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Diversification with international assets and cryptocurrencies using Black-Litterman

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Abstract

Purpose – The aim of the study was to analyze the performance of Black-Litterman (BL) portfolios using a views estimation procedure that simulates investor forecasts based on technical analysis.

Design/methodology/approach – Ibovespa, S&P500, Bitcoin and interbank deposit rate (IDR) indexes were respectively considered proxies for the national, international, cryptocurrency and fixed income stock markets. Forecasts were made out of the sample aiming at incorporating them in the BL model, using several portfolio weighting methods from June 13, 2013 to August 30, 2022.

Findings – The Sharpe, Treynor and Omega ratios point out that the proposed model, considering only variable return assets, generates portfolios with performances superior to their traditionally calculated counterparts, with emphasis on the risk parity portfolio. Nonetheless, the inclusion of the IDR leads to performance losses, especially in scenarios with lower risk tolerance. And finally, given the impact of turnover, the naive portfolio was also detected as a viable alternative.

Practical implications – The results obtained can contribute to improve investors practices, specifically by validating both the performance improvement – when including foreign assets and cryptocurrencies –, and the application of the BL model for asset pricing.

Originality/value – The main contributions of the study are: performance analysis incorporating cryptocurrencies and international assets in an uncertain recent period; the use of a methodology to compute the views simulating the behavior of managers using technical analysis; and comparing the performance of portfolio management strategies based on the BL model, taking into account different levels of risk and uncertainty.

Keywords Portfolios, Black-Litterman, Cryptoassets

Paper type Research paper

1. Introduction

The portfolio theory developed by Markowitz (1952), although constituting a great advance in finance, proved to be complex in terms of empirical applicability. Aware of this limitation, Black and Litterman (1992) developed an asset allocation model incorporating individual investor *views* into market expectations for asset returns. In this way, it would be possible to obtain more efficient portfolios than the market, if the views were correct and the investor had strong confidence in them (O'Toole, 2013), which would challenge the market efficiency hypothesis in the short term. This model has been the focus of studies in recent works such as those by Allaj (2020) and Kolm, Ritter, and Simonian (2021), which recognized its relevance for active management purposes.

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With the intensification of the information flow at global level, concomitant to the evolution of financial systems, there was an expansion of the scope for investments. For that matter, the diversification of portfolios with international assets has become a feasible possibility (Santos & Coelho, 2010). Additionally, cryptocurrencies have become more popular since 2013, being considered a viable option from the point of view of diversification (Batista & Alves, 2021). Notwithstanding, the discussion about the inclusion of these assets for diversification purposes in the literature has not been exhausted, especially regarding emerging markets, which generally have a lower level of maturity. Therefore, this theme is relevant for contemporary works.

The aim of the study was to analyze the performance of Black-Litterman (BL) portfolios using a *views* estimation procedure that simulates investor forecasts based on technical analysis. The specific objectives were to analyze the efficiency gain in different risk tolerance scenarios, as well as to compare the approaches of Markowitz and BL, under different portfolio management strategies, composed of both national and international assets, cryptocurrencies and fixed income.

Regarding the contributions of this study, it is worth noting firstly, that the performance gain of different investment strategies when using BL is verified empirically, providing continuity to studies such as the one by Bessler, Opfer, and Wolff (2017). Secondly, this work uses a procedure to compute the *views* that simulates the decision-making process of managers by employing technical analysis indicators, as in Kara, Ulucan, and Atici (2019), bringing the results of the study closer to what happens in real decision-making. Additionally, different levels of market risk, three uncertainty parameters and five strategies for building portfolios were considered. Finally, as Portelinha, Campani, and Roquete (2021) and Neto and Colombo (2021), this work deepens the discussion on the use of international assets and cryptocurrencies as a possibility of global risk hedging in the Brazilian market, but contemplating a more comprehensive period of analysis, with simulation of investments outside the sample as well as the use of robust statistical tests for the differences in the Sharpe ratio (SR).

The results point to the validation of the *views* elaboration strategy and BL for different asset allocation strategies. Notwithstanding, the inclusion of fixed income, although contributing to the reduction of risk, ended up eroding the returns of the portfolios to a greater extent, generating efficiency losses. Finally, the naive and equal risk contribution portfolios proved to be viable investment options within the spectrum of the study for more risk-averse investors.

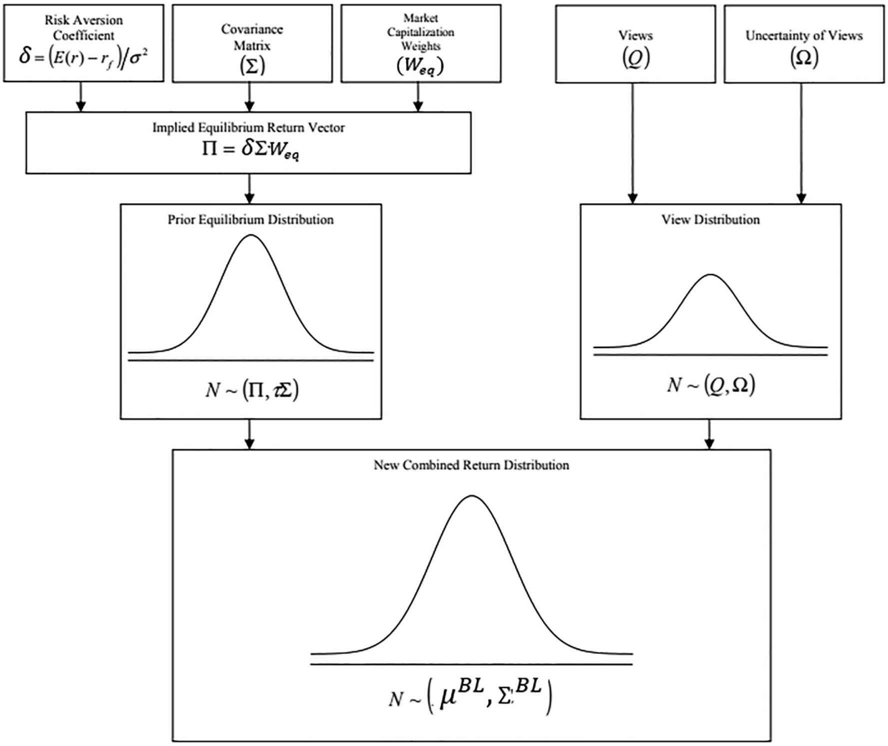
2. Theoretical review of the literature

2.1 Black-Litterman model

The BL model can provide greater flexibility for asset pricing, based on a model guided by market balance and neutrality (usually considered in portfolio theory applications) and incorporates, through Bayesian principles, drivers based on the individual expectations of the investors (Black & Litterman, 1992). In this way, the model is a weighted sum of market expectations with individual expectations, thus being a model for arbitrage. Figure 1 summarizes the idea behind this methodology.

Taking market equilibrium into account, all investors would have their portfolios close to the market portfolio, that is, the weights of the assets that compose the vector W would be equal to the weight of the assets of the market portfolio, W_{eq}. Therefore, according to Black (1989), the risk premium in equilibrium Π assumes the same expectations of the agents, as well as the demand for assets would equal their supply, taking into consideration the risk aversion δ and the correlation matrix of returns Σ (see Equation (1)).

$$\Pi = \delta \Sigma W_{eq} \tag{1}$$



Source(s): Adapted from Idzorek (2007)

Figure 1.
Illustration of the Black-Litterman methodology

According to the model, in equilibrium, we have that $\mu = \Pi + \epsilon^e$, in which ϵ^e is a vector of random variables with mean 0 and standard deviation equal to the multiplication of Σ by a scalar τ, which relates to the uncertainty of expectations in market equilibrium, normally assuming values between 0.025 and 0.300 (Bessler *et al.*, 2017). Therefore, the expected return $E(R)$ follows a normal distribution, according to Equation (2).

$$E(R) \sim N(\Pi, \tau\Sigma) \tag{2}$$

Complementarily, Black and Litterman (1992) define a vector of *views* Q to incorporate the K individual expectations about market assets. Besides that, the authors define P as a KxN *link* matrix to relate the *views* to the N assets of the portfolio.

