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Learners' Misconceptions and Errors when Solving Inequalities

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Abstract

This study explored Grade 12 learners' misconceptions and errors when solving inequalities. A test on Inequalities was administered to a randomly selected sample of 50 Grade 12 learners in Sekhukhune District, South Africa. A rubric was used to guide the assessment and scoring of learners' scripts. Ten (10) learners were purposively selected based their test responses for interviews to explain their errors, misconceptions and reasoning. Results indicated that learners' errors are due to misunderstandings from prior learning and insufficient mathematical content knowledge. Misconceptions and errors recorded from learners' work include: learners solved inequalities as equations, treated inequality signs as an equal sign, and multiplying both sides of inequalities involving fractions by a variable. Learners had challenges in presenting solutions of inequalities using graphical and number lines. The study recommends that teachers should make an effort to understand learners' thought processes and use this understanding to anticipate learners' misconceptions and errors and prescribe remediation corrective strategies.

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INTRODUCTION

'Inequalities' is one of challenging topics in the high school mathematics curricula in South Africa (Vishal, 2012). Fremont (2012) defines inequalities as a mathematical statement that compares two quantities built from expressions using one or more of the symbols ($<$; $>$; $\leq$; $\geq$). Learners perform poorly in the topic in national examinations (Makonye & Matuku, 2016). The understanding of principles and uses of inequalities is one of the fundamental requirements in mathematics (Almog & Ilany, 2012). Pachpatte (2006) reports that inequalities play a role in most branches of mathematics and have wide applications in other topics such as trigonometry, linear programming, functions, financial mathematics and calculus. It is therefore important for learners to endow them with meaning. To solve inequalities successfully, learners should possess the ability to think critically and solve the problem carefully,meticulously and cautiousilly (Sholihah et al, 2017). However, a study conducted by El-khateeb, 2016) reveal that learners make multiple errors when solving inequalities. A number of researchers concur that some of these errors originate from learners' misconceptions from prior learning (Schnepper & McCoy, 2013; Qian & Lehman, 2017). These

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misconceptions hinder and retard their performance and learning of the subject (Booth et al., 2014). This study sought to highlight sources of these errors and misconceptions which learners commit when inequalities problems and the ways in which those learning. It is important for teachers to familiarise themselves. Furthermore, the study sought to highlight the ways in which those errors and misconceptions interfere with the solution processes. This helps to raise teachers' level of awareness of learners' common misconceptions and errors and correct them during instruction.

Blanco and Garotte (2007) noted that learners make very serious errors and misconceptions when solving inequalities. Solving inequalities refers to finding all values of the variable for which the inequality is true or that satisfy the inequality (Larson, Hostetler & Edwards, 2008). Solving inequalities helps learners with algebraic thinking necessary to develop logical, systematic, and critical thinking skills (Dollo, 2018).Vaiyavutjamai and Clements (2006) identify numerous fossilised misconceptions that guide learners' thinking with respect to linear inequalities. They identified two common misconceptions: the first is related to a tendency to give the answer to the corresponding equation; and the second relates to an expectation that there is only one real-number value of the variable that makes the inequality true. These researchers conclude that a major part of the difficulty that learners experience with inequalities arises from the semantic complexity of inequality tasks. Solving equations is considered to be an important topic in studying properties and applications of functions, which require learners to be aware and to understand different methods of finding the solution set in the different types of inequalities (El-khateeb, 2016). According to Tarraf, Hejase and Hejase (2018) creating intuitive understanding of the concept of inequality and its properties is necessary. Boero and Bazzini (2004) also add that teaching inequalities without taking due account of the concept of function 'implies a 'trivialisation' of the subject, resulting in a sequence of routine procedures which are not easy for learners to understand' (p. 140). Furthermore, Balomenou, Komis and Zacharos (2017) report that learners' solutions to inequalities are incorrect due to misapplications of the 'balance method' that is used when solving equations. The balance method solves equations by doing the same operation on both sides of the equal sign.

