

# ANALYTICAL EVALUATION OF NONLINEAR EFFECTS IN MC-CDMA SIGNALS

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## ABSTRACT

In this paper we present an analytical tool for the performance evaluation of nonlinear effects in MC-CDMA signals. This tool takes advantage of the Gaussian-like behavior of MC-CDMA signals with a large number of subcarriers and employs results on memoryless nonlinear devices with Gaussian inputs so as to characterize statistically the signals at the output of the nonlinear device. This characterization is then used for an analytical computation of the SIR levels (Signal-to-Interference Ratio) and the BER performance (Bit Error Rate)<sup>1</sup>.

A set of numerical results is presented and discussed, showing the accuracy of the proposed analytical BER performance analysis.

## KEYWORDS

MC-CDMA, nonlinear effects, Gaussian processes.

## 1 Introduction

The MC-CDMA schemes (Multicarrier Code Division Multiple Access) [1, 2] combine a CDMA scheme with an OFDM modulation (Orthogonal Frequency Division Multiplexing) [3], so as to allow high transmission rates over severe time-dispersive channels without the need of a complex receiver implementation. Since the spreading is made in frequency domain, the time synchronization requirements are much lower than with conventional direct sequence CDMA schemes. Moreover, the diversity effect inherent to the spreading allows good performances for high code rates, as well as good uncoded performances.

As with other multicarrier schemes, the MC-CDMA signals have strong envelope fluctuations and high PMEPR values (Peak-to-Mean Envelope Power Ratio) which makes them very prone to nonlinear effects. These nonlinear effects can be both intentional (such as the ones inherent to a nonlinear signal processing for reducing the envelope fluctuations, as in [4]-[7]) or not (such as the ones inherent to a nonlinear power amplification [8, 9]).

The performance evaluation of a nonlinear transmission usually resorts to Monte-Carlo simulations that require a long computation time. For this reason, analytical approaches have been proposed for the performance

evaluation of nonlinear effects in OFDM transmission [5]-[7],[10].

In this paper we present an analytical approach for the performance evaluation of MC-CDMA signals in the presence of nonlinear effects. For this purpose, we take advantage of the Gaussian-like behavior of MC-CDMA signals with a large number of subcarriers and employ results on memoryless nonlinear devices with Gaussian inputs so as to characterize statistically the signals at the output of the nonlinear device [7, 11]. This characterization is then used for an analytical computation of the SIR levels (Signal-to-Interference Ratio) and the BER performance (Bit Error Rate) of MC-CDMA schemes in the presence of nonlinear effects.

This paper is organized as follows: the MC-CDMA schemes considered in this paper are described in sec. 2. The analytical characterization of the transmitted signals in the presence of nonlinear effects is made in sec. 3 and used in sec. 4 for analytical performance evaluation purposes. A set of performance results is presented in sec. 5 and sec. 6 is concerned with the conclusions and final remarks of this paper.

## 2 Systems Description

In this paper we consider the downlink transmission (i.e., the transmission from the BS (Base Station) to the MT (Mobile Terminal)) within MC-CDMA systems employing frequency-domain spreading, although our approach could also be employed in the uplink transmission. A constant spreading factor  $K$  is assumed for all users (the extension to VSF schemes (Variable Spreading Factor) [12] is straightforward). The frequency-domain block to be transmitted by the BS is an interleaved version of the block  $\{S_k; k = 0, 1, \dots, N - 1\}$ <sup>2</sup>, where  $N = KM$ , with  $K$  denoting the spreading factor and  $M$  the number of data symbols for each user. The frequency-domain symbols are given by

$$S_k = \sum_{p=1}^{K_U} \xi_p S_{k,p}, \quad (1)$$

<sup>2</sup>Typically, the transmitted frequency-domain block is generated by submitting the block  $\{S_k; k = 0, 1, \dots, N - 1\}$  to a rectangular interleaver with dimensions  $K \times M$ , i.e., the different chips associated to a different data symbol are uniformly spread within the transmission band.

