# An Iterative Frequency-Domain Decision Feedback Receiver for CDMA Systems

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Abstract - Unless high-complexity receiver structures are employed, conventional DS-CDMA (Direct Sequence Code Division Multiple Access) schemes can have a poor performance in severe time-dispersive channels, since the orthogonality between users is lost.

In this paper we propose an iterative frequency-domain decision feedback equalizer for the downlink transmission within DS-CDMA systems employing block-transmission techniques, with an appropriate cyclic extension appended to each block.

Our performance results show that the proposed receiver structure has excellent performance, close to the single-user MFB (Matched Filter Bound), even for severe time-dispersive scenarios and/or in the presence of strong interfering channels.

# I. Introduction

The DS-CDMA wireless systems (Direct Sequence Code Division Multiple Access) [1] are known to show high capacities and good performances in moderately time-dispersive channels. For these reasons, several DS-CDMA systems have been adopted in recent years, namely the IS-95 standard and, more recently, the UMTS (Universal Mobile Telecommunications System) system [2].

For severe time-dispersive channels the loss of orthogonality between users can lead to a significant performance degradation, unless high-complexity receiver structures are employed (e.g., a RAKE receiver with a large number of fingers [3] or a multipath interference cancelation receiver [4]).

Block transmission techniques, with appropriate cyclic extensions and employing FDE techniques (Frequency-Domain Equalization), have been shown to be suitable for high data rate transmission over severe time-dispersive channels. In fact, due to the Fast Fourier Transform (FFT) implementation, the number of operations per symbol grows logarithmically with the block duration (and, therefore, the ISI (Inter-Symbol Interference) span), instead of linearly (at least) with the ISI span, as with conventional time-domain equalizers.

OFDM (Orthogonal Frequency Division Multiplexing) is the most popular modulation based on this technique [5]. Single Carrier (SC) modulation using FDE is an alternative approach based on this principle [6]. Like with OFDM modulations, the data blocks are preceded by a cyclic prefix, long enough to cope with the channel length. The received signal is transformed to frequency domain, equalized in frequency domain and then transformed back to time domain. The overall implementation complexities for SC schemes with FDE and OFDM schemes are similar. However, the SC schemes have lower envelope fluctuations than the corresponding OFDM schemes, while offering similar, or even better, performances [7]-[9].

A promising receiver structure for DS-CDMA systems was recently proposed [10], [11]. As with OFDM modulations and SC modulations with FDE, an appropriate cyclic extension is append to each block and an FDE is employed. The FDE techniques are especially interesting for the downlink transmission (i.e., from the base station to the mobile terminal) in DS-CDMA wireless systems employing orthogonal spreading, since all codes are affected by the same multipath channel, which simplifies the receiver implementation.

Since conventional FDE schemes can be regarded as infinite-length linear equalizers, we can have significant noise enhancement, especially when a ZF (Zero Forcing) equalizer is employed in channels with deep in-band notches, which can lead to significant performance degradation. For this reason, an MMSE (Minimum Mean-Squared Error) FDE equalizer is usually preferable [12].

It is well-known that an MMSE FDE does not perform an ideal channel inversion. Therefore, when this type of equalizer is employed in DC-CDMA systems we are not able to fully orthogonalize the different spreading codes. This means that we can have severe interference when different powers are attributed to the different users.

It is widely accepted that DFEs (Decision Feedback Equalizers) can significantly outperform linear equalizers, provided that the error propagation problem is limited. For this reason, a hybrid time-frequency SC-DFE was proposed in [9], employing a frequency-domain feedforward filter and a timedomain feedback filter. This hybrid time-frequency-domain DFE has a better performance than a linear FDE. However, as with conventional, time-domain DFEs, it can suffer from error propagation, especially when the feedback filters have a large number of taps. A promising IB-DFE (Iterative Block DFE) approach for SC transmission was proposed in [13] and extended to transmit/diversity scenarios in [14]. Within these IB-DFE schemes, both the feedforward and the feedback parts are implemented in the frequency domain. Since the feedback loop takes into account not just the hard-decisions for each block, but also the overall block reliability, the error propagation problem is significantly reduced. Consequently, the IB-DFE techniques offer much better performances than the non-iterative methods [13], [14]. In fact, the IB-DFE schemes can be regarded as low complexity turbo equalizers [15] since the feedback loop uses the equalizer outputs instead of the channel decoder outputs.

