^{fg}, b_{ii1}^{fg} < b_i < \frac{B}{2}(a_i + c_i h_{i1}), c_i < c_{ii1}^{fg}, b_{t-i1}^{fg} < b_{-i} < \frac{B}{2}(a_{-i} + c_{-i} h_{-i1})$	$r_i^{fg*} = r_i^{fgu},$ $r_{-i}^{fg*} = r_{-i}^{fgu}$
$f < \min \{f_{ii3}^{fg}, f_{ii4}^{fg}, f_{t-i3}^{fg}, f_{t-i4}^{fg}\}, g < \min \{g_{ii3}^{fg}, g_{t-i3}^{fg}\}, a_i > a_{ii1}^{fg}, a_{-i} > a_{t-i1}^{fg}, b_{ii1}^{fg} < b_i < \frac{B}{2}(a_i + c_i h_{i1}), c_i \geq c_{ii1}^{fg}, b_{t-i1}^{fg} < b_{-i} < \frac{B}{2}(a_{-i} + c_{-i} h_{-i1})$	$r_i^{fg*} = 0,$ $r_{-i}^{fg*} = r_{-i}^{fgu}$
$f < \min \{f_{ii3}^{fg}, f_{ii4}^{fg}, f_{t-i3}^{fg}, f_{t-i4}^{fg}\}, g \geq \max \{g_{ii3}^{fg}, g_{t-i3}^{fg}\}, c_i < c_{ii2}^{fg}, a_i < a_{ii3}^{fg}, b_{ii1}^{fg} < b_i < \frac{B}{2}(a_i + c_i (h_{i1} + 2k_i)), b_{t-i1}^{fg} < b_{-i} < \frac{B}{2}(a_{-i} + c_{-i} (h_{-i1} + 2k_{-i}))$	$r_i^{fg*} = 1,$ $r_{-i}^{fg*} = 1$
$f < \min \{f_{ii3}^{fg}, f_{ii4}^{fg}, f_{t-i3}^{fg}, f_{t-i4}^{fg}\}, g \geq \max \{g_{ii3}^{fg}, g_{t-i3}^{fg}\}, c_i \geq c_{ii2}^{fg}, b_{ii1}^{fg} < b_i < \frac{B}{2}(a_i + c_i (h_{i1} + 2k_i)), b_{t-i1}^{fg} < b_{-i} < \frac{B}{2}(a_{-i} + c_{-i} (h_{-i1} + 2k_{-i}))$	$r_i^{fg*} = 0,$ $r_{-i}^{fg*} = 1$
$f \geq \max \{f_{ii3}^{fg}, f_{t-i3}^{fg}\}, g \geq \max \{g_{ii3}^{fg}, g_{t-i3}^{fg}\}, c_i < c_{ii2}^{fg}, a_i < a_{ii3}^{fg}, b_i \geq \frac{B}{2}(a_i + c_i (h_{i1} + 2k_i)), b_{-i} \geq \frac{B}{2}(a_{-i} + c_{-i} (h_{-i1} + 2k_{-i}))$	$r_i^{fg*} = 1,$ $r_{-i}^{fg*} = 1$
$f \geq \max \{f_{ii3}^{fg}, f_{t-i3}^{fg}\}, g \geq \max \{g_{ii3}^{fg}, g_{t-i3}^{fg}\}, c_i \geq c_{ii2}^{fg}, b_i \geq \frac{B}{2}(a_i + c_i (h_{i1} + 2k_i)), b_{-i} \geq \frac{B}{2}(a_{-i} + c_{-i} (h_{-i1} + 2k_{-i}))$	$r_i^{fg*} = 0,$ $r_{-i}^{fg*} = 1$
$f_{ii1}^{fg} \leq f \leq \min \{f_{ii2}^{fg}, f_{t-i3}^{fg}, f_{t-i4}^{fg}\}, g > \max \{g_{ii1}^{fg}, g_{ii2}^{fg}, g_{t-i3}^{fg}\}, c_{-i} < c_{t-i2}^{fg}, a_{-i} < a_{t-i3}^{fg}, b_i > \max \left\{ \frac{Bc_i k_i N_i}{2(N_i + N_{-i})}, \frac{Bc_i k_i}{3} \right\}, b_{t-i1}^{fg} < b_{-i} < \frac{B}{2}(a_{-i} + c_{-i} (h_{-i1} + 2k_{-i})),$	$r_i^{fg*} = 1,$ $r_{-i}^{fg*} = 1$
$f_{ii1}^{fg} \leq f \leq \min \{f_{ii2}^{fg}, f_{t-i3}^{fg}, f_{t-i4}^{fg}\}, g > \max \{g_{ii1}^{fg}, g_{ii2}^{fg}, g_{t-i3}^{fg}\}, c_{-i} \geq c_{t-i2}^{fg}, b_i > \max \left\{ \frac{Bc_i k_i N_i}{2(N_i + N_{-i})}, \frac{Bc_i k_i}{3} \right\}, b_{t-i1}^{fg} < b_{-i} < \frac{B}{2}(a_{-i} + c_{-i} (h_{-i1} + 2k_{-i}))$	$r_i^{fg*} = 1,$ $r_{-i}^{fg*} = 0$
$g > \max \{g_{ii1}^{fg}, g_{ii2}^{fg}\}, f_{ii1}^{fg} \leq f < \min \{f_{ii2}^{fg}, f_{t-i5}^{fg}\}, b_{-i} < \min \{b_{t-i1}^{fg}\}, b_i > \max \left\{ \frac{Bc_i k_i N_i}{2(N_i + N_{-i})}, \frac{Bc_i k_i}{3} \right\}$	$r_i^{fg*} = 1,$ $r_{-i}^{fg*} = 0$
$b_i < \min \{b_{ii1}^{fg}\}, f < \min \{f_{ii5}^{fg}\}$	$r_i^{fg*} = 0$

Table 6.3: Optimal  $f^*$  and  $IA_i^{fg*}$  for “fg” case.

$(f^*, g^*)$	$IA_i^{fg*}, i=1,2$	Conditions	Equilibrium
$(f_{ii1}^{fg}, \max\{g_{ii1}^{fg}, g_{ii2}^{fg}\})$	$B \sum_{i=1}^2 \left( \frac{(1-f^*)(1-g^*)}{2} + \left( \frac{f^* N_i}{N_1 + N_2} \right) (h_{i1} - k_i) \right)$	$b_i > \max \left\{ \frac{Bc_i k_i N_i}{2(N_i + N_{-i})}, \frac{Bc_i k_i}{3} \right\}$	$r_i^{fg*} = 1$
$(0, \max\{g_{ii3}^{fg}, g_{t-i3}^{fg}\})$	$B \frac{(1-g^*)}{2} \sum_{i=1}^2 (h_{i1} - k_i)$	$b_{ii1}^{fg} < b_i < \frac{B}{2}(a_i + c_i(h_{i1} + 2k_i))$ , $c_i < c_{ii2}^{fg}$ , $a_i < a_{ii3}^{fg}$ , $b_{t-i1}^{fg} < b_{-i} < \frac{B}{2}(a_{-i} + c_{-i}(h_{-i1} + 2k_{-i}))$	$r_i^{fg*} = 1$ , $r_{-i}^{fg*} = 1$
$(\min\{f_{t13}^{fg}, f_{t14}^{fg}, f_{t23}^{fg}, f_{t24}^{fg}\}, [\max\{g_{t13}^{fg}, g_{t23}^{fg}\}, 1])$	$B \left( \frac{(1-f^*)}{2} + \frac{f^* N_1}{N_1 + N_2} \right) h_{11}$	$h_{11} \leq h_{t1}^{fg}$ , $c_1 \geq c_{t12}^{fg}$ , $b_{ii1}^{fg} < b_i < \frac{B}{2}(a_i + c_i(h_{i1} + 2k_i))$	$r_1^{fg*} = 0$ , $r_2^{fg*} = 1$
$(0, [\max\{g_{t13}^{fg}, g_{t23}^{fg}\}, 1])$	$Bh_{11}$	$h_{11} > h_{t1}^{fg}$ , $c_1 \geq c_{t12}^{fg}$ , $b_{ii1}^{fg} < b_i < \frac{B}{2}(a_i + c_i(h_{i1} + 2k_i))$	
	$Bh_{21}$	$h_{11} \leq h_{t1}^{fg}$ , $c_2 \geq c_{t22}^{fg}$ , $b_{ii1}^{fg} < b_i < \frac{B}{2}(a_i + c_i(h_{i1} + 2k_i))$	$r_1^{fg*} = 1$ , $r_2^{fg*} = 0$
$(\min\{f_{t13}^{fg}, f_{t14}^{fg}, f_{t23}^{fg}, f_{t24}^{fg}\}, [\max\{g_{t13}^{fg}, g_{t23}^{fg}\}, 1])$	$B \left( \frac{(1-f^*)}{2} + \frac{f^* N_2}{N_1 + N_2} \right) h_{21}$	$h_{11} > h_{t1}^{fg}$ , $c_2 \geq c_{t22}^{fg}$ , $b_{ii1}^{fg} < b_i < \frac{B}{2}(a_i + c_i(h_{i1} + 2k_i))$	
$(\max\{f_{ii3}^{fg}, f_{t-i3}^{fg}\}, \max\{g_{ii3}^{fg}, g_{t-i3}^{fg}\})$	$B \sum_{i=1}^2 \left( \frac{(1-f^*)(1-g^*)}{2} + \frac{f^* N_i}{N_1 + N_2} \right) (h_{i1} - k_i)$	$b_i \geq \frac{B}{2}(a_i + c_i(h_{i1} + 2k_i))$ , $c_i < c_{ii2}^{fg}$ , $a_i < a_{ii3}^{fg}$ , $b_{-i} \geq \frac{B}{2}(a_{-i} + c_{-i}(h_{-i1} + 2k_{-i}))$	$r_i^{fg*} = 1$ , $r_{-i}^{fg*} = 1$

Table 6.3: Continued.

$(f^*, g^*)$	$IA_i^{fg^*}, i=1,2$	Conditions	Equilibrium
$(1, [\max\{g_{i13}^{fg}, g_{i23}^{fg}\}, 1])$	$B \frac{N_1}{N_1 + N_2} h_{11}$	$h_{11} \leq h_{11}^{fg}, c_1 \geq c_{i12}^{fg},$ $b_i \geq \frac{B}{2}(a_i + c_i(h_{i1} + 2k_i))$	$r_1^{fg^*} = 0,$ $r_2^{fg^*} = 1$
$(\max\{f_{i13}^{fg}, f_{i23}^{fg}\}, [\max\{g_{i13}^{fg}, g_{i23}^{fg}\}, 1])$	$B \left( \frac{(1-f^*)}{2} + \frac{f^* N_1}{N_1 + N_2} \right) \times h_{11}$	$h_{11} > h_{11}^{fg}, c_1 \geq c_{i12}^{fg},$ $b_i \geq \frac{B}{2}(a_i + c_i(h_{i1} + 2k_i))$	
$(1, [\max\{g_{i13}^{fg}, g_{i23}^{fg}\}, 1])$	$B \frac{N_2}{N_1 + N_2} h_{21}$	$h_{11} > h_{11}^{fg}, c_2 \geq c_{i22}^{fg},$ $b_i \geq \frac{B}{2}(a_i + c_i(h_{i1} + 2k_i))$	$r_1^{fg^*} = 1,$ $r_2^{fg^*} = 0$
$(\max\{f_{i13}^{fg}, f_{i23}^{fg}\}, [\max\{g_{i13}^{fg}, g_{i23}^{fg}\}, 1])$	$B \left( \frac{(1-f^*)}{2} + \frac{f^* N_2}{N_1 + N_2} \right) h_{21}$	$h_{11} \leq h_{11}^{fg}, c_2 \geq c_{i22}^{fg},$ $b_i \geq \frac{B}{2}(a_i + c_i(h_{i1} + 2k_i))$	
$(f_{i11}^{fg}, \max\{g_{ii1}^{fg}, g_{ii2}^{fg}, g_{ii3}^{fg}\})$	$B \sum_{i=1}^2 \left( \frac{(1-f^*)(1-g^*)}{2} + \frac{f^* N_i}{N_1 + N_2} \right) (h_{i1} - k_i)$	$b_i > \max \left\{ \frac{Bc_i k_i N_i}{2(N_i + N_{-i})}, \frac{Bc_i k_i}{3} \right\}$ $c_{-i} < c_{i-2}^{fg}, a_{-i} < a_{i-3}^{fg},$ $b_{i-1}^{fg} < b_{-i} < \frac{B}{2}(a_{-i} + c_{-i}(h_{-i1} + 2k_{-i}))$	$r_i^{fg^*} = 1,$ $r_{-i}^{fg^*} = 1$
$(\min\{f_{i12}^{fg}, f_{i23}^{fg}, f_{i24}^{fg}\}, [\max\{g_{i11}^{fg}, g_{i12}^{fg}, g_{i23}^{fg}\}, 1])$	$B \left( \frac{(1-f^*)}{2} + \frac{f^* N_2}{N_1 + N_2} \right) h_{21}$	$b_i > \max \left\{ \frac{Bc_i k_i N_i}{2(N_1 + N_2)}, \frac{Bc_i k_i}{3} \right\},$ $c_2 \geq c_{i22}^{fg}, h_{11} > h_{11}^{fg},$ $b_{i21}^{fg} < b_2 < \frac{B}{2}(a_2 + c_2(h_{21} + 2k_2))$	$r_1^{fg^*} = 1,$ $r_2^{fg^*} = 0$
$(f_{i11}^{fg}, [\max\{g_{i11}^{fg}, g_{i12}^{fg}, g_{i23}^{fg}\}, 1])$	$B \left( \frac{(1-f^*)}{2} + \frac{f^* N_2}{N_1 + N_2} \right) h_{21}$	$b_i > \max \left\{ \frac{Bc_i k_i N_i}{2(N_1 + N_2)}, \frac{Bc_i k_i}{3} \right\},$ $c_2 \geq c_{i22}^{fg}, h_{11} \leq h_{11}^{fg},$ $b_{i21}^{fg} < b_2 < \frac{B}{2}(a_2 + c_2(h_{21} + 2k_2))$	

Table 6.3: Continued.