Learners' errors and misconceptions reflect their thinking (Gardee & Brodie, 2015). The determination to mitigate errors and misconceptions provides a sense of confidence that has the ability to achieve the desired goals of problem-solving (Solihah, Hendriana & Maya, 2018). From a constructivist perspective, errors are sensible and reasonable to learners. They illuminate important aspects of learners' reasoning, both valid and not valid. Errors reflect the manner in which learners reason and the processes through which they attempt to construct their own knowledge (Herold & Aspire, 2014). Olivier (1989) defined errors as wrong answers that learners arrived at dues to poor planning of the solution algorithm. He further assets that errors are systematic in that they repeated regularly in the same circumstances. Errors are also symptoms of erroneous conceptual structures or schemas that a learner makes in the process of trying to understand a mathematical concept. Misconceptions are the underlying beliefs and principles in the cognitive structure that are the cause of systematic conceptual errors (Ndlovu, Amin &

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Samuel, 2017). Errors can be used by teachers to provide learners with epistemological access to mathematics and contribute to developing learners' conceptual understanding (Brodie, Jina & Modau, 2009). Errors and misconceptions also provide evidence of incomplete and partially acquired mathematical knowledge (Godden, 2012). Teachers can use errors to afford learners epistemological access to mathematics and contribute to developing learners' conceptual understanding (Gardee & Brodie, 2015). However, it is important for teachers to know the source of errors portray during the solution process so that they plan the instruction accordingly. According to Bray and Santagata (2014) errors and misconceptions can be a springboard on which to design instruction. In addition, Hansen, Drews, Dudgeon, Lawton and Surtees (2017) mentioned that understanding learners' errors and misconceptions can help teachers to select instructional and remedial strategies that can engage with learners' misconceptions.In order to help learners unlearn the misconceptions and relearn the correct conceptions, it is important for teachers to be aware of the sources of of these errors and misconceptions as well as their formation.

A number of studies have been conducted on learners' errors and misconceptions in the various branches of algebra (Almog & Ilany, 2012; Mathaba & Bayaga, 2019). Comparatively, few studies address the issue of learners' misconceptions in inequalities. This dearth in research about learners' errors and misconceptions with inequalities accounts for the lack of pedagogical strategies to address the challenges. Without adequate knowledge of learners' understanding of inequalities, the teacher may underestimate the complexity of the learning. Having knowledge of learners' errors is essential for the process of remediating the error, without which teachers simply re-teach without engaging with the mathematical source of the error (Shalem, Aspire & Sorto, 2014). Focusing on errors and misconceptions, as evidence of mathematical thinking on the part of learners, helps teachers to understand how learners think, and consequently adjust their teaching approaches. Failure to take cognisant of learners' errors and misconceptions obscures teachers from using proper remedial and corrective strategies (Saputro, Suryadi, Rosjanuardi & Kartasasmita, 2018).

Teachers must be aware of learners' potential misconceptions before they even start teaching, and are prepared to address them as they arise during the lessons. Making errors is a significant part of the learning process, if these errors are dealt with diagnostically, they can result in meaningful learning (Tulis, Steuer & Dresel, 2016). Most student errors are not of an accidental character, but are attributable to individual problem solving strategies and rules from previous experience in the mathematics classroom (Sarwadi & Shahrill, 2014). This study is an attempt to fill this gap by exploring and examining learners' methods and use of relevant concepts and processes of solving inequalities,that is, their errors, misconceptions and their sources. A study conducted by Younger and Cobbett (2014) in the Eastern Caribbean state of Antigua and Barbuda found that some of the reasons why learners make mistakes when solving inequalities include insufficient time, carelessness, not understanding a mathematical concept at all, confusing different concepts or failing to transition from object-oriented thinking to process-oriented thinking. Botty, Yusof, Shahrill and Mahadi (2015) also reported that learners' lack of knowledge of negative integers, careless mistakes

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and poor basics in solving algebraic equations compounded their attempts to solve inequalities.

