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CONTEMPORARY PLANETARY GEARBOXES AND THEIR CALCULATION

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A B S T R A C T

The increased demands for driver comfort and stringent pollution control measures have resulted in a revival of planetary gearboxes for road applications, due to their possibility to change the transmission ratio under load in synchronism with engine operation. Modern boxes provide as many transmission ratios as possible from the least possible number of simple component planetary gear trains (PGTs) by providing links between elements of multiple component PGTs. The application conditions decide to prioritize either the maximum number of transmission ratios, or ruggedness and reliability. Power circulation, hollow shafts, or complex planet carrier arrangements are avoided if possible. This paper deals with multispeed complex PGTs composed of at least two interconnected simple component PGTs controlled by brakes and clutches. Several variants of complex PGTs and the placement of brakes and clutches on external shafts of the gear trains are examined, and the transmission ratio functions derived. The kinematics of multi speed gear trains are obtained as combinations of two or more two-speed gear trains. An analysis of several contemporary gearbox layouts is provided together with the transmission ratio functions, together with an overview of the procedure for the calculation of creation of multi-speed gear trains is given.

1. INTRODUCTION

Internal combustion engines (ICEs), regardless of their operating cycle, have been used in road and off-road vehicle applications since the late 19th century as a replacement for horse and steam drawn vehicles. Unlike a steam engine, the ICE is unable to self-start from standstill, and therefore it requires a transmission to match the speed and torque of the engine to the revolutions of the wheels (Syzrantsev & Syzrantseva, 2022). The transmission must also appropriately split between the final drive and the gearbox. Depending on the application, the gearbox must have an adequate number of transmission ratios to enable the whole power range of the engine to be used. The transmission must also be built with a high internal efficiency to meet environmental demands and reduce operating costs. It must also be robust, reliable, and low maintenance, and

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should not have adverse reactions to extreme climatic conditions.

Until the early 2000s, mechanical transmissions were used almost exclusively in motor vehicles. They are robust, simple to manufacture and maintain, and can be built with as many gear ratios as required. Their main downside is the relatively high skill required to operate a manual transmission, and the requirement of a clutch used to disconnect the engine from the gearbox while changing gears and to “slip” on starting from standstill. This results in wear, and it essentially mandates a very large transmission ratio for the bottom gear and the reverse gear as they must both be able to move the vehicle from standstill with the clutch slipping, resulting in only a quarter of the normally available engine available for rolling off. Vehicle manufacturers have been aware of this since the introduction of the Ford Model T, which used a planetary gear train (PGT) very similar to the modern Ravigneaux to obtain two forward and one reverse gear selected by depressing a pedal. The transmission used a wet plate clutch.

Several designs of manual planetary-based boxes followed, however the first true automatic gearboxes, able to change gears without reducing or interrupting engine power output, appeared just before World War II. These boxes are either torque converter based with a simple planetary stage for reverse and low gear or use a fluid coupling with three or four forward gears derived by a planetary gear train. All those boxes exhibit low fuel efficiency due to the lack of a lock-up clutch in the hydrodynamic elements, and efforts were made to combat this by providing “split power”, i.e., connecting some transmission elements directly to the engine flywheel. On the other hand, having a fluid stage means that the first gear can have a smaller transmission ratio, especially when combined with a torque converter.
Automatics following modern design principles will appear only in the 1960s with the widespread adoption of the Simpson gear train combined with the three-element torque converter equipped with a lock-up clutch. Further development was in the form of adding overdrive and/or low gear trains to existing gearsets and the addition of electronic shifting controls. From this point, gear trains have evolved in two directions. The first direction is to extract the greatest possible number of gear ratios from a gearset by adding brakes and clutches and is most common for passenger car transmissions. The other design direction prioritizes ruggedness and reliability and prefers to add component geartrains to achieve extra gear ratios. In this case, power circulation is avoided whenever possible, and the use of hollow shafts, especially for sun gears, is reduced to a minimum. It should also be mentioned that manual transmissions have evolved into automated manuals, however they still have clutch wear issues and require very large low gear ratios.

Current designs of PGT based gearboxes use a torque converter stage and can withstand power inputs of about 600 kW, the actual limit being the centrifugal loads inside the gearbox.

Modern planetary gearboxes essentially use combinations of two and three-carrier gear trains controlled by clutches, with a single-carrier stage usually used either as a low gear or overdrive stage (Arnaudov & Karaivanov, 2013, 2019; Jelaska, 2012; Kudriavtsev & Kirdyiashev, 1977; Looman, 1996; Müller, 1998; Tkachenko, 2003; Sanjin Troha, 2011; Sanjin Troha, Vrcan, Karaivanov, & Isametova, 2020).
It is well known that gearboxes using application of PGTs have considerable advantages in comparison to conventional boxes, with expanded possibilities for application (Karaivanov & Troha, 2021; Pavlovic & Fragassa, 2020; Stefanović-Marinović, Vrcan, Troha, & Milovančević, 2022; Vrcan, Stefanović-Marinović, Tica, & Troha, 2022).

Two-carrier PGTs with four external shafts composed of two PGTs of the basic type were primarily considered for the purposes of the research presented in this article, extended to more component PGTs where appropriate. An overview of the internal structure of the researched gear trains is given, and all schemes and layout variants are systematized. Due to the large number of permutations involved, a software program for numerical simulation and calculation of PGT parameters was developed to determine the internal workings and the most important basic parameters of the component gear trains and the whole gear train. The acceptable transmission solutions analysed in this paper were generated using this specially developed computer program. The rule of torques will be used on some gear train examples to demonstrate the calculation procedure and to point out specific design solutions.

2. THE PLANETARY GEARBOX

Planetary gear trains with shifting capabilities are created by connecting several component trains and by adding conveniently placed brakes and clutches. For example, it is possible to extract 6-12 transmission ratios from two simple PGTs just by using clutches and brakes (Kim, 2007; Simpson, 1951, 1953, 1959, 1962), and such layouts may be even interconnected in various ways. Therefore, it is very important to explore the shifting capabilities of each layout and to have tools for the selection of the most appropriate layout. The basic simple gear train for all PGTs researched in this paper is the 2k-h, type A according to Kudryavtsev, or $\overline{\mathrm{AI}}$ according to Arnaudov and Karaivanov. This simple PGT can be very easily represented by a Wolf-Arnaudov symbol. These symbols mark each shaft in a different way, i.e., the sun shaft 1 is marked by a thin line, the ring gear shaft 3 with a thick line, and the planet carrier shaft S by two parallel lines (Figure 1). The relationships between the torques acting on the simple PGT shafts are laid out in equations 1-3. The shaft torques are given as functions of the ideal torque ratios which are the basic value when calculating the transmission ratios of complex planetary gearboxes. It should be noted that the equations in Figure 1 do not take efficiencies into consideration (1):

$$\eta_0 = \eta_{13(S)} = \eta_{31(S)} = 1\,. \tag{1}$$

The torque ratio is defined by (2):

$$t = \frac{T_3}{T_1} = \frac{T_{D\max}}{T_{D\min}} = \left|\frac{z_3}{z_1}\right| > +1. \tag{2}$$

The relations between the torques present in the simple PGT are defined by the equations (3):

$$T_1 : T_3 : T_S = T_{D\min} : T_{D\max} : T_\Sigma = \\ T_1 : t \cdot T_1 : -(1+t) \cdot T_1 = +1 : +t : -(1+t)\,. \tag{3}$$

The following conditions (4) also apply:

$$T_1 \equiv T_{D\min} < T_3 \equiv T_{D\max} < |T_S| \equiv |T_\Sigma|\,. \tag{4}$$

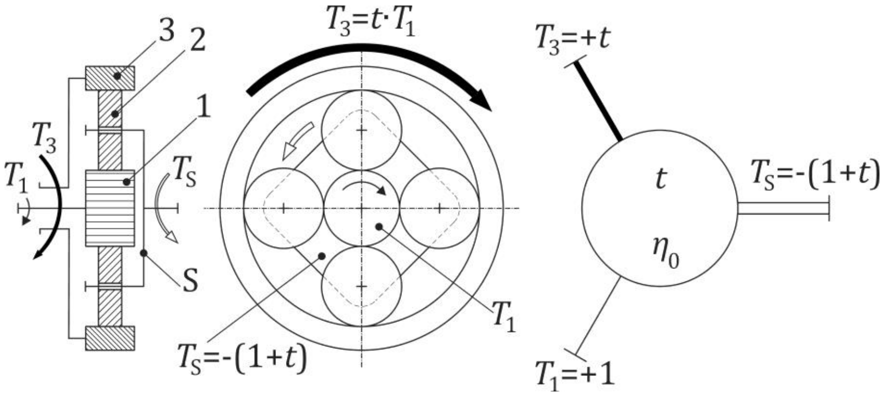

Figure 1. The most used simple planetary gear train, 2k-h or 1A1

3. TWO-CARRIER PGTS WITH TWO CONNECTING AND FOUR EXTERNAL SHAFTS

The characteristics of two-carrier PGTs with two connecting and four external shafts have been extensively analysed in (Sanjin Troha, 2011), with brakes placed on two external shafts, while the remaining two shafts are used to connect the power source and the load. The reactive member is determined by activation of the respective brake, thus changing the both the direction of the power flow through the gear train, and the transmission ratio.

