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Employing behavioural portfolio theory for sustainable investment: Examining drawdown risks and ESG factors

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ABSTRACT
This study uses behavioural portfolio theory (BPT) within the Markowitz Portfolio Theory framework to enhance portfolio management by focusing on sustainability and risk mitigation during market downturns. It selects portfolios to hedge against market lows using Conditional Drawdown at Risk (CDaR) and Expected Regret of Drawdown (ERoD). These measures help choose securities that perform well during a market decline. This study applies drawdown-based risk metrics to assist institutional investors and fiduciaries in making informed investment and fund management decisions. By merging BPT with Markowitz's mean–variance framework, selected investments are maintained above a safety threshold, contributing to the portfolio's overall quality and sustainability. Additionally, by incorporating an Environmental, Social, and Governance (ESG) preference function, the findings suggest that BPT built portfolios meet traditional performance standards and align with socially responsible investment principles, thereby offering higher utility and alignment with investor values focused on sustainable investing.

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Capital Asset Pricing Model (CAPM); drawdown; CDaR; ERoD; behavioural portfolio theory (BPT); ESG

1. Introduction

1.1. Portfolio selection

Any conjecture that implies a study on either portfolio theory or needful financial risk management considers the capital asset pricing model (CAPM) introduced by Sharpe (1964, 1992) as a foundational model. Its foundation is the portfolio optimisation of the mean-variance problem (Markowitz, 1991). A security's anticipated or expected return vis a vis the market's anticipated excess returns are linked by the Standard Beta (β). Specifically, in the context of portfolio management, β has been a crucial indication of asset performance. Individual companies are generally ranked and analysed according to how much they vary from the market, with the overall market having β of 1.0 as a benchmark. The popular Markowitz-defined mean-variance portfolio optimisation problem attempts to prune the variance section of a selected portfolio to maximise returns.

As an alternative to variance, other non-symmetric considerations of risk profiles have been proposed, particularly the Conditional Value-at-Risk (CVaR), which describes the conditional

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expected loss exceeding the value-at-risk (VaR), which was established initially for continuous distributions by Rockafellar and Uryasev (2000) and further introduced for discrete distributions in subsequent work (Rockafellar & Uryasev, 2002). The concept of semi-variance can also be included in the list of non-symmetric risk measures, which involves calculating the portfolio's potential downside risk by observing the dispersion of all instances in a collection of data that deviate from the mean or target value.

However, variance, CVaR, semi-variance, and several other non-symmetric risk indicators have significant inherent limitations; for instance, they are static features that do not consider prolonged successive portfolio losses. Moreover, these risk measurements do not account for worst-case losses, are difficult to calculate for large portfolios, and are nonadditive (in the case of CVaR). Therefore, instead of static measurements, risk functions and measures that are dynamic to drawdowns are frequently utilised in effective portfolio management. The modern objective of fund managers is to create risk-averse portfolios that offer minimal drawdown. Maximum drawdown is the most popular drawdown feature. However, from a practical standpoint, this measure is not an ideal risk indicator because it considers only one particular market event on a historical sample path involving prices. Several limitations of previous risk methods have given rise to another risk measure, known as the Conditional Drawdown-at-Risk (CDaR). In effect, the CDaR method averages a specific percentage segment of the largest portfolio drawdowns across an investment period (Chekhlov et al. 2005); this method has also been adopted in this study.

The CDaR refers to the CVaR of a portfolio's cumulative drawdown observations. The most important quality of CDaR is its ability to include all the necessary significant characteristics of measures that exhibit a deviation, including convexity, non-negative homogeneity, non-negativity, and invariant characteristics for constant translation. Chekhlov et al. (2005) comprehensively explained these factors. Notably, within the financial sector, systemic dependencies were found using CDaR.

Additionally, this study introduces the Expected Regret of Drawdown (ERoD), a new drawdown-based risk measurement method. ERoD refers to the average drawdowns greater than a certain threshold. The mean of losses that surpass a certain set threshold is termed the drawdowns' observed expected regret; at times, it is also referred to as the Low Partial Moment. Good hedges against market drawdowns are shown by ERoD β. Securities that deliver a negative and low ERoD are helpful when building a portfolio. Comparing ERoD β to CDaR β, ERoD β has a conceptual edge. Moreover, as ERoD β is calculated based on the observed drawdowns that cross a defined boundary, there's often a limit imposed over the sample size of the examined drawdowns.

The ERoD method, its calculations, and equivalence with CVaR were established by Testuri and Uryasev (2004) and considered for modelling the necessary drawdown betas. Further research involving ERoD β and portfolio optimisation was carried out by Ding and Uryasev (2022). However, CDaR β only considers an undetermined proportion of the greatest drawdowns.

1.2. Portfolio optimisation and characteristic analysis of behavioural portfolio theory in close connection to ESG preference

Owing to the convenience of probabilistic modelling for a variety of scenarios, behavioural asset allocation models have recently attracted increasing attention. Numerous investors and fiduciaries have been debating the advantages of using behavioural models over traditional mean-variance models, vis a vis the trade-offs involved. In this study, we compare portfolios or asset allocations produced by the behavioural portfolio theory (BPT) developed by Shefrin and Statman (2000) with mean-variance theory (MVT) (Markowitz, 1952). A portfolio or asset allocation that corresponds to the minimum risk provided by a dedicated return is referred to as the optimal portfolio. Markowitz's mean-variance optimisation uses variance as the risk measure, and illustrating a graph is commonly known as the efficient frontier, which depicts the set of portfolios that are optimal and present the lowest risk for a given level of expected return, or vice-versa. However, not only do statistical relations and terms define a portfolio's integrity, but some other typical characteristics concerning behavioural aspects such as investor sentiment, the period for investment, and the objective and appetite for the riskiness of investment are also important. Shefrin and Statman (2000) created the behavioural portfolio theory (BPT) as an alternative model of portfolio choice to better capture these various characteristics of behavioural investment aspects. The basis of BPT stands in stark contrast to Markowitz's mean-variance theory. Two important studies in this area include Roy's (1952) model, which defines the risk variable as the chance of investment ruin, and Lopes's (1987) study of fear and hope in investors' minds driving their investment decisions. The model proposed by Roy (1952) incorporated a safety-first constraint in its formulation that aimed to prevent investors' wealth from decreasing in most scenarios (i.e. states of nature of the portfolio at different time intervals).

The first stage of our analysis relating to portfolio selection involves selecting stocks from a population based on the CDaR and ERoD drawdown risk measures. We chose stocks that exhibited an inverse movement to market decline and analysed their behavioural aspects. The necessary steps and calculations for finding such stocks are elaborated upon in subsequent studies. The first stage of BPT involves computing the proportion of portfolios that pass the BPT model's first safety constraint. To guarantee that the results are exhaustive, we considered various subsistence levels and also took into account various maximum allowable probabilities of ruin with various degrees of subsistence. These findings demonstrate the existence of behavioural portfolios in various market environments. However, these specific findings also test the idea that safety constraints should alert investors to unfavourable investment conditions and advise them to leave the market. The second stage involves analysing the portfolio's sustainability and responsibility traits. We conducted a comparative study of the utility derived from the environmental, social, and governance (ESG) Preference function that falls under the BPT. Portfolios that were not under the ambit of the BTP were defined using a suitable ESG preference function. The results show that behaviourally apt portfolios (i.e. portfolios falling inside the BPT efficient frontier) have higher utility derived from the ESG Preference function than their counterparts outside the BPT efficient frontier during non-pandemic hit time horizons. The results indicate that the pandemic era has had a negative impact on the 'sustainability' and portfolio losses for a given probability of portfolios with less utility derived from the ESG Preference function that leads to irrational investments.

This study establishes the relationship between portfolio selection and portfolio optimisation in the context of sustainable and responsible investments. The study finds an important application of holding a portfolio that is drawdown-averse, performs inversely to the market when it is in decline, and has sustainable investment attached to the portfolio, ensured by the probability of safety that investment will generate positive returns and has better utility than any other non-safe portfolio. Thus, we confirm that the constructed portfolio is socially responsible and averse to market downturns.

This research integrates the portfolio selection methodology and optimisation techniques. The integration is unique in the sense that it achieves a safe investment by calculating the chances of the investment not decreasing and subsequently measuring the utility derived from an ESG preference function for portfolios that are both safe and drawdown-averse.

The remainder of this paper is structured as follows: Section 2 reviews the existing related literature. Section 3 describes the dataset and relevant methodologies. Section 4 presents the empirical outcomes and results. Section 5 presents the discussion, insights, and conclusions.

2. Literature review

The foundational idea of effective portfolio management is propounded by the famous CAPM introduced by (Sharpe, 1964; Lintner, 1965), and (Mossin, 1966). The CAPM constitutes a materially essential idea in contemporary finance. It aligns with modern theories that involve portfolio management and finds applications in security valuation, performance evaluation of different asset classes, and portfolio risk management. CAPM may be analysed from different perspectives; one is dependent on the fact that 'risk' assumes the value of variance, which, in turn, is evident when the mean-variance portfolio problem, as defined by Markowitz, is reformulated for fulfilling optimality conditions. Another outlook of CAPM is that it involves linearity and relates the predicted returns or gains of an equity or security and a diverse market portfolio with considerable depth by incorporating Asset β as the slope of the linear function. Thus, it may be assumed that analysing market scenarios during drawdowns requires risk metrics and measures that are sensitive to drawdowns. This study deals with robust risk measurement techniques that include drawdowns. Grossman and Zhou (1993) conducted the earliest fundamental venture to work out a portfolio optimisation problem featuring a continuous-time environment, maintaining a constraint on the portfolio's relative drawdown consisting of a single risky asset. Furthermore, Cvitanic and Karatzas (1994) extended the findings of Grossman and Zhou from a single asset to several risky assets in portfolio constituents.

In addition, several other reviews attempted to create strong foundational and implementational knowledge about portfolios, vis-a-vis their reaction to drawdowns, consisting of capital allocation strategies with a drawdown constraint propounded by Meucci (2010), mean-variance portfolio problems with drawdown constraints by Alexander and Baptista (2006), and drawdown modelling and estimation conducted by Leal and Mendes (2005) and López and Peijan, (2004). The principal basis and scope of the study were conducted structurally and methodologically, following Chekhlov et al. (2005). Furthermore, Chekhlov et al. (2005) proposed the notion of CDaR for simple single-path portfolios and extended the definition of CDaR for multisample paths in their follow-up work.

The ERoD method, its calculations, and equivalence with CVaR were established by Testuri and Uryasev (2004) and considered for modelling the necessary drawdown betas. Further research involving ERoD and portfolio optimisation was conducted by Ding and Uryasev (2022).

Roy (1952) and Shefrin and Statman (2000) introduced BPT as the primary basis for its inspection features. Roy (1952) was among the first to discuss a very important proposition of the well-being technique. The basic premise is that investors want to reduce the probability of failure or ruin. Telser (1955) further expanded Roy's concept by suggesting an appropriate threshold for the likelihood of ruins. Notably, when the likelihood of ruin is below a predetermined threshold β, a portfolio is protected. In addition, other studies (e.g., Pfiffelmann et al., 2016) examined and compared portfolios produced by both BPT and MVT.

Post-COVID-19 pandemic Ethical Investing, also called socially responsible investing (SRI), has gained considerable attention (Singh, 2020; Broadstock et al., 2021). SRI screens companies based on (ESG) practices (Renneboog et al., 2008). Investors can choose to invest in companies with good ESG practices or exclude those with poor ESG practices (Meunier & Ohadi, 2023; Lagerkvist et al., 2020). SRI is a growing trend, as more investors look to invest in companies that have a positive impact on the world (Bollen, 2007).

The literature provides various perspectives on the economic viability of SRI in the stock market. Advocates of SRI highlight the 'doing good while doing well' view that there is a positive relationship between social and financial performance and that investors can earn higher returns by choosing high-rated SRI stocks (Brogi, & Lagasio, 2019). This is based on the 'good management hypothesis', which suggests that meeting the requirements of major stakeholders, such as ensuring product enhancement or job security, can lead to higher financial performance as a result of continued business or firm loyalty (Cornell & Shapiro, 1987; McGuire et al., 1988). Environmentally friendly companies are less likely to be fined or sued, because they are more likely to comply with environmental regulations. They are also more likely to be considered responsible corporate citizens, which can lead to increased sales. Additionally, socially responsible companies are more likely to have higher employee morale because they are more likely to provide good working conditions and benefits. Companies with good governance are likely to be seen as reliable and trustworthy, which may result in increased sales. They are more likely to attract and retain top talent, leading to increased innovation and productivity (Brekke & Nyborg, 2004). Giese et al. (2019)

found that companies with strong ESG practices have a positive impact on their valuation and performance, as they tend to have lower capital costs, higher valuations, higher profitability, and lower exposure to tail risk. Friede et al. (2015) analysed over 2200 individual stocks and found that approximately 90% of them showed a non-negative relationship between ESG and corporate financial performance.

