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# Solving the coupled Schrödinger -Korteweg- de-Vries system by modified variational iteration method with genetic algorithm

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**ABSTRACT:** A system of nonlinear partial differential equations was solved using a modified variational iteration method (MVIM) combined with a genetic algorithm. The modified method introduced an auxiliary parameter (p) in the correction functional to ensure convergence and improve the outcomes. Before applying the modification, the traditional variational iteration method (VIM) was used firstly. The method was applied to numerically solve the system of Schrödinger-KdV equations. By comparing the two methods in addition to some of the previous approaches, it turns out the new algorithm converges quickly, generates accurate solutions and shows improved accuracy. Additionally, the method can be easily applied to various linear and nonlinear differential equations.

**Keywords:** coupled Schrödinger–KdV equation, Genetic Algorithm, Lagrange multiplier, Variational iteration method, modified variational iteration method.

## 1. INTRODUCTION

In recent years, nonlinear differential equations have been widely used as models to explain physical phenomena in a variety of scientific disciplines, particularly in biology, physics, chemical, optics, plasma waves, etc. [1–4]. One of the important nonlinear PDEs is known as Schrodinger-KdV equations are frequently used to simulate the nonlinear dynamics of one-dimensional Langmuir and ion acoustic waves traveling at ion acoustic speeds. The Schrodinger-KdV equation system has been resolved by several authors using a variety of methods, including the variational iteration method [5], the modified variational iteration method [6], the homotopy perturbation method [7], the optimal homotopy asymptotic method [8], the new iterative method [9], the modified laplace decomposition method [10], the compact finite difference scheme [11], the Runge-Kutta structure-preserving methods [12], the (G′/G)-expansion technique [13], the differential transform method [14] etc. The (VIM) is one of the most straightforward and efficient methods for locating approximations of (PDEs), and most authors have used it to produce a range of numerical results. [15–18], It demonstrates the suitability of this approach for a wide range of technical and scientific applications. If a solution exists, the VIM present convergent approximations of the precise findings; otherwise, only a few approximations can be used numerically.

The goal of this research is to solve the coupled Schrödinger-Korteweg-de Vries system using the (VIM) standard and a new modification of (VIM) using a genetic algorithm. The authors will support their findings by comparing their results to those of other approaches and by showing the solutions in 3D graphics.

This essay's sections are organized as follows: Section (2) presents the basic VIM concept. A modification for VIM is shown in the next section (3). In section (4), a definition of a genetic algorithm is provided. The modified VIM methods, along with a genetic strategy, are used in section (5) to solve a system of PDEs and show solutions through graphing. In the last section (6), several conclusions are made.

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## 2. BASIC CONCEPTS OF VIM

Suppose we have the nonlinear partial differential equation (NPDE) given below:

$$L\left(u\left(x\right)\right]+N\left(u\left(x\right)\right]=c\left(x\right) \tag{1}$$

where $L[u]$ is a linear term and $N\left(u\right]$ is a nonlinear term, and $c$ is the source term.

The approximate solution $u_{n+1}\left(x\right)$ of Eq. (1) can be obtained as [19]

$$u_{n+1}\left(x\right)=u_n\left(x\right)+\int_0^t \lambda(\tau)\left(L\left(u_n\left(\tau\right)\right\}+N\left(\widetilde{u_n\left(\tau\right)}\right\}-c\left(\tau\right)\right]d\tau \tag{2}$$

where $\lambda$ known as the Lagrange multiplier (LM), $\check{u}_n$ is a term being restricted, which in turn gives $\delta\check{u}_n$ =0 and gives the following LM:

$$\lambda=\frac{(-1)^m(\tau-t)^{m-1}}{(m-1)!},\quad m\geq 1 \tag{3}$$

Substituting Eq. (3) into Eq. (2), the following formula will be obtained:

$$u_{n+1}\left(x\right)=u_n\left(x\right)+\int_0^t \frac{(-1)^m(\tau-t)^{m-1}}{(m-1)!}\left(L\left(u_n\left(\tau\right)\right\}+N\left(u_n\left(\tau\right)\right\}-c(\tau)\right]d\tau, \tag{4}$$

