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Which Solow Model – Homogeneous Technology-, Heterogeneous Technology-, or Human Capital-Augmented – Best Explains OECD Growth? Fresh Evidence from Bayesian Monte Carlo Simulations

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ABSTRACT

The Solow growth model significantly impacts growth econometrics. However, its primary issue arises when applied to OECD samples. While incorporating human capital accumulation improves the model's goodness-of-fit, it remains low for this subset. Furthermore, augmenting the model with various technology variables leads to different results. In light of doubts regarding multicollinearity within the frequentist framework, this study aims to determine which modified model specification well explores the OECD growth pattern. By employing Monte Carlo simulation within the Bayesian panel non-linear framework, our findings suggest that the human capital-augmented Solow growth model with homogeneous technology best elucidates economic growth in OECD countries, aligning with the productivity convergence hypothesis.

INTRODUCTION

Understanding economic growth, one of the oldest and most significant research agendas, is crucial for the well-being of future generations. That is why growth economists endeavor to explain the process of economic growth and cross-country differences in average income. The most basic but popular growth model, Solow model (Solow, 1956), representative of the first period of modern growth theory, posits that economic growth through physical capital accumulation is constrained by diminishing returns over time, leading to a steady state. At the steady state, per capita income increases at the speed of exogenous technical progress. The main critique of the Solow model is its low explanatory power for the set of OECD countries, where the capital share in income is high (MRW, 1992). Many subsequent studies have attemptted to enhance the canonical Solow model by incorporating various explanatory variables (MRW, 1992; Knowles and Owen, 1995; Park and Prat, 1996; Felipe and McCombie, 2005; Abu-Qarn, 2019). MRW (1992) introduced human capital as an independent predictor in the model, while others augmented it with technology variables (Islam, 1995; Nonneman and Vanhoudt, 1996; Lee et al., 1997; Felipe and McCombie, 2005; Abu-Qarn, 2019).

Nevertheless, including new independent variables in the frequentist analyses poses challenges due to highly correlated predictors. The intricate relationship between fixed investment and technology is influenced by various factors such as positive correlation and reverse causality (Lucas, 1988; Romer, 1990; MRW, 1992; Islam, 1995; Jones, 1995; Acemoglu, 2008; Jing Liu et al., 2022). Physical investments enhance the adoption of new technologies and innovations, resulting in faster technology advancement. On the other hand, higher technology growth can prompt increased investment as firms seek to remain competitive by adopting the latest technologies. Similarly, the relationship between human capital, savings, and population is complex (MRW, 1992)—more savings or lower population growth causes higher human capital via more significant income. Hence, including all these independent variables in a single growth model can potentially cause multicollinearity, leading to severe statistical issues (Jaya et al., 2019; Pesaran and Smith, 2019). Multicollinearity makes it difficult or even impossible to isolate the independent impacts of the predictors on the dependent variable. Furthermore, the coefficient estimates of correlated variables become biased and difficult to interpret accurately, while the standard errors can be inflated, potentially leading to misleading statistical significance tests.

To mitigate multicollinearity, one approach is to use ordinary techniques such as variable selection or dimension reduction methods to eliminate one of the correlated variables. Alternatively, if the theoretical importance of all the independent variables is significant and one wishes to include them in the model, Bayesian methods that can handle multicollinearity effectively need to be explored. Bayesian methods offer some advantages in dealing with multicollinearity compared to traditional frequentist methods (Block et al., 2011; Jaya et al., 2019; Pesaran and Smith, 2019). Bayesian methods provide a more flexible and principled framework for modeling complicated relationships among variables. Here are some ways the Bayesian approach can help overcome multicollinearity issues:

(i) Prior specification: In Bayesian analysis, one can include informative priors that express our beliefs about the relationships between variables before observing the data. One can guide the model towards plausible parameter estimates by incorporating specific prior knowledge, even when multicollinearity is present. Informative priors can help stabilize the estimation process and provide more realistic parameter estimates.

(ii) Sampling-based estimation: Bayesian inferences rely on Markov Chain Monte Carlo (MCMC) techniques to obtain the posterior distribution of parameters. These techniques allow for efficient sampling from the joint posterior distribution, which can be helpful when dealing with multicollinearity, especially in high-dimensional settings.

(iii) Prioritizing parameter uncertainty: Bayesian inference naturally provides a posterior distribution for each parameter, including variance and covariance estimates. This means that instead of point estimates, one gets a range of likely values, which helps understand the uncertainty around the parameter estimates, mainly when multicollinearity is present.

(iv) Non-linear models: Bayesian methods are more amenable to handling non-linear relationships between variables. When multicollinearity arises due to interactions between variables, Bayesian modeling can easily incorporate these non-linear effects, allowing for a more flexible representation of the underlying relationships.

With all the above arguments in mind, the research focuses on determining which of the following models – the canonical or human capital-augmented Solow models augmented by homogeneous and heterogeneous technology – best accounts for income disparity across OECD countries. A Bayesian non-linear framework through MCMC simulations is applied to an OECD panel to achieve this goal. The assumption of fixed technological progress, depreciation rates, and country-specific rates for these variables will be incorporated into the canonical and augmented Solow models. Our research contributes to the growth area in the following ways: first, the adoption of thoughtful Bayesian estimation allows for coding the interactions among variables, disentangling individual effects of the predictor variables on the response; the research concludes that the Solow model augmented with country-specific human capital accumulation and identical technology performs best in depicting the economic growth of OECD countries; second, the study demonstrates the advantages of the Bayesian approach over frequentist inference in handling complex growth models.

1. CANONICAL SOLOW GROWTH MODEL, MRW SPECIFICATION, AND RELATED EMPIRICS

1.1 MRW specification of canonical Solow model

The canonic Solow growth model includes labor-augmenting technical progress as an exogenous variable, known as Harrod-neutral technical progress. The Cobb-Douglas production function is utilized:

$$Q_{it} = AK_{it}^{\alpha}(LE)_{it}{}^{1-\alpha}. \qquad (1)$$

where $Q, K,$ and L are income, physical capital, and raw labor, respectively. E is labor-augmenting technical progress; LE is effective labor; α is income elasticity concerning physical capital ($0 < \alpha < 1$); and i, t are country and time, respectively.