In this paper we consider the downlink transmission (i.e., the transmission from the BS (Base Station) to the MT (Mobile Terminal)) within a DS-CDMA system employing block transmission techniques, with an appropriate cyclic extension appended to each block. An iterative receiver structure is proposed which can be regarded as an IB-DFE receiver especially designed for DS-CDMA signals. The proposed receiver combines a feedforward part and a feedback part, both implemented in the frequency domain. The feedback loop uses the block estimates from the previous iteration and, as with IB-DFE receivers, we take into account the overall block reliability so as to avoid error propagation. To improve the performance, we estimate not just the data symbols associated to the desired user but also the data symbols from the other users. Therefore, our receiver can be regarded as a frequencydomain multiuser detection technique with parallel interference cancelation.

This paper is organized as follow: the block transmission CDMA schemes considered here are described in sec. II. In sec. III we describe the IB-DFE receiver structure proposed in this paper. Sec. IV presents a set of performance results and sec. V is concerned with the conclusions of the paper.

#### II. Block Transmission DS-CDMA

In this paper we consider the downlink transmission of DS-CDMA systems employing block transmission techniques with frequency-domain equalization. The BS transmits simultaneously the data blocks for P users. For the sake of simplicity, it is assumed that all users have the same spreading factor and the same data rate; however, these techniques can be easily extended to both VSF schemes (Variable Spreading Factor) and multicode schemes [2]. The block of chips transmitted by the BS block is  $\{s_n; n = 0, 1, ..., N-1\}$ , where N = KM, with K denoting the spreading factor and M denote the number of data symbols for each user. The overall "chip" symbols  $s_n$  are given by

$$s_n = \sum_{p=1}^{P} \xi_p s_{n,p} \tag{1}$$

where  $\xi_p$  is an weighting coefficient, proportional to the transmitted power for the *p*th user, and  $s_{n,p} = c_{n,p}d_{\lfloor n/K \rfloor,p}$  is the *n*th chip for the *p*th user ( $\lfloor x \rfloor$  denotes 'larger integer not higher that *x*'), with  $\{d_n; n = 0, 1, \ldots, M - 1\}$  denoting the block of data symbols associated to the *p*th user and  $\{c_{n,p}; n = 0, 1, \ldots, N - 1\}$  denoting the corresponding spreading sequence. Throughout this paper it is assumed that  $c_{n,p} = \pm \sqrt{2}/2 \pm j/\sqrt{2}/2$ ,  $n = mK, mK+1, \ldots, mK+K-1$ , is the product of an *N*-length Hadamard-Walsh sequence with

a pseudo-random QPSK sequence (Quadrature Phase Shift Keying), common to all users of the BS.

An appropriate cyclic extension is appended to each block transmitted by the BS. This cyclic extension is removed at the receiver. The time-domain block at the receiver input is  $\{y_n; n = 0, 1, \ldots, N - 1\}$ . The corresponding frequency-domain block is  $\{Y_k; k = 0, 1, \ldots, N - 1\} = \text{DFT} \{y_n; n = 0, 1, \ldots, N - 1\}$ . If the cyclic extension is longer than the overall channel impulse response, it can be shown that

$$Y_k = H_k S_k + N_k,\tag{2}$$

where  $H_k$  and  $N_k$  denote the channel frequency response and the noise term for the *k*th frequency, respectively, and  $\{S_k; k = 0, 1, \ldots, N-1\}$  = DFT  $\{s_n; n = 0, 1, \ldots, N-1\}$  can be regarded as the frequency-domain transmitted block.

For an ideal AWGN channel  $H_k$  is constant for all k and the different users are orthogonal. Therefore, data block associated to the *p*th user can be estimated by "de-spreading" the received time-domain block  $\{y_n; n = 0, 1, ..., N - 1\}$ , i.e.,

$$\tilde{d}_{m,p} = \sum_{n=mK}^{mK+K} y_n c_{n,p}^* = \xi_p K d_{m,p} + \nu_m^{eq}, \qquad (3)$$

with  $\nu_m^{eq}$  denoting the equivalent noise component.

However, this approach is not adequate for time-dispersive channels, since the orthogonality between users is lost. In the following section we present a frequency domain receiver structure which is appropriate for severe time-dispersive channels.

