Proceedings of the XXVII Iberian Latin American Congress on Computational Methods in Engineering
September 3 to 6, 2006 - Belém, Pará - BRAZIL.

DESIGN OF LAMINATED STRUCTURES USING PIEZOELECTRIC MATERIALS

José M. Simões Moita
José Herskovits
jmoita@ualg.pt
jose@optimize.ufrj.br
Universidade do Algarve, Escola Superior de Tecnologia, Campus da Penha, Faro, Portugal.
COPPE/UFRJ-Universidade Federal do Rio de Janeiro, Caixa Postal 68503, 21945-970, Rio de Janeiro, Brazil.
Cristóvão M. Mota Soares
Carlos A. Mota Soares
cmmsoares@alfa.ist.utl.pt
carlosmotasoares@dem.ist.utl.pt
IDMEC/IST-Instituto Superior Técnico, A, Rovisco Pais, 1049-001, Lisboa Codex, Portugal.

***Abstract**. Composite structures incorporating piezoelectric sensors and actuators are increasingly becoming important due to the offer of potential benefits in a wide range of engineering applications such as vibration and noise suppression, shape control and precision positioning. This paper presents a finite element formulation based on the classical laminated plate theory for laminated structures with integrated piezoelectric layers or patches, acting as actuators. The finite element model is a single layer triangular nonconforming plate/shell element with 18 degrees of freedom for the generalized displacements, and one electrical potential degree of freedom for each piezoelectric element layer or patch, which are surface bonded on the laminate. An optimization of the patches position is performed to maximize the piezoelectric actuators efficiency as well as, the electric potential distribution is search to reach the specified structure transverse displacement distribution (shape control). A gradient based algorithm is used for this purpose. The model is applied in the optimization of illustrative laminated plate cases, and the results are presented and discussed.*

***Keywords:** Finite Elements, Piezoelectric Actuators, Optimization.*

1. INTRODUCTION

Composite structures incorporating piezoelectric sensors and actuators are increasingly becoming important due to the offer of potential benefits in a wide range of engineering applications such as vibration and noise suppression, shape control and precision positioning. The optimal location of the piezoelectric patches, in order to maximize the piezoelectric actuators efficiency, is one of the tasks to be considered in the analysis of the piezolaminated structures.

A significant number of works in the fields of analysis, control and optimization in composite structures integrating piezoelectric material had been carried out. One of the pioneering works is due to Allik and Hughes (1970) who carried out the variational formulation and developed a solid finite element for vibration analysis. Tzou and Tseng (1990) presented a finite element formulation for plates and shells containing integrated distributed piezoelectric sensors and actuators applied to control advanced structures. In more recent works Chen et al. (1996) developed a finite element based on the first order displacement field for dynamic analysis of plates where the vibration active control is obtained with the actuators potential been given by an amplified signal of the sensors potential which arises an active damped system, and Samanta et al. (1996) developed an eight-nodded finite element for the active vibration control of laminated plates with piezoelectric layers acting as distributed sensors and actuators. The active control capability is studied using a simple algorithm with negative velocity feedback. Also Lam et al. (1997) and Moita et al. (2005) developed finite element models based on the classical laminated theory and higher order shear deformation theory, respectively, for the active control of composite plates containing piezoelectric sensors and actuators using the Newmark method, Bathe (1982), to calculate the dynamic response of laminated structures. Active vibration control is obtained through actuators potential, which is given by an amplified signal of the sensors potential.

Most of past work in the area of adaptive structures has focused on the analysis of structures with sensors and actuators, and the corresponding associate control system. Very few works have focused on the development of methodologies for the optimization of laminated structures incorporating sensors and actuators, to enhance their performance. A model for the optimization of the induced-strain actuator location and configuration for active vibration control had been proposed by Liang et al. (1995). Batra and Liang (1997) used a three-dimensional linear theory of elasticity to find the optimal location of an actuator on a simple-supported rectangular laminated plate with embedded PZT layers. The optimal design is obtained by fixing the applied voltage and the size of the actuator and moving it around in order to find the maximum out-of-plane displacement. More recently, Franco Correia et al. (2000) presented refined finite element models based on higher order displacement fields applied to the optimal design of laminated composite plates with embedded or surface bonded piezoelectric actuators and sensors, and Achuthan et al. (2001) study the shape control of composite laminated beams with nonlinear piezoelectric patch actuators developing a finite element model.

In the present work a discrete model based on the classical laminated plate theory is developed. A three-nodded flat triangular finite element is used, with 18 mechanical degrees of freedom and one electric degree of freedom per piezoelectric layer or patch of the finite element. The optimal placement of piezoelectric actuators patches as well as the electric potential distribution to perform the structure shape control is carried out using a gradient-based algorithm methodology, in association with algorithm FAIPA, Herskovits et al. (2005). The model is applied in the optimization of illustrative laminated plate cases, and the results are presented and discussed.

