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MODELLING AND SIMULATION OF THREE-PHASE POWER TRANSFORMERS

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Abstract: This paper presents the development and implementation of a digital simulation model of a three-phase, three-leg, power transformer. The proposed model, implemented in MATLAB environment, is based on the physical concept of representing windings as mutually coupled coils, and it is intended to study its adequacy to incorporate, at a later stage, the influences of the occurrence of windings inter-turn short-circuits. Both simulation and laboratory tests results, obtained so far, for a three-phase, 6 kVA transformer, demonstrate the adequacy of the model under normal operating conditions.

List of Symbols

$[B]$ = susceptance matrix, p.u.

$[B_m]$ = magnetising susceptance matrix, p.u.

b_{dm} = direct magnetising susceptance, p.u.

b_{hm} = homopolar magnetising susceptance, p.u.

$[I]$ = vector of currents flowing through the inductive elements, A

$[I_f]$ = vector of currents entering the transformer by its terminals, A

i_{dm} = direct component magnetising current, p.u.

i_{hm} = homopolar component magnetising current, p.u.

$[L]$ = inductance matrix, H

$[R]$ = resistance matrix, representing the load losses, Ω

$[R_{fe}]$ = resistance matrix, representing the no-load losses, Ω or p.u.

$[V]$ = vector of voltages between the terminals of each winding, V

$[X_{sc}]$ = short-circuit reactance matrix, Ω or p.u.

$[Z]$ = generic impedance matrix, p.u.

z_d = direct component impedance, p.u.

z_h = homopolar component impedance, p.u.

z_S = self impedance of a phase or leg, p.u.

z_M = mutual impedance among the three phases, p.u.

1.- INTRODUCTION

Transformers constitute the largest single component of the transmission and distribution equipment market. In 1995, the estimated value of the world market for power transformers was US$11.85 billions [1]. Therefore it is quite obvious the need for the development of on-line diagnostic techniques, that would aid in transformers maintenance. A survey of the most important methods, actually in use, for condition monitoring and diagnostics of power transformers, presented in [2], stresses the need for the development of new diagnostic techniques, which can be applied without taking transformers out of service, and which can also provide a fault severity criteria, in particular for determining transformers winding faults.

Preliminary experimental results, presented in [2] and [3], concerning the use of the Park's Vector Approach, has demonstrated the effectiveness of this non-invasive technique for diagnosing the occurrence of inter-turn short-circuits in the windings of operating three-phase transformers. The on-line diagnosis is based on identifying the appearance of an elliptic pattern, corresponding to the transformer supply current Park's Vector representation, whose ellipticity increases with the severity of the fault and whose major axis orientation is associated to the faulty phase.

In order to obtain a deeper knowledge in the study of in-

ter-turn short-circuits occurrence, and also to acquire a generalised perspective of this phenomenon, it becomes necessary to develop a digital computer simulation model of three-phase power transformers, which is the scope of this paper.

During the last decades a number of transformer models have been developed, which have been used for the simulation of a wide range of steady-state and transient phenomena [4].

Most of the models were developed for use in the Electromagnetic Transients Program (EMTP) environment, namely, the Saturable Transformer Model [5], the Geometrical Model (GMTRAN) [6] and the Admittance Matrix Model (BCTRAN) [7-9], among others. The programming language of EMTP is FORTRAN-77. Such a program has a closed architecture and uses a large number of code lines to satisfy requirements ranging from low level data manipulation to the actual solution mathematics which eventually become diluted and almost impossible to visualise [10].

To overcome these difficulties, the three-phase transformer model presented in this paper was implemented in the MATLAB environment.

The three-phase transformer model described here is based on the physical concept of representing windings as mutually coupled coils; with this approach a three-phase transformer simply becomes a system of 6 coupled coils. The admittance matrix for the coupled coils can easily be derived from the commonly available test data [7].

