Research of the movement of solutions in waterless dry horizons

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Abstract. The movement of solutions in waterless dry layers and the influence of irrigated rock on the radius the filtration flow and the nature of its saturation were established in the paper. The liquid flow curves were constructed for various distances from the source axis. The developed methodology of non-stationary filtration mode during underground leaching determined the filtration coefficients in laminar and turbulent modes. It is necessary to take into account the physical properties of both the rock mass and the filtered leaching solutions. Research has established that the shape of the spreading of solutions in the system of interaction between the injection and discharge wells (the edge boundaries of the streamline) must be considered as a rhombus, and the spreading angle is formed depending on the degree of clogging of the pore volume.

1 Introduction

Issues of the theory of the movement of liquids in drained productive layers, which could be used in engineering calculations of hydrodynamic parameters using the method of underground leaching, are still very poorly developed.

In this regard, the choice of leaching parameters in the design and development of ore deposits is most often made without detailed scientific and technical justification based on empirical dependencies.

A number of experts recommend using the work of V.V. for these purposes. Ivakin, in which a mathematical solution was obtained to the spatial problem of steady motion of a free flow in a homogeneous porous medium, directed from the center of a point source, taking into account the deforming force of gravity.

As a result of the assumptions made, he was able to derive the depression curve equation and find an expression for the filter flow current function [1].

V.M. Nasberg, analyzing the theoretical conclusions of V.V. Ivakina, came to the conclusion that in the equation of the depression curve the pressures do not coincide with their geometric parameters above the reference plane, so he proposed his solution, in which the noticed shortcomings are eliminated by introducing a fictitious flow into the calculation schem.

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2 Materials and methods

Received by V.V. Ivakin and V.M. Nasberg's solutions provide mainly a qualitative characteristic of the depression surface of the infiltration flow. These authors proved that when water is injected into dry porous materials, the depression surface of the filtration flow has the shape of an ellipsoid of revolution, which at a certain depth passes from the source into a circular cylinder with a radius determined from the expression

$$\mathbf{R} = \sqrt{\frac{\mathbf{Q}}{\boldsymbol{\pi}\mathbf{k}}}, \quad (1)$$

which Q – liquid consumption, m^3/day; k – filtration coefficient, m/day.

Our experiments showed that the calculated radius of the depression surface of the infiltration flow is significantly smaller than the experimental one, therefore formula (1) can only be used for approximate engineering calculations [2].

I.R. Richards and E.S. Childs extended Darcy's formula to an unsaturated flow, as a result of which they were able to describe the flow velocity q using the equation

$$q = K(W)\text{grad}\eta(W) + K(W)z, \quad (2)$$

$$\eta = -P(W)_\gamma \quad (3)$$

which K – hydraulic conductivity, m^2/day;

W – saturation as a fraction of the porosity of the medium, %;

η – capillary pressure, MPa;

p – pressure MPa;

γ – liquid density, g/cm^3;

z – a unit vector, parallel to the axis.

The equation expresses a linear relationship between the specific fluid flow rate, averaged over a certain volume, and the hydraulic gradient, when the value of hydraulic conductivity depends on saturation.

The work proposes a mathematical model that takes into account the movement of free and bound moisture, as well as the transition of the first to the second:

$$\frac{\partial q_m}{\partial y} + \gamma(a - u) = \frac{\partial u}{\partial t}, \quad (4)$$

$$\frac{\partial q_g}{\partial y} - \gamma(a - u) = \frac{\partial W}{\partial t}, \quad (5)$$

which q_m, q_g are, respectively, the specific consumption of bound and free moisture per unit area, m3/hour;

W – respectively, the content of cohesive and free moisture in the soil, %;

a – molecular moisture capacity, %;

γ – moisture transfer coefficient;

u – coordinate;

t – time, hour.

By some simplifications of the model proposed in the work, linear equations can be obtained that are suitable for studying the movement of moisture in rocks [3]. Thus, the linearized moisture transfer equation for the problem of intrasoil leaching from horizontal and spherical point drains under the condition of constant initial moisture content W0 and known flow rate on the drain

$$Q(t) = Q_0 + Q_1 \exp(-\lambda t), \quad (Q_1 < 0, \lambda > 0), \qquad \lambda = \beta/2\sqrt{D},$$

which D – capillary diffusion coefficient;

β – constant coefficient determined experimentally.