It is important to distinguish between an error and a misconception. Luneta and Makonye (2010) define an error as a mistake, slip, and blunder or inaccuracy and deviation from accuracy while Hansen et al.(2014) interpret a misconception as the misapplication of a rule, an overgeneralisation or under generalisation of an alternative conception of a mathematical concept. Misconceptions are misunderstandings about mathematical ideas which learners entertain and which usually lead to errors occurring (Mulungye, O'Connor & Ndethiu, 2016). Misconceptions are the most serious kind of errors which teachers should need to urgently rectify in a learning situation. Errors may occur for a variety of underlying reasons, ranging from the careless mistake (less serious) to errors resulting from misconceptions (more serious) (Hansen, et al., 2020). Learners also make errors if they do not understand what is required of them in a mathematical task (Moru et al., 2014). Errors are systematic and regular, pervasive and persistent across contexts, hence it is important to identify their sources and correct them as they surface. Errors occur at a deeper conceptual level, hence correcting conceptual misunderstandings goes a long way towards addressing them. Khalid and Embong (2019) argues that the underlying cause of errors is misconception. A misconception occurs when a learner's conception conflicts the accepted meanings and understandings in mathematics (Egodawatte & Stoilescu, 215). Errors can be easily identified on a learner's work while a misconception can only be realised by probing further on the supplied response. A misconception represents a deeper lack of understanding, which cannot be solved simply by providing the correct answer in its place, but can recur if not adequately addressed. Misconceptions are built over time, leaving learners with significant gaps in their mathematical understanding that will carry on into later years.

Errors and misconceptions also arise from inconsistent, alternative interpretations of mathematical ideas (Anthony & Walshaw, 2009). According to Anderson (2002) learners' difficulties with inequalities can be attributed to lack of necessary proficiency or knowledge while McNeil and Alibali (2005) argue that misconceptions from earlier learning constrain later learning. Luneta (2015) contends that errors are a result of lack of conceptual understanding while Sarwadi and Shahrill (2014) attribute learners' errors to failure to understand the concepts on which procedures are based. Instead of dismissing errors and misconceptions as wrong thinking, teachers should conceive them as natural and normal as part of a learner's conceptual development. For example, learners hold the belief that multiplication always makes something bigger which is not true when dealing with proper fractions (Bulgar, 2009). Another type of error diagnosed by Botty et al.(2015) was learners' generalisation of their knowledge of solving equations to solve inequalities. Mamba (2013) indicates some ways in which the equation model incorrectly influences learners' approach to solving inequalities: failure to reverse the inequality sign when multiplying or dividing by a negative number; multiplying both sides of a rational inequality by the denominator; solving the quadratic inequality in the same way as solving the equation and overgeneralisation of the multiplicative rule.

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Learners' errors and misconceptions can be used as building blocks for developing deeper understandings. Teachers should embrace errors and misconceptions during teaching and learning. Effective teaching entails identifying what the learner knows and can do and build on these proficiencies rather than identifying and filling gaps(Gibbons, Brown & Niebling, 2018)). Learners' thinking and reasoning becomes visible through misconceptions and errors they commit. Misconceptions and errors must be utilised as opportunities for further learning. Learners' misconceptions and errors must be revealed and used as building blocks for teaching. Mdaka (2014) reiterates that teachers should inculcate a learning culture of encouraging learners to make mistakes as part of problem solving, because errors are seen as part of learning. Competent teachers skilfully make connections for the learners, and ensure that relevant and meaningful tasks provide an appropriate challenge to the learner. Karsenty, Arcavi and Hadas (2007) maintain that knowledge of learners' errors and misconceptions can contribute to teachers' pedagogical content knowledge. Constructivist teaching takes into account learners' current conceptions and builds from there (Brandon & All, 2010).

The type of inequalities in the South African secondary school curriculum include linear, quadratic and rational (Salihu,2017). Most curricula worldwide encourage learners to represent situations that involve equations and inequalities, and that they should distinguish meanings of equivalent forms of expressions, equations and inequalities and solve them fluently (Verikios & Farmaki, 2010). The use of graphs to solve inequalities is highly effective; however, this may not be the case with South Africa since learners lack graphical skills (Phage, 2015). Errors and misconceptions about mathematics concepts affect further learning, due to the hierarchy of mathematics knowledge structure; therefore, it is important to correct these misconceptions for future learning. Son (2013) argued that important insights into the nature of cognitive skills and its acquisition can be gained by examining misconceptions and errors.

The inequalities cut across a number of topics of the South African Grade 12 mathematics curriculum. Therefore, solving problems successfully with minimum errors is an important part of the mathematics curriculum (Yusniawati et al., 2018). The inability to solve mathematical problems is influenced by lack of accuracy in selecting appropriate algorithms or problem-solving models (Yuliana et al., 2019). However, the Department of Basic Education's diagnostic report (2021) highlights that South African Grade 12 learners encounter challenges with inequalities. They could not distinguish between quadratic inequalities and equations and tend to use the word "and" and "or" in the context of inequalities interchangeably. This indicates that they possess and portray some errors and misconceptions in this concept. Therefore, there is a need to explore the sources of these learners' errors and misconceptions on the topic of inequalities and the ways in which they affect their solution process. The study was guided by the following research questions:

a. What are the errors and misconceptions exhibited by learners when solving inequalities?
b. In what ways do these errors and misconceptions interfere with the learners' solution processes?