## III. Receiver Structure

Fig. 1 presents the receiver structure that we are considering in this paper which can be regarded as the extension of an IB-DFE [13] receiver to DS-CDMA systems. For a given iteration i, the frequency-domain samples at the output of the IB-DFE are given by

$$\tilde{S}_{k}^{(i)} = F_{k}^{(i)} Y_{k} - B_{k}^{(i)} \hat{S}_{k}^{(i-1)}$$
(4)

where  $\{F_k^{(i)}; k = 0, 1, \dots, N - 1\}$  and  $\{B_k^{(i)}; k = 0, 1, \dots, N - 1\}$  denote the feedforward and the feedback coefficients, respectively.  $\{\hat{S}_k^{(i-1)}; k = 0, 1, \dots, N - 1\}$  denotes the DFT of the overall chip estimates from the previous iteration  $\{\hat{s}_n^{(i-1)}; n = 0, 1, \dots, N - 1\}$ , which are given by

$$\hat{s}_{n}^{(i-1)} = \sum_{p=1}^{P} \xi_{p} \hat{s}_{n,p}^{(i-1)}$$
(5)

where  $\hat{s}_{n,p}^{(i-1)} = \hat{d}_{\lfloor n/K \rfloor,p}^{(i-1)} c_{n,p}$ , with  $\hat{d}_m^{(i-1)}$  denoting the estimate of  $d_{m,p}$  for the (i-1)th iteration, obtained from

$$\tilde{d}_{m,p}^{(i-1)} = \sum_{n=mK}^{mK+K} \tilde{s}_n^{(i-1)} c_{n,p}^*,$$
(6)

with the block of time-domain samples at the equalizer output in the (i-1)th iteration given by  $\{\tilde{s}_n^{(i-1)}; n = 0, 1, \dots, N-1\}$ = IDFT  $\{\tilde{S}_k^{(i-1)}; k = 0, 1, \dots, N-1\}$ .



Fig. 1. IB-DFE receiver structure.

The forward and backward IB-DFE coefficients,  $\{F_k^{(i)}; k = 0, 1, \ldots, N-1\}$  and  $\{B_k^{(i)}; k = 0, 1, \ldots, N-1\}$ , respectively, are chosen so as to maximize the "signal-to-noise plus interference and residual ISI ratio". It can be shown that the optimum feedforward and feedback coefficient are given by

$$B_k^{(i)} = \rho^{(i-1)} \left( F_k^{(i)} H_k - \gamma^{(i)} \right)$$
(7)

and

$$F_k^{(i)} = \frac{SNR \cdot H_k^*}{1 + SNR(1 - (\rho^{(i-1)})^2)|H_k|^2},$$
(8)

respectively, with

$$SNR = \frac{E[|s_n|^2]}{2\sigma_N^2},\tag{9}$$

with  $\sigma_N^2$  denoting the variance of the real and imaginary parts of the noise component in the received time-domain samples  $y_n$ ,

$$\gamma^{(i)} = \frac{1}{N} \sum_{k=0}^{N-1} F_k^{(i)} H_k \tag{10}$$

and the correlation coefficient

$$\rho^{(i)} = \frac{E[\hat{s}_n^{(i)} s_n^*]}{E[|s_n|^2]}.$$
(11)

It should be noted that, for the first iteration (i = 0), we do not have any information about  $S_k$  and the correlation coefficient is zero. This means that  $B_k^{(0)} = 0$  and

$$F_k^{(0)} = \frac{SNR \cdot H_k^*}{1 + SNR |H_k|^2},$$
(12)

corresponding to the optimum frequency-domain equalizer coefficients under the MMSE criterion (Minimum Mean-Squared Error) [7], [12]. After the first iteration, and if the residual BER is not too high, at least for the users with higher transmit power, we have  $d_{m,p} = d_{m,p}$  for most of the data symbols, leading to  $\hat{s}_n \approx s_n$  and  $\hat{S}_k \approx S_k$ . Consequently, we can use the feedback coefficients to eliminate a significant part of the residual intersymbol interference.

The blockwise reliability  $\rho^{(i)}$ , which is a key parameter for the good performance of the proposed receiver, can be estimated as described in Appendix A.