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where  $\xi_p$  is an appropriate weighting coefficient that accounts for the power control in the downlink (the power associated to the  $p$ th user is proportional to  $|\xi_p|^2$ ) and  $S_{k,p} = C_{k \text{ Mod } k,p} A_{\lfloor k/K \rfloor,p}$  is the  $k$ th chip for the  $p$ th user ( $\lfloor x \rfloor$  denotes 'larger integer not higher than  $x$ '), where  $\{A_{k,p}; k = 0, 1, \dots, M-1\}$  is the block of data symbols associated to the  $p$ th user and  $\{C_{k,p}; k = 0, 1, \dots, K-1\}$  is the corresponding spreading sequence. An orthogonal spreading is assumed throughout this paper, with  $C_{k,p}$  belonging to an QPSK constellation (Quadrature Phase Shift Keying). Without loss of generality, it is assumed that  $|C_{k,p}| = 1$ .

As with conventional OFDM, an appropriate cyclic extension is appended to each block transmitted by the BS. At the receiver, the cyclic extension is removed and the received samples are passed to the frequency domain, leading to the block is  $\{Y_k; k = 0, 1, \dots, N-1\}$ .

It can be shown that, when the cyclic extension is longer than the overall channel impulse response, the samples  $Y_k$  can be written as

$$Y_k = H_k S'_k + N_k, \quad (2)$$

where  $H_k$  and  $N_k$  denote the channel frequency response and the noise term for the  $k$ th frequency, respectively. Since the orthogonality between users is lost in frequency selective channels, an FDE (Frequency-Domain Equalizer) is required before the despreading operation [2, 13].

### 3 Nonlinear Effects in MC-CDMA Signals

In the previous section an ideal linear transmitter was assumed. However, the MC-CDMA signal has large envelope fluctuations which makes them very prone to nonlinear effects. In this section we present an appropriate model of the transmitted signals when a nonlinear transmitter is considered. For the sake of simplicity, the transmitter structure depicted in fig. 1 is assumed, where the nonlinear device is modeled as a bandpass memoryless nonlinearity [8] operating on an oversampled version of the MC-CDMA burst. In fact, adding  $N' - N$  zeros to the original frequency-domain block (i.e.,  $N' - N$  idle subcarriers), followed by an IDFT operation is equivalent to generate a sampled version of the MC-CDMA signal, with an oversampling factor  $M_{Tx} = N'/N$ . An optional frequency-domain filtering procedure, through the set of multiplying coefficients  $G_k, k = 0, 1, \dots, N'-1$ , can also be considered so as to reduce the out-of-band radiation levels inherent to the nonlinear operation [5].

The transmitter structure of fig. 1 is similar to the nonlinear signal processing schemes proposed in [5, 6] for reducing the PMEPR of OFDM signals while maintaining the spectral efficiency of conventional OFDM schemes. This transmitter model could also be employed for evaluating the impact of a nonlinear power amplification: we just have to consider an oversampling factor large enough

and to omit the frequency-domain filtering subsequent to the nonlinear operation.

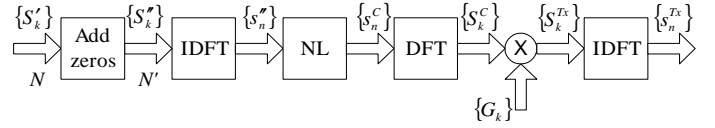


Figure 1. Transmitter model considered in this paper.

When the number of subcarriers is high ( $N \gg 1$ ) the time-domain coefficients  $s''_n$  can be approximately regarded as samples of a complex Gaussian process. If  $E[S_k] = 0$  and  $E[S_k S_{k'}^*] = 2\sigma_S^2 \delta_{k,k'}$  ( $\delta_{k,k'} = 1$  for  $k = k'$  and 0 otherwise), with  $\sigma_S^2 = \frac{1}{2} E[|S_k|^2]$  ( $E[\cdot]$  denotes "ensemble average"), then it can be easily demonstrated that  $E[s''_n] = 0$  and

$$E[s''_n s''_{n'}^*] = R_s(n - n') = 2\sigma^2 \frac{\text{sinc}((n - n')N/N')}{\text{sinc}((n - n')/N')} \exp\left(-\frac{j\pi(n - n')}{N'}\right) \quad (3)$$

( $n, n' = 0, 1, \dots, N'-1$ ), with  $\sigma^2 = \frac{N}{(N')^2} \sigma_S^2$ .