2. PIEZOELECTRIC LAMINATES. CONSTITUTIVE EQUATIONS.

Assuming that a piezoelectric composite plate consists of several layers, including the piezoelectric layers, the constitutive equation for an orthotropic layer of the laminate substrate, is

$$\overline{s} = \overline{\mathbf{Q}}\,\overline{e} \tag{1}$$

and the constitutive equations of a deformable piezoelectric material, coupling the elastic and the electric fields are given by, Tiersten (1969)

$$\overline{s} = \overline{\mathbf{Q}}\,\overline{e} - \overline{\mathbf{e}}\,\overline{\mathbf{E}} \tag{2}$$

$$\overline{\mathbf{D}} = \overline{\mathbf{e}}^{T}\,\overline{e} + \overline{\mathbf{p}}\,\overline{\mathbf{E}} \tag{3}$$

where $\overline{s} = [\sigma_{xx}\ \sigma_{yy}\ \sigma_{xy}]^T$ is the elastic stress vector and $\overline{e} = [\varepsilon_{xx}\ \varepsilon_{yy}\ \gamma_{xy}]^T$ the elastic strain vector, which components are associated with the displacement field of the classical laminated plate theory, $\overline{Q}$ the elastic constitutive matrix, $\overline{e}$ the piezoelectric stress coefficients matrix, $\overline{\mathbf{E}} = [\overline{E}_x\ \overline{E}_y\ \overline{E}_z]^T$ the electric field vector, $\overline{\mathbf{D}} = [\overline{D}_x\ \overline{D}_y\ \overline{D}_z]^T$ the electric displacement vector and $\overline{p}$ the dielectric matrix, in the element local system (*x,y,z*) of the laminate. $\overline{Q}_{ij}, \overline{e}_{ij}, \overline{p}_{ij}$ are functions of ply angle α for the k^{th} layer, and are given explicitly in Reddy (2004).

The electric field vector is the negative gradient of the electric potential ϕ, which is assumed to be applied in the thickness t_k direction, where it can vary linearly, i.e.

$$\overline{\mathbf{E}} = -\nabla\phi \qquad ; \qquad \overline{\mathbf{E}} = \{0\quad 0\quad E_z\}^T \tag{4}$$

where

$$E_z = -\phi / t_k \tag{5}$$

The constitutive equations (2) and (3) can be written in the form

$$\hat{s} = \begin{Bmatrix} \overline{\sigma} \\ \overline{D} \end{Bmatrix} = \begin{bmatrix} \overline{Q} & \overline{e} \\ \overline{e}^T & -\overline{p} \end{bmatrix} \begin{Bmatrix} \overline{e} \\ -\overline{E} \end{Bmatrix} = \hat{\mathbf{C}}\,\hat{e} \tag{6}$$

3. FINITE ELEMENT FORMULATION

The flat triangular finite element model has three nodes and six degrees of freedom per node, the displacements u_0, v_0, w_0 and rotations $\theta_x, \theta_y, \theta_z$. The introduction of fictitious stiffness coefficients K_{θ_z}, corresponding to rotations θ_z, to eliminate the problem of a singular stiffness matrix, for which the elements are coplanar or near coplanar, is required.. The element local displacements, slopes and rotations are expressed in terms of nodal variables through shape functions $\mathbf{N}_i$ given in terms of area co-ordinates, Zienckiewicz (1977). The displacement field can be represented in matrix form as

$$\mathbf{u} = \mathbf{Z}\,(\sum_{i=1}^{3} \mathbf{N}_i\ \mathbf{d}_i) = \mathbf{Z}\,\mathbf{N}\,\mathbf{a} \qquad ; \qquad \mathbf{d} = \sum_{i=1}^{3} \mathbf{N}_i\ \mathbf{d}_i = \mathbf{N}\,\mathbf{a} \tag{7}$$

with

$$\mathbf{d}_i = \{u_{0i}\ v_{0i}\ w_{0i}\ \theta_{xi}, \theta_{yi}, \theta_{zi}\}^T \qquad ; \qquad \mathbf{a} = \{\mathbf{d}_1 \quad \mathbf{d}_2 \quad \mathbf{d}_3\}^T \tag{8}$$

$$\mathbf{Z} = \begin{bmatrix} 1 & 0 & 0 & 0 & -z & 0 \\ 0 & 1 & 0 & z & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{9}$$

and the strain field as follows

$$\{\bar{\varepsilon}\} = \{\bar{\varepsilon}_m + z\,\bar{\varepsilon}_b\} = \left\{\sum_{i=1}^{3} \left(\mathbf{B}_i^m + z\,\mathbf{B}_i^b\right)\mathbf{d}_i\right\} = \mathbf{B}^{mb}\ \mathbf{a} \tag{10}$$