As previously mentioned, these PGTs are composed of two interconnected simple component gear trains of the 1A1 (or 2k-h) type, as those types offer most advantages in actual use. Analysis has shown that these PGTs can be used for a broad spectrum of combinations of transmission rations and can be used in a wide range of applications, ranging from lifting devices and machine tools main drives to main propulsion gearboxes for fishing boats and pleasure craft and railway vehicle main gearboxes or reversing drives.

A basic layout of a two-carrier PGT with two connecting and four external shafts may be seen in Figure 2. Figure 3 shows the possible layouts for two-carrier PGTs with two connecting and four external shafts, while Figure 4 shows the layout variants according to the convention in (Vrcan, Stefanović-Marinović, et al., 2022; Vrcan, Troha, & Marković, 2022). Furthermore, the letters A and B in Fig. 4. are used to display the energy flow and denote the layout variant. Besides that, the locations of brakes Br1 and Br2 are marked, and any variant can be applied to any layout.

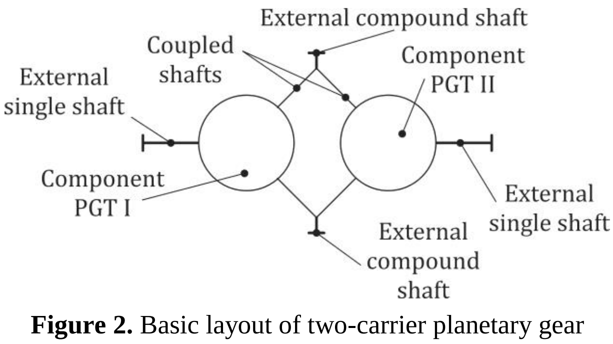

Figure 2. Basic layout of two-carrier planetary gear train

The layout variant nomenclature using the cardinal points must be explained at this point. For example, S36WN(S/E) used for a PGT with two brakes means layout S36, power input on the western external shaft, power output on the northern external shaft, and (S/E) means that the southern and eastern external shafts have brakes mounted on them. Depending on the brake, which is activated, the layout variant is denoted as S36WN(S) or S36WN(E). The remaining unnamed shaft rotates freely and transmits no torque.

Figure 5 displays the kinematic scheme of the S12WS(N/E) planetary gearset. This gearset will be subject to structural analysis, the results of which will be displayed in Figure 6.

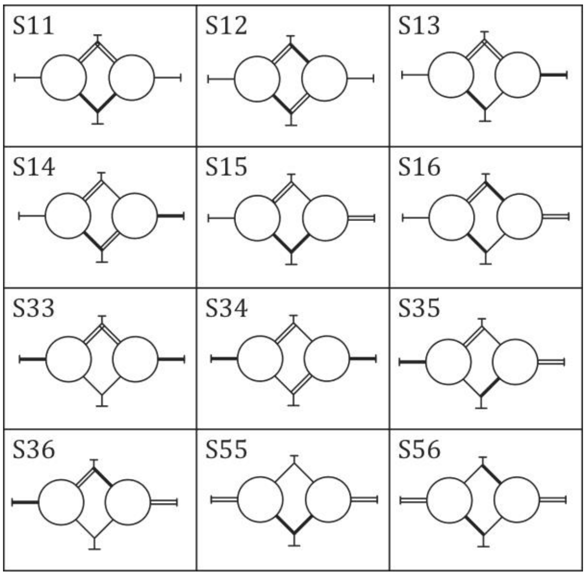

Figure 3. Possible layouts of two-carrier PGTs with four external shafts

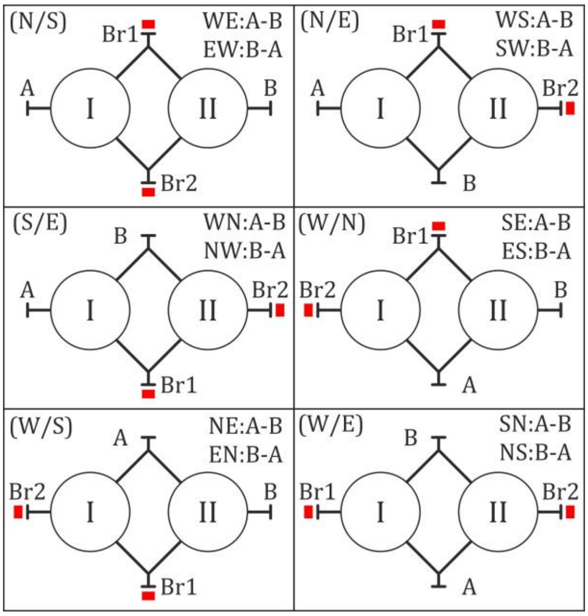

Figure 4. Possible layout variants of two-carrier PGTs

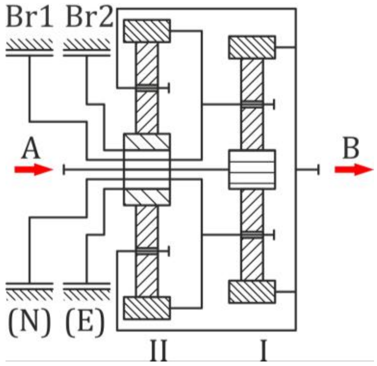

Figure 5. Kinematic scheme of the S12WS(N/E) PGT

The image on the left shows the operation of the gearset with brake Br1 (N) on as S12WS(N), while the right image shows the operation of the gearset with brake Br2 (E) on as S12WS(E). The ideal torques have been determined for all shafts, and transmission ratios i_{Br1} and i_{Br2} have been calculated as a function of the ideal torque ratios of the component PGTs t_{I} and t_{II}. The ideal torque ratio is numerically equal to the ratio of the number of teeth of the ring gear and the sun gear for each component PGT (5,6):

$$t_{\mathrm{I}} = \left| \frac{z_{3\mathrm{I}}}{z_{1\mathrm{I}}} \right|. \tag{5}$$

$$t_{\mathrm{II}} = \left| \frac{z_{3\mathrm{II}}}{z_{1\mathrm{II}}} \right|. \tag{6}$$

The analysis of the transmission ratio functions shows that the PGT with brake Br1 on works like a multiplicator with the output shaft running in the opposite direction to the input shaft, while with brake Br2 on the PGT operates like a reducer with the output shaft rotating in the same direction as the input shaft. It should be also noted that with brake Br1 on only geartrain I operates, while true two-carrier operation is achieved with brake Br2 on, unfortunately with power circulation present.

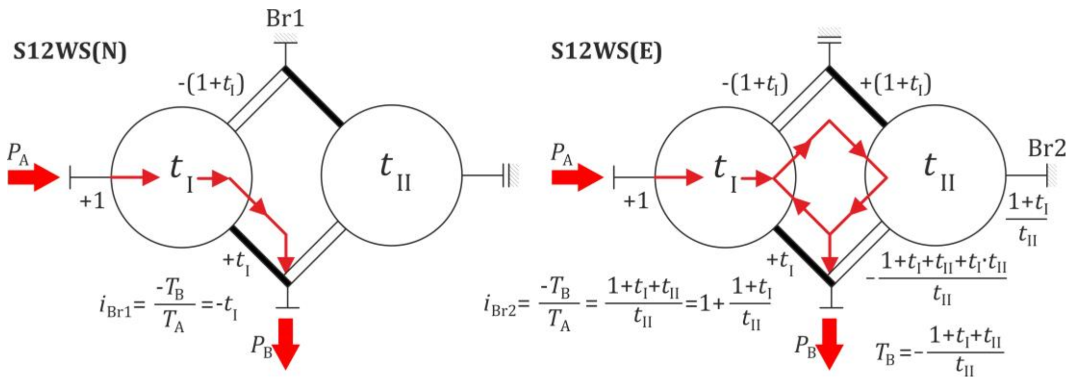

Figure 6. Determining the transmission ratio functions for the S12WS(N/E) gear train with brake Br1 (N) on and with brake Br2 (E) on

The kinematic scheme of complex planetary gear train S12NS(W/E) is displayed in Figure 7.