For Q1=0 these solutions give:

1.Line source:

$$4\pi D(W - W_0)/Q_0 = -E(-r^2/4Dt). \quad (6)$$

2.Point source:

$$4\pi D(W - W_0)/Q_0 = \frac{1}{r}\text{erfc}(r/2\sqrt{Dt}), \qquad (7)$$

which D – average capillary diffusion coefficient;

W – maximum possible content of cohesive moisture in the soil, %;

W_0 – initial humidity, %.

r – distance from the source to the point at which humidity is measured, m.

A numerical method for solving the equation of unsaturated flow in soil was developed by I.R. Philip. To estimate z in a vertical flow, he proposes a solution in the form

$$z = ft^{1/2} + ft + \psi t^{3/2} + \omega t^2 + \ldots\ldots + f_m t^{m/2}, \qquad (8)$$

where $\varphi, f, \psi, \omega, f_m$ – functions of W.

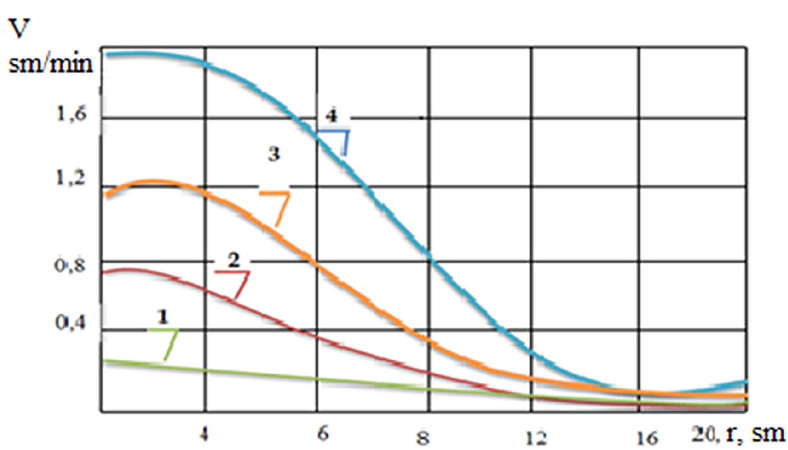
It should be noted that, despite the many solutions to problems of filtration at incomplete saturation proposed by different authors, there is still no method for determining irrigation parameters for the purposes of underground leaching, with the help of which it would be possible to make engineering calculations that meet the requirements of practice.

All known methods contain, as a rule, one or more dimensionless deterministic parameters, the value and method of determination of which are unknown, which does not allow their use for practical purposes. As a result of research on large-scale laboratory installations, the main empirical patterns have been identified formation of infiltration flow in conditions that are as close as possible to natural ones, and the factors influencing it during ore leaching have been established.

Thus, the filtration flow, which is formed during injection into dried horizons through a point source, as well as during complete filtration, has the shape of a cylinder limited at the top by a dome, and the latter is formed when the ore space is not completely saturated with liquid [3].

It is clear from the curves that the degree of saturation of the rock within the boundaries of the flow decreases with distance from its axis, approaching natural moisture. This suggests that the speed of the elementary flow at the source can even reach the filtration coefficient for a given medium, and near its outer boundary it decreases to minimum values.

These curves also indicate that the magnitude of the linear velocity of elementary flows at any point inside the figure changes in direct proportion to the total volumetric flow rate. Consequently, with an increase in the latter, the radius (diameter) of the filtration flow will actually remain constant or change very slightly.



1-100 cm^3/min and 36%; 2-100 cm^3/min, and 44%; 3-400 cm^3/min and 36%; 4-150 cm^3/min and 44%

Fig. 1. Dependence of the speed of liquid movement in weakly watered ores on the distance to the axis of the liquid source at liquid flow and volumetric porosity of the rock mass.

