Application of 2D Variational Mode Decomposition Method in Seismic Signal Denoising

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Abstract—**Seismic data are typical nonlinear and nonstationary data. In the acquisition and processing of seismic data, many factors interfere with it. Seismic data contain both effective waves and random noises, seriously affecting the quality of seismic data and not conducive to the goal of fine interpretation of subsequent seismic data. Therefore, studying new seismic data denoising methods is beneficial for improving the quality of seismic data and plays a very important role in subsequent seismic data interpretation. In this paper, the principle of variational mode decomposition (VMD) and 2D-VMD is introduced in detail, and the seismic profile with a simple signal and fault model is denoised. Compared to traditional empirical mode decomposition (EMD), the 2D-VMD method has the best seismic data denoising effect. The test results of the synthesised signal show that the 2D-VMD method has a signal-to-noise ratio of 47.14 dB after denoising, which is higher than the signal-to-noise ratio after EMD and VMD denoising, indicating that it has a better denoising effect. The VMD and 2D-VMD methods are applied to the denoising of actual seismic data. The application results show that the 2D-VMD method can effectively improve the quality of the seismic data, enhance the continuity and reliability of the seismic data, and is conducive to the fine interpretation of subsequent seismic data.**

Index Terms—**2D variable mode decomposition; Random noise; Denoising; Forward modelling.**

I. INTRODUCTION

Seismic exploration is one of the important geophysical exploration means for the development of mineral resources and engineering construction [1]. The process of seismic exploration is often affected by various factors, mainly noise caused by the natural environment, system noise, and secondary noise generated by data acquisition instruments. In the process of seismic signal processing, how to effectively remove these interferences and improve the quality of seismic exploration and improve the reliability of seismic data is an urgent problem in seismic exploration [2]–[5]. At present, existing noise suppression methods include median filtering, principal component analysis, f-x domain predictive

filtering, independent component analysis, polynomial fitting, wavelet transform, nonlocal mean, empirical mode decomposition (EMD), variational mode decomposition (VMD), etc. [6], [7]. In addition to the above methods, there have been many new methods in recent years. Geetha, Hota, and Karras [8] used honey badger optimisation algorithm to optimise the wavelet transform method; compared with the traditional denoising method, the denoising effect of this method is more obvious. Zhu, Sui, Li, Li, Gu, and Wang [9] used frequency domain singular value decomposition (FSVD) to denoise the microseismic mine signal. The results show that the signal-to-noise ratio is improved by more than 10 dB after FSVD processing. Tan, Hu, and Ohno [10] used the generalised S-transform and nonlinear complex diffusion for seismic signal denoising [10]. As a result, the signal-to-noise ratio increased.

Variational mode decomposition (VMD) is a new adaptive and nonrecursive signal processing method proposed by Dragomiretskiy and Zosso [11]. Compared to classical empirical mode decomposition (EMD), it has the advantages of strong noise resistance [12], [13]. In 2015, Dragomiretskiy and Zosso [14] proposed two-dimensional variational mode decomposition (2D-VMD). Similar to conventional VMD, the 2D-VMD constraint function introduced a penalty term and a Lagrange multiplier to transform the equation into an unconstrained equation. The saddle point of the extended Lagrange equation was obtained by alternating direction method of multipliers (ADMM), the decomposed two-dimensional modal components and the corresponding frequency vectors can be obtained. Fang, Wen, Gu, Liu, and Zhang [7] studied the seismic data noise removal method based on VMD. The research results show that the VMD method has excellent noise suppression and amplitude retention performance. Gao, Ma, Dong, Lu, and Liu [15] proposed a joint algorithm that combined 2D-VMD with the correlation coefficient and used it for image reconstruction. The algorithm has high image reconstruction accuracy and improved image quality. Chang and Gao [16] proposed a 2D-VMD-based data denoising algorithm, which can remove noise and retain effective components in the original signal to

the maximum extent. Xue and He [17] proposed a data denoising strategy that effectively combines 2D-VMD and bilateral filtering methods. Na [18] used genetic algorithm to optimise the 2D-VMD denoising method and applied this algorithm to the study of denoising of medical ultrasound images. Feng, Liu, Liu, Zhou, and Zhao [19] used the adaptive Wiener filtering method to process the signal after decomposition of 2D-VMD and reconstruct the signal, which improved the details after the image denoising. Zhu, Zhang, Zhao, and Chen [20] proposed an improved 2D-IVMD for target recognition of carrier-free ultra-wideband (UWB) radar.

Seismic signal denoising is an important step in seismic data processing. 2D-VMD technology has achieved good application results in image processing and other fields of denoising. However, the research of 2D-VMD in seismic data denoising is still relatively limited, and the application effect also needs further verification. Therefore, this paper introduces the 2D-VMD method into the denoising of seismic data. The testing of simple signals and synthetic seismic data demonstrates the effectiveness of the method, revealing that the 2D-VMD method can be used to denoise seismic signals. The application results show that VMD and 2D-VMD methods have strong denoising ability and can effectively improve the signal-to-noise ratio and data quality. Compared to the VMD method, 2D-VMD can improve data quality more effectively, enhance the continuity of seismic events, and is conducive to the fine interpretation of subsequent seismic data.