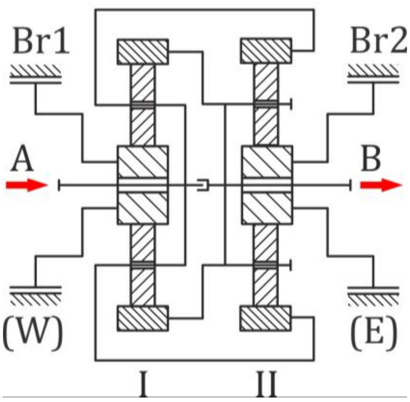

Figure 7. Kinematic scheme of the S12NS(W/E) PGT

This gear train retains the same structure of S12WS, however in this case external shafts N and S are the respective power input and output, while the brakes are placed on external shafts W and E.

The results of the structural analysis of this PGT are displayed in Figure 8. The image on the left shows the operation of the gearset with brake Br1 (W) on as S12NS(W), while the right image shows the operation of the gearset with brake Br2 (E) on as S12NS(E).

It is possible to deduct from the transmission ratio functions that with brake Br1 on the gear train works like a multiplier, with both the input and output shafts turning in the same direction, while with brake Br2 on the gear train works like a reduction gear with both shafts also turning in the same direction. The difference from the case of the S12WS(N/E) train is that with either brake on only one component PGT transmits power, and there is no case of true two-carrier operation or power circulation. It should be noted that this kind of analysis can be performed for any layout variant discussed in this paper.

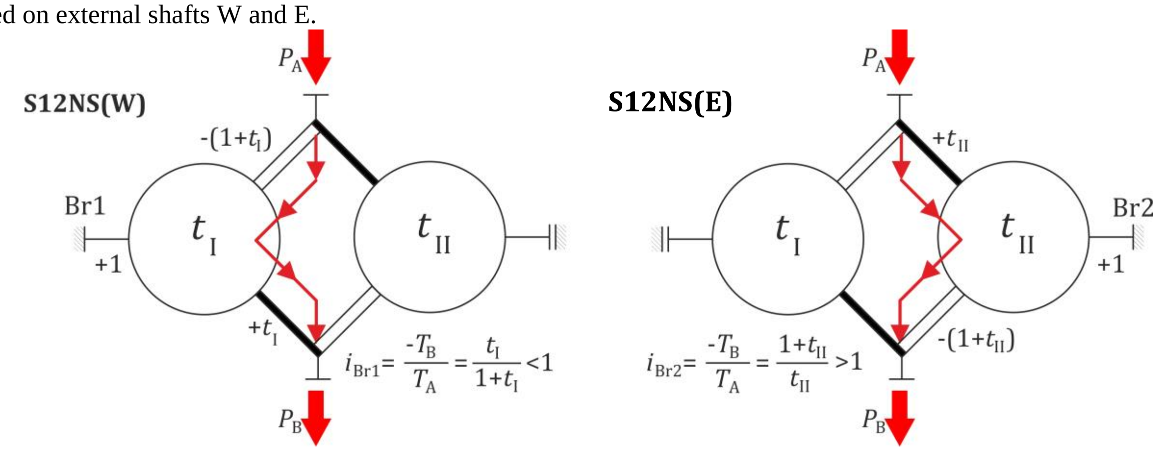

Figure 8. Determining the transmission ratio functions for the S12NS(W/E) gear train with brake Br1 (W) on and with brake Br2 (E) on

4. MULTIVARIANT PLANETARY GEAR TRAIN

It is possible to increase the number of possible transmission ratios of a planetary gear train composed of two or more component trains by adding a control system able to change the input and/or output shaft of the gearbox, as the change of shaft essentially means that the PGT has changed into a different layout variant.

Assuming that only two-carrier PGTs are taken into consideration, as each layout variant can provide two transmission ratios, each compound PGT could provide up to six different transmission ratios. For example, just by enabling the power source and driven machine to be connected to two different shafts (two-variant compound PGT) it is possible to create four transmission ratios, and by expanding this to three (three-variant) it is possible to create up to six transmission ratios.

The kinematic layout and structural scheme of such a PGT, S12NS(W/E)-WS(N/E) is shown in Figure 9. This PGT combines the PGTs from Figure 5 and 7 in one single package. It should be noted that for multivariant PGTs a naming convention has been accepted that the main layout variant with the input shaft closest to the north external shaft is named first, followed by the next external input shaft in the anti-clockwise direction. The four transmission ratios that this gear train is capable of are achieved with the clutch – brake combinations S1-Br2, S1-Br3, S2-Br1 and S2-Br3. Therefore, it is obvious that multivariant PGTs can be successfully used in practical applications, so their possibilities must be thoroughly investigated.

Kinematic analysis of the layout variants has pointed out that it is not possible to combine all variants within one scheme, but only those having a common input or output shaft, and that shaft must remain permanently coupled to the power source or powered machine.

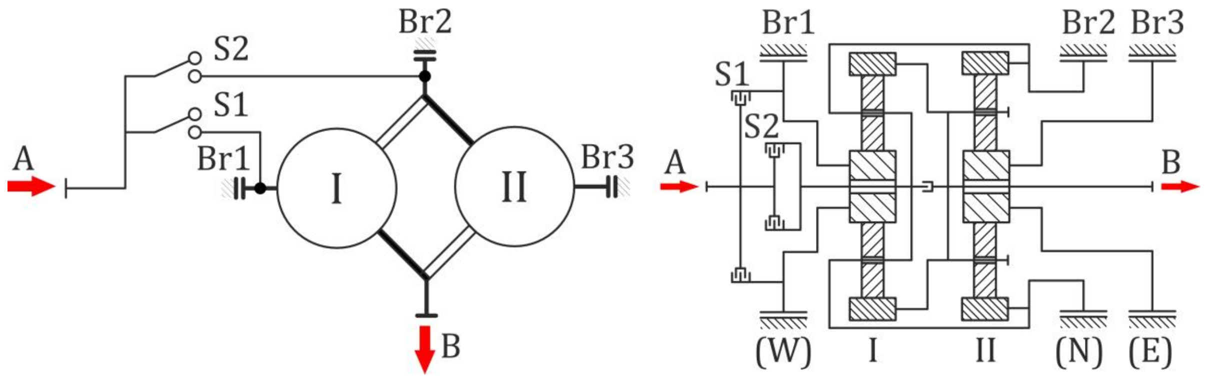

Figure 9. Structural scheme (left) and kinematic scheme (right) of the two-shaft, four-layout gear train S12NS(W/E)-WS(N/E) gear train

Table 1 lists all the possible combinations of two layout variants, giving a total of 12 gear train pairs and their kinematic inversions (driving and driven machine shaft swaps). The analysis of symbolic representations (Fig. 4) has shown that only four pairs of three layout variants make sense from a design standpoint, and those are listed in Table 2. The other combinations cannot be built as their layout variants do not share a common input or output shaft. The sets in table 2 are marked blue for shared input and red for shared output.

Table 1. Theoretical combinations of two different layout variants within the same scheme for multivariant PGTs

No.	Combination	Kinematic inverse
1	WE(N/S), WS(N/E)	EW(N/S), SW(N/E)
2	WE(N/S), WN(S/E)	EW(N/S), NW(S/E)
3	EW(N/S), ES(N/W)	WE(N/S), SE(N/W)
4	EW(N/S), EN(W/S)	WE(N/S), NE(W/S)
5	WS(N/E), WN(S/E)	SW(N/E), NW(S/E)
6	SW(N/E), ES(N/W)	WS(N/E), SE(N/W)
7	SW(N/E), NS(W/E)	WS(N/E), SN(W/E)
8	NW(S/E), EN(W/S)	WN(S/E), NE(W/S)
9	NW(S/E), NS(W/E)	WN(S/E), SN(W/E)
10	ES(N/W), EN(W/S)	SE(N/W), NE(W/S)
11	SE(N/W), SN(W/E)	ES(N/W), NS(W/E)
12	NE(W/S), NS(W/E)	EN(W/S), SN(W/E)

Table 2. Feasible combinations of three different layout variants within the same scheme for multivariant PGTs

No.	Combination	Kinematic inverse
1	WE(N/S), WS(N/E), WN(S/E)	EW(N/S), SW(N/E), NW(S/E)
2	EW(N/S), ES(N/E), EN(W/S)	WE(N/S), SE(S/E), NE(W/S)
3	SE(N/W), SN(W/E), SW(N/E)	ES(N/W), NS(W/E), WS(N/E)
4	NE(W/S), NW(S/E), NS(W/E)	EN(W/S), WN(S/E), SN(W/E)

5. KINEMATIC CHARACTERISTICS OF MULTIVARIANT PLANETARY GEAR TRAINS

The operating conditions of all variants of two-carrier compound PGTs with four external shafts have been determined using the transmission ratio functions derived according to the procedure used in Figures 6 and 8 (Sanjin Troha, 2011). This procedure involves determining whether the PGT is operating as a reducer or a multiplier, and whether the input and output shaft rotate in the same direction or not.