Another element used in the model is the Ω, a matrix that reflects the uncertainties of Q, calculated from the diagonal of $P(\tau\Sigma)P'$. Therefore, the BL defines $E(R) \sim N(\mu^{BL}, \Sigma^{BL})$, where μ^{BL} is the BL excess return, calculated according to Equation (3). It is worth mentioning that μ^{BL} is a weighting factor by the uncertainty of expected returns, from both the *views* and the market equilibrium. It is also noticed that if the investor does not have divergent opinions from market expectations, $\mu^{BL} = \Pi$.

$$\mu^{BL} = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right] \tag{3}$$

After its publication, BL started to be considered in several studies, aiming both to analyze the properties of the modeling and to propose modifications for its implementation. In this way, the validity of studies on such modeling is reinforced, as well as its relevance for active management of portfolios.

Allaj (2013), for example, considers that performing the Markowitz optimization process working with Σ^{BL} – which corresponds to the conditional variance of BL – may not be the most appropriate approach. The author proposes the use of the unconditional covariance matrix, calculated with the sum of $\Sigma^{BL} + \Sigma$, thus allowing to include the effects of the randomness of the returns to the uncertainties of the parameters. This adaptation has been used in recent works, such as Allaj (2020) and Kolm *et al.* (2021).

The elaboration of Q also generates great interest in the literature, either for the systemic and feasible replication of *views* for simulations, as well as for being an alternative from the point of view of a manager. Among the most used alternatives are the utilization of Generalized AutoRegressive Conditional Heteroskedasticity (GARCH) models (see Duqi, Franci, and Torluccio (2014) and Harris, Stoja, and Tan (2017)), the dynamic regression models (see Fernandes, Street, Fernandes, and Valladão (2018) and Kolm *et al.* (2021)), or even approaches that combine the two techniques, such as Kara *et al.* (2019). Further, Neto and Colombo (2021, p. 94) used the "sample averages of asset returns" as *views*.

2.2 Diversification with international assets and cryptocurrencies

Still on Markowitz's theory, an efficient portfolio is one in which it is not possible to increase profitability without an increase of risk. In this sense, an alternative to improve the performance of national portfolios would be to include international assets whose correlations with national assets are low or negative (Lewis, 1999).

As explained by Santos and Coelho (2010), international diversification can help reduce local systemic risks. Furthermore, the existence of economic cycles and local crises, whose effects affect the financial market, can be circumvented with this strategy, especially when investments from emerging and developed countries are combined (Bhutto, Ahmed, Streimikiene, Shaikh, & Streimikis, 2020).

The positive effects of the inclusion of cryptocurrencies in portfolios are similarly verified. Works such as those carried out by Batista and Alves (2021) and Platanakis and Urquhart (2020) identified performance gains after the inclusion of Bitcoins. One of the aspects of this market that makes it attractive for portfolio management concerns is its low transaction costs, as well as the level of regulation (Kim, 2017). In addition to this, Bouri, Molnár, Azzi, Roubaud, and Hagfors (2017) and Elsayed, Gozgor, and Yarovaya (2022), identified that Bitcoin (BTC) can be used to *hedge* in moments of global uncertainty in the short term. Portelinha *et al.* (2021) state that, although there are studies that prove the performance gain with the inclusion of cryptocurrencies, this class is still rarely used by managers in the elaboration of portfolios. Such phenomenon is explained by Hu, Parlour, and Rajan (2019) as tracing to volatility and speculative behavior of cryptocurrencies.

3. Methodology

3.1 Indicators, database and portfolios

Four asset classes were selected for the composition of the portfolios. For national variable income assets, the Ibovespa (IBOV) was selected. Regarding the cryptocurrency market, it was decided to include BTC which, in this class, is the asset with the highest capitalization and liquidity, as well as a correlation of about 0.96, with a cryptoasset market index, theoretical and non-negotiable, according to the work of Trimborn and Härdle (2018). Studies such as those by Platanakis and Urquhart (2020) and Bouri *et al.* (2017) also used BTC as a proxy for the cryptocurrency market due to above mentioned characteristics.

In order to incorporate international assets, American assets were selected. And, to avoid problems of disparity in the opening and closing of exchanges, as well as the exchange rate issue, a national Exchange-Traded Fund (ETF) was used; whose *benchmark* is the S&P500, the IVVB11, as in Neto and Colombo (2021). Market indexes and BTC data were collected from YahooFinance, all in national currency. Finally, to measure fixed income assets we have considered interbank deposit rate (IDR), which is used as an index for various fixed income assets and fixed income investment funds. This data was collected from Brasil Central Bank.

The *proxy* for the risk-free rate r_f was the one published by the Brazilian Center for Research in Financial Economics of the University of São Paulo (NEFIN, 2017), following Cavalcante-Filho, De-Losso, and Santos (2021), which is computed from the 30-day IDR Swap. The main reason for this choice was the fact that it is a rate negotiated on B3 with daily availability. Additionally, a national r_f was used, since the objective of the study is to analyze portfolios from the perspective of a Brazilian and noninternational investor, which is the most recurrent case in the literature.

In this study, five methodologies were selected to calculate portfolios, which are: the naive portfolio (NP), tangent (Tang), minimum variance (MinV), parity of risk (ParR) and VolT, according to Pflug, Pichler, and Wozabal (2012) and Harvey *et al.* (2018). Equations (4)–(8) illustrate the formula for calculating portfolio weights according to aforementioned methodologies, respectively.

$$W^{NP} = \frac{1}{N} \tag{4}$$

$$W^{Tang} = \frac{\Sigma^{-1}\mu}{1_{\mathrm{N}}\Sigma^{-1}\mu} \tag{5}$$

$$\mathrm{W}^{\mathrm{MinV}} = \frac{\Sigma^{-1}1'_N}{1_{\mathrm{N}}\Sigma^{-1}1'_N} \tag{6}$$

$$w_i^{ParR} = \frac{\frac{1}{\sigma_i}}{\sum\frac{1}{\sigma_i}} \tag{7}$$

$$w_i^{VolT} = \frac{\left(\frac{1}{\sigma_i}\right)^{\eta}}{\sum\left(\frac{1}{\sigma_i}\right)^{\eta}} \tag{8}$$

In these equations, N is the number of available assets, μ is the excess return vector, Σ is the covariance matrix, 1_{N} is a vector of N ones (1), σ_i is the standard deviation of asset i, η is the adjustment parameter for the change in weights in relation to changes in the volatility of assets over time, defined as 4 based on empirical studies (Iquiapaza, Vaz, & Borges, 2016).

The data period goes from 06/13/2013 to 08/31/2022, with the choice of the initial and final period, respectively, justified by the beginning of BTC trading and the availability of r_f until the date of the study.

3.2 View prediction models

Following an adaptation of the methodology by Kara *et al.* (2019), initially seven technical analysis indicators were calculated for the IBOV, S&P500 and BTC: average true range (ATR), average direction index (ADX), simple moving average (SMA), exponential moving average (EMA), convergence and divergence of moving averages (MACD) – considering moving windows for short, long term and for construction of the signal of, respectively, 10, 90 and 30 days –, hurst exponent R/S (HURST) and relative strength index (RSI). Starting from a moving window of 126 days, autoregressive–moving-average (ARMA)-GARCH models were adjusted to forecast the indicators at t+1.