The approximations $u_n\left(x,t\right)$ , $n>0$ of solution $u\left(x,t\right)$ will be obtained using the obtained LM and any initial function $u_0\left(x,t\right)$, consequently, the exact solution may be obtained by using

$$u\left(x,t\right)=\lim_{n\to\infty} u_n\left(x,t\right) \tag{5}$$

## 3. MODIFIED VARIATIONAL ITERATION METHOD

The approximate solution $u_{n+1}\left(x\right)$ of Eq. (1) for given initial condition $u_0\left(x\right)$ can be found to clarify the MVIM as follows:

$$u_{n+1}\left(x\right)=u_n\left(x\right)+p\int_0^t \lambda(\tau)\left(N\left(\widetilde{u_n\left(\tau\right)}\right\}-c\left(\tau\right)\right]d\tau \tag{6}$$

where $p$ is an auxiliary term which is used to ensures convergence to the exact solution. While $\lambda$ is the LM, which can be found by the equation (3). Substituting Eq. (3) into Eq. (6). Then, we have the iterative algorithm for Eq. (1):

$$\begin{cases} u_0(x) \text{ is an initial approximation} \\ u_{n+1}(x)=u_n(x)+p\int_0^t \frac{(-1)^m(\tau-t)^{m-1}}{(m-1)!}\left[N\left\{\widetilde{u_n(\tau)}\right\}-c(\tau)\right]d\tau \end{cases} \tag{7}$$

The genetic algorithm can be used to improve the results of partial differential equations. It works by selecting the best value for the parameter p in order to get the most accurate result.

## 4. GENETIC ALGORITHMS (GA)

**Definition**

An optimization method called a genetic algorithm takes its cues from natural selection and evolution based on Darwin's Principle.

**Operators for (GA)** [20]

A genetic algorithm uses three operators including selection, crossover, and mutation to solve complicated problems by simulating the evolutionary process.

**The basic steps of a (GA)** [21]

1. An initial population of potential answers to the problem being answered is first created by the algorithm. A chromosome, which houses a collection of genes that encode each person's genetic information, is used to symbolize each member of the population. The population size and chromosomal distribution are determined by the issue at hand.

2. Based on how well it resolves the problem at hand, the fitness of each member of the population is assessed. Each member of the population is given a fitness score by the fitness function, which is specified in terms of the problem being solved.

3. To produce the next generation of progeny, the fittest people are chosen for reproduction. One of the preceding selection techniques, like fitness-proportional selection or tournament selection, is used in the selection process.

4. Crossover is the process in which the chosen individuals' genetic material is merged to produce new children.

5. In order to increase genetic diversity, the new children go through a process called mutation, when random alterations are incorporated into their genetic makeup.

6. The next generation of people is created by the new offspring, who replace the least healthy members of the current population. By doing this, it is made sure that the population size is consistent throughout the process.

7. When a good solution is found, the algorithm stops, or when the allotted number of generations has been reached. According on the final population of people's fitness scores, the answer is often picked.

## 5. NUMERICAL EXAMPLE

Consider the model of coupled Schrödinger–KdV equation in the form [9]:

$$\begin{cases} u_t - v_{xx} - vw = 0, \\ v_t + u_{xx} + uw = 0, \\ w_t + 6ww_x + w_{xxx} - 2uu_x - 2vv_x = 0 \end{cases} \tag{1}$$

With the following initial conditions:

$$\begin{cases} u(x,0) = cos(x), \\ v(x,0) = sin(x), \\ w(x,0) = \frac{3}{4} \end{cases} \tag{2}$$

The exact solutions of Eq. (1) is given by [9]

$$\begin{cases} u(x,t) = cos\left(x + \frac{t}{4}\right), \\ v(x,t) = sin\left(x + \frac{t}{4}\right), \\ w(x,t) = \frac{3}{4}. \end{cases} \tag{3}$$

To solve the system (1) by means of the standard VIM, we construct the following correction functional as

$$\begin{cases} u_{n+1} = u_n + \int_0^t \lambda_1(\tau)\left((u_t)_n - (v_n)_{xx} - v_n w_n\right] d\tau, \\ v_{n+1} = v_n + \int_0^t \lambda_2(\tau)\left((v_t)_n + (u_n)_{xx} + u_n w_n\right] d\tau, \\ w_{n+1} = w_n + \int_0^t \lambda_3(\tau)\left((w_t)_n + 6w_n(w_n)_x + (w_n)_{xxx} - 2u_n(u_n)_x - 2v_n(v_n)_x\right] d\tau \end{cases} \tag{4}$$

where $\lambda_1(\tau)$, $\lambda_2(\tau)$ and $\lambda_3(\tau)$ are general Lagrange multipliers and $u_n(x,t), v_n(x,t)$ *and* $w_n(x,t)$ denote restricted variations i.e. $\delta u_n = \delta v_n = \delta w_n = 0$