This data was used to synthetise the vectorized data on the operating conditions of all theoretically possible two-variant and three-variant PGTs. The data is vectorized using vectors codified in the norm of AB CD EF GH, where each position has its assigned meaning (Table 3), and each vector position can assume a value increasing from 0 to 9. The data for theoretically possible two-variant PGTs is given in (Table 4), while the data for three-variant PGTs is given in (Table 5). A designer can use these tables to obtain information about the theoretical capabilities of each layout variant quickly and restrict their choices to those variants that can match the required operating conditions. The concepts of "confirmed numbers" will be explained here. In most layout variants, the direction of rotation of the output shaft in relation to the input shaft and whether the PGT will operate as a reducer or a multiplicator does not depend on the ideal torque ratios of the component PGTs but exclusively on the linkages between the PGT elements. In such cases, the vectorization provides complete data on the operating regime. However, some variants exist on which the operating regime also depends on the component PGT torque ratios, and such variants are excluded from Tables 4 and 5.

Table 3. Characterization vector breakdown for multivariant PGTs

Pos.	**Combination Kinematic inverse**
A	Largest confirmed number of positive ransmission ratios
B	Largest confirmed number of negativetransmission ratios
C	Largest confirmed number of reductiontransmission ratios
D	Largest confirmed number of multiplicationtransmission ratios
E	Largest confirmed number of positive reductiontransmission ratios
F	Largest confirmed number of negative reductiontransmission ratios
G	Largest confirmed number of positivemultipication transmission ratios
H	Largest confirmed number of negativemultiplication transmission ratios

Table 4. Vectorized operating condition kinematic data for two-variant PGTs

	Variants			
Layout	WE(N/S), WS(N/E)	WE(N/S), WN(S/E)	WE(N/S), SE(N/W)	WE(N/S), NE(W/S)
S11	21 11 01 00	30 11 10 00	21 11 00 01	30 11 00 10
S12	13 20 11 00	22 20 20 00	22 02 00 20	13 02 00 11
S13	22 40 22 00	31 40 31 00	31 40 31 00	40 31 30 10
S14	13 40 13 00	22 40 22 00	22 22 02 20	13 31 03 10
S15	13 40 13 00	31 40 31 00	31 40 31 00	31 31 21 10
S16	13 40 13 00	31 40 31 00	31 40 31 00	31 31 21 10
S33	21 02 00 01	30 02 00 10	30 20 10 00	30 20 10 00
S34	13 22 11 02	22 31 21 01	22 13 01 21	13 22 02 11
S35	13 22 11 02	31 31 30 01	31 31 30 01	31 22 20 11
S36	13 11 10 01	31 30 30 00	31 30 30 00	31 21 20 10
S55	30 02 00 10	30 02 00 10	30 20 10 00	30 20 10 00
S56	40 13 10 30	31 22 11 20	31 22 20 11	40 31 30 10
	WS(N/E), WN(S/E)	WS(N/E), SE(N/W)	WS(N/E), SN(W/E)	WN(S/E), NE(W/S)
S11	11 22 11 00	02 22 01 01	21 31 21 00	20 22 10 10
S12	31 40 31 00	31 22 11 20	31 31 21 10	31 22 20 11
S13	13 40 13 00	13 40 13 00	22 40 22 00	31 31 21 10
S14	31 40 31 00	31 22 11 20	31 31 21 10	31 31 21 10
S15	22 40 22 00	22 40 22 00	13 31 12 01	40 31 30 10
S16	22 40 22 00	22 40 22 00	13 40 13 00	40 31 30 10
S33	11 04 00 11	11 11 10 01	21 22 20 01	20 22 10 10
S34	31 31 30 01	31 13 10 21	31 22 20 11	31 31 21 10
S35	22 31 21 01	22 31 21 01	13 22 11 02	40 31 30 10
S36	22 21 20 01	22 21 20 01	13 21 11 01	40 31 30 10
S55	20 04 00 20	20 22 10 10	12 04 00 12	20 22 10 10
S56	31 13 01 30	31 13 10 21	22 13 01 21	31 31 21 10
	WN(S/E), SN(W/E)	SE(N/W), NE(W/S)	SE(N/W), SN(W/E)	NE(W/S), SN(W/E)
S11	30 31 30 00	11 22 00 11	21 31 20 01	30 31 20 10
S12	40 31 30 10	31 04 00 31	40 13 10 30	31 13 10 21
S13	31 40 31 00	31 31 21 10	31 40 31 00	40 31 30 10
S14	40 31 30 10	31 13 01 30	40 13 10 30	31 22 11 20
S15	31 31 30 01	40 31 30 10	31 31 30 01	31 22 20 11
S16	31 40 31 00	40 31 30 10	31 40 31 00	31 31 21 10
S33	30 22 20 10	20 40 20 00	30 40 30 00	30 40 30 00
S34	40 31 30 10	31 13 01 30	40 13 10 30	31 22 11 20
S35	31 31 30 01	40 31 30 10	31 31 30 01	31 22 20 11
S36	31 40 31 00	40 31 30 10	31 40 31 00	31 31 21 10
S55	12 04 00 12	20 40 20 00	12 22 10 02	12 22 10 02
S56	13 22 02 11	31 31 30 01	13 22 11 02	22 31 21 01

Table 5. Vectorized operating condition kinematic data for two-variant PGTs

Layout	Variants			
	WE(N/S), WS(N/E), WN(S/E)	WE(N/S), SE(S/E), NE(W/S)	WS(N/E), SE(N/W), SN(W/E)	WN(S/E), NE(W/S), SN(W/E)
S11	31 22 11 00	31 22 00 11	22 42 21 01	40 42 30 10
S12	33 40 31 00	33 04 00 31	51 33 21 30	51 33 30 21
S13	33 60 33 00	51 51 41 10	33 60 33 00	51 51 41 10
S14	33 60 33 00	33 33 03 30	51 33 21 30	51 42 31 20
S15	33 60 33 00	51 51 41 10	33 51 32 01	51 42 40 11
S16	33 60 33 00	51 51 41 10	33 60 33 00	51 51 41 10
S33	31 04 00 11	40 40 20 00	31 42 30 01	40 42 30 10
S34	33 42 31 02	33 24 02 31	51 24 20 31	51 42 31 20
S35	33 42 31 02	51 42 40 11	33 42 31 02	51 42 40 11
S36	33 31 30 01	51 41 40 10	33 41 31 01	51 51 41 10
S55	40 04 00 20	40 40 20 00	22 24 10 12	22 24 10 12
S56	51 24 11 40	51 42 40 11	33 24 11 22	33 42 22 11

6. METHODICS FOR THE SYNTHESIS OF MULTIVARIANT PLANETARY GEAR TRAINS

The transmission ratio functions of the obtainable two-speed variants are the basis for the kinematic synthesis of multivariant PGTs, since the PGTs are essentially combinations of two or three layout variants. The relations of the transmission ratios *i* to the ideal torque ratios t_I and t_{II} for each transmission ratio of every variant are given in (Sanjin Troha, 2011). A graphical representation of the transmission ratio functions of a two-variant PGT will provide four stacked surfaces that share a common domain with the independent variables t_I and t_{II}. The graphical representation of the transmission ratios that can be obtained from a two-variant PGT that can provide four transmission ratios is provided in Figure 10 (left). The intervals of the required transmission ratios I_1, I_2, I_3 and I_4 which satisfy the condition (7) are shown on the z-axis in Figure 10 (right).