# IV. Performance Results

In this section, we present a set of performance results concerning the proposed receiver structure. We consider the downlink transmission within a DS-CDMA system with spreading factor K = 16 and M = 64 data symbols for each user, corresponding to blocks with length KM = 512, plus an appropriate cyclic extension. QPSK constellations, with Gray mapping, are employed. We consider the power delay profile type C for the HIPERLAN/2 (HIgh PERformance Local Area Network) [16], with uncorrelated Rayleigh fading on the different paths (similar results were obtained for other severely time-dispersive channels). The duration of the useful part of the block is  $5\mu$ s and the CE has duration  $1.25\mu$ s. We consider uncoded BER performances under perfect synchronization and channel estimation conditions. We have P = K = 16 users. corresponding to a fully loaded scenario. It is assumed that the receiver knows the spreading codes for all users, as well as the corresponding attributed powers.

Let us first assume that there is no power control at the BS, i.e., the all users have the same power (this means that  $\xi_p$  is constant). Fig. 2 shows the impact of the number of iterations on the average BER (identical for all users). For the sake of comparisons, we also include the corresponding MFB performance (Matched Filter Bound). From this figure, we can observe that our iterative receiver structure allows a significant improvement on the BER performance: the required  $E_b/N_0$  for BER=10<sup>-4</sup> is about 15dB for the first iteration (corresponding to a conventional linear FDE), dropping to about 10dB after three iterations. Moreover, the resulting BER performances can be very close to the MFB after just a few iterations (about 0.5dB after four iterations).

Let us consider now a scenario where the BS attributes different powers to the different users. We will consider two classes of users, denoted by  $C_L$  and  $C_H$ . It is assumed that the average power attributed to a class  $C_H$  user is 10dB above the average power attributed to a class  $C_L$  user (this can correspond to a situation where the  $C_L$  users are closer to the BS than the  $C_H$  users). It is also assumed that half users belong to each class, i.e, we have 8  $C_L$  users and 8  $C_H$  users. Clearly, the  $C_L$  users face strong interference conditions. Figs. 3 and 4 show the BER performances for



Fig. 2. Average BER for iterations 1 to 4, as well as the corresponding MFB performance.

the  $C_L$  and  $C_H$  users, respectively. Once again, the iterative detection procedure allows significant performance gains. By comparing figs. 3 and 4, we note that the performance of low power users asymptotically approaches the MFB when we increase the number of iterations; however, for high power users, the BER at  $10^{-4}$  is still about 2dB from the MFB. This can be explained from the fact that the BER is much lower for high-power users, allowing an almost perfect interference cancelation of their effects on low-power users; therefore, the corresponding performances can be very close to the MFB. The higher BERs for the low-power users preclude an appropriate interference cancelation when we detect high-power users (see also figure 5, where the BERs are expressed as a function of the  $E_b/N_0$  of low-power users, 10dB above the  $E_b/N_0$  of high-power users).

# V. Conclusions

An iterative frequency-domain decision feedback equalizer was proposed for the downlink of DS-CDMA systems employing block-transmission techniques with appropriate cyclic extension. For the first iteration, the proposed receiver reduces to a conventional FDE employed in DS-CDMA systems; the subsequent iterations provide increasingly improved performances.

Our performance results show that the proposed receiver is appropriate for DS-CDMA wireless systems, with performances very close to the single-user MFB after just a few iterations, even for fully loaded systems, in severe timedispersive channels and/or in the presence of strong interfering codes.

It should be noted that the proposed receiver is very flexible. For a given system, the number of iterations can depend on the type of receiver and/or the quality requirements for that service: a small number of iterations for low-cost receivers and/or services with lower quality requirements; a larger number of



Fig. 3. BER performance for the low-power users (class  $C_L$ ).



Fig. 4. BER performance for the high-power users (class  $C_H$ ).

iterations for services with high quality requirements. Since the receiver structure is independent on the adopted spreading factor, it can easily be extended to VSF schemes (Variable Spreading Factor) [2]; it an also be employed with multicode schemes [2].

#### Appendix

#### A. Computation of the Correlation Factor

The correlation factor is crucial for the good performance of the proposed receivers. In fact, it can be regarded as a blockwise measure of the reliability of the estimates used in the feedback loop.