The electric field is given by

$$\mathbf{E} = -\mathbf{B}^{\phi}\ \phi \tag{11}$$

where $\mathbf{B}^{\phi}$ is the electric field – potential relations matrix given by

$$\mathbf{B}^{\phi} = \begin{bmatrix} 1/t_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1/t_{NPL} \end{bmatrix} \tag{12}$$

The static equations of a laminated composite plate can be derived from the virtual work principle, which is given as follows:

$$\int_{t_1}^{t_2} \left\{ \sum_{K=1}^{N} \left(\int_{A^e} \int_{h_{k-1}}^{h_k} \delta\,\hat{e}^T\ \hat{C}_k\ \ {}_t\hat{e}^L\ dz\ \ dA^e \right) - \left(\int_V \mathbf{f}\ \delta\mathbf{u}\ dV + \right.\right.$$

$$\left.\left. \int_S \mathbf{T}\ \delta\mathbf{u}\ dS + \sum_i \mathbf{F}_i\ \delta\mathbf{u}_i + \int_S \mathbf{Q}\ \ \delta\phi\ dS \right) \right\} dt = 0 \tag{13}$$

Entering the equations (7) to (11) into equation (13), we have

$$\int_{t_1}^{t_2}\left[\sum_{K=1}^{N}\left(\int_A \int_{h_{k-1}}^{h_k} \delta\begin{Bmatrix} a \\ \phi \end{Bmatrix}^T \begin{bmatrix} \mathbf{B}^{mb} & 0 \\ 0 & \mathbf{B}^{\phi} \end{bmatrix}^T \begin{bmatrix} \overline{\mathbf{Q}} & \overline{\mathbf{e}} \\ \overline{\mathbf{e}}^T & -\overline{\mathbf{p}} \end{bmatrix} \begin{bmatrix} \mathbf{B}^{mb} & 0 \\ 0 & \mathbf{B}^{\phi} \end{bmatrix} \begin{Bmatrix} a \\ \phi \end{Bmatrix} dz\, dA \right. \right. -$$

$$\left. + \int_V \delta\{a\}^T \mathbf{N}^T \mathbf{f}\, dV + \int_S \delta\{a\}^T \mathbf{N}^T \mathbf{T}\, dS + \delta\{a\}^T \mathbf{F}_c + \int_S \mathbf{Q}\, \delta\{\phi\}\, dS \right] dt = 0 \tag{14}$$

To the first and second terms of first member of Eq. (13), corresponds the element stiffness, which are defined by

$$\mathbf{K} = \begin{bmatrix} K_{uu} & K_{u\phi} \\ K_{\phi u} & K_{\phi\phi} \end{bmatrix} = \sum_{K=1}^{N} \int_A \int_{h_{k-1}}^{h_k} \begin{bmatrix} \mathbf{B}^{mb} & 0 \\ 0 & \mathbf{B}^{\phi} \end{bmatrix}^T \begin{bmatrix} \overline{\mathbf{Q}} & \overline{\mathbf{e}} \\ \overline{\mathbf{e}}^T & -\overline{\mathbf{p}} \end{bmatrix}_k \begin{bmatrix} \mathbf{B}^{mb} & 0 \\ 0 & \mathbf{B}^{\phi} \end{bmatrix} dz\ dA \tag{15}$$

To the third term of Eq. (13), corresponds the external mechanical force vector $\mathbf{F}_{ext}^{mec}$ and the applied electric charge vector F^{ele}.

The element stiffness matrix as well as external load vector are initially computed in the local coordinate system attached to the element. To solve general structures, local - global transformations are needed, Zienckiewicz (1977). After these transformations the assembled system of equations for laminated structures with integrated piezoelectric layers or patches, acting as actuators, is:

$$\begin{bmatrix} K_{uu} & K_{u\phi} \\ K_{\phi u} & K_{\phi\phi} \end{bmatrix} \begin{Bmatrix} \mathbf{q} \\ \phi \end{Bmatrix} = \begin{Bmatrix} F_{ext}^{mec} \\ F^{ele} \end{Bmatrix} \tag{16}$$

This system of equations can be written in the following form:

$$[K_{uu}]\{q\} = \{F_{ext}^{mec} - K_{u\phi}\, \phi\} \tag{17}$$

$$[K_{\phi u}]\{q\} + [K_{\phi\phi}]\{\phi\} = \{F^{ele}\} \tag{18}$$

In practice, voltage is specified as input to the actuators, and we than write:

$$[K_{uu}]\{q\} = \{F_{ext}^{mec} - F^{(A)}\} \tag{19}$$

where $\{F^{(A)}\} = \{-K_{u\phi}\, \phi\}$ is an additional force due to the voltage applied to the actuators.