Its stationary conditions can be obtained as:

$$\begin{cases} \lambda'_i(\tau) = 0, \\ 1 + (\lambda_i(\tau)\rfloor_{\tau=t} = 0 \end{cases}, \tag{5}$$

So, the Lagrange multiplier can be identified as follows

$$\lambda_1 = \lambda_2 = \lambda_3 = -1 \tag{6}$$

Then the formulas (4) becomes in the following:

$$\begin{cases} u_{n+1} = u_n - \int\limits_0^t ((u_t)_n - (v_n)_{xx} - v_n w_n]\, d\tau, \\ v_{n+1} = v_n - \int\limits_0^t ((v_t)_n + (u_n)_{xx} + u_n w_n]\, d\tau, \\ w_{n+1} = w_n - \int_0^t ((w_t)_n + 6w_n(w_n)_x + (w_n)_{xxx} - 2u_n(u_n)_x - 2v_n(v_n)_x]\, d\tau \end{cases} \tag{7}$$

By using the equations (7) and (2), we can obtain the following different approximations:

$$u_0\,(x,t) = cos(x)$$

$$v_0\,(x,t) = sin(x)$$

$$w_0\,(x,t) = \frac{3}{4}$$

$$u_1\,(x,t) = cos(x) - \frac{1}{4} t sin(x)$$

$$v_1\,(x,t) = sin(x) + \frac{3}{4} t cos(x)$$

$$w_1\,(x,t) = \frac{3}{4}$$

$$u_2\,(x,t) = cos(x) - \frac{1}{4} t sin\,(x) - \frac{1}{32} t^2 cos(x)$$

$$v_2\,(x,t) = sin(x) + \frac{1}{4} t cos(x) - \frac{1}{32} t^2 sin(x)$$

$$w_2\,(x,t) = \frac{3}{4}$$

Now, to solve the system (1) with the initial conditions (2) by means of the MVIM, the following correction functional will be constructed as

$$\begin{cases} u_{n+1} = u_n + p_1 \int\limits_0^t \lambda_1\,(\tau)\,(-(v_n)_{xx} - v_n w_n]\, d\tau, \\ v_{n+1} = v_n + p_2 \int\limits_0^t \lambda_2\,(\tau)\,((u_n)_{xx} + u_n w_n]\, d\tau, \\ w_{n+1} = w_n + p_3 \int_0^t \lambda_3\,(\tau)\,(6w_n(w_n)_x + (w_n)_{xxx} - 2u_n(u_n)_x - 2v_n(v_n)_x]\, d\tau \end{cases} \tag{8}$$

From the Eq. (6), the formulas of variational iteration can be obtained

$$\begin{cases} u_{n+1} = u_n - p_1 \int\limits_0^t (-(v_n)_{xx} - v_n w_n]\, d\tau, \\ v_{n+1} = v_n - p_2 \int\limits_0^t ((u_n)_{xx} + u_n w_n]\, d\tau, \\ w_{n+1} = w_n - p_3 \int_0^t (6w_n(w_n)_x + (w_n)_{xxx} - 2u_n(u_n)_x - 2v_n(v_n)_x]\, d\tau \end{cases} \tag{9}$$

The following different approximations can be taken by utilizing the relations shown in Eqs. (9) and (2)

$$u_0(x,t) = cos(x)$$

$$v_0(x,t) = sin(x)$$

$$w_0(x,t) = \frac{3}{4}$$

$$u_1(x,t) = cos(x) - \frac{1}{4}tp_1 sin(x)$$

$$v_1(x,t) = sin(x) + \frac{1}{4}tp_2 cos(x)$$

$$w_1(x,t) = \frac{3}{4}$$

$$u_2(x,t) = cos(x) - \frac{1}{2}tp_1 sin(x) - \frac{1}{32}t^2 p_1 p_2 cos(x)$$

$$v_2(x,t) = sin(x) + \frac{1}{2}tp_2 cos(x) - \frac{1}{32}t^2 p_1 p_2 sin(x)$$

$$w_2(x,t) = \frac{3}{4} + \frac{1}{24}t^3 {p_1}^2 p_3 sin(x) cos(x) - \frac{1}{24}t^3 {p_2}^2 p_3 sin(x) cos(x) - \frac{1}{4}t^2 p_1 p_3 cos^2(x) + \frac{1}{4}t^2 p_1 p_3 sin^2(x) \\ -\frac{1}{4}t^2 p_2 p_3 sin^2(x) + \frac{1}{4}t^2 p_2 p_3 cos^2(x)$$