Opponents of SRI argue that firms that invest in socially responsible activities are at a competitive disadvantage compared with firms that do not, which can impact investment performance (Knoll, 2002; Barnett, 2007). The managerial opportunism hypothesis proposes that managers may seek to maximise their gains during times of economic prosperity while attempting to appease shareholders through social activities during times of poor financial performance (Posner & Schmidt, 1992). Studies have found that portfolios with low sustainability performance outperform their peers with high sustainability performance (Brammer et al., 2006; Hong & Kacperczyk, 2009). In addition, social screens may eliminate large and stable blue-chip companies, leaving only smaller and more volatile companies with lower return potential. Additionally, SRI may eliminate or favour certain industries, limiting diversification by shrinking portfolios (Ortas et al., 2014). Some studies find that SRI neither adds nor destroys value in terms of risk-adjusted returns because corporate social responsibility is not priced by the market (Hamilton & Statman, 1993; Statman, 2006; Humphrey et al., 2012). Others show that ESG investment is not well integrated into investor sentiment in Asian economies (Dhasmana et al., 2023; Gutsche et al., 2021).

2.1. Research gaps

Much research covering portfolio optimisation subject to various types of risk measures (symmetric, nonsymmetric, and robust) has been conducted. Market decline has several macroeconomic repercussions, and investors act cautiously during these periods. This requires a portfolio that is market drawdown-averse, stable, sustainable, and responsible. Existing literature comprehensively touched upon the behavioural aspects of the portfolio vis a vis its inherent risks. However, there is an opportunity to examine the accompanying correlation between the 'sustainability factor' represented through the utility derived from the ESG Preference function and the BPT portfolios.

Acknowledging the existing literature and the scope for further analysis, we identified the following research gaps:

RQ1: Do the portfolios constructed from drawdown-averse stocks satisfy the safety-first constraint; if so, how does the proportion of portfolios satisfying the safety-first constraint effectively vary with the changing probability of ruin?

The primary goal is to obtain stocks that are inversely correlated with the market and gain value when the market declines. This is achieved by minimising the Conditional Drawdown-at-Risk at various levels of significance and attaining the drawdown beta from the modified CAPM. Subsequently, we study the safety proposition of our investment in drawdown-averse stocks and deduce whether the majority of portfolios pass the safety-first constraint by varying the chance that the investment will fall. This study was conducted using a BPT framework with safety-first constraints.

RQ2: Portfolios constructed from drawdown-averse stocks within the BPT efficient frontier have a higher utility derived from the ESG Preference function than portfolios outside the BPT efficient frontier, which essentially indicates socially responsible and stable investing.

The research proposition essentially indicates the effectiveness of our portfolio in displaying socially responsible and stable investments.

3. Data and methodology

Our dataset comprises the daily stock price data of 500 US companies that constitute the S&P500 index and their ESG scores. Daily price data were obtained from the Bloomberg Terminal. ESG data and the data required for the creation of the Fama-French factors were also taken from Bloomberg, comprising the recent Bloomberg E, S, and G scores for S&P500 companies. The proxy for market-related calculations and benchmarks is the S&P500 Index. The main purpose of selecting this particular dataset was to determine the associated depth and diversity of the companies present within the lot. It comprises a large cap that trades 500 stocks across several industry sectors, which essentially represent the strength of the corporate economy. The analysis was conducted over 10 years, from December 2012 to December 2022. For the characteristic analysis of the BPT, we employed a simulation method, discussed further in the following section, where the time horizon spanned 10 years (i.e. 1 December 2012–1 December 2022). Importantly, prior to analysing the dataset, we needed to address the theoretical background behind portfolio optimisation during drawdowns, along with CAPM with both CDaR and ERoD and draw BPT-efficient frontiers.

3.1. Conditional Drawdown-at-Risk (CDaR)

Let us assume $r_1, \ \ldots., r_T$ are the rates of return of a security or stock for T time instances in succession, such that r_i denotes the rate of return for period i, $i = 1, \ \ldots, T$. We define w_t as the rate of return (cumulative) of equity for time t. w_t may be considered in an uncompounded sense and is defined by $w_t = \sum_{i=1}^{t} r_i, \ \ i = 1, \ \ldots, \ T$. For complexity constraints and ease of understanding, we employed the cumulative rate of return (uncompounded) in the subsequent study.

The security's or equity's drawdown at a time instance t along τ*–window* refers to a loss in value of the equity from the highest point of the cumulative rate of return curve that is experienced within the interval $[t_\tau, t]$, where $t_\tau = t - \tau$ for $t > \tau$. Formally, it is defined as

$$d_t = \max_{t_\tau \le k \le t} w_k - w_t \tag{1}$$

$$t_\tau = \max\{t - \ \tau, \ 1\}, \ \ t = 1, \ldots.., \ T, \ \ \ \tau \in \{1, \ \ldots, \ T\}$$

How the instrument behaves when downside market conditions, such as drawdowns, appear over time is generally designated by three drawdown risk functions or measures. They are *Maximum Drawdown*, $MaxDD(w) = \max_{1 \le t \le T} d_t$, *Average Drawdown*, $AvDD(w) = \frac{1}{T}\sum_{t=1}^{T} d_t$, and Conditional Drawdown − at − Risk. CDaR is defined and stated in the below definitions.

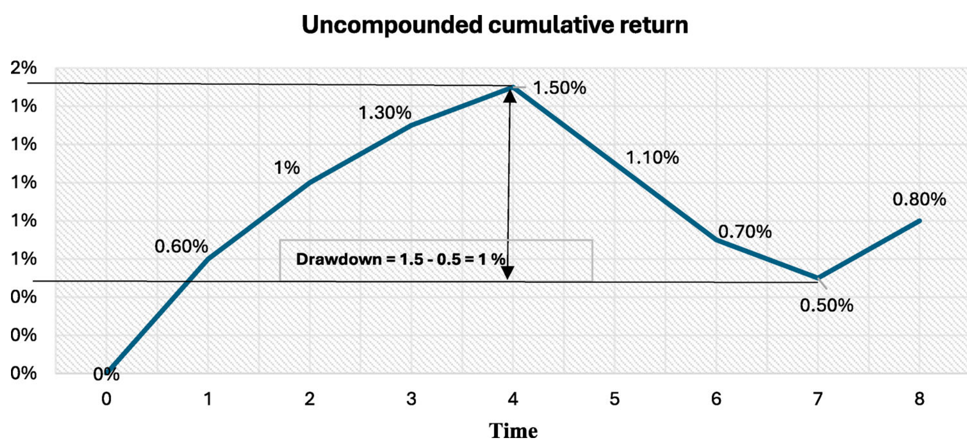

Figure 1. The graph represents an uncompounded cumulative rate of return. Here, we have the lookup window τ *assuming the value* 8, for $t = 7$, *and* w_5 *is calculated to be* 0.5%. w_t *achieves the highest value of* 1.5% in the interval [0, 8] preceding $t = 7$ *and it* occurs at *time instance* 4. Therefore d_5, *the largest drawdown in the interval* [0, 8] *is equal to* 1.5% − 0.5% = 1%.

3.1.1. CdaR on a single sample path

CdaR on a single sample path for α ∈ [0,1) such that α. T is an integer symbolised by $D_{\propto}(w)$, representing the mean of $(1 - \propto) \times 100\%$ greatest drawdowns, and given by the expression,

$$D_{\propto}(w) = \sum\nolimits_{t=1}^{T} q_t^* d_t, \qquad q_t^* = 1/((1 - \propto)T) \tag{2}$$

Let us assume that there are a total of S sample paths for chronology $r_1, .., r_T$ with probability $p_s > 0$ and $r_{s1}, \ldots, r_{sT}$ correspond to a particular sample path s, $s = 1, \ldots, S$. The uncompounded cumulative growth rate is computed as $w_{st} = \sum_{k=1}^{t} r_{sk}$, and the compounded cumulative rate of return is $w_{st} = \prod_{k=1}^{t}((1 + r_{sk}) - 1)$. The drawdowns are now calculated as $d_{st} = \max\limits_{t_\tau \leq k \leq t} w_{sk} - w_{st}$.

3.1.2. CdaR computation for multiple sample paths

Given $\alpha \in [0, 1)$, CdaR involving multiple sample paths, $D_{\propto}(w)$, which is also said to be the average of $(1 - \propto) \times 100$ % of drawdowns is defined by,

$$\begin{aligned} D_{\propto}(w) &= \max_{\{q_{st}\} \in Q} \sum\nolimits_{s=1}^{S} \sum\nolimits_{t=1}^{T} p_s q_{st} d_{st}, \\ where\ Q &= \left\{ \{q_{st}\}_{s,t=1}^{S,T} \middle| \sum\nolimits_{s=1}^{S} \sum\nolimits_{t=1}^{T} p_s q_{st} = 1, \quad 0 \leq q_{st} \leq \frac{1}{(1 - \propto)T} \right\} \end{aligned} \tag{3}$$

The two special cases for both settings (single and multiple sample path) where $\propto = 1$ and $\propto = 0$, represent maximum drawdown and average drawdown respectively.

3.2. Expected Regret of Drawdown (EroD)

The EroD for portfolio x with threshold ϵ, is the expected regret of the random variable $D(w(x))$, that is,

$$ERoD_{\varepsilon}(w(x)) = E[(D(w(x)) - \varepsilon)^+] \tag{4}$$

The equation above calculates EroD by taking the expected value or mean of drawdown events that cause the instrument's value to fall above a particular threshold ε. Simplifying the EroD equation for multiple sample paths yields:

$$ERoD_{\varepsilon}(w(x)) = \frac{1}{T} \sum_{s=1}^{S} \sum_{t=1}^{T} p_s(d_{st}(x) - \varepsilon)^+ \tag{5}$$

3.3. Portfolio optimisation using CdaR and EroD

Suppose we have a portfolio that consists of n securities or equities, and x_i denotes the relative position held for the i^{th} stock in the portfolio (positive or negative). We assume that the portfolio does not contain any risk-free instruments (e.g. government 10Y bonds) and $\sum x_i$ is unconstrained. The generated profit (loss) of the positions in portfolio $x = (x_1, \ldots, x_n)$ over period t in sample path s is given by $\sum_{i=1}^{n} r_{st}^i x_i$. This profit or loss when measured on an accumulated uncompounded cumulative basis over t periods results in the expression $\sum_{k=1}^{t} \left(\sum_{i=1}^{n} r_{sk}^i x_i \right) \equiv \sum_{i=1}^{n} (\sum_{k=1}^{t} r_{sk}^i) x_i \equiv \sum_{i=1}^{n} w_{st}^i x_i \equiv w_{st}^P(x)$.