II. BASIC THEORY

EMD decomposition is an effective method for data analysis. First, the original signal is interpolated by a cubic spline, and then the envelope is calculated. A new sequence can be obtained by subtracting the envelope mean value from the original data, repeat the above processing process for the new sequence until it is a monotone function. At this time, this sequence is the first intrinsic mode function (IMF), and then this component is extracted from the original signal; by taking the remaining residual term as a new original signal and repeating the above process, the original data can be decomposed into n IMF. The process of EMD is as follows [21].

The original signal is interpolated by a cubic spline, and the envelope mean is calculated. A new data sequence can be obtained by subtracting the original signal, i.e.,

$$x(t) - m_1 = h_1. \quad (1)$$

Then repeat the above process, including

$$h_1 - m_{11} = h_{11}, \quad (2)$$

where m_{11} is the upper and lower envelope mean of h_1. Repeat the appeal process k times until h_{1k} is a monotone function, i.e.,

$$h_{1(k-1)} - m_{1k} = h_{1k}. \quad (3)$$

The eigenmode function obtained by the first decomposition is represented by c_1,

$$c_1 = h_{1k}. \quad (4)$$

It will be extracted from the original data, and the other is the residual term, namely

$$r_1 = x(t) - c_1. \quad (5)$$

After that, take the residual item as a new sequence and repeat the above method to obtain a new residual item, namely

$$r_2 = r_1 - c_2. \quad (6)$$

Repeat the above steps until the residual meets the decomposition requirements. At this point, the signal decomposition is completed, and the following results are obtained

$$x(t) = c_1 + c_2 + c_3 + \ldots\ldots + c_n + r_n. \quad (7)$$

In this way, the original signal is decomposed into IMF and residual terms through the above steps.

EMD is more suitable for time-frequency analysis of nonlinear and nonstationary signals, such as seismic data, but it still has some problems, such as end effect and mode aliasing.

It decomposes the original signal by a single model to obtain multiple eigen functions, and then removes the residual based on the threshold criterion, and reconstructs the signal to obtain the reconstructed signal. It is an adaptive method with good decomposition ability for nonlinear seismic signals, which can decompose signals into IMF [10]. Furthermore, it has a reliable mathematical theoretical foundation and can better solve the problem of pattern mixing in EMD methods. The VMD decomposition steps are as follows [22]:

1. Calculate each intrinsic mode function through the Hilbert transform and solve for the marginal spectrum of each mode function.
2. Estimate the centre frequency of each mode based on exponential hybrid modulation and modulate the spectrum of each mode to the corresponding baseband.
3. Estimate the bandwidth of each modal function based on the Gaussian smoothness and gradient squared criterion of the demodulated signal.

We obtained a variational constrained problem and continued to solve the unconstrained problem using Lagrange multipliers and penalty function terms and operators. Using the iterative relationship of the function, we obtain the IMF. The specific construction steps are as follows.

Calculate the unilateral spectrum of the seismic data signal according to the first step

$$\left[\delta(t) + \frac{j}{\pi t} \right] * u_k(t), \quad (8)$$

where δ is the Dirichlet impulse function and $*$ is the convolution symbol. Calculate the centre frequency and response baseband according to step 2

$$\left[\left(\delta(t) + \frac{i}{\pi t} \right) * u_k(t) \right] e^{-j\omega_k t}. \quad (9)$$

Calculate the square L2 norm of the above demodulation signal gradient and estimate the signal bandwidth. By introducing constraints, an optimal variational model is constructed to minimise the sum of the aggregation bandwidth of each component

$$\min_{\{u_k\},\{\omega_k\}}\left\{\sum_{k=1}^{k}\left\|\partial_t\left(\delta(t)+\frac{j}{\pi t}\right)*u_k(t)\right]e^{-j\omega_k t}\right\|\right\}, \quad (10)$$

where K is the number of components, $u_k = \{u_1, u_2, ..., u_k\}\omega_k = \{\omega_1, \omega_2, ..., \omega_k\}$.

4. By introducing the Lagrangian multiplication operator and the sum of quadratic penalty factors, continuous iteration is achieved. Then

$$\sum_{k}\left(\frac{\left\|\hat{u}_k^{n+1}-\hat{u}_k^n\right\|_2^2}{\left\|\hat{u}_k^n\right\|_2^2}\right)<\varepsilon. \quad (11)$$

5. By repeatedly repeating the above process, the K IMF components were ultimately obtained.

All generalised VMD properties are integrated to define the two-dimensional extension of the VMD. From the definition of two-dimensional analytic function, the minimisation of this function is the following

$$\left|\begin{array}{c}\min_{\{u_k\},\{\omega_k\}}\left\{\sum_{k}\alpha_k\left\|\nabla\left[u_{AS,k}(x)e^{-j(\omega_k,x)}\right]\right\|_2^2\right\}\\ s.t. \forall x: \sum_{k}u_k(x)=f(x),\end{array}\right| \quad (12)$$

of which $\{u_k\} = \{u_1, ..., u_k\}, \{\omega_k\} = \{\omega_1, ..., \omega_k\}$.