Compare the numerical results $u_1(x,t)$, $u_2(x,t)$, $v_1(x,t)$, $v_2(x,t)$ and the absolute errors using the standard VIM with the exact solutions within the interval $-10 \leq x \leq 10$ where $h = 0.1$ in tables 1 and 3. The numerical results $u_1(x,t)$, $u_2(x,t)$, $v_1(x,t)$, $v_2(x,t)$ and the absolute errors obtained by applying the MVIM-GA compared with the exact solutions within the interval

$-10 \leq x \leq 10$ where $h = 0.1$ are given in tables 2 and 4. Compare the improved variational iteration approach with the MVIM-GA for some iterations of the solution in Table 5.

### 5.1 ILLUSTRATE SOLUTIONS BY GRAPHING

In the figures 2 and 6, the solutions $u_3(x,t)$ *and* $v_2(x,t)$ are plotted by standard VIM where $0 \leq t \leq 1$, $-5 \leq x \leq 5$. In the figures 3 and 7, the plot in 3D for the solutions $u_3(x,t)$ *and* $v_2(x,t)$ by MVIM-GA where $0 \leq t \leq 1$, $-5 \leq x \leq 5$. The exact solutions $u(x,t)$ *and* $v(x,t)$ are plotted in the figures 1 and 5 for the same interval. The absolute errors using the MVIM-GA solution presented in the figures 4 and 8.

## 6. CONCLUSION

In this research, the system of nonlinear (PDEs) by two ways has been solved. In the first part, the standard variational iteration method has been used. The modified VIM with genetic algorithm utilized in the second part to solve . By comparing the results, the VIM with genetic algorithm is more accurate than the standard VIM and the modified VIM with genetic techniques is more accurate than the two methods that presented in the first and second parts. Furthermore, it has been shown that the present method is straightforward to finding approximate solutions in many other fields. Afterwards, this notable method could be more efficiently used to find linear and nonlinear partial differential equations which orderly emerge in engineering, physics, and other technological areas.

**Table 1.** **Comparison between the VIM, the exact solution and the absolute errors for some iterations of the VIM solution**

| t | x | $u_E$ | $u_{1(VIM)}$ | $\|u_E - u_{1(VIM)}\|$ | $u_{2(VIM)}$ | $\|u_E - u_{2(VIM)}\|$ |
|---|---|---|---|---|---|---|
| 0.1 | -6 | 0.94500536 | 0.94619951 | 1.194E-03 | 0.94499229 | 1.307E-05 |
| | -3 | -0.98170220 | -0.98293649 | 1.234E-03 | -0.98169090 | 1.113E-05 |
| | 0 | 0.99875026 | 1 | 1.249E-03 | 0.99875000 | 2.604E-07 |
| | 3 | -0.99580832 | -0.99704849 | 1.240E-03 | -0.99581100 | 2.681E-06 |
| | 6 | 0.97293527 | 0.97414106 | 1.205E-03 | 0.97294084 | 5.570E-06 |
| 0.5 | -4 | -0.74289782 | -0.74824393 | 5.346E-03 | -0.74313734 | 2.395E-04 |
| | -1 | 0.64099685 | 0.64548617 | 4.489E-03 | 0.64126506 | 2.682E-04 |
| | 2 | -0.52626633 | -0.52980901 | 3.542E-03 | -0.52655786 | 2.915E-04 |
| | 5 | 0.40100258 | 0.40352771 | 2.525E-03 | 0.40131160 | 3.090E-04 |
| | 8 | -0.26771276 | -0.26916981 | 1.457E-03 | -0.26803309 | 3.203E-04 |
| 1 | -10 | -0.94757980 | -0.97507680 | 2.749E-02 | -0.94885582 | 1.276E-03 |
| | -7 | 0.89300634 | 0.91814890 | 2.514E-02 | 0.89458945 | 1.583E-03 |
| | -4 | -0.82055935 | -0.84284424 | 2.228E-02 | -0.82241788 | 1.858E-03 |
| | 4 | -0.44608748 | -0.46444299 | 1.835E-02 | -0.44401663 | 2.070E-03 |
| | 7 | 0.56792417 | 0.58965560 | 2.173E-02 | 0.56609615 | 1.828E-03 |
| | 10 | -0.67839385 | -0.70306625 | 2.467E-02 | -0.67684526 | 1.548E-03 |