3.3.1. Portfolio optimisation problem with CdaR

Optimising a portfolio involving CdaR can be formulated in two equivalent ways:

(a) minimising the CdaR of the respective portfolios over a horizon of T periods subject to a lower-bound constraint on the portfolio's expected rate of return at time T

$$\min_{x} CDaR_{\propto}(w^{p}(x)) \quad such\ that \quad \sum\nolimits_{s=1}^{S} p_s w_{sT}^{p}(x) \geq \delta \tag{6}$$

(b) Maximising the expected rate of return of the respective portfolio at time T subject to an upper-bound constraint on the CdaR of the portfolio over the horizon of T periods.

$$\max_{x} \sum\nolimits_{s=1}^{S} p_s w_{sT}^{p}(x) \quad such\ that \quad CDaR_{\propto}(w^{p}(x)) \leq \vartheta \tag{7}$$

3.3.2. Portfolio optimisation problem with EroD

Compared to optimising a portfolio involving CdaR, EroD portfolio optimisation can be formulated as follows:

(a)
$$\min_{x} ERoD_{\varepsilon}(w^{p}(x)) \quad such\ that \quad \sum\nolimits_{s=1}^{S} p_s w_{sT}^{p}(x) \geq \delta \tag{8}$$

(b)
(c)
$$\max_{x} \sum\nolimits_{s=1}^{S} p_s w_{sT}^{p}(x) \quad such\ that \quad ERoD_{\varepsilon}(w^{p}(x)) \leq \vartheta \tag{9}$$

The above defined formulations for solving a portfolio optimisation problem using CdaR(α) and EroD methods resemble the mean-variance optimisation issue determined by Markowitz. The only difference occurring in the drawdown methods is that variance is substituted with α – CdaR and EroD (ε)

3.4. Finding Requisite Optimality Conditions for portfolio optimisation with CdaR and EroD measures using CAPM

In accordance with the necessary optimality condition for CAPM formulation, proposed in (Zabarankin et al. 2014) paper, Beta (β) accounting for the drawdown risk measure CdaR is calculated. We assume $w^{M} = w(x^{*})$ is the optimal array of the portfolio's cumulative potential growth obtained by solving (6) or (7). The requisite conditions of optimality used in the CAPM formulation for the solution (x^{*}) complying th equations (6) and (7) are defined as

$$\sum_{s=1}^{S} p_s w_{sT}^{i} = \beta_{CDaR}^{i} \sum_{s=1}^{S} p_s w_{sT}^{M} \tag{10}$$

$$\beta_{CDaR}^{i} = \frac{\sum_{s=1}^{S} \sum_{t=1}^{T} p_s q_{st}^{*} (w_{s,\tau(s,t)}^{i} - w_{st}^{i})}{CDaR_{\alpha}(w^{M})} \tag{11}$$

The most recent maximum present in the historical data in the cumulative returns has an index, that is, $\tau(s, t)$, and is given by,

$$\tau(s, t) = max\left\{k \mid 1 \leq k \leq t,\ w_{sk}^{M} = \max_{1 \leq i \leq t} w_{si}^{M}\right\}$$

Explanation of terms occurring in the CdaR (β) equation:

- $\sum_{s=1}^{S} p_s w_{sT}^{M} = cumulative\ expected\ return\ of\ the\ market$
- $\sum_{s=1}^{S} p_s w_{sT}^{i} = cumulative\ expected\ return\ of\ the\ instrument\ i$

The necessary optimal conditions in the CAPM formulation involving EroD is stated as:

$$\sum_{s=1}^{S} p_s w_{sT}^{i} = \beta_{ERoD}^{i} \sum_{s=1}^{S} p_s w_{sT}^{M} \tag{12}$$

$$\beta_{ERoD}^{i} = \frac{\frac{1}{T}\sum_{s=1}^{S} \sum_{t=1}^{T} p_s q_{st}^{*} (w_{s,\tau(s,t)}^{i} - w_{st}^{i})}{\tilde{E}_{\varepsilon}(w^M)} \tag{13}$$

Explanation of terms occurring in EroD (ε) equation:

- $\tilde{E}_{\epsilon}(w^M) = \frac{1}{T}\sum_{s=1}^{S}\sum_{t=1}^{T} p_s q_{st}^{*}(w_{s,\tau(s,t)}^{M} - w_{st}^{M}) =$ *ERoD with threshold ε for return w^M*
- $\beta_{ERoD}^{i} =$ *ERoD Beta relating the total expected cumulative return of the optimal portfolio (market)and total expected cumulative return of the security i*
- $\tau(s, t) =$ *index of the most recent historic maximum in the cumulative returns*
- $d_{st}^{M} = w_{s,\tau(s,t)}^{M} - w_{st}^{M} =$ *drawdowns of the optimal portfolio*
- $q_{st}^{*} = (d_{st}^{M} \geq \varepsilon) =$ *indicator function which is equal to 1 for $d_{st}^{M} \geq \varepsilon$, and 0 otherwise.*

We examine the potential hedging performance of the stocks included in the market portfolio using drawdown betas. The S&P500 Index is employed as a stand-in for the market portfolio and as a proxy for an ideal portfolio. The results are summarised in Table 1. It represents all stocks that have either the CdaR ($\alpha = 90\%$), EroD ($\varepsilon = 50\%$), and Average Drawdown Beta negative. Notably, stocks with low beta values, which generally represent the degree of agreement between the stock and the market when the market declines, are of particular interest. Whether CdaR betas offer more detail on hedging capabilities than normal betas is a subject of debate.

3.5. Characteristic analysis of BPT in close connection with the utility derived from the ESG Preference function

For the BPT and ESG analyses, we considered monthly stock return data. The 80 companies presented in Table 2, with at least one drawdown risk measure as negative, were selected as the population for the character analysis of BPT. Notably, processing monthly calendar returns is challenging because there are particular public holidays and non-trading days. To simplify the procedure, we presume that a month would have 20 trading days. We used the stock values paid by the companies to calculate the monthly stock returns daily using a rolling window methodology. The formula is as follows:

$$R_{t,i} = \log P_{t,i} - \log P_{t-20,i}$$

$R_{t,i}$ = The monthly return on stock i for day t

$P_{t,i}$ = Price of stock i for day t

Therefore, a Return matrix stated as an 'R matrix' is constructed containing 986 monthly returns of the 80 US S&P500 stocks spanned over four years (December 2018 – December 2022) selected after the CdaR and EroD drawdown optimisation.

$$R = \begin{bmatrix} R_{1,1} & R_{1,2} & \cdots & R_{1,80} \\ R_{2,1} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ R_{986,1} & \cdots & \cdots & R_{986,80} \end{bmatrix}$$

Table 1. Overview of the reviewed literature. The table includes 80 companies that are a part of the S&P 500 global index having potential hedging capability against the market. The table includes 80 companies that are a part of the S&P 500 global index having potential hedging capability against the market.

S. No.	Author	Objective	Inference and findings
1	Mio, Fasan, & Scarpa 2023	The analogy between ESG metrics/ratings and Socially Responsible Investing	- ESG metrics and utility reflection of ESG ratings are true representations of Socially Responsible Investing (SRI)
2	Prol & Kim 2022	examining the risk-return performance of optimised equity portfolios that incorporate environmental, social, and governance (ESG) criteria in the New York Stock Exchange (NYSE)	- Optimised ESG equity portfolios can generate higher risk-adjusted returns compared to traditional portfolios, indicating the potential financial benefits of integrating ESG factors into investment strategies
3	Goldberg & Mouti 2022	Analyzing the predicting power of ESG data for forecasting returns and maximum drawdowns	- Existence of predictive power in ESG data for predicting portfolio returns and maximum drawdown - Incorporating company-specific fundamental datasets and ESG data assists in forecasting drawdowns and profitability
4	Avramov, Cheng, Lioui, & Tarelli 2022	Studying the variability and uncertainties in the ESG ratings	- Higher uncertainty in ESG ratings leads to higher market risk
5	Rui Ding & Uryasev 2022	Introduction of a new dynamic investment performance measure of risk known as Expected Regret of Drawdown (EroD)	- Calculation of EroD β similar to the CdaR β calculated in Zabarankin et al. 2014 - EroD β is more sensitive to market drawdowns compared to Standard Beta
6	Li Chen, Lipei Zhang, Jun Huang, Helu Xiao, & Zhongbao Zhou 2021	Incorporating ESG criteria into portfolio optimisation	- Inclusion of Maximization of ESG of the Investors in the portfolio optimisation problem - Resulting portfolio represents a socially responsible portfolio
7	Rodriguez, Gomez, & Contreras 2021	The research proposes a diversified behavioural portfolio as an alternative to Modern Portfolio Theory	- MPT's assumes presence of rational investors and normally distributed returns - Research suggests incorporating behavioural biases in portfolio construction.
8	Pfiffelmann et al. 2016	Comparing Behavioural Portfolio Theory (BPT) with Markowitz Mean-Variance (MV) portfolio theory	- Simulation study to compare BPT portfolios with MV portfolio theory and locate them in the MV efficient frontier
9	Zabarankin et al. 2014	Portfolio Optimisation with Conditional Drawdown-at-Risk (CdaR) as the risk metric.	- Substitution of portfolio variance with CdaR as the risk measure - Calculation of CdaR β for CAPM formulation in the single and multi-sample path setting - Analysis of CdaR β to find hedge positions during market drawdowns
10	Das & Statman 2013	Comparative study on the effect of including options and other structured products in the portfolio on BPT and MV portfolio theory	- Options and structured products belong in optimum behavioural portfolios but not in optimal mean-variance portfolios. - Risk metric is defined by the probability of failing to cross the threshold level of the objective associated with a mental account sub-portfolio, or by the expected shortfall.
11	Das et al. 2010	Introduction of Mental Accounting (MA) Framework integrated with features of MV portfolio theory and BPT	- Equivalence relation between MVT, MA, along with VaR as the risk measure - Adherence to investor's goals and aspirations when MVT and BPT are unified with the MA framework
12	Alexander & Baptista 2006	Characterization of optimal portfolios in mean-variance or mean tracking error volatility models when a Maximum	- The inclusion of the Maximum Drawdown constraint increases the optimal portfolio's standard deviation and

(*Continued*)

Table 1. Continued.

S. No.	Author	Objective	Inference and findings
		Drawdown constraint is present	tracking error volatility, therefore, fails to beat the benchmark
13	Mendes & Leal 2005	Statistical 30odelling using Extreme Value Theory to understand the severity and duration of the maximum drawdown.	- Maximum likelihood estimation of Extreme Value Theory for 30odelling the maximum drawdown. - Maximum drawdown is perhaps related to the GARCH volatility of daily returns.
14	Chekhlov et al. 2005	Conditional Drawdown constraint for multi-sample path	- Extension of single path approach presented in Chekhlov et al. 2005 to multi-sample path concerning portfolios
15	Lopez De Prado & Peijan 2004	Assessing the Loss potential of Hedge Funds incorporating three market risk indicators – VaR, Drawdown, and Time Under-The-Water	- There exists a notable difference in Drawdown and VaR risk metrics for investments. - VaR is ineffective and unable to capture all dimensions of market risk when normality conditions and/or time-independence assumptions do not hold true.
16	Testuri & Uryasev 2004	Comparison of portfolio optimisation techniques with expected regret and Conditional Value-at-Risk	- Understanding the equivalent mathematical relation between Conditional Value-at-Risk and Expected Regret for the portfolios
17	Chekhlov et al. 2004	Introduction of a new one-parameter risk metric called Conditional Drawdown	- Formulation of Portfolio optimisation and effective portfolio management subject to Conditional Drawdown constraint
18	Shefrin and Statman 2000	Development of positive behavioural portfolio theory	- Distinction between behavioural portfolio theory and mean-variance portfolio theory
19	Cvitanic & Karatzas 1994	Portfolio optimisation problem subject to drawdown constraints	- Extension of Grossman and Zhou's drawdown analysis of single risky asset to multiple risky assets
20	Grossman & Zhou 1993	Maximize the asymptotic long-run growth rate of one's Wealth under a condition that ensures the local stability of that wealth	- First introduction of drawdown as a risk measure in portfolio optimisation - Only limited to a single risky asset
21	Sharpe 1964, Lintner 1965, and Mossin 1966	Identifying the principal relationship between the systemic risk and the expected return of an investment	- Birth of asset pricing theory named the Capital Asset Pricing Model (CAPM) - Identification of optimal investment policy for investors using preference function made up of expected value and standard deviation of future wealth
22	Telser 1955	Maximizing expected income in the futures market incorporating the Safety-first constraint	- Profitability scenarios due to hedging in the futures market based on different positions held (unhedged, log-hedged, short-hedged)
23	Roy 1952	Introduction of the concept of the Safety-first approach	- Measuring the Investor's portfolio risk in terms of probability of ruin rather than the variance of portfolio

$R_{1,1}$ denotes the monthly return on the first stock over the time horizon beginning from the 3 December 2018 to the 31 December 2018. Similarly, $R_{2,2}$ is the monthly return of the second stock over the time period starting on the 4 December 2018 and ending on the 2 January 2019.

Our goal is to calculate the expected portfolio returns using a historical sequence of ex-post returns as inputs. Thus, we used a rolling sample method and create potential outcomes (states of nature) for each date t (*from* $t = 251$ *to* $t = 986$). In total, we conducted 736 simulations (*from* $t = 251$ *to* $t = 986$), from which we first chose 50 stocks at random from the 80 that comprised our selection. Prior to date t, we chose the 250 monthly returns of these 50 stocks

Table 2. Average drawdown β is represented as AvgDD_beta, CdaR-90% β is represented as CdaR90_beta, EroD β is represented as EroD_beta, and Standard β is represented as beta.