Suppose that labor-augmenting technical progress has a rate of g. In this scenario, the effect of technical progress on production is similar to that of raw labor. For instance, if the growth rate of labor productivity (*E*) is 0.02, it means that in period t, 100 workers would produce the same output as 102 workers did in the previous period (t-1).

Additionally, if the population (*L*) and labor productivity (*E*) grow at a rate of n and g, respectively, effective labor (*LE*) would increase at a higher rate, specifically $(n + g)$, which exceeds the growth rate of raw labor (*L*). This indicates that the combined effect of population growth and labor productivity growth results in a higher growth rate of effective labor, leading to a more significant impact on production.

In the Solow model, augmented with Harrod-neutral technical progress, k^E there is a ratio between physical capital and effective labor ($k^E = \frac{K}{LE}$), and q^E is the income per unit of effective labor ($q^E = \frac{Q}{LE}$). Applying $LE_{it} = L_{it}E_{it}$ leads to:

$$q_{it} = q_{it}^E E_{it} = q_{it}^E e^{gt}. \qquad (2)$$

$$k_{it} = k_{it}^E E_{it} = k_{it}^E e^{gt}. \qquad (3)$$

Dividing (1) by LE, we obtain:

$$q_{it}^E = A(k_{it}^E)^{\alpha}. \qquad (4)$$

Hence, the fundamental growth equation is rewritten:

$$\Delta k_{it}^E = sq_{it}^E - (n + g + \delta)k_{it}^E. \qquad (5)$$

Or

$$\Delta k_{it}^E = sA(k_{it}^E)^{\alpha} - (n + g + \delta)k_{it}^E. \qquad (6)$$

where Δk_{it}^E represents the increment of physical capital, and the variables $s, \delta, n,$ and g represent the fraction of savings (physical investment) in income, depreciation rate, growth rate of population, and technology growth rate, respectively.

The right-hand side of (6) consists of two terms: the first term represents investment per unit of effective labor, and the second term represents "break-end investment" – an amount of investment that compensates for the depreciated part of the capital stock – along with the growth rate of effective labor $(n + d)$. As a result, when investment per unit of effective labor matches the "break-end investment," k^E

increases. Conversely, if they do not match, k^E decreases. This interplay between investment and effective labor drives changes in k^E over time.

The model reaches the steady state when $\Delta k_{it}^E = 0$. Transforming (2) and (3), we obtain:

$$k^* = \left(\frac{sA}{n+g+\delta)}\right)^{\frac{1}{1-\alpha}} e^{gt}. \quad (7)$$

$$q^* = A^{\frac{1}{1-\alpha}} \left(\frac{s}{n+g+\delta)}\right)^{\frac{\alpha}{1-\alpha}} e^{gt}. \quad (8)$$

where k^* and q^* are per capita capital and income at the steady state, respectively. We take logarithms to obtain:

$$lnq = a + \frac{\alpha}{1-\alpha}\ln(s) - \frac{\alpha}{1-\alpha}(n+g+\delta) \quad (9)$$

At the steady state, (8) indicates that per capita income gains continuous growth at a speed of g. This sustained growth is primarily attributed to the impact of labor-augmenting technical progress, which counteracts the law of diminishing returns to capital, a factor that could otherwise hinder per capita income growth. The gradual accumulation of raw labor, coupled with technological advancements, allows effective labor to expand in tandem with physical capital, leading to an increase in the ratio between capital and labor at a rate of g. As a result, both capital and income experience growth at a combined rate of $(n+g)$, while the ratio between capital and labor and per capita income rise at a rate of g. The steady-state reflects a dynamic equilibrium where various factors work harmoniously to sustain economic growth.

When MRW (1992) introduced human capital stock (H) into the production function, (1) becomes:

$$Q_{it} = K_{it}^{\alpha} H_{it}^{\beta} (A_{it} L_{it})^{1-\alpha-\beta} \quad (10)$$

where β is income elasticity concerning human capital $(0 < \beta < 1)$, MRW hold the assumption of decreasing returns to all capital: $\alpha + \beta < 1$.

Transforming (10) and taking logarithms, we obtain:

$$lnq = a + \frac{\alpha}{1-\alpha-\beta}\ln(s_k) + \frac{\beta}{1-\alpha-\beta}\ln(s_h) - \frac{\alpha+\beta}{1-\alpha-\beta}(n+g+\delta) \quad (11)$$

where s_k and s_h are the share of physical and human capital investment in income, respectively.

1.2 Related empirics

MRW (1992) conducted one of the most influential works revitalizing the canonical Solow growth model. Their study focused on specifying the Solow model and testing its predictions. To estimate their regression, making an identifying restricted assumption of homogeneous technology across countries (g+δ=0.05), the authors employed a simple frequentist technique (Ordinary Least Squares) with data spanning from 1960 to 1985 for three distinct subsets of countries: the first dataset comprised 98 countries, the second included 75 countries, and the third was limited to 22 OECD countries. The outcomes of their analysis were mixed. The Solow model elucidated more than half of the income per capita variation among diverse countries, except for a specific subset – the OECD economies. The outcomes were satisfactory in the initial two subsamples, showing an R-squared value of 0.59 and implying an elasticity of physical capital (α) of 0.6. Nevertheless, the results for the OECD subsample were considerably less fulfilling. The estimated coefficient of $\ln(\mathrm{n} + 0.05)$ was found to be statistically insignificant despite having the correct negative sign. Furthermore, the R-squared value for the OECD countries was extremely low, amounting to only 0.06. These findings highlighted potential limitations of the standard Solow model when applied to the OECD economies. To improve the Solow model's explanatory power, MRW (1992) chose to include human capital in the analysis, due to which, for the OECD subsample, R-squared increases from 0.06 to 0.24, a low for a well-specified model level, while implied α decreases from 0.36 to 0.14 and implied β obtains a value of 0.37. Suggesting that multicollinearity is implicit in their augmented model, MRW (1992) stated:

"Human-capital accumulation may be correlated with saving rates and population growth rates; this would imply that omitting human-capital accumulation biases the estimated coefficients on saving and population growth."