The above equation is used as an objective function to evaluate the modal bandwidth. The bandwidth and the square criterion of the gradient are evaluated. At the same time, to maintain the fidelity of the signal, it will be adjusted to the respective estimated centre frequency through the mixing index.

By adding a quadratic penalty term and Lagrange multiplier to deal with reconstruction constraints, the optimisation method is the alternating direction method of multipliers (ADMM).

The 2D-VMD algorithm is continuously updated and optimised in the frequency domain, and uses the results obtained from the inverse Fourier transform to obtain the final result. The specific process of the 2D-VMD algorithm can be described as follows [23]:

1. Initialize, and $\{\hat{u}_k^0\}\{\hat{\omega}_k^0\}, \{\lambda^0\}$ n;
2. It is updated in frequency domain by ADMM;
3. Update, where λ

$$\hat{\lambda}^{n+1}(\omega)=\hat{\lambda}^n(\omega)+\tau\left(\hat{f}(\omega)-\sum_{k}\hat{u}_k^{n+1}(\omega)\right); \quad (13)$$

4. Until, stop the iteration, $\frac{\sum_{k}\left\|\hat{u}_k^{n+1}-\hat{u}_k^n\right\|_2^2}{\left\|\hat{u}_k^n\right\|_2^2}<K_e$.

III. DENOISING TEST OF SIMPLE SIGNAL AND FAULT MODEL

An ordinary simple cosine signal is established and the noise is added to it. The cosine signal is obtained as shown in Fig. 1. Using EMD and VMD to denoise the signal, the results are shown in Fig. 2.

It can be seen from the results in Fig. 2 that both VMD and EMD methods can denoise the signal when ratio noise is added. The signal-to-noise ratio of seismic signals using the VMD method has been improved. The error analysis in Fig. 3 shows that VMD has a better effect and can improve the signal-to-noise ratio of data.
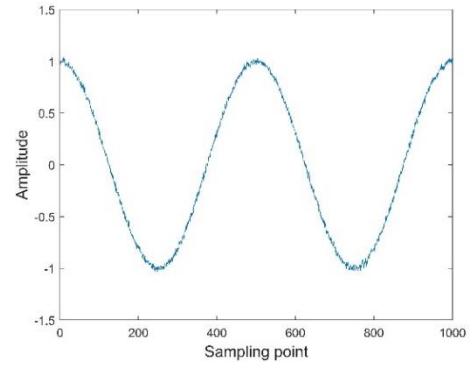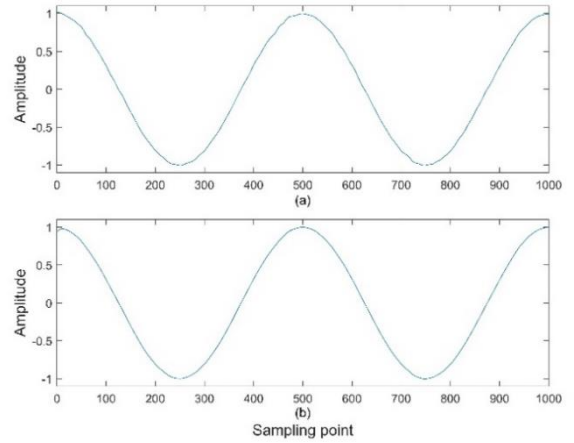

Fig. 1. Signal with noise.

Fig. 2. Simple signal reconstruction diagram of the EMD and VMD methods.
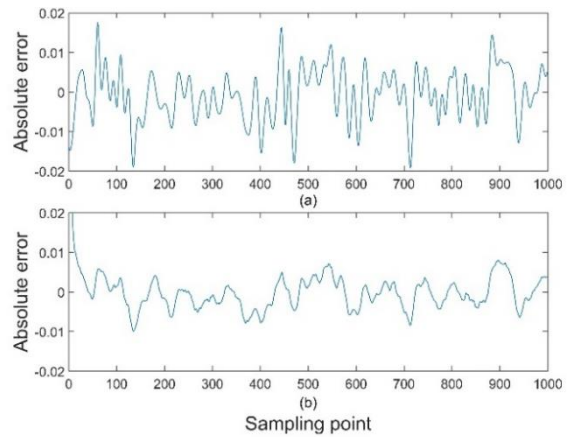

Fig. 3. Error analysis.

