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## HIGHER ORDER MODEL FOR ANALYSIS OF MAGNETO-ELECTRO-ELASTIC PLATES

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**Abstract.** *In this paper is presented an higher-order model for static and free vibration analyses of magneto-electro-elastic plates, which allows the analysis of thin and thick plates. The finite element model is a single layer triangular plate/shell element with 24 degrees of freedom for the generalized mechanical displacements. Two degrees of freedom are introduced per each element layer, one corresponding to the electrical potential and the other for magnetic potential. Solutions are obtained for different laminations of the magneto-electro-elastic plate, as well as for the purely elastic plate as a special case.*

## 1 INTRODUCTION

In the recent years the study of smart structures has attracted significant researchers. The use of smart materials, such as piezoelectric and/or piezomagnetic materials, in the form of layers or patches embedded and/or surface bonded on laminated composite structures, can provide the so-called adaptive structures. In these cases, the structure behaviour is not defined by not only geometry and material properties, but also electric and magnetic fields that are applied to the structures, because the piezomagnetic materials have the ability of converting energy from one form to the other (among magnetic, electric, and mechanical energies).

Static and dynamic analysis of structures with piezoelectric material had been the objective of a considerable number of studies. However, magneto-electro-elastic structures only recently had been aim of investigation. Pan [1], and Pan and Heyliger [2], presented an exact solution for the analysis of simply supported magnet-electro-elastic laminated plates, regarding static and free vibration behaviour, respectively. Wang et al. [3] developed a mixed analytical solution for magneto-electro-elastic plates. Lage et al. [4, 5] developed a partial mixed layerwise finite element model for the static and free vibration analysis of magneto-electro-elastic laminated plate structures. Chen et al. [6] developed an analytical solution for free vibration problem of simply supported rectangular plates with general inhomogeneous material properties along the thickness direction. Ramirez et al. [7] present an approximate solution for the free vibration problem of two-dimensional magneto-electro-elastic laminates, by combining a discrete layer approach with the Ritz method.

In this paper we present a finite element model, based in the third-order shear deformation theory, for static and free vibration analyses of plate structures integrating piezoelectric/piezomagnetic layers. This model using a higher order displacement field, allows to take into account transverse shear stresses, and is suitable for the analysis of highly anisotropic structures ranging from high to low length-to-thickness ratios. This approach leads to better displacements results when compared to classical Kirchoff or even first order shear deformation theories based models [8]. A simple and efficient three-node triangular fat plate element is used. The formulation introduces one electric potential and one magnetic potential degree of freedom for each piezoelectric and piezomagnetic layer of the finite element. Results obtained with the proposed finite element model are presented and discussed for static and free vibration cases.

## 2 THIRD-ORDER SHEAR DEFORMATION PLATE THEORY. DISPLACEMENTS AND STRAINS.

The assumed displacement field, for the numerical higher order finite element model, is a third order expansion in the thickness coordinate for the in-plane displacements and a constant transverse displacement, conjugated with the condition that the transverse shear stresses vanish on the top and bottom faces, which is equivalent to the requirement that the corresponding strains be zero on these surfaces [8].

$$\begin{aligned}
 u(x, y, z) &= u_0(x, y) - z \theta_y(x, y) + z^3 c_1 \left[ \theta_y(x, y) - \frac{\partial w_0}{\partial x} \right] \\
 v(x, y, z) &= v_0(x, y) + z \theta_x(x, y) + z^3 c_1 \left[ -\theta_x(x, y) - \frac{\partial w_0}{\partial y} \right] \\
 w(x, y, z) &= w_0(x, y)
 \end{aligned} \tag{1}$$

where  $u_0, v_0, w_0$  are displacements of a generic point in the middle plane of the laminate referred to the local axes - x,y,z directions,  $\theta_x, \theta_y$  are the rotations of the normal to the middle plane, about the x axis (clockwise) and y axis (anticlockwise),  $\partial w_0/\partial x, \partial w_0/\partial y$  are the slopes of the tangents of the deformed mid-surface in x,y directions,  $\theta_z$  is the rotation about the local z axis, which does not enter in the formulation in the local coordinate system, and  $c_1 = 4/3h^2$ , with h the total thickness of the laminate.

The strains components associated with the displacements in equation (1) are conveniently represented as:

$$\begin{Bmatrix} \bar{\boldsymbol{\epsilon}}_m + z \bar{\boldsymbol{\epsilon}}_b + z^3 \bar{\boldsymbol{\epsilon}}_b^* \\ \bar{\boldsymbol{\epsilon}}_s + z^2 \bar{\boldsymbol{\epsilon}}_s^* \end{Bmatrix} = \mathbf{Z} \bar{\boldsymbol{\epsilon}} \quad (2)$$

where  $\mathbf{Z}$  is an appropriate matrix containing powers of z.