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Theoretical framework

Research into learners' errors and misconceptions is a key strand of constructivist research. Brodie (2005) highlighted that errors are systematic and consistent across time and place, resistant to instruction, and extremely reasonable when viewed from the perspective of the learner. Thus, this study is guided by a combination of the cognitive constructivism (Cobb & Bauersfeld, 1995) and notions of concept image and concept definition (Tall & Vinner, 1981). Constructivism is an epistemology, or a theory that explains how people know what they know (Bodner, 1986). According to constructivists, problem solving is at the heart of learning, thinking and development. As learners solve problems and discover the consequences of their actions, through reflecting on their experiences, they construct their own understanding. Learning is an active process that requires a change in the learner. This is achieved through the activities the learner engages in, including the consequences of those activities, and through reflection. The two theories help to describe or explain the state of learners' thinking and how learners make sense of mathematical concepts. Learners' errors and misconceptions can better be understood in terms of how they learn. Understanding is a cognitive process so it is worthwhile to consider the theoretical underpinnings of this study in terms of cognitive factors. Hence, errors and misconceptions are embedded in the cognitive realm. Constructivists argue that a learner's mind is the primary unit of analysis in learning (Piaget, 1968). Constructivists posit that a learner constructs knowledge through a cognising mind driven by self-regulation (Wong, 2013). As learners encounter circumstances that are at variance with their current understanding, they develop tension and anxiety, called cognitive conflict, a state of perturbation. This perturbation is a state of mental disequilibrium, driving individuals to explore the problem in relation to their prior understanding. This revision forces learners to think systematically and carefully in order to reconcile and settle their disturbed state. Reconciling this state results in learning.

From a constructivist perspective misconceptions are crucially important to learning and teaching, because misconceptions form part of a learner's conceptual structure that will interact with new concepts, and influence new learning, mostly in a negative way, because misconceptions generate errors. Misconceptions are generated during the process of accommodating new knowledge. The process of accommodating new knowledge is more challenging than assimilating knowledge into existing schema (Wray & Medwell, 2008). By attempting to assimilate knowledge that we should accommodate, we tend to 'overgeneralise' new knowledge based on prior correct knowledge (Ball, Lubienski & Mewborn, 2001). We apply knowledge that is correct in one domain to another in which it no longer works (Smith, DiSessa & Roschelle, 1993). This is why errors are not random; they have some grounding in learners' prior knowledge. A constructivist framework suggests that errors are sensible and reasonable to learners and that they illuminate important aspects of learners' reasoning, both valid and not valid.

State of the literature

According to El-khateeb (2016) the most errors when solving linear inequalities are due to lack of basic algebraic operations and deletion as well as solving fractional and absolute value inequalities. Giltas and Tatar (2011) urged

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that it is the responsibility of teachers to identify learners' misconceptions with inequalities and use these challenges in planning lessons that mitigate misconceptions. Ndlovu (2019) found that the use of an integrated approach (graphic and algebraic) proved to be an effective learning strategy for solving quadratic inequalities in a graphing calculator mediated classroom. The approach allows learners to visualise and interpret the graphs and their properties (for example; zeros, intervals, axis of symmetry, concavity and domain) displayed on the screens of the graphic computers. Bicer et al. (2014) provided evidence that even teachers misconstrue that only one value makes an inequality correct, and treating inequality's solution in the same way as an equation solution; and lack of understanding what an inequality question asks the problem solver to find, having trouble with representing inequalities' solutions graphically. There is a dearth in knowledge about the source of learners' errors and misconceptions when solving inequalities. This study is an attempt to fill this gap by exploring learners' errors and misconceptions when solving inequalities and the ways do these errors and misconceptions interfere with the learners' solution processes.