It is well-known that the output of a bandpass memoryless nonlinear device with a Gaussian input can be written as the sum of two uncorrelated components [14]: a useful one, proportional to the input, and a self-interference one, i.e.,

$$s_n^C = \alpha s_n'' + d_n, \quad (4)$$

where  $E[s_n'' d_n^*] = 0$  and  $\alpha = E[s_n^C s_n^{C*}] / E[|s_n''|^2]$ .

The average power of the useful component is  $P_{NL}^S = |\alpha|^2 \sigma^2$ , and the average power of the self-interference component is given by  $P_{NL}^I = P_{NL} - P_{NL}^S$ , where  $P_{NL}$  denotes the average power of the signal at the nonlinearity output, given by

$$P_{NL} = \frac{1}{2} E[|s_n^C|^2] = \frac{1}{2} E[|g_C(R)|^2] = \frac{1}{2} \int_0^{+\infty} |g_C(R)|^2 \frac{R}{\sigma^2} \exp\left(-\frac{R^2}{2\sigma^2}\right) dR. \quad (5)$$

It can be shown [11] that

$$E[s_n^C s_{n'}^{C*}] = R_s^C(n - n') = \sum_{\gamma=0}^{+\infty} 2P_{2\gamma+1} \frac{(R_s(n - n'))^{\gamma+1} (R_s^*(n - n'))^\gamma}{(R_s(0))^{2\gamma+1}}, \quad (6)$$

where the coefficient  $P_{2\gamma+1}$  denotes the total power associated to the IMP (Inter-Modulation Product) of order  $2\gamma + 1$ , which can be obtained as described in [7, 11].

Since  $R_s^C(n - n') = |\alpha|^2 R_s(n - n') + E[d_n d_{n'}^*]$ , it can be easily recognized that  $P_1 = |\alpha|^2 \sigma^2$  and  $E[d_n d_{n'}^*] = R_d(n - n')$  is obtained by using  $\sum_{\gamma=1}^{+\infty}$  instead of  $\sum_{\gamma=0}^{+\infty}$  in the right-hand side of (6). The total self-interference power is

$$P_{NL}^I = \frac{1}{2} R_d(0) = \sum_{\gamma=1}^{+\infty} P_{2\gamma+1} = P_{NL} - P_{NL}^S. \quad (7)$$

Having in mind (4) and the signal processing chain in fig. 1, the frequency-domain block  $\{S_k^{CF} = S_k^C G_k; k = 0, 1, \dots, N' - 1\}$  can obviously be decomposed into useful and self-interference components:

$$S_k^{CF} = \alpha S_k'' G_k + D_k G_k, \quad (8)$$

where  $\{D_k; k = 0, 1, \dots, N' - 1\}$  is the DFT of  $\{d_n; n = 0, 1, \dots, N' - 1\}$ . Clearly,  $E[D_k] = 0$  and

$$E[D_k D_{k'}^*] = \begin{cases} N' G_d(k), & k = k' \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

$(k, k' = 0, 1, \dots, N' - 1)$ , where  $\{G_d(k); k = 0, 1, \dots, N' - 1\}$  denotes the DFT of the block  $\{R_d(n); n = 0, 1, \dots, N' - 1\}$  ( $R_d(n - n') = E[d_n d_{n'}^*]$ ); Moreover,  $D_k$  exhibits quasi-Gaussian characteristics for any  $k$ , provided that the number of subcarriers is high enough. Clearly,  $E[S_k^C S_{k'}^{C*}] = 0$  for  $k \neq k'$  and  $E[|S_k^C|^2] = N' G_s^C(k)$ , where  $\{G_s^C(k) = |\alpha|^2 G_s(k) + G_d(k); k = 0, 1, \dots, N' - 1\}$  denotes the DFT of  $\{S_k^C(n); k = 0, 1, \dots, N' - 1\}$  (given by (6)), with  $\{G_s(k); k = 0, 1, \dots, N' - 1\} = \text{DFT} \{R_s(n); n = 0, 1, \dots, N' - 1\}$ . Therefore,  $E[S_k^{CF} S_{k'}^{CF*}] = 0$  for  $k \neq k'$ , and  $E[|S_k^{CF}|^2] = |G_k|^2 E[|S_k^C|^2] = N' |G_k|^2 G_s^C(k)$ .