4. OPTIMAL DESIGN

A general structural optimization problem can be stated as

$$\mathit{min}\ \{\Omega(\mathbf{b})\}$$

$$\begin{aligned} \textit{subject to:} \quad & b_i^l \le b_i \le b_i^u & i = 1,\dots,n \\ & \Psi_j(\mathbf{q},\mathbf{b}) \le 0 & j = 1,\dots,m \end{aligned} \tag{20}$$

where $\Omega(\mathbf{b})$ is the objective function, $\mathbf{b}$ is the vector of design variables b_i, $\Psi_j(\mathbf{q},\mathbf{b})$ are the *m* inequality behavioral constraint equations, b_i^l and b_i^u are respectively, the lower and upper limits of the design variables and n is the total number of design variables.

If the objective function and/or the constraint equations are continuous functions of the design variables, mathematical programming techniques, Herskovits et al. (2005), requiring only the computation of $\Omega(\mathbf{b})$, $\Psi_j(\mathbf{q},\mathbf{b})$ and their gradients, provides a general, flexible and efficient formulation for engineering design problems. Here the optimization problems are solved by using a feasible directions non-linear interior point algorithm FAIPA, Herskovits et al. (2005).

In this work, first we search for the optimal core lamination sequence, which leads to the minimum strain energy of the plate

$$U = \frac{1}{2} q^T K_{uu} \, q \tag{21}$$

$$\frac{dU}{d\alpha_i} = -\frac{1}{2} q^T \frac{\partial K_{uu}}{\partial \alpha_i} \, q \tag{22}$$

For the optimal location of the piezoelectric patches we consider that the patches can only assume the positions corresponding to the finite element mesh discretization. The optimal location of the piezoelectric actuator discrete patches is to be found in order to achieve maximum piezoelectric actuator performance. The present approach assumes that the shape of plate structure is described by the transverse displacement w. Let w_{d_i} and w_{a_i}, respectively, represent the desired transverse displacement and the actual transverse displacement corresponding to node i. In order to control the shape of the plate structure, we minimize an objective function defined by the mean-squared error between the desired and the achieved shape, defined by the transverse displacement w in a certain number of nodes, np:

$$\Omega(\mathbf{b}) = \sum_{i=1}^{np} \left(w_{d_i} - w_{a_i} \right)^2 \tag{23}$$

The positional optimization is here obtained by referring the patch position by its thickness, i.e., we make the similitude between the position relevancy and the thickness of the piezoelectric patches The derivative of objective function in order to general design variables are:

$$\frac{d\Omega}{db} = -\sum_{i=1}^{np} 2 \left(w_{d_i} - w_{a_i} \right) \frac{\partial w_{a_i}}{\partial b} \tag{24}$$

In static type situations, where the vectorial distance to the outer surface of piezoelectric patches are taken as the design variables, we have:

$$\frac{\partial w_{a_i}}{\partial h_0} = [K_{uu}]^{-1} \left(\frac{\partial \{F^A\}}{\partial h_0} - \frac{\partial [K_{uu}]}{\partial h_0} \{q\} \right) \tag{25}$$

Here the stiffness matrix is constant, and the force due to the voltage applied to the actuators was defined as $\{F^{(A)}\} = \lfloor K_{u\phi} \rfloor \{\phi\}$ where

$$\lfloor K_{u\phi} \rfloor = \int_A \left((h_1 - h_0)\mathbf{B}^m \ \bar{\mathbf{e}}\ \mathbf{B}^\phi + \frac{h_1^2 - h_0^2}{2}\ \mathbf{B}^b\ \bar{\mathbf{e}}\ \mathbf{B}^\phi \right) dA \tag{26}$$

Then, being $\mathbf{B}^\phi = \dfrac{1}{h_1 - h_0}$ we have

$$\{F^{(A)}\} = \int_A \left(\mathbf{B}^m + \frac{h_1 + h_0}{2} \mathbf{B}^b \right) \bar{\mathbf{e}}\, dA\, \{\phi\} \tag{27}$$

and its derivative with respect to the vectorial distance h_0 is given by:

$$\frac{\partial \{F^{(A)}\}}{\partial h_0} = \int_A \frac{1}{2} \mathbf{B}^b\, \bar{\mathbf{e}}\, dA\, \{\phi\} \tag{28}$$

Thus, finally we have

$$\frac{d\Omega}{dh_0} = -\sum_{i=1}^{np} 2\left(w_{d_i} - w_{a_i}\right)[K_{uu}]^{-1} \int_A \frac{1}{2}\mathbf{B}^b\, \bar{\mathbf{e}}\, dA\, \{\phi\} \tag{29}$$