To compare and analyse with existing denoising methods in recent years, the EMD, VMD, and 2D-VMD methods were used to perform comparative denoising analysis on the

signals tested in [24]. First, establish three sine signals (*s1*, *s2*, *s3*):

$$s1 = sin(2 * pi * t),$$
$$s2 = 0.5 * sin(24 * pi * t),$$
$$s3 = 0.125 * sin(288 * pi * t),$$

and one Gaussian white noise signal (*s4*)

$$s4 = wgn(2000, 1, -35)',$$

$t = 0.001:0.001:2.$

Figure 4 shows four signal components. Figure 5 shows the signal without noise (*s1 + s2 + s3*). Figure 6 shows a signal with noise (*s1 + s2 + s3 + s4*), with an initial signal-to-noise ratio of 32.9360 dB.
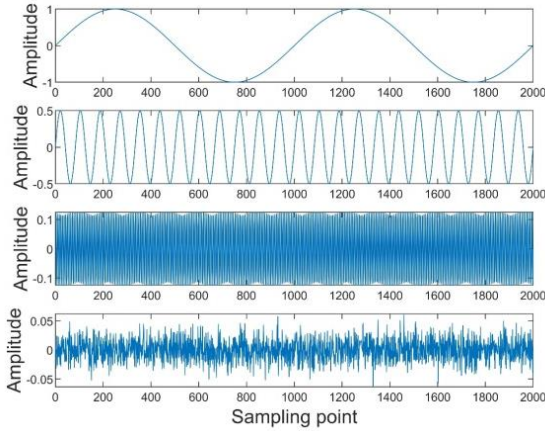

Fig. 4. Four signal components.
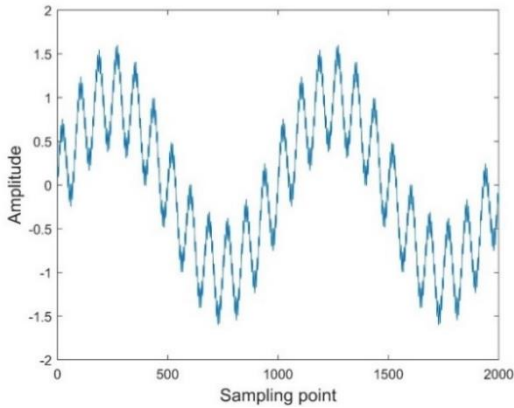
Fig. 5. Synthetic signal without noise.
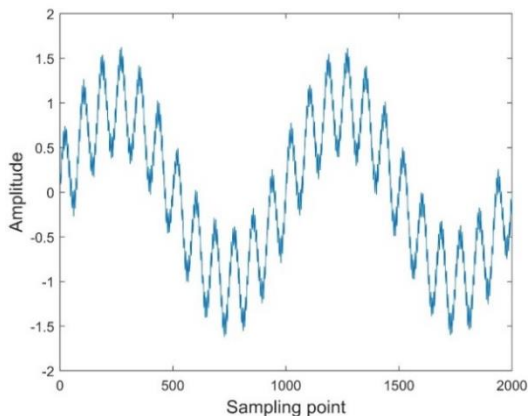
Fig. 6. Synthetic signal with noise.

Figures 7 to 9 show the results of the decomposition of EMD, VMD, and 2D-VMD. Figures 10 to 12 show the results of EMD, VMD, and 2D-VMD denoising.

The test results of the synthesised signal show that the 2D-VMD method has a signal-to-noise ratio of 47.14 dB after denoising, which is higher than the signal-to-noise ratio after EMD and VMD denoising, indicating that it has a better denoising effect.

There is a significant modal aliasing effect in the EMD decomposition results and there is a significant improvement in the VMD decomposition results compared to the EMD. First, low-frequency signals are decomposed, and then high-frequency signals are decomposed.
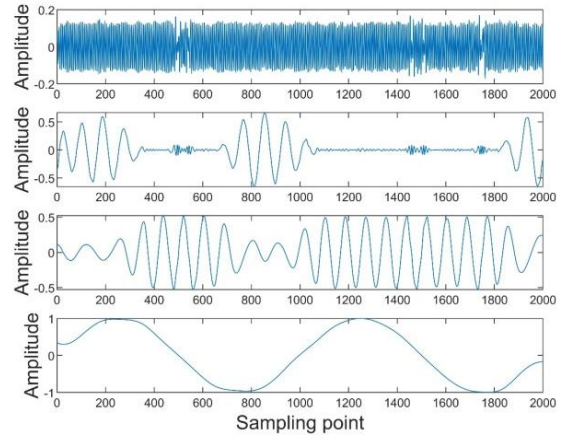