RESEARCH METHOD

Research design

The study used a mixed methods research design. It utilised a combination of a test and mini-depth interviews with respondents who were purposefully sampled based on the errors and misconceptions recorded from their scripts. This was done to add more detail and explanation to some of their responses. A content analysis design was utilised to identify learners' errors and misconceptions when solving inequalities. Karlsson and Sjøvaag (2016) describe content analysis as a method of studying and analysing communication in a systematic, objective, and quantitative manner for the purpose of measuring variables. As for Hsieh and Shannon (2005), it is a research methodology that utilises a set of procedures to make valid inferences from text. The data for the study were obtained from learners' responses to the test administered by the researchers. A scoring rubric was used to assess learners' errors and misconceptions. Codes were developed in order to create categories of errors and misconceptions.

Participants

Participants for this study consisted of 50 randomly selected Grade 12 learners from Mathematics and Science enrichment and supplementary instruction school. The learners were enrolled at different schools scattered in the Sekhukhune District of Limpopo province but attended enrichment classes on weekends and holidays. Learners were taught the topic 'inequalities' and three tests were administered fortnightly. A rubric was used to assess learners' work. Permission to engage learners was sought from the Provincial Department of Education in Limpopo, the circuits, principals and supplementary instruction centre managers. Participants were issued with consent forms written in plain language statements that clearly describe the aim of the research and the nature of involvement of participants. Participants were also informed of their rights and any risks associated with participation. At all times the researchers observed the welfare of the participants and respected the dignity and personal privacy of the individuals. Letter

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codes (L1 – L50) were used to identify participants so that they remain anonymous throughout the study.

Research tools

The data collection tools for this study consisted of a test and personal interviews in order to assess learners' knowledge of, and approaches to solving, absolute value inequalities. The test was pilot-tested to ensure validity and reliability (Sullivan, 2011). The test consisted of six inequality tasks which can be solved algebraically and graphically. The test questions were aligned with what was taught and discussed in class with the intention to test learners' understanding. The test was administered after learners had studied inequalities. It contained questions from four main areas of algebra: variables, algebraic expressions, equations, and word problems. A rubric was used to assess and score learners' scripts. Conceptual errors and misconceptions were observed and recorded during marking. Ten (10) learners who were purposely selected based on the misconceptions identified in their work were interviewed to shed more light and probe on their misconceptions and their reasoning. The purpose of the interviews was to discuss the learners' approaches to the tasks in order to fully comprehend their ways of thinking and identify the causes of their errors and misconceptions (Gill, Stewart, Treasure & Chadwick, 2008). During the interviews, learners were asked to explain their thinking while they were solving the same problems again. Probing and prompting questions were posed to obtain in-depth information about the nature and origin of those errors and misconceptions (Egodawatte, 2011).

Data analysis

All 50 test papers were assessed using a rubric. All errors and misconceptions from learners' incorrect and partial answers were considered. Questions that were not attempted by the learners were not considered for analysis since no errors and misconceptions could be identified from these responses. Inductive analysis was used to categorise learners' errors and misconceptions into themes. These themes were further confirmed and validated using interviews. Ten (10) participants were selected to participate in the interviews based on the errors and misconceptions recorded in their written responses. Five (5) participants who skipped some questions due to lack of conceptual knowledge of inequalities and procedural fluency were also purposively interviewed in order to ascertain their level of competency with inequalities. Samples of learners' errors and misconception were reproduced and illustrated for reporting purposes in the ensuing sections

RESULTS AND DISCUSSION

The misconceptions from the results of the study can be grouped into the following:

a. Misconceptions with interpretation of inequality symbols
b. Misconceptions due to failure to understand the question
c. Misconceptions with algebraic processes
d. Misconceptions with multiplying both sides of an inequality by a variable
e. (v)Misconceptions with interpretation of word problems
f. Misconceptions with applying knowledge from other topics
g. Misconceptions with reversing the inequality sign
h. Misconceptions with modulus inequality

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Interviews

Interviews were conducted with 15 learners who were purposively selected based on their written answers, errors and misconceptions identified from their work. Learners who presented non-standard answers, either correct or incorrect partial answers or those who did respond to some of the questions due to their level of difficulty or learners who gave only their final result and did not elaborate upon their process of solving the task. The purpose of the interviews was to discuss the learners' approaches to the tasks in order to fully comprehend their ways of thinking and identify the causes of both their correct and incorrect solutions.