Furthermore, the debate surrounding cross-country income disparities among OECD countries has prompted growth economists to reconsider the old approach, assuming that the term "A" (representing total factor productivity, TFP) is the same across all countries. Jorgenson (1995) recommended not specifying homogeneous technology variables in growth models, suggesting that this assumption may need to capture the reality of technological differences between nations accurately. Islam (1995) further emphasizes that assuming homogeneous technology can lead to an omitted variable bias. The growth model may fail to capture crucial factors influencing income differences by overlooking the variations in technology levels across countries. Prescott (1998) offers a different perspective, arguing that differences in savings rates might not be as significant as TFP when explaining income disparities. He proposes that the focus should shift towards developing a TFP theory to understand better the sources of economic growth and variations in income levels. Numerous subsequent studies (Islam, 1995; Lee et al., 1997; Felipe and McCombie, 2005; Abu-Qarn, 2019) have explored variations in technology levels and rates across countries and replicated the Solow model employing various frequentist methods. However, the empirical results obtained from these studies have been mixed, leading to divergent conclusions. Notably, adopting a panel approach, Islam (1995) found that the fit of the Solow model considerably improved for the OECD subset and concluded: "The present paper advocates and implements a panel data approach to deal with this issue. The panel data framework makes it possible to allow for differences of the above-mentioned type in the form of unobservable individual "country effects"." Similarly, Felipe and McCombie (2005) found a significant improvement in the model's explanatory power when taking into account variations in technology across OECD countries. By contrast, Abu-Qarn (2019) revealed that incorporating heterogeneous technology did not produce a better model fit for three data samples, including OECD countries. However, similar to MRW (1992), adding the human capital variable notably improved the goodness-of-fit. This discrepancy in the mentioned findings highlights the complicated connections between population, savings, human capital, technology, and income. By our suggestion, the primary reason for the contradictory findings is that incorporating heterogeneous technology or human capital into the Solow model creates a close association between physical investment and productivity variables, as well as between human capital investment and savings and population. These dimensions are strongly correlated in general. The high correlation between these variables can create difficulties for frequentist methods in the mentioned studies. This correlation may lead to issues such as multicollinearity, making it challenging to disentangle the independent effects of the predictors on income.

In summary, empirical investigations incorporating human capital and technology variations across OECD countries in the Solow growth model have yielded conflicting outcomes. Some studies show improved explanatory power with the inclusion of differences in technology, while others indicate limited improvement. The high correlation between population growth, physical and human capital investment, and technology poses challenges for frequentist methods. The current study implements Bayesian non-linear estimation with specific priors on a panel of OECD countries to address this significant challenge when incorporating country-specific human capital, technology, and depreciation rates. By selecting specific priors, Bayesian analysis offers a more flexible approach to handle multicollinearity and allows for a better understanding of parameter uncertainties and non-linear relationships in a growth model. As a result, the study will provide more reliable and robust evidence on the best version of the Solow growth model.

2. BAYESIAN MCMC SIMULATIONS AND DATA

2.1 Bayesian MCMC simulations

Since the 1990s, the application of Bayesian approaches in various fields, from genetics to macroeconomics, has sparked a revolution in data analysis. However, growth econometrics has been relatively absent from using Bayesian methods. In this article, we aim to introduce growth researchers to the thoughtful (with specific informative priors) Bayesian methodology as an effective tool to tackle model uncertainty arising from statistical issues. By doing so, we can address some of the limitations of the frequentist approach and benefit from a more flexible and intuitive framework for handling uncertainty. The most significant advantage of Bayesian analysis is its ability to incorporate specific prior information or beliefs about the parameters of interest. Unlike the frequentist approach, which does not formally include prior information, Bayesian analysis can utilize this additional knowledge, resulting in more efficient estimation from the available data. By producing a posterior distribution of the parameters, Bayesian analysis enables researchers to express uncertainty in parameter estimates and make probabilistic statements about the parameter values. On the contrary, frequentist methods often rely on point estimates and confidence intervals, which may not fully capture the extent of uncertainty. So, Bayesian analysis is well-suited for handling model complexity and hierarchies, making it a valuable tool for dealing with high-dimensional data and intricate relationships between variables. Bayesian analysis proves to be effective in dealing with multicollinearity, a common issue in regression analysis, by incorporating specific prior information and providing a full posterior distribution of the parameters. In contrast, frequentist methods may struggle with such complexity and can lead to issues like overfitting or underfitting. In frequentist regression, multicollinearity can result in unstable coefficient estimates and inflated standard errors (Block et al., 2011; Jaya et al., 2019; Pesaran and Smith, 2019; Thach et al., 2019).

The study employs MCMC simulations within a Bayesian non-linear regression model with specific priors on the elasticity parameters (α, β) to assess the canonical Solow model and its augmented versions. In the canonical and human capital-augmented Solow models, we incorporate constant exogenous technology variables along with heterogeneous technology and depreciation rates. Thus, we need to evaluate four Solow models: two canonical Solow models and two human capital-augmented Solow models with homogeneous and heterogeneous technology variables. To estimate the Bayesian growth models based on equation (11), we suppose that all the OECD countries in the sample have reached their steady state.

To assess the canonical Solow model, the study adopts the assumptions of MRW (1992), with a technology growth rate of 0.02 and a depreciation rate of 0.03. Various assumptions regarding technology and depreciation rates are applied to both the canonical and augmented versions of the Solow model. For comparison and selection among the four Bayesian models, Bayesian information criteria such as Deviance Information Criterion (DIC), log of marginal likelihood (log(ML)), Bayes factor measured in log metric (log(BF)), and posterior probabilities of models (P(M/y)) are analyzed. A Bayesian model performs better if the DIC is smaller while the remaining statistics are larger. Furthermore, visual tools such as observed vs. fitted plots, residual vs. fitted plots, and predictive interval plots are employed to compare the goodness-of-fit between the best Bayesian and frequentist growth models. In conjunction with the graphical tools, we utilize standard metrics such as Mean Squared Error (MSE), (Root Mean Squared Error) (RMSE), and Mean Absolute Error (MAE). The smaller the metric value is, the more precise the model predictions become.