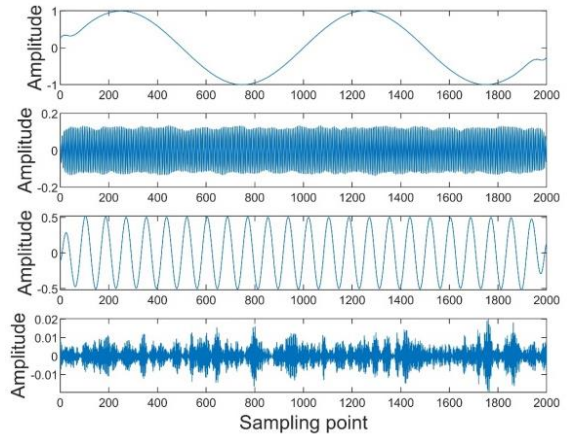Fig. 7. Results of the EMD decomposition.
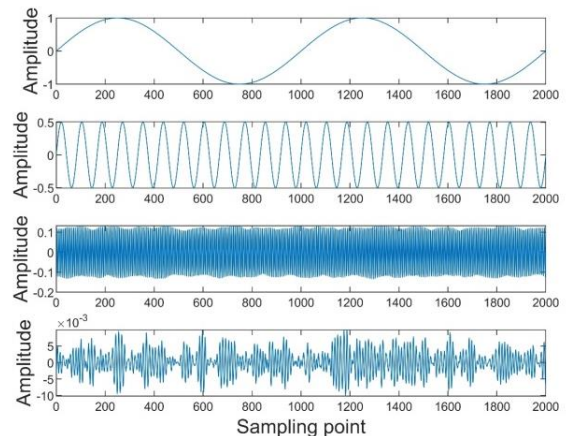

Fig. 8. Results of the VMD decomposition.

Fig. 9. Results of the 2D-VMD decomposition.

The reconstruction effect of the EMD is poor, and there is a

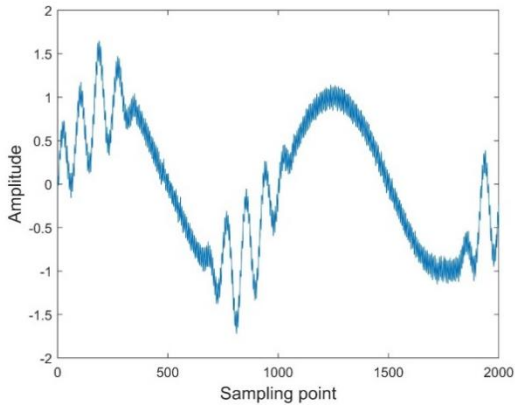significant error between it and the original signal.

Fig. 10. The denoising results of EMD.

The signal-to-noise ratio after EMD reconstruction is 8.368 dB, the signal-to-noise ratio after VMD reconstruction is 43.5622 dB, and the signal-to-noise ratio after 2D-VMD reconstruction is 47.1409 dB. The signal-to-noise ratio has been significantly improved, which is beneficial for denoising seismic signals.
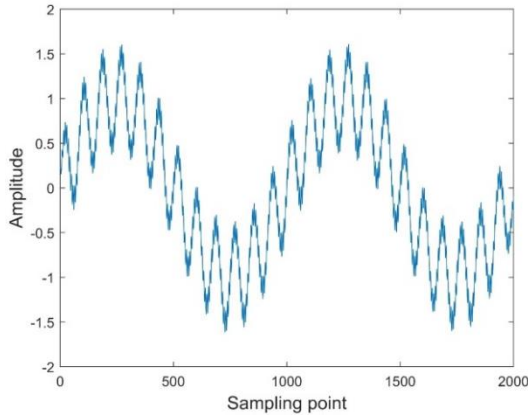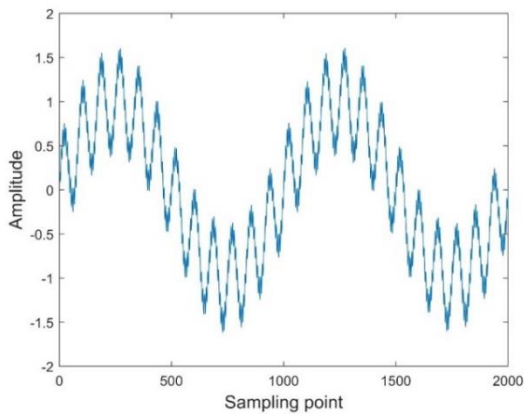

Fig. 11. The denoising results of VMD.

Fig. 12. The denoising results of 2D-VMD.

A fault is a structure in which the earth's crust is fractured under stress and the rock blocks on both sides of the fault face undergo significant relative displacement. Fault interpretation is an important part of seismic data interpretation. To illustrate the effect of the denoising method proposed in this paper, the seismic profile-containing fault model is used for comparative analysis. Establish a simple fault model as shown in Fig. 13. Two faults are located near trace 60. First, add 35 % signal-to-noise ratio noise to each

signal in the model to get the one-dimensional noise fault model as shown in Fig. 14. Then use the EMD method, VMD method, and 2D-VMD method to denoise this fault model. The results are shown in Figs. 15–17.
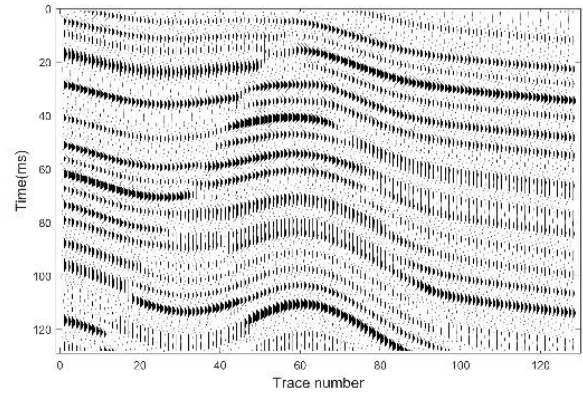