$(f^*, g^*)$	$IA_i^{fg^*}, i=1,2$	Conditions	Equilibrium
$\left( f_{i21}^{fg}, \left[ \max \left\{ g_{i21}^{fg}, g_{i22}^{fg}, g_{i13}^{fg}, 1 \right\} \right] \right)$	$B \left( \frac{(1-f^*)}{2} + \frac{f^* N_1}{N_1 + N_2} \right) h_{11}$	$b_i > \max \left\{ \frac{Bc_i k_i N_i}{2(N_1 + N_2)}, \frac{Bc_i k_i}{3} \right\}$ , $c_1 \geq c_{i12}^{fg}, h_{11} > h_{i1}^{fg}$ , $b_{i11}^{fg} < b_1 < \frac{B}{2}(a_1 + c_1(h_{11} + 2k_1))$	$r_1^{fg^*} = 0$ , $r_2^{fg^*} = 1$
$\left( \min \left\{ f_{i22}^{fg}, f_{i13}^{fg}, f_{i14}^{fg} \right\}, \left[ \max \left\{ g_{i21}^{fg}, g_{i22}^{fg}, g_{i13}^{fg}, 1 \right\} \right] \right)$	$B \left( \frac{(1-f^*)}{2} + \frac{f^* N_1}{N_1 + N_2} \right) h_{11}$	$b_i > \max \left\{ \frac{Bc_i k_i N_i}{2(N_1 + N_2)}, \frac{Bc_i k_i}{3} \right\}$ , $c_1 \geq c_{i12}^{fg}, h_{11} \leq h_{i1}^{fg}$ , $b_{i11}^{fg} < b_1 < \frac{B}{2}(a_1 + c_1(h_{11} + 2k_1))$	
$\left( \min \left\{ f_{i12}^{fg}, f_{i25}^{fg} \right\}, \left[ \max \left\{ g_{i11}^{fg}, g_{i12}^{fg} \right\}, 1 \right] \right)$	$B \left( \frac{(1-f^*)}{2} + \frac{f^* N_2}{N_1 + N_2} \right) h_{21}$	$h_{11} > h_{i1}^{fg}, b_2 < \min \left\{ b_{i21}^{fg} \right\}$ , $b_1 > \max \left\{ \frac{Bc_1 k_1 N_1}{2(N_1 + N_2)}, \frac{Bc_1 k_1}{3} \right\}$	$r_1^{fg^*} = 1$ , $r_2^{fg^*} = 0$
$\left( f_{i11}^{fg}, \left[ \max \left\{ g_{i11}^{fg}, g_{i12}^{fg} \right\}, 1 \right] \right)$	$B \left( \frac{(1-f^*)}{2} + \frac{f^* N_2}{N_1 + N_2} \right) h_{21}$	$h_{11} \leq h_{i1}^{fg}, b_2 < \min \left\{ b_{i21}^{fg} \right\}$ , $b_1 > \max \left\{ \frac{Bc_1 k_1 N_1}{2(N_1 + N_2)}, \frac{Bc_1 k_1}{3} \right\}$	
$\left( f_{i21}^{fg}, \left[ \max \left\{ g_{i21}^{fg}, g_{i22}^{fg} \right\}, 1 \right] \right)$	$B \left( \frac{(1-f^*)}{2} + \frac{f^* N_1}{N_1 + N_2} \right) h_{11}$	$h_{11} \leq h_{i1}^{fg}, b_1 < \min \left\{ b_{i11}^{fg} \right\}$ , $b_2 > \max \left\{ \frac{Bc_2 k_2 N_2}{2(N_1 + N_2)}, \frac{Bc_2 k_2}{3} \right\}$	$r_1^{fg^*} = 0$ , $r_2^{fg^*} = 1$
$\left( \min \left\{ f_{i22}^{fg}, f_{i15}^{fg} \right\}, \left[ \max \left\{ g_{i21}^{fg}, g_{i22}^{fg} \right\}, 1 \right] \right)$	$B \left( \frac{(1-f^*)}{2} + \frac{f^* N_1}{N_1 + N_2} \right) h_{11}$	$h_{11} > h_{i1}^{fg}, b_1 < \min \left\{ b_{i11}^{fg} \right\}$ , $b_2 > \max \left\{ \frac{Bc_2 k_2 N_2}{2(N_1 + N_2)}, \frac{Bc_2 k_2}{3} \right\}$	
$(0, [0,1])$	$\frac{B}{2}(h_{11} + h_{21})$	$b_i < \min \left\{ b_{i1}^{fg} \right\}$	$r_i^{fg^*} = 0$

Table 6.4: Conditions for existence of Nash equilibriums for “g” case.

	Conditions	Equilibrium
1	$g > \max \{g_{ii1}^g, g_{ii2}^g\}, b_i > \frac{Bc_i k_i}{2}, i=1,2$	$r_i^{g*} = 1$
2	$g < \min \{g_{ii3}^g, g_{t-i3}^g\}, a_i < a_{ii1}^g, N_{-i} > N_{t-i1}^g, c_i < c_{ii1}^g, i, -i=1,2,$ $i \neq -i, b_{ii1}^g < b_i < \frac{B}{2}(a_i + c_i h_{i1}), b_{t-i1}^g < b_{-i} < \frac{B}{2}(a_{-i} + c_{-i} h_{-i1})$	$r_i^{g*} = r_i^{gu},$ $r_{-i}^{g*} = r_{-i}^{gu}$
3	$g < \min \{g_{ii3}^g, g_{t-i3}^g\}, b_{ii1}^g < b_i < \frac{B}{2}(a_i + c_i h_{i1}), c_i \geq c_{ii1}^g,$ $b_{t-i1}^g < b_{-i} < \frac{B}{2}(a_{-i} + c_{-i} h_{-i1})$	$r_i^{g*} = 0,$ $r_{-i}^{g*} = r_{-i}^{gu}$
4	$g \geq \max \{g_{ii3}^g, g_{t-i3}^g\}, b_i > b_{ii1}^g, c_i < c_{ii2}^g, a_i < a_{ii2}^g, b_{-i} > b_{t-i1}^g$	$r_i^{g*} = 1,$ $r_{-i}^{g*} = 1$
5	$g \geq \max \{g_{ii3}^g, g_{t-i3}^g\}, b_i > b_{ii1}^g, c_i \geq c_{ii2}^g, b_{-i} > b_{t-i1}^g$	$r_i^{g*} = 0,$ $r_{-i}^{g*} = 1$
6	$g > \max \{g_{ii1}^g, g_{ii2}^g, g_{t-i3}^g\}, b_i > \frac{Bc_i k_i}{2}, c_{-i} < c_{t-i2}^g, a_{-i} < a_{t-i2}^g,$ $b_{t-i1}^g < b_{-i} < \frac{B}{2}(a_{-i} + c_{-i}(h_{-i1} + 2k_{-i}))$	$r_i^{g*} = 1,$ $r_{-i}^{g*} = 1$
7	$g > \max \{g_{ii1}^g, g_{ii2}^g, g_{t-i3}^g\}, b_i > \frac{Bc_i k_i}{2}, c_{-i} \geq c_{t-i2}^g,$ $b_{t-i1}^g < b_{-i} < \frac{B}{2}(a_{-i} + c_{-i}(h_{-i1} + 2k_{-i}))$	$r_i^{g*} = 1,$ $r_{-i}^{g*} = 0$
8	$g > \max \{g_{ii1}^g, g_{ii2}^g\}, b_{-i} < \min \{b_{t-i1}^g\}, b_i > \frac{Bc_i k_i}{2}$	$r_i^{g*} = 1,$ $r_{-i}^{g*} = 0$
9	$b_i < \min \{b_{ii1}^g\}$	$r_i^{g*} = 0$

Table 6.5: Optimal  $g^*$  and  $IA_i^{g^*}$  for “g” case.

$g^*$	$IA_i^{g^*}$	Conditions	Equilibrium
$\max \{g_{ii1}^g, g_{ii2}^g\}$	$B \frac{(1-g^*)}{2} \times \sum_{i=1}^2 (h_{i1} - k_i)$	$b_i > \frac{Bc_i k_i}{2}$	$r_i^{g^*} = 1$
$\max \{g_{ii3}^g, g_{t-i3}^g\}$	$B \frac{(1-g^*)}{2} \times \sum_{i=1}^2 (h_{i1} - k_i)$	$b_i > b_{ii1}^g, c_i < c_{ii2}^g, a_i < a_{ii2}^g, b_{-i} > b_{t-i1}^g$	$r_i^{g^*} = 1,$ $r_{-i}^{g^*} = 1$
$\left[ \max \{g_{ii3}^g, g_{t-i3}^g\}, 1 \right]$	$Bh_{i1}$	$b_i > b_{ii1}^g, c_i \geq c_{ii2}^g, b_{-i} > b_{t-i1}^g$	$r_i^{g^*} = 0,$ $r_{-i}^{g^*} = 1$
$\max \{g_{ii1}^g, g_{ii2}^g, g_{t-i3}^g\}$	$B \frac{(1-g^*)}{2} \times \sum_{i=1}^2 (h_{i1} - k_i)$	$b_i > \frac{Bc_i k_i}{2}, c_{-i} < c_{t-i2}^g, a_{-i} < a_{t-i2}^g,$ $b_{t-i1}^g < b_{-i} < \frac{B}{2} (a_{-i} + c_{-i} (h_{-i1} + 2k_{-i}))$	$r_i^{g^*} = 1,$ $r_{-i}^{g^*} = 1$
$\left[ \max \{g_{ii1}^g, g_{ii2}^g, g_{t-i3}^g\}, 1 \right]$	$Bh_{-i1}$	$b_i > \frac{Bc_i k_i}{2}, c_{-i} \geq c_{t-i2}^g,$ $b_{t-i1}^g < b_{-i} < \frac{B}{2} (a_{-i} + c_{-i} (h_{-i1} + 2k_{-i}))$	$r_i^{g^*} = 1,$ $r_{-i}^{g^*} = 0$
$\left[ \max \{g_{ii1}^g, g_{ii2}^g\}, 1 \right]$	$Bh_{-i1}$	$b_{-i} < \min \{b_{t-i1}^g\}, b_i > \frac{Bc_i k_i}{2}$	$r_i^{g^*} = 1,$ $r_{-i}^{g^*} = 0$
$[0, 1]$	$B(h_{i1} + h_{-i1})$	$b_i < \min \{b_{ii1}^g\}$	$r_i^{g^*} = 0$

Table 6.6: Conditions for existence of Nash equilibriums for “ $f$ ” case.

Conditions	Equilibrium
$f \leq \min \{f_{i1}^f\}, a_i > a_{i1}^f, b_i \leq \frac{B}{2}(a_i + c_i h_{i1}), i=1,2$	$r_i^{f*} = 0$
$f_{t-i1}^f < f \leq f_{i1}^f, a_i > a_{i1}^f, a_{-i} \leq a_{t-i1}^f, b_i \leq b_{i1}^f, b_i \leq \frac{B}{2}(a_i + c_i h_{i1}),$ $b_{-i} > b_{t-i1}^f, b_{-i} \leq \frac{Bc_{-i}k_{-i}N_{-i}}{N_i + N_{-i}}, n_{-i1} \leq n_{t-i1}^f, n_{-i2} + n_{-i3} > N_i,$ $i, -i=1,2, i \neq -i$	$r_i^{f*} = 0,$ $r_{-i}^{f*} = 0$
$f \leq \min \{f_{i1}^f, f_{t-i2}^f\}, a_i \leq a_{i1}^f, b_i > \frac{B}{2}(a_i + c_i h_{i1}), n_{i1} \leq n_{i1}^f,$ $n_{i2} + n_{i3} > N_{-i}, b_{-i} \leq \frac{B}{2}(a_{-i} + c_{-i}(h_{-i1} + k_{-i}))$	$r_i^{f*} = 1,$ $r_{-i}^{f*} = 0$
$f_{i1}^f < f \leq f_{t-i2}^f, a_i > a_{i1}^f, b_i > b_{i2}^f,$ $b_i > \frac{Bc_i k_i N_i}{N_i + N_{-i}}, b_{-i} \leq \frac{B}{2}(a_{-i} + c_{-i}(h_{-i1} + k_{-i}))$	
$f > \max \{f_{i2}^f\}, b_i > \frac{Bc_i k_i N_i}{N_i + N_{-i}}$	$r_i^{f*} = 1$

Table 6.7: Optimal  $f^*$  and  $IA_L^f$  \* for “ $f$ ” case.

Sub-case 1: Includes the case where (0,0) and (1,1) exist together as multiple equilibria and unique equilibrium. If  $a_i > a_{i1}^f$ ,  $\frac{Bc_i k_i N_i}{N_1 + N_2} < b_i \leq \frac{B}{2}(a_i + c_i h_{i1})$ ,  $i = 1, 2$ ,  $b_{i1}^f < b_1 \leq b_{i2}^f$

$f^*$	$IA_L^f$ *	Conditions	Equilibrium
$f_{i22} - \varepsilon$	$B \left( \frac{h_{11} + h_{21}}{2} \right)$	$\max \{h_{21}, h_{21} + k_1 - k_2\} < h_{11} \leq h_{i2}$ , $N_1 > N_2$	(0,0)
0		$\max \{h_{21}, h_{21} + k_1 - k_2\} < h_{11}$ , $N_1 \leq N_2$	(0,0)
1	$B \left( \frac{N_1(h_{11} - k_1)}{N_1 + N_2} + \frac{N_2(h_{21} - k_2)}{N_1 + N_2} \right)$	$\max \{h_{21}, h_{21} + k_1 - k_2, h_{i2}\} < h_{11}$ , $N_1 > N_2$	(1,1)

Sub-case 2: Includes the case where (0,1) and (1,0) exist together as multiple equilibria.

If  $a_i > a_{i1}^f$ ,  $\frac{Bc_i k_i N_i}{N_1 + N_2} < b_i \leq \frac{B}{2}(a_i + c_i h_{i1})$ ,  $b_1 > b_{i2}^f$ ,  $b_2 > b_{i22}^f$

$f^*$	$IA_L^f$ *	Conditions	Equilibrium
$f_{i12} - \varepsilon$	$B \left( h_{11} + \frac{f^* N_2}{N_1 + N_2} (h_{21} - k_2 - h_{11}) \right)$	$h_{11} > h_{21} + \frac{f_{i12} N_2 k_2 - f_{i11} N_1 k_1}{N_1(1 - f_{i11}) + N_2(1 - f_{i12})}$ , $N_1 \leq N_2$	(0,1)
$f_{i11} + \varepsilon$		$h_{11} \leq h_{21} + \frac{f_{i12} N_2 k_2 - f_{i11} N_1 k_1}{N_1(1 - f_{i11}) + N_2(1 - f_{i12})}$ , $N_1 \leq N_2$	(1,0)
0	$B \left( \frac{h_{11} + h_{21}}{2} \right)$	$N_1 > N_2$	(0,0)

Sub-case 3: Includes the case where no equilibrium and unique equilibrium exist. If

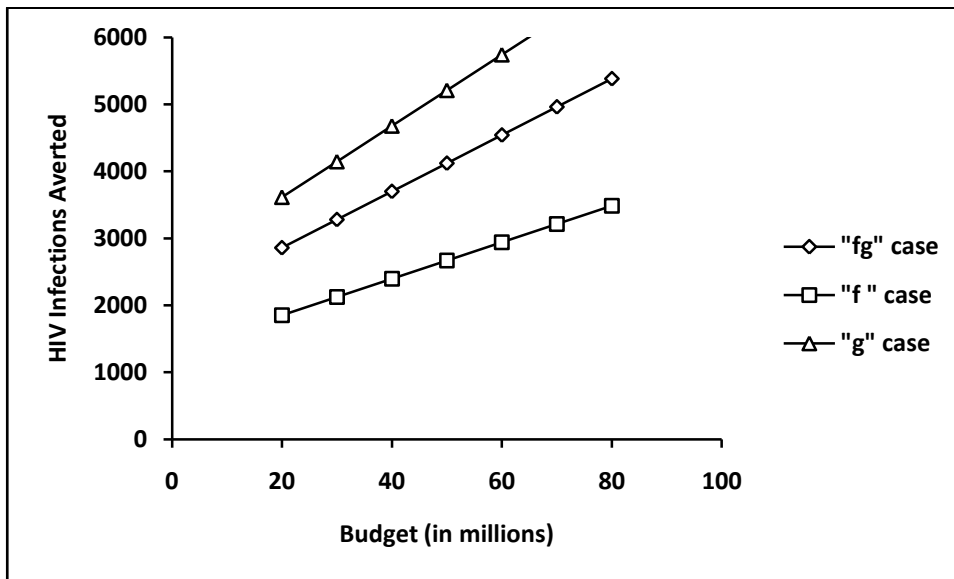
$a_1 > a_{i11}^f$ ,  $a_2 \leq a_{i21}^f$ ,  $b_{i12}^f < b_1 \leq b_{i11}^f$ ,  $\frac{Bc_i k_i N_i}{N_1 + N_2} < b_i \leq \frac{B}{2}(a_i + c_i h_{i1})$ ,  $b_2 > b_{i21}^f$ ,  $n_{21} \leq n_{i2}^f$ ,

$n_{22} + n_{23} > N_1$

$f^*$	$IA_L^f$ *	Conditions	Equilibrium
$f_{i22} - \varepsilon$	$B \left( h_{21} + \frac{f^* N_1}{N_1 + N_2} (h_{11} - k_1 - h_{21}) \right)$	$\frac{h_{11} - f_{i22} N_1 (h_{11} - k_1)}{N_2 + N_1 (1 - f_{i22})} < h_{21} \leq h_{11} - k_1$	(1,0)
0		$h_{21} \leq \frac{h_{11} - f_{i22} N_1 (h_{11} - k_1)}{N_2 + N_1 (1 - f_{i22})}$	(1,0)



Figure 6.1: HIV infections averted versus budget for both regions for all three cases.