DIC is calculated as:

$$DIC = D(\bar{\theta}) + 2p_D \quad (12)$$

where $D(\bar{\theta})$ is the posterior mean deviance, and p_D is the effective number of parameters in a model.

Log(ML) is measured by:

$$\log P(y) = log \int p(y|\theta)p(\theta)d\theta \quad (13)$$

where $P(y)$ is the marginal likelihood of the data y, $p(y|\theta)$ is the likelihood function, and $p(\theta)$ is the prior distribution of the parameters θ. Note that the integral is taken over the parameter space θ.

As the ratio of the marginal likelihoods under two competing models, M_1 and M_2, BF is calculated as:

$$BF = \frac{p(y|M_1)}{p(y|M_2)} \quad (14)$$

where $p(y|M_1)$ and $p(y|M_2)$ are the marginal likelihoods of the data under models M_1 and M_2, respectively.

As the average of the absolute differences between the observed values and the predicted values, MAE is calculated as:

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i| \quad (15)$$

where n is the number of observations (data points), y_i is the actual (observed) value for observation i, and $\hat{y}_i$ is the predicted value for observation i.

MSE and RMSE measure the average squared difference between the predicted and the observed values. MSE is measured by:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2 \quad (16)$$

RMSE is the square root of the MSE:

$$RMSE = \sqrt{MSE} \quad (17)$$

We set a default configuration for the MCMC sample during our simulation process. We discard the initial 2,500 burn-in iterations to ensure the stability and convergence of the algorithm. Additionally, to effectively assess MCMC convergence in a high-dimensional regression setting, we use a thinning rate of 50. This allows us to reduce the number of iterations while retaining the sample's representativeness. Consequently, 502,451 iterations are executed within the MCMC algorithm, providing robust and reliable analysis.

2.2 Data

Our investigation utilizes a panel dataset encompassing 28 OECD countries from 1970 to 2019. The dataset is designed to explore the connections between various fundamental inputs and their impact on aggregate income. The dataset comprises essential variables: physical investment share in GDP (s_k), human capital investment share in GDP (s_h), population growth rate (n), productivity (TFP) growth rate (g), depreciation rate (δ), and per capita income (q). To set a specific informative prior for the elasticity parameters (α, β), we restrict its possible values between 0 and 0.5, adopting a uniform(0,0.5) prior distribution. This mildly informative prior choice aims to introduce a regularization effect on the posterior distribution, enhancing the stability and reliability of the parameter estimates. To check estimation robustness, we run one more simulation with a uniform(0,1) prior for parameters α, β.

Data on per capita income, TFP, depreciation rate, and population are sourced from version 10.01 of the Penn World Tables. At the same time, information on the physical and human capital investment share in GDP is gathered from the World Bank's World Development Indicators. By employing a thoughtful Bayesian estimation with informative prior settings relying on these comprehensive datasets, the study gains more profound insights into the linkages between the crucial variables and their implications for economic growth in the OECD economies.

3. BAYESIAN SIMULATION OUTCOMES AND INTERPRETATION

3.1 Convergence diagnosis

Before performing Bayesian inferences, it is necessary to inspect the convergence of MCMC chains to ensure the robustness of the Bayesian analysis. Our investigation has carried out thorough diagnostic tests to assess the convergence of the MCMC chains in relation to our models. The results for the best-fitting Bayesian model, recorded in Appendix A, indicate reasonable diagnostic graphs. Specifically, the trace plots show no discernible trends and rapidly approach constant mean and variance values, indicating favorable convergence. Additionally, the autocorrelation plots exhibit acceptable patterns, and the histograms resemble the shape of probability distributions (Appendices B and C). Overall, the MCMC chains of our model demonstrate good mixing, suggesting no serious convergence issues. It can be confidently concluded that the MCMC chains have effectively converged to the target distribution. This provides a solid foundation for performing Bayesian inferences and drawing reliable conclusions about the parameters and relationships in the model.

3.2 Goodness-of-fit comparison between homogeneous technology-, heterogeneous technology-, human capital-augmented Solow growth models

Table 1 demonstrates the performance of the four Solow growth models estimated performing MCMC simulations. According to the Bayesian information criteria estimates, the Solow model augmented with human capital and homogeneous technology is most preferable for the OECD sample. The finding is consistent mainly with MRW (1992). The main reason for this finding is productivity convergence arising in the advanced world (Bernard and Jones, 1996; Sadik, 2008; Mendez, 2020). The productivity convergence hypothesis posits that adopting technologies within groups of industrialized countries is more likely and faster due to the low costs of importing them from a few centers. The estimated values of constant, α, and β for this model do not differ considerably from those of the canonical and human capital-augmented Solow models with heterogeneous technology. The estimated value of the constant is around 10, while those of α and β are approximately 0.01 and 0.02, respectively. However, the canonical Solow model stands apart, with estimated values of the constant and α equal to 8.06 and 0.26, respectively. Notably, augmentation with human capital improves the Solow model's goodness-of-fit, aligning with previous studies (MRW, 1992; Abu-Qarn, 2019). Conversely, by adding country-specific technology variables, the Solow model produces conflicting results: the model fit increases in Islam (1995) and Felipe and McCombie (2005) but decreases in Abu-Qarn (2019) and our study. Regarding the estimated values for α, the results from our human capital-augmented Solow models are similar to those by Abu-Qarn (2019) but much lower compared to MRW (1992) and Islam (1995). It is noteworthy that our estimated values of β are much lower compared to MRW (1992), Islam (1995), and Abu-Qarn (2019).