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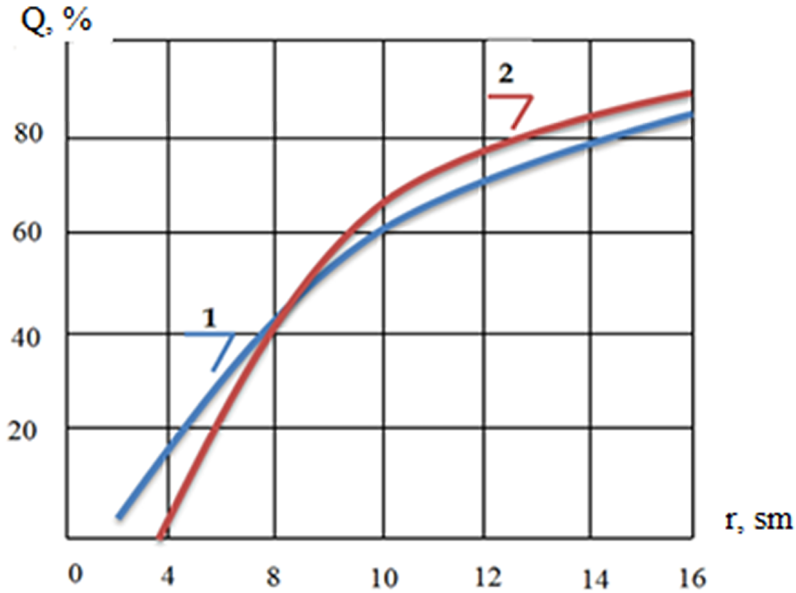
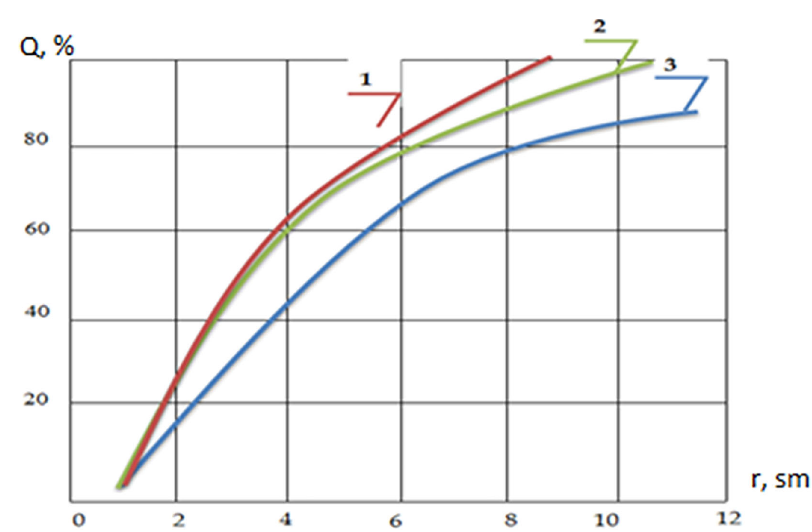
As a result of the research, the influence of the size of the irrigated rock on the radius of the filtration flow and the nature of its saturation was established, on the basis of which the curves in Figure 1 were constructed. However, the structure of the flow changes and the liquid is more evenly distributed in the ore, which is more homogeneous in granulometric composition. In such ore, even a slight change in density affects the structure of liquid distribution inside the infiltration flow (Figure 2).



1-0-5 mm, 2-5-10 mm

Fig. 2. Dependence of the total fluid flow rate in the infiltration flow on the distance to its formation axis.



1-0-5 mm, 2-3-10 mm

Fig. 3. Dependence of total fluid flow in the infiltration flow from the distance to its formation axis.

The patterns of formation of filtration flow during the injection of solutions into non-watered rocks, revealed as a result of experimental studies, form the basis of the developed method [6].

The loss of hydraulic pressure of a solution when filtering it through a productive formation of a certain thickness m can be represented as the following analytical relationship [7].

$$\Delta H = \lambda \frac{L}{d_{cannel}} \cdot \tag{9}$$

which ΔH – the loss of pressure of the solution flow when filtering it through the formation, m;

L – filtration path length, m;

$d_{channel}$ – averaged diameter of curved channels along the productive layer, m;

$\vartheta_{channel}$ – real (true) speed of movement of solutions in pore channels, m/s;

λ – coefficient of hydraulic resistance.

Let's transform the expression (9) in the form of parameters characterizing the solid constituent layer.