Respondents indicated that they lack prior knowledge of solving equations and extend it to solving inequalities. Most of the participants reported a lack of understanding that inequalities have an infinite number of solutions which are not restricted to whole numbers. Most learners struggle with the idea that inequalities can be solved such that the variable is on the left or right of the relation. This may result in learners misunderstanding whether the solution is less than or greater than the boundary found. Learners' responses indicate that they regarded the process of solution as a sequence of routine actions, not worth spending much time on. The interviewees mentioned that they had no strategy when they saw the task. One said that he thought of equations, and went on solving the task, while keeping in mind the exception of not dividing or multiplying by a negative number without reversing the sign. This is consistent with Ward (2016) who concluded that learners' conception of inequalities are meaningless strings of symbols to which certain well-defined procedures are routinely applied.

Misconceptions with interpretation of inequality symbols

Content analysis of learners' answer scripts revealed that they do not have a strong background understanding of inequality symbols and their meanings. It is noteworthy that some learners do not know how to compare numbers using inequalities. This is evident from learner A, who presented her final answer as shown in Table 1.

Table 1. Misconceptions of inequality symbols (Mutodi, 2019)

Learner L6	**Learner L9**
$5x^2 + 4 > 21x$	$5x^2 + 4 > 21x$
$5x^2 - 21x + 4 > 0$	$5x^2 - 21x + 4 > 0$
$(5x - 1)(x - 4) > 0$	Critical values
$(5x - 1) = 0 \ \ or \ (x - 4) = 0$	$x = \frac{-b \ \pm \sqrt{b^2 - 4ac}}{2a}$
$5x = 1 \ \ or \ \ x = 4$	$x = \frac{-(-21) \ \pm \sqrt{(-21)^2 - 4(5)(4)}}{2(5)}$
$x = \frac{1}{5} \ \ or \ \ x = 4$	$= \frac{21 \ \pm \sqrt{361}}{2(5)}$
Critical values	$\frac{1}{5} \ \ or \ \ x = 4$
$\frac{1}{5} \ \ or \ \ x = 4$	
$\frac{1}{5} \ > \ x > 4$	

The written responses in Table 1 indicate that participants changed the given inequality symbol to an equal-sign in almost each of the non-linear inequalities, and solved the problem as an equation instead of an inequality. The written responses above suggest that the learners L6 and L9 know how to solve quadratic inequalities

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up to critical values. The learners treated the inequality as a quadratic equation and correctly found the critical values. However, the learners unsuccessfully used these values to find values of x for which the inequality holds. The learners seemed not to have reflected on the meaning of the symbol '>'. Learner L6 did not realise that the answer $\frac{1}{5} > x > 4$ is meaningless since it implies that $\frac{1}{5} > 4$, which is not true.

The learner also mentioned the phrase 'critical values' towards the end of the working, while the things below that title do not represent critical values. The transition from '$(5x-1)(x-4) > 0$ to $5x-1=0$ or $x-4=0$ was not explained. The learner knows the procedure and words associated with the topic but lacks conceptual meaning. Thus learners conceive critical values as answers to inequalities. Learner L9 has a partially completed answer and this means that the learner misconstrue $x=4$ and $x=\frac{1}{5}$ as the answers to the inequality. This is consistent with Luneta and Makonye (2010) and Ndlovu (2019) who both concluded that learners conceive critical values as solutions to inequalities and therefore provide partial solutions to inequalities.

Misconceptions due to failure to understand the question

Table 2. Failure to understand the questions (Mutodi, 2019)

Learner L21	**Learner L17**	**Learner L26**
$\frac{x-3}{x^2-8x-20} > 0$.	$\frac{x-3}{x^2-8x-20} > 0$	$\frac{x-3}{x^2-8x-20} > 0$
$\frac{x-3}{(x-10)(x+2)} > 0$	$\frac{x-3}{(x-10)(x+2)} > 0$	LCD: $x^2-8x-20$
$x-3=0$ or $x-10=0$	$x = -2$	$\frac{x-3}{x^2-8x-20}$ x $\frac{x^2-8x-20}{1} > 0$
and $x+2=0$	-2 is the smallest	$x-3 > 0$
$x=3$, $x=10$ and	integer	$x > 3$
$x=-2$		
Hence the smallest value of x is -2.		