#	Symbol	Name	Sector	CdaR90_beta	AvgDD_beta	EroD_beta	beta
1	ABMD	Abiomed	Health Care	−0.618	−1.657	−1.598	0.953
2	ATVI	Activision Blizzard	Communication Services	0.161	−0.385	−0.339	0.862
3	ADBE	Adobe	Information Technology	0.458	−0.113	−0.088	1.222
4	LNT	Alliant Energy	Utilities	0.404	−0.217	−0.139	0.592
5	GOOGL	Alphabet (Class A)	Communication Services	0.324	−0.153	−0.119	1.073
6	AMZN	Amazon	Consumer Discretionary	−0.219	−1.445	−1.333	1.027
7	AEE	Ameren Corp	Utilities	0.144	−0.398	−0.338	0.634
8	AEP	American Electric Power	Utilities	0.267	−0.305	−0.22	0.522
9	AMT	American Tower	Real Estate	0.089	−0.476	−0.38	0.801
10	AWK	American Water Works	Utilities	0.046	−0.732	−0.632	0.618
11	ATO	Atmos Energy	Utilities	0.014	−0.819	−0.72	0.621
12	AZO	AutoZone	Consumer Discretionary	0.037	−0.118	−0.101	0.741
13	CPB	Campbell Soup	Consumer Staples	−0.164	−0.305	−0.296	0.41
14	CHD	Church & Dwight	Consumer Staples	0.003	−0.469	−0.398	0.495
15	CINF	Cincinnati Financial	Financials	0.802	−0.124	−0.068	1.02
16	CTXS	Citrix Systems	Information Technology	−0.272	−0.262	−0.28	0.795
17	CLX	Clorox	Consumer Staples	−0.624	−0.664	−0.652	0.328
18	CME	CME Group	Financials	0.275	−0.017	0.017	0.854
19	CMS	CMS Energy	Utilities	0.155	−0.534	−0.455	0.548
20	ED	Consolidated Edison	Utilities	0.222	−0.285	−0.241	0.452
21	STZ	Constellation Brands	Consumer Staples	0.775	−0.089	0.032	0.861
22	CPRT	Copart	Industrials	1.028	−0.02	0.1	0.969
23	CCI	Crown Castle	Real Estate	0.228	−0.348	−0.253	0.74
24	DXCM	DexCom	Health Care	−0.433	−1.625	−1.352	1.056
25	DLR	Digital Realty Trust	Real Estate	−0.049	−0.593	−0.523	0.688
26	DG	Dollar General	Consumer Discretionary	−0.001	−0.037	−0.009	0.588
27	DPZ	Domino's Pizza	Consumer Discretionary	−0.333	−0.701	−0.633	0.644
28	DTE	DTE Energy	Utilities	0.473	−0.15	−0.069	0.715
29	DUK	Duke Energy	Utilities	0.335	−0.046	0.02	0.579
30	EBAY	eBay	Consumer Discretionary	0.199	−0.079	−0.036	0.84
31	EW	Edwards Lifesciences	Health Care	0.036	−1.328	−1.128	0.998
32	EA	Electronic Arts	Communication Services	0.368	−0.076	−0.004	0.849
33	LLY	Eli Lilly & Co	Health Care	−0.31	−0.663	−0.482	0.742
34	EQIX	Equinix	Real Estate	0.135	−0.533	−0.374	0.846
35	EVRG	Evergy	Utilities	0.207	−0.776	−0.685	0.691
36	ES	Eversource Energy	Utilities	0.136	−0.229	−0.179	0.615
37	EXR	Extra Space Storage	Real Estate	−0.094	−0.985	−0.918	0.614
38	FISV	Fiserv	Information Technology	0.493	−0.286	−0.219	1.045
39	GIS	General Mills	Consumer Staples	−0.012	0.08	0.081	0.457
40	GPN	Global Payments	Information Technology	0.783	−0.131	−0.121	1.234
41	HRL	Hormel	Consumer Staples	−0.604	−1.071	−1.073	0.486
42	IDXX	Idexx Laboratories	Health Care	0.591	−0.198	−0.079	0.945
43	JKHY	Jack Henry & Associates	Information Technology	0.179	−0.286	−0.22	0.785
44	KMB	Kimberly-Clark	Consumer Staples	0.12	−0.169	−0.131	0.528
45	KR	Kroger	Consumer Staples	−0.051	0.246	0.267	0.427
46	MKTX	MarketAxess	Financials	−1.353	−1.839	−1.834	0.768
47	MKC	McCormick & Company	Consumer Staples	−0.144	−0.635	−0.557	0.628
48	MCD	McDonald's	Consumer Discretionary	0.088	−0.229	−0.249	0.739
49	MAA	Mid-America Apartments	Real Estate	0.445	−0.326	−0.258	0.762
50	MPWR	Monolithic Power Systems	Information Technology	0.26	−0.271	−0.17	1.472
51	MSI	Motorola Solutions	Information Technology	0.539	−0.351	−0.323	0.902
52	NDAQ	Nasdaq	Financials	0.22	−0.155	−0.148	0.941
53	NFLX	Netflix	Communication Services	0.262	−0.568	−0.556	1.082
54	NEM	Newmont	Materials	−0.091	0.458	0.36	0.36
55	NEE	NextEra Energy	Utilities	0.238	−0.497	−0.362	0.669
56	NKE	Nike	Consumer Discretionary	0.419	−0.01	0	0.948
57	NI	NiSource	Utilities	0.286	−0.291	−0.251	0.705
58	NOC	Northrop Grumman	Industrials	0.354	−0.277	−0.206	0.765
59	NVDA	Nvidia	Information Technology	0.654	−0.753	−0.527	1.501
60	NVR	NVR	Consumer Discretionary	0.604	−0.225	−0.143	0.927
61	ORLY	O'Reilly Automotive	Consumer Discretionary	0.081	−0.513	−0.436	0.849
62	PNW	Pinnacle West Capital	Utilities	0.307	−0.315	−0.265	0.663

(*Continued*)

Table 2. Continued.

#	Symbol	Name	Sector	CdaR90_beta	AvgDD_beta	EroD_beta	beta
63	POOL	Pool Corporation	Consumer Discretionary	0.194	−0.63	−0.543	0.897
64	PGR	Progressive Corporation	Financials	0.024	−0.549	−0.569	0.75
65	PSA	Public Storage	Real Estate	−0.15	−0.921	−0.907	0.575
66	O	Realty Income Corporation	Real Estate	0.343	−0.408	−0.35	0.818
67	REGN	Regeneron Pharmaceuticals	Health Care	−0.581	−0.017	0.077	0.915
68	ROL	Rollins	Industrials	0.086	−0.436	−0.306	0.771
69	SBAC	SBA Communications	Real Estate	0.288	−0.003	0.124	0.759
70	SO	Southern Company	Utilities	0.174	−0.496	−0.433	0.602
71	SBUX	Starbucks	Consumer Discretionary	0.155	−0.125	−0.187	0.994
72	TTWO	Take-Two Interactive	Communication Services	−0.012	−0.231	−0.198	0.909
73	TYL	Tyler Technologies	Information Technology	0.222	−0.642	−0.571	0.896
74	TSN	Tyson Foods	Consumer Staples	0.478	−0.214	−0.254	0.694
75	VRSN	Verisign	Information Technology	0.05	−0.778	−0.723	0.957
76	VRSK	Verisk Analytics	Industrials	0.461	−0.092	−0.024	0.856
77	WRB	W. R. Berkley Corporation	Financials	0.807	−0.002	0.056	0.898
78	WEC	WEC Energy Group	Utilities	0.003	−0.594	−0.533	0.528
79	WST	West Pharmaceutical Services	Health Care	0.227	−0.547	−0.388	0.803
80	XEL	Xcel Energy	Utilities	0.015	−0.512	−0.443	0.577

and found matrix R^*.

$$R^* = \begin{bmatrix} R^*_{t-250,1} & R^*_{t-250,2} & \cdots & R^*_{t-250,80} \\ R^*_{t-249,1} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ R^*_{t-1,1} & \cdots & \cdots & R^*_{t-1,80} \end{bmatrix}$$

Then, we constructed another matrix θ, containing 1000 snapshots or states of nature for our portfolio of 50 stocks at a considered date t. We chose a row of matrix R at random to model the first condition of nature or the return realised at the conclusion of our single period. To obtain 1000 states of nature for our 50 stocks, we repeated this procedure 1000 times. Notably, the structure of the correlations among various stocks is not changed by arbitrarily choosing a row of matrices R^*.

$$\theta = \begin{bmatrix} \theta_{1,1} & \theta_{1,2} & \cdots & \theta_{1,50} \\ \theta_{2,1} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \theta_{1000,1} & \cdots & \cdots & \theta_{1000,\ 50} \end{bmatrix}$$

$\theta_{i,j}$ is the monthly return of stock j if the state of nature I occurs.

The above-defined process for constructing the matrix R^* and the theta matrix was repeated in 736 simulations.

3.6. Generation of portfolios

An investor has an unlimited number of portfolio options to choose from for each date t. However, for our study, we built a set of 1,000 portfolios. The purchaser selects one portfolio from available 1,000 options. The maximum number of stocks that an investor may own is capped at 50. Notably, the number of stocks in a portfolio guarantees a high degree of diversification. Furthermore, a sample of 1,000 portfolios without short sales was created using the methods described below to provide the closest representation of the real-world decision-making process for investors.

As we assume a portfolio of 50 stocks, the weight associated with a given stock is equal to k/n *with* $k = 0, 1, I, n$ ($n = 50$ *in our case*) where the most diversified portfolio seems to correspond to each weight equal to 1/n, whereas the least diversified portfolio corresponds to the investors' decision to invest all of their wealth in just one security or stock. Based on this, we can model all possible portfolio compositions by considering all possible integer decompositions of the number n, which in turn may be determined through a dynamic programming approach, given the premise that the weight associated with various stocks is equal to k/n. For example, for n = 5, the total number of integer decompositions is seven.

$$I_1 = \begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad I_2 = \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad I_3 = \begin{bmatrix} 3 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad I_4 = \begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad I_5 = \begin{bmatrix} 2 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad I_6 = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} \quad I_7 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

After obtaining all the possible integer decompositions of the number n, we randomly selected 1000 decompositions to construct our portfolio, and again randomly shuffled the weights to remove selection bias, finally dividing the whole vector by n. To illustrate our case where n = 50, 204,226 integer decompositions are possible in total. We then randomly chose 1, 000 decompositions from the 204, 226 potential integer decompositions, rearranged these vectors randomly, and divided each element by n = 50 to convert them into weights.

We used the same 1,000 weight vectors to create 1,000 portfolios for each simulation; however, 50 randomly selected stocks differed for each simulation.

3.7. The efficient frontier of the BPT optimal portfolio

Our goal in this part of the study is to locate the BPT optimal portfolio within the mean-variance space using empirical analysis, for which we used 1,000 produced portfolios to estimate the efficient frontier empirically. We first determine whether another portfolio with greater expected returns and lower variance exists for each portfolio. Notably, a portfolio is deemed to be located on the efficient frontier if no other portfolio is found within the sample, with both a higher expected return and lower variance. The portfolios on the efficient frontier are denoted by S. The BPT optimal portfolio satisfies the following set of relationships:

$$\max E(W)$$

$$subject\ to\ P\,(W < A) < \ \alpha$$

W = terminal wealth distribution of the investor at the end of the time period
A = Aspiration Level
α = Extent of ruin or Acceptable probability of ruin

3.8. ESG preference function: Comparing ESG score for portfolios falling inside BPT efficient frontier with portfolios outside the BPT efficient frontier

An ESG preference function (Pedersen et al. 2021) that reflects the utility derived from the ESG Preference function inherited by a portfolio is defined and stated (below) in such a way that it incorporates a generalised way of including the average ESG scores of the stocks in the portfolios. Importantly, utility seems to be penalised more if the investor has a stock with a low average ESG score.

$$e\,(x, s) = \ e_1 \frac{x^T s}{x^T 1} - \ e_2 \frac{x^T diag\left(\frac{1}{s_{avg,\ 1}}, \ \ldots, \ \frac{1}{s_{avg,\ n}}\right)x}{(x^T 1)^2}$$

x = vector of weights of the portfolio
$s_{avg},\ i$ = Average ESG score of the company i present in the portfolio calculated by taking the arithmetic mean of the yearwise Bloomberg ESG score (period-4yrs (2018-2022))

s = vector composed of s_{avg}, i, i.e., $(s_{avg,\ 1}, s_{avg,\ 2}, \ldots, s_{avg,\ n})$
$e\ (x, s)$ = ESG preference function
$e_1, e_2 \in \mathbb{R}$ are real-valued parameters

For all 1000 generated portfolios, the utility derived from the ESG Preference function is calculated. We then analysed the proportion of simulations (out of 736) in which the BPT-efficient portfolios have higher average utility than the portfolios outside the BPT-efficient frontier.