Clearly, the total power of the useful and self-interference components of the transmitted signals are  $P_{Tx}^S = \sum_k E[|\alpha S_k'' G_k|^2]$  and  $P_{Tx}^I = \sum_k E[|D_k G_k|^2]$ , respectively. We can also define the power of the self-interference component in the in-band region as

$$P_{Tx,IB}^I = \sum_{k \text{ in-band}} E[|D_k G_k|^2] \quad (10)$$

When  $G_k = 1$  for the  $N$  in-band subcarriers,  $P_{Tx}^S = P_{NL}^S$ . If we also have  $G_k = 0$  for the  $N' - N$  out-of-band subcarriers then  $P_{Tx,IB}^I = P_{Tx}^I$ .

The "signal-to-interference ratio" (SIR) for the transmitted signals is

$$SIR_{Tx} = \frac{P_{Tx}^S}{P_{Tx}^I} \leq SIR_{NL} = \frac{P_{NL}^S}{P_{NL}^I}, \quad (11)$$

where  $SIR_{NL}$  denotes the SIR at the output of the non-linear device; the SIR for the in-band region is

$$SIR_{Tx,IB} = \frac{P_{Tx}^S}{P_{Tx,IB}^I}. \quad (12)$$

We can also define a SIR for each subcarrier, given by

$$SIR_k = \frac{E[|\alpha S_k''|^2]}{E[|D_k|^2]}. \quad (13)$$

Without oversampling, (3) leads to  $R_s(n - n') = 2\sigma^2 \delta_{n, n'}$ . From (6),  $R_s^C(n - n') = 2 \sum_{\gamma=0}^{+\infty} P_{2\gamma+1} = 2P_1 + 2 \sum_{\gamma=1}^{+\infty} P_{2\gamma+1}$  for  $n = n'$  and  $R_s^C(n - n') = 0$  for  $n' \neq n$ ; therefore,

$$SIR_k = \frac{P_1}{\sum_{\gamma=1}^{+\infty} P_{2\gamma+1}} = SIR_{NL} = SIR_{Tx} = SIR_{Tx,IB}, \quad (14)$$

which is independent of  $k$ , when  $M_{Tx} = 1$ . For  $M_{Tx} > 1$  (i.e., when  $N' > N$ ),  $R_s(n - n') \neq 2\sigma^2 \delta_{n, n'}$  and  $SIR_k$  is a function of  $k$ , since  $E[|D_k|^2]$  depends also on  $k$ .

## 4 Analytical Performance Evaluation

### 4.1 SIR Levels and BER Performance

From (2), the frequency-domain samples at the receiver are given by  $Y_k = S_k^{Tx} H_k + N_k$ , provided that the guard interval is long enough.

For an ideal Gaussian channel, the detection of the  $k$ th symbol transmitted by the  $p$ th user is based on the "despread" samples

$$\tilde{A}_{k,p} = \sum_{k' \in \Psi_k} Y_{k'} C_{k',p}^* = \alpha \xi_p K A_{k,p} + D_{k,p}^{eq} + N_{k,p}^{eq}, \quad (15)$$

with  $\Psi_k$  denoting the set of frequencies used to transmit the  $k$ th data symbol. As referred above, for a frequency-selective channel we need to perform an FDE previous to the "despreading" operation. In (15),

$$D_{k,p}^{eq} = \sum_{k' \in \Psi_k} D_{k'} C_{k',p}^* \quad (16)$$

and

$$N_{k,p}^{eq} = \sum_{k' \in \Psi_k} N_{k',p} C_{k',p}^* \quad (17)$$

denote the equivalent self-interference and noise terms for detection purposes, respectively.