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## Chapter 7

### Summary, Conclusions, and Future Directions

The primary purpose of this research is to investigate the impact of incentives in encouraging optimal allocation in a multi-level HIV prevention funds allocation process. We first examined practical as well as theoretical models in the literature in Chapters 2 and 3, respectively. We then developed a series of models to investigate the impact of incentives in a two-level decision making process. To our knowledge, these models are the first to consider the impact of incentives in a HIV prevention funds allocation process.

Three different model frameworks were developed in three chapters. In Chapter 4, we consider a two-level decision-making process in which an upper-level decision maker (UD) allocates funds to a single lower-level decision maker (LD) who then distributes funds to three programs. Here the UD uses an incentive scheme to encourage the LD to reduce the fraction of funds reserved for equity by making the amount received by the LD dependent on this fraction. In particular, the UD may withhold funds to encourage an allocation that is more efficient. We illustrate this process with an example using data from California, U.S.

In Chapter 5, we extend the model developed in Chapter 4 to include information asymmetry. We assume that there is an UD who is allocating funds to a single LD who then distributes funds to two programs. We consider 2 sources of information asymmetry. In case 1, the preferences of the LD with respect to allocating funds proportionally are unknown to the UD. In case 2, the number of infections prevented per dollar is known to the LD, but is unknown to the UD. We illustrate this arrangement with an example using data from California, U.S.

In Chapter 6, we extend the model developed in Chapter 4 to include multiple LDs. Here we examine the impact of two different types of incentives within and between the lower levels. The UD sets two levels of incentives that maximize the total number of

infections averted and the LDs simultaneously set the fraction of the funds received to be reserved for proportional allocation. We examine three cases: one in which the UD provides two types of incentives both between regions and within regions; a second in which the UD provides incentives only between regions, and a third in which the UD provides incentives only within regions. We illustrate the use of these models using data from California and New York, U.S.

### **Managerial Insights**

Our results provide many useful insights for policy makers about how to design incentive schemes that help to encourage effective utilization of limited funds and significantly improve health outcomes at the lower level. We list some of these as follows:

- If the LD strongly prefers optimal or proportional allocation then that person will choose to allocate optimally or proportionally even without incentives.
- Incentives may be beneficial if the LD has less preference for optimal allocation since that person may switch from optimal to proportional allocation without incentives.
- Incentives may not be needed for a larger budget and bigger size of the risk group with highest number of infections averted/dollar, or a larger budget and higher number of infections averted/ dollar in a program since the LD will always elect to allocate optimally even without incentives.
- The LD is less likely to choose a proportional allocation as that person's preferences for the budget and the number of infections averted increases.
- Incentives can encourage an optimal allocation under asymmetric information. However, there is a loss of efficiency (LOE) when we compare an asymmetric information case with a full information case.

- The loss of efficiency (LOE) increases with the increase in the budget, the number of infections prevented per dollar, the level of incentive given, size of the risk group, and the probability that the LD is of high type.
- If fewer infections are prevented per dollar in both the regions or if both the LDs have lower preferences for proportional allocation then full incentive is given to encourage both the LDs to choose optimal allocation. In this case, there is no loss of efficiency.
- Both types of incentives, between and within the regions encourage optimal allocation of the entire budget by the LD with higher preferences for infections prevented/dollar if both the LDs have moderate preferences for proportional allocation. However, if both the LDs have higher preferences for proportional allocation then incentives are given only between regions to encourage optimal allocation.
- No incentives are given within regions if both the LDs have moderate preferences for proportional allocation and infections prevented/dollar in a region with larger infected population is higher than the other region.

### **Directions for Future Research**

We identified several possible directions for future research. Short time horizon models were developed which do not incorporate epidemic dynamics. Thus, models for use with respect to longer time horizons could be developed that would take into account epidemic dynamics which may then lead to different allocation decisions.

We assumed that the benefits of intervention scale linearly with respect to amounts invested which is a common assumption in cost-effectiveness analysis. However, this assumption may not always be valid. I would like to develop a model that incorporates a production function, which is defined as a function that links the number of dollars invested in a program with the number of HIV/AIDS infections prevented.

Different types of incentive schemes were considered under both symmetric and asymmetric information cases, incorporating the LD's concerns about equity and the UD's decision regarding the strength of the incentives to help demonstrate the significance of incentives in encouraging an optimal allocation of HIV prevention funds. However, our primary purpose is to show the significance of incentives and our analysis could be generalized for various other incentive schemes by adjusting for the upper and lower-level utility functions.

We assume that there is a single decision maker at the upper level. However, in practice, there are multiple stakeholders including donors and advocacy groups taking part in the decision making process and each having its own priorities, objectives, and limitations. I would like to develop a model in which multiple UDs take part in the decision making process and have their own priorities. The objective function in the model could be a weighted sum of the priorities to be maximised subject to given constraints. The analysis of such a model may lead to different analyses.

A feasible incentive scheme can be formulated to encourage optimal allocation if data is available to the UD. However, in practice, the required data is often unavailable. We consider 2 cases of information asymmetry. There could be various other factors present at the lower level impacting decisions at the upper level and which are unknown to the upper level such as local politics and social considerations. I would like to examine the impact of incentives under such asymmetric information. A signalling model can be formulated which incorporates such asymmetric information about various factors at the lower level.

Another possible extension could be a sequential game employing imperfect information in which the game includes an UD who is allocating funds to multiple LDs over multiple periods. Incomplete information regarding decisions made at the lower-levels is available at the upper level. This would lead to a complex model in which *a priori* information about the lower level could be incorporated to make decision at the upper level.

## APPENDICES

### 8. Proof of Chapter 4

#### Proof of Proposition 4.1

Proof: (i) The lower-level utility function is  $aZ + br + cIA$ . When  $Z = B(1 - rf)$  and  $IA = B(1 - rf)(h_1 - rk)$  are substituted into this we obtain:  $L_L : \max_r U_L = aZ + br + cIA$

$$= aB(1 - rf) + br + cB(1 - rf)(h_1 - rk)$$

$$\text{s.t. } 0 \leq r \leq 1$$

The second derivative of  $U_L$  with respect to  $r$  is given by,

$$U_L'' = \frac{d}{dr} \left[ aB(-f) + b + cB(-f(h_1 - rk) - k(1 - rf)) \right] = 2cBfk \geq 0.$$

Since  $U_L'' \geq 0$ ,  $U_L$  is a convex function of  $r$  and the optimal solution is either  $r^* = 1$  or 0.

$$(ii) U_L(0) = aB + cBh_1 \text{ and } U_L(1) = aB(1 - f) + b + cB(1 - f)(h_1 - k).$$

$$\text{If } U_L(1) \geq U_L(0) \text{ then } r^* = 1 \text{ and if } U_L(1) < U_L(0) \text{ then } r^* = 0. \quad (8.1)$$

Note that  $B(a + c(h_1 - k)) > 0$  because  $h_1 > k$ .

$$\text{If } f \leq f_i^L = \frac{b - cBk}{B(a + c(h_1 - k))} \text{ then } aBf + cBfh_1 - cBfk \leq b - cBk$$

$$\text{Thus, } 0 \leq -aBf + b - cBfh_1 + cBfk - cBk.$$

Adding  $aB + cBh_1$  to each side, we obtain

$$aB + cBh_1 \leq aB(1 - f) + b + cB(1 - f)(h_1 - k). \quad (8.2)$$

$$\Rightarrow U(0) \leq U(1).$$

Thus,  $U(0) \leq U(1)$  if  $f \leq f_i^L$ .

(iii) This follows by reversing the inequality  $f \leq f_i^L$  in part (ii). ■

#### Proof of Corollary 4.1:

Proof: When  $b > b_{ut}^L$ , then  $b > b_{ut}^L = B(a + ch_1) \Rightarrow b - cBk > B(a + ch_1 - ck) \Rightarrow$

$$\frac{b - cBk}{B(a + c(h_1 - k))} = f_i^L > 1.$$

From Proposition 1, part (iii) we know that  $f \leq f_i^L \Rightarrow r^* = 1$ . Since  $0 \leq f \leq 1$  and  $f_i^L > 1$ , then  $b > b_{ut}^L \Rightarrow r^* = 1$ .

Next, consider the case when  $b < b_{ut}^L = cBk \Rightarrow b - cBk < 0 \Rightarrow \frac{b - cBk}{B(a + c(h_1 - k))} = f_i^L < 0$ .

From Proposition 1, part (ii) we know that  $f > f_i^L \Rightarrow r^* = 0$ . ■

**The proof of Proposition 4.2 is straightforward and omitted.** ■

### Proof of Proposition 4.3

Proof: Recall that the lower-level optimization model is

$$L_M: \max_r U_M(r) = Z^a r^b I A^c = (B(1 - rf))^{a+c} r^b (h_1 - rk)^c$$

$$\text{s.t. } 0 \leq r \leq 1$$

The first derivative of  $U_M(r)$  is

$$U_M'(r) = B^{a+c} (1 - rf)^{a+c-1} r^{b-1} (h_1 - rk)^{c-1} Q(r) \quad (8.3)$$

where,  $Q(r) = r^2(a + b + 2c)kf - r((a + b + c)h_1f + (b + c)k) + bh_1 = a_q r^2 + b_q r + c_q$ .

We solve for  $r^*$  by setting,  $U_M'(r) = 0$ .

Equation (8.3) has 5 roots given by  $r = \frac{1}{f}$ ,  $r = 0$ ,  $r = \frac{h_1}{k}$  and the two roots of the

quadratic equation  $Q(r)$ . The first three roots result in  $U_M(r) = 0$ . We thus focus on

the two roots of  $Q(r)$ . We first show that  $Q(r)$  has two positive real roots.

$$\begin{aligned} b_q^2 - 4a_q c_q &= ((a + b + c)h_1f + (b + c)k)^2 - 4(a + b + 2c)kfbh_1 \\ &= ((a + b + c)h_1f)^2 + 2(a + b + c)h_1f(b + c)k + ((b + c)k)^2 - 4(a + b + 2c)kfbh_1 \\ &= ((a + b + c)h_1f)^2 - 2(a + b + c)h_1f(b + c)k + ((b + c)k)^2 + 4(a + b + c)h_1f(b + c)k - 4(a + b + 2c)kfbh_1 \\ &= ((a + b + c)h_1f - (b + c)k)^2 + 4((a + b + c)(b + c) - (a + b + 2c)b)h_1fk \end{aligned}$$



$$= \left( (a+b+c)h_1f - (b+c)k \right)^2 + 4(c(a+c))h_1fk > 0 \quad (8.4)$$

The inequality in (8.4) follows from the fact that both terms are strictly positive. Thus,  $Q(r)$  has two real roots. Let  $r^l$  and  $r^u$ ,  $r^l < r^u$ , be the two roots of  $Q(r)$  defined by,

$$r^u = \frac{(a+b+c)h_1f + (b+c)k + \sqrt{\left( (a+b+c)h_1f + (b+c)k \right)^2 - 4(a+b+2c)kfbh_1}}{2(a+b+2c)kf}$$

$$r^l = \frac{(a+b+c)h_1f + (b+c)k - \sqrt{\left( (a+b+c)h_1f + (b+c)k \right)^2 - 4(a+b+2c)kfbh_1}}{2(a+b+2c)kf} \quad (8.5)$$

Since  $a, b, c, h_1, k > 0$  and  $f \geq 0$ , and from (8.4)  $r^l > 0$  and by definition  $0 < r^l < r^u$ .

Thus,  $Q(r)$  has two positive real roots.

$U_M(r)$  is continuous on  $[0, m]$  and is differentiable on  $(0, m)$ , where  $m = \min\left(\frac{1}{f}, \frac{h_1}{k}\right)$ .

Since  $U_M(0) = U_M(m) = 0$ , then by Rolle's Theorem  $\exists r \in (0, m)$  s.t.  $U'(r) = 0$ . Since

$r^l < r^u$ , this value of  $r$  is  $r^l$ . If  $r^l < 1$  then  $r^* = r^l$ , otherwise,  $r^* = 1$ . Thus,

$$r^* = \min\{r^l, 1\}. \quad \blacksquare$$

Prior to stating the result for a multiplicative function, we define some new notations.

$$f_{i2}^M = \frac{-(b+c)k}{(a+b+c)h_1 - 2(a+b+2c)k}, m_1 = \frac{2(a+b+2c)}{a+b+c}, m_2 = \frac{2a+b+3c}{a+b+c},$$

$$m_3 = \frac{a+b+2c}{a+b+c}, n_i = \frac{h_1n_1(n_2+n_3)(m_i-1)}{h_1n_1+m_i(h_2n_2+h_3n_3)}, i=1,2,3.$$

#### Proof of Corollary 4.2:

Proof: By using L'Hopital's rule,  $\lim_{f \rightarrow 0} r^l = \frac{bh_1}{(b+c)k}$ . (8.6)

We next show that  $b > b_i^M \Rightarrow 0 < f_{i1}^M < 1$ .

$$b > b_i^M \Rightarrow bh_1 - (b+c)k > 0$$

$$(a+c)(h_1-k) + bh_1 - (b+c)k > bh_1 - (b+c)k > 0 \quad (8.7)$$

$$(a+b+c)h_1 - (a+b+2c)k > 0 \quad (8.8)$$

$f_{i1}^M > 0$  and  $f_{i1}^M < 1$  follows immediately from (8.7).

Thus,  $b > b_i^M \Rightarrow 0 < f_{i1}^M < 1$ . (8.9)

i-a) We first show that if  $b > b_i^M$  and  $0 \leq f \leq f_{i1}^M$  then  $r^l > 1$ , which implies  $r^* = 1$ .

If  $f = 0$  then  $\lim_{f \rightarrow 0} r^l = \frac{bh_1}{(b+c)k} > 1$ , which implies  $r^* = 1$ .

Now suppose,  $f \leq f_{i1}^M = \frac{bh_1 - (b+c)k}{(a+b+c)h_1 - (a+b+2c)k}$ ,

$f((a+b+c)h_1 - (a+b+2c)k) \leq bh_1 - (b+c)k$  where inequality follows from (8.8).