The components of the vector  $\bar{\boldsymbol{\epsilon}}$  are given by:

$$\begin{aligned} \bar{\boldsymbol{\epsilon}}_m &= \left\{ \boldsymbol{\epsilon}_{x0} \ \boldsymbol{\epsilon}_{y0} \ \boldsymbol{\epsilon}_{xy0} \right\}^T = \left\{ \frac{\partial u_0}{\partial x}, \frac{\partial v_0}{\partial y}, \frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} \right\}^T \\ \bar{\boldsymbol{\epsilon}}_b &= \left\{ k_x, k_y, k_{xy} \right\}^T = \left\{ -\frac{\partial \theta_y}{\partial x}, \frac{\partial \theta_x}{\partial y}, \frac{\partial \theta_x}{\partial x} - \frac{\partial \theta_y}{\partial y} \right\}^T \\ \bar{\boldsymbol{\epsilon}}_b^* &= \left\{ k_x^*, k_y^*, k_{xy}^* \right\}^T = c_1 \left\{ \frac{\partial \theta_y}{\partial x} - \frac{\partial^2 w_0}{\partial x^2}, -\frac{\partial \theta_x}{\partial y} - \frac{\partial^2 w_0}{\partial y^2}, -\frac{\partial \theta_x}{\partial x} + \frac{\partial \theta_y}{\partial y} - 2 \frac{\partial^2 w_0}{\partial x \partial y} \right\}^T \\ \bar{\boldsymbol{\epsilon}}_s &= \left\{ \phi_x, \phi_y \right\}^T = \left\{ -\theta_y + \frac{\partial w_0}{\partial x}, \theta_x + \frac{\partial w_0}{\partial y} \right\}^T \\ \bar{\boldsymbol{\epsilon}}_s^* &= \left\{ \psi_x, \psi_y \right\}^T = \left\{ c_2 \left( \theta_y - \frac{\partial w_0}{\partial x} \right), c_2 \left( -\theta_x - \frac{\partial w_0}{\partial y} \right) \right\}^T \end{aligned} \quad (3)$$

where  $c_2 = 4/h^2$

### 3 MAGNETO-ELECTRO-ELASTIC LAMINATES. CONSTITUTIVE EQUATIONS

For an anisotropic magneto-electro-elastic solid, the constitutive equations for each layer, coupling mechanical, electrical and magnetic fields, are as follows [1]

$$\begin{aligned} \bar{\boldsymbol{\sigma}} &= \bar{\mathbf{C}} \bar{\boldsymbol{\epsilon}} - \bar{\mathbf{e}} \bar{\mathbf{E}} - \bar{\mathbf{q}} \bar{\mathbf{H}} \\ \bar{\mathbf{D}} &= \bar{\mathbf{e}}^T \bar{\boldsymbol{\epsilon}} + \bar{\boldsymbol{\epsilon}} \bar{\mathbf{E}} + \bar{\mathbf{d}} \bar{\mathbf{H}} \\ \bar{\mathbf{B}} &= \bar{\mathbf{q}}^T \bar{\boldsymbol{\epsilon}} + \bar{\mathbf{d}} \bar{\mathbf{E}} + \bar{\boldsymbol{\mu}} \bar{\mathbf{H}} \end{aligned} \quad (4)$$

where  $\bar{\sigma} = [\bar{\sigma}_{xx} \ \bar{\sigma}_{yy} \ \bar{\sigma}_{xy}]^T$  is the elastic stress vector and  $\bar{\varepsilon} = [\varepsilon_{xx} \ \varepsilon_{yy} \ \gamma_{xy}]^T$  the elastic strain vector,  $\bar{E} = [\bar{E}_x \ \bar{E}_y \ \bar{E}_z]^T$  the electric field vector,  $\bar{H} = [\bar{H}_x \ \bar{H}_y \ \bar{H}_z]^T$  the magnetic field vector,  $\bar{D} = [\bar{D}_x \ \bar{D}_y \ \bar{D}_z]^T$  the electric displacement vector,  $\bar{B} = [\bar{B}_x \ \bar{B}_y \ \bar{B}_z]^T$  the magnetic induction vector,  $\bar{C}$  the elastic constitutive matrix,  $\bar{e}$  the piezoelectric matrix,  $\bar{q}$  the piezomagnetic matrix,  $\bar{d}$  the magnetoelectric matrix,  $\bar{\varepsilon}$  is the dielectric matrix, and  $\bar{\mu}$  is the magnetic permeability matrix, in the element local coordinate system.

For the third-order shear deformation plate theory those matrices take the following forms

$$\bar{C} = \begin{bmatrix} \bar{C}_{11} & \bar{C}_{12} & \bar{C}_{16} & 0 & 0 \\ \bar{C}_{12} & \bar{C}_{22} & \bar{C}_{26} & 0 & 0 \\ \bar{C}_{16} & \bar{C}_{26} & \bar{C}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{C}_{44} & \bar{C}_{45} \\ 0 & 0 & 0 & \bar{C}_{45} & \bar{C}_{55} \end{bmatrix} \quad \bar{e} = \begin{bmatrix} 0 & 0 & \bar{e}_{31} \\ 0 & 0 & \bar{e}_{32} \\ 0 & 0 & \bar{e}_{36} \\ \bar{e}_{14} & \bar{e}_{24} & 0 \\ \bar{e}_{15} & \bar{e}_{25} & 0 \end{bmatrix} \quad \bar{q} = \begin{bmatrix} 0 & 0 & \bar{q}_{31} \\ 0 & 0 & \bar{q}_{32} \\ 0 & 0 & \bar{q}_{36} \\ \bar{q}_{14} & \bar{q}_{24} & 0 \\ \bar{q}_{15} & \bar{q}_{25} & 0 \end{bmatrix} \quad (5)$$