Clearly, the power of the self-interference term,  $D_{k,p}^{eq}$ , is

$$P_{k,p}^{I,eq} = \sum_{k' \in \Psi_k} E[|D_{k'}|^2] = \frac{2P_{Tx,IB}^I}{M} \zeta_k, \quad (18)$$

with

$$\zeta_k = \frac{M \sum_{k' \in \Psi_k} E[|D_{k'}|^2]}{\sum_{k' \text{ in-band}} E[|D_{k'}|^2]} \quad (19)$$

(it is assumed that  $|C_{k,p}| = 1$ ). Since we are employing orthogonal spreading sequences, the "useful" component for detection purposes of the  $p$ th user is

$$\alpha \sum_{k' \in \Psi_k} S_{k'} C_{k',p}^* = \alpha \xi_p A_{k,p} \sum_{k' \in \Psi_k} |C_{k',p}|^2 = \alpha \xi_p K A_{k,p}. \quad (20)$$

By assuming  $E[|A_{k,p}|^2] = 1$ , the power of the "useful" component for detection purposes when detecting the  $p$ th user is

$$\begin{aligned} P_{k,p}^{S,eq} &= |K \alpha \xi_p|^2 = \frac{K |\alpha \xi_p|^2}{\sum_{p'} \xi_{p'}^2} \sum_{k' \in \Psi_k} E[|S_{k'}|^2] = \\ &= \frac{K |\alpha \xi_p|^2}{K_U \xi^2} \sum_{k' \in \Psi_k} E[|S_{k'}|^2] = \frac{K}{K_U} \eta_{\xi,p} \frac{2P_{Tx}^S}{M}, \end{aligned} \quad (21)$$

with

$$\overline{\xi^2} = \frac{1}{K_U} \sum_{p'} \xi_{p'}^2 \quad (22)$$

and

$$\eta_{\xi,p} = \frac{\xi_p^2}{\overline{\xi^2}}. \quad (23)$$

Therefore, the corresponding signal-to-self-interference ratio for detection purposes is

$$SIR_{k,p}^{eq} = \frac{P_{k,p}^{S,eq}}{P_{k,p}^{I,eq}} = \frac{K}{K_U} \eta_{\xi,p} SIR_{Tx,IB} \zeta_k^{-1}. \quad (24)$$

From (24), it is clear that the equivalent SIR for detection purposes increases when we decrease the number of users, for a given spreading factor  $K$ . This is a consequence of the samples of the self-interference component,  $D_k C_{k,p}^*$ , being uncorrelated, contrarily to the useful components. We can also note that the equivalent SIR for detection purposes is not the same for the different users: the users with smaller attributed powers (i.e., the users that are closer to the BS and/or have better propagation conditions) have worse  $SIR_{k,p}^{eq}$  levels, and, consequently, a larger performance degradation due to the nonlinear effects.

Since the self-interference components  $D_k$  are approximately Gaussian-distributed at the subcarrier level [7],  $D_k^{eq}$  is also approximately Gaussian-distributed, even when the number of users is small. Therefore, if the data symbols are selected from a QPSK constellation under a Gray mapping rule (the extension to other constellations is straightforward), the BER for an ideal Gaussian channel is approximately given by

$$BER_{k,p} = Q\left(\sqrt{SNR_{k,p}^{eq}}\right), \quad (25)$$

where  $Q(\cdot)$  denotes the well-known Gaussian error function and  $SNR_{k,p}^{eq}$  denotes an equivalent signal-to-noise ratio for the detection of the  $k$ th data symbol, for the  $p$ th user. This ratio is given by

$$SNR_{k,p}^{eq} = \frac{P_{k,p}^{S,eq}}{P_{k,p}^{I,eq} + P_{k,p}^{N,eq}}, \quad (26)$$

where  $P_{k,p}^{N,eq} = E[|N_k^{eq}|^2] = KE[|N_k|^2] = 2KN_0$ , with  $N_0$  denoting the PSD of the channel noise. It can be shown that

$$SNR_{k,p}^{eq} = 2\eta_S \eta_{\xi,p}^I \frac{E_{b,p}}{N_0}, \quad (27)$$

where

$$E_{b,p} = \frac{P_{Tx}^S + P_{Tx}^I}{2K_U M} \eta_{\xi,p} \quad (28)$$

denotes the average bit energy for the  $p$ th user,

$$\eta_S = \frac{P_{Tx}^S}{P_{Tx}^S + P_{Tx}^I} \quad (29)$$

and

$$\eta_{k,p}^D = \left(1 + 2 \frac{K_U}{K \eta_{\xi,p}} \cdot \frac{P_{Tx,IB}^I \zeta_k}{P_{Tx}^S + P_{Tx}^I} \cdot \frac{E_b}{N_0}\right)^{-1}. \quad (30)$$

Clearly, the degradation factor  $\eta_S$  is associated to the useless power spent in the transmitted self-interference; the degradation factor  $\eta_{k,p}^D$  is due to the fact that the equivalent, quasi-Gaussian self-interference  $D_k^{eq}$  is added to the Gaussian channel noise.