This inequality is rearranged as follows.

$$\begin{aligned} (a+b+2c)fk - ((a+b+c)fh_1 + (b+c)k) &\geq -bh_1 \\ 4((a+b+2c)fk)^2 - 4((a+b+c)fh_1 + (b+c)k)(a+b+2c)fk &\geq -4bh_1(a+b+2c)fk \\ ((a+b+c)fh_1 + (b+c)k)^2 + 4((a+b+2c)fk)^2 - 4((a+b+c)fh_1 + (b+c)k)(a+b+2c)fk \\ &\geq ((a+b+c)fh_1 + (b+c)k)^2 - 4bh_1(a+b+2c)fk \\ \left[ (a+b+c)fh_1 + (b+c)k - 2(a+b+2c)fk \right]^2 &\geq ((a+b+c)fh_1 + (b+c)k)^2 - 4bh_1(a+b+2c)fk \end{aligned} \quad (8.10)$$

In order to show that the term inside the square bracket is positive so that we do not have to worry about absolute value we separately consider the 2 sub-cases:  $n_1 > n_{i1}$  or  $n_1 \leq n_{i1}$ .

i-a-1) Let  $n_1 > n_{i1} = \frac{h_1n_1(n_2+n_3)(m_1-1)}{h_1n_1+m_1(h_2n_2+h_3n_3)}$ . Then

$$\begin{aligned} n_1(h_1n_1+m_1(h_2n_2+h_3n_3)) &> h_1n_1(n_2+n_3)(m_1-1) \\ \Rightarrow h_1(n_1+(n_2+n_3)(1-m_1)) &< -m_1(h_2n_2+h_3n_3), \\ \Rightarrow h_1(n_1+n_2+n_3) &> m_1(h_1n_2+h_1n_3-h_2n_2-h_3n_3) \\ \Rightarrow h_1 &> m_1 \left( (h_1-h_2)\frac{n_2}{N} + (h_1-h_3)\frac{n_3}{N} \right), \text{ where } N = n_1+n_2+n_3 \end{aligned}$$

Thus,  $h_1 - m_1k > 0$

$$\Rightarrow h_1(a+b+c) - 2k(a+b+2c) > 0. \quad (8.11)$$

$$\text{Thus, } f_{i_2}^M = \frac{-(b+c)k}{(a+b+c)h_1 - 2(a+b+2c)k} < 0.$$

$$\text{Since } f \geq 0 \text{ and } f_{i_2}^M < 0 \Rightarrow f > f_{i_2}^M.$$

$$\text{Thus, } f((a+b+c)h_1 - 2(a+b+2c)k) > -(b+c)k$$

$$\Rightarrow (a+b+c)fh_1 + (b+c)k - 2(a+b+2c)fk > 0 \quad (8.12)$$

Thus, the term inside the square bracket in (8.10) is positive. From (8.10) we get,

$\Rightarrow$

$$(a+b+c)fh_1 + (b+c)k - 2(a+b+2c)fk \geq \sqrt{((a+b+c)fh_1 + (b+c)k)^2 - 4bh_1(a+b+2c)fk} \quad (8.13)$$

$$\Rightarrow r^l = \frac{(a+b+c)h_1f + (b+c)k - \sqrt{((a+b+c)h_1f + (b+c)k)^2 - 4(a+b+2c)kfbh_1}}{2(a+b+2c)kf} \geq 1$$

$$\text{Since, } r^u > r^l \Rightarrow r^* = 1. \quad (8.14)$$

i-a-2) Let  $n_1 \leq n_{i_1}$

$$n_1 \leq n_{i_1} \Rightarrow h_1(a+b+c) - 2k(a+b+2c) < 0 \Rightarrow f_{i_2}^M > 0$$

There are 2 sub-cases to consider:  $n_1 \leq n_{i_2}$  and  $n_1 > n_{i_2}$ .

$$\text{i-a-2-1) Let } n_1 \leq n_{i_2} = \frac{h_1n_1(n_2+n_3)(m_2-1)}{h_1n_1+m_2(h_2n_2+h_3n_3)}. \quad (8.15)$$

On reversing inequality in (8.11) we get  $h_1 - m_2k \leq 0 \Rightarrow h_1(a+b+c) - k(2a+b+3c) \leq 0$ .

Thus,

$$h_1(a+b+c) \leq k(2a+2b+4c-b-c)$$

$$\Rightarrow h_1(a+b+c) - 2k(a+b+2c) \leq -k(b+c)$$

$$\Rightarrow 1 \geq \frac{-k(b+c)}{h_1(a+b+c) - 2k(a+b+2c)} = f_{i_2}^M. \quad (8.16)$$

Thus,  $0 < f_{i_2}^M \leq 1$ . From (8.9), we have  $0 < f_{i_1}^M < 1$ .

Next we show that  $f_{i_1}^M < f_{i_2}^M$ .

$$\begin{aligned}
& \text{Let } (b(h_1 - k) - kc)^2 + k^2c(a+c) + (a+c)(h_1 - k)^2 > 0 \\
& \Rightarrow b^2(h_1 - k)^2 - 2bkc(h_1 - k) + k^2c^2 + k^2c(a+c) + (a+c)(h_1 - k)^2 > 0 \\
& \Rightarrow b(a+b+c)(h_1 - k)^2 - 2bkc h_1 + bk^2c + k^2c(a+b+2c) > 0 \\
& \Rightarrow bh_1((a+b+c)h_1 - 2k(a+b+2c)) + bk^2(a+b+2c) + k^2c(a+b+2c) > 0 \\
& \Rightarrow bh_1((a+b+c)h_1 - 2k(a+b+2c)) + (b+c)k^2(a+b+2c) > 0 \\
& \Rightarrow \\
& -(b+c)k((a+b+c)h_1 - k(a+b+2c)) < (bh_1 - (b+c)k)((a+b+c)h_1 - 2k(a+b+2c))
\end{aligned}$$

From (8.8), we know that  $h_1(a+b+c) - k(a+b+2c) > 0$  and

$$h_1(a+b+c) - 2k(a+b+2c) < 0.$$

$$\begin{aligned}
& \frac{-(b+c)k}{(a+b+c)h_1 - 2k(a+b+2c)} > \frac{bh_1 - (b+c)k}{(a+b+c)h_1 - k(a+b+2c)} \\
& \Rightarrow f_{i1}^M < f_{i2}^M \tag{8.17}
\end{aligned}$$

Next we show that  $n_{i1} > n_{i2}$ ,

$$\text{Let } (m_1 - 1)h_1n_1 + m_2(m_1 - 1)(h_2n_2 + h_3n_3) > (m_2 - 1)h_1n_1 + (m_2 - 1)m_1(h_2n_2 + h_3n_3)$$

Because,  $m_1 - 1 > m_2 - 1$  and  $(m_1 - 1)m_2 > (m_2 - 1)m_1$

$$\begin{aligned}
& \frac{(m_1 - 1)}{h_1n_1 + m_1(h_2n_2 + h_3n_3)} > \frac{(m_2 - 1)}{h_1n_1 + m_2(h_2n_2 + h_3n_3)} \\
& \frac{h_1n_1(n_2 + n_3)(m_1 - 1)}{h_1n_1 + m_1(h_2n_2 + h_3n_3)} > \frac{h_1n_1(n_2 + n_3)(m_2 - 1)}{h_1n_1 + m_2(h_2n_2 + h_3n_3)}
\end{aligned}$$

Thus,  $n_{i1} > n_{i2}$ . (8.18)

Thus,  $0 < f \leq f_{i1}^M < f_{i2}^M$

$$f < f_{i2}^M = \frac{-k(b+c)}{h_1(a+b+c) - 2k(a+b+2c)}$$

$$f[h_1(a+b+c) - 2k(a+b+2c)] > -k(b+c)$$

$$h_1f(a+b+c) + k(b+c) - 2kf(a+b+2c) > 0$$

The term inside the square bracket in Eq (8.10) is positive, thus, from (8.14)  $r^* = 1$ .

(8.19)

i-a-2-2) Let  $n_1 > n_{i2} \Rightarrow f_{i2}^M > 1$  which follows by reversing the inequality in (8.15).

Thus,  $0 < f \leq f_{i1}^M < 1 < f_{i2}^M$ , from (8.19) we know that  $r^* = 1$ .

Therefore, for each of the three conditions, i.e.  $b > b_i^M, 0 \leq f \leq f_{i1}^M$ , and  $n_1 > n_{i1}$  or

$b > b_i^M, 0 \leq f \leq f_{i1}^M, n_1 \leq n_{i1}$ , and  $n_1 > n_{i2}$  or  $b > b_i^M, 0 \leq f \leq f_{i1}^M, n_1 \leq n_{i1}$ , and  $n_1 \leq n_{i2}$ ,

we get  $r^* = 1$ .

This completes the proof of (i) part a. Now we examine (i) part b. There are two sub-cases.

i-b-1) Note that  $f_{i1}^M < f \leq 1$  and  $n_1 > n_{i1}$  then the direction of the inequality in (8.14) is

reversed. Thus,  $r^* = r^l < 1$ .

(8.20)

From (8.13),

$$(a+b+c)fh_1 + (b+c)k - 2(a+b+2c)fk < \sqrt{((a+b+c)fh_1 + (b+c)k)^2 - 4bh_1(a+b+2c)fk}$$

(8.21)

$$\begin{aligned} & 2((a+b+c)fh_1 + (b+c)k) - 2(a+b+2c)fk \\ & < (a+b+c)fh_1 + (b+c)k + \sqrt{((a+b+c)fh_1 + (b+c)k)^2 - 4bh_1(a+b+2c)fk} \\ & \frac{2((a+b+c)fh_1 + (b+c)k) - 2(a+b+2c)fk}{2(a+b+2c)fk} \\ & < \frac{(a+b+c)fh_1 + (b+c)k + \sqrt{((a+b+c)fh_1 + (b+c)k)^2 - 4bh_1(a+b+2c)fk}}{2(a+b+2c)fk} = r^u \end{aligned}$$

$$\frac{(a+b+c)fh_1 + (b+c)k}{(a+b+2c)fk} - 1 < r^u$$

From (8.12),  $(a+b+c)fh_1 + (b+c)k - 2(a+b+2c)fk > 0$

$$\frac{(a+b+c)fh_1 + (b+c)k}{(a+b+2c)fk} \geq 2$$

$$\frac{(a+b+c)fh_1 + (b+c)k}{(a+b+2c)fk} - 1 \geq 1 \quad (8.22)$$

$$\text{Thus, } 1 \leq \frac{(a+b+c)fh_1 + (b+c)k}{(a+b+2c)fk} - 1 < r^u \Rightarrow 1 < r^u. \quad (8.23)$$

i-b-2) Let  $n_1 \leq n_{i_1}$

$$n_1 \leq n_{i_1} \Rightarrow h_1(a+b+c) - 2k(a+b+2c) < 0 \Rightarrow f_{i_2}^M > 0$$

There are 2 sub-cases to consider:  $n_1 \leq n_{i_2}$  and  $n_1 > n_{i_2}$ .

i-b-2-1)  $n_1 \leq n_{i_2} \Rightarrow f_{i_2}^M < 1$  from (8.16).

Thus,  $0 < f_{i_2}^M < 1$ .

From (8.17)  $f_{i_1}^M < f_{i_2}^M$ .

Thus, either  $f_{i_1}^M \leq f \leq f_{i_2}^M$  or  $f_{i_1}^M < f_{i_2}^M < f \leq 1$  case exist.

If  $f_{i_1}^M \leq f \leq f_{i_2}^M$ , we know from (8.20) and (8.23)  $r^* = r^l < 1$  and  $r^u > 1$ , respectively.

(8.24)

If  $f_{i_1}^M < f_{i_2}^M < f \leq 1$  we show that  $r^* = r^l < 1$  and  $r^u > 1$ . Let

$$f_{i_2}^M = \frac{-k(b+c)}{h_1(a+b+c) - 2k(a+b+2c)} < f$$

$$f[h_1(a+b+c) - 2k(a+b+2c)] < -k(b+c)$$

$$h_1f(a+b+c) + kf(b+c) - 2k(a+b+2c) < 0$$

The term inside the square bracket in Equation (8.10) is negative, thus,

$$-(a+b+c)fh_1 - (b+c)k + 2(a+b+2c)fk < \sqrt{((a+b+c)fh_1 + (b+c)k)^2 - 4bh_1(a+b+2c)fk} \quad (8.25)$$

$$(a+b+c)fh_1 + (b+c)k - \sqrt{((a+b+c)fh_1 + (b+c)k)^2 - 4bh_1(a+b+2c)fk} <$$

$$2((a+b+c)fh_1 + (b+c)k) - 2(a+b+2c)fk$$

$$\frac{(a+b+c)fh_1 + (b+c)k - \sqrt{((a+b+c)fh_1 + (b+c)k)^2 - 4bh_1(a+b+2c)fk}}{2(a+b+2c)fk} < \frac{(a+b+c)fh_1 + (b+c)k}{(a+b+2c)fk} - 1$$

$$r^l < \frac{(a+b+c)fh_1 + (b+c)k}{(a+b+2c)fk} - 1$$

The direction of the inequality in (8.22) is reversed. Thus,

$$\frac{(a+b+c)fh_1+(b+c)k}{(a+b+2c)fk} - 1 < 1.$$

Thus,  $r^* = r^l < 1$ . (8.26)

$$-(a+b+c)fh_1 - (b+c)k + 2(a+b+2c)fk < \sqrt{\left((a+b+c)fh_1 + (b+c)k\right)^2 - 4bh_1(a+b+2c)fk}$$

$$1 < r^u = \frac{(a+b+c)fh_1 + (b+c)k \sqrt{\left((a+b+c)fh_1 + (b+c)k\right)^2 - 4bh_1(a+b+2c)fk}}{2(a+b+2c)fk}$$

Thus,  $r^u > 1$ . (8.27)

i-b-2-2) Let  $n_1 > n_{i2} \Rightarrow f_{i2}^M > 1$  from (8.17), thus,  $f_{i1}^M < f \leq 1$

From (8.24),  $r^* = r^l < 1$  and  $r^u > 1$ .

This completes the proof of part (i) and we now consider part (ii).

ii) Let  $b \leq b_i^M \Rightarrow bh_1 - (b+c)k \leq 0$ .

If  $f = 0$  then from (8.6)  $r^* = r^l = \frac{bh_1}{(b+c)k} < 1$ .

We next show that if  $0 < f \leq 1$  then also  $r^* = r^l < 1$ .

There are two sub-cases to consider:  $n_1 \leq n_{i3}$  and  $n_1 > n_{i3}$ .

ii-1) Let  $n_1 \leq n_{i3} \Rightarrow h_1 - m_3k < 0 \Rightarrow (a+b+c)h_1 - (a+b+2c)k < 0 \Rightarrow f_{i1}^M > 1$ .

$h_1(a+b+c) - (a+b+2c)k < 0 \Rightarrow h_1(a+b+c) - (a+b+2c)2k < 0 \Rightarrow f_{i2}^M > 0$ .

On reversing inequalities in (8.17),  $f_{i2}^M < f_{i1}^M$ , thus,  $0 < f_{i2}^M < 1 < f_{i1}^M$ .

Thus, there are two possibilities either  $0 < f \leq f_{i2}^M$  or  $f_{i2}^M < f \leq 1$ .

If  $0 < f \leq f_{i2}^M$  then the term inside square bracket is positive and from (8.20)  $r^* = r^l < 1$ .

If  $f_{i2}^M < f \leq 1$  then the term inside square bracket is negative, thus, from (8.26)

$r^* = r^l < 1$ .

ii-2) Let  $n_1 > n_{i3} \Rightarrow (a+b+c)h_1 - (a+b+2c)k > 0$ .

Thus,  $f_{i1}^M < 0$  and  $0 < f \leq 1 \Rightarrow f > f_{i1}^M$ .

Recall from (8.18) that  $n_{i1} > n_{i2}$ . There are 3 sub-cases to consider:

ii-2-1) Let  $n_1 > n_{i1} \Rightarrow h_1(a+b+c) - 2k(a+b+2c) > 0 \Rightarrow f_{i2}^M < 0$ . Similar proof as part a-1 of (i).

ii-2-2) Let  $n_{i2} < n_1 \leq n_{i1} \Rightarrow f_{i2}^M > 1$ . Thus, from (8.20),  $r^* = r^l < 1$ .

ii-2-3) Let  $n_1 \leq n_{i2} < n_{i1} \Rightarrow h_1(a+b+c) - 2k(a+b+2c) > 0 \Rightarrow 0 < f_{i2}^M < 1$ . Thus, either  $0 < f \leq f_{i2}^M$  or  $f_{i2}^M < f \leq 1$  exist.