After that, a support vector regression (SVR) model was used pursuant to Kara *et al.* (2019), and a linear regression (LR) model was applied to relate indicator forecasts with asset prices (Equation (9)). In order to identify what would be the best strategy for out-of-sample forecasts, the results of the two model regressions were also analyzed including the quotation of the indicator in period t−1 (Equation (10)).

$$Cotation_t = \beta_1\widehat{ATR}_t + \beta_2\widehat{ADX}_t + \beta_3\widehat{EMA}_t + \beta_4\widehat{SMA}_t + \beta_5\widehat{MACD}_t + \beta_6\widehat{HURST}_t + \beta_7\widehat{RSI}_t \tag{9}$$

$$\begin{aligned} Cotation_t = {} & \beta_1\widehat{ATR}_t + \beta_2\widehat{ADX}_t + \beta_3\widehat{EMA}_t + \beta_4\widehat{SMA}_t + \beta_5\widehat{MACD}_t \\ & + \beta_6\widehat{HURST}_t + \beta_7\widehat{RSI}_t + \beta_8 Cotation_{t-1} \end{aligned} \tag{10}$$

Afterward, for the selection of the best alternative, the root mean squared error and the mean absolute percentage errors were analyzed. The returns of the forecasts of the last period in each moving window were considered as the components of Q at t+1.

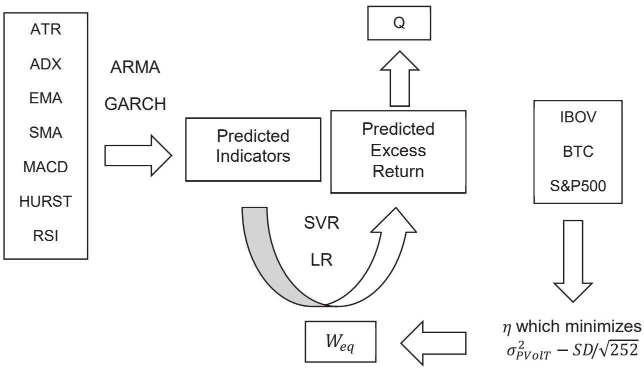
3.3 Market equilibrium portfolio

In order to calculate the W_{eq} vectors for BL, a volatility timing (VolT) portfolio created with the historical returns of the IBOV, S&P500 and BTC was considered as a market index. For this purpose, a η was calculated so that maximized the approximation of the portfolio's volatility with 8% p.y., 15% p.y. and 20% p.y., in order to verify whether the investor's risk profile would affect the results, similarly to Fernandes *et al.* (2018).

Figure 2 summarizes the previously discussed methodology.

3.4 Black-Litterman's portfolios

Based on these inputs, it was possible to estimate δ, Ω and Σ, calculating $\delta = \mu^{Mkt}/\sigma^2_{Mkt}$ while limiting the values to the interval between 2 and 4, which according to Engle (2012) is the usual spectrum of risk aversion. Therefore, using BL the expected equilibrium returns were identified. Finally estimates of μ^{Π} e μ^{BL} were created by replicating this process for all sample elements with a 60-day moving average.



Source(s): Elaborated by the authors

Figure 2.
Methodology for calculating views and equilibrium weights

Regarding Σ, for the market equilibrium, the historical covariance was considered, while for BL we have chosen to use GARCH modeling with constant correlations in the estimation window. This option is justified as the use of historical variance is expected by most investors, so that the inclusion of GARCH modeling can be identified as a particularity of individual expectations. For the purpose of this work, we have opted for daily portfolios rebalancing. Finally, values of 0.1625, 0.025 and 0.300 were tested for τ, these being the average, minimum and maximum values used in the literature.

3.5 IDR procedures and inclusion

The investment in fixed income was included through an adaptation of the M2 Index by Modigliani and Modigliani (1997). Defining $w_{pt} = \sigma_{Mkt}/\sigma_p$, the weight of the portfolio p, depending on the portfolio and market risk (σ_p *e* σ_{Mkt}) and $1 - w_{pt}$ invested in the IDR. That portfolio return ($r^*_{i,t}$) was calculated according to Equation (11). This strategy is in line with the idea of Canner, Mankiw, and Weil (1997), according to which the ideal situation for a given investor would be to have a percentage of their investments in risk-free assets, and the other part in mutual funds, represented here by the market portfolio.

$$r^*_{i,t} = w_{pt} * r_{p,t} + (1 - w_{pt}) * r_{IDR,t} \tag{11}$$

It should be noted, however, that originally the M2 Index uses the r_f rate to calculate the balance of portfolios, so that the variance of the portfolios is equal to that of the market. Nonetheless, here we have decided for IDR rate, as it is more compatible with fixed income funds than r_f. Besides, and due to the option for not allowing short selling, w_{pt} range was limited to 0 and 1.

3.6 Procedures and performance evaluation

In the study carried out, short sales were prohibited in order to both limit variation in asset weights, as well as avoid portfolio leverage, this complying to the aim of forming feasible and replicable portfolios. In a way, such an approach is comparable to the proposal by Batista and Alves (2021).

It is also noted that a restriction was included for W^{Tang}, to fluctuate between 30% and 45%, aiming to avoid high portfolio concentration in just one asset, as well as ensuring the presence of the three asset classes in all periods.

Finally, performance analyses of the portfolios formed were carried out, considering the average return, standard deviation, SR, beta, Treynor ratio (TR), omega ratio (OR), tracking error (TE) and turnover (TO) (DeMiguel & Nogales, 2009; Jensen, 1968; Sharpe, 1964; Treynor, 1996; Keating & Shadwick, 2002).

We have applied the test proposed by Ledoit and Wolf (2008) to examine the SR difference among the various portfolios.

4. Data analysis

4.1 Out-of-sample projections

Based on the calculated technical indicators, Equations (9) and (10) were used to make price projections for the IBOV, S&P500 and BTC using SVR and LR models. After analyzing the forecast error, the model selected as the most appropriate was the LR considering the quotation lag, and the forecasts generated by this model were those used for the composition of Q.

4.2 Weights of market portfolios

Firstly, regarding the annualized average, the values were 13.4%, 22.3% and 63.5%, for IBOV, S&P500 and BTC, respectively, as shown in Table 1. Thus, it is observed that the

	Mean	Median	Standard deviation	Variation coefficient	Sharpe ratio	Omega ratio
IBOV	0.134	0.276	0.260	1.939	0.264	1,105
SP500	0.223	0.152	0.215	0.967	0.731	1,190
BTC	0.635	0.664	0.725	1.142	0.786	1,261
Market Risk 8%	0.215	0.360	0.178	0.828	0.773	1,251
Market Risk 15%	0.287	0.373	0.191	0.665	1.076	1,297
Market Risk 20%	0.407	0.387	0.229	0.563	1.389	1,332
Ledoit and Wolf's Sharpe Ratio Difference Test Statistics						
	IBOV	*SP500*	*BTC*	*Market Risk 8%*	*Market Risk 15%*	*Market Risk 20%*
IBOV						
SP500	−0.889					
BTC	−1.248	−0.564				
Market Risk 8%	1.808*	0.877	0.877			
Market Risk 15%	2.189**	1.567	0.459	−1.472		
Market Risk 20%	2.189**	1.806*	1.439	−1.469	−0.941	

Note(s): 1: *** indicates significance at 1%, ** indicates significance at 5% and * indicates significance at 10%
2: IBOV: Ibovespa; SP500: IVVB11 an ETF whose benchmark is the S&P500. BTC: Bitcoin. Market is a portfolio made up of Ibovespa, S&P500 and BTC. The percentages of 8%, 15% and 20% represent ex-ante Market risk levels. Data refer to the period between 11/18/2016 and 08/30/2022
Source(s): Elaborated by the authors

Table 1. Basic descriptive statistics of annualized portfolio and index returns

inclusion of foreign assets as well as cryptocurrencies can be interesting to optimize the returns of a national investor, since the IBOV presented the lowest result.

Regarding volatility, it is possible to see that the standard deviation of BTC was more than twice as big as that of stock markets, reflecting the greater risk of cryptocurrencies. When analyzing SR, it is clear that this risk is partially offset by the higher average return, indicating a greater efficiency of BTC compared to IBOV and SP500. However, the Ledoit and Wolf test points out that there is no statistical difference between the considered indicators.