$$i_1 \in I_1, i_2 \in I_2, i_3 \in I_3, i_4 \in I_4 \,. \tag{7}$$

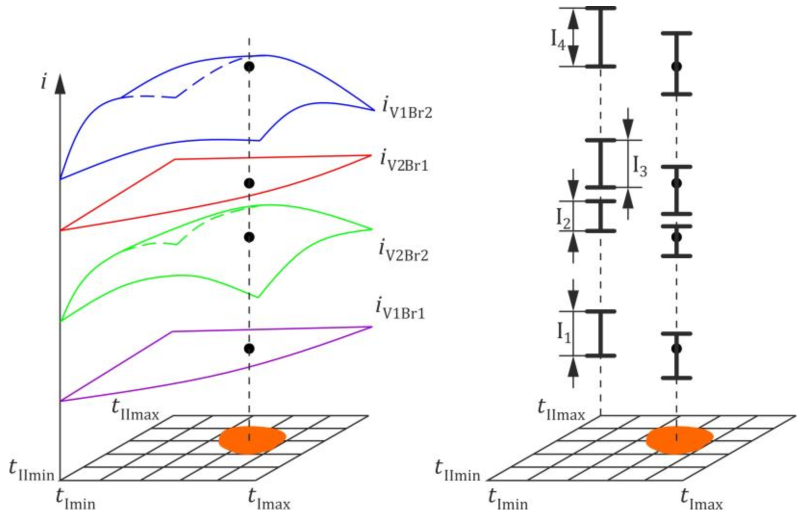

Figure 10. Domain search procedure for detecting the combination of variants that fulfils the required transmission ratio intervals. Left: obtainable intervals. Right: overlap of the required transmission ratio intervals and layout variant capabilities

A software program was developed that determines the values of the transmission ratio functions for every possible combination of ideal torque ratios and verifies whether the value falls within the demanded ranges of ratios I_1, I_2, I_3 and I_4. If such pairs of ideal torque ratios exist, the software lists them as solutions. The solutions are then evaluated according to additional relevant criteria, i.e., diameter of component PGTs, equivalent efficiency ratio etc. (Stefanović-Marinović, Vrcan, Troha, & Milovančević, 2022; Sanjin Troha, Karaivanov, & Vrcan, 2022; Sanjin Troha, Vrcan, Stefanović-Marinović, & Sedak, 2022).

A three-speed multivariant PGT will be synthetised to demonstrate operation of the program, using the test data presented in Table 6.

Table 6. Test data for the synthesis of a three-speed multivariant PGT

Data	Value
Transmission ratio 1	$2,4 \leq i_{k1} \leq 2,6$
Transmission ratio 2	$1,35 \leq i_{k2} \leq 1,45$
Transmission ratio 3	$-2,7 \leq i_{k3} \leq -2,6$
Sun gear tooth number	$z_{1I} = z_{1II} = 18$
Input torque	$T_A = 50$ Nm

The software program has found six combinations (Table 7) that provide two solutions each in the required intervals. In addition to the data on ideal torque ratios, transmission ratios and ring gear pitch circle diameters, the program has determined the exact variant and brake to be used. If the criteria of the minimal outside diameter of the component PGTs is applied, the S36WN(S/E)-SN(W/E) gear train with t_I = 2,5 and t_{II} =2,6667 presents an optimal solution.

Table 7. Valid solution combinations for three-speed PGT synthesis. The optimal solution is in light blue.

Scheme LV	t_I	t_{II}	i_{k1}	i_{k2}	i_{k3}	z_{3I}	z_{3II}	d_{3I} [mm]	d_{3II} [mm]
S12 EN(W/S)-SN(W/E)	2,3333	2,6667	2,571 EN(W)	1,428 SN(E)	-2,666 EN(S)	42	48	94,5	96
S12 EN(W/S)-SN(W/E)	2,5	2,6667	2,466 EN(W)	1,4 SN(E)	-2,666 EN(S)	45	48	101,25	96
S12 WS(N/E)-NS(W/E)	2,6667	2,3333	2,571 WS(E)	1,428 NS(E)	-2,666 WS(N)	48	42	96	94,5
S12 WS(N/E)-NS(W/E)	2,6667	2,5	2,466 WS(E)	1,4 NS(E)	-2,666 WS(N)	48	45	96	101,25
S36 WN(S/E)-SN(W/E)	2,3333	2,6667	2,571 WN(E)	1,428 WN(S)	-2,666 SN(E)	42	48	63	96
S36 WN(S/E)-SN(W/E)	2,5	2,6667	2,466 WN(E)	1,4 WN(S)	-2,666 SN(E)	45	48	61,87	96

The scheme and layout variant data supplied by the program was used to create the structural and kinematic scheme in Figure 11. It must be said that the most convenient procedure is to create the structural scheme first, and then proceed to create the kinematic scheme (Leistner, Lörsch, & Wilhelm, 1987; Müller, 1998; Tica, Vrcan, Troha, & Marinković, 2023; S. Troha, Vrcan, Stefanović–Marinović, & Sedak, 2023; Vrcan, Ivanov, Alexandrov, & Isametova, 2022).

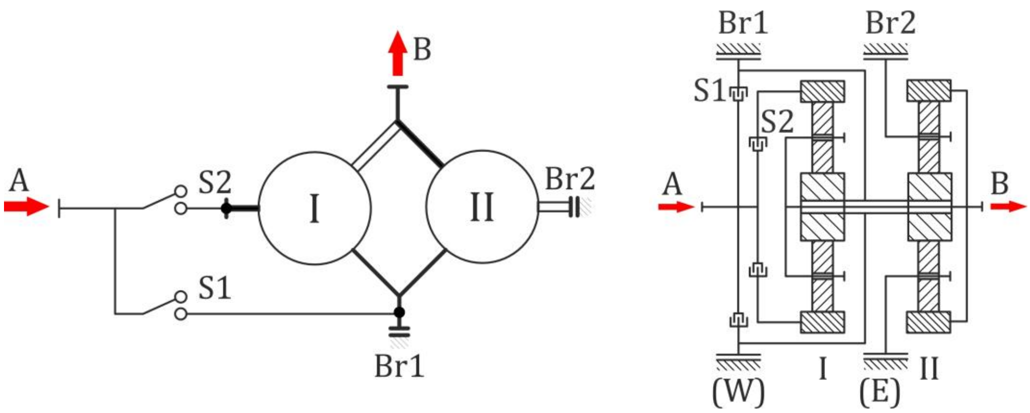

Figure 11. Structural (left) and kinematic scheme (right) of the two-variant PGT S36WN(S/E)-SN(W/E)

7. CALCULATION OF THE LOAD FUNCTIONS OF THE PLANETARY GEAR TRAIN ELEMENTS

The torques acting on the basic elements of the PGT can be determined by separately analysing each component PGT. The procedure begins by selecting a convenient sun gear and assigning it the torque of +1. The torques acting on the other elements are then easily calculated by means of equations (2) and (3). After the analysis of a component PGT is complete, the values are transferred to connecting shafts while taking care that the torque receives an opposite sign when entering the next planetary gearset. For example, if two planetary sets are joined by their sun gears, the torque on the second sun gear will be equal to that on the first gear, but of opposite sign.

In case there are several shafts linked at one point, it becomes a nodal point, and it is important that the sum of torques at each point equals zero. Furthermore, the torques on the input and output members are equal to the sum of all torques related to the respective gear train member. After the calculation is complete, the overall transmission ratio is then calculated using the input torque T_A and output torque T_B (8):

$$i = -\frac{T_B}{T_A} \qquad (8)$$

When dealing with complex, multi-carrier gear trains the complexity of the calculation depends on the starting point. Sometimes several starts might be needed to avoid situations with two unknowns at nodal points. For all torques to be expressed as a function of the input torque, every calculated torque must be multiplied with the reciprocal value of the input torque, which is then reduced to one. All further torques are then calculated by multiplying with the input torque.

The torque on any locked element is an external torque, and it can be used as a sanity check for the calculations as the sum of torques across the shafts of any component PGTs must be equal to zero.

7.1 The Simpson gearset

The calculation procedure will be now demonstrated on the example of a three-speed, two-variant PGT commonly called the “Simpson gearset”. The gearset is well known in automatic transmission design since the 1950s and the abandonment of split-torque transmissions with the introduction of torque converter lockup clutches.

Essentially it is a S36WN(S/E)-S36SN(W/E) gear train, however the S36SN(W) variant is not possible due to kinematics limiting brake placement. The transmission in this form provides two gears in which the input and output shafts rotate in the same direction, and one gear in which the output shaft rotates in the opposite direction. In automotive applications, a direct drive gear is obtained by connecting the W and S shafts together, while an extra multiplication gear was achieved by adding a simple output gearset (Figure 12), which can be easily calculated separate from the main gear train.