## 4.2 Especial Cases and Simplified Formulas

For most cases of interest, the analytical approach for obtaining the SIR levels and the BER performances described above can be simplified with only a very slight decrease in its accuracy.

As it was referred in the previous section, if there is no oversampling before the nonlinear operation (i.e., for  $M_{Tx} = N'/N = 1$ ), then  $P_{Tx,IB}^I = P_{Tx}^I = P_{NL}^I$ ,  $P_{Tx}^S/P_{Tx}^I = SIR_{Tx} = SIR_{NL}$  and  $\zeta_k = 1$ . Therefore,

$$\eta_S = \frac{SIR_{NL}}{1 + SIR_{NL}} \quad (31)$$

and  $\eta_{k,p}^D$ , which becomes independent of  $k$ , is given by

$$\eta_{k,p}^D = \left(1 + 2 \frac{K_U}{K \eta_{\xi,p}} \cdot \frac{1}{1 + SIR_{NL}} \cdot \frac{E_b}{N_0}\right)^{-1}. \quad (32)$$

It can easily be verified that when the "chips" associated to a given data block are uniformly spread in the transmission band (i.e., for a rectangular interleaver with size  $K \times M$ ) then  $\zeta_k \approx 1$ , provided that the spreading factor is not too low.

Let us assume now that  $M_{Tx} > 1$ . To obtain an approximate formulas for the SIR levels that does not require the computation of all IMPs, we will assume that the total self-interference power is associated to the IMP of order 3, i.e.,  $P_3 = P_{NL}^I$  and  $P_{2\gamma+1} = 0$ ,  $\gamma > 1$  (this is a slightly pessimistic assumption relatively to the in-band self-interference levels). In that case, it can be shown that the average power of the self-interference component for the  $N$  in-band subcarriers is

$$\begin{aligned} \frac{1}{N} \sum_{k \text{ in-band}} E[|D_k|^2] &\approx \\ &\approx \frac{\kappa(M_{Tx})}{N} \sum_k E[|D_k|^2] = \frac{\kappa(M_{Tx})}{N} 2P_{NL}^I, \end{aligned} \quad (33)$$

with

$$\begin{aligned} \kappa(M_{Tx}) &= \\ &= \begin{cases} \frac{1}{3}(-M_{Tx}^3 + 6M_{Tx}^2 - 12M_{Tx} + 10), & 1 \leq M_{Tx} < 2 \\ \frac{2}{3}, & M_{Tx} \geq 2. \end{cases} \end{aligned}$$

This means that, for  $G_k = 1$  for the  $N$  in-band subcarriers and 0 for the  $N' - N$  out-of-band subcarriers,

$$\eta_S \approx \frac{SIR_{NL}/\kappa(M_{Tx})}{1 + SIR_{NL}/\kappa(M_{Tx})} \quad (34)$$

and  $\eta_{k,p}^D$  is almost independent of  $k$  and given by

$$\eta_{k,p}^D = \left( 1 + 2 \frac{K_U}{K \eta_{\xi,p}} \cdot \frac{1}{1 + SIR_{NL}/\kappa(M_{Tx})} \cdot \frac{E_b}{N_0} \right)^{-1} \quad (35)$$

For  $M_{Tx} \geq 2$ ,  $\kappa(M_{Tx}) = 2/3$  and we have a gain of 3/2 (i.e., approximately 1.8dB) in the equivalent SIR levels relatively to the case where there is no oversampling ( $M_{Tx} = 1$ ).