Starting from the historical data of these series, the W_{eq} were calculated using the VolT model, adjusting the *ex ante* volatility of the resulting portfolio to values of 8% p.y., 15% p.y. and 20% p.y. Figure 3 shows the evolution of the generated weights.

It is possible to verify that as greater volatility is allowed, there are increases in the inclusion of cryptocurrency in the portfolio. Additionally, during COVID-19 crisis period, between March and November 2020, market portfolios were predominantly composed of the S&P500. Eventually, it is possible to see that at the end of 2021, with the fall of BTC, a greater share of the IBOV was included in the market portfolios, and at the risk levels of 8% and 15% there was almost no participation of BTC.

It is also observable that the combination of the three asset classes provides a reduction in the standard deviation when compared to the individual assets, corroborating what is expected according to the portfolio theory and reinforcing the conclusions of Bhutto *et al.* (2020) and Bouri *et al.* (2017) on the potential for hedging and expanding portfolio returns with the inclusion of international assets and cryptocurrencies. The analysis of the coefficient of variation of the market portfolios indicates that, although a portfolio with a maximum risk of 20% presents a higher standard deviation, the return shows a proportionally greater increase, so that this one proved to be the most attractive when considering the risk-return ratio.

It is also possible to verify that the generated portfolio, considering the volatility of 8%, presents a lower performance than BTC. Nevertheless, the other portfolios generated, when

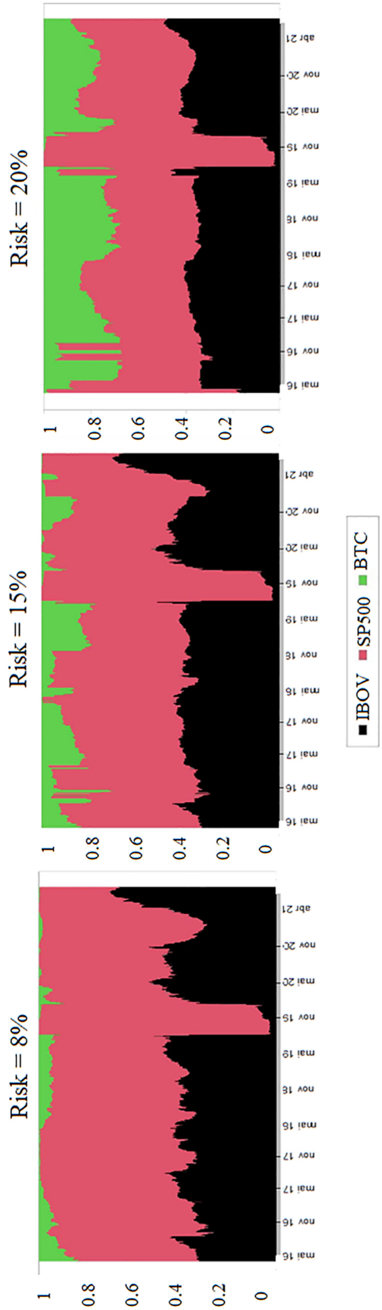


Figure 3.
Market equilibrium weighting with different risks levels

Note(s): IBOV: Ibovespa; SP500: IVVB11, an ETF whose benchmark is the S&P500. BTC: Bitcoin. Market is a portfolio made up of Ibovespa, S&P500, and BTC. The percentages of 8%, 15%, and 20% represent ex-ante Market risk levels. Data refer to the period between 11/18/2016 and 08/30/2022

Source(s): Elaborated by the authors considering greater risk tolerance, start to have a better performance by both OR and SR. And finally, the Ledoit and Wolf test indicates a significant increase in SI for all three market simulations when compared to IBOV, and for the market portfolio with a risk of 20% compared to the S&P500. The other comparisons were not significant.

4.3 Portfolio performance analysis

Based on the procedures that were highlighted in the methodology section, the weights of the selected portfolios were calculated for each of the three market risk configurations, separating them into two groups: equilibrium (Eq), which correspond to portfolios according to the portfolio theory and BL. This division was made to verify the performance of the BL methodology for different risk levels.

For the proposed – methodology, the variations of τ did not generate significant oscillations for the weights of the portfolios. For the analyses presented hereinafter, the results correspond to portfolios formed with $\tau = 0.1625$, corresponding to the average of the values used in the literature, as well as the value adopted by Platanakis and Urquhart (2020) and Neto and Colombo (2021).

Starting the analyses, Figure 4 shows the weights of the Tang, MinV, ParR and VolT portfolios, considering a risk of 20%. Regarding Tangs, it is visible that the formulation is in a kind of balance between the three asset classes, consistent with the restrictions imposed. However, the participation of the S&P500 is, in general, predominant in all analyzed periods. BTC showed an increase in share in times of great market uncertainty, especially when there are IBOV declines.

Regarding the MinVs, it can be seen that they are portfolios with a lower share of BTC, reflecting the greater volatility of this asset. It is noteworthy, however, that in MinV_BL, not only there is greater presence of cryptocurrencies, but also of foreign assets, signaling that, despite the increase in portfolio volatility compared to MinV_Eq, BTC and SP500 can be included in portfolios that aim to minimize variance.

It was also identified that American assets were predominant in VolT_Eq during the peak of the COVID-19 crisis. As for VolT_BL and ParR, greater variations in weights can be seen, particularly in moments of market stress. Moreover, with the BTC value decrease at the end of the sample, a reduction in the participation of this class in these portfolios is clearly noted.

When it comes to portfolio performance, Table 2 summarizes the results (Appendix 1 contains the comparison of the indicators for the three values of τ). From the point of view of risk-adjusted return using SR and TR, it can be seen that in general portfolios considering BL have higher indices than their equilibrium counterparts. The only exception is the VolT strategy, in which only the TR considering a market with a risk of 20% has shown superior modeling performance. And, in line with the observation of Neto and Colombo (2021), investors with an aggressive profile can benefit more from the inclusion of cryptocurrencies and international assets in their portfolios.

Complementarily, the Ledoit and Wolf test has identified that in general the SRs were not statistically different (see Appendix 2). The only recurring exception is the ParR_BL strategy, which had a better SR in some comparisons, which corroborates the analyses regarding the superiority of its performance at all analyzed market risk levels. This verification is in line with what was showcased by Harvey *et al.* (2018), that such portfolios, by restricting volatility, reduce the probability of extreme results and consequently end up generating good SR and TR results.

As for the OR, it is noted that BL versions presented higher indices than their counterparts in equilibrium. Furthermore, ParR_BL and VolT_Eq remained as strategies with better indexes, which demonstrate robustness in the results.

It is interesting to note that, although simple, the NP strategy presents relatively good SR and TR when compared to Eq and BL portfolios at different levels of risk. Thus, for more