2.- MODEL DEVELOPMENT

The complete circuit-model of a three-phase, three-leg, two-winding transformer, with a star-star winding connection is shown in Fig. 1. The inductive elements include mutual couplings among all windings. The resistances in series with the inductive elements represent the load losses, whereas parallel resistances represent the no-load losses [6]. The basic voltage and current equations of the transformer model can be written as follows:

$$[V] = [R] \cdot [I] + \frac{d}{dt}[L] \cdot [I] \tag{1}$$

$$[I_f] = [I] + [R_{fe}]^{-1} \cdot [V] \tag{2}$$

where, for the case of a three-phase, two-winding transformer, vectors $[V]$, $[I]$ and $[I_f]$ have six elements and the square matrices $[R]$, $[R_{fe}]$ and $[L]$ have 6×6 dimensions.
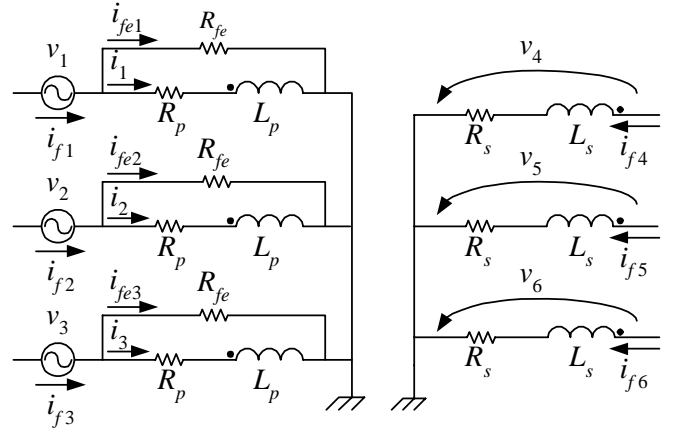

Fig. 1: Circuit-model of a three-phase two-winding transformer.

The diagonal matrix $[R]$ is easily obtained from the short-circuit test.

The precision of the model is strongly dependent on the accuracy of the inductance matrix $[L]$. The direct calculation of the $[L]$ matrix must be done with very high accuracy, especially if the exciting current is low, to make certain that the short-circuit inductances are still reasonably accurate [7-9]. To avoid the ill-conditioning problems of the $[L]$ matrix, an alternative formulation can be used, which consists in the inverse inductance matrix, $[L]^{-1}$, direct calculation.

In order to simplify the notation, the inverse inductance matrix will be expressed as a function of the modulus of the susceptance matrix, $[B]$:

$$[L]^{-1} = \omega \cdot [B] \tag{3}$$

being ω the angular frequency. As a first approach, the exciting current is ignored. Thus, matrix $[B]$ is built only from the short-circuit reactances, leading to [7-9]:

$$[B] = \begin{bmatrix} [X_{sc}]^{-1} & -[X_{sc}]^{-1} \\ -[X_{sc}]^{-1} & [X_{sc}]^{-1} \end{bmatrix} \text{ p.u.} \tag{4}$$

where $[X_{sc}]$ is the short-circuit reactance matrix. As in any other three-phase power system, the matrix $[X_{sc}]$ has the form:

$$[Z] = \begin{bmatrix} z_S & z_M & z_M \\ z_M & z_S & z_M \\ z_M & z_M & z_S \end{bmatrix} \text{ p.u.} \tag{5}$$

with:

$$z_S = \frac{1}{3}\left(z_h + 2 \cdot z_d\right) \text{ p.u.} \qquad (6)$$

and

$$z_M = \frac{1}{3}\left(z_h - z_d\right) \text{ p.u.} \qquad (7)$$

where z_S is the self impedance of a phase or leg and z_M is the mutual impedance among the three phases or legs. These self and mutual impedances are related to the direct and homopolar values, z_d and z_h respectively. With the values of the reactances of the direct and homopolar short-circuit tests inserted in (6) and (7), the matrix $[X_{sc}]$ is constructed using (5), and consequently $[B]$ is determined through (4).

It is now necessary to include the effect of the linear magnetisation reactance. From the values of the p.u. exciting currents and the p.u. no-load losses in the direct and homopolar excitation tests, it is possible to determine the direct and homopolar magnetising currents, i_{dm} and i_{hm}, respectively, which flow in the magnetisation reactance. The reciprocals of the direct and homopolar p.u. magnetising reactances (b_{dm} and b_{hm}, respectively), become:

$$b_{dm} = i_{dm} \text{ p.u. and } b_{hm} = i_{hm} \text{ p.u.} \qquad (8)$$

The magnetising matrix, $[B_m]$, is then calculated applying the general formulas of (5) through (7). This matrix is added to one of the diagonal sub-matrices of $[B]$ [9].