The MPT is a predominantly normative theory, whereas the BPT is a predominantly positive theory. This study shows the sustainability of investment by analysing the utility derived from the ESG preference function and achieving safety from the downside effects of our portfolio. Positive economics theory aligned with BPT explains investors' desire for a stable portfolio. However, the implementation of the BPT with suitable constraints and objective functions is inherited from the underlying MPT. The objective function here is to maximise returns, and the only constraint is to avoid the loss of investment up to a certain level. The optimisation problem framework of the MPT is limited; thus, the integration of the MPT and BPT occurs with a positive inline theory. The utility derived from the ESG preference function is an extended analysis that is used to infer the sustainable nature of behaviourally appropriate portfolios. BPT essentially involves considering specific utility functions to achieve the desired goals. Our study suggests that portfolios that pass the safety-first constraint and constitute the Pareto front have higher utility, as derived from the ESG preference function. This concludes our two-step analysis for deriving sustainable, responsible, and stable investments.

An extended analysis of the utility derived from the ESG preference function is performed to determine the possibility of safe and sustainable investment. The study shows that the average utility derived from the ESG preference function for portfolios that follow BPT and lie on the BPT efficient frontier is in excess of the proportion of portfolios that do not follow BPT. This authenticates the portfolio losses for a given probability under the BPT study and provides a value proposition for responsible investing based on better ESG utility of the portfolios following the safety-first constraint and having maximum returns.

4. Empirical results

4.1. Portfolio selection

Table 3 consists of stocks that have all the drawdown betas, that is, CdaR ($\alpha = 90\%$), ErоD ($\varepsilon = 50\%$), and Average Drawdown Beta negative. Stocks with a negative average drawdown have good hedging properties and thereby generate non-negative returns in most situations, particularly when markets experience a downside. Most stocks (as shown in Table 2) do have positive standard β, which is not motivating for hedging purposes because of the good performance of the market itself. However, these stocks and company shares have negative Average Drawdown Betas, which in turn, designates that on average, they hedge the market effectively when examined over a larger time horizon. Moreover, the standard beta and its respective outcomes on returns are applicable over the entire historical data, irrespective of whether the market is performing well or bad, whereas CdaR and EroD β account only for market drawdowns and are, therefore, unaffected by market's upside performance. This is the primary understandable difference between CdaR and EroD β and the standard β.

Consequently, the best hedging stock offers positive payoffs for both little and major market declines, resultantly, it delivers a negative value of CdaR β across all confidence levels, and negative EroD beta at some threshold. Table 3 shows the equities that have the maximum hedging capacity among the 500 companies that are a part of the S&P 500 index.

An important observation from both Tables 2 and 3 is that the companies that feature in the consumer discretionary and consumer staples industries hedge the market against drawdowns

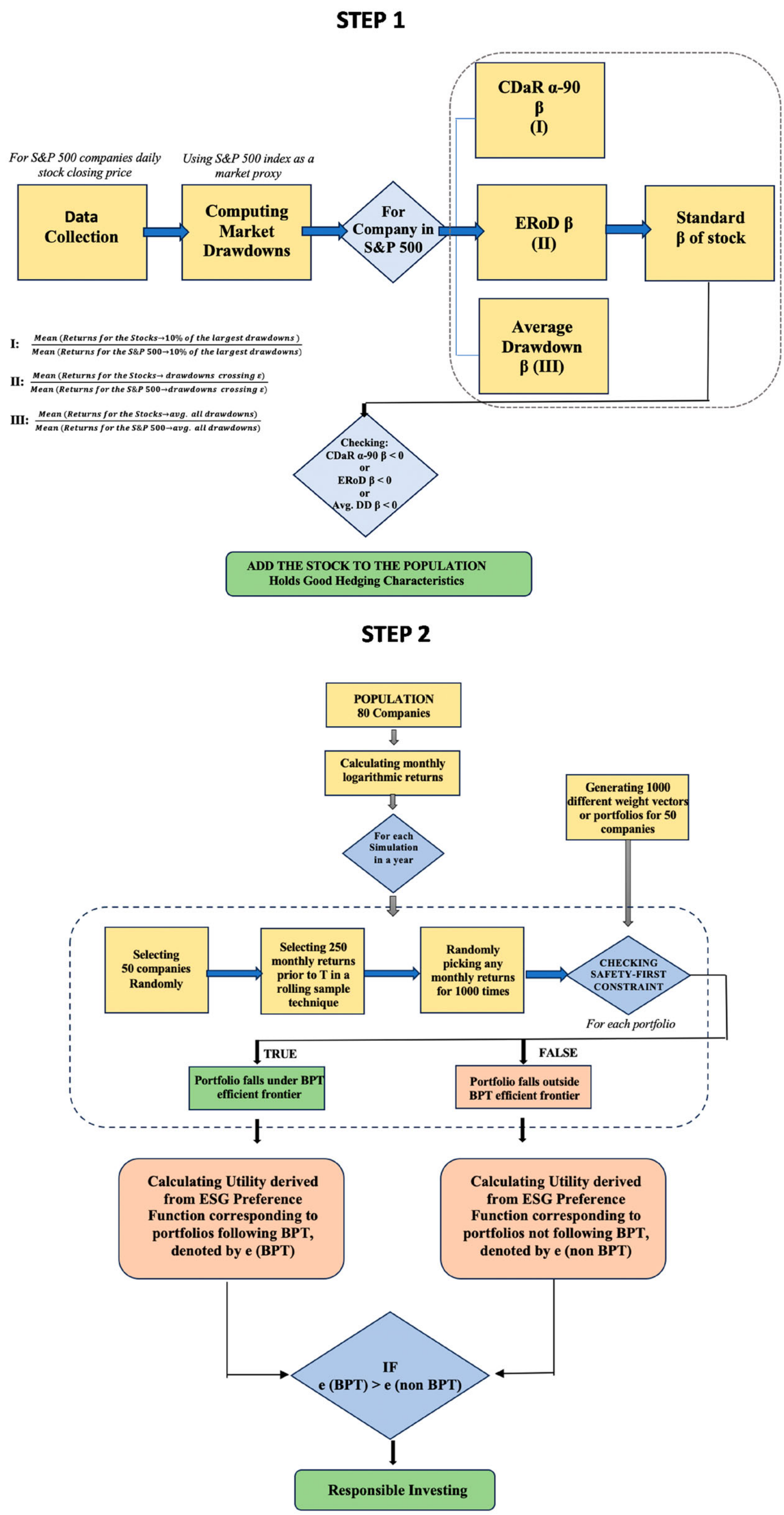

Chart 1. Model flowchart

Table 3. Average drawdown β is represented as AvgDD_beta, CDaR-90% β is represented as CDaR90_beta, ERoD β is represented as ERoD_beta, and Standard β is represented as beta. The table represents companies' part of the S&P 500 global index having the best hedging capability against the market. All drawdown betas for these 16 companies are negative. The table represents companies' part of the S&P 500 global index having the best hedging capability against the market. All drawdown betas for these 16 companies are negative.

#	Symbol	Name	Sector	CDaR90_beta	AvgDD_beta	ERoD_beta	beta
1	ABMD	Abiomed	Health Care	−0.618	−1.657	−1.598	0.953
2	AMZN	Amazon	Consumer Discretionary	−0.219	−1.445	−1.333	1.027
3	CPB	Campbell Soup	Consumer Staples	−0.164	−0.305	−0.296	0.41
4	CTXS	Citrix Systems	Information Technology	−0.272	−0.262	−0.28	0.795
5	CLX	Clorox	Consumer Staples	−0.624	−0.664	−0.652	0.328
6	DXCM	DexCom	Health Care	−0.433	−1.625	−1.352	1.056
7	DLR	Digital Realty Trust	Real Estate	−0.049	−0.593	−0.523	0.688
8	DG	Dollar General	Consumer Discretionary	−0.001	−0.037	−0.009	0.588
9	DPZ	Domino's Pizza	Consumer Discretionary	−0.333	−0.701	−0.633	0.644
10	LLY	Eli Lilly & Co	Health Care	−0.31	−0.663	−0.482	0.742
11	EXR	Extra Space Storage	Real Estate	−0.094	−0.985	−0.918	0.614
12	HRL	Hormel	Consumer Staples	−0.604	−1.071	−1.073	0.486
13	MKTX	MarketAxess	Financials	−1.353	−1.839	−1.834	0.768
14	MKC	McCormick & Company	Consumer Staples	−0.144	−0.635	−0.557	0.628
15	PSA	Public Storage	Real Estate	−0.15	−0.921	−0.907	0.575
16	TTWO	Take-Two Interactive	Communication Services	−0.012	−0.231	−0.198	0.909

the most. FMCG, Consumer Discretionary, and Staple firms offer products and services related to the necessities of a larger population. During tough market times and rising inflation, consumers are generally much more mindful of their spending and often seek the necessary demand fulfilment. Thus, these companies sustain their businesses through the liquid demand and supply states prevailing in the market; therefore, their stock prices flourish during drawdowns.

4.2. Benchmarking the BPT in relation to ESG-consideration with more conventional systematic risk factors: Evidence from 3-Factor and 5-Factor Fama-French model

Factor models are statistical models that attempt to explain complex events by using a small number of underlying causes or factors. The CAPM, compares the returns of a stock or portfolio with the returns of the entire market using only one variable. By contrast, the Fama-French model uses three variables.

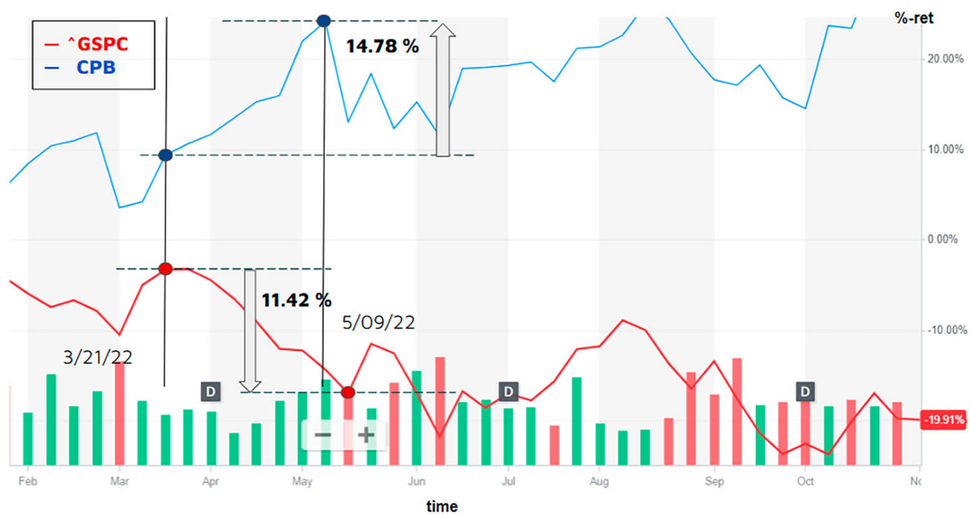

Figure 2. CPB (Campbell Soup) raised 14.78% cumulative returns during the period 21 March 22 to 9 May 22 (< 2 months) when the market (S&P 500 index) experienced an 11.42% decline during the same period.

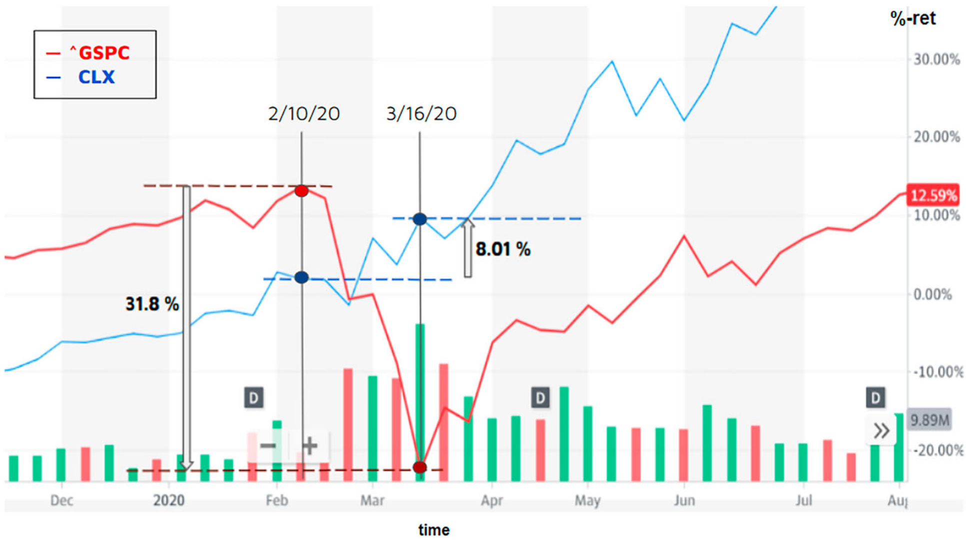

Figure 3. CLX (Clorox) raised 8.01% cumulative returns during the period 10 February 2020 to 16 March 2020 (< 2 months), when the market (S&P 500 index) experienced a whopping 31.08% decline during the same period.