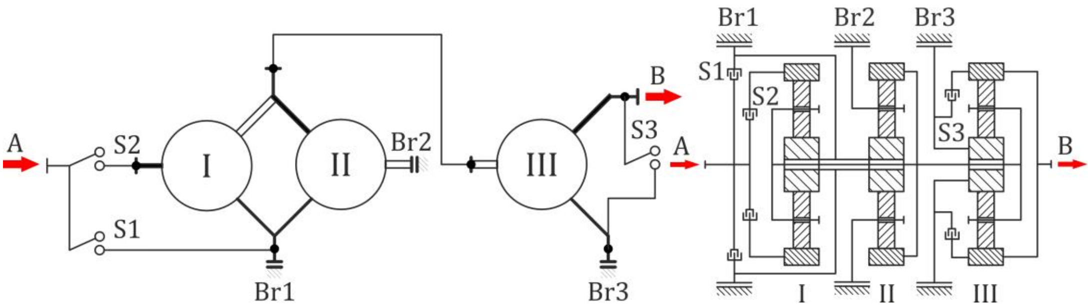

Figure 12. Structural (left) and kinematic scheme (right) of the Simpson gearset, expanded to 4 “forward” and one “reverse” gear

It should be also mentioned that there are two valid placements for gearset 3, either before gearset I or after gearset II. The position behind gearset II is most common, however more recent designs place the gearset before gearset I. This option has the potential to be used as a pre-gearbox to extract six forward and two reverse gears, however the authors are unaware of such a solution being deployed. The highest transmission ratio is achieved in layout variant S36WN(E). This is achieved by closing clutch S1 and applying brake Br2 (Figure 13).

In this case, the power enters through the ring gear of PGT I and then splits over to PGT II before re-joining at shaft N through planet carrier I and ring gear II.

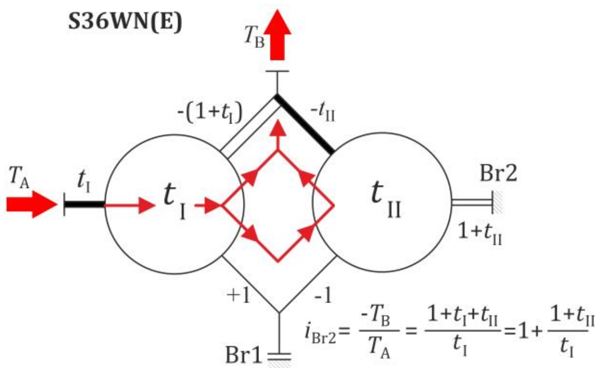

Figure 13. Kinematic scheme of the gearset shifted into S36WN(E) mode

The transmission ratio is given by equation (9):

$$i_{Br2} = -\frac{T_B}{T_A} = \frac{1+t_I+t_{II}}{t_I} = 1+\frac{1+t_{II}}{t_I} \qquad (9)$$

The second highest transmission ratio is achieved by layout variant S36WN(S), which is achieved by closing clutch S2 and applying brake Br1 (Figure 14). In this case, power is transmitted through ring gear I to the planet carrier I with sun gear I locked. The gearset effectively operates in single-PGT mode.

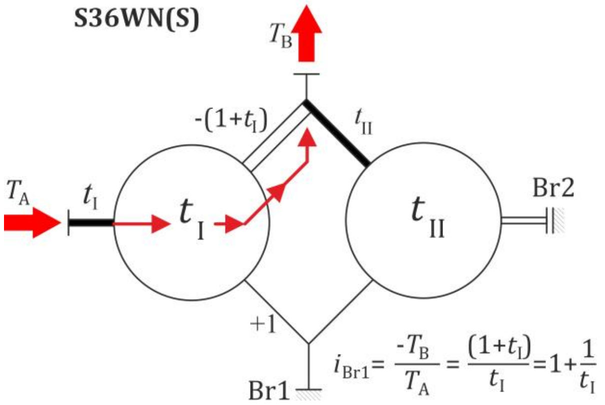

Figure 14. Kinematic scheme of the gearset shifted into S36WN(S) mode

The transmission ratio is given by equation (10):

$$i_{Br1} = -\frac{T_B}{T_A} = \frac{1+t_I}{t_I} = 1 + \frac{1}{t_I} \qquad (10)$$

The gear train has a transmission ratio in which the output shaft rotates in the opposite direction to the input shaft. This is achieved with layout variant S36SN(W) (Figure 15):

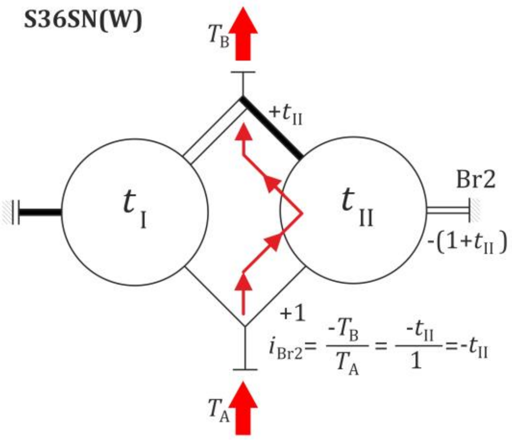

Figure 15. Kinematic scheme of the gearset shifted into S36SN(W) mode

For this variant, clutch S1 is engaged and brake Br2 is on. Power is transmitted from sun gear II to ring gear II with carrier II held stationary. This causes the output element to rotate in the direction opposite to the input shaft. The transmission ratio is given by equation (11):

$$i_{Br2} = -\frac{T_B}{T_A} = -\frac{t_{II}}{1} = -t_{II} \qquad (11)$$

The gearset is capable of another transmission ratio as layout S36SN(E) (Figure 16.). In this mode, the power enters the gearset through external shaft S, enters PGT I via the sun gear, and exits PGT I to external shaft N via planet carrier I.

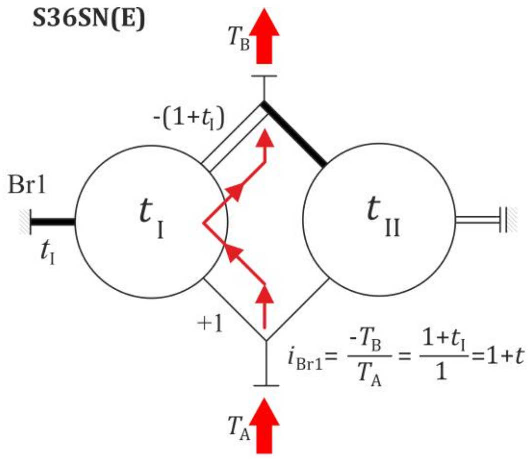

Figure 16. Kinematic scheme of the gearset shifted into S36SN(E) mode

The transmission ratio for this case given by equation (12):

$$i_{Br1} = -\frac{T_B}{T_A} = \frac{1+t_{II}}{1} = 1 + t_{II} \qquad (12)$$

However, this transmission ratio is not kinematically practicable as it renders impossible the installation of clutch S1 (Figure 12).

The gear train has a direct drive mode that is achieved by closing simultaneously the clutches S2 and S1 with both brakes released. In this mode the ring and sun gears of PGT I are locked together and cause the whole gear train to rotate in unison.

PGT III is present only in newer iterations of the gearbox. It was added to provide a transmission ratio in which the output shaft would rotate faster than the input shaft. The transmission ratio for this stage is given by equation (13):

$$i_3 = -\frac{T_B}{T_A} = \frac{t_{III}}{1+t_{III}} \qquad (13)$$

The gear train is normally held locked by clutch S3, connecting the sun and ring gears, however when engaging fourth gear this clutch is released together with brake Br3 engaging to lock the sun gear III, increasing the speed of the output shaft.

To conclude, even if the original concept is somewhat dated, the transmission is robust and proven to a point that a variant of the Simpson gearset combined with the 4th gear PGT was developed for Porsche as a the ZF4HP22HL automatic with manual control (Tiptronic) gearbox (Garrett, Newton, & Steeds, 2001; Sclater, 2011).

7.2 The ZF HP500 gearbox family

The ZF HP500 family of gearboxes is interesting for study as it is a family of gearboxes built for mid to high power applications in city and highway buses.

The gear train is clearly designed for high loads, as most of the shifting is done by three or four brakes depending on the model, and two or three clutches. The base model extracts six forward gears and one reverse gear from a three-PGT gearset (Figure 17).