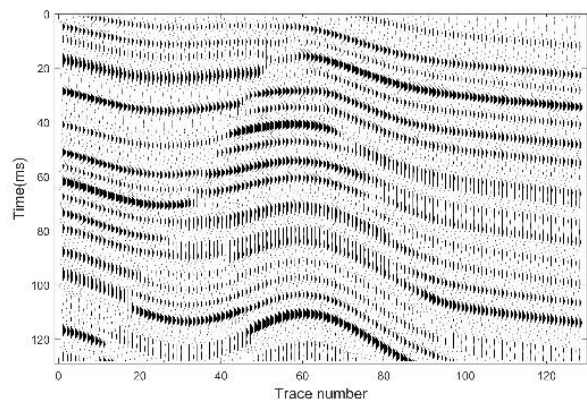Fig. 13. Simple fault model.

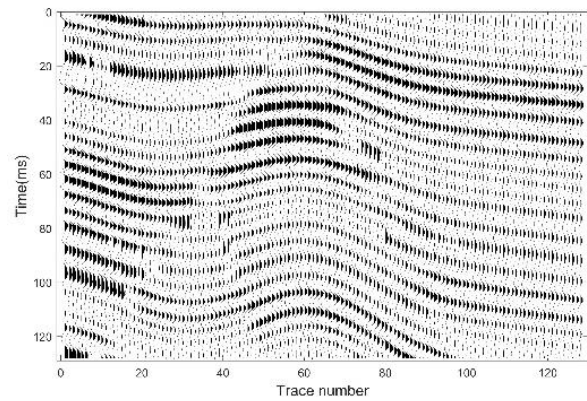Fig. 14. One-dimensional noise fault model.

Fig. 15. Reconstruction of the one-dimensional noise fault model by EMD.
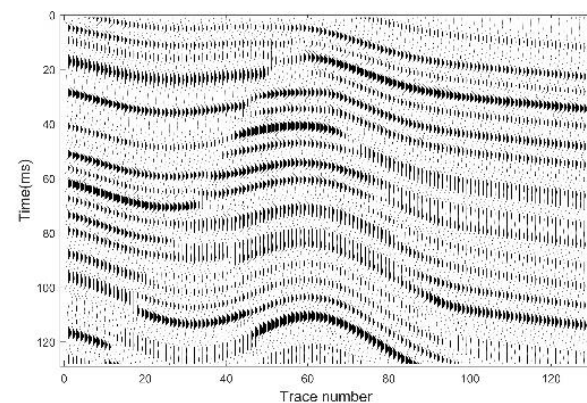

Fig. 16. Reconstruction diagram of the one-dimensional noise fault model of the VMD method.
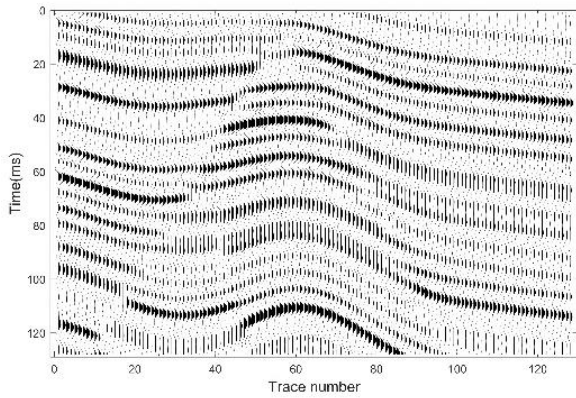
Fig. 17. Simple model reconstruction of the 2D-VMD noise method.

As shown in Figs. 15–17, it can be seen that the denoising effect of VMD is better than that of EMD when testing the fault model, while the effect of 2D-VMD is good, but the difference between it and ordinary VMD is not obvious.