**Table 2.** **Comparison between the MVIM-GA, the exact solution and the absolute errors for some iterations of the solution**

| t | x | $u_E$ | $u_{1(GVIM-III)}$ $p_1$ = 0.99973 $p_2$ = 0.9957 $p_3$ = 1 | $(u_E - u_1\|$ | $u_{2(GVIM-III)}$ $p_1$ = 0.52129 $p_2$ = 0.49998 $p_3$ = 1 | $(u_E - u_2\|$ |
|---|---|---|---|---|---|---|
| 0.1 | -6 | 0.95288558 | 0.95318678 | 3011E-04 | 0.95280925 | 7.633E-05 |
| | -3 | -0.98615550 | -0.98646544 | 3.099E-04 | -0.98623364 | 7.813E-05 |
| | 0 | 0.99968751 | 1 | 3.124E-04 | 0.99971855 | 3.104E-05 |
| | 3 | -0.99321077 | -0.99351954 | 3.087E-04 | -0.99329008 | 7.931E-05 |
| | 6 | 0.96685490 | 0.96715378 | 2.988E-04 | 0.96679327 | 6.163E-05 |
| 0.5 | -4 | -0.74289782 | -0.74821839 | 5.320E-03 | -0.74354106 | 6.432E-04 |
| | -1 | 0.64099685 | 0.64545777 | 4.461E-03 | 0.64086474 | 1.321E-04 |
| | 2 | -0.52626633 | -0.52977832 | 3.511E-03 | -0.53380138 | 7.535E-03 |
| | 5 | 0.40100258 | 0.40349535 | 2.492E-03 | 0.40805399 | 7.051E-04 |
| | 8 | -0.26771276 | -0.26913642 | 1.423E-03 | -0.26713940 | 5.733E-04 |
| 1 | -10 | -0.94757980 | -0.97504008 | 2.746E-02 | -0.94403381 | 3.545E-03 |
| | -7 | 0.89300634 | 0.91810455 | 2.509E-02 | 0.90200212 | 8.995E-03 |
| | -4 | -0.82055935 | -0.84279316 | 2.223E-02 | -0.82557659 | 5.017E-03 |
| | 4 | -0.44608748 | -0.46449408 | 1.840E-02 | -0.44106301 | 5.024E-03 |
| | 7 | 0.56792417 | 0.59294508 | 2.502E-02 | 0.57652157 | 8.597E-03 |
| | 10 | -0.67839385 | -0.70310297 | 2.470E-02 | -0.69044105 | 1.204E-02 |

**Table 3. Comparison between the VIM, the exact solution and the absolute errors for some iterations of the VIM solution**

| t | x | $v_E$ | $v_{1(VIM)}$ | $(v_E - v_{1(VIM)}\|$ | $v_{2(VIM)}$ | $(v_E - v_{2(VIM)}\|$ |
|---|---|---|---|---|---|---|
| 0.1 | -6 | 0.30332994 | 0.30341975 | 8.981E-05 | 0.30333243 | 2.495E-06 |
| | -3 | - 0.16582314 | - 0.16586982 | 4.667E-05 | -0.16582572 | 2.575E-06 |
| | 0 | 0.024997395 | 0.02500000 | 2.604E-06 | 0.02500000 | 2.604E-06 |
| | 3 | 0.11632867 | 0.11637019 | 4.151E-05 | 0.11632609 | 2.580E-06 |
| | 6 | -0.25532642 | -0.25541124 | 8.481E-05 | -0.2553239 | 2.504E-06 |
| 0.5 | -4 | 0.66940482 | 0.67509704 | 5.692E-03 | 0.66918452 | 2.203E-04 |
| | -1 | - 0.76754350 | -0.77393319 | 6.389E-03 | -0.76735920 | 1.842E-04 |
| | 2 | 0.85031978 | 0.85727907 | 6.959E-03 | 0.85017518 | 1.446E-04 |
| | 5 | - 0.91607692 | -0.92346650 | 7.389E-03 | -0.91597490 | 1.020E-04 |
| | 8 | 0.96349876 | 0.97117074 | 7.671E-03 | 0.96344138 | 5.738E-05 |
| 1 | -10 | 0.31951919 | 0.33425322 | 1.473E-02 | 0.31725256 | 2.266E-03 |
| | -7 | -0.45004407 | -0.46851103 | 1.846E-02 | -0.44798020 | 2.063E-03 |
| | -4 | 0.57156131 | 0.59339159 | 2.183E-02 | 0.56974151 | 1.819E-03 |
| | 4 | -0.89498935 | -0.92021340 | 2.522E-02 | -0.89656332 | 1.573E-03 |
| | 7 | 0.82308087 | 0.84546216 | 2.238E-02 | 0.82493133 | 1.850E-03 |
| | 10 | -0.73469843 | -0.75378899 | 1.909E-02 | -0.73678833 | 2.089E-03 |