To include excitation losses, three coupled resistance branches must be added across the terminals of one side (R_{fe} in Fig. 1). Therefore, only the first diagonal 3×3 sub-matrix of the magnetising resistance matrix $[R_{fe}]$ has non-zero values. Using again the general formulas of (5) through (7), this sub-matrix is constructed, being their diagonal and off-diagonal elements obtained from the p.u. no-load losses [9].

After the conversion of p.u. values to physical values, the three-phase, three-leg, two-winding transformer model, with star-star winding connection, is complete.

3.- LINEAR MODEL VALIDATION

For the model validation a three-phase, three-leg transformer, of 6 kVA, 220/127 V, was used. The transformer has two-windings per phase on the primary and on the secondary side, having, one of each, been modified by addition of a number of tappings connected to the coils, for each of the three phases, allowing for the introduction of different percentages of shorted turns at several locations in the winding, for subsequent studies of inter-turn short-circuits occurrence.

The direct and homopolar short-circuit and excitation tests were performed with the transformer having a star-star winding connection, without neutral wire, in order to obtain the matrices elements.

The transformer digital model, with $Y/y0$ connections, was initially tested under no-load conditions. With the secondary side open-circuited, a symmetrical three-phase voltage system, with transformer nominal values, was applied to the primary side. The experimental primary exciting currents are shown in Fig. 2(a), which presents highly distorted waveforms. Since the transformer non-linear effects are neglected, the simulated primary currents are three-phase symmetrical vectors, Fig. 2(b).

The transformer model, with $Yn/yn0$ connections, was then tested under steady-state load conditions. A three-phase, star connected, balanced resistor of 107 Ω per phase, was coupled to the secondary side, being the primary side energised by a symmetrical three-phase voltage system, with transformer nominal values. The experimental and simulated
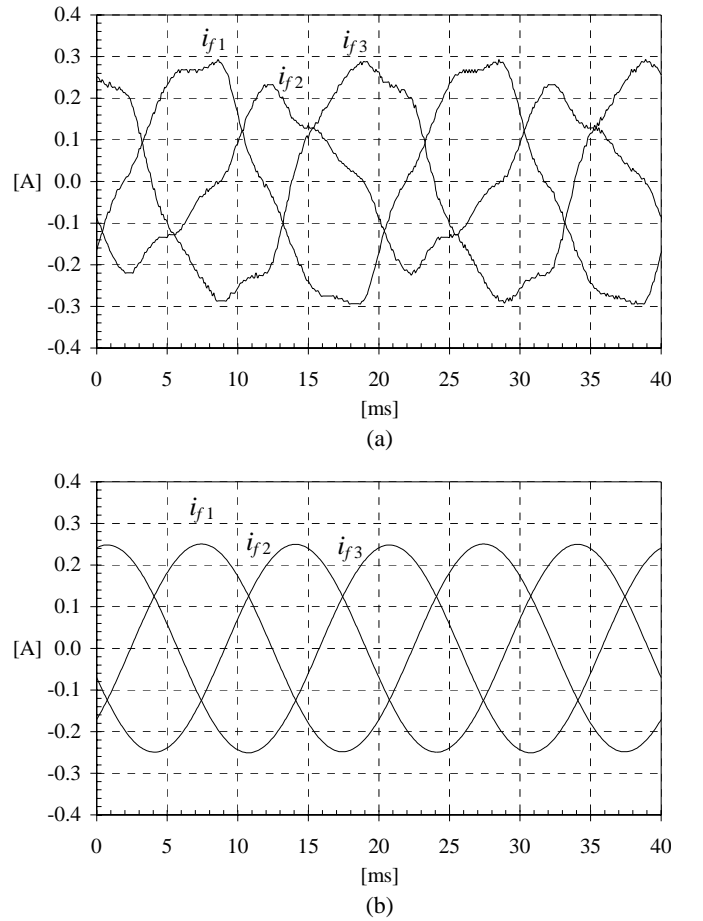

(a)

(b)

Fig. 2: No-load test waveforms: (a) primary side experimental currents; (b) primary side simulated currents.

primary side current waveforms are shown in Fig. 3(a) and Fig. 3(b), respectively. It is obvious the minor amplitude of the experimental central leg winding current in relation to the other phase currents, which results from the higher inductance of this leg [3]. Since the saturation effects have not been considered, so far, in the model, the simulation current waveforms do not present this characteristic, being imperative to include this phenomenon in the digital model.