$$\bar{\varepsilon} = \begin{bmatrix} \bar{\varepsilon}_{11} & \bar{\varepsilon}_{12} & 0 \\ \bar{\varepsilon}_{21} & \bar{\varepsilon}_{22} & 0 \\ 0 & 0 & \bar{\varepsilon}_{33} \end{bmatrix} \quad \bar{d} = \begin{bmatrix} \bar{d}_{11} & \bar{d}_{12} & 0 \\ \bar{d}_{21} & \bar{d}_{22} & 0 \\ 0 & 0 & \bar{d}_{33} \end{bmatrix} \quad \bar{\mu} = \begin{bmatrix} \bar{\mu}_{11} & \bar{\mu}_{12} & 0 \\ \bar{\mu}_{21} & \bar{\mu}_{22} & 0 \\ 0 & 0 & \bar{\mu}_{33} \end{bmatrix}$$

The electric and the magnetic field vectors are the negative gradient of the electric and magnetic potentials, which are assumed to be applied and varying linearly in the thickness direction, i.e.

$$\begin{aligned} \bar{E} &= -\nabla \phi \\ \bar{E} &= \{0 \ 0 \ E_z\}^T \\ E_z &= -\phi/t_p \end{aligned} \quad (6)$$

$$\begin{aligned} \bar{H} &= -\nabla \psi \\ \bar{H} &= \{0 \ 0 \ H_z\}^T \end{aligned} \quad (7)$$

$$H_z = -\psi/t_m$$

The constitutive Eq. (4) can be written in the synthetic form

$$\hat{\sigma} = \hat{C} \hat{\varepsilon} \quad (8)$$

where we define for magneto-electro-elasticity

$$\hat{\sigma} = \begin{Bmatrix} \bar{\sigma} \\ \bar{D} \\ \bar{B} \end{Bmatrix} \quad \hat{C} = \begin{bmatrix} \bar{C} & \bar{e} & \bar{q} \\ \bar{e}^T & -\bar{\varepsilon} & -\bar{d} \\ \bar{q}^T & -\bar{d}^T & -\bar{\mu}^T \end{bmatrix} \quad \hat{\varepsilon} = \begin{Bmatrix} \bar{\varepsilon} \\ -\bar{E} \\ -\bar{H} \end{Bmatrix} \quad (9)$$

#### 4 FINITE ELEMENT FORMULATION

The dynamic equations of a laminated composite plate can be derived from the Hamilton's principle:

$$\int_{t_1}^{t_2} \left\{ \sum_{K=1}^N \left( \int_{A^e} \int_{h_{k-1}}^{h_k} \delta \hat{\boldsymbol{\varepsilon}}^T \hat{\mathbf{C}}_k \hat{\boldsymbol{\varepsilon}} dz dA^e - \int_{A^e} \int_{h_{k-1}}^{h_k} \delta \dot{\mathbf{u}}^T \rho_k \dot{\mathbf{u}} dz dA^e \right) - \left( \int_S \mathbf{T} \delta \mathbf{u} dS + \int_S \mathbf{Q}^\phi \delta \phi dS + \int_S \mathbf{Q}^\psi \delta \psi dS \right) \right\} dt = 0 \quad (10)$$

The non-conforming higher order triangular finite element model with three nodes and eight mechanical degrees of freedom per node, is used in this work. By assuming that  $c_1=0$  and that sections before deformation remain plane after deformation and perpendicular to the middle surface, i.e. by neglecting the transverse shear deformation, a Kirchhoff laminated finite element model with 6 degrees of freedom per node (CPT) is also easily obtained. The introduction of fictitious stiffness coefficients  $K_{\theta_z}$ , corresponding to rotations  $\theta_z$ , to eliminate the problem of a singular stiffness matrix, for which the elements are coplanar or near coplanar, is required. The element local displacements, slopes and rotations are expressed in terms of nodal variables through shape functions  $\mathbf{N}_i$  given in terms of area co-ordinates  $L_i$ , Zienkiewics [9]. The displacement field can be represented in matrix form as:

$$\mathbf{u} = \mathbf{Z} \left( \sum_{i=1}^3 \mathbf{N}_i \mathbf{d}_i \right) = \mathbf{Z} \mathbf{N} \mathbf{a} \quad ; \quad \mathbf{d} = \sum_{i=1}^3 \mathbf{N}_i \mathbf{d}_i = \mathbf{N} \mathbf{a} \quad (11)$$

$$\mathbf{Z} = \begin{bmatrix} 1 & 0 & 0 & 0 & -z^3 c_1 & 0 & -z + z^3 c_1 & 0 \\ 0 & 1 & 0 & -z^3 c_1 & 0 & +z - z^3 c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (12)$$

$$\mathbf{d}_i = \left\{ u_0 \quad v_0 \quad w_0 \quad -\frac{\partial w}{\partial y} \quad \frac{\partial w}{\partial x} \quad \theta_x \quad \theta_y \quad \theta_z \right\}_i \quad (13)$$

and the strain field as follows:

$$\bar{\boldsymbol{\varepsilon}} = \sum_{i=1}^3 \mathbf{B}_i^{\text{mec}} \mathbf{d}_i = \mathbf{B}^{\text{mec}} \mathbf{a}^e \quad (14)$$