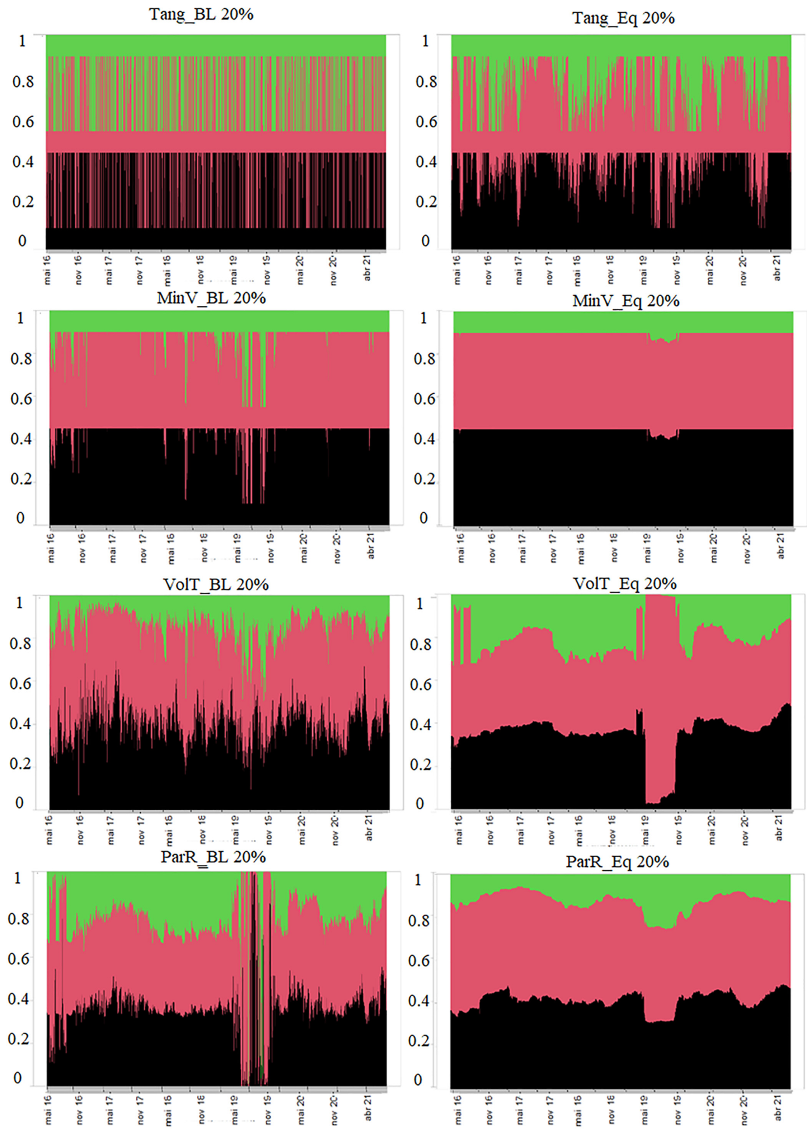


Figure 4.
Weights of the equilibrium and Black-Litterman portfolios

Note(s): Eq: considers historical mean and sample covariance as inputs. BL: considers as inputs returns and covariances using views in the Black-Litterman model; Tang: tangent portfolio; MinV: minimum variance portfolio; ParR: portfolio with equal risk contribution; VolT: volatility timing portfolio; Market is a portfolio consisting of Ibovespa, S&P500, and BTC. The percentages of 20% represent ex-ante market's risk levels. Data refer to the period from 11/18/2016 to 08/30/2022

Source(s): Elaborated by the authors

Portfolio	Return	Standard deviation	Sharpe ratio	Beta	Treynor ratio	Omega ratio	Tracking error	Turnover
Market 8%	0.215	0.178	0.773	1.000	0.138	1.251	0.000	
Tang_Eq 8%	0.253	0.209	0.832	1.038	0.168	1.255	0.098	0.033
Tang_BL 8%	0.428	0.305	1.107	1.084	0.311	1.293	0.237	0.423
MinV_Eq 8%	0.260	0.198	0.907	1.004	0.179	1.274	0.086	0.010
MinV_BL 8%	0.296	0.208	1.030	0.992	0.216	1.308	0.110	0.040
ParR_Eq 8%	0.294	0.203	1.047	1.021	0.208	1.303	0.090	0.012
ParR_BL 8%	0.482	0.250	1.551	1.034	0.375	1.383	0.169	0.101
VolT_Eq 8%	0.401	0.235	1.328	1.068	0.293	1.327	0.139	0.018
VolT_BL 8%	0.325	0.210	1.148	1.006	0.240	1.332	0.110	0.115
Market 15%	0.287	0.191	1.076	1.000	0.205	1.297	0.000	
Tang_Eq 15%	0.292	0.228	0.923	1.075	0.196	1.267	0.101	0.068
Tang_BL 15%	0.418	0.305	1.076	1.177	0.279	1.287	0.209	0.424
MinV_Eq 15%	0.260	0.198	0.907	0.955	0.189	1.274	0.079	0.010
MinV_BL 15%	0.297	0.208	1.032	0.950	0.226	1.308	0.102	0.040
ParR_Eq 15%	0.294	0.203	1.047	0.960	0.221	1.303	0.088	0.012
ParR_BL 15%	0.482	0.250	1.552	1.087	0.357	1.384	0.141	0.101
VolT_Eq 15%	0.401	0.235	1.328	1.117	0.280	1.327	0.102	0.018
VolT_BL 15%	0.326	0.210	1.150	0.950	0.254	1.332	0.107	0.115
Market 20%	0.407	0.229	1.389	1.000	0.318	1.332	0.000	
Tang_Eq 20%	0.348	0.269	0.977	1.076	0.244	1.265	0.109	0.121
Tang_BL 20%	0.416	0.305	1.068	1.125	0.290	1.286	0.166	0.424
MinV_Eq 20%	0.260	0.198	0.907	0.766	0.235	1.274	0.107	0.010
MinV_BL 20%	0.297	0.208	1.032	0.774	0.277	1.309	0.120	0.040
ParR_Eq 20%	0.294	0.203	1.047	0.791	0.269	1.303	0.104	0.012
ParR_BL 20%	0.482	0.250	1.551	0.998	0.389	1.383	0.102	0.101
VolT_Eq 20%	0.401	0.235	1.328	1.023	0.305	1.327	0.024	0.018
VolT_BL 20%	0.326	0.210	1.150	0.783	0.309	1.332	0.121	0.115
NP	0.454	0.296	1.225	1.192	0.304	1.298	0.123	0.015

Note(s): Eq: considers historical mean and sample covariance as inputs. BL: considers as inputs returns and covariances using *views* in the Black-Litterman model; Tang: tangent portfolio; MinV: minimum variance portfolio; ParR: portfolio with equal risk contribution; VolT: volatility timing portfolio; NP: naive weighted portfolio. Market is a portfolio consisting of Ibovespa, S&P500, and BTC. The percentages of 8%, 15% and 20% represent ex-ante market's risk levels. Data refer to the period from 11/18/2016 to 08/30/2022
Source(s): Elaborated by the authors

Table 2. Annualized results of portfolios with IBOV, SP500 and BTC

conservative profiles, NP is a viable alternative, validating previous studies such as those by Pflug *et al.* (2012) and Iquiapaza *et al.* (2016), who comment on this investment strategy. Conversely, the greater the investor's risk tolerance, the better the performance of the market proxy, confirming the importance of considering different levels of risk in the context of the study.

It is also important to point out that the restrictions imposed on the W^{Tang} impacted the maximization of the SR. This restriction contributed to the fact that these portfolios were not generated from combinations of assets that provided an interesting risk-return ratio compared to the other alternatives.

The TE results indicate that portfolios that take market equilibrium into account present greater proximity of returns with their respective market benchmarks. This fact, along with the previous performance analyses, corroborates the importance of the proposed BL modeling for the elaboration of portfolios.

Further, the turnover analysis – TO, indicates lower reallocation rates for Eq portfolios compared to their counterparts. This phenomenon indicates that, although BL portfolios present better performance than market equilibrium, fees and brokerage portfolios, in addition to transaction costs, can erode the performance of these portfolios to a greater degree, especially Tang, which exceeded the average of 40%. Moreover, higher TO may also be the reflex of difficulties in the formation of portfolios, given possible supply and demand limitations, which reveals one fragility of this strategy.

All in all, results indicate that BL contributes to the construction of more efficient portfolios. Besides that, it was observed that in general BL portfolios carry a greater volume of BTC than their counterparts, causing an increase in their volatility, corroborating the results of Hu *et al.* (2019). However, SR and TR indicate that the increase in returns is greater than the increase in risk, corroborating studies such as those by Harvey *et al.* (2018), Platanakis and Urquhart (2020) and Elsayed *et al.* (2022) on the performance increase when using BL. Nevertheless, these portfolios present an increase in TO, which can result in higher transaction costs, as well as difficulties in forming portfolios if assets do not present sufficient liquidity.