Then the loss of pressure of the productive flow during filtration through the hydraulic radius of the pore channels in the zone.

$$\Delta H = \lambda \frac{L}{4R} \cdot \frac{\vartheta^2_{\text{channel}}}{2g}, \tag{10}$$

R – hydraulic radius of pore channels in the productive formation, m;

$$R = \frac{\omega}{\lambda} = \frac{\omega \cdot L}{x \cdot L} = \frac{V}{c}, \tag{11}$$

which ω – the cross-sectional area of the flow along the pore layer, m^2;

η – wetted perimeter of pore channels, m;

V – volume of the pore layer, m^3;

c – total area of the lateral surface of the pore channels, m^2.

The volume of the pore layer can be calculated from the dependence:

$$V = \frac{V_0 \cdot h}{1-\varepsilon}, \tag{12}$$

which V_0 – the volume of particles, the folded filtration layer, m^3;

h – number of particles in the pore layer:

ε – porosity of the layer, fractions of units.

$$\delta = \pi \cdot d^2_{\text{T}} \cdot n \tag{13}$$

which d_{T} – the diameter of the particles of the solid composing the zone, mm;

δ – total area of the lateral surface of particles, m^2.

Combining expressions (9), (10) and (11), we obtain the dependence for the hydraulic radius of the pore (filter) channel

$$R = \frac{V}{c} = \frac{V_k \cdot h \cdot \varepsilon}{\pi \cdot d^2_{\text{T}} \cdot h} = \frac{\pi \cdot d^3_{\text{T}}}{6} \cdot \frac{\varepsilon}{\pi \cdot d^2_{\text{T}}(1-\varepsilon)} = \frac{d-\varepsilon}{6(1-\varepsilon)}, \tag{14}$$

or

$$d = 4R = \frac{2}{3} \frac{d_{\text{T}} \cdot \varepsilon}{(1-\varepsilon)} f,$$

which φ – the shape coefficient of the solid composing zone (according to experimental data for sand φ=0.88). The true velocity of the productive solution in the pore channel and the filtration rate are related as follows

$$\vartheta_{\text{channel}} = \frac{\vartheta_f}{\varepsilon}. \tag{15}$$

According to Ergun's experience, the coefficient of hydraulic resistance during filtration most accurately describing the filtration process is equal to

$$\lambda = \frac{133}{Re_{channel}} + 2{,}34, \tag{16}$$

which Re_{channel} – Reynold's number for the pore channel;

$$Re_{\text{channel}} = \frac{\vartheta_{channel} \cdot d_{\text{channel}}}{V}. \tag{17}$$

Taking into account expressions (14) and (16), after some algebraic transformations, expression (17) will take the form

$$\text{Re}_{\text{channel}} = \frac{2}{3} \cdot \frac{\text{F}}{1-\varepsilon} \cdot \frac{\vartheta_{\text{f}} \cdot \text{d}_{\text{T}}}{\nu} = \frac{2}{3} \cdot \frac{\text{F}}{1-\varepsilon} \cdot \text{Re}, \tag{18}$$

which $\vartheta_{ф}$ – filtration rate (fictitious, used in hydrogeological calculations), m/s;

Re – Reynolds number for the filtered productive layer.

$$Re = \frac{\vartheta_f \cdot d_T}{\nu_0}, \quad (19)$$

which ν_0 – the kinematic viscosity of the productive solution, m²/s (for water ν=1.01·10^{-6}).

When the volume changes [7] concentration of salts in the productive solution, kinematic viscosity is recalculated for the specific salt composition of the solution, can be determined in the expression

$$\nu_1 = \nu_0 \left[1 + \frac{2 \cdot 5 \cdot M}{2(1-1{,}35)M}\right]^2, \quad (20)$$

which M – the volumetric concentration of salts in the productive solution.