The electric and magnetic fields are given by:

$$\mathbf{E} = -\mathbf{B}^\phi \phi \quad (15)$$

$$\mathbf{H} = -\mathbf{B}^\psi \psi$$

where  $\mathbf{B}^\phi$  and  $\mathbf{B}^\psi$  are given by

$$\mathbf{B}^\phi = \begin{bmatrix} 1/t_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1/t_p \end{bmatrix} \quad ; \quad \mathbf{B}^\psi = \begin{bmatrix} 1/t_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1/t_m \end{bmatrix} \quad (16)$$

Entering the equations (11) to (15) into equation (10), we have

$$\int_{t_1}^{t_2} \left\{ \sum_{K=1}^N \left( \int_{A^e} \int_{h_{k-1}}^{h_k} \delta \begin{Bmatrix} \mathbf{a} \\ \phi \\ \psi \end{Bmatrix}^T \begin{bmatrix} \mathbf{B}^{\text{mec}} & 0 & 0 \\ 0 & \mathbf{B}^\phi & 0 \\ 0 & 0 & \mathbf{B}^\psi \end{bmatrix}^T \begin{bmatrix} \bar{\mathbf{C}} & \bar{\mathbf{e}} & \bar{\mathbf{q}} \\ \bar{\mathbf{e}}^T & -\bar{\mathbf{e}} & -\bar{\mathbf{d}} \\ \bar{\mathbf{q}}^T & -\bar{\mathbf{d}}^T & -\bar{\boldsymbol{\mu}}^T \end{bmatrix} \begin{bmatrix} \mathbf{B}^{\text{mec}} & 0 & 0 \\ 0 & \mathbf{B}^\phi & 0 \\ 0 & 0 & \mathbf{B}^\psi \end{bmatrix} \begin{Bmatrix} \mathbf{a} \\ \phi \\ \psi \end{Bmatrix} dz dA^e - \right. \\
 \left. \int_{A^e} \int_{h_{k-1}}^{h_k} \delta \begin{Bmatrix} \dot{\mathbf{a}} \\ \dot{\phi} \\ \dot{\psi} \end{Bmatrix}^T \rho_k \begin{Bmatrix} \dot{\mathbf{a}} \\ \dot{\phi} \\ \dot{\psi} \end{Bmatrix} dz dA^e \right) - \left( \int_S \delta \{\mathbf{a}\}^T \mathbf{N}^T \mathbf{T} dS + \int_S Q^\phi \delta \phi dS + \int_S Q^\psi \delta \psi dS \right) \Big\} dt = 0 \quad (17)$$

To the first and second terms of first member of Eq.(17), correspond, respectively, the element stiffness and mass matrices, which are defined by

$$[\mathbf{K}] = \sum_{k=1}^N \int_A \int_{h_{k-1}}^{h_k} \begin{bmatrix} \mathbf{B}^{\text{mec}^T} \bar{\mathbf{C}} \mathbf{B}^{\text{mec}} & \mathbf{B}^{\text{mec}} \bar{\mathbf{e}} \mathbf{B}^\phi & \mathbf{B}^{\text{mec}^T} \bar{\mathbf{q}} \mathbf{B}^\phi \\ \mathbf{B}^{\phi^T} \bar{\mathbf{e}} \mathbf{B}^{\text{mec}} & \mathbf{B}^{\phi^T} \bar{\mathbf{e}} \mathbf{B}^\phi & \mathbf{B}^{\phi^T} \bar{\mathbf{d}} \mathbf{B}^\psi \\ \mathbf{B}^{\psi^T} \bar{\mathbf{q}} \mathbf{B}^{\text{mec}} & \mathbf{B}^{\psi^T} \bar{\mathbf{d}} \mathbf{B}^\phi & \mathbf{B}^{\psi^T} \bar{\boldsymbol{\mu}} \mathbf{B}^\psi \end{bmatrix} dz dA = \begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\phi} & \mathbf{K}_{u\psi} \\ \mathbf{K}_{\phi u} & \mathbf{K}_{\phi\phi} & \mathbf{K}_{\phi\psi} \\ \mathbf{K}_{\psi u} & \mathbf{K}_{\psi\phi} & \mathbf{K}_{\psi\psi} \end{bmatrix} \quad (18)$$

$$[\mathbf{M}] = \begin{bmatrix} \mathbf{M}_{uu} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (19)$$

$$\text{with } \mathbf{M}_{uu} = \int_A \mathbf{N}^T \left( \sum_{k=1}^n \rho_k \int_{h_{k-1}}^{h_k} \mathbf{Z}^T \mathbf{Z} dz \right) \mathbf{N} dA$$

To the third term of Eq.(17), corresponds the applied force vector, the mechanical force vector  $\mathbf{F}^{\text{mec}}$ , electric charge vector  $\mathbf{F}^{\text{ele}}$ , and magnetic charge vector  $\mathbf{F}^{\text{mag}}$ .