**Table 4. Comparison between the MVIM-GA, the exact solution and the absolute errors for some iterations of the solution**

| t | x | $v_E$ | $v_{1(GVIM-III)} p_1 = 0.99973 p_2 = 0.9957 \ p_3 = 1$ | $(v_E - v_1\|$ | $v_{2(GVIM-III)} p_1 = 0.52129 p_2 = 0.49998 \ p_3 = 1$ | $(v_E - v_2\|$ |
|---|---|---|---|---|---|---|
| 0.1 | -6 | 0.30332994 | 0.30331653 | 1.340E-05 | 0.30339603 | 6.609E-06 |
| | -3 | - 0.16582314 | - 0.16576339 | 5.974E-05 | -0.16585733 | 3.419E-05 |
| | 0 | 0.024997395 | 0.02489750 | 9.989E-05 | 0.02499900 | 1.604E-06 |
| | 3 | 0.11632867 | 0.11647661 | 1.479E-05 | 0.11632609 | 3.101E-05 |
| | 6 | -0.25532642 | -0.25551445 | 1.880E-04 | -0.25532944 | 3.020E-06 |
| 0.5 | -4 | 0.66940482 | 0.67544837 | 6.043E-03 | 0.66855930 | 8.455E-04 |
| | -1 | - 0.76754350 | -0.77422360 | 6.680E-03 | -0.76692248 | 6.210E-04 |
| | 2 | 0.85031978 | 0.85750275 | 7.182E-03 | 0.85102963 | 7.098E-04 |
| | 5 | - 0.91607692 | -0.92361896 | 7.542E-03 | -0.91651535 | 4.384E-04 |
| | 8 | 0.96349876 | 0.97124894 | 7.750E-03 | 0.96315693 | 3.418E-05 |
| 1 | -10 | 0.31951919 | 0.33515523 | 1.563E-02 | 0.32983065 | 1.031E-02 |
| | -7 | -0.45004407 | -0.46932148 | 1.927E-02 | -0.45316752 | 3.123E-03 |
| | -4 | 0.57156131 | 0.59409425 | 2.253E-02 | 0.57723409 | 5.672E-03 |
| | 4 | -0.89498935 | -0.91951073 | 2.452E-02 | -0.91404283 | 1.905E-02 |
| | 7 | 0.82308087 | 0.84465171 | 2.157E-02 | 0.82010357 | 2.977E-03 |
| | 10 | -0.73469843 | -0.75288699 | 1.818E-02 | -0.73934964 | 4.651E-03 |

**Table 5. Comparison the absolute errors between the GVIM-III with the modified variational iteration algorithm in the 1st iteration of solution**

| $x$ | t | $AE$ for $u(x,t)$ [6] | $AE$ for$-u_{(GVIM-III)}$ | $AE$ for $v(x,t)$ [6] | $AE$ for $v_{(GVIM-III)}$ |
|---|---|---|---|---|---|
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.04 | 0.04 | 3.918E-04 | 2.413E-04 | 2.003E-02 | 4.994E-03 |
| 0.08 | 0.08 | 1.334E-03 | 9.643E-04 | 4.028E-02 | 9.951E-03 |
| 0.12 | 0.12 | 2.476E-03 | 2.165E-03 | 6.097E-02 | 1.483E-02 |
| 0.16 | 0.16 | 3.471E-03 | 3.838E-03 | 8.230E-02 | 1.960E-02 |
| 0.20 | 0.20 | 3.972E-03 | 5.975E-03 | 1.044E-01 | 2.423E-02 |