For example, with an increase in the volume concentration of the solid to 30 g/l (M = 0,03), the kinematic viscosity of the productive solutions will be

$$\nu_1 = \nu_0 \left[1 + \frac{2 \cdot 5 \cdot M}{2(1 - 1{,}35 \cdot M)}\right]^2 = 1{,}01 \cdot 10^{-6} \left[1 + \frac{2 \cdot 5 \cdot 0{,}03}{2(1 - 1{,}35 \cdot 0{,}03)}\right]^2 = 1{,}09 \cdot 10^{-6} м^2/s.$$

Taking into account expression (18), the value of the hydraulic resistance coefficient (16) will take the form.

$$\lambda = \frac{4}{3}\left(\frac{150(1-\varepsilon)\cdot\nu}{\vartheta_f \cdot d_T \cdot f} + 1{,}75\right). \quad (21)$$

Next, using expressions (9) and (10), we obtain the equation for pressure loss during solution filtration through the productive formation,

$$\Delta H = \lambda \frac{3}{2} \cdot \frac{1-\varepsilon}{\varepsilon^3 \cdot \phi} \cdot \frac{L}{d_{channel}} \cdot \frac{\vartheta^2_{channel}}{2g}, \quad (22)$$

or taking into account

$$\Delta H = \left(\frac{2 \cdot 150(1-\varepsilon)^2 \cdot \nu}{\vartheta_f \cdot d_T \cdot \phi^2 \cdot \varepsilon^3}\right)\frac{L}{d_T} \cdot \frac{\vartheta_f}{2g} + \left(\frac{2 \cdot 1{,}75(1-\varepsilon)}{\phi \cdot \varepsilon^3}\right)\frac{L}{d_T} \cdot \frac{\vartheta_f^2}{2g},$$

which the filtration coefficient:

a) for laminar mode

$$K_{FL} = \frac{\varepsilon^3 \cdot d_T \cdot f^2 \cdot g}{(1-\varepsilon)^2 \cdot 150 \cdot \nu}, \quad (23)$$

b) for turbulent mode

$$K_{FT} = \sqrt{\frac{\varepsilon^3 \cdot d_t \cdot F \cdot g}{(1-\varepsilon) \cdot 1{,}75 \cdot 2}}. \quad (24)$$

Let's consider the spreading of a productive solution along a pore layer with a configuration of the outer boundaries of streamlines, which corresponds to a diamond-shaped shape (Figure 4).

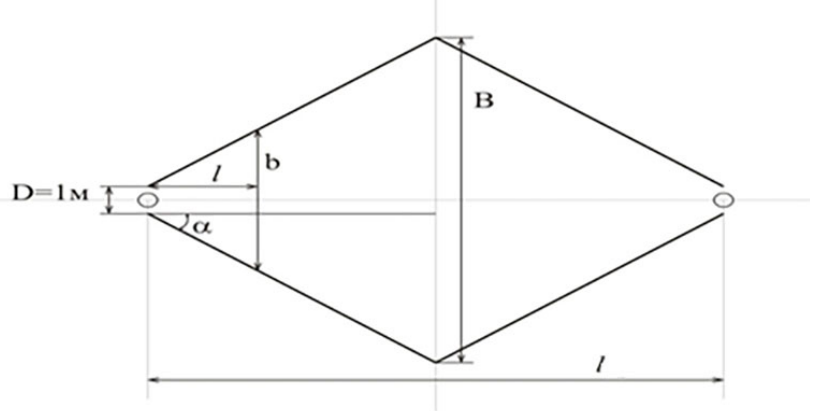


Fig. 4. Scheme of spreading of the edge lines of solution flow.

Current filtration flow width (b)

1. $tg\alpha = \frac{(B-1)}{2} \cdot \frac{2}{l_{wel}} = \frac{B-1}{l_{wel}}, \frac{b-1}{l}; b = l\frac{B-1}{l_{crd}} + 1; b = l \cdot tg\alpha + 1;$

2. $$\vartheta_{\phi} = \frac{Q_f}{\omega} = \frac{Q_f}{b \cdot m} = \frac{Q_f}{\left(l\left(\frac{B-1}{l_{wel}}\right)+1\right) \cdot m}, \text{ filtration speed,} \tag{25}$$

3. $$\frac{\Delta H}{l} = \left(\frac{150(1-\varepsilon)^2 \cdot \nu}{\varepsilon^3 \cdot d_T^2 \cdot f^2 \cdot g}\right) \cdot \vartheta_f, \text{ (for laminar filtration mode).} \tag{26}$$