From Eq. (17), yields the element equilibrium equations. To solve general structures, local-global transformations [9] are carried out for the element stiffness and mass matrices as well as for the load vector which are initially computed in the local coordinate system attached to the element. After the local-global transformations, the assembled system of equations for static analysis is

$$\begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\phi} & \mathbf{K}_{u\psi} \\ \mathbf{K}_{\phi u} & \mathbf{K}_{\phi\phi} & \mathbf{K}_{\phi\psi} \\ \mathbf{K}_{\psi u} & \mathbf{K}_{\psi\phi} & \mathbf{K}_{\psi\psi} \end{bmatrix} \begin{Bmatrix} \mathbf{q} \\ \phi \\ \psi \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}^{\text{mec}} \\ \mathbf{F}^{\text{ele}} \\ \mathbf{F}^{\text{mag}} \end{Bmatrix} \quad (18)$$

and for free vibration analysis is

$$\left( \begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{u\phi} & \mathbf{K}_{u\psi} \\ \mathbf{K}_{\phi u} & \mathbf{K}_{\phi\phi} & \mathbf{K}_{\phi\psi} \\ \mathbf{K}_{\psi u} & \mathbf{K}_{\psi\phi} & \mathbf{K}_{\psi\psi} \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{M}_{uu} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right) \begin{Bmatrix} \mathbf{q} \\ \phi \\ \psi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad (20)$$

where  $\{\mathbf{q}\}$ ,  $\{\phi\}$  and  $\{\psi\}$  are respectively, in the global coordinate system, the generalized mechanical displacement, electric potential and magnetic potential vectors.

## 5 APPLICATIONS

### Simply-supported square plate with fixed ends.

A simply-supported square plate with fixed ends, made of piezoelectric material BaTiO<sub>3</sub> (called B) and magnetostrictive material CoFe<sub>2</sub>O<sub>4</sub> (called F), with different stacking sequences, is analysed by the finite element method using the third order shear deformation theory. The side dimension is  $L=1\text{m}$  and the thickness of each layer is  $h=0.1\text{ m}$  (case I) or  $0.01\text{m}$  (case II). The properties of piezoelectric material are  $C_{11}=C_{22}=166 \times 10^9 \text{ N/m}^2$ ,  $C_{12}=77 \times 10^9 \text{ N/m}^2$ ,  $C_{66}=44.5 \times 10^9 \text{ N/m}^2$ ,  $C_{44}=C_{55}=43 \times 10^9 \text{ N/m}^2$ ,  $e_{31}=e_{32}=-4.4 \text{ C/m}^2$ ,  $e_{15}=e_{24}=11.6 \text{ C/m}^2$ ,  $\epsilon_{11}=\epsilon_{22}=11.2 \times 10^{-9} \text{ C}^2/\text{N m}^2$ ,  $\epsilon_{33}=12.6 \times 10^{-9} \text{ C}^2/\text{N m}^2$ ,  $\mu_{11}=\mu_{22}=5 \times 10^{-6} \text{ N s}^2/\text{C}^2$ ,  $\mu_{33}=10 \times 10^{-6} \text{ N s}^2/\text{C}^2$ . The properties of the magnetostrictive material are  $C_{11}=C_{22}=286 \times 10^9 \text{ N/m}^2$ ,  $C_{12}=173 \times 10^9 \text{ N/m}^2$ ,  $C_{66}=56.5 \times 10^9 \text{ N/m}^2$ ,  $C_{44}=C_{55}=45.3 \times 10^9 \text{ N/m}^2$ ,  $q_{31}=q_{32}=580.3 \text{ N/Am}$ ,  $q_{15}=q_{24}=550 \text{ N/Am}$ ,  $\epsilon_{11}=\epsilon_{22}=0.08 \times 10^{-9} \text{ C}^2/\text{N m}^2$ ,  $\epsilon_{33}=0.093 \times 10^{-9} \text{ C}^2/\text{N m}^2$ ,  $\mu_{11}=\mu_{22}=-590 \times 10^{-6} \text{ N s}^2/\text{C}^2$ ,  $\mu_{33}=157 \times 10^{-6} \text{ N s}^2/\text{C}^2$ . The mass density for both materials is  $\rho=1600 \text{ kg/m}^3$ .

### 5.1 Static analysis

In the static analysis is considered the plate of case I, with total thickness  $H=0.3\text{ m}$ , and stacking sequence (B/F/B), subjected to a sinusoidal mechanical traction applied at its top surface  $t_z = \sin(\pi x/a) \sin(\pi y/a)$ . The boundary conditions are as follows:

$$u(x, y, z) = v(x, y, z) = w(x, y, z) = 0 \quad \text{for } x=0 ; x=L \text{ and } y=0 ; y=L$$

$$\phi(x, y, \pm h/2) = \psi(x, y, \pm h/2) = 0$$

The present solution was obtained using a (8x8) element mesh (128 triangular elements). The results shown here are obtained for the node of coordinates  $x=0.75\text{ m}$ ,  $y=0.25\text{ m}$  (displacements), or for the gauss point nearest of this node (stresses) or for the element that contains this gauss point (electric potential, magnetic potential, electric displacement and magnetic induction). A transverse displacement  $w=5.40 \times 10^{-12}\text{ m}$  was obtained at mentioned node, while Lage et al. [4], that uses a partial mixed layerwise finite element model, and  $w$  is a cross-thickness variable, it take values  $5.39 \times 10^{-12}\text{ m} < w < 5.87 \times 10^{-12}$ . The present cross-thickness distribution of in-plane displacement  $u$  evaluated at the same node is shown in Fig. 1, where is compared with results from Lage et al. [4].