Table 1. Posterior summary of the canonical and human capital-augmented Solow models

Bayesian models	*Canonical Solow growth models*		*Human capital-augmented Solow growth models*	
Technology variables	*homogeneous technology*	*heterogeneous technology*	*homogeneous technology*	*heterogeneous technology*
Specification of technology and depreciation rates	$g = 0.02$, $\delta = 0.03$	$g = tfp$, $\delta = delta$	$g = 0.02$, $\delta = 0.03$	$g = tfp$, $\delta = delta$
Number of OECD countries	28	28	28	28
MCMC sample size	10000	10000	10000	10000
Constant	8.060	10.107	10.229	10.307
Implied α	0.263 (0.006)* [0.201,0.292]**	0.033 (0.004) [0.002,0.091]	0.013 (0.002) [0.000,0.045]	0.013 (0.001) [0.000,0.044]
Implied β			0.017 (0.005) [0.001,0.066]	0.021 (0.003) [0.001,0.068]
DIC	2703.028	2752.757	1123.334	1124.512
Log(ML)	-1364.732	-1390.287	-576.6644	-576.7437
Log(BF)	.	-25.55514	788.0674	787.9881
P(M/y)	0.0000	0.0000	0.5198	0.4802
Error variance	0.303	0.313	0.215	0.215

Note: *denotes Markov chain standard error (MCSE), ** denotes PPI (Posterior Probability Interval) representing a 95% probability that a mean coefficient lies between two values in the population.

Source: Calculations by the author

3.3 Goodness-of-fit comparison between frequentist and Bayesian growth models

This subsection compares the performance of frequentist and Bayesian inferences in estimating the human capital-augmented Solow growth model with homogeneous technology. This task is carried out via observed vs. fitted plots, residual plots, and predictive interval plots. As evident from the first type of diagnostic plots (Figure 1a), notable differences emerge between the predictive performance of the Bayesian and frequentist models. The yellow (Bayesian) points align more closely with the diagonal line (y = x) compared to the blue (frequentist) ones. This indicates that the Bayesian model's predicted values match the observed values better than the frequentist model.

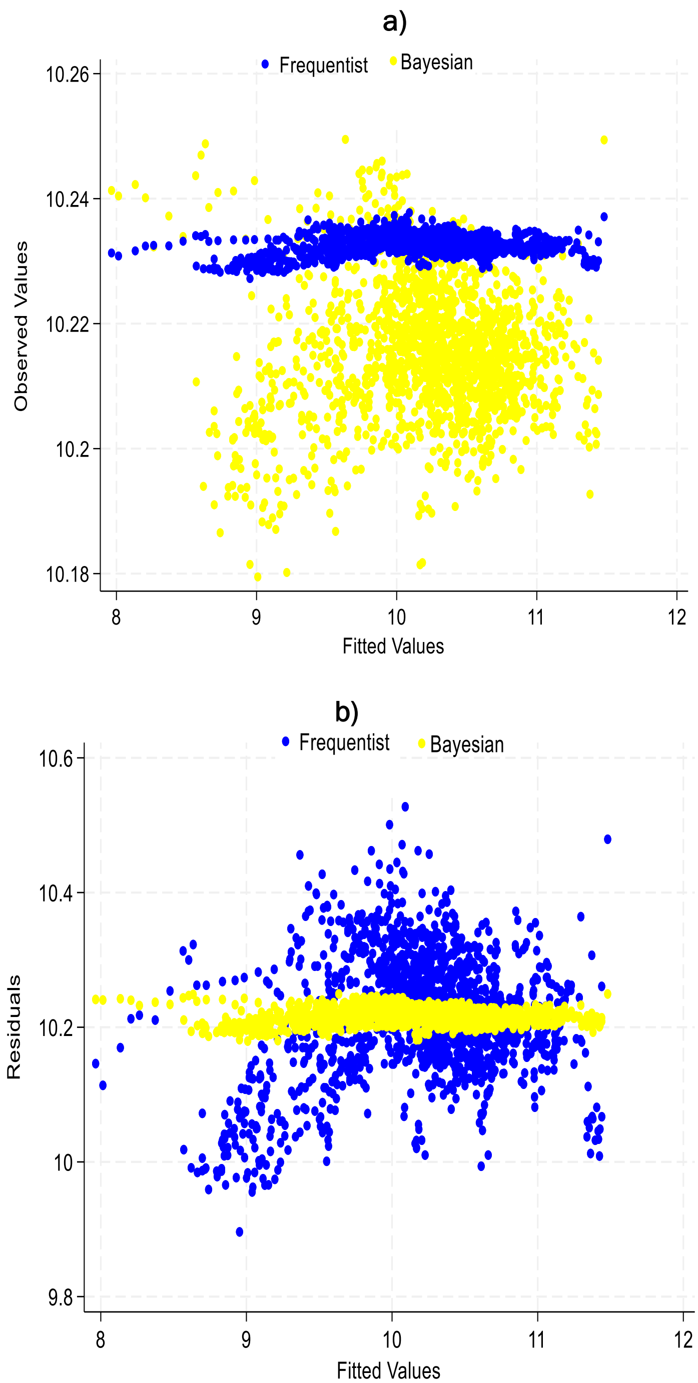

Figure 1. Bayesian vs. frequentist: a) Observed vs. fitted plot, b) Residual plot

Source: own

Furthermore, the second type of plot (Figure 1b) exhibits that, compared to the frequentist residual (blue) plot, the residuals of the Bayesian (yellow) plot are more evenly spread across the range of predicted values, which indicates that the Bayesian model's predictions are unbiased and have no systematic errors.