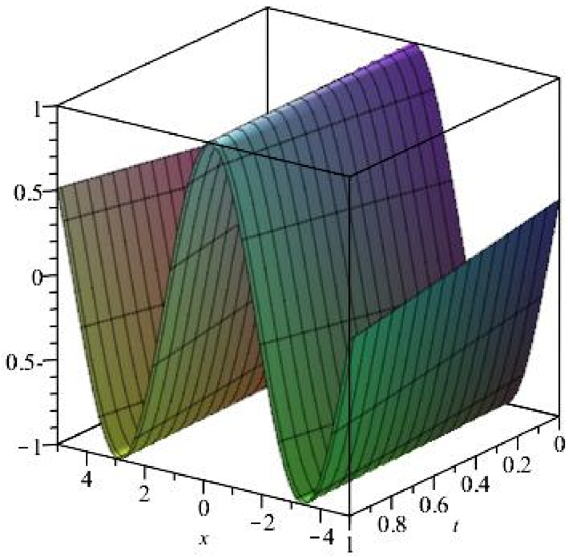


**FIGURE 1.** $u_{Exact}(x,t)$

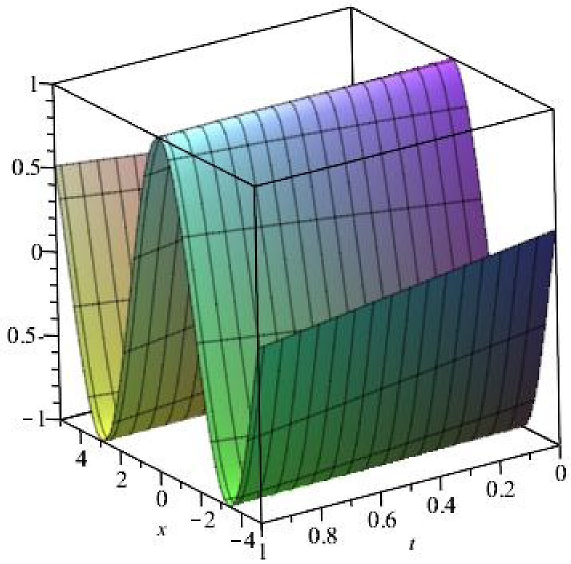


**FIGURE 2.** $u_{VIM}(x,t)$

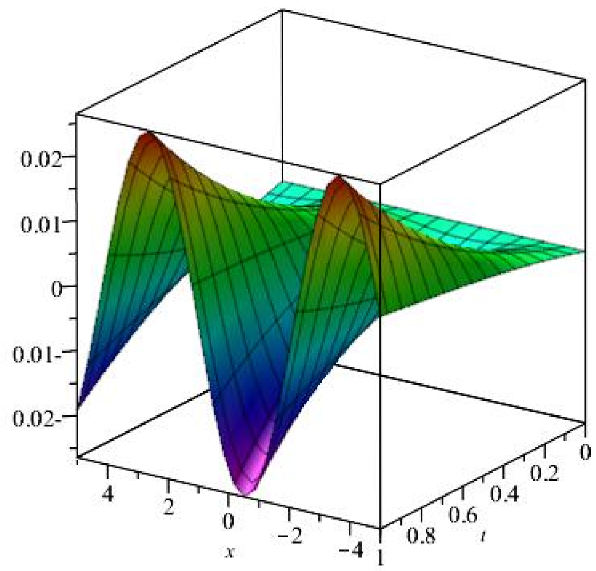


**FIGURE 3.** $u_{MVIM-GA}(x,t)$

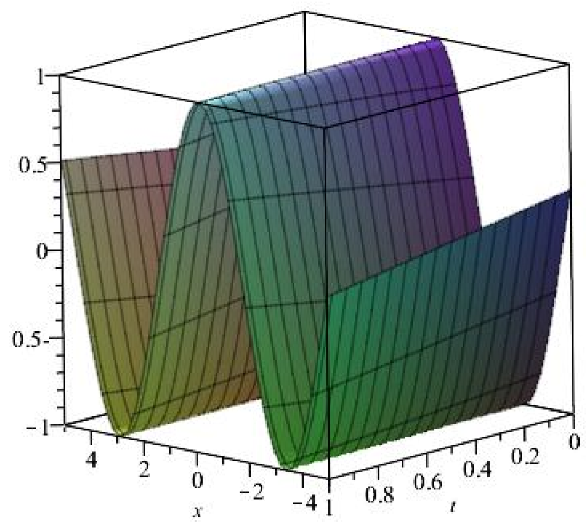


**FIGURE 4.** $(u_{Exact} - u_{MVIM-GA}|$

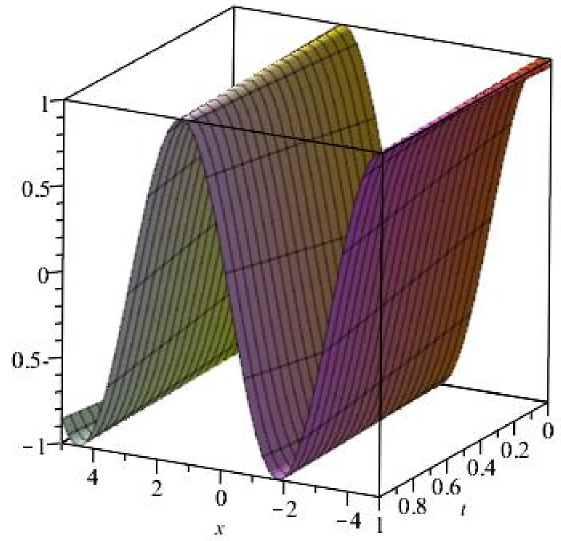


**FIGURE 5.** $v_{Exact}(x,t)$

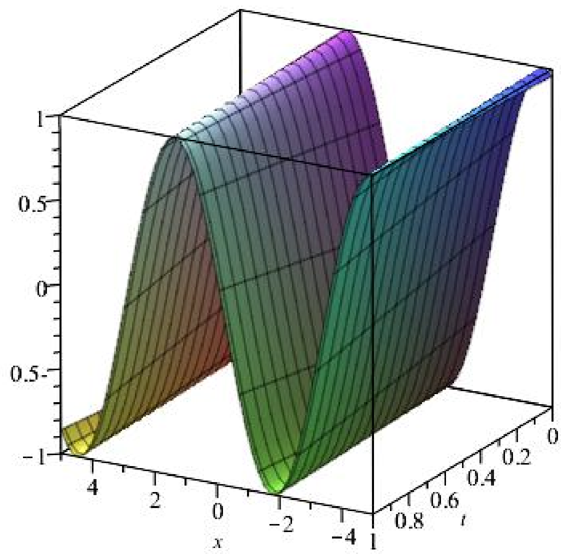