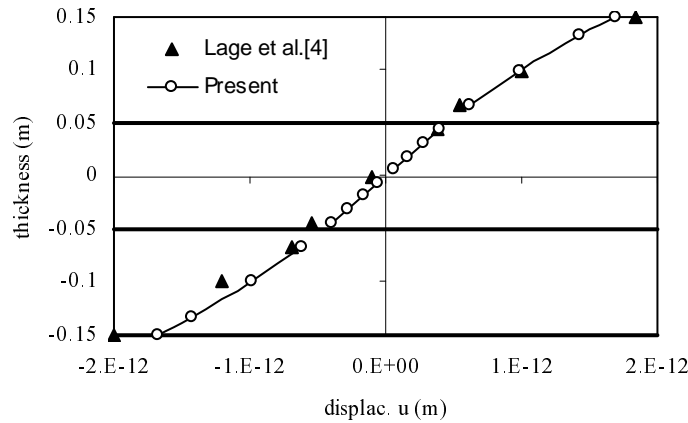


Fig. 1.  $u$  displacement distribution across plate thickness

Also the present cross-thickness distributions of stresses are presented in Figs 2-3. Again, the present results are compared with those taken from the graphics of Lage et al. [4].

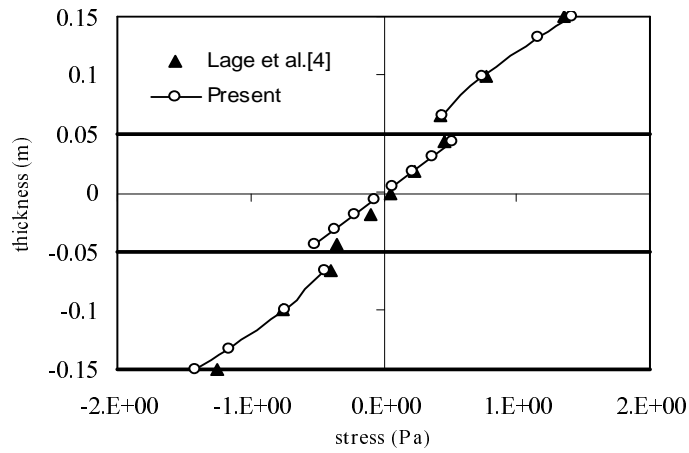


Fig. 2.  $\sigma_{xx}$  stress distribution across plate thickness

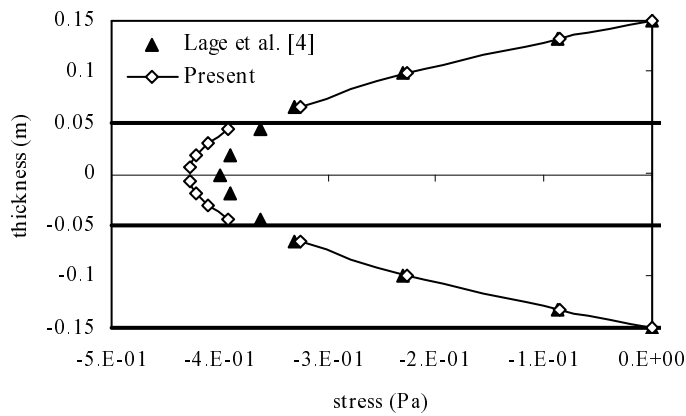


Fig. 3.  $\sigma_{xz}$  stress distribution across plate thickness

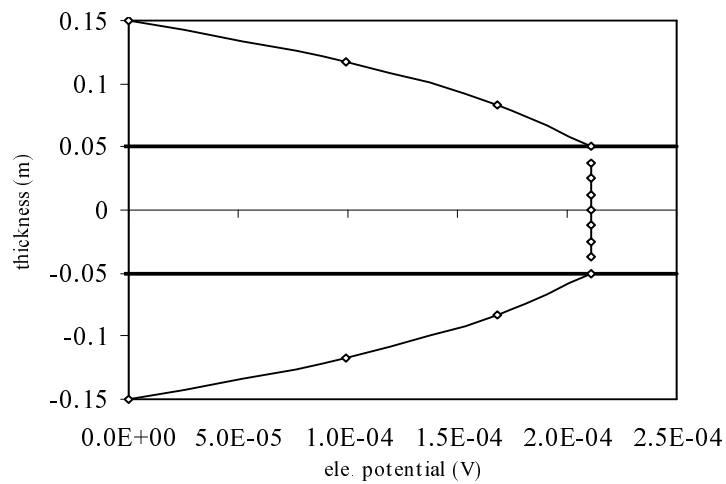


Fig. 4. Electric potential distribution across plate thickness



In Fig. 4-7, the cross-thickness distributions of electric and magnetic potentials as well as the electric displacement and the magnetic induction, evaluated at the element that contains the previous referred gauss point, are presented.