An expansion of the CAPM is the Fama-French three-factor and five-factor models. Three factors are used in the three-factor Fama–French model to explain stock returns: (1) market risk (represented by MKT), (2) small-cap companies' outperformance compared to large-cap companies (represented by SMB), and (3) high book-to-market value companies' outperformance compared to low book-to-market value companies (HML). In addition to the current three-factor Fama–French model, exposure to two additional components is a part of the five-factor model. The return differential between profitable and unprofitable companies (as indicated by the RMW) and between

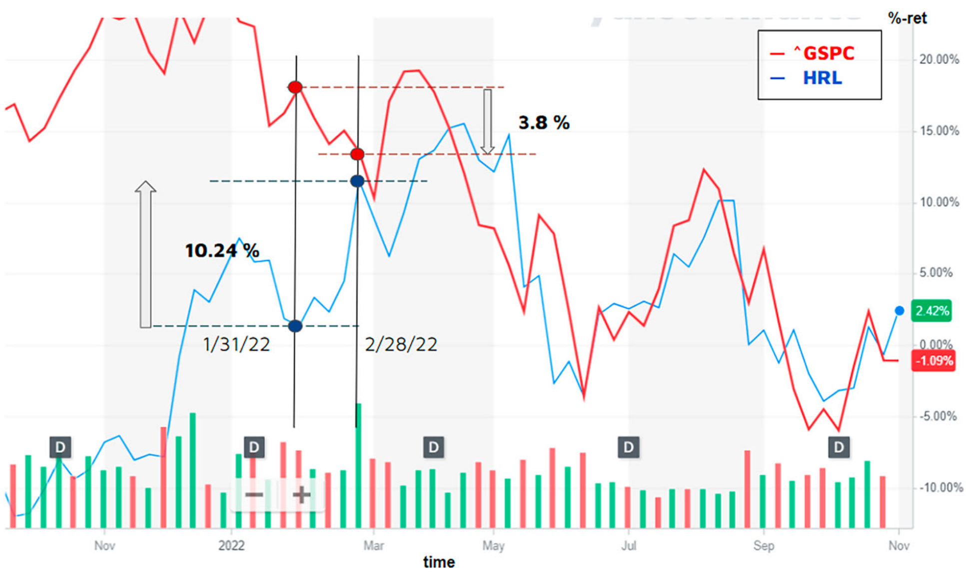

Figure 4. HRL (Hormel) raised 10.24% cumulative returns during the period 31 January 2022 to 28 February 2022 (<2 months) when the market (S&P 500 index) experienced a 3.8% decline during the same period.

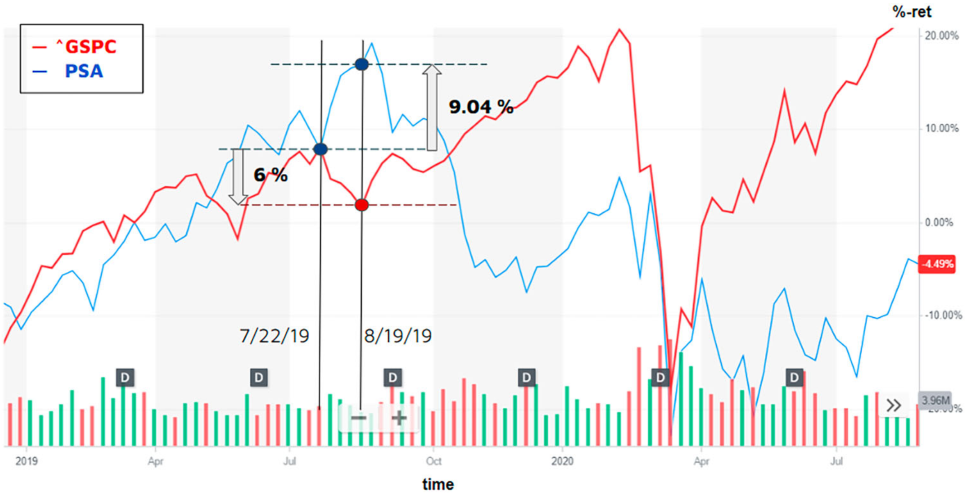

Figure 5. PSA (Public Storage) raised 9.04% cumulative returns during the period 22 July 2019 to 19 August 2019 (< 1 month) when the market (S&P 500 index) experienced a 6% drop in returns during the same period.

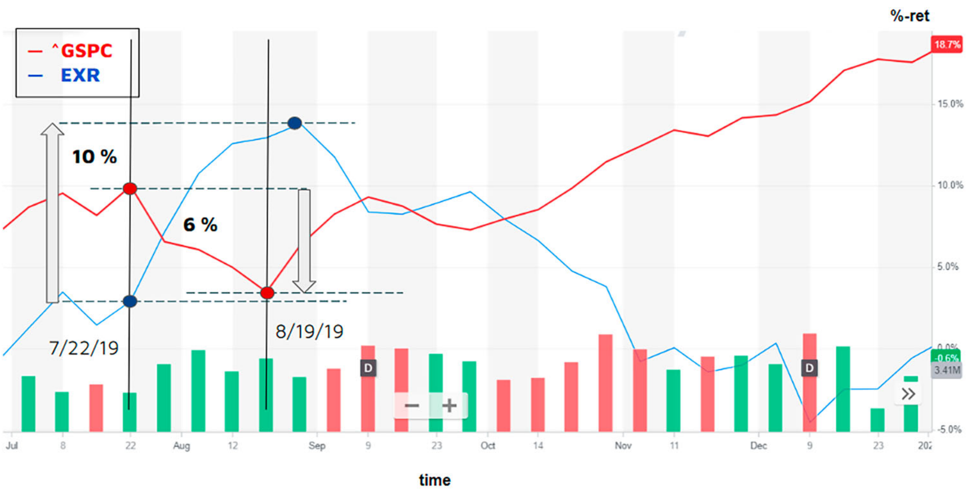

Figure 6. EXR (Extra Space Storage) raised 10% cumulative returns during the period 22 July 2019 to 19 August 2019 (< 1 month) when the market (S&P 500 index) experienced a 6% drop in returns during the same period.

companies that invest aggressively and those that invest conservatively (as indicated by the CMA) are explained by the new components.

General Representation of 3-factor and 5-factor Fama-French model:

1. 3-Factor Fama-French model:

$$r = R_f + \beta_1 * MKT + \beta_2 * SMB + \beta_3 * HML + \epsilon$$

2. 5-Factor Fama-French model:

$$r = R_f + \beta_1 * MKT + \beta_2 * SMB + \beta_3 * HML + \beta_4 * RMW + \beta_5 * CMA + \epsilon$$

Finding expected returns using 3-factor and 5-factor Fama-French model over the period 2018–2023 to benchmark with BPT:

Step 1: Running time-series regression for each stock in the portfolio (comprising of 80 stocks which form our population for BPT-analysis) to generate the **factor loadings for each stock**, that is, β_i's corresponding to each factor. The regression form is:

3-Factor Fama-French: $$r_i = \ constant + \ \beta_{MKT} * MKT_i + \beta_{SMB} * SMB_i + \ \beta_{HML} * HML_i + \ \epsilon_i \quad (for\ i^{th}\ stock)$$

5-Factor Fama-French: $$r_i = \ constant + \ \beta_{MKT} * MKT_i + \ \beta_{SMB} * SMB_i + \ \beta_{HML} * HML_i + \beta_{RMW} * RMW_i + \ \beta_{CMA} * CMA_i + \ \epsilon_i \quad (for\ i^{th}\ stock)$$

Here β_K. is the OLS estimate of factor loadings corresponding to Factor K generated on running the regression with historical returns as the dependent variable and Factors as the independent variable.

Step 2: Running cross-sectional regression over the 80 stocks in the portfolio to find the factor risk premia, that is, the portfolio risk associat with a particular factor. Once we know the factor loadings, we run a cross-sectional regression on the stocks in our portfolio to estimate the values of the factor risk premia. The regression form is:

3-Factor Fama-French: $$E(R_p) = \ const + RiskPremia_{MKT} * \beta_{MKT} + RiskPremia_{SMB} * \beta_{SMB} + \ RiskPremia_{HML} * \beta_{HML} + \epsilon_i$$

5-Factor Fama-French: $$E(R_p) = \ const + RiskPremia_{MKT} * \beta_{MKT} + RiskPremia_{SMB} * \beta_{SMB} + \ RiskPremia_{HML} * \beta_{HML} + \ RiskPremia_{RMW} * \beta_{RMW} + \ RiskPremia_{CMA} * \beta_{CMA} + \ \epsilon_i$$

Here, $RiskPremia_K$ is the OLS estimate of the factor risk premia generated by running a cross-sectional regression over the stocks in our portfolio, with the expected value of historical returns for each stock as the dependent variable and factor loadings obtained from Step 1 as the independent variable.

4.3. Empirical results corresponding to behavioural portfolio theory and ESG Scores

An analysis of the BPT characteristics involves the selection of all 80 stocks (Table 2), the primary reason for which is diversified selection (Table 3 is a subset of Table 2). Then, we assess the percentage of portfolios meeting the first safety constraint for various aspirations and subsistence for the characteristic study of behavioural portfolios. If more than 1,000 (1-α) of the 1,000 potential outcomes that were considered return more than the aspiration level S, a portfolio is deemed to have survived the safety test.

Furthermore, it may be noted that the proportion of portfolios passing the safety-first constraint is 19% when the maximum allowable probability of ruin is equal to 0.2, and the expected end wealth level is the same as the starting wealth, but it rises to 90% when the maximum allowable probability of ruin α is equal to 0.35. As the urge for investors decreases with α, this result seems to be quite normal. When α declines, the investor must be certain of additional natural circumstances. Conversely, if α grows, the speculator must be certain of fewer natural conditions. Therefore, it may be affirmed that when α is high, it is very likely that a portfolio would meet the safety-first constraint. Notably, to generate the findings mentioned above, we considered only taking into account hypothetical situations in which at least one of the portfolio weight distributions satisfied the first safety constraint.

Comparison with 3-Factor and 5-Factor Fama French Model:

3-Factor Fama-French average returns: $E(R_p) = \ 0.36\%$

5-Factor Fama-French average returns: $E(R_p) = \ 3.01\%$

Table 4. Factor loadings for 3-Factor Fama-French model for each stock calculated using historical data corresponding to factors and daily returns.