**FIGURE 6.** $v_{VIM}\,(x,t)$

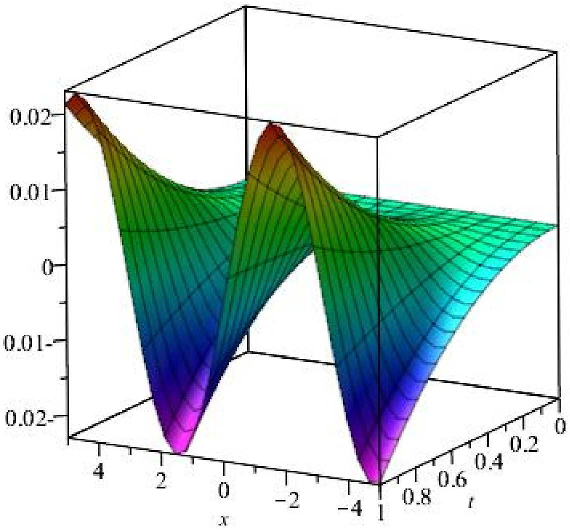


**FIGURE 7.** $v_{MVIM-GA}(x,t)$

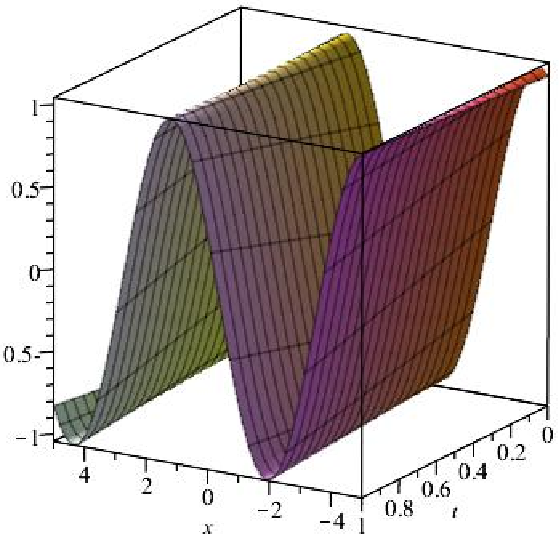


**FIGURE 8.** $(v_{Exact} - v_{MVIM-GA}|$

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## CONFLICTS OF INTEREST

The author declares no conflict of interest.

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