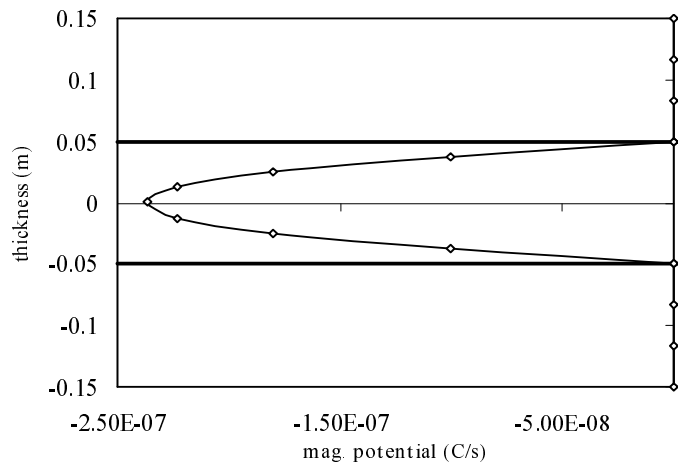


Fig. 5. Magnetic potential distribution across plate thickness

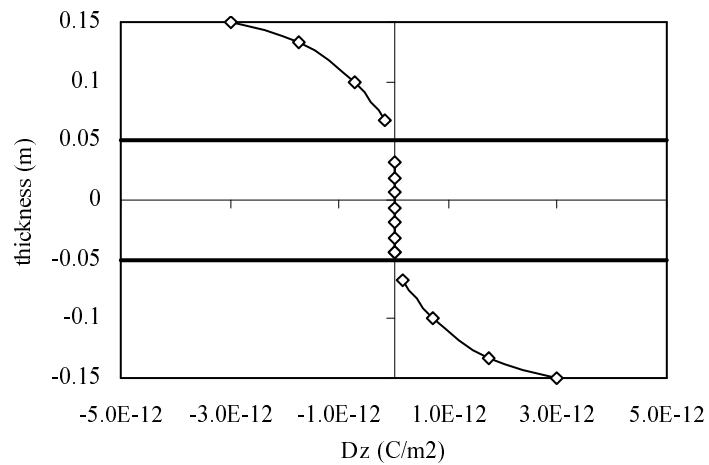


Fig. 6. Electric displacement  $D_z$  distribution across plate thickness

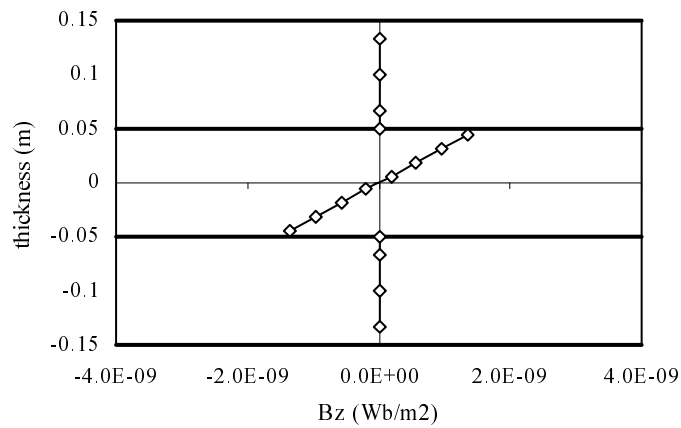


Fig. 7. Magnetic induction  $B_z$  distribution across plate thickness

## 5.2 Free-vibrations analysis

The simple supported square plate of case II is analysed. The side is equal  $L=1$  m but now the total thickness is  $H=0.04$  m (each lamina with 0.01 m of thickness) and the stacking sequence is (B/F/F/B). The results obtained using a (6x6) element mesh (72 triangular elements) are shown in Table 1, and compared with those obtained in Lage et al [5]. Elastic plate means that the piezoelectric and magnetic properties are not considered - set to zero. A very good approximation is observed between the results obtained by using the layerwise and single layer models.

Mode	elastic plate		magneto-electro-elastic plate	
	Present	Lage et al. [5]	Present	Lage et al. [5]
1	2452.8	2498.5	2462.5	2542.9
2	6226.9	6281.0	6251.0	6391.0
3	6311.7	6281.0	6335.2	6391.0
4	10295.5	10017.4	10331.7	10192.0

Table 1. Natural Frequencies (rad/s):  $L=1$  m ;  $H=0.04$  m for B/F/F/B plate

Secondly the plate of case I with different stacking sequences is analysed. The results obtained are given in Table 2. In Fig 8 are shown the first six mode shapes of (F/B/F) plate stacking sequence.