The computation of  $\eta_S$  and  $\eta_{k,p}^D$  involves only two integrals inherent to  $\alpha$  and  $P_{NL}$ . If the nonlinearity corresponds to an ideal envelope clipping, i.e., when

$$g_C(R) = \begin{cases} R, & R \leq s_M \\ s_M, & R > s_M, \end{cases} \quad (36)$$

with  $s_M$  denoting the clipping level, which is a very common situation, these two integrals can be written in a closed form:

$$\alpha = 1 - \exp\left(-\frac{s_M^2}{2\sigma^2}\right) + \frac{\sqrt{2\pi}s_M}{2\sigma} Q\left(\frac{s_M}{\sigma}\right) \quad (37)$$

and

$$P_{NL} = \sigma^2 \left( 1 - \exp\left(-\frac{s_M^2}{2\sigma^2}\right) \right). \quad (38)$$

## 5 Performance Results

In this paper we present a set of performance results concerning the performance evaluation of nonlinear effects in MC-CDMA signals. It is assumed that the MC-CDMA signals have a spreading factor  $K = 64$  and each user has  $M = 16$  data symbols per block, corresponding to MC-CDMA blocks with length  $N = KM = 1024$ , plus an appropriate cyclic extension. The same power is attributed to each user (i.e.,  $\xi_p = 1$  for all users). We consider the transmitter structure depicted in fig. 1, where an ideal envelope clipping, operating on a sampled version of the MC-CDMA signal, is adopted for reducing the envelope fluctuations of the transmitted signals (unless otherwise stated, an oversampling factor  $M_{Tx} = 2$  is assumed). However, as it was already referred, our analytical approach could easily be extended for other nonlinear devices, namely those associated to a nonlinear power amplification (in fact, since we have almost the same SIR levels regardless of the oversampling factor  $M_{Tx}$ , provided that  $M_{Tx} \geq 2$ , our performance results are still valid of a perfectly linearized power amplifier, modeled as an ideal envelope clipping).

Fig. 2 shows the impact of the normalized clipping level  $SIR_{Tx,IB}$  when  $M_{Tx} = 1$  or 2. We also include the

approximate  $SIR_{Tx,IB}$  formula that is obtained by using (33), with  $M_{Tx} \geq 2$ . From this figure, it is clear that our approximate formula for  $SIR_{Tx,IB}$  is very accurate, especially for moderate clipping levels.

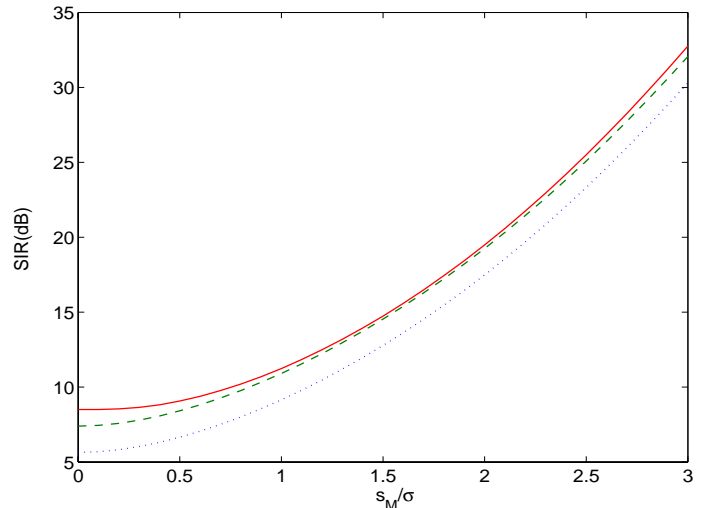


Figure 2.  $SIR_{Tx,IB}$  when  $M_{Tx} = 1$  (dotted line) or 2 (solid line) and  $3SIR_{NL}/2$  (dashed line).

Let us consider an ideal AWGN channel. The theoretical BER performances, together with the corresponding simulated results, are depicted in figs. 3 and 4. The number of users is  $K_U = K$  in fig. 3 (i.e., a fully loaded system) and  $K/2$  in fig. 4. Clearly, our analytical approach is very accurate. The approximate BER formulas are slightly pessimistic for low clipping levels; for moderate and high clipping levels they are very accurate (this is in accordance with fig. 2). It should be noted that the increase in the robustness against nonlinear effects when  $K_U < K$  (implicit in the formulas of sec. 4) was confirmed by the simulations.