If  $0 < f \leq f_{i2}^M$  then from (8.20),  $r^* = r^l < 1$  and if  $f_{i2}^M < f \leq 1$  then from (8.25),

$$r^* = r^l < 1. \quad \blacksquare$$

#### Proof of Proposition 4.4

Proof: Before proving parts (i) and (ii) of proposition 4, we first show that  $IA_M(r^*(f))$

is an increasing function in  $f$ . Clearly,  $\frac{dIA_M(r^l(f))}{df} = \frac{\partial IA_M(r^l)}{\partial r^l} \times \frac{\partial r^l(f)}{\partial f}$ .

$IA_M(r^*(f))$  is a quadratic function in  $r$  having roots  $\frac{h_1}{k}$  and  $\frac{1}{f}$ .  $IA_M(r^*(f))$  is

decreasing in  $r \in [0, 1]$  since both  $\frac{h_1}{k}$  and  $\frac{1}{f}$  are greater than 1. Before, we show that  $r^l$  is

decreasing in  $f$ , we define some terms:

$$\phi_1 = (a+b+c)h_1$$

$$\phi_2 = (b+c)k$$

$$\phi_3 = 4(a+b+2c)kbh_1$$

$$\phi_4 = 2(a+b+2c)k$$

Therefore,  $r^l = \frac{\phi_1 f + \phi_2 - \sqrt{(\phi_1 f + \phi_2)^2 - f \phi_3}}{f \phi_4}$ .

On simplification,  $\frac{dr^l}{df} = \frac{2\phi_1\phi_2 f + 2\phi_2^2 - f\phi_3 - 2\phi_2\sqrt{(\phi_1 f + \phi_2)^2 - f\phi_3}}{2f^2\phi_4\sqrt{(\phi_1 f + \phi_2)^2 - f\phi_3}}$ , we now show that

$r^l$  is decreasing in  $f$ .

Let  $0 < 4ac + 4c^2$

$$4(a+b+2c)kbh_1 < 4(a+b+c)kh_1(b+c)$$



$$\phi_3 - 4\phi_1\phi_2 < 0$$

$$4f^2\phi_1^2\phi_2^2 + 4\phi_2^4 + f^2\phi_3^2 + 8f\phi_1\phi_2^3 - 4f\phi_1\phi_2\phi_3 - 4f\phi_2^2\phi_3 \\ < 4f^2\phi_1^2\phi_2^2 + 8f\phi_1\phi_2^3 + 4\phi_2^4 - 4f\phi_2^2\phi_3$$

$$(2\phi_1\phi_2f + 2\phi_2^2 - f\phi_3)^2 < \left(2\phi_2\sqrt{(\phi_1f + \phi_2)^2 - f\phi_3}\right)^2$$

$$\frac{dr^l}{df} = \frac{2\phi_1\phi_2f + 2\phi_2^2 - f\phi_3 - 2\phi_2\sqrt{(\phi_1f + \phi_2)^2 - f\phi_3}}{2f^2\phi_4\sqrt{(\phi_1f + \phi_2)^2 - f\phi_3}} < 0.$$

Thus,  $IA_M(r^l(f))$  is an increasing function in  $f$ . (8.28)

(i) For  $0 \leq f \leq f_{t1}^M$ ,  $IA_M(r^* = 1)$  is an increasing function in  $f$ . Thus,  $f^* = 0$

$$\text{and } IA_M(r^* = 1) = B(h_1 - k). \quad (8.29)$$

For  $f_{t1}^M < f \leq 1$ ,  $IA_M(r^* = r^l) = B(1 - r^l f)(h_1 - r^l k)$  is an increasing function in  $f$ , from

$$(8.28). \text{ Thus, } f^* = 1 \text{ and } IA_M(r^* = r^l) = B(1 - r^l)(h_1 - r^l k).$$

We next compare  $IA_M(r^* = 1)$  and  $IA_M(r^* = r^l)$ .

$$\Rightarrow B(h_1 - k) = B(1 - r^l)(h_1 - r^l k)$$

$$\Rightarrow (r^l)^2 k - r^l(h_1 + k) + k = 0$$

The function  $(r^l)^2 k - r^l(h_1 + k) + k$ , is a quadratic function having minimum at

$$\left(\frac{h_1 + k}{2k}, 1 - \left(\frac{h_1 + k}{2k}\right)^2\right) \text{ where } \frac{h_1 + k}{2k} > 1 \text{ and } 1 - \left(\frac{h_1 + k}{2k}\right)^2 < 0. \text{ The graph of the quadratic}$$

function passes through two points  $(k, 0)$  and  $(k - h_1, 1)$ , where  $k - h_1 < 0$ . The roots,

$r_{ii}, i = 1, 2$  are given below,

$$\text{Let } r_{ii} = \frac{(h_1 + k) \pm \sqrt{(h_1 + k)^2 - 4k^2}}{2k}, i = 1, 2$$

where,  $r_{i1} \in [1, \infty]$  and  $r_{i2} \in [0, 1]$ .

We next show that if  $h_1 < h_t$  then  $(r^l)^2 k - r^l(h_1 + k) + k > 0$ .

Let  $h_1 < h_t$

$\Rightarrow$

$$h_1 < \frac{(a+b+2c)\sqrt{(h_1+k)^2-4k^2} - \sqrt{((a+b+c)h_1+(b+c)k)^2-4(a+b+2c)kbh_1} - k(a+c)}{c}$$

$\Rightarrow$

$$-\sqrt{((a+b+c)h_1+(b+c)k)^2-4(a+b+2c)kbh_1} > h_1c + k(a+c) - (a+b+2c)\sqrt{(h_1+k)^2-4k^2}$$

Rearranging and dividing by  $2k$ ,

$$\frac{(a+b+c)h_1+(b+c)k - \sqrt{((a+b+c)h_1+(b+c)k)^2-4(a+b+2c)kbh_1}}{2(a+b+2c)k} > \frac{(h_1+k) - \sqrt{(h_1+k)^2-4k^2}}{2k}$$

$$\Rightarrow r^l > r_{i2} \text{ and thus, } (r^l)^2 k - r^l(h_1+k) + k > 0.$$

$$\Rightarrow B(h_1 - k) > B(1 - r^l)(h_1 - r^l k)$$

$$\Rightarrow IA_M(r^* = 1) > IA_M(r^* = r^l).$$

Thus, from (8.29)  $f^* = 0$  is optimal. Similarly, if  $h_1 > h_t$  then  $(r^l)^2 k - r^l(h_1+k) + k < 0$ ,

thus,  $f^* = 1$  is optimal.

(ii) Note that if  $b \leq b_t^M$  then  $r^* = r^l < 1$  and from (8.28),  $IA_M(r^l(f))$  is an increasing

function in  $f$ , thus,  $f^* = 1$ . ■

## 9. Proof of Chapter 5

### Lower-level

#### Case 1 (Linear Utility Function)

From Proposition 4.1 in Chapter 4, we can show that for problem  $L_{1L}$  with two sub-populations:

(i)  $U_L(r_{1j})$ ,  $j = H, L$  is a convex function of  $r_{1j}$  and therefore the optimal solution is either  $r_{1j}^* = 0$  or 1.

(ii) For a high type LD:

(a) If  $f \leq f_{t1}^{bL}$  then  $r_{1H}^* = 1$ .

(b) If  $f > f_{t1}^{bL}$  then  $r_{1H}^* = 0$ .

(iii) For a low type LD:

(a) If  $f \leq f_{t2}^{bL}$  then  $r_{1L}^* = 1$ .

(b) If  $f > f_{t2}^{bL}$  then  $r_{1L}^* = 0$ . ■

From Corollary 4.1 in Chapter 4 we get,

When  $b^j > b_{tt}^{bL}$ ,  $j = H, L$  then  $r_{1j}^* = 1$  and when  $b^j \leq b_{tt}^{bL}$  then  $r_{1j}^* = 0$ . ■

From the conditions listed above we get Table 5.3. In Table 5.3, if  $b^j \leq b_{tt}^{bL}$ ,  $j = H, L$  then any  $f^* \in [0, 1]$ , resulting in  $r_{1j}^* = 0$  and  $E[IA] = Bh_1$ . Similarly, under other conditions given in Table 5.3, optimal solution can or cannot exist. The reason that an optimal solution cannot exist is given in the footnote.

#### Case 1 (Multiplicative Utility Function)

From Proposition 4.3 and Corollary 4.2 in Chapter 4, we can show that for problem  $L_{1M}$

with two sub-populations,  $r_{1j}^* = \min\{r_{1j}^l, 1\}$ .

For a high type LD:

i) If  $b^H > b_{t1}^{bM}$  then

a) If  $0 \leq f < f_{t1}^{bM}$  then  $r_{1H}^* = 1$ .

b) If  $f_{t1}^{bM} \leq f \leq 1$  then  $r_{1H}^* = r_{1H}^l < 1$ .

ii) If  $b^H \leq b_{t1}^{bM}$  then  $r_{1H}^* = r_{1H}^l < 1$ .

For a low type LD:

i) If  $b^L > b_{t1}^{bM}$  then

a) If  $0 \leq f < f_{t2}^{bM}$  then  $r_{1L}^* = 1$ .

b) If  $f_{t2}^{bM} \leq f \leq 1$  then  $r_{1L}^* = r_{1L}^l < 1$ .

ii) If  $b^L \leq b_{t1}^{bM}$  then  $r_{1L}^* = r_{1L}^l < 1$ . ■

From the conditions listed above we get Table 5.4. In Table 5.4, if  $b^j \leq b_{t1}^{bM}$ ,  $j = H, L$

then  $f^* = 1$  is optimal, resulting in  $r_{1j}^* = r_{1j}^l < 1$  and  $E[IA] = B(1 - r_{1H}^l)(h_1 - r_{1H}^l k)p + B(1 - r_{1L}^l)(h_1 - r_{1L}^l k)q$ , which can be written as below:

$$\begin{aligned} E[IA] &= 0, \text{ if } f \in [0, f_{t2}^{bL}) \\ &= B(1 - r_{1H}^l f)(h_1 - r_{1H}^l k)p + B(1 - r_{1L}^l f)(h_1 - r_{1L}^l k)q, \text{ if } f \in [f_{t2}^{bL}, f_{t1}^{bL}), \text{ where} \\ &\quad f_{t2}^{bL} < f_{t1}^{bL} \\ &= B(1 - r_{1H}^l f)(h_1 - r_{1H}^l k), \text{ if } f \in [f_{t1}^{bL}, 1] \end{aligned}$$

A convex combination of  $B(1 - r_{1H}^l f)(h_1 - r_{1H}^l k)$  and  $B(1 - r_{1L}^l f)(h_1 - r_{1L}^l k)$  is a convex function.  $B(1 - r_{1H}^l f)(h_1 - r_{1H}^l k)$  is an increasing function in  $f$ , therefore,  $f^* = 1$  is optimal. Similarly, under other conditions given in Table 5.4, optimal solution can or cannot exist. We next solve one of the cases specified in Table 5.4 and other cases can be solved in a similar manner.

If  $b^j > b_{t1}^{bM}$ ,  $j = H, L$  then

- $f^* = 0$  is optimal, resulting in  $E[IA] = B(h_1 - k)$ .
- $f^* = 1$  is optimal, resulting in

$$E[IA] = B(1 - r_{1H}^l f)(h_1 - r_{1H}^l k)p + B(1 - r_{1L}^l f)(h_1 - r_{1L}^l k)q.$$

We next show that, if  $b^j > b_{t1}^{bM}$ ,  $j = H, L$ ,  $p > p_{t1}^{bM}$  then  $f^* = 1$  is optimal, resulting in

$$E[IA] = B(1 - r_{1H}^l f)(h_1 - r_{1H}^l k)p + B(1 - r_{1L}^l f)(h_1 - r_{1L}^l k)q.$$

Let  $p > p_{i1}^{bM} = \frac{(h_1 - k) - (1 - r_{1L}^l)(h_1 - r_{1L}^l k)}{(1 - r_{1H}^l)(h_1 - r_{1H}^l k) - (1 - r_{1L}^l)(h_1 - r_{1L}^l k)}$ , where

$(1 - r_{1H}^l)(h_1 - r_{1H}^l k) - (1 - r_{1L}^l)(h_1 - r_{1L}^l k) > 0$  since  $h_1 > k \frac{r_{1L}^l(1 - r_{1L}^l) - r_{1H}^l(1 - r_{1H}^l)}{(r_{1H}^l - r_{1L}^l)}$ , where

$k \frac{r_{1L}^l(1 - r_{1L}^l) - r_{1H}^l(1 - r_{1H}^l)}{(r_{1H}^l - r_{1L}^l)}$  is an infinitesimally small quantity than  $h_1$ .

$$p > \frac{(h_1 - k) - (1 - r_{1L}^l)(h_1 - r_{1L}^l k)}{(1 - r_{1H}^l)(h_1 - r_{1H}^l k) - (1 - r_{1L}^l)(h_1 - r_{1L}^l k)}$$

This inequality is rearranged as follows.

$$\Rightarrow E[IA] = B(1 - r_{1H}^l)(h_1 - r_{1H}^l k)p + B(1 - r_{1L}^l)(h_1 - r_{1L}^l k)q > E[IA] = B(h_1 - k)$$

Therefore, if  $b^j > b_{i1}^{bM}$ ,  $j = H, L$ ,  $p > p_{i1}^{bM}$  then  $f^* = 1$  is optimal, resulting in

$$E[IA] = B(1 - r_{1H}^l)(h_1 - r_{1H}^l k)p + B(1 - r_{1L}^l)(h_1 - r_{1L}^l k)q.$$

### Case 2 (Linear Utility Function)

At the program level, if  $h_1 \leq h_2$  then  $E[IA] = B(1 - r_{2H}^l f)(h_2^H - r_{2H}^l k_2)$  and if  $h_1 > h_2$  then

$E[IA] = B(1 - r_{2L}^l f)(h_1 - r_{2L}^l k_1)$ . From Proposition 4.1, we can show that for problem

$L_{2L}$  with two sub-populations:

(i)  $U_L(r_{2j}^*)$ ,  $j = L, M$  is a convex function of  $r_{2j}$  and the optimal solution is either  $r_{2j}^* = 0$  or 1.

(ii) For a high type LD:

a) If  $f \leq f_{i1}^{HL}$  then  $r_{2H}^* = 1$ .

b) If  $f > f_{i1}^{HL}$  then  $r_{2H}^* = 0$ .

(iii) For a low type LD:

a) If  $f \leq f_{i2}^{HL}$  then  $r_{2L}^* = 1$ .

b) If  $f > f_{i2}^{HL}$  then  $r_{2L}^* = 0$ . ■

From Corollary 4.1 we can show that:

(i) For a high type LD:

a) If  $h_2^H \leq h_{t2}^{hL}$  and  $b > b_t^{hL}$  then  $r_{2H}^* = 1$ .

b) If  $h_2^H > h_{t1}^{hL}$  then  $r_{2H}^* = 0$ .

(ii) For a low type LD:

a) If  $h_1 \leq h_{t2}^{hL}$  and  $b > b_t^{hL}$  then  $r_{2L}^* = 1$ .

b) If  $h_2^L \leq h_{t3}^{hL}$ , and  $c > c_t^{hL}$  then  $r_{2L}^* = 0$ . ■

We next show that if  $h_1 \leq h_{t2}^{hL}$  and  $b > b_t^{hL}$  then  $r_{2L}^* = 1$ .

Let  $h_1 \leq h_{t2}^{hL}$ , where  $h_{t2}^{hL} > 0$  since  $b > b_t^{hL}$ .

$$h_1 \leq h_{t2}^{hL} = \left( \frac{b}{B} - a \right) \frac{1}{c} \Rightarrow b > B(a + ch_1) \Rightarrow b - cBk_1 > B(a + ch_1 - ck_1) \Rightarrow$$

$$\frac{b - cBk_1}{B(a + c(h_1 - k_1))} = f_{t2}^{hL} > 1. \text{ We know that } f \leq f_{t2}^{hL} \Rightarrow r_{2L}^* = 1. \text{ Similarly we obtain other}$$

conditions.