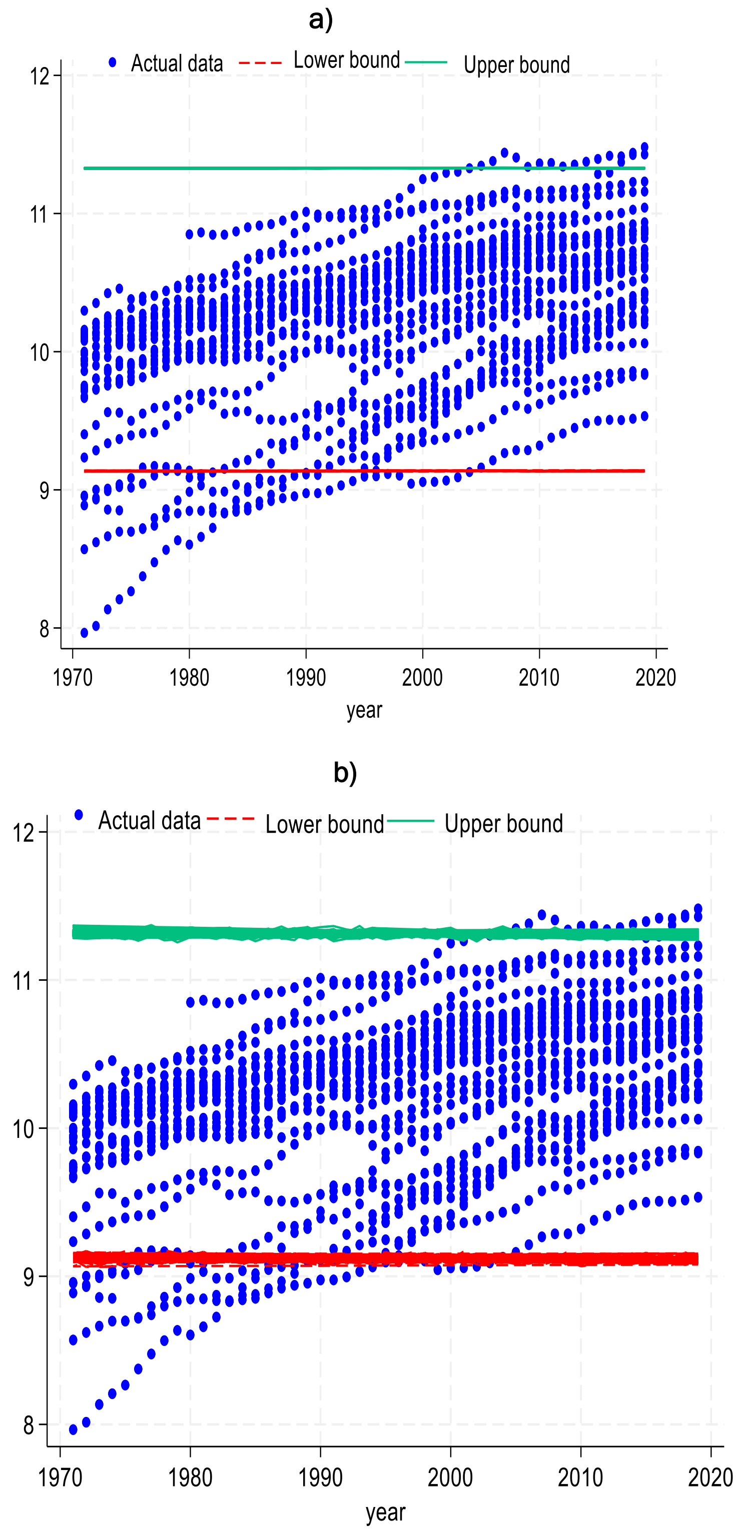

Figure 2. Actual data (GDP per capita) within predictive intervals: a) Frequentist model, b) Bayesian model

Source: own

Figure 2 compares the number of observations (actual data) of the variable GDP per capita within the predictive intervals between the Bayesian and frequentist human capital-augmented Solow models with homogeneous technology. A closer examination of the diagrams reveals only minimal disparity. However, the numeric results presented in Table 2, with values of 0.948 and 0.951 against 0.935, clearly indicate an advantage of the Bayesian models over the frequentist one. Moreover, other metrics such as MSE, RMSE, and MAE also favor the Bayesian models over the frequentist one. Based on these findings, we can conclude that the Bayesian approach outperforms the frequentist approach, particularly in addressing multicollinearity inherent in complex growth models.

Table 2. Comparing the frequentist and Bayesian augmented Solow growth models

Human capital-augmented Solow growth models	*Bayesian approach*		*Frequentist approach*
Prior for α, β	Uniform(0,0.5)	Uniform(0,1)	
Specification of technology and depreciation rates	$g = 0.02$, $\delta = 0.03$	$g = 0.02$, $\delta = 0.03$	$g = 0.02$, $\delta = 0.03$
Number of OECD countries	28	28	28
Constant	10.229	10.287	10.472
Implied α	0.013	0.010	0.005
Implied β	0.017	0.012	0.006
MSE	0.303	0.303	0.312
RMSE	0.550	0.550	0.559
MAE	0.436	0.431	0.437
Percentage of observations within predictive intervals	0.948	0.951	0.935

Source: Calculations by the author

Furthermore, Table 2 shows that varying hyperparameters of the uniform prior do not alter the estimates of the elasticities α and β. Additionally, we observe no significant distinction in the estimated values of the constant, α, and β between the Bayesian and frequentist models.

CONCLUSION

Departing from the canonical Solow growth model, which poorly explains economic growth in the OECD countries, numerous studies have attempted to incorporate human capital and technology variables into the model. Nonetheless, the findings have been inconclusive and sometimes contradictory. The primary reason for the mixed results may be the issue of multicollinearity arising from the high correlation between population growth, human and physical capital investment, and technological progress. This statistical challenge is one that traditional frequentist methods struggle to handle effectively. In contrast, the Bayesian approach offers a more flexible and robust solution to address this correlation problem. Our study involved a sequence of MCMC simulations within a Bayesian non-linear framework, analyzing a panel dataset encompassing 28 OECD economies. The results indicate that the Solow growth model, when supplemented with heterogeneous human capital, homogeneous technology, and homogeneous depreciation rates, best explores the OECD growth pattern, which aligns with the productivity convergence hypothesis.

Based on the research findings, the study proposes the adoption of the augmented Solow model estimation results using the thoughtful Bayesian approach as a reliable empirical foundation for informing growth policies. Policymakers can gain a more accurate understanding of the dynamics of economic growth and make informed decisions to promote sustainable development and prosperity in the OECD context.