Also the optimal voltage distribution that minimizes de previous objective function can be evaluated. For this type of optimization, the design variables are the voltages of each piezoelectric patch. As we have

$$\frac{d\Omega}{d\phi} = -\sum_{i=1}^{np} 2\left(w_{d_i} - w_{a_i}\right)\frac{\partial w_{a_i}}{\partial \phi} \tag{30}$$

and

$$[K_{uu}]\{q\} = \lfloor K_{u\phi} \rfloor \{\phi\} \tag{31}$$

$$\{q\} = [K_{uu}]^{-1} \lfloor K_{u\phi} \rfloor \{\phi\} \tag{32}$$

the analytical sensitivity of component q_j of displacement vector is given by, Moita et al. (2000)

$$\frac{\partial q_j}{\partial \{\phi\}} = Z^T\, [K_{uu}]^{-1} \lfloor K_{u\phi} \rfloor \tag{33}$$

where $Z = \dfrac{\partial q_j}{\partial q_k} = \begin{cases} 0 \text{ if } k \neq j \\ 1 \text{ if } k = j \end{cases}$ (34)

Thus, the analytical sensitivity of a specific displacement of vector $\{q\}$, $w_{a_i} = q_j$, is given by:

$$\frac{\partial w_{a_i}}{\partial \{\phi\}} = Z^T \left[K_{uu}\right]^{-1} \left[K_{u\phi}\right] \tag{35}$$

Thus the derivatives of objective function in order to the design variables are obtained as follows:

$$\frac{d\Omega}{d\,\phi} = -\sum_{i=1}^{np} 2\left(w_{d_i} - w_{a_i}\right) Z^T \left[K_{uu}\right]^{-1} \left[K_{u\phi}\right] \tag{36}$$

5. NUMERICAL APPLICATIONS

5.1 Simply-supported square plate subjected to uniform distributed load.

A simply-supported square ($a \times a$) laminated plate, having the initial lamination sequence of $[0^o/90^o/0^o]$, integrating piezoelectric actuator patches made of PZT, bonded on upper surface, is considered. The material properties of the substrate layers are: $E_1 = 172.5\,\text{GPa}$, $E_2 = 6.9\,\text{GPa}$, $G_{12} = 3.45\,\text{GPa}$, $\nu_{12} = 0.25$. The material and piezoelectric properties of PZT are: $E_1 = E_2 = 63$ GPa, $G_{12} = 24$ GPa, $\nu_{12} = 0.30$, $e_{31} = e_{32} = -22.86\,\text{C/m}^2$, $p_{33} = 1.5 \times 10^{-8}$ F/m. The side dimension is a = 0.18 m and the thickness of the substrate layers and PZT are 0.002 m and 0.0001 m, respectively. The plate is modeled by a (6x6) element mesh, 72 triangular elements. The plate, Figure 1, is divided in 9 groups of elements, each one with 8 triangular elements. Each pair of contiguous triangular elements forms a piezoelectric patch. By using a feasible directions non-linear interior point algorithm, first we search for the optimal core lamination sequence, which leads to the minimum strain energy of the plate at any load level. The optimal lamination sequence is found to be $[45^o/-45^o/45^o]$, which is in close agreement with an available alternative analytical solution, Pederson (1987).

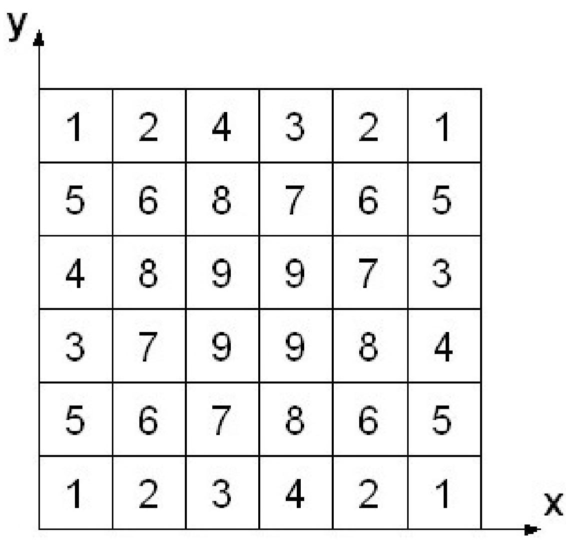

Figure 1. Plate divided into 9 groups of elements.

Next we pretend to investigate the optimal position of the piezoelectric actuators patches, in order to maximize the control of the plate deformation shape. The optimal positions of the patches are obtained as represented in Figure 2, i.e. the central plate elements. In order to control the shape of the plate, making use of all patches, the optimal voltage distribution is given in Table 1.