4.4 Performance analysis of portfolios with IDR

With the aim of analyzing the effects of the inclusion of fixed income assets, the IDR used as a proxy for a fixed income asset was incorporated into the portfolios through an adaptation of the M2 Index. The results of the portfolios are shown in Table 3. In general, a reduction in the risk of the portfolios is perceived when adding the IDR, which was expected given the low volatility of fixed income and the metric used for its inclusion in the portfolios. Among the portfolios that show the greatest reductions, Tang_BL and NP stand out, which can be understood by the similarity of their configuration to that of the market portfolios, according to their betas.

Proportionally, however, it was possible to verify a greater reduction in return than in risk. For this reason, the SR, TR and OR of these portfolios were lower than those of the portfolios generated considering only variable return assets. Furthermore, these portfolios present higher TO values compared to those previously generated. However, it should be noted that for the portfolios analyzed above, generally the ParR and VolT strategies performed better in some comparisons of the Ledoit and Wolf test (see Appendix 3).

Therefore, it is possible to conclude that according to this methodology, the inclusion of the IDR, although effective in reducing the volatility of portfolios, ends up eroding return of the assets to a greater degree, in addition to increasing the volume of fluctuations in the weights with each recalibration.

In this way, it can be said that the portfolios with the four classes analyzed showed a loss of efficiency and performance compared to portfolios with IBOV, SP500 and BTC. This finding is more evident in portfolios that consider a volatility tolerance of 8% p.y., in line with the results of Harvey *et al.* (2018).

5. Conclusion

The aim of the study was to analyze the performance of portfolios composed of national, international and cryptocurrency assets of investors in an emerging market, using BL. To this end, a proposal was made to simulate agents' forecasts, using technical analysis indicators of asset prices, through a hybrid approach, using time series modeling to capture trend patterns of technical indicators, and then using RL and SVR for analysts' price forecasts for one period ahead, this being the Q component of the model. In order to avoid high asset TO rates, as well as the extreme leverage of the portfolios, short selling was not allowed in the portfolios.

The analysis of the market portfolio for different risk levels considering variable income assets shows a great concentration of weights of the IBOV and S&P500, and at the peak of the

Table 3. Annualized results of portfolios with IBOV, SP500 and BTC, IDR included

Portfolio	Return	Standard deviation	Sharpe ratio	Beta	Treynor ratio	Omega ratio	Tracking error	Turnover
Market 8%	0.215	0.178	0.773	1.000	0.138	1.251	0.000	
Tang_Eq 8%	0.198	0.182	0.671	0.902	0.136	1.250	0.088	0.157
Tang_BL 8%	0.220	0.193	0.737	0.800	0.178	1.269	0.135	0.565
MinV_Eq 8%	0.209	0.179	0.737	0.892	0.148	1.270	0.086	0.139
MinV_BL 8%	0.232	0.182	0.843	0.856	0.180	1.290	0.104	0.181
ParR_Eq 8%	0.235	0.172	0.910	0.846	0.185	1.295	0.088	0.171
ParR_BL 8%	0.307	0.208	1.080	0.879	0.255	1.350	0.138	0.221
VolT_Eq 8%	0.250	0.179	0.955	0.851	0.201	1.315	0.099	0.148
VolT_BL 8%	0.266	0.190	0.981	0.889	0.209	1.315	0.106	0.201
Market 15%	0.244	0.184	0.898	0.705	0.234	1.304	0.144	0.171
Tang_Eq 15%	0.287	0.191	1.076	1.000	0.205	1.297	0.000	
Tang_BL 15%	0.211	0.195	0.689	0.923	0.145	1.267	0.084	0.174
MinV_Eq 15%	0.243	0.200	0.822	0.824	0.199	1.287	0.128	0.602
MinV_BL 15%	0.213	0.185	0.738	0.876	0.156	1.274	0.083	0.101
ParR_Eq 15%	0.245	0.186	0.891	0.840	0.198	1.308	0.100	0.147
ParR_BL 15%	0.242	0.176	0.928	0.816	0.200	1.303	0.089	0.130
VolT_Eq 15%	0.331	0.217	1.136	0.924	0.267	1.384	0.127	0.221
VolT_BL 15%	0.267	0.193	0.968	0.907	0.206	1.327	0.086	0.147
Market 20%	0.274	0.192	1.008	0.851	0.227	1.332	0.106	0.179
Tang_Eq 20%	0.268	0.195	0.960	0.789	0.238	1.298	0.131	0.171
Tang_BL 20%	0.407	0.229	1.389	1.000	0.318	1.332	0.000	
MinV_Eq 20%	0.295	0.229	0.930	0.914	0.233	1.265	0.095	0.216
MinV_BL 20%	0.294	0.217	0.976	0.814	0.261	1.286	0.120	0.595
ParR_Eq 20%	0.228	0.189	0.796	0.723	0.208	1.274	0.111	0.066
ParR_BL 20%	0.277	0.191	1.028	0.711	0.276	1.309	0.120	0.100
VolT_Eq 20%	0.275	0.181	1.078	0.700	0.278	1.303	0.108	0.089
VolT_BL 20%	0.408	0.235	1.359	0.917	0.348	1.383	0.107	0.190
NP	0.327	0.212	1.147	0.903	0.269	1.327	0.052	0.124

Note(s): Eq: considers historical mean and sample covariance as inputs. BL: considers as inputs returns and covariances using *views* in the Black-Litterman model; Tang: tangent portfolio; MinV: minimum variance portfolio; ParR: portfolio with equal risk contribution; VolT: volatility timing portfolio; NP: naive weighted portfolio. Market is a portfolio consisting of Ibovespa, S&P500, and BTC. The percentages of 8%, 15% and 20% represent ex-ante market's risk levels. Data refer to the period from 11/18/2016 to 08/30/2022
Source(s): Elaborated by the authors

COVID-19 crisis, there was a greater concentration in the American ETF. It is also noticeable that, given the high volatility of BTC, its weight in the portfolio was not predominant at any time. Evidence of its ability to hedge against times of market stress was observed. However, after the beginning of the downward movement of BTC, its participation in the portfolios decreased, which opens the door to investigate whether the hedging power is related only in moments of higher returns in this market.

The analyses of the portfolios with the inclusion of views generally present superior performance to equilibrium portfolios, for the three risk profiles analyzed. Moreover, the variations tested in the τ parameter did not change the results of the generated portfolios, which confirm the robustness of BL studies for values oscillating within the limit usually used in the literature.

In a general way, ParR_BL was identified as being the best asset allocation strategy option. However, for more risk-averse profiles, the NP was also presented as a viable option, while more audacious profiles may choose to follow the benchmark of the proposed market.

In conclusion, when including IDR, it is generally possible to notice a performance deterioration of the generated portfolios. Therefore, it was possible to identify not only the efficiency of BL, but also of the proposed strategy for the elaboration of Q.

As a limitation of this work, we can mention four points. First, it was decided to use proxies for the analyzed markets, instead of analyzing a set of assets of each class. Besides that, given the inexistence of a market portfolio that considers such investments; it was decided to use the VolT modeling to calculate the market portfolio with three levels of risk. Additionally, in this study, currency effects were disregarded when using an ETF for the United States (US) market and BTC values in Brazil's currency. Finally, portfolios formed were prepared without considering short sales.

Therefore, for further studies, it is suggested to first replicate the study with specific assets instead of proxies for the market, as well as expanding the international market to other countries. Also, it is suggested to explore the sensitivity of the results to different recalibrations, in alternative ways of computing the market portfolio, of analyzing the effects of foreign exchange and other assets, and testing other ways of including fixed income. Finally, allowing short sales in the analysis can provide relevant information regarding the superiority of BL when compared to traditional models.

References

Allaj, E. (2013). The Black–Litterman model: A consistent estimation of the parameter tau. *Financial Markets and Portfolio Management*, *27*(2), 217–251.

Allaj, E. (2020). The Black–Litterman model and views from a reverse optimization procedure: An out-of-sample performance evaluation. *Computational Management Science*, *17*(3), 465–492.

Batista, D. T., & Alves, C. F. (2021). Análise do impacto do Bitcoin na eficiência de uma carteira diversificada para investidores brasileiros. *Revista Brasileira de Gestão de Negócios*, *23*(2), 353–369.