From the conditions listed above we get Table 5.5. Under conditions given in Table 5.5, optimal solution can or cannot exist. The reasons are provided in the footnote.

### Case 2 (Multiplicative Utility Function)

From Proposition 4.3 and Corollary 4.2, we can show that for problem  $L_{2M}$  with two

sub-populations,  $r_{2j}^* = \min\{r_{2j}^l, 1\}$ .

For a high type LD:

i) If  $h_2^H > h_{t1}^{hM}$  then

a) If  $0 \leq f < f_{t1}^{bM}$  then  $r_{2H}^* = 1$ .

b) If  $f_{t1}^{bM} \leq f \leq 1$  then  $r_{2H}^* = r_{2H}^l < 1$ .

ii) If  $h_2^H \leq h_{t1}^{hM}$  then  $r_{2H}^* = r_{2H}^l < 1$ .

For a low type LD:

i) If  $h_2^L > h_{t2}^{hM}$  then

a) If  $0 \leq f < f_{t2}^{bM}$  then  $r_{2L}^* = 1$ .

b) If  $f_{t2}^{bM} \leq f \leq 1$  then  $r_{2L}^* = r_{2L}^l < 1$ .

ii) If  $h_2^L \leq h_{t2}^{hM}$  then  $r_{2L}^* = r_{2L}^l < 1$ . ■

From the conditions listed above we get Table 5.6. For each condition specified in table 5.6, optimal solutions can or cannot exist. The reasons are provided in the footnote.

### Upper-level

To calculate the loss of efficiency (LOE) in the asymmetric information case, first of all, based on the optimal  $f^*$  value the optimal  $r^*$  and  $IA^*$  can be obtained from condition specified at the lower level for each type of LD. Thus,  $E[IA^A]$  can be calculated for each optimal value of  $f^*$ , where superscript A presents asymmetric information case.

We calculate  $E[IA^F]$  for the full information case from Chapter 4. Thus,

$$LOE = E[IA^F] - E[IA^A].$$

Table 5.4: Upper-level problem for linear utility function at the lower level when  $b$  is unknown.

	$b^L \leq b_{it}^{bL}$	$b^L > b_{ut}^{bL}$	$b_{it}^{bL} < b^L \leq b_{ut}^{bL}$
$b^H \leq b_{it}^{bL}$	$E[IA] = Bh_1,$ $f^* \in [0, 1],$ $r_{1j}^* = 0, j = H, L,$ LOE = 0	Cannot exist <sup>1</sup>	Cannot exist <sup>1</sup>
$b^H > b_{ut}^{bL}$	$E[IA] = B(h_1 - k)p + Bh_1q,$ $f^* = 0,$ $r_{1H}^* = 1, r_{1L}^* = 0,$ LOE = 0	$E[IA] = B(h_1 - k),$ $f^* = 0,$ $r_{1j}^* = 1, j = H, L,$ LOE = 0	If $p > p_i^{bL}$ then $E[IA] = B(h_1 - k),$ $f^* = 0,$ $r_{1j}^* = 1, j = H, L,$ If $p \leq p_i^{bL}$ then $E[IA] = B(1 - f_{i2}^{bL})(h_1 - k)p + Bh_1q,$ $f^* = f_{i2}^{bL},$ $r_{1H}^* = 1, r_{1L}^* = 0,$ LOE $\geq 0$
$b_{it}^{bL} < b^H \leq b_{ut}^{bL}$	$E[IA] = Bh_1,$ $f^* \in [f_{i1}^{bL}, 1],$ $r_{1j}^* = 0, j = H, L,$ <sup>2</sup> LOE = 0	Cannot exist <sup>1</sup>	$E[IA] = Bh_1,$ $f^* \in [\max\{f_{i1}^{bL}, f_{i2}^{bL}\}, 1],$ $r_{1j}^* = 0, j = H, L,$ <sup>3</sup> LOE = 0

<sup>1</sup>Cannot exist because we have assumed  $b^H > b^L$ , which implies  $b_{ut}^{bL} > b_{it}^{bL}$ .

<sup>2</sup>Other case is not considered because if  $b^L \leq b_{it}^{bL}$  and  $b_{it}^{bL} < b^H \leq b_{ut}^{bL}$  then  $E[IA]$  when  $f \geq f_{i1}^{bL}$  is higher than  $E[IA]$  when  $f < f_{i1}^{bL}$ .

<sup>3</sup>Other 3 cases are not considered because 1)  $f_{i1}^{bL} > f_{i2}^{bL}$ , 2) if  $b_{it}^{bL} < b^L \leq b_{ut}^{bL}, b_{it}^{bL} < b^H \leq b_{ut}^{bL}$  and  $f \geq f_{i2}^{bL}$  then  $E[IA]$  when  $f \geq f_{i2}^{bL}$  is higher than  $E(IA)$  when  $f < f_{i2}^{bL}$ , and 3) if  $b_{it}^{bL} < b^L \leq b_{ut}^{bL}, b_{it}^{bL} < b^H \leq b_{ut}^{bL}$  and  $f < f_{i1}^{bL}$  then  $E[IA]$  when  $f \geq f_{i1}^{bL}$  is higher than  $E[IA]$  when  $f < f_{i1}^{bL}$ , respectively.



Table 5.5: Upper level problem for the multiplicative utility function at the lower level when  $b$  is unknown.

	$b^L > b_{i1}^{bM}$	$b^L \leq b_{i1}^{bM}$
$b^H > b_{i1}^{bM}$	<p>If <math>p &gt; p_{i1}^{bM}</math> then</p> $E[IA] = B(1 - r_{iH}^l)(h_1 - r_{iH}^l k) p + B(1 - r_{iL}^l)(h_1 - r_{iL}^l k) q,$ $f^* = 1,$ $r_{ij}^* = r_{ij}^l < 1, j = H, L.$ <p>If <math>p \leq p_{i1}^{bM}</math> then</p> $E[IA] = B(h_1 - k),$ $f^* = 0,$ $r_{ij}^* = 1, j = H, L,$ $\text{LOE} \geq 0.$	<p>If <math>p &gt; p_{i2}^{bM}</math> then</p> $E[IA] = B(1 - r_{iH}^l)(h_1 - r_{iH}^l k) p + B(1 - r_{iL}^l)(h_1 - r_{iL}^l k) q,$ $f^* = 1,$ $r_{ij}^* = r_{ij}^l < 1, j = H, L.$ <p>If <math>p \leq p_{i2}^{bM}</math> then</p> $E[IA] = B(h_1 - k) p + B \frac{h_1 c}{b^L + c} q,$ $f^* = 0,$ $r_{iH}^* = 1, r_{iL}^* = r_{iL}^l < 1,$ $\text{LOE} \geq 0.$
$b^H \leq b_{i1}^{bM}$	Cannot exist. <sup>4</sup>	$E[IA] = B(1 - r_{iH}^l)(h_1 - r_{iH}^l k) p + B(1 - r_{iL}^l)(h_1 - r_{iL}^l k) q,$ $f^* = 1,$ $r_{ij}^* = r_{ij}^l < 1, j = H, L,$ $\text{LOE} = 0$

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<sup>4</sup> Cannot exist because  $b^H > b^L$ .

Table 5.6: Upper level problem for linear utility function at the lower level when  $h$  is unknown.

	$h_2^L \leq h_{t_3}^{hL}$	$h_1 \leq h_{t_2}^{hL}$	$h_2^L > h_{t_3}^{hL}, h_1 > h_{t_2}^{hL}$
$h_2^H > h_{t_1}^{hL}$	$E[IA] = Bh_2^H p + Bh_1 q,$ $f^* \in [0, 1],$ $r_{2j}^* = 0, j = H, L,$ LOE = 0.	$E[IA] = Bh_2^H p + B(h_1 - k_1) q,$ $f^* = 0,$ $r_{2H}^* = 0, r_{2L}^* = 1,$ LOE = 0.	$E[IA] = Bh_2^H p + Bh_1 q,$ $f^* \in [f_{t_2}^{hL}, 1],$ $r_{2j}^* = 0, j = H, L,$ <sup>5</sup> LOE = 0.
$h_2^H \leq h_{t_2}^{hL}$	Cannot exist <sup>6</sup>	$E[IA] = B(h_2^H - k_2) p + B(h_1 - k_1) q,$ $f^* = 0,$ $r_{2j}^* = 1, j = H, L,$ LOE = 0.	Cannot exist <sup>6</sup>
$h_{t_2}^{hL} < h_2^H \leq h_{t_1}^{hL}$	$E[IA] = Bh_2^H p + Bh_1 q,$ $f^* \in [f_{t_1}^{hL}, 1],$ $r_{2j}^* = 0, j = H, L,$ <sup>7</sup> LOE = 0.	If $p > p_t^{hL}$ then $E[IA] = Bh_2^H p + B(1 - f_{t_1}^{hL})(h_1 - k_1) q,$ $f^* = f_{t_1}^{hL}, r_{2H}^* = 0, r_{2L}^* = 1.$ If $p \leq p_t^{hL}$ then $E[IA] = B(h_2^H - k_2) p + B(h_1 - k_1) q,$ $f^* = 0, r_{2j}^* = 1, j = H, L,$ LOE $\geq 0.$	$E[IA] = Bh_2^H p + Bh_1 q,$ $f^* \geq [\max\{f_{t_1}^{hL}, f_{t_2}^{hL}\}, 1],$ $r_{2j}^* = 0, j = H, L,$ <sup>8</sup> LOE = 0.

<sup>5</sup>Not considered because if  $h_2^H > h_{t_1}^{hL}, h_2^L > h_{t_3}^{hL}$ , and  $h_1 > h_{t_2}^{hL}$  then  $E[IA]$  when  $f \geq f_{t_2}^{hL}$  is higher than  $E[IA]$  when  $f < f_{t_2}^{hL}$ .

<sup>6</sup>Cannot exist because  $h_2^H > h_1$ .

<sup>7</sup>Not considered because if  $h_{t_2}^{hL} < h_2^H \leq h_{t_1}^{hL}, h_2^L > h_{t_3}^{hL}, h_1 > h_{t_2}^{hL}$  and  $f \geq f_{t_1}^{hL}$  then  $E[IA]$  when  $f \geq f_{t_2}^{hL}$  is higher than  $E[IA]$  when  $f < f_{t_2}^{hL}$ .

<sup>8</sup>Other 3 cases are not considered because 1) if  $h_{t_2}^{hL} < h_2^H \leq h_{t_1}^{hL}, h_2^L > h_{t_3}^{hL}, h_1 > h_{t_2}^{hL}$  and  $f \geq f_{t_1}^{hL}$  then  $E[IA]$  when  $f \geq f_{t_2}^{hL}$  is higher than  $E[IA]$  when  $f < f_{t_2}^{hL}$ , 2) if  $h_{t_2}^{hL} < h_2^H \leq h_{t_1}^{hL}, h_2^L > h_{t_3}^{hL}, h_1 > h_{t_2}^{hL}$  and  $f \geq f_{t_1}^{hL}$  then  $E[IA]$  when  $f \geq f_{t_2}^{hL}$  is higher than  $E[IA]$  when  $f < f_{t_2}^{hL}$ , and 3) if  $h_{t_2}^{hL} < h_2^H \leq h_{t_1}^{hL}, h_2^L > h_{t_3}^{hL}, h_1 > h_{t_2}^{hL}$  and  $f \geq f_{t_1}^{hL}$  then  $E[IA]$  when  $f \geq f_{t_2}^{hL}$  is higher than  $E[IA]$  when  $f < f_{t_2}^{hL}$ , respectively.

Table 5.7: Upper level problem for the multiplicative utility function at the lower level when  $h$  is unknown.

	$h_2^L > h_{i2}^{hM}$	$h_2^L \leq h_{i2}^{hM}$
$h_2^H > h_{i1}^{hM}$	<p>If <math>p &gt; p_{i1}^{hM}</math> then</p> $E[IA] = B(1 - r_{2H}^l)(h_2^H - r_{2H}^l k_2)p + B(1 - r_{2L}^l)(h_1 - r_{2L}^l k_1)q,$ $f^* = 1,$ $r_{2j}^* = r_{2j}^l < 1, j = H, L.$ <p>If <math>p \leq p_{i1}^{hM}</math> then</p> $E[IA] = B(h_2^H - k_2)p + B(h_1 - k_1)q,$ $f^* = 0,$ $r_{2j}^* = 1, j = H, L,$ $\text{LOE} \geq 0.$	<p>If <math>p &gt; p_{i2}^{hM}</math> then</p> $E[IA] = B(1 - r_{2H}^l)(h_2^H - r_{2H}^l k_2)p +$ $B(1 - r_{2L}^l)(h_1 - r_{2L}^l k_1)q,$ $f^* = 1,$ $r_{2j}^* = r_{2j}^l < 1, j = H, L.$ <p>If <math>p \leq p_{i2}^{hM}</math> then</p> $E[IA] = B(h_2^H - k_2)p + B \frac{ch_1}{(b+c)} q,$ $f^* = 0,$ $r_{2H}^* = 1, r_{2L}^* = r_{2L}^l < 1,$ $\text{LOE} \geq 0.$
$h_2^H \leq h_{i1}^{hM}$	<p>If <math>p &gt; p_{i3}^{hM}</math> then</p> $E[IA] = B(1 - r_{2H}^l)(h_2^H - r_{2H}^l k_2)p + B(1 - r_{2L}^l)(h_1 - r_{2L}^l k_1)q,$ $f^* = 1,$ $r_{2j}^* = r_{2j}^l < 1, j = H, L.$ <p>If <math>p \leq p_{i3}^{hM}</math> then</p> $E[IA] = B \frac{ch_2^H}{(b+c)} p + B(h_1 - k_1)q,$ $f^* = 0,$ $r_{2H}^* = r_{2H}^l, r_{2L}^* = 1,$ $\text{LOE} \geq 0.$	$E[IA] = B(1 - r_{2H}^l)(h_2^H - r_{2H}^l k_2)p +$ $B(1 - r_{2L}^l)(h_1 - r_{2L}^l k_1)q,$ $f^* = 1,$ $r_{2j}^* = r_{2j}^l < 1, j = H, L,$ $\text{LOE} = 0.$

## 10. Proof of Chapter 6

### Two Incentives (“fg” Case)

#### Proof for Proposition 6.1:

(i) The lower-level utility function is  $a_i Z_i + b_i r_i^{fg} + c_i I A_i, i = 1, 2$ . When

$$Z_i = B \left[ \frac{(1 - r_i^{fg} + r_{-i}^{fg})(1 - f)(1 - r_i^{fg} g)}{2} + \frac{f N_i}{N_i + N_{-i}} \right], i, -i = 1, 2, i \neq -i, \text{ and}$$

$I A_i = Z_i (h_{i1} - r_i^{fg} k_i)$  are substituted into this we obtain:  $U_i = a_i Z_i + b_i r_i^{fg} + c_i I A_i$

$$= (a_i + c_i (h_{i1} - r_i^{fg} k_i)) B \left[ \frac{(1 - r_i^{fg} + r_{-i}^{fg})(1 - f)(1 - r_i^{fg} g)}{2} + \frac{f N_i}{N_i + N_{-i}} \right] + b_i r_i^{fg}$$

$$= a_{ci} (r_i^{fg})^3 + b_{ci} (r_i^{fg})^2 + c_{ci} r_i^{fg} + d_{ci}$$

where,  $a_{ci} = -\frac{(1-f)}{2} B c_i k_i g < 0, b_{ci} = \frac{(1-f)}{2} B (c_i k_i (1 + g (1 + r_{-i}^{fg})) + g (a_i + c_i h_{i1})) > 0,$

$$c_{ci} = b_i - \frac{(1-f)}{2} B \left( (1 + g (1 + r_{-i}^{fg})) (a_i + c_i h_{i1}) + c_i k_i (1 + r_{-i}^{fg}) \right) - \frac{f B c_i k_i N_i}{(N_i + N_{-i})},$$

$$d_{ci} = \left( \frac{(1-f)}{2} (1 + r_{-i}^{fg}) + \frac{f N_i}{N_i + N_{-i}} \right) (a_i + c_i h_{i1}) B > 0.$$