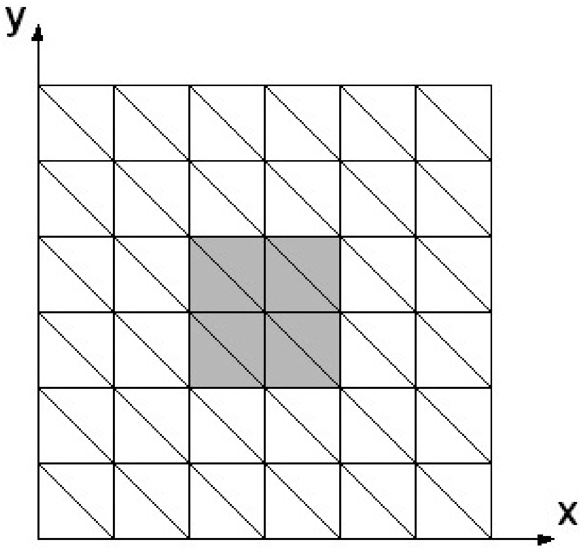

Figure 2. Optimal patch locations

Table.1. Optimal voltage distribution

Group nº	1	2	3	4	5	6	7	8	9
Voltage	-169.1	-139.6	-186.6	-160.6	-139.6	-273.3	-366.4	-436.9	-531.5

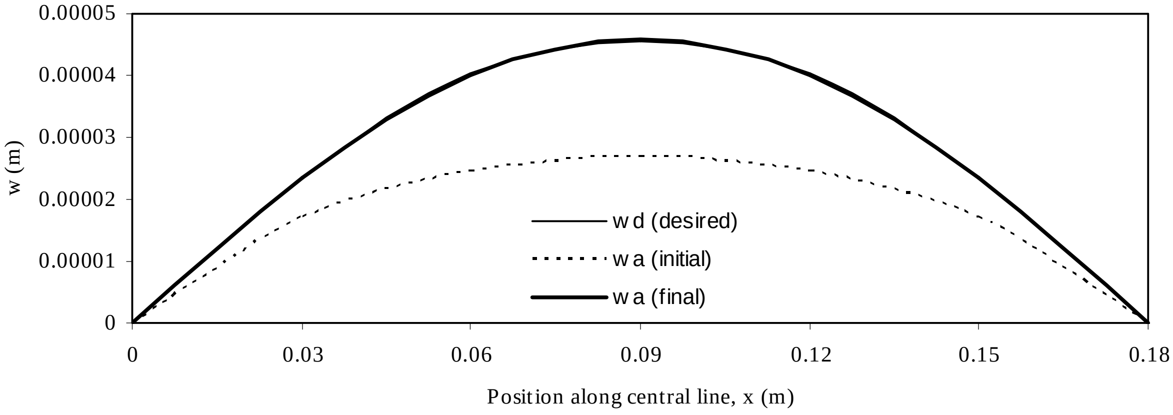

Figure 3. Central line deflections.

In Figure 3 are shown the central line deflection for mechanical load of 15000 N/m^2 (desired curve), the central line deflection for initial actuation with all the patches with same voltage (-180 V), and the achieved central line deflection obtained with the voltage distribution given in Table 1.

5.2 Rectangular panel subjected to uniform distributed load.

A rectangular plate made of S-Glass/Epoxy with the lamination sequence of [0º/45º/-45º/0º] with piezoceramic actuator patches PC5K (lead zirconate titanate), is considered. The plate is simply supported along the shorter sides and free at the longer sides, and is subjected to a uniformly distributed load of 1000 N/m^2. The material properties of the S-Glass/Epoxy layers are $E_1 = 55\,\text{GPa}$, $E_2 = 16\,\text{GPa}$, $G_{12} = 7.6\,\text{GPa}$, $\nu_{12} = 0.25$. The thickness of outside layers is 1 mm each and the thickness of inside layers is 0.5 mm each. The material and electric properties of PC5K are $E_1 = E_2 = 60.24$ GPa, $G_{12} = 23$ GPa, $\nu_{12} = 0.31$, $e_{31} = e_{32} = -26.72\,\text{C/m}^2$, $p_{33} = 5.04\,\text{x}\,10^{-8}$ F/m. The thickness of the piezoceramic patches is 0.5 mm. The plate is divided in 7 groups of elements. Here two cases are considered: in the first one for each group of elements correspond an actuator patch, and for the second case only three actuator patches are bonded to the core of the laminated as represented in Figure 4.