Bessler, W., Opfer, H., & Wolff, D. (2017). Multi-asset portfolio optimization and out-of-sample performance: An evaluation of Black–Litterman, mean-variance, and naïve diversification approaches. *The European Journal of Finance*, *23*(1), 1–30.

Bhutto, S. A., Ahmed, R. R., Streimikiene, D., Shaikh, S., & Streimikis, J. (2020). Portfolio investment diversification at global stock market: A Cointegration analysis of emerging BRICS(P) group. *Acta Montanistica Slovaca*, *25*(1), 57–69.

Black, F. (1989). Universal hedging: Optimizing currency risk and reward in international equity portfolios. *Financial Analysts Journal*, *45*(4), 16–22.

Black, F., & Litterman, R. (1992). Global portfolio optimization. *Financial Analysts Journal*, *48*(5), 28–43.

Bouri, E., Molnár, P., Azzi, G., Roubaud, D., & Hagfors, L. I. (2017). On the hedge and safe haven properties of Bitcoin: Is it really more than a diversifier? *Finance Research Letters*, *20*, 192–198.

Canner, N., Mankiw, N. G., & Weil, D. N. (1997). An asset allocation puzzle. *American Economic Review*, *87*(1), 181–191.

Cavalcante-Filho, E., De-Losso, R., & Santos, J. C. S. (2021). Which factors matter to investors? Evidence from Brazilian mutual funds. *Revista Brasileira de Gestão de Negócios*, *23*, 63–80.

DeMiguel, V., & Nogales, F. J. (2009). Portfolio selection with robust estimation. *Operations Research*, *57*(3), 560–577.

Duqi, A., Franci, L., & Torluccio, G. (2014). The Black–Litterman model: The definition of views based on volatility forecasts. *Applied Financial Economics*, *24*(19), 1285–1296.

Elsayed, A. H., Gozgor, G., & Yarovaya, L. (2022). Volatility and return connectedness of cryptocurrency, gold, and uncertainty: Evidence from the cryptocurrency uncertainty indices. *Finance Research Letters*, *47*, 102732.

Engle, R. (2012). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business and Economic Statistics*, *20*(3), 339–350.

Fernandes, B., Street, A., Fernandes, C., & Valladão, D. (2018). On an adaptive Black–Litterman investment strategy using conditional fundamentalist information: A Brazilian case study. *Finance Research Letters*, *27*, 201–207.

Harris, R. D. F., Stoja, E., & Tan, L. (2017). The dynamic Black–Litterman approach to asset allocation. *European Journal of Operational Research*, *259*(3), 1085–1096.

Harvey, C. R., Hoyle, E., Korgaonkar, R., Rattray, S., Sargaison, M., & Van Hemert, O. (2018). The impact of volatility targeting. *The Journal of Portfolio Management*, *45*(1), 14–33.

Hu, A. S., Parlour, C. A., & Rajan, U. (2019). Cryptocurrencies: Stylized facts on a new investible instrument. *Financial Management*, *48*(4), 1049–1068.

Iquiapaza, R. A., Vaz, G. F. C., & Borges, S. L. (2016). Portfolio evaluation of volatility timing and reward to risk timing investment strategies: The Brazilian case. *Revista de Finanças Aplicadas*, *7*(2), 1–19.

Jensen, M. C. (1968). The performance of mutual funds in the period 1945-1964. *The Journal of Finance*, *23*(2), 389–416.

Kara, M., Ulucan, A., & Atici, K. B. (2019). A hybrid approach for generating investor views in Black–Litterman model. *Expert Systems with Applications*, *128*, 256–270.

Keating, C., & Shadwick, W. F. (2002). A universal performance measure. *Journal of Performance Measurement*, *6*(3), 59–84.

Kim, T. (2017). On the transaction cost of Bitcoin. *Finance Research Letters*, *23*(1), 300–305.

Kolm, P. N., Ritter, G., & Simonian, J. (2021). Black-Litterman and beyond: The Bayesian Paradigm in investment management. *The Journal of Portfolio Management*, *47*(5), 91–113.

Ledoit, O., & Wolf, M. (2008). Robust performance hypothesis testing with the Sharpe ratio. *Journal of Empirical Finance*, *15*(5), 850–859.

Lewis, K. K. (1999). Trying to explain home bias in equities and consumption. *Journal of Economic Literature*, *37*(2), 571–608.

Markowitz, H. (1952). Portfolio selection. *The Journal of Finance*, *7*(1), 77–91.

Modigliani, F., & Modigliani, L. (1997). Risk-adjusted performance. *Journal of Portfolio Management*, *23*(2), 45–54.

NEFIN (2017). *Metodologia*. FEA-USP. Available from: https://nefin.com.br/resources/NEFIN_methodology.pdf

Neto, O. D., & Colombo, J. A. (2021). The impact of cryptocurrencies on the performance of multi-asset portfolios: Evidence from Brazil. *Revista Brasileira de Financas*, *19*(4), 86–130.

O'Toole, R. (2013). The Black–Litterman model: A risk budgeting perspective. *Journal of Asset Management*, *14*(1), 2–13.

Pflug, G. C., Pichler, A., & Wozabal, D. (2012). The 1/N investment strategy is optimal under high model ambiguity. *Journal of Banking and Finance*, *36*(2), 410–417.

Platanakis, E., & Urquhart, A. (2020). Should investors include Bitcoin in their portfolios? A portfolio theory approach. *The British Accounting Review*, *52*(4), 100837.

Portelinha, M., Campani, C. H., & Roquete, R. (2021). The impacts of cryptocurrencies in the performance of Brazilian stocks' portfolios. *Economics Bulletin*, *41*(3), 1919–1931.

Santos, J. O. dos, & Coelho, P. A. (2010). Análise da relação risco e retorno em carteiras compostas por índices de bolsa de valores de países desenvolvidos e de países emergentes integrantes do bloco econômico BRIC. *Revista Contabilidade and Finanças*, *21*, 23–37.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. *The Journal of Finance*, *19*(3), 425–442.

Treynor, J. L. (1996). How to rate management investment funds. *Harvard Business Review*, *43*(1), 63–75.

Trimborn, S., & Härdle, W. K. (2018). CRIX an Index for cryptocurrencies. *Journal of Empirical Finance*, *49*, 107–122.

Further reading

Idzorek, T. (2007). A step-by-step guide to the Black-Litterman model: incorporating user-specified confidence levels. In Satchell, S. (Ed.), *Forecasting Expected Returns in the Financial Markets* (*1*, pp. 17–38). Academic Press. ISBN 9780750683210. doi: 10.1016/B978-075068321-0.50003-0.