4.- SATURATION EFFECTS

Core saturation behaviour can be determined from the open-circuit magnetisation curves of the transformer [11]. In each integration step, the magnetising matrix, $[B_m]$, is now updated using the inductances calculated from the magnetisation curves of the correspondent columns of the transformer. Since the homopolar magnetisation curve has an almost constant slope [12], it can be approximated to the case of a linear inductance, as previously. A direct 5 segment piece-wise linear inductance, for each leg of the transformer,
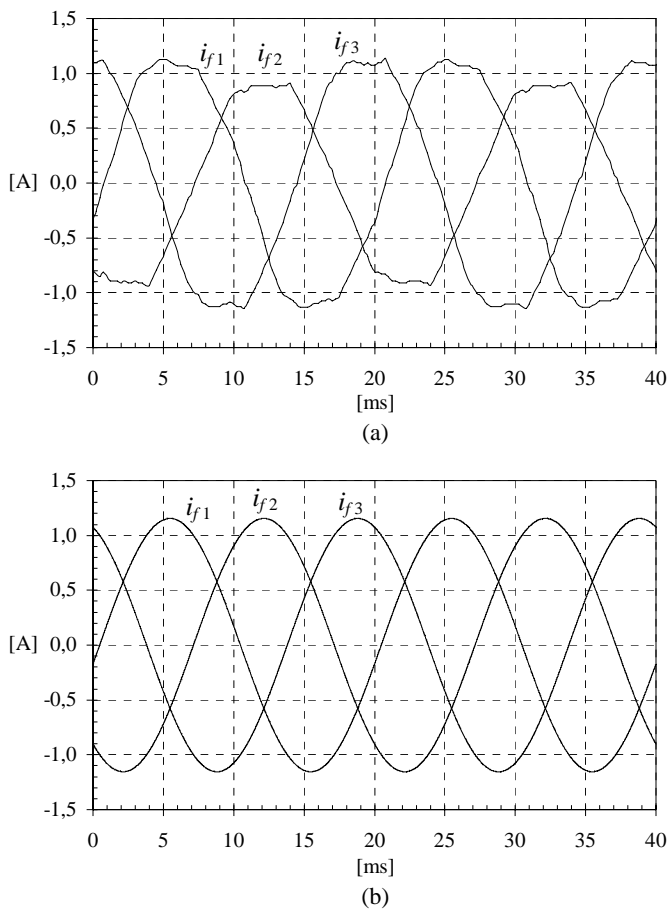
(a)

(b)

Fig. 3: Steady-state load conditions waveforms: (a) primary side experimental currents; (b) primary side simulated currents.

was then used to model the saturation effects, as presented in Fig. 4. The no-load losses are still modelled as a linear resistance.

The values of the magnetising currents are determined from the values of the exciting currents and no-load losses, for each point of the magnetisation curves. The $V_{RMS} - I_{RMS}$ points of the magnetisation curves are then converted to a flux–current maximum values characteristics in order to obtain the magnetising inductance [9].

Since the magnetisation curves are different for each column of the transformer, (5) is no longer valid, because the system becomes unbalanced. To overcome this difficulties the diagonal elements of $[B_m]$ were calculated, in each integration step, using the inductance of the correspondent direct magnetisation curve, being the off-diagonal elements calculated using the average of the correspondents direct magnetising inductances of the two columns involved. With this approximation, it is possible, however roughly, to take also into account the magnetic circuit asymmetries.