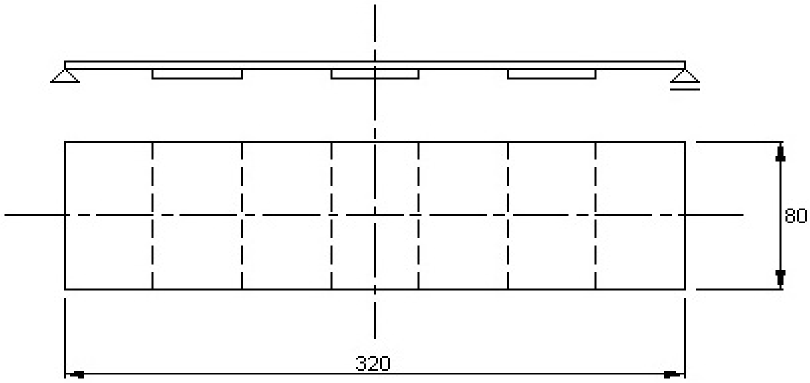

Figure 4. Rectangular panel with three piezoelectric actuator patches

The shape of the plate is defined by the transversal displacements w_i along the longer central line of the plate. The objective is to find the appropriate electric voltages that should be applied to the seven or three actuators patches in order to minimize the mean-squared error between the actual shape and the desired shape of the plate. The results obtained for the first case are shown in Figure 5 where the central line deflection corresponding to a mechanical load of 500 N/m^2 (desired curve), and the achieved central line deflection obtained with the voltage distribution given in Table 2, are represented.

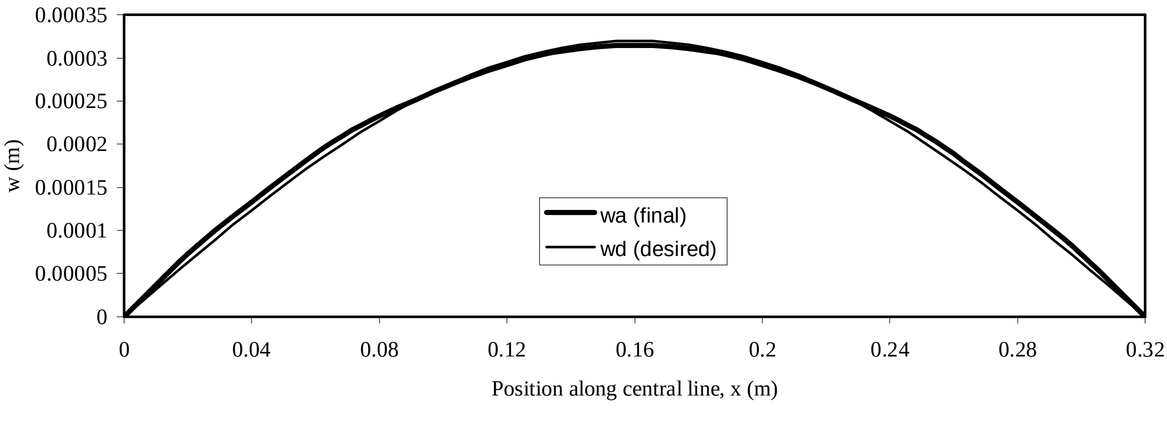

Figure 5. Central line deflections

Table 2. Voltage distribution

Group nº	1	2	3	4	5	6	7
Voltage	-115.1	-78.5	-167.0	-222.4	-167.0	-78.5	-115.1

For the second case, in Figure 6 are shown the central line deflection corresponding to a mechanical load of 250 N/m^2 (desired curve), and the achieved central line deflection obtained with the voltage distribution given in Table 3.