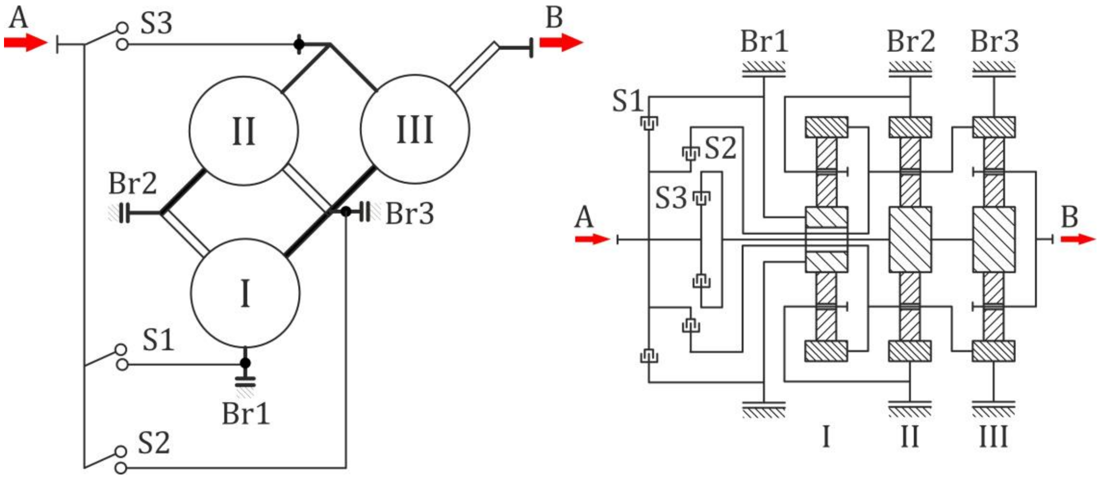

Figure 17. Structural (left) and kinematic scheme (right) of the three-carrier, six-speed ZF 6HP500 gearbox

The gearbox is obviously built for reliability as most interconnections are made through planet carriers and ring gears, with sun gears II and III sharing a common shaft.

The same gearbox exists with four (4HP500) and five (5HP500) gears. For five gears, sixth gear is disabled in the control system, while the clutch connecting the input shaft to rings I and III and carrier 2 is deleted in the four-gear version (Figure 18).

First gear is achieved by engaging clutch S3 and brake Br3, effectively engaging the gearbox in single carrier mode over PGT III, and obtaining a transmission ratio of (14):

$$i_1 = 1 + t_{\mathrm{III}} \tag{14}$$

Second gear engages clutch S3 and brake Br2 to send PGTs II and III into two-carrier mode as S36SE(W). The gear ratio equals (15):

$$i_2 = \frac{1 + t_{\mathrm{II}} + t_{\mathrm{II}} t_{\mathrm{III}} + t_{\mathrm{III}}}{1 + t_{\mathrm{II}} + t_{\mathrm{III}}} \tag{15}$$

For third gear, clutch S3 is engaged with brake Br1 and PGTs I, II and III operate in three-carrier mode. The gear ratio equals (16):

$$i_3 = \frac{1 + t_{\mathrm{I}} + t_{\mathrm{II}} + t_{\mathrm{III}} + t_{\mathrm{I}} t_{\mathrm{III}} + t_{\mathrm{II}} t_{\mathrm{III}}}{1 + t_{\mathrm{I}} + t_{\mathrm{I}} t_{\mathrm{II}} + t_{\mathrm{III}}} \tag{16}$$

Fourth gear activates clutches S3 and S2 at the same time, resulting in PGT III being locked in unison, providing direct drive with $i_4 = 1$.

Fifth gear returns to three-carrier mode, with brake Br1 and clutch S2 engaged (17):

$$i_5 = \frac{t_{\mathrm{II}} + t_{\mathrm{III}} + t_{\mathrm{I}} t_{\mathrm{III}}}{t_{\mathrm{I}} t_{\mathrm{II}} - t_{\mathrm{II}} - 1 - t_{\mathrm{I}} + t_{\mathrm{III}} + t_{\mathrm{I}} t_{\mathrm{III}}} \tag{17}$$

Sixth gear is achieved by activating brake Br2 with clutch S2 at the same time, sending PGTs II and III into two-carrier mode as S36NE(W). The gear ratio equals (18):

$$i_6 = \frac{1 + t_{\mathrm{III}}}{1 + t_{\mathrm{II}} + t_{\mathrm{III}}} \tag{18}$$

Reverse is obtained by activating brake Br3 and clutch S1. This causes PGT I, II and III to operate as three serially connected simple PGTs. Carrier I outputs to ring II, carrier II is locked, which causes sun II to turn sun III opposite to sun I. The gear ratio equals (19):

$$i_{\mathrm{R}} = -\frac{1 + t_{\mathrm{I}} + t_{\mathrm{III}} + t_{\mathrm{I}} t_{\mathrm{III}}}{t_{\mathrm{II}}} \tag{19}$$

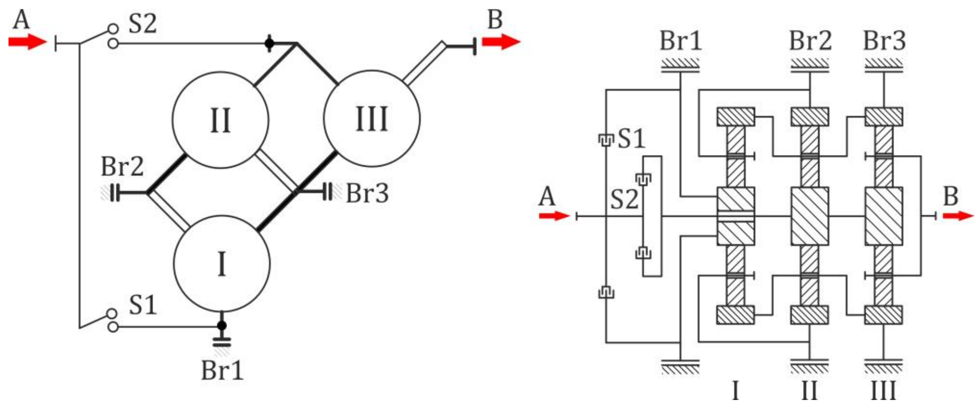

Figure 18. Structural (left) and kinematic scheme (right) of the three-carrier, four-speed ZF 4HP500 gearbox

The gearbox is also manufactured in a high-capacity variant with four gearsets and six forward gears. This variant has an extra gearset IV with its carrier mated to carrier III and an extra brake. This gearset IV is used as first gear, and the gears from 1 to 5 of a standard gearbox become gears 2 to 6 in the four gearset model (Figure 19).

The first gear calculation in this case is numerically equal to calculation for the three-gearset variant (20):

$$i_1 = 1 + t_{IV} \tag{20}$$

It is interesting that this gearset is capable of seven gears, but the seventh gear remains disabled in the control unit.

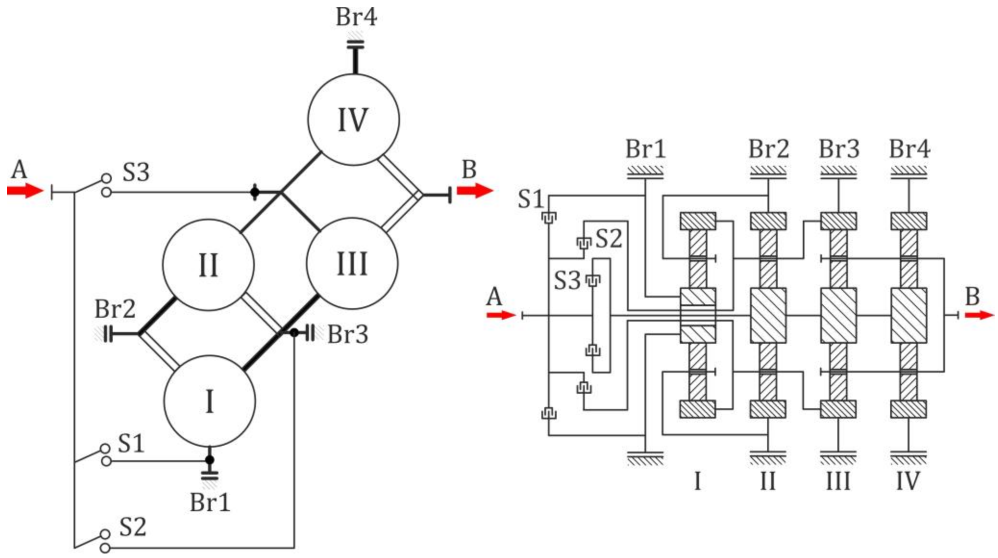

Figure 19. Structural (left) and kinematic scheme (right) of the four-carrier, six-speed ZF 6HP500 gearbox

7.3 The ZF 8HP gearbox family

The ZF 8HP family of gearboxes is interesting as a contemporary gearbox family for mid to high powered road applications and has recently been updated for mild hybrid applications using a “pancake” motor generator bolted to the engine flywheel.

The 8HP family of gearboxes represents a common trend in planetary gearbox design that is characterized by the abandonment of Lepelettier and Ravigneaux sets due to the high torque outputs of electrically assisted internal combustion engines.