As with any empirical research, our study has limitations that should be acknowledged. One limitation is the potential for omitted variable bias, as there might be other unobserved factors influencing economic growth that should have been accounted for in our model. Additionally, the OECD sample may not fully represent the diverse global economic landscape, which could affect the generalizability of our findings. Despite these limitations, our study contributes to understanding economic growth dynamics by showcasing the benefits of adopting a thoughtful Bayesian approach and considering the effects of heterogeneous factors in the Solow growth model. Further research and exploration of more comprehensive datasets could enhance the accuracy and scope of our findings.

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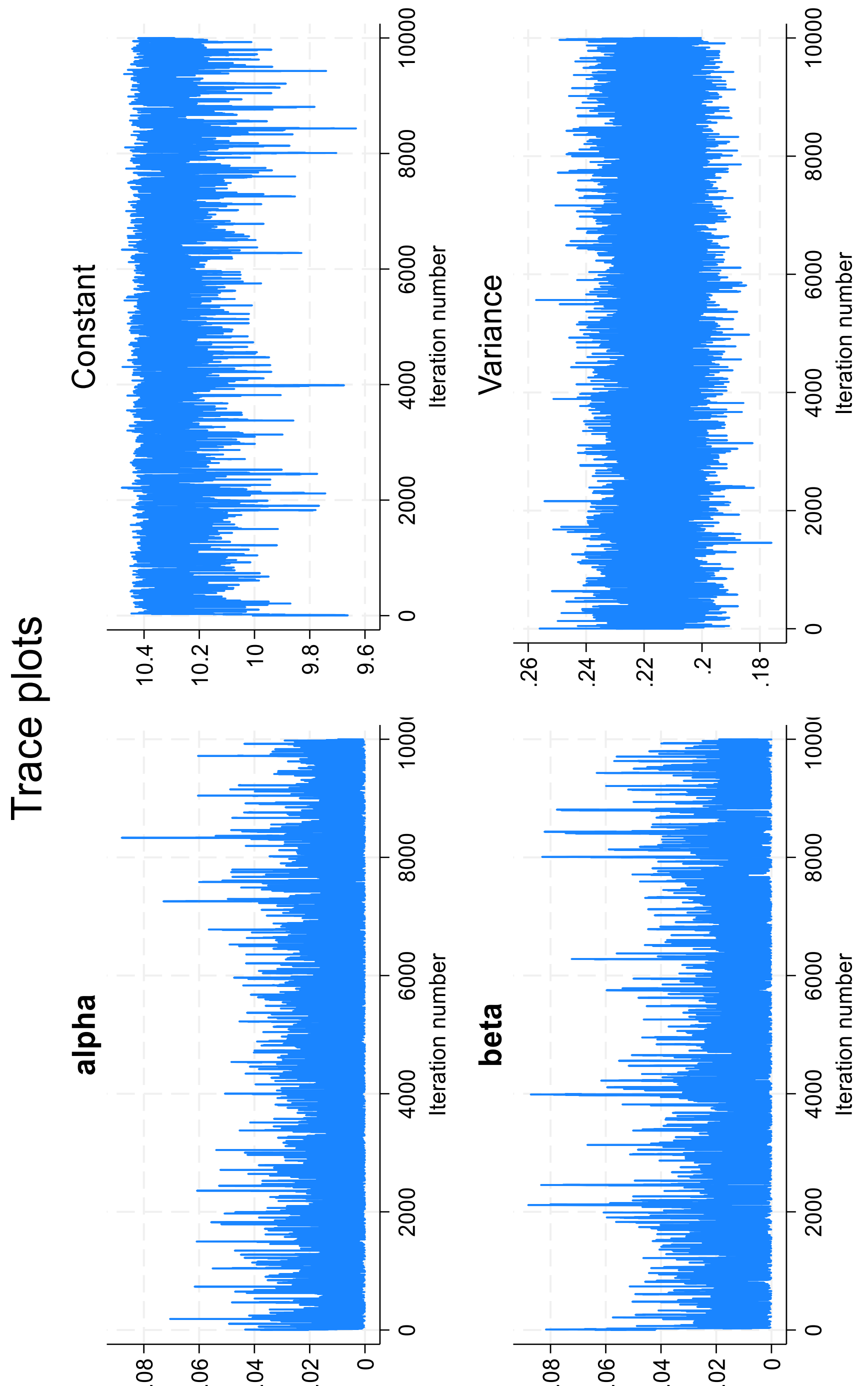
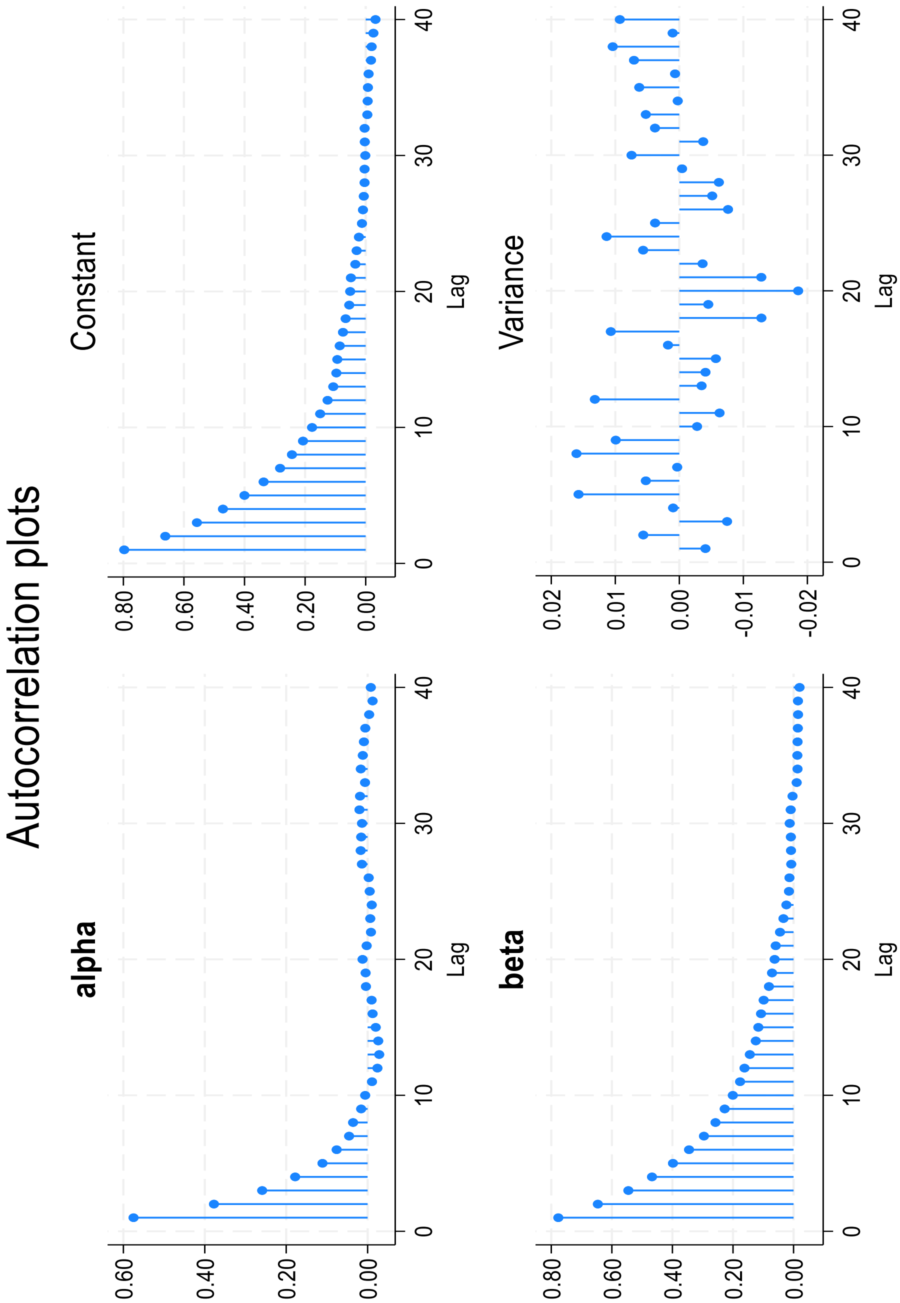
APPENDICES