The first derivative of  $U_i$  is,

$$U_i' = 3a_{ci} (r_i^{fg})^2 + 2b_{ci} r_i^{fg} + c_{ci} \quad (10.1)$$

Setting,  $U_i'(r_i) = 0$  gives two roots,

$$r_i^{fg^{gu}} = \frac{-2b_{ci} + \sqrt{4b_{ci}^2 - 12a_{ci}c_{ci}}}{6a_{ci}}, r_i^{fg^{gl}} = \frac{-2b_{ci} - \sqrt{4b_{ci}^2 - 12a_{ci}c_{ci}}}{6a_{ci}} \text{ of the quadratic equation}$$

(10.1). If  $c_{ci} > 0$  then  $r_i^{fg^{gu}} > 0$  and  $r_i^{fg^{gl}} < 0$ . If  $c_{ci} \leq 0$  then  $r_i^{fg^{gu}} > r_i^{fg^{gl}} > 0$  and if  $c_{ci} \leq 0$  then imaginary roots exist resulting in  $r_i^{fg^*} = 0$ . Thus, if  $c_{ci} > 0$  then  $r_i^{fg^*} = \min \{ r_i^{fg^{gu}}, 1 \}$ , if  $b_i \geq b_{i1}^{fg}$  then  $c_{ci} \leq 0$  resulting in  $r_i^{fg^*} = r_i^{fg^{gu}}$  or 0 or 1 and if  $b < b_{i1}^{fg}$  then  $c_{ci} \leq 0$  resulting in  $r_i^{fg^*} = 0$ . ■

We next show that if  $b_i \geq b_{i1}^{fg}$  then  $4b_{ci}^2 - 12a_{ci}c_{ci} > 0$  and if  $b < b_{i1}^{fg}$  then  $4b_{ci}^2 - 12a_{ci}c_{ci} \leq 0$ .

$$\begin{aligned} \text{Let : } 4b_{ci}^2 - 12a_{ci}c_{ci} &= 4 \frac{(1-f)^2 B^2}{4} \left( g(a_i + c_i h_{i1}) + c_i k_i (1 + g(1 + r_{-i}^{fg})) \right)^2 - \\ &12g \frac{(1-f)}{2} B c_i k_i \left( -b_i + \frac{(1-f)}{2} B \left( (a_i + c_i h_{i1}) (1 + g(1 + r_{-i}^{fg})) + c_i k_i (1 + r_{-i}^{fg}) \right) + \frac{c_i k_i B f N_i}{N_i + N_{-i}} \right) \\ &= (1-f)^2 B^2 \left( g(a_i + c_i h_{i1}) + c_i k_i (1 + g(1 + r_{-i}^{fg})) \right)^2 - \\ &- b_i 6g(1-f) B c_i k_i + 6g(1-f) B c_i k_i \left( \frac{(1-f)}{2} B \left( (a_i + c_i h_{i1}) (1 + g(1 + r_{-i}^{fg})) + c_i k_i (1 + r_{-i}^{fg}) \right) + \frac{c_i k_i B f N_i}{N_i + N_{-i}} \right) \end{aligned}$$

Thus, if  $b_i \geq b_{i1}^{fg}$  then  $4b_{ci}^2 - 12a_{ci}c_{ci} > 0$  and if  $b < b_{i1}^{fg}$  then  $4b_{ci}^2 - 12a_{ci}c_{ci} \leq 0$ . (10.2)

(ii) To obtain the conditions listed in Table 2, we show that if  $r_{-i}^{fg} < r_{t-i}^{fg}$ ,

$$g > \max \left\{ g_{i1}^{fg}, g_{i2}^{fg} \right\}, f_{i1}^{fg} \leq f \leq f_{i2}^{fg}, \text{ and } b_i > \max \left\{ \frac{B c_i k_i N_i}{2(N_i + N_{-i})}, \frac{B c_i k_i}{3} \right\}, \text{ then } r_i^{fg} \geq 1.$$

First of all, we show that, if  $r_{-i}^{fg} < r_{t-i}^{fg}$ ,  $g > g_{i1}^{fg}$ ,  $f \geq f_{i1}^{fg}$ , and  $b_i > \frac{B c_i k_i N_i}{2(N_i + N_{-i})}$  then

$$c_{ci} > 0.$$

$$\text{Let } r_{-i}^{fg} < r_{t-i}^{fg} = \frac{\frac{2b_i}{(1-f)B} - (a_i + c_i h_{i1})(1-g) - \frac{c_i k_i f N_i}{(1-f)(N_i + N_{-i})} - c_i k_i}{c_i k_i + (a_i + c_i h_{i1})g}, \text{ where}$$

$$\frac{2b_i}{(1-f)B} - (a_i + c_i h_{i1})(1-g) - \frac{c_i k_i f N_i}{(1-f)(N_i + N_{-i})} - c_i k_i > 0 \text{ since } g > g_{i1}^{fg}, g_{i1}^{fg} \leq 1 \text{ since}$$

$$f \geq f_{i1}^{fg} \text{ and } f_{i1}^{fg} < 1 \text{ since } b_i > \frac{B c_i k_i N_i}{2(N_i + N_{-i})}.$$

$$\Rightarrow r_{-i}^{fg} < \frac{b_i - (1-f) \frac{B}{2} (a_i + c_i h_{i1})(1-g) - c_i k_i \left( \frac{f B N_i}{(N_i + N_{-i})} - (1-f) \frac{B}{2} \right)}{(1-f) \frac{B}{2} (c_i k_i + (a_i + c_i h_{i1})g)}$$

This inequality is rearranged as follows.

$$\Rightarrow c_{ci} = b_i - \frac{(1-f)}{2} B \left( (1 + g(1 + r_{-i}^{fg})) (a_i + c_i h_{i1}) + c_i k_i (1 + r_{-i}^{fg}) \right) - \frac{f B c_i k_i N_i}{(N_i + N_{-i})} > 0.$$

We next show that  $r_{i-1}^{fg} < r_{i-2}^{fg}$  since

$$\begin{aligned} & \frac{b_i}{(1-f)B} - \frac{(a_i + c_i(h_{i1} - k_i))}{2}(1-g) - \frac{c_i k_i f N_i}{(1-f)(N_i + N_{-i})} > \\ & \frac{2b_i}{(1-f)B} - (a_i + c_i h_{i1})(1-g) - \frac{c_i k_i f N_i}{(1-f)(N_i + N_{-i})} - c_i k_i \text{ and} \\ & \frac{(a_i + c_i(h_{i1} - k_i))}{2} g + \frac{c_i k_i}{2}(1-g) < c_i k_i + (a_i + c_i h_{i1}) g. \end{aligned}$$

$$r_{i-2}^{fg} = \frac{\frac{2b_i}{(1-f)B} - (a_i + c_i(h_{i1} - k_i))(1-g) - \frac{2c_i k_i f N_i}{(1-f)(N_i + N_{-i})}}{(a_i + c_i(h_{i1} - k_i))g + c_i k_i(1-g)} > 0, \text{ because}$$

$$\frac{2b_i}{(1-f)B} - (a_i + c_i(h_{i1} - k_i))(1-g) - \frac{2c_i k_i f N_i}{(1-f)(N_i + N_{-i})} > 0 \text{ since } g > g_{ii2}^{fg}, g_{ii2}^{fg} \leq 1 \text{ since}$$

$$f \leq f_{ii2}^{fg}, \text{ where, clearly } f_{ii2}^{fg} > 0.$$

$$r_{-i}^{fg} \leq r_{i-2}^{fg} \Rightarrow$$

$$r_{-i}^{fg} \left( \frac{(a_i + c_i(h_{i1} - k_i))}{2} g + \frac{c_i k_i}{2}(1-g) \right) (1-f)B \leq b_i - (1-f)B \frac{(a_i + c_i(h_{i1} - k_i))}{2}(1-g) - \frac{Bc_i k_i f N_i}{(N_i + N_{-i})}$$

This inequality is rearranged as follows.

$$\begin{aligned} & \Rightarrow 3 \frac{(1-f)}{2} B c_i k_i g - 2 \frac{(1-f)}{2} B (c_i k_i (1+g(1+r_{-i}^{fg})) + g(a_i + c_i h_{i1})) \\ & \leq b_i - \frac{(1-f)}{2} B \left( (1+g(1+r_{-i}^{fg})) (a_i + c_i h_{i1}) + c_i k_i (1+r_{-i}^{fg}) \right) - \frac{f B c_i k_i N_i}{(N_i + N_{-i})} \end{aligned}$$

$$\Rightarrow 3(-a_{ci}) - 2b_{ci} \leq c_{ci}$$

$$\Rightarrow b_{ci}^2 + 9(-a_{ci})^2 - 6(-a_{ci})b_{ci} \leq b_{ci}^2 + 3(-a_{ci})c_{ci}, \text{ since } a_{ci} < 0$$

$$\Rightarrow (-b_{ci})^2 + 9(-a_{ci})^2 + 6(-a_{ci})(-b_{ci}) \leq b_{ci}^2 + 3(-a_{ci})c_{ci}, \text{ since } b_{ci} > 0$$

$$\Rightarrow ((-b_{ci}) + 3(-a_{ci}))^2 \leq b_{ci}^2 - 3a_{ci}c_{ci}$$

Since  $a_{ci} < 0$  and  $b_{ci} > 0$ , if  $c_{ci} > 0$  then  $b_{ci}^2 - 3a_{ci}c_{ci} > 0$  and if  $c_{ci} \leq 0$  then from Eq. (10.2)

$$b_{ci}^2 - 3a_{ci}c_{ci} > 0 \text{ if } b_{ci} \geq b_{ii1}^{fg}.$$

$$\Rightarrow -b_{ci} + 3(-a_{ci}) \leq \sqrt{b_{ci}^2 - 3a_{ci}c_{ci}}$$

$$\Rightarrow 1 \leq r_i^{fgu} = \frac{-b_{ci} - \sqrt{b_{ci}^2 - 3a_{ci}c_{ci}}}{-3(-a_{ci})} \tag{10.3}$$

Thus, if  $r_{-i}^{fg} < r_{t-i1}^{fg}$ ,  $g > g_{ii1}^{fg}$ ,  $f \geq f_{ii1}^{fg}$ , and  $b_i > \frac{Bc_i k_i N_i}{2(N_i + N_{-i})}$  then  $c_{ci} > 0$  and if  $r_{-i}^{fg} \leq r_{t-i2}^{fg}$ ,

$$g > g_{ii2}^{fg}, f \leq f_{ii2}^{fg}, \text{ where } f_{ii1}^{fg} < f_{ii2}^{fg} \text{ since } b_i > \frac{Bc_i k_i}{3}, \text{ then } r_i^{fgu} \geq 1. \quad (10.4)$$

Therefore, if  $c_{ci} > 0$  then  $r_i^{fgu} \geq 1$  since  $r_{t-i1}^{fg} < r_{t-i2}^{fg}$ . (10.5)

We next show that, if  $f < f_{ii3}^{fg}$  and  $b_i < \frac{B}{2}(a_i + c_i(h_{i1} + 2k_i))$  then  $r_{t-i1}^{fg} < 1$ .

$$\text{Let } f < f_{ii3}^{fg} = \frac{a_i + c_i(h_{i1} + 2k_i) - \frac{2b_i}{B}}{a_i + c_i(h_{i1} + 2k_i) - \frac{c_i k_i}{N_i + N_{-i}}}, \text{ where clearly, } a_i + c_i(h_{i1} + 2k_i) - \frac{c_i k_i}{N_i + N_{-i}} > 0$$

$$\text{and } a_i + c_i(h_{i1} + 2k_i) - \frac{2b_i}{B} > 0 \text{ if } b_i < \frac{B}{2}(a_i + c_i(h_{i1} + 2k_i)).$$

$$\text{Thus, } f \left( a_i + c_i(h_{i1} + 2k_i) - \frac{c_i k_i}{N_i + N_{-i}} \right) < a_i + c_i(h_{i1} + 2k_i) - \frac{2b_i}{B}$$

$$\Rightarrow \frac{2b_i}{B} - \frac{c_i k_i}{N_i + N_{-i}} + (a_i + c_i h_{i1})g < (a_i + c_i(h_{i1} + 2k_i))(1-f) + (a_i + c_i h_{i1})g$$

$$\Rightarrow r_{t-i1}^{fg} = \frac{\frac{2b_i}{(1-f)B} - (a_i + c_i h_{i1})(1-g) - \frac{c_i k_i f N_i}{(1-f)(N_i + N_{-i})} - c_i k_i}{c_i k_i + (a_i + c_i h_{i1})g} < 1.$$

Thus, if  $f < f_{ii3}^{fg}$  and  $b_i < \frac{B}{2}(a_i + c_i(h_{i1} + 2k_i))$  then  $r_{t-i1}^{fg} < 1$ . (10.6)

From equation (10.4), if  $r_{-i}^{fg} < r_{t-i1}^{fg}$ ,  $g > g_{ii1}^{fg}$ ,  $f \geq f_{ii1}^{fg}$ , and  $b_i > \frac{Bc_i k_i N_i}{2(N_i + N_{-i})}$  then  $c_{ci} > 0$ ,

where  $r_{t-i1}^{fg} > 1$  since, clearly  $f_{ii1}^{fg} > f_{ii3}^{fg}$ .

Similarly, we can show that

1. if  $b_i \geq b_{ii1}^{fg}$ ,  $r_{-i}^{fg} > r_{t-i1}^{fg}$ ,  $f < f_{ii3}^{fg}$  and  $b_i < \frac{B}{2}(a_i + c_i(h_{i1} + 2k_i))$  then  $c_{ci} \leq 0$ , where

$$b_{ii1}^{fg} < \frac{B}{2}(a_i + c_i(h_{i1} + 2k_i)) \text{ and } r_{t-i1}^{fg} < 1. \quad (10.7)$$

2. if  $b_i > b_{ii1}^{fg}$ ,  $r_{-i}^{fg} > r_{t-i1}^{fg}$ ,  $f \geq f_{ii3}^{fg}$ , and  $b_i \geq \frac{B}{2}(a_i + c_i(h_{i1} + 2k_i))$  then  $c_{ci} \leq 0$ , where

$$r_{t-i1}^{fg} > 1. \quad (10.8)$$

We next show that  $b_{ii1}^{fg} < \frac{B}{2}(a_i + c_i h_{i1}) < \frac{B}{2}(a_i + c_i(h_{i1} + 2k_i))$ .