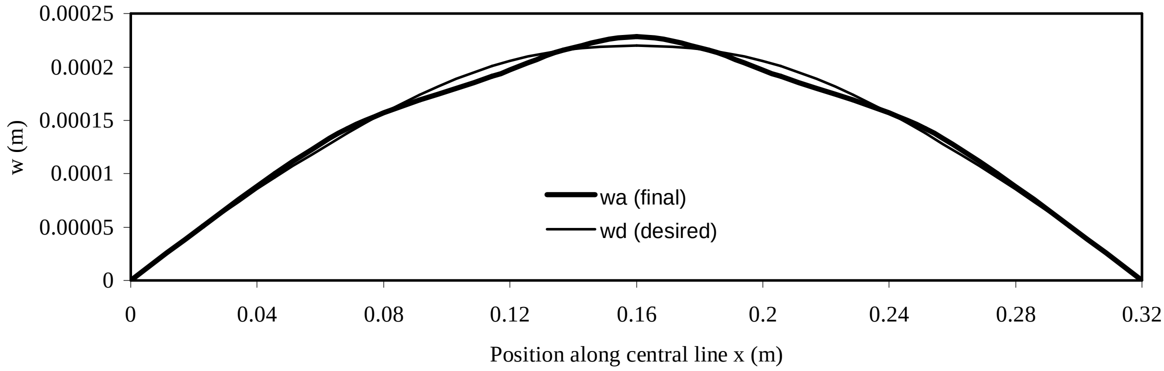

Figure 6. Central line deflections

Table 3. Voltage distribution for three actuators patches

Group nº	1	2	3
Voltage	-171.5	-289.8	-171.5

6. CONCLUSIONS

The shape control capability of composite structures covered with piezoelectric layers or patches is investigated, using the finite element method. A finite element based on the Kirchhoff classical theory, has been used. The present model has been validated in Moita et al**.** (2000), where the solutions for deflection and sensed voltage in a bimorph beam, are compared with the solutions obtained by other authors. The core optimization had been performed in order to minimize the strain energy of the plate. Also the patch position and the structure shape optimizations had been performed in order to maximize the effect a defined set of actuators. In all of these optimizations a gradient-based algorithm had been used. From the Fig. 5 and Fig. 6 we can observe that the case of patches covering an entire layer give a much better shape control.

Acknowledgments

The authors thank the financial support of POCTI/FEDER, Fundação para a Ciência e Tecnologia (FCT), POCI/EME/56316/2004, POCI(2010)/FEDER, FP6-2003-NMD-TI3/G3, Contract 13517- Project CASSEM, and CNPq (Brazil) and FARPEJ (Brazil)

REFERENCES

Achuthan A., Keng A. K., & Ming W. C., 2001. Shape control of coupled nonlinear piezoelectric beams. *Smart Material Structures*, vol. 10, pp. 914-924.

Allik, H., Hughes T., 1970. Finite Element Method for Piezoelectric Vibration. *International Journal for Numerical Methods Engineering*, vol. 2, pp. 151-157.

Bathe K. J., 1982. *Finite Element Procedures in Engineering Analysis*. Prentice-Hall Inc, Englewood Cliffs, New Jersey.

Batra R. C., Liang X. Q., 1997. The vibration of a rectangular laminated elastic plate with embedded piezoelectric sensors and actuators. *Computers and Structures,* vol. 63, n. 2, pp. 203-216.

Chen C. Q., Wang X. M., & Shen Y. P., 1996. Finite element approach of vibration control using self-sensing piezoelectric actuators. *Computers and Structures*, vol. 60, n. 3, pp. 505-512.

Franco Correia V. M., Mota Soares C. M., & Mota Soares C. A., 2000. Refined models for the optimal design of adaptive structures using simulated annealing. *Composite Structures*, vol. 54, pp. 161-167.

Herskovits J., Mappa P., Goulard E., & Mota Soares C.M., 2005. Mathematical programming models and algorithms for engineering design optimization. *Computer Methods in Applied Mechanics and Engineering*, vol. 194, pp 3244-3268.

Lam K. Y., Peng X. Q., Liu G. R., & Reddy J. N., 1997. A finite element model for piezoelectric composite laminates. *Smart Material Structures*, vol. 6, pp. 583-591.

Liang C., Sun F. P., & Rogers C.A., 1995. Determination of design of optimal actuator location and configuration based on actuator power factor. *Journal of Intelligent Material Systems and Structures*, vol. 6, pp. 456-464.

Moita, J. S., Barbosa, J. I, Mota Soares**,** C. M, & Mota Soares, C. A., 2000. Sensitivity Analysis and Optimal Design of Geometric Non-Linear Laminated Plates and Shells. *Computers and Structures*, vol. 76, pp. 407-420.

Moita J. S., Mota Soares C. M., & Mota Soares C. A., 2005. Active control of forced vibrations in adaptive structures, using a higher order model. *Composite Structures*, vol. 71, pp. 349-355.

Reddy J. N., 2004. *Mechanics of laminated composite plates and Shells*. CRC Press, 2 nd Edition, New York.

Pedersen P., 1987. On sensitivity analysis and optimal design of specially orthotropic laminates. *Eng. Optimization*, vol. 11, pp. 305-316.

Samanta B., Ray M. C., & Bhattacharyya R., 1996. Finite element model for active control of intelligent structures. *AIAA Journal*, vol. 34, n. 9, pp. 1885-1893.

Tiersten H. F., 1969. *Linear Piezoelectric Plate Vibrations*. Plenum Press, New York.

Tzou H. S., Tseng C. I., 1990. Distributed Piezoelectric Sensor/Actuator Design for Dynamic Mesuarement/Control of Distributed Parametric Systems: A Piezoelectric Finite Element Approach. *Journal of Sound and Vibration*, vol. 138, pp. 17-34.

Zienckiewicz O. C., 1977. *The finite element method in engineering sciences*. McGraw-Hill, 3rd ed., London.