Stock Ticker	F1 (MKT)	F2 (SMB)	F3 (HML)
ABMD	0.7061459	−1.7982321	−0.1798496
ATVI	−0.4065323	0.23784798	−0.0500213
ADBE	0.07523146	−1.0746655	−0.0140039
LNT	0.09516842	−0.2806525	−0.3859637
GOOGL	0.32137787	−1.3302362	−0.0718488
AMZN	0.35522942	−0.2450898	−0.3633947
AEE	0.04830748	−0.4145238	−0.3841115
AEP	0.09326268	−0.4080751	−0.452504
AMT	0.14968788	−0.3165708	−0.4366032
AWK	0.30769851	−0.7782169	−0.1971177
ATO	0.24976421	−0.6518978	−0.2709817
AZO	−0.1996942	−0.1966167	0.25634217
CPB	0.123065	0.00903451	0.06288619
CHD	0.29474846	−0.3243226	−0.6239384
CINF	−0.1522022	−1.6430968	0.64973112
CTXS	−0.1244879	0.24198621	−0.0055262
CLX	0.20725843	−0.5453104	−0.3638093
CME	0.10646304	−0.3299407	−0.4126329
CMS	0.17638052	−0.1740179	−0.4800238
ED	−0.0644547	−0.1646004	0.13030497
STZ	−0.0997925	−0.8151156	0.12130567
CPRT	0.25201356	−0.4527439	−0.5114795
CCI	0.0624659	−1.5565969	0.76721137
DXCM	0.27523969	−1.0048399	−0.5054098
DLR	0.31478259	−0.3474308	−0.0333639
DG	0.44530769	−0.5727043	0.16919388
DPZ	0.17177542	−0.4177949	−0.339699
DTE	0.12320783	−0.2897829	−0.4695941
DUK	0.21715828	−1.1833146	−0.5950432
EBAY	0.14798976	−0.7725986	−0.2565782
EW	−0.4857809	−0.1488427	0.49030385
EA	0.25162926	−0.7422446	0.1321176
LLY	0.30978767	−0.792938	−0.4436469
EQIX	−0.0764118	−0.5750612	−0.101468
EVRG	0.059016	−0.1533004	−0.4678619
ES	0.09578295	−0.3507382	−0.5821069
EXR	0.09978295	0.09778295	0.10178295
FISV	0.10578295	0.09778295	0.10478295
GIS	0.09678295	0.10578295	0.09678295
GPN	0.01778385	0.1480425	−0.1120987
HRL	−0.5649248	0.24701264	0.599172
IDXX	0.34791793	−0.0952583	−0.5948761
JKHY	0.37352316	−2.1402101	0.29587619
KMB	0.03586907	−0.4965991	0.16679134
KR	0.18125993	−0.391804	−0.0881307
MKTX	0.62277051	−0.9708707	−0.2882832
MKC	0.16993217	−1.1677633	0.31075365
MCD	0.36945845	−0.5184491	−0.3943647
MAA	−0.0028071	−0.1942543	0.10724831
MPWR	0.31519764	−1.0882861	−0.2158784
MSI	0.08617201	−0.8825192	−0.3397336
NDAQ	0.29945886	−0.5978717	−0.1909105
NFLX	0.03599763	−0.7648135	0.16392244
NEM	−0.1817039	−0.7946689	0.63509906
NEE	0.42353916	−1.6515804	−0.0267992
NKE	0.29851245	−0.3012168	−0.3892201
NI	0.28157052	−1.0910569	−0.3980371
NOC	0.14940937	−0.492797	−0.2410059
NVDA	0.03306326	−0.5943019	0.51592659
NVR	1.02764041	−2.131728	−0.6157399
ORLY	0.16075624	−0.5124448	0.09023207
PNW	−0.0421811	−0.4061703	0.46259331

(*Continued*)

Table 4. Continued.

Stock Ticker	F1 (MKT)	F2 (SMB)	F3 (HML)
POOL	0.28779503	−0.5175605	0.09427927
PGR	−0.0685603	−1.0200632	−0.1186472
PSA	−0.2942029	0.12570942	0.08985828
O	0.1716354	−0.4445585	−0.4430556
REGN	0.11155977	−0.6124491	−0.2065868
ROL	0.141487	−0.8464836	0.20297383
SBAC	0.13187645	−0.9752179	0.58321187
SO	0.27723374	−0.3744772	−0.5359467
SBUX	0.02625744	0.01027393	−0.4126115
TTWO	0.22646788	−0.5618244	0.48945803
TYL	−0.2419124	−0.1259122	−0.053569
TSN	0.1325347	−0.6276235	−0.564147
VRSN	0.16997002	−0.9384132	−0.2819353
VRSK	0.1555468	−0.6973168	0.18802385
WRB	0.2824387	−0.6366637	−0.2704188
WEC	−0.1909723	−0.8938296	0.40389636
WST	0.07975966	−0.0525258	−0.4307742
XEL	0.63091298	−0.7980326	−0.4197699

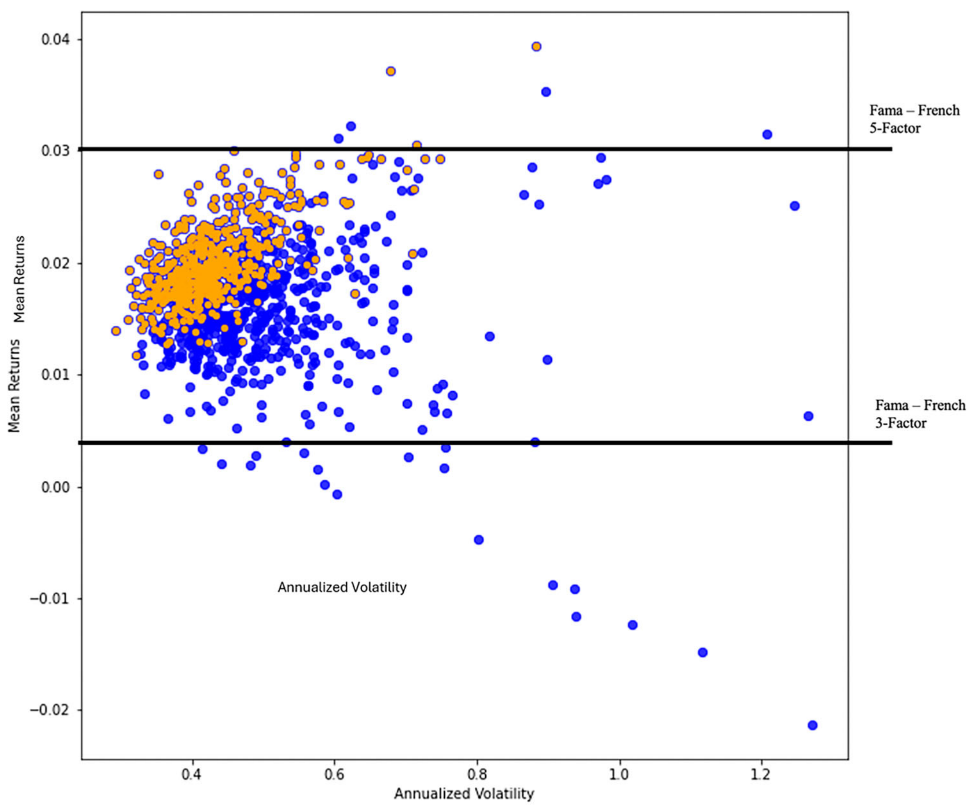

Figure 7. Portfolios in agreement with the safety-first constraint for $\alpha = 0.25$ *and* $S = W$.

4.4. ESG score and responsible portfolios

Based on the ESG preference function, for each simulation, we calculated the utility derived from the ESG Preference function of the portfolios falling within the ambits of the BPT efficient frontier and the utility derived from the ESG Preference function of the portfolios outside the BPT efficient

Table 5. Factor loadings for 5-Factor Fama-French model for each stock calculated using historical data corresponding to factors and daily returns.

Stock Ticker	F1 (MKT)	F2 (SMB)	F3 (HML)	F4 (RMW)	F5 (CMA)
ABMD	0.02107902	−0.0046113	0.01922996	0.18898755	−0.0071211
ATVI	0.03558236	0.03200375	−0.0096396	0.12494136	−0.0369888
ADBE	0.00533562	−0.0708112	0.0681461	0.04277957	−0.066467
LNT	0.00759159	0.01589311	−0.05072	0.05055676	0.00614424
GOOGL	0.02228045	−0.0170264	0.01017364	0.10570618	−0.0032625
AMZN	−0.000388	−0.0322521	0.0337234	0.02951436	−0.0174538
AEE	0.00808576	−0.0026743	−0.0381513	0.02484802	0.01041471
AEP	0.0085193	0.01233033	−0.0494838	−0.0030931	0.04443725
AMT	0.01319112	−0.0106639	−0.0111887	0.04289439	0.02933801
AWK	0.00489645	−0.0039618	−0.0042947	0.03485594	0.01687999
ATO	0.00399255	−0.0251741	−0.0125943	0.01070934	−0.010717
AZO	0.00072983	0.01463334	−0.0383216	−0.0382881	0.00049001
CPB	−0.0053581	0.01115901	−0.0119743	0.01492483	−0.0204759
CHD	0.00069846	−0.0059108	−0.0215157	−0.0120309	0.01760434
CINF	0.01396568	−0.0412269	0.01860869	0.0992527	−0.0476964
CTXS	0.00083828	0.03355838	−0.0417416	0.02801345	0.05163575
CLX	0.01172807	−0.0512232	0.03534796	0.0454916	−0.0251708
CME	0.00161346	−0.005865	−0.0281639	−0.002613	−0.0020609
CMS	−0.0072651	−0.0155499	−0.0237323	−0.0722422	0.03217666
ED	−0.0159201	−0.0680104	0.08043194	−0.0714622	−0.086501
STZ	0.0044558	−0.0436704	0.05168285	−0.0210377	−0.0858928
CPRT	−0.0046947	−0.0496339	0.03049687	−0.0314915	0.00462746
CCI	0.01956691	−0.0405388	0.05451163	0.08193856	−0.0105435
DXCM	0.00901686	−0.0269672	−0.0272094	0.08902602	0.07564704
DLR	−0.0094488	−0.0330349	0.04519224	−0.0908413	−0.0373135
DG	−0.025218	−0.065485	0.0758029	−0.0796208	−0.0272474
DPZ	0.00729657	0.00517721	−0.0239295	0.02779749	0.00799634
DTE	0.00012581	0.00477107	−0.0157684	−0.0474388	0.04067439
DUK	0.03861317	0.02062635	−0.0324326	0.1599959	0.0368649
EBAY	0.00855966	−0.0547885	0.08206898	0.04262784	−0.1244554
EW	0.01165069	0.0066117	0.00745902	0.08919805	−0.0319304
EA	0.01892992	−0.0203331	0.03973324	0.1717186	0.04079778
LLY	0.00873501	0.00679907	−0.0184671	0.05007112	0.04910238
EQIX	0.01114278	−0.0115542	−0.0421444	0.06911957	0.00850517
EVRG	0.01292586	0.00369936	−0.0277506	0.06112002	0.03352108
ES	0.01979895	−0.0310261	−0.0093675	0.0489704	0.03095269
EXR	0.02979895	0.02897391	0.02063252	0.0989704	0.12095269
FISV	0.08979895	0.01897391	0.02063252	0.0889704	0.08095269
GIS	0.06979895	0.00897391	0.00063252	0.1089704	0.13095269
GPN	0.00151837	0.00051705	−0.0215578	−0.0132359	0.00180362
HRL	−0.0119798	−0.0426501	0.10544113	−0.0838036	−0.1809582
IDXX	0.009109	−6.63E−05	−0.0216039	0.06142139	0.02668882
JKHY	0.01334736	−0.0281163	0.04903291	0.03584882	−0.0594818
KMB	0.00459355	−0.0501091	0.054227	−0.0025293	−0.0846077
KR	−0.0011544	−0.0107318	0.00957482	0.02895774	−0.0220765
MKTX	0.00150341	−0.0079482	−0.0080619	0.09673277	0.02740164
MKC	0.01795501	0.00993039	0.01141542	0.0808251	−0.0412956
MCD	−0.0083805	−0.0329768	0.01879665	0.02214243	−0.0409828
MAA	−0.008709	−0.007126	0.01537729	−0.0665589	−0.0134336
MPWR	0.00357618	−0.0263123	0.01651311	0.01914047	0.00619357
MSI	0.00997898	−0.0141739	0.01313906	0.07047077	−0.033602
NDAQ	0.00593409	−0.0269151	0.0220486	0.03036486	−0.045272
NFLX	0.01697303	−0.0089626	−0.0089319	0.08635658	−0.0001537
NEM	0.03005051	0.00047609	−0.0053659	0.0845061	−0.0223003
NEE	−0.0050885	0.00947025	−0.0567813	0.05029935	0.05246728
NKE	0.01193151	−0.0049414	−0.0164216	0.06315433	0.04052962
NI	−0.0034291	−0.0725806	0.09927906	−0.056803	−0.0676054
NOC	−0.0012183	−0.0182291	−0.0114823	−0.0340011	−0.0112374
NVDA	−0.0012684	−0.0164297	0.00592944	0.02273979	−0.0431189
NVR	−0.0089332	−0.1175216	0.09653121	−0.0301266	−0.0318575
ORLY	−0.0058886	−0.0123657	0.00847156	0.02966287	0.05141707
PNW	−0.0052929	−0.0139637	−0.0032287	−0.0338309	−0.0196813

(*Continued*)

Table 5. Continued.