Let

$$\begin{aligned} & \frac{B}{2} \left( a_i + c_i \left( h_{i1} \left( 1 - \frac{(1+g(1+r_{-i}^{fgu}))}{3} (1-f) \right) - \frac{2fk_i N_i}{N_i + N_{-i}} \right) \right) + \frac{B(1-f)}{2} \left( \frac{g^2(a_i + c_i h_{i1})^2}{3gc_i k_i} \right) + \\ & \frac{B(1-f)c_i k_i}{2} \left( \frac{(1-g(1+r_{-i}^{fgu}))^2 + g(1+r_{-i}^{fgu})}{3g} \right) > 0 \end{aligned}$$

On rearranging the terms,

$$\begin{aligned} \Rightarrow \frac{B}{2}(a_i + c_i h_{i1}) > b_{ii1}^{fg} &= \frac{fBc_i k_i N_i}{(N_i + N_{-i})} + \\ & \frac{(1-f)}{2} B \left( (1+g(1+r_{-i}^{fgu}))(a_i + c_i h_{i1}) + c_i k_i (1+r_{-i}^{fgu}) - \frac{(c_i k_i (1+g(1+r_{-i}^{fgu})) + g(a_i + c_i h_{i1}))^2}{3gc_i k_i} \right). \end{aligned}$$

We next show that, if  $r_{-i}^{fg} > r_{t-i2}^{fg}$ ,  $g < g_{ii3}^{fg}$ ,  $f < f_{ii4}^{fg}$ ,  $a_i > a_{ii1}^{fg}$ , and  $b_i < \frac{B(a_i + c_i h_{i1})}{2}$  then

$$r_i^{fgu} < 1, \text{ where } r_{t-i2}^{fg} < 1. \quad (10.9)$$

$$\text{Let } g_{ii3}^{fg} = \frac{a_i + c_i h_{i1}}{c_i k_i} + \frac{1}{(1-f)} \left( \frac{2fN_i}{N_i + N_{-i}} - \frac{2b_i}{Bc_i k_i} \right) > 0 \text{ since } f < f_{ii4}^{fg} = \frac{a_i + c_i h_{i1} - \frac{2b_i}{B}}{a_i + c_i h_{i1} - \frac{2N_i c_i k_i}{N_i + N_{-i}}}$$

$$\Rightarrow f \left( \frac{2N_i}{N_i + N_{-i}} - \frac{a_i + c_i h_{i1}}{c_i k_i} \right) > \frac{2b_i}{Bc_i k_i} - \frac{a_i + c_i h_{i1}}{c_i k_i}, \text{ where } \frac{2N_i}{N_i + N_{-i}} - \frac{a_i + c_i h_{i1}}{c_i k_i} < 0 \text{ since}$$

$$a_i > a_{ii1}^{fg} = \frac{2N_i c_i k_i}{N_i + N_{-i}} - c_i h_{i1}.$$

$$\Rightarrow \frac{2fN_i}{N_i + N_{-i}} - \frac{f(a_i + c_i h_{i1})}{c_i k_i} > \frac{2b_i}{Bc_i k_i} - \frac{a_i + c_i h_{i1}}{c_i k_i}$$

$$\Rightarrow g_{ii3}^{fg} = \frac{2fN_i}{(1-f)(N_i + N_{-i})} + \frac{a_i + c_i h_{i1}}{c_i k_i} - \frac{2b_i}{Bc_i k_i (1-f)} > 0.$$



$$\text{Let } g < g_{ii3}^{fg} = \frac{a_i + c_i h_{i1}}{c_i k_i} + \frac{1}{(1-f)} \left( \frac{2fN_i}{N_i + N_{-i}} - \frac{2b_i}{Bc_i k_i} \right)$$

$$\Rightarrow$$

$$\frac{c_i k_i}{2} g + g \frac{a_i + c_i (h_{i1} - k_i)}{2} < g \frac{a_i + c_i (h_{i1} - k_i)}{2} + \frac{a_i + c_i h_{i1}}{2} + \frac{c_i k_i f N_i}{(1-f)(N_i + N_{-i})} - \frac{b_i}{B(1-f)}$$

This inequality can be rearranged as follows.

$$\Rightarrow \frac{b_i}{B(1-f)} - \frac{c_i k_i f N_i}{(1-f)(N_i + N_{-i})} - \frac{a_i + c_i (h_{i1} - k_i)}{2} (1-g) < \frac{c_i k_i}{2} (1-g) + \frac{a_i + c_i (h_{i1} - k_i)}{2} g$$

$$\Rightarrow r_{t-i2}^{fg} = \frac{\frac{b_i}{B(1-f)} - \frac{c_i k_i f N_i}{(1-f)(N_i + N_{-i})} - \frac{a_i + c_i (h_{i1} - k_i)}{2} (1-g)}{\frac{c_i k_i}{2} (1-g) + \frac{a_i + c_i (h_{i1} - k_i)}{2} g} < 1$$

Let  $r_{-i}^{fg} > r_{t-i2}^{fg}$  then from equation (10.3) we get  $r_i^{fgu} < 1$ .

Therefore, if  $r_{-i}^{fg} > r_{t-i2}^{fg}$ ,  $f < \min \{ f_{ii3}^{fg}, f_{ii4}^{fg} \}$ ,  $g < g_{ii3}^{fg}$ ,  $a_i > a_{ii1}^{fg}$ , and

$b_{ii1}^{fg} < b_i < \frac{B}{2} (a_i + c_i h_{i1})$  then  $r_i^{fgu} < 1$  where  $r_{t-i2}^{fg} < 1$ . Similarly, from equation (10.7) if

$r_{-i}^{fg} > r_{t-i2}^{fg}$ ,  $f < \min \{ f_{ii3}^{fg}, f_{ii4}^{fg} \}$ ,  $b_{ii1}^{fg} < b_i < \frac{B}{2} (a_i + c_i (h_{i1} + 2k_i))$ , and  $g \geq g_{ii3}^{fg}$  then  $r_i^{fgu} \geq 1$ ,

where  $r_{t-i2}^{fg} \geq 1$  from equation (10.9) since  $g \geq g_{ii3}^{fg}$ ,  $g_{ii3}^{fg} < 1$  since

$b_i < \frac{B}{2} (a_i + c_i (h_{i1} + 2k_i))$ . Similarly, from equation (10.8) if  $r_{t-i1}^{fg} < r_{-i} < r_{t-i2}^{fg}$ ,  $f \geq f_{ii3}^{fg}$ ,

$b_i \geq \frac{B}{2} (a_i + c_i (h_{i1} + 2k_i))$ , and  $g \geq g_{ii3}^{fg}$  then  $r_i^{fgu} \geq 1$ .

If  $b_i < b_{ii1}^{fg}$ , where  $b_{ii1}^{fg} > 0$  since  $f < f_{ii5}^{fg}$ ,  $f_{ii5}^{fg} > 1$ .

We next show that, in Table 5, if  $f < \min \{ f_{ii3}^{fg}, f_{ii4}^{fg}, f_{t-i3}^{fg}, f_{t-i4}^{fg} \}$ ,  $g < \min \{ g_{ii3}^{fg}, g_{t-i3}^{fg} \}$ ,

$b_{ii1}^{fg} < b_i < \frac{B}{2} (a_i + c_i h_{i1})$ ,  $b_{t-i1}^{fg} < b_{-i} < \frac{B}{2} (a_{-i} + c_{-i} h_{-i1})$ ,  $c_i < c_{ii1}^{fg}$ ,  $a_{ii1}^{fg} < a_i < a_{ii2}^{fg}$ , and

$N_{-i}^{fg} > N_{t-i1}^{fg}$  then  $U(r_i^{fg*} = 0, r_{-i}^{fg*} = r_{-i}^{fgu}) < U(r_i^{fg*} = r_i^{fgu}, r_{-i}^{fg*} = r_{-i}^{fgu})$ .

Let  $c_{ii1}^{fg} > c_i$

$\Rightarrow$

$$\begin{aligned}
& b_i - \frac{(1-f)}{2} B a_i (1 + g(1 + r_{-i}^{fgu} - r_i^{fgu})) \\
& \overline{\left( \frac{(1-f)}{2} B \left( (h_{i1} - r_i^{fgu} k_i) (1 + g(1 + r_{-i}^{fgu} - r_i^{fgu})) + k_i (1 + r_{-i}^{fgu}) (1 - r_i^{fgu} g) \right) + \frac{f B k_i N_i}{(N_i + N_{-i})} \right)} > c_i \\
& \text{where, } b_i - \frac{(1-f)}{2} B a_i (1 + g(1 + r_{-i}^{fgu} - r_i^{fgu})) > 0 \text{ since } a_i < a_{ii2}^{fg}. \\
& \Rightarrow b_i - \frac{(1-f)}{2} B a_i (1 + g(1 + r_{-i}^{fgu} - r_i^{fgu})) > \\
& c_i \left( \frac{(1-f)}{2} B \left( (h_{i1} - r_i^{fgu} k_i) (1 + g(1 + r_{-i}^{fgu} - r_i^{fgu})) + k_i (1 + r_{-i}^{fgu}) (1 - r_i^{fgu} g) \right) + \frac{f B k_i N_i}{(N_i + N_{-i})} \right) \\
& \Rightarrow \\
& r_i^{fgu} b_i > r_i^{fgu} \frac{(1-f)}{2} B \left( (1 + g(1 + r_{-i}^{fgu} - r_i^{fgu})) (a_i + c_i (h_{i1} - r_i^{fgu} k_i)) + c_i k_i (1 + r_{-i}^{fgu}) (1 - r_i^{fgu} g) \right) + \\
& \frac{r_i^{fgu} f B c_i k_i N_i}{(N_i + N_{-i})} \\
& \Rightarrow r_i^{fgu} b_i - \frac{r_i^{fgu} f B c_i k_i N_i}{(N_i + N_{-i})} + \left( \frac{(1-f)}{2} B (1 + r_{-i}^{fgu}) + \frac{f B N_i}{(N_i + N_{-i})} \right) (a_i + c_i h_{i1}) - \\
& r_i^{fgu} \frac{(1-f)}{2} B \left( (1 + g(1 + r_{-i}^{fgu} - r_i^{fgu})) (a_i + c_i (h_{i1} - r_i^{fgu} k_i)) + c_i k_i (1 + r_{-i}^{fgu}) (1 - r_i^{fgu} g) \right) > \\
& \left( \frac{(1-f)}{2} B (1 + r_{-i}^{fgu}) + \frac{f B N_i}{(N_i + N_{-i})} \right) (a_i + c_i h_{i1}) \\
& \Rightarrow U(r_i^{fg*} = r_i^{fgu}, r_{-i}^{fg*} = r_{-i}^{fgu}) > U(r_i^{fg*} = 0, r_{-i}^{fg*} = r_{-i}^{fgu})
\end{aligned}$$

Thus, if  $f < \min \{f_{ii3}^{fg}, f_{ii4}^{fg}, f_{t-i3}^{fg}, f_{t-i4}^{fg}\}$ ,  $g < \min \{g_{ii3}^{fg}, g_{t-i3}^{fg}\}$ ,  $b_{ii1}^{fg} < b_i < \frac{B}{2} (a_i + c_i h_{i1})$ ,

$b_{t-i1}^{fg} < b_{-i} < \frac{B}{2} (a_{-i} + c_{-i} h_{-i1})$ ,  $c_i < c_{ii1}^{fg}$  and  $a_{ii1}^{fg} < a_i < a_{ii2}^{fg}$ , then  $r_i^{fg*} = r_i^{fgu}$ ,  $r_{-i}^{fg*} = r_{-i}^{fgu}$ ,

where  $a_{ii1}^{fg} < a_{ii2}^{fg}$  since  $N_{-i}^{fg} > N_{t-i1}^{fg}$ . Similarly, we can show that, if  $f < \min \{f_{ii3}^{fg}, f_{ii4}^{fg}\}$ ,

$g \geq \max \{g_{ii3}^{fg}\}$ ,  $b_{ii1}^{fg} < b_i < \frac{B}{2} (a_i + c_i (h_{i1} + 2k_i))$ ,  $c_i < c_{ii2}^{fg}$  and  $a_i < a_{ii3}^{fg}$  then

$U(r_i^{fg*} = 0, r_{-i}^{fg*} = 1) < U(r_i^{fg*} = 1, r_{-i}^{fg*} = 1)$ , where  $c_{ii2}^{fg} > 0$  since  $a_i < a_{ii3}^{fg}$ .  $\blacksquare$

### Proof for Proposition 6.2:

The upper-level utility function is  $IA = \sum_{i=1}^2 IA_i$

$$= B \sum_{\substack{i,-i=1 \\ i \neq -i}}^2 \left[ \frac{(1-r_i^{fg} + r_{-i}^{fg})(1-f)(1-r_i^{fg}g)}{2} + \frac{fN_i}{N_i + N_{-i}} \right] (h_{i1} - r_i^{fg}k_i)$$

We next solve the upper-level problem.

$$IA_i^m : \max_{f,g} \sum_i IA_i(r_i^m * (f, g)), i=1,2, m=fg, f, g \quad (10.10)$$

$$\text{s.t. } 0 \leq f \leq 1 \quad (10.11)$$

$$0 \leq g \leq 1 \quad (10.12)$$

$$r_i^m * (f, g) = \arg \max (L_i(r_i^m)) \quad (10.13)$$

If  $r_1^m * (f, g) = 0, r_2^m * (f, g) = 1$  then  $IA$  is independent of  $g$  and is an increasing function of  $f$  if  $h_{11} > h_{11}^{fg}$ . Similarly, if  $r_1^m * (f, g) = 1, r_2^m * (f, g) = 0$  then  $IA$  is independent of  $g$  and is an increasing function of  $f$  if  $h_{11} \leq h_{11}^{fg}$ . If  $r_i^m * (f, g) = 1$  then clearly  $IA$  is a decreasing function of  $f$  and  $g$ . If  $r_i^m * (f, g) = 0$  then clearly  $IA$  is independent of  $g$  and is a decreasing function of  $f$ . If  $r_i^m * (f, g) = r_i^{fgu} * (f, g) < 1$  then from the Second Partial Derivative Test we can show that there neither exist a local max or a local min. We next show that if  $D = IA_{ff}IA_{gg} - (IA_{fg})^2 < 0$ , where  $IA_{ff}$  is the second partial derivative of  $IA$  with respect to  $f$  then a saddle point at  $(f^*, g^*)$  exist, where,  $(f^*, g^*)$  is obtained by equating first derivative of  $IA$  with respect to  $f$  and  $g$  to zero as below.

$$IA_f = \frac{\partial IA}{\partial f} = B \sum_{\substack{i,-i=1 \\ i \neq -i}}^2 \left[ -\frac{(1-r_i^{fg} + r_{-i}^{fg})(1-r_i^{fg}g)}{2} + \frac{N_i}{N_i + N_{-i}} \right] (h_{i1} - r_i^{fg}k_i) = 0$$

$$\Rightarrow g^* = \frac{\sum_{\substack{i,-i=1 \\ i \neq -i}}^2 \left[ \frac{(1-r_i^{fg} + r_{-i}^{fg})}{2} - \frac{N_i}{N_i + N_{-i}} \right] (h_{i1} - r_i^{fg}k_i)}{\sum_{\substack{i,-i=1 \\ i \neq -i}}^2 r_i^{fg} \frac{(1-r_i^{fg} + r_{-i}^{fg})}{2} (h_{i1} - r_i^{fg}k_i)}$$

$$IA_g = \frac{\partial IA}{\partial g} = -B(1-f) \sum_{i=1}^2 r_i^{fg} (h_{i1} - r_i^{fg} k_i) = 0$$

$$\Rightarrow f^* = 1$$

Clearly,  $IA_{ff} = 0$ ,  $IA_{gg} = 0$ , and  $IA_{gf} = B \sum_{i=1}^2 r_i^{fgu} (h_{i1} - r_i^{fgu} k_i) > 0$ . Thus,  $D < 0$  and neither

a local max nor local min exists. Thus, a saddle point at  $(f^*, g^*)$  exist.

If  $r_1^m * (f, g) = r_1^{fgu} (f, g) < 1$ ,  $r_2^m * (f, g) = 0$  then we substitute  $r_1^m * (f, g) = 0$  in the above equations and show that neither local max nor local min exist. ■

Similarly, by substituting  $g = 0$  and  $f = 0$  in the results of “fg” case, we obtain

Proposition 6.3, 6.4 of “g” case and Proposition 6.5 and 6.6 of “f” case. ■

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