The 8HP family of gearboxes is characterized by four component gearsets, of which I and II have interconnected suns, ring II is permanently connected to sun II, sun III is permanently connected to sun IV, and carrier I is permanently connected to ring IV. There are two brakes, Br1 acting on sun I and II, and brake Br2 acting on ring I. Carrier II is permanently connected to the input shaft, while clutch S3 connects the input shaft to the link between ring III and sun IV. This link is connected to sun III and ring II by clutch S1. Finally, clutch S2 connects planet carrier III to planet carrier IV. It should be said that most of the shifting is done by clutches, unlike commercial vehicle transmissions where most of the shifting is done by brakes. The structural and kinematic schemes may be seen in Figure 20.

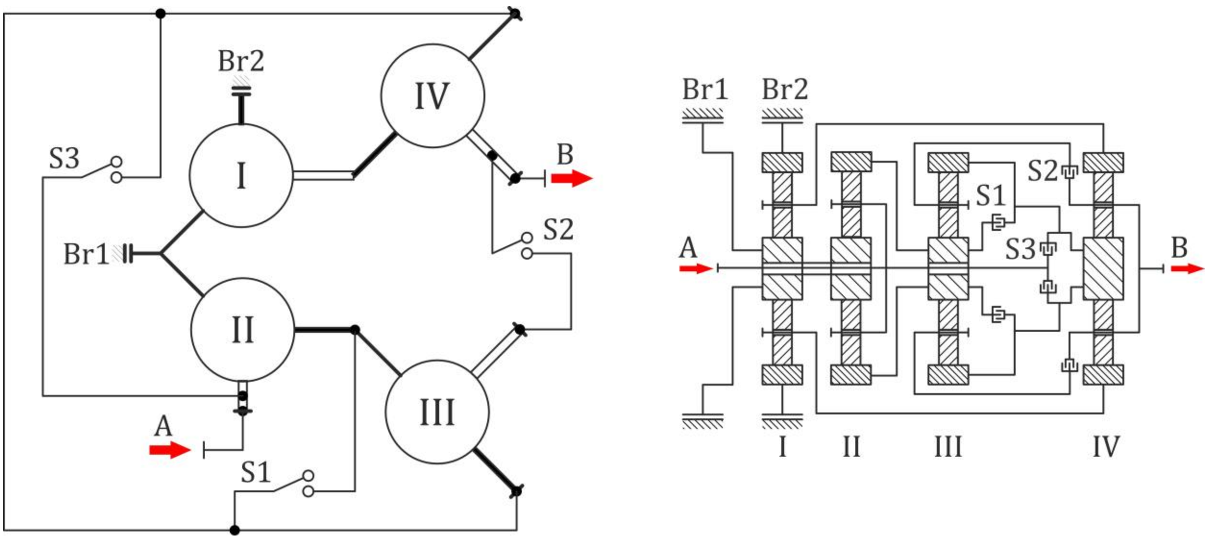

Figure 20. Structural (left) and kinematic scheme (right) of the four-carrier, eight-speed ZF 8HP70 gearbox

First gear is achieved by engaging brakes Br1 and Br2, to lock ring IV, while clutch S3 connects sun gear IV to the input shaft, effectively engaging the gearbox in single carrier mode over PGT IV, and obtaining a transmission ratio of (21):

$$i_1 = 1 + t_{IV} \tag{21}$$

In second gear, brakes Br1 and Br2 engage to lock sun II and ring gear IV. Input is via carrier II to ring II, and clutch S1 completes the connection to the sun gear IV. The PGT is effectively a combination of two linearly joined PGTs, and the gear ratio equals (22):

$$i_2 = \frac{t_{II}\left(1 + t_{IV}\right)}{1 + t_{II}} \tag{22}$$

In third gear, brake Br2 is engaged stopping ring gear I. Clutch S3 connects sun gear IV to the input shaft, while clutch S1 connects ring gear II to the input shaft, causing PGT 2 to turn like a block. This in turn causes PGTs I and IV to operate in two-carrier mode as S36SE(W). The gear ratio equals (23):

$$i_2 = \frac{1 + t_I + t_I t_{IV} + t_{IV}}{1 + t_I + t_{IV}} \tag{23}$$

In fourth gear, brake Br2 holds ring gear I, while the application of clutches S1 and S2 causes gearsets III and IV to rotate like a block, while also connecting carrier I to ring gear II. Input is through carrier II, and there is a constant connection between suns I and II. This in turn causes PGTs I and II to operate in two-carrier mode as S36EN(W). The gear ratio equals (24):

$$i_4 = \frac{1 + t_I + t_{II}}{1 + t_{II}} \tag{24}$$

When fifth gear is engaged, brake Br2 locks ring gear I. Clutch S3 connects sun gear IV and ring gear III to the input shaft, and clutch S2 connects carrier III to carrier IV. Carrier II then drives the sun gears II and I which in turn drive carrier I, operating the gear train in true four-carrier mode. The gear ratio equals (25):

$$i_5 = \frac{1 + t_{IV} + t_{II} t_{IV}\left(1 + t_{III}\right)}{1 + t_I + t_{IV}\left(1 + t_{III}\left(1 + t_{II}\right)\right)} \tag{25}$$

In sixth gear, clutches S1, S2 and S3 engage to connect PGTs III and IV as a block to the input shaft. This results in the PGT being locked in direct drive with i_6 = 1.

For seventh gear, brake Br1 locks sun gears I and II, while clutch S3 connects ring gear III to the input shaft. Input is through carrier II which drives ring gear II and in turn sun gear III. Output is through gearset IV which rotates as a block with carrier III. This results in gearsets II and III operating as a two-carrier train S16NE(W). The gear ratio equals (26):

$$i_7 = \frac{t_{II}\left(1 + t_{III}\right)}{t_{II}\left(1 + t_{III}\right) + 1} \tag{26}$$

Eighth gear is achieved with Br1 holding sun gears I and II, with power flow from carrier II to ring II. Clutches S1 and S2 engage to cause PGTs III and IV to rotate in unison with ring gear II. The transmission ratio equals (27):

$$i_8 = \frac{t_{II}}{1 + t_{II}} \tag{27}$$

Reverse is obtained by activating brakes Br1 and Br2. to hold down sun gears I and II. Clutch S2 engages to connect ring gear III to the output shaft, causing a reversal of rotation. Ring gear III is connected to sun IV, driving PGT IV in the opposite direction of the input shaft. Analysis shows that in this configuration, the gear train is PGT II operating as a carrier to ring multiplier connected to trains III and IV operating as two-carrier train S13WN(E). The gear ratio equals (28):

$$i_{\mathrm{R}} = \frac{-t_{\mathrm{I}} t_{\mathrm{III}} \left(1+t_{\mathrm{IV}}\right) + t_{\mathrm{II}} \left(1+t_{\mathrm{III}}\right)}{1+t_{\mathrm{II}}} \quad (28)$$

8. CONCLUSION

This paper deals with the calculation procedures for the calculation of two-carrier, two-variant switching PGTs. These PGTs enable the creation of gearboxes with not more than four transmission ratios plus direct drive by means of a two-carrier gear train. Extended analysis of the properties of two and three variant trains has been performed, enabling this procedure to be extended to three-variant PGTs providing a maximum of six transmission ratios plus direct drive.

As there is a lot of PGTs fulfilling these general characteristics, the design of PGTs requires methodical approach or software support to avoid selecting a suboptimal solution. The kinematic characteristics of multivariant planetary gear trains are thoroughly analysed, and a procedure for their classification is proposed, together with the methods for planetary gear synthesis. As this procedure covers the operating regimes of every gear drive, this enables the designer to select only the variants that fulfil the application demands. Experience has shown that it is best practice to create kinematic schemes from structural symbols instead of attempting to assemble the gear train from zero. The torque method is an important tool in these calculations. It has been presented in this paper and thoroughly explained on a Simpson gearset. Furthermore, another two contemporary gearsets have been analysed to illustrate the procedure, the first a heavy-duty commercial gearbox, and the other a somewhat light duty box for hybrid motor vehicles. Both PGTs have been analysed to demonstrate the difference in the design approach and reduced to their actual operating units for every transmission ratio, effectively proving that most gearsets operate in two-carrier mode at maximum in most cases, and that operation with more than three PGTs is generally avoided by designers, most probably due to the complexity of the calculation. pass to multiple shafts only on few occasions. Four-carrier mode is usually restricted to few occasions for gearsets that must extract many transmission ratios from a small number of gearsets.

It can be concluded that the methods presented in this paper are applicable to various gear trains, including those with two and more than two carriers, and the methods for selecting the optimal gear train configurations presented in this paper may be successfully applied in future computer programs.

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