To test the denoise effect of 2D-VMD on 2D images, add 2D Gaussian noise as shown in Fig. 18 to the existing fault model and then use three methods to denoise the noisy model, and the results are shown in Figs. 19–21.
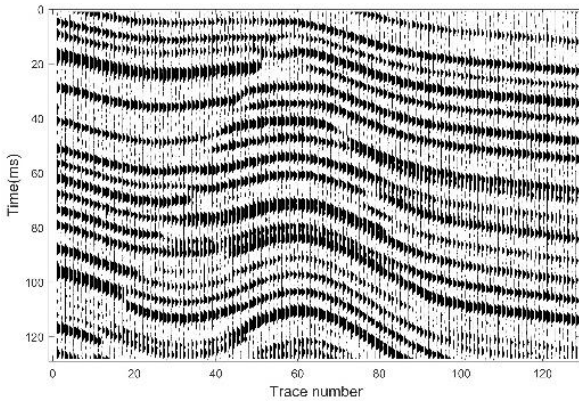

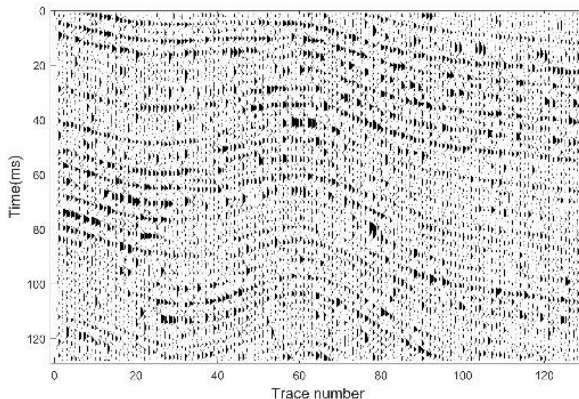Fig. 18. Two-dimensional Gaussian noise fault model.

Fig. 19. Reconstruction of the two-dimensional noise fault model by the EMD method.

From Figs. 19–21, it can be seen that when testing the fault model with two-dimensional Gaussian noise, the recognition of the original model after EMD denoising is greatly reduced. The effect of VMD denoising is better than that of EMD, but there is still a lot of noise interference on the image; compared to the above two traditional methods, the denoising effect of 2D-VMD is more obvious, and the original in-phase
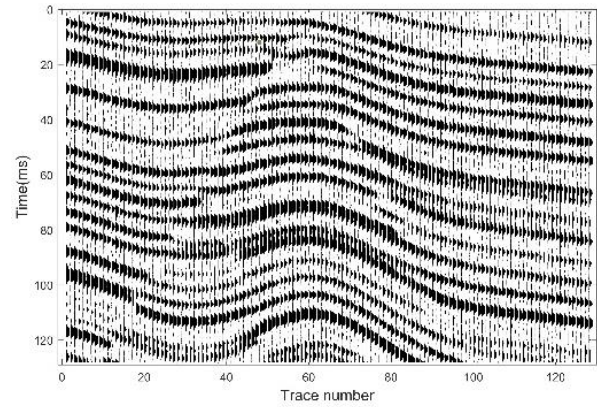
axis is clearer.

Fig. 20. Reconstruction diagram of the two-dimensional noise fault model of the VMD method.
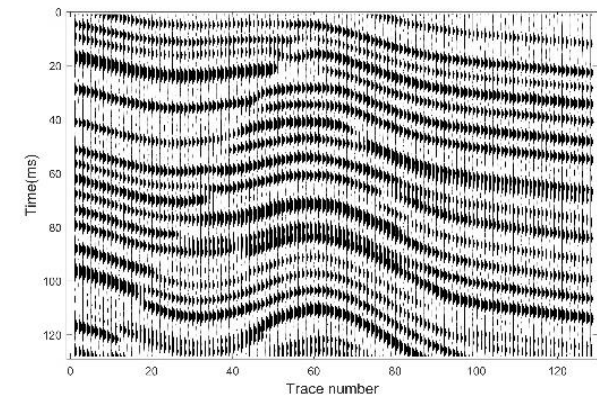

Fig. 21. 2D-VMD 2D noise method simple model reconstruction results.