## 6 Conclusions and Final Remarks

In this paper we presented an analytical tool for the performance evaluation of nonlinear effects in MC-CDMA signals. For this purpose, we took advantage of the Gaussian-like behavior of MC-CDMA signals with a large number of subcarriers and employed well-known results on memoryless nonlinear devices with Gaussian inputs so as to characterize statistically the signals at the output of the nonlinear device.

We also included analytical, exact formulas for the BER computation, as well as low complexity, approximate formulas which require only the evaluation of two integrals (if the nonlinear device corresponds to an ideal envelope clipping, these two integrals can be written in a

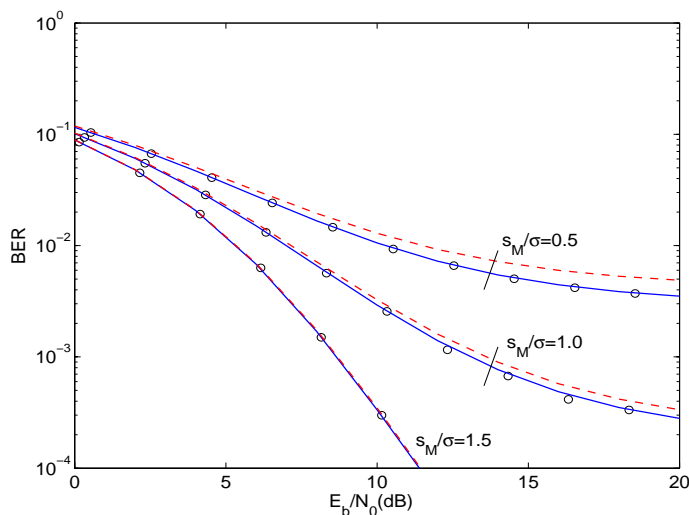


Figure 3. Exact (solid line) and approximate (dashed line) BER formulas when  $K_U = K$  (the (o) denote the simulated results).

closed form). Since the OFDM signals can be regarded as MC-CDMA signals with  $K = 1$  and  $M = N$ , our low-complexity SIR and BER expressions can also be used for evaluating the nonlinear effects in OFDM schemes.

It was shown that the ratio between the number of used channels and the spreading factor has a key influence on the robustness of a given MC-CDMA scheme to nonlinear effects since. The higher this ratio the lower the robustness to nonlinear effects. The spreading provides a diversity effect over the nonlinear interference.

It should be noted that, by using our statistical characterization of the signals at the output of the nonlinear device we can simplify Monte-Carlo simulations: due to the Gaussian nature of the self-interference component, we do not need to simulate the nonlinear operation (we just need to modify the noise variance to include both the channel noise and the self-interference component).

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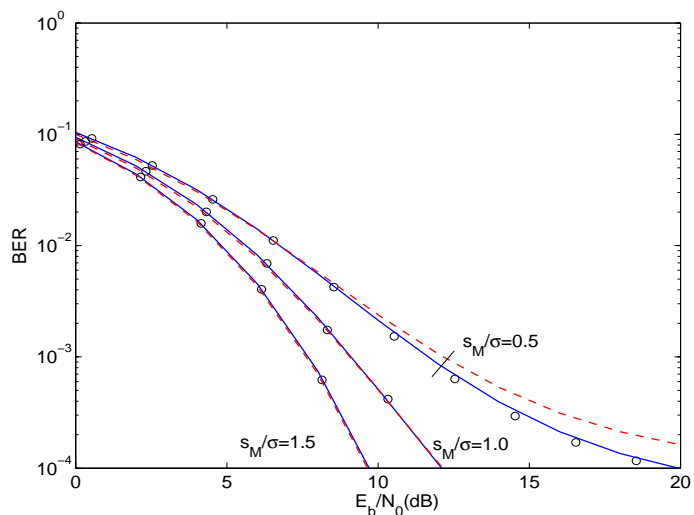


Figure 4. Exact (solid line) and approximate (dashed line) BER formulas when  $K_U = K/2$  (the (o) denote the simulated results).

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