5.- NON-LINEAR MODEL VALIDATION

In the no-load test, the saturation effects and the asymmetries in the magnetic core of the transformer are outlined. For the $Y/y0$ winding connections, the no-load test was again simulated, with the saturation effects included in the model. The simulated primary currents (Fig. 5), in which the core saturation effects are obvious, are in relatively good agreement with the experimental ones (Fig. 2(a)).

The simulated direct magnetisation curves were obtained trough several excitation tests. The flux-current characteristics, plotted in Fig. 6, follow the real curves accurately.
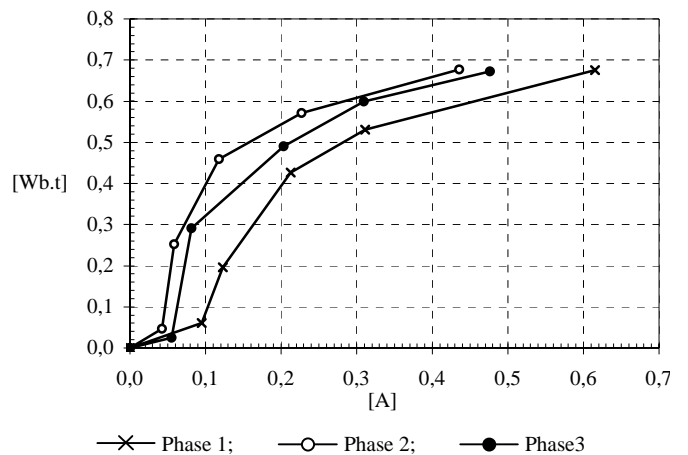
Fig. 4: Experimental direct magnetisation curves.
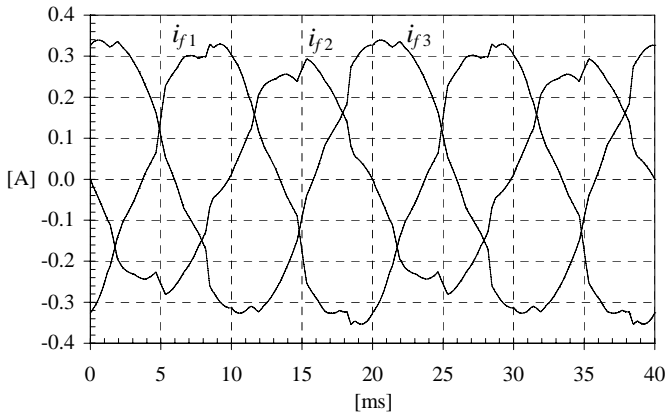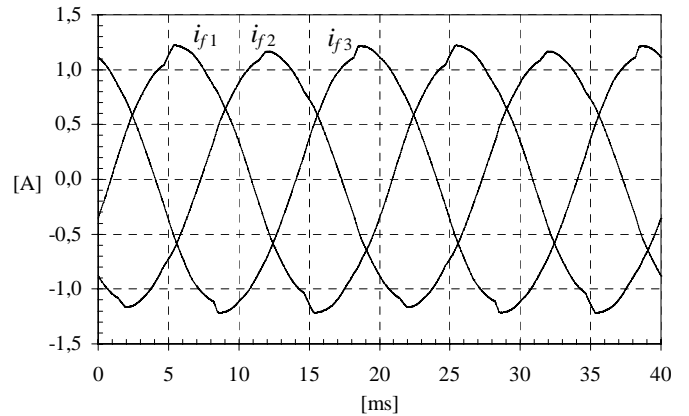
Fig. 5: Primary side simulated exciting currents.

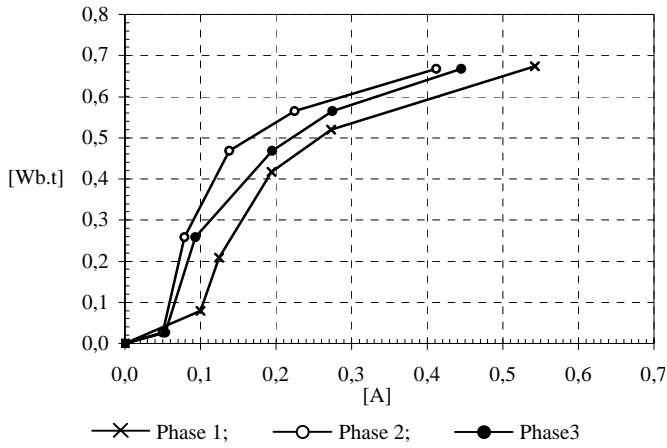Fig. 7: Primary side simulated currents under steady-state load conditions.