Appendix 1

Table A1.
Comparison of performance indicators for tau variations

Portfolio	Return	Standard deviation	Sharpe ratio	Beta	Treynor ratio	Omega ratio	Tracking error	Turnover
τ = 0.1625 X τ = 0.025								
Tang_Eq 8%	–	–	–	–	–	–	–	–
MinV_Eq 8%	–	–	–	–	–	–	–	–
VolT_Eq 8%	–	–	–	–	–	–	–	–
ParR_Eq 8%	–	–	–	–	–	–	–	–
Tang_BL 8%	–	–	0.001	−0.001	–	–	–	–
MinV_BL 8%	–	–	−0.001	0.001	–	–	–	–
VolT_BL 8%	–	–	–	–	–	–	–	–
ParR_BL 8%	–	–	–	–	–	–	–	–
NP 8%	–	–	–	–	–	–	–	–
Market 8%	–	–	–	–	–	–	–	–
Tang_Eq 15%	−0.001	–	−0.003	–	−0.001	−0.001	–	–
MinV_Eq 15%	–	–	–	–	–	–	–	–
VolT_Eq 15%	–	–	–	–	–	–	–	–
ParR_Eq 15%	–	–	–	–	–	–	–	–
Tang_BL 15%	−0.004	–	−0.013	–	−0.003	−0.004	–	0.002
MinV_BL 15%	0.001	–	0.001	–	–	–	–	–
VolT_BL 15%	0.001	–	0.004	–	–	–	–	–
ParR_BL 15%	–	–	0.002	–	–	–	–	–
NP 15%	–	–	–	–	–	–	–	–
Market 15%	–	–	–	–	–	–	–	–
Tang_Eq 20%	−0.001	–	−0.003	–	−0.001	–	–	–
MinV_Eq 20%	–	–	–	–	–	–	–	–
VolT_Eq 20%	–	–	–	–	–	–	–	–
ParR_Eq 20%	–	–	–	–	–	–	–	–
Tang_BL 20%	–	–	0.001	–	–	–	–	–
MinV_BL 20%	0.001	–	0.001	–	–	0.001	−0.001	–
VolT_BL 20%	0.001	–	0.003	0.001	0.001	–	–	–
ParR_BL 20%	–	–	–	–	–	–	–	–
NP 20%	–	–	–	–	–	–	–	–
Market 20%	–	–	–	–	–	–	–	–
τ = 0.3 X τ = 0.1625								
Tang_Eq 8%	–	–	–	–	–	–	–	–
MinV_Eq 8%	–	–	–	–	–	–	–	–
VolT_Eq 8%	–	–	–	–	–	–	–	–

(*continued*)

Portfolio	Return	Standard deviation	Sharpe ratio	Beta	Treynor ratio	Omega ratio	Tracking error	Turnover
ParR_Eq 8%	–	–	–	–	–	–	–	–
Tang_BL 8%	–	–	0.001	–	0.001	–	–	–
MinV_BL 8%	0.001	–	0.004	–	0.001	0.001	–	–
VolT_BL 8%	0.001	–	0.003	0.001	–	0.001	–	–
ParR_BL 8%	–	–	–	–	–	–	–	–
NP 8%	–	–	–	–	–	–	–	–
Market 8%	–	–	–	–	–	–	–	–
Tang_Eq 15%	0.001	–	0.001	–	–	0.001	–	–
MinV_Eq 15%	–	–	–	–	–	–	–	–
VolT_Eq 15%	–	–	–	–	–	–	–	–
ParR_Eq 15%	–	–	–	–	–	–	–	–
Tang_BL 15%	–	–	0.001	–	–	–	–	–
MinV_BL 15%	–	–	–	–	–	–	–	–
VolT_BL 15%	–	–	–	–	–	0.001	–	–
ParR_BL 15%	–	–	−0.001	–	–	–	–	–
NP 15%	–	–	–	–	–	–	–	–
Market 15%	–	–	–	–	–	–	–	–
Tang_Eq 20%	0.001	–	0.003	–	0.001	–	–	–
MinV_Eq 20%	–	–	–	–	–	–	–	–
VolT_Eq 20%	–	–	–	–	–	–	–	–
ParR_Eq 20%	–	–	–	–	–	–	–	–
Tang_BL 20%	–	–	0.001	–	–	0.001	–	–
MinV_BL 20%	–	–	–	–	–	−0.001	–	–
VolT_BL 20%	–	–	–	–	−0.001	–	–	–
ParR_BL 20%	–	–	–	–	−0.001	–	–	–
NP 20%	–	–	–	–	–	–	–	–
Market 20%	–	–	–	–	–	–	–	–

Note(s): Eq: considers historical mean and sample covariance as inputs. BL: considers as inputs returns and covariances using *views* in the Black-Litterman model; Tang: tangent portfolio; MinV: minimum variance portfolio; ParR: portfolio with equal risk contribution; VolT: volatility timing portfolio; NP: naive weighted portfolio. Market is a portfolio consisting of Ibovespa, S&P500 and BTC. The percentages of 8%, 15% and 20% represent ex-ante market's risk levels. Data refer to the period from 11/18/2016 to 08/30/2022
Source(s): Elaborated by the authors

Table A1.

Appendix 2

	Portfolio 1	Portfolio 2	p-value	SR portfolio 1	SR portfolio 2
Risk 8% p.y.	Tang_Eq 8%	ParR_Eq 8%	0.0279	0.8315	*1.0473*
	Tang_Eq 8%	VolT_BL 8%	0.0699	0.8315	*1.1486*
	Tang_Eq 8%	ParR_BL 8%	0.0559	0.8315	*1.5510*
	MinV_Eq 8%	ParR_BL 8%	0.0878	0.9072	*1.5510*
	ParR_BL 8%	Mercado 8%	0.0978	1.5510	*0.7734*
Risk 15% p.y.	MinV_Eq 15%	VolT_BL 15%	0.0998	0.9072	*1.1505*
	Tang_Eq 15%	ParR_BL 15%	0.0439	0.9223	*1.5516*
	MinV_Eq 15%	ParR_BL 15%	0.0878	0.9072	*1.5516*
	Tang_BL 15%	ParR_BL 15%	0.0898	1.0764	*1.5516*
Risk 20% p.y.	Tang_Eq 20%	VolT_Eq 20%	0.0659	0.9790	*1.3280*
	Tang_Eq 20%	ParR_BL 20%	0.0619	0.9790	*1.5511*
	MinV_Eq 20%	ParR_BL 20%	0.0878	0.9072	*1.5511*
	Tang_BL 20%	ParR_BL 20%	0.0818	1.0683	*1.5511*
	Tang_Eq 20%	Mercado 20%	0.0519	0.9790	*1.3891*

Note(s): 1: Only the results of the tests with a p-Value lower than 10% are presented in the table; other comparisons were not significant
2: Eq: considers historical mean and sample covariance as inputs. BL: considers as inputs returns and covariances using *views* in the Black-Litterman model; Tang: tangent portfolio; MinV: minimum variance portfolio; ParR: portfolio with equal risk contribution; VolT: volatility timing portfolio; NP: naive weighted portfolio. Market is a portfolio consisting of Ibovespa, S&P500 and BTC. The percentages of 8%, 15% and 20% represent ex-ante market's risk levels. Data refer to the period from 11/18/2016 to 08/30/2022
Source(s): Elaborated by the authors

Table A2.
Ledoit and Wolf's test for annualized portfolio Sharpe ratio difference

Appendix 3

	Portfolio 1	Portfolio 2	p-value	SR portfolio 1	SR portfolio 2
Risk 8% p.y.	Tang_Eq 8%	ParR_Eq 8%	0.0719	0.6709	*0.9102*
	Tang_Eq 8%	VolT_BL 8%	0.0559	0.6709	*0.9821*
	MinV_Eq 8%	VolT_BL 8%	0.0818	0.7367	*0.9821*
Risk 15% p.y.	Tang_Eq 15%	ParR_BL 15%	0.0539	0.6889	*0.9288*
	Tang_Eq 15%	VolT_BL 15%	0.0858	0.6889	*1.0088*
	MinV_Eq 15%	VolT_BL 15%	0.0679	0.7390	*1.0088*
Risk 20% p.y.	MinV_Eq 20%	MinV_BL 20%	0.0539	0.7959	*1.0283*
	MinV_Eq 20%	VolT_BL 20%	0.0459	0.7959	*1.1166*

Note(s): 1: Only the results of the tests with a p-Value lower than 10% are presented in the table; other comparisons were not significant
2: Eq: considers historical mean and sample covariance as inputs. BL: considers as inputs returns and covariances using *views* in the Black-Litterman model; Tang: tangent portfolio; MinV: minimum variance portfolio; ParR: portfolio with equal risk contribution; VolT: volatility timing portfolio; NP: naive weighted portfolio. Market is a portfolio consisting of Ibovespa, S&P500 and BTC. The percentages of 8%, 15% and 20% represent ex-ante market's risk levels. Data refer to the period from 11/18/2016 to 08/30/2022
Source(s): Elaborated by the authors

Table A3.
Ledoit and Wolf's test for differences in annualized Sharpe ratios of portfolios with IDR

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