IV. APPLICATION EXAMPLES

Most of the study area is covered by a quaternary system, and only bedrock is exposed in the gully. According to surface observation, borehole exposure and rock coal seam correlation results, the strata from old to new include Majiagou Formation of the Middle Ordovician system, Benxi Formation of the Middle Carboniferous system, Taiyuan Formation of the Upper Carboniferous system, Shanxi Formation of the Lower Permian system, the Lower Shihezi Formation, Upper Shihezi Formation of the Upper Permian system, Shiqianfeng Formation, Neogene, and Quaternary. The Midwest of the exploration area is covered by aeolian sand and alluvial proluvial deposits, with a thickness of 2 m–5 m in general. The Neogene red beds and bedrock are mainly exposed near Dagou, and the Neogene red beds are also directly exposed on the surface in the northern slope of the exploration area and the central and western regions of the exploration area. The thickness of the overburden in the exploration area varies greatly and violently, and the shallow seismic geological conditions are general. The noise of the seismic signal in this area is relatively developed at any time, which is not conducive to the fine interpretation of subsequent seismic data. Therefore, the seismic data denoising is one of the most important tasks. The actual seismic data of the stacked profile are shown in Fig. 22. After VMD and 2D-VMD denoising, the reconstructed seismic records of the actual study area can be obtained as shown in Figs. 23 and 24.
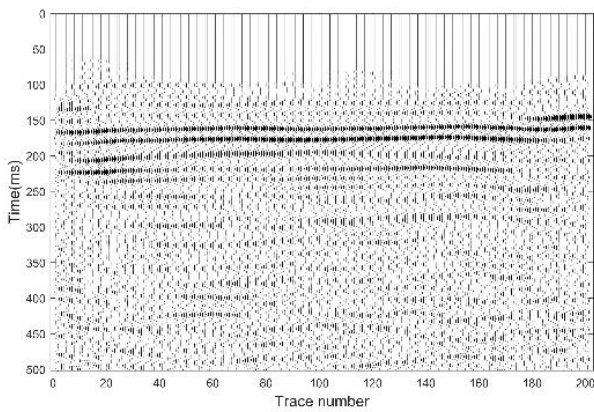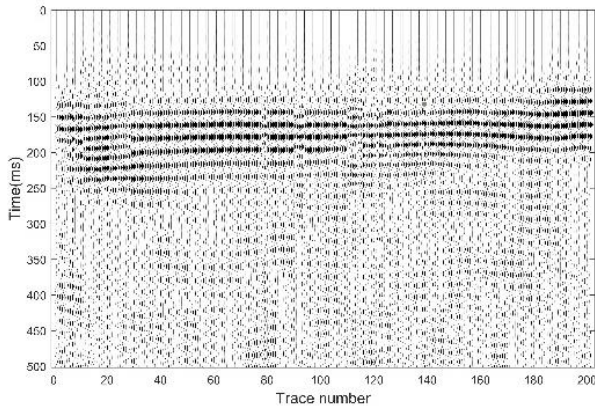
Fig. 22. Actual seismic data.

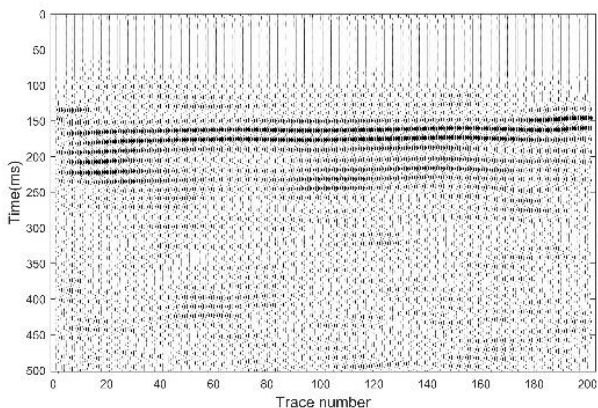Fig. 23. Reconstruction of actual seismic data by the VMD method.

Fig. 24. Reconstruction of actual seismic data by the 2D-VMD method.

From Figs. 23 and 24, we can see that although the VMD method has a certain denoising effect, there are obvious signal distortion and modal aliasing near the 20th and 120th signals. After 2D-VMD reconstruction, the same direction axis between 150 ms and 200 ms is continuous and clear, and the signal-to-noise ratio is improved so that each seismic record becomes smooth and can better reflect the basic form of the formation. Therefore, 2D-VMD also has a good denoising effect in actual seismic records.

V. DISCUSSION

There are a series of noise caused by natural environment, system noise, and secondary noise generated by data acquisition instruments in seismic signals, which seriously affect the quality of seismic signals and are not conducive to processing and interpretation of subsequent seismic data. In this paper, the 2D-VMD method is applied to seismic signal

denoising, and it can be seen that the signal-to-noise ratio is significantly improved compared with traditional VMD and EMD. However, there are still some shortcomings when 2D-VMD is applied to seismic signal denoising:

The denoising effect of 2D-VMD will obviously be affected by several input parameters, such as the number of reconstructed modes, error control parameters, etc. The modification of these parameters will make the denoising effect of 2D-VMD develop in a better or worse direction. Currently, the specific influence of these parameters on the denoising effect has not been clearly studied.

The efficiency of 2D-VMD in processing actual seismic data is significantly lower than that of VMD and EMD, indicating that there is still some room for optimisation in the algorithm of 2D-VMD.

In view of the above problems, future research work will focus on the research of the 2D-VMD algorithm, mainly including the following two points:

1. Control the influence of input parameters on denoising effect in a favourable direction;

2. The algorithm is further optimised to improve denoising efficiency in practical work.

VI. CONCLUSIONS

When processing a single signal, both EMD and VMD have a better denoising effect, but the effect of VMD is better than that of EMD.

By adding one-dimensional and two-dimensional Gaussian white noise to the simple synthetic fault seismic record model, the denoising results show that the VMD method has better noise resistance and obvious denoising effect than the EMD method, but the denoising effect of two-dimensional noise is significantly lower than that of the 2D-VMD method.

The VMD and 2D-VMD algorithms are applied to denoising of actual seismic signals. The application results show that the 2D-VMD method can effectively improve the signal-to-noise ratio of seismic signals and enhance the continuity of the seismic event, providing reliable basic data for subsequent seismic data interpretation.

CONFLICTS OF INTEREST

The authors declare that they have no conflicts of interest.

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