—✕— Phase 1;   —○— Phase 2;   —●— Phase3

Fig. 6: Simulated direct magnetisation curves.

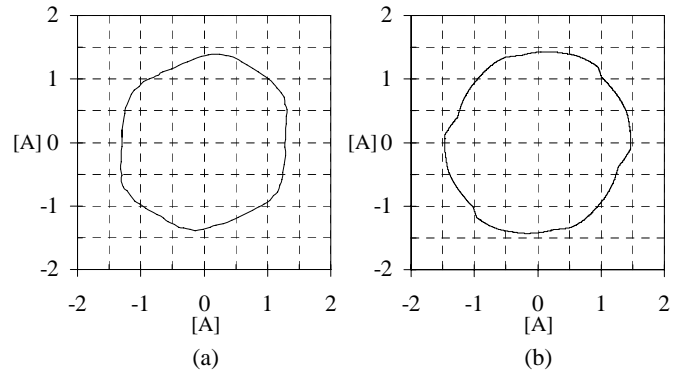(a)   (b)

Fig. 8: Primary side current Park's Vector patterns under steady-state load conditions: (a) experimental; (b) simulated.

The previous steady state load test was again simulated with the saturation effects included in the model. The simulated primary side current waveforms are shown in Fig. 7. In spite of the fact that the central leg winding current is slightly lower than the other phase currents, the simulated primary currents system is not so unbalanced as the experimental one, (Fig. 3(a)).

The experimental primary side current Park's Vector pattern, under steady state load conditions, shown in Fig 8(a), differs slightly from the circular locus expected for ideal conditions, due to, among others, the lower amplitude of the central leg primary winding current, with respect to the others primary windings currents. The corresponding simulated Park's Vector pattern, shown in Fig. 8(b), is in relatively good agreement with the experimental one.

6.- DISCUSSION

From the previously presented tests, it is obvious that the

transformer model behaviour is strongly dependent on the saturation effects, which are included in the algorithm with the aid of the symmetrical components theory, with a linear homopolar magnetising inductance and a piecewise direct inductance. However, an operating point variation in the homopolar magnetisation curve does not imply a corresponding variation in the direct magnetisation curve, and vice-versa, since the effects are separated, in the model, by the symmetrical components theory, which, in reality, are no longer uncoupled. These factors, in conjunction with the non-linear behaviour of the iron and hysteresis losses, are the major causes for the model inaccuracies, despite its satisfactory overall performance. Furthermore, the introduction of winding inter-turn short-circuits, using the approach suggested in [3], results in a more unbalanced impedance system, in which two coupled groups of asymmetries will exist, dealing with, among others, local saturation points in the magnetic circuit and several different leakage fluxes. The study of these phenomena with the symmetrical components theory is not suitable. Therefore, although the model behaviour is still reasonable accurate under normal operating conditions, in which the transformer is taken as a "black box", it can not be

used for winding inter-turn short-circuits analysis. For this type of studies, an alternative formulation must be considered, which should be based in direct phase quantities.

7.- CONCLUSIONS

This paper describes the development and implementation of a digital simulation model of a three-phase, three-leg, two-winding transformer. The elements of the model matrices can be calculated from the available test data concerning direct and homopolar, short-circuit and excitation tests, which constitute the main advantage of the model.

Both simulation and laboratory tests results demonstrate the adequacy of the model, under normal operating conditions. However, it seems that the model is not suitable to study winding inter-turn short-circuits occurrence, being an alternative formulation suggested.

Further work is currently in progress, using the suggested alternative formulation, concerning the modelling and simulation of the behaviour of three-phase transformers, under the influence of winding inter-turn short circuits. Additionally, some refinements of the new proposed model will be introduced, with the aim of dealing with the simultaneous occurrence of winding faults, unbalanced supply voltages, or even the surrounding presence of power electronics equipment.

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