A: Human capital-augmented Solow model



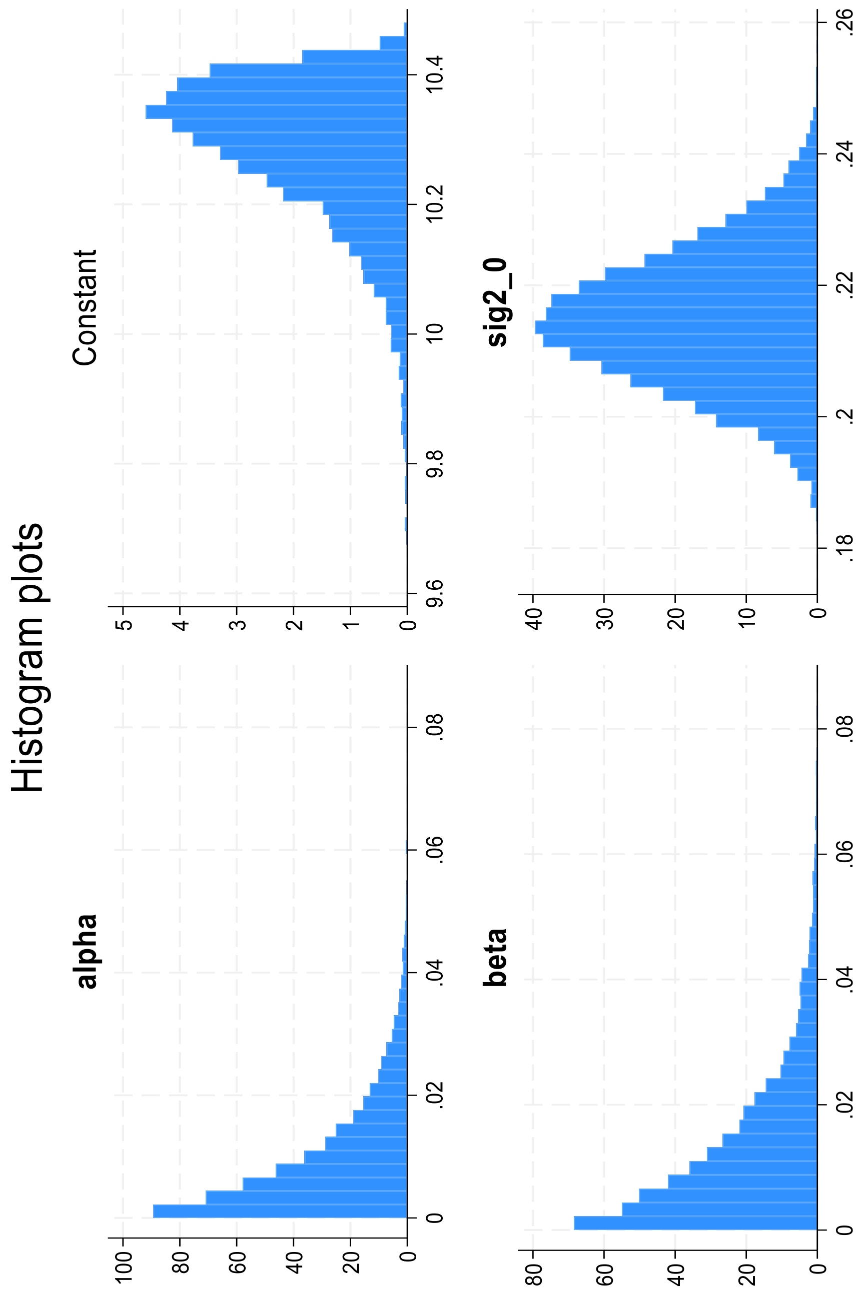
Source: Calculations by the author

B: Human capital-augmented Solow model



Source: Calculations by the author

C: Human capital-augmented Solow model



Source: Calculations by the author