Stock Ticker	F1 (MKT)	F2 (SMB)	F3 (HML)	F4 (RMW)	F5 (CMA)
POOL	−0.0180431	0.01886747	−0.0283844	−0.0427942	−0.0184399
PGR	0.0008656	−0.0696184	0.02815328	−0.001381	0.01127811
PSA	0.00731156	−0.0097091	−0.0225422	0.03225766	−0.0241934
O	0.00720683	−0.0276534	0.01198714	−0.0105459	0.02549018
REGN	0.00795995	−0.049548	0.02454899	0.03380624	−0.018645
ROL	0.01904086	−0.0045316	−0.029643	0.12891425	0.03709719
SBAC	0.00023816	−0.0047344	0.01554635	0.00961047	−0.0296869
SO	−0.0045585	−0.0414627	0.02687225	−0.0269439	−0.0131774
SBUX	−0.0015116	−0.0104669	−0.0035768	−0.0124587	−0.0208576
TTWO	−0.0162213	−0.0408118	0.06835944	−0.0125874	−0.1136414
TYL	0.01902002	−0.043194	0.04230977	0.05274286	−0.0471569
TSN	0.02406914	−0.0414892	0.04489251	0.01608178	−0.0372206
VRSN	0.00710704	−0.0235998	0.00508781	0.04385289	−0.0501229
VRSK	−0.0028739	−0.0662373	0.03573621	0.00249593	−0.0548918
WRB	−0.0085405	−0.0610818	0.04593461	−0.0300013	−0.0890614
WEC	0.01322845	−0.0361542	0.00708438	0.01031018	−0.0386011
WST	0.00038351	0.01329295	−0.0394634	−0.0279185	0.03761014
XEL	−0.0057798	−0.0255115	0.02895515	0.06597884	0.00136906

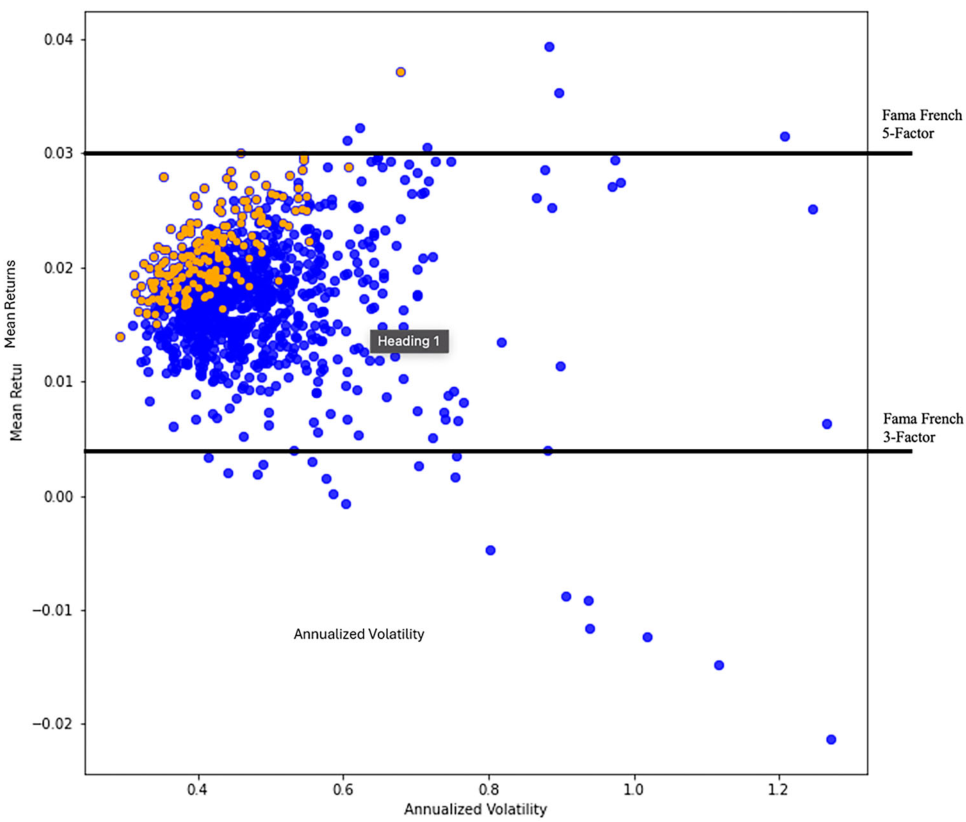

Figure 8. Portfolios in agreement with the safety-first constraint for $\alpha = 0.20$ *and* $S = W$.

frontier. Our analysis stretched across 10 years, starting from December 2012 to December 2022, and subsequently divided the total number of simulations across these 10 years to obtain a bigger picture of the portfolio losses for a given probability of investors' investments in drawdown-averse

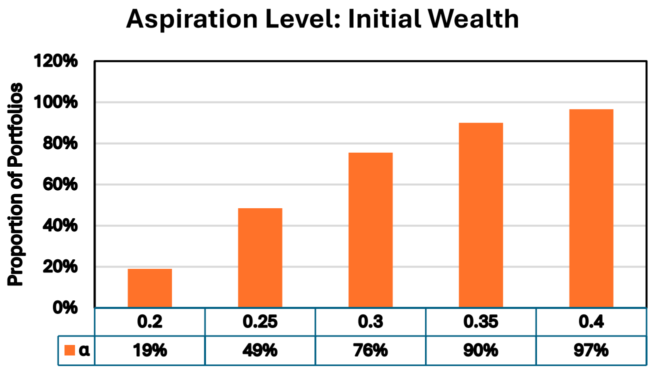

Figure 9. Number of portfolios out of 1,000 portfolios satisfying the safety-first constraint.

Table 6. BPT characteristic and Fama-French portfolio.

	Proportion of BPT portfolios greater than	
BPT characteristic	3-Factor Fama French	5-Factor Fama French
$\alpha = 0.25$ ***and*** $\boldsymbol{S} = \boldsymbol{W}$	100%	10.20%
$\alpha = 0.20$ ***and*** $\boldsymbol{S} = \boldsymbol{W}$	100%	15.79%

portfolios. Another important reason for segregating the ESG analysis across years is to observe the effect of COVID-19 on responsible investment.

According to the tabulated results shown above, behaviourally apt portfolios consisting of draw-down-opposed companies were in direct compatibility with socially responsible investing, represented by the utility derived from the ESG Preference function in nine of the 10 years. Pre- and post-pandemic periods (i.e. time horizons spanning years 2019 and 2022), witnessed a strong tuning between BPT and utility derived from the ESG Preference function (DF3) (%-simulations > 60%). However, responsible and rational investing witnessed a hit during the immensely affected periods of the pandemic (2020 and 2021). There has been crucial growth in concordance with the BPT and utility derived from the post-pandemic ESG Preference function, which represents a restoration of responsible investing.

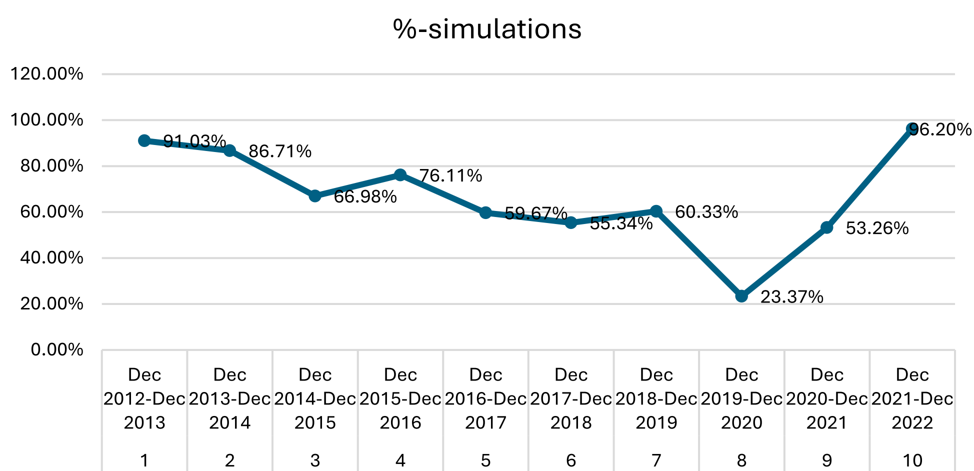

Figure 10. %-simulations which result in higher utility derived from the ESG preference function for BPT efficient portfolios than Non BPT efficient portfolios.

Calculations for the Conditional Drawdown-at-Risk were performed at various levels of significance to ensure robust outcomes. CDaR-beta is measured along with the standard beta to obtain drawdown-averse stocks. This helps ensure that stocks co-move negatively with the market during downturns. Furthermore, the BPT optimisation problem was carried out through a large number of simulations and by varying the probability of ruin to achieve robust results. The study also benchmarks the results with fundamental 3- and 5-Factor Fama French models.

5. Conclusion

To learn more about securities and instruments that have the necessary hedging capacity when the market experiences a downside, we extend the concept of the CAPM with drawdown measures framed in an earlier work of Zabarankin et al. (2014). We used CdaR at various confidence levels (α). We established the EroD as a new dynamic risk measurement variable. Moreover, both single- and multiple-sample-path settings for holding positions of stocks present in the portfolio for CAPM formulation were used to establish the requisite optimality requirements for effective portfolio management. This setting attempts to reduce or constrain the CdaR and EroD measures of risk hovering over the portfolio's growth rate over a certain period. The confidence level α, where $\alpha \in [0, 1]$, controls the proportion of the worst drawdowns that needs to be administered for CdaR- β calculation, with special cases of $\alpha = 1 \; and \; \alpha = 0$ stating maximum and average drawdowns, respectively. The EroD β assesses portfolio performance during market declines that are greater than a predetermined ε. The EroD β accounts for nonzero dips and drawdowns in the dataset used for small positive ε values.

We thereby concluded that both CdaR and EroD β are more sensitised towards market dips than the Standard Betaβ, and include several demonstrations for a few equities present in S&P 500 companies. Notably, while Standard Beta remains positive, both CdaR and EroD β may be negative for some equities. Consequently, despite having a positive Standard Beta, these equities show good returns when the market declines.

In the second part of the study, we analyse portfolios created from selected US equities under the condition that assures investors' wealth prosperity against the probability of ruin. This study primarily aims to understand the characteristics of an optimal BPT portfolio. According to their findings, BPT models advise investors when to engage in and leave the market based on their financial goals. It should be mentioned that the asset allocation produced by a behavioural optimal portfolio falls on the Markowitz space efficient frontier in approximately 80% of cases.

Additionally, this study resolves an important hypothesis concerning the utility derived from a portfolio's ESG Preference function. The utility derived from the ESG Preference function directly communicates the portfolio's sustainability and stability. We conclude with an upright finding that the portfolios present in the BPT optimal efficient frontier show higher utility than those outside the BPT efficient frontier for years unaffected by the pandemic. However, BPT and its derived utility did not seem to go hand in hand during the most affected pandemic period. Thus, it may be assumed that both the utility derived from the ESG Preference function and BPT are effectively synchronised when macroeconomic conditions are not drastically affected, thereby providing insights into the responsibleness, sustainability, and stability of the portfolio.

Integrating BPT with the framework of MPT helps in understanding the incorporation of probabilistic constraints instead of algebraic or statistical constraints and portfolio efficiency based on investment safety. By employing a simulation study, we tend to ensure that the majority of portfolios that follow the safety-first constraint and features in the Pareto front have a higher utility derived from the ESG preference function. This study proposes a stable and responsible investment not only in times of normal market conditions but also during market downturns. The theoretical implications involve optimising returns conditional on probabilistic scenarios that serve the purpose of socially responsible investments for a longer investment horizon.

Notably, some sectors featuring these stocks had commonalities. These include stocks from FMCG, consumer discretionary firms, and staple firms, which are largely associated with essential consumer requirements. Their stock prices are not affected by market decline because of the continued demand for such products, and can therefore be used as a good investment possibility.

Businesses and institutions indulging in ESG often enjoy a competitive advantage. The products and services offered by these firms are welcomed by the consumer base, as they promote sustainability. ESG financing promotes sustainability in business operations. Businesses that effectively incorporate ESG principles in their operations find opportunities for cost savings, less resource waste, lower energy usage, and significantly lower overall operational costs. Recently, investors have been extensively applying various analytical techniques to these non-financial factors to identify material risks and growth opportunities. Therefore, ESG investors may be understood as potential 'value-based' investors and are thereby more interested in the larger investment horizon, and less worried about the momentary and short-lived returns. They valued time and associations with companies with ESGs inherent in their mandates.

Disclosure statement

No potential conflict of interest was reported by the authors.

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