Dependence of the pressure gradient on the hydrodynamic parameters of the productive layer

$$\Delta H = \left(\frac{150(1-\varepsilon)^2 \cdot \nu}{\varepsilon^3 \cdot d_T^2 \cdot f^2 \cdot g}\right) l \cdot \vartheta_f. \tag{27}$$

The absolute value of the pressure gradient during non-stationary laminar filtration:

$$\Delta H = \left(\frac{150 \cdot (1-\varepsilon)^2 \cdot \nu}{\varepsilon^3 \cdot d_T^2 \cdot f^2 \cdot g}\right) \cdot l \frac{Q_f}{\omega}, \tag{28}$$

which ω – the cross-sectional area of the filtration flow in the volume of the productive layer, m^2;
m –thickness of layer, m.

$$\omega = b \cdot m = \left(l\left(\frac{B-1}{l_{wel}}\right) + 1\right) \cdot m, \tag{29}$$

$$\Delta H = \left(\frac{150(1-\varepsilon)^2 \cdot \nu}{\varepsilon^3 \cdot d_T^2 \cdot f^2 \cdot g} \cdot \frac{Q_f}{m} \int_0^{l_{crd}/2} \frac{l}{l\left(\frac{B-1}{l_{wel}}\right)+1} dl\right). \tag{30}$$

Let's consider the integral within $0 \div l_{wel}/2$,

$$\int_0^{l_{wel}/2} \frac{l}{l\left(\frac{B-1}{l_{wel}}\right)+1} dl = \left(\frac{l_{wel}}{B-1}\right)^2 \left[1 + \frac{B-1}{l_{wel}} \cdot l - ln\left(1 + \frac{B-1}{l_{wel}} l\right)\right] =$$

$$\left(\frac{l_{wel}}{B-1}\right)^2 \left[\frac{B-1}{2} - ln\left(1 + \frac{B-1}{2}\right)\right], \tag{31}$$

$$\Delta H = 2 \cdot \frac{150(1-\varepsilon)^2 \cdot \nu \cdot Q_f}{\varepsilon^3 \cdot d_T^2 \cdot f^2 \cdot g \cdot 3600 \cdot m} \cdot \left(\frac{l_{wel}}{B-1}\right)^2 \cdot \left[\frac{B-1}{2} - ln\left(1 + \frac{B-1}{2}\right)\right]. \tag{32}$$

3 Results

From the hydrodynamics of groundwater it is known that the steady-state regime of liquid filtration in the pore volume of a rock mass is characterized by continuity of flow; from the hydrodynamics of groundwater it is known that the steady-state regime of liquid filtration in the pore volume of a rock mass is characterized by continuity of flow. The research results are presented in Table 1.

Table 1. Hydrodynamic parameters of solution filtration.

ΔH, m	m, m	l, m	b, m	ε, average value	ν, m^2/s	D_{channl} m	f	q, m^3/ hour	g, m^2/ sec
0.783068	10	50	15	0.4	0.000001	0.001	0.88	2	9.8
1.127618	15	60	15	0.4	0.00001	0.001	0.88	3	9.8
1.578796	20	65	15	0.4	0.00001	0.001	0.88	4	9.8
1.873746	25	63	15	0.4	0.00001	0.001	0.88	5	9.8

where
l – current distance from the injection well to the pumping well, m;
2 – maximum spreading width of the filtration flow, m.

For turbulent filtration of productive solutions through the formation, the expression for pressure loss during unsteady filtration will take the form:

$$\Delta H_T = 2\sqrt{\frac{(1-\varepsilon)\cdot 1{,}75\cdot 2}{\varepsilon^3\cdot d_T\cdot f\cdot g}\cdot\frac{Q_f}{3600\cdot m}\cdot\left(\frac{l_{\text{wel}}}{b-1}\right)^2\cdot\left[\frac{B-1}{2}-ln\left(1+\frac{B-1}{2}\right)\right]}, \quad (33)$$

4 Conclusion

The developed methodology of non-stationary filtration mode during underground leaching determined the filtration coefficients in laminar and turbulent modes; it is necessary to take into account the physical properties of both the rock mass and the filtered leaching solutions. Research has established that the shape of the spreading of solutions in the system of interaction between the injection and discharge wells (the edge boundaries of the streamline) must be considered as a rhombus, and the spreading angle is formed depending on the degree of clogging of the pore volume.

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