From (11), and assuming uncorrelated data blocks, it can be



Fig. 5. As in figs. 3 and 4, but with the BER performances as a function of the  $E_b/N_0$  of low-power users.

easily shown that

$$\rho = \sum_{p=1}^{P} \xi_p^2 \frac{E[\hat{d}_{n,p} d_{n,p}^*]}{E[|d_{n,p}|^2]} = \sum_{p=1}^{P} \xi_p^2 \rho_p, \tag{13}$$

with

$$\rho_p = \frac{E[d_{n,p}d_{n,p}^*]}{E[|d_{n,p}|^2]} \tag{14}$$

(for the sake of notation simplicity, we ignore the dependence with the iteration order i).

If the  $d_{n,p}$  belong to a QPSK constellation, it can be shown that [14]

$$\rho_p = 1 - 2P_{b,p}, \tag{15}$$

with  $P_{b,p}$  denoting the average BER for the *p*th user. This BER rate can be estimated during the training phase, using reference blocks. We can also employ the method proposed in [14] for estimating  $\rho_p$ .

#### References

- A. Viterbi, CDMA: Principles of SS Communication, Addison Wesley, 1995.
- [2] T. Ojamperä and R. Prasad, Wideband CDMA for Third Generation Mobile Communications, Artech House Publ., 1998.
- [3] J. Proakis, *Digital Communications*, McGraw-Hill Science/Engineering/Math; 4th edition, 2000.
- [4] K. Higuchi, A. Fujiwara, and M. Sawahashi, "Multipath Interference Canceller for High-Speed Packet Transmission with Adaptive Modulation and Coding Scheme in W-CDMA Forward Link", *IEEE J. on Sel. Areas in Comm.*, Vol. 20, No. 2, pp. 419–432, Feb. 2002.
- [5] L.Cimini Jr., "Analysis and Simulation of a Digital Mobile Channel using Orthogonal Frequency Division Multiplexing", *IEEE Trans. on Comm.*, Vol. 33, No. 7, July 1985.

- [6] H.Sari, G.Karam and I.Jeanclaude, "An Analysis of Orthogonal Frequency-division Multiplexing for Mobile Radio Applications", *In Proc. IEEE Vehic. Tech. Conf., VTC'94*, pp. 1635–1639, Stockholm, June 1994.
- [7] A. Gusmão, R. Dinis, J. Conceição, and N. Esteves, "Comparison of Two Modulation Choices for Broadband Wireless Communications", *IEEE VTC'00 (Spring)*, Tokyo, Japan, May 2000.
- [8] P. Montezuma and A. Gusmão, "A Pragmatic Coded Modulation Choice for Future Broadband Wireless Communications", *IEEE VTC'01 (Spring)*, Rhodes, Greece, May 2001.
- [9] D.Falconer, S.Ariyavisitakul, A.Benyamin-Seeyar and B.Eidson, "Frequency Domain Equalization for Single-Carrier Broadband Wireless Systems", *IEEE Comm. Mag.*, Vol. 4, No. 4, pp. 58–66, April 2002.
- [10] S. Barbarossa and F. Cerquetti, "Simple Space-Time Coded SS-CDMA Systems Capable of Perfect MUI/ISI Elimination", *IEEE Comm. Letters*, Vol. 5, No. 12, pp. 471–473, Dec. 2001.
- [11] K. Baum, T. Thomas, F. Vook, V. Nangia, "Cyclic-Prefix CDMA: An Improved Transmission Method for Broadband DS-CDMA Cellular Systems", *IEEE WCNC*, pp. 183–188, 2002.
- [12] A.Gusmão, R.Dinis and N.Esteves, "On Frequency-domain Equalization and Diversity Combining for Broadband Wireless Communications", *IEEE Trans. on Comm.*, Vol. 51, No. 7, pp. 1029–1033, July 2003.
- [13] N. Benvenuto and S. Tomasin, "Block Iterative DFE for Single Carrier Modulation", *IEE Elec. Let.*, Vol. 39, No. 19, pp. 1144– 1145, Sep. 2002.
- [14] R. Dinis, A. Gusmão, and N. Esteves, "On Broadband Block Transmission over Strongly Frequency-Selective Fading Channels", *Proc. Wireless 2003*, Calgary, Canada, July 2003.
- [15] M. Tüchler, R. Koetter and A. Singer, "Turbo Equalization: Principles and New Results", *IEEE Trans. on Comm.*, Vol. 50, May 2002.
- [16] ETSI, "Channel models for HIPERLAN/2 in Different Indoor Scenarios", ETSI EP BRAN 3ERI085B, pp. 1-8, March 1998.