Mode	elastic plate				magneto-electro-elastic plate			
	B only	F only	B/F/B	F/B/F	B only	F only	B/F/B	F/B/F
1	12600.01	14882.24	12751.45	14738.18	12631.77	14905.91	12783.17	14761.57
2	24926.81	27965.02	25211.44	27664.41	24964.69	27987.31	25249.50	27686.12
3	25147.20	28198.62	25433.88	27895.27	25185.42	28221.14	25472.28	27917.20
4	34176.64	37478.54	34566.57	37048.92	34210.58	37496.29	34601.21	37065.91
5	39204.94	42568.46	39657.88	42059.00	39234.02	42582.32	39687.79	42072.00
6	39963.98	43475.44	40422.48	42960.06	39995.27	43490.76	40454.58	42974.49

Table 2. Natural Frequencies (rad/s) :  $L=1$  m ;  $H=0.3$  m – HSDT

## 6 CONCLUSIONS

In this paper a finite element model based on a higher-order plate theory is developed for the static and free vibration analyses of magneto-electro-elastic plates. This model allows to obtain in static analysis the through-thickness distributions of primary variables- mechanical displacements, electric potentials and magnetic potentials-as well as for electric displacements and magnetic inductions. Some results were compared with those obtained by Lage et al. [4] that use a partial mixed layerwise finite element model, and good agreement is found. It should be observed that the layerwise theory allows to the cross-thickness variation of the transversal displacement  $w$ , and the corresponding strain  $\epsilon_{zz}$ , which is relevant in the stresses results, but essentially in the electric and magnetic potentials as well in electric displacement and magnetic induction. In the free vibration analysis, the results obtained by using the two different models, shown in Table 1, are of the same magnitude, but the quite difference be-

tween the layerwise approach and the single layer formulation of the present model, justifies the discrepancies found between the results.

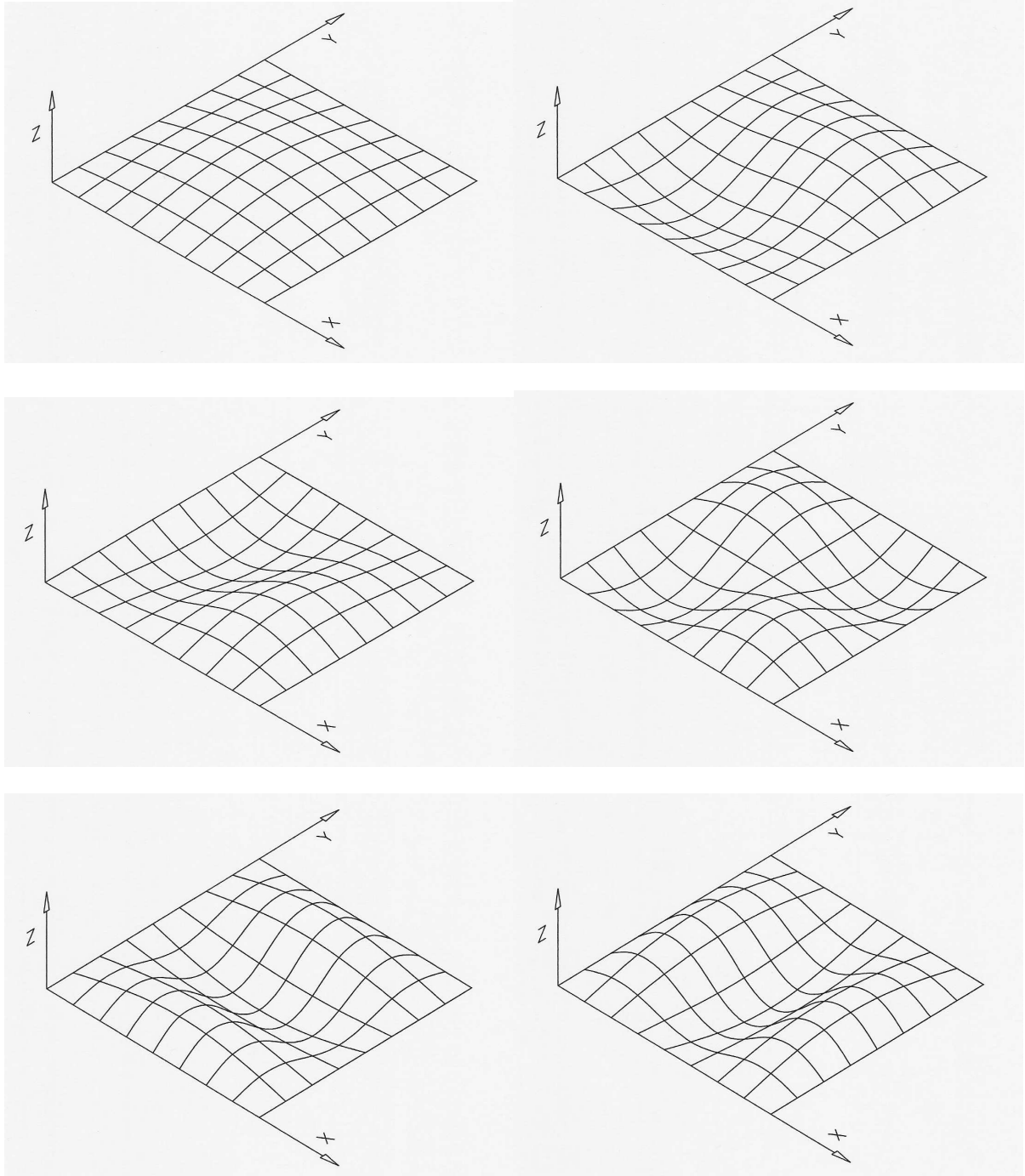


Figure 8. The first six mode shapes of F/B/F plate

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