The most common misconception exhibited by learners (64%) in solving the $\frac{x-3}{x^2-8x-20} > 0$ was to find the values of x and then compare the values based on the magnitude of the numbers. Learners factorised the denominator and equated the linear factors to zero. They obtained the values $x=3$, $x=10$ and $x=-2$. From this set of solutions $x=-2$ is the smallest value, hence it was the most common answer. Another common answer to this problem was $x>3$ (Learner L26), which was obtained by multiplying both sides of the inequality by the denominator (another misconception), $x^2-8x-20>0$ leading to $x-3>0$.

This was finally simplified to give $x>3$. After making this error, learners were supposed to have $x=4$ as their final answer since it is the minimum integer value of $x>3$, however, this was not the case. It can be concluded that learners misunderstood the question based on the responses. Failure to interpret inequality

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questions correctly was also indicated as one of the obstacles by Anggoro and Prabawanto (2019).

Misconceptions with algebraic processes

Table 3. Incorrect algebraic processes (Mutodi, 2019)

Learner L35	Learner L7	Learner L28
$x^3 - 4x^2 + x + 6 < 0$ $x(x^2 - 4x + 1 + 6) < 0$ $x < 0 \ or \ x^2 - 4x + 7 < 0$ $x < 0$; $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(7)}}{2(1)}$ $x = 0 \ ; \ x = \frac{4 \pm \sqrt{16-28}}{2}$ $x = 0 \ ; \ x = \frac{4 \pm \sqrt{-12}}{2}$ $x = 0 \ \ or \ \ 2 + \sqrt{3}$ or $2 - \sqrt{3}$	$2^{2x} - 6.2^x > 16$ $2^2.2^x - 6.2^x > 2^4$ $2^x(2^2 - 6) > 2^4$ $2^x(4 - 6) > 2^4$ $2^x(-2) > 2^4$ $\frac{2^x(-2)}{-2} > \frac{2^4}{-2}$ $2^x > -8$ $x > -4$	$x(x^2 - 4x + 1 + 6) < 0$ $(x-1)(x^2 - 5x + 6) < 0$ $(x-1)(x-2)(x-3) < 0$ $(x-1) < 0 \ or$ $(x-2) < 0$ or $(x-3) < 0$ $x < 1, \ x < 2 \ and \ x < 3$

Another source of misconception observed was failure to carry out algebraic processes such as factorisation. Learner L35 incorrectly factorised the cubic function. Learner L7 incorrectly factorised an exponential function while Learner L28 incorrectly factorised the cubic function using synthetic division. The first three terms of the expression have a common factor x while the fourth term is a constant, but learner L35 forced to create two factors which are all wrong. Learner L7 also struggled to factorise the exponential function. Hence learners failed to apply inequalities in other topics or algebraic processes. The responses indicate that learners do not understand the concept of an inequality and cannot distinguish it from an equation. These findings are similar to what Botty, Yusof, Shahrill and Mahadi (2015) found in their study.

This could also be the result of learners being offered incomplete or a distorted conceptual explanation when the topic was introduced to them for the first time. An instructional suggestion would be to make sure these concepts are explained by presenting not only the complete definition and examples of what it is, but also by showing examples of what it is not, following this by why it is so or why it is not so. These types of misconceptions can be corrected and prevented by carefully selecting and presenting examples that would challenge assumptions of learners.

An instructional suggestion to prevent and remedy the misconceptions with indices is to provide learners with explanations of rules of indices with examples rather than just giving the learners a list of rules that need to be memorised. If the rules are explained with examples and counter examples, that would help learners develop a deep understanding of the concept. It is recommended that teachers carefully choose examples that would not lead learners to develop misconceptions.

Table 4. Misconceptions of multiplying an inequality by a variable (Mutodi, 2019)

Learner L6	Learner L9	Learner L21
$\frac{6}{x-2} - 1 \geq -x - 3$ CLD = $x - 2$	$x - \frac{2}{x} > 5$	$x - \frac{2}{x} > 5$ $x^2 - 2 > 5x$

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$6-1(x-2) \geq -x-3(x-2)$	$x-2>5x$	$x^2-5x-2>0$
$6-x-2) \geq -x^2-2x-3x+6$		$x=\frac{5\pm\sqrt{-5^2}-4(1)(-2)}{2(1)}$
$-x-2) \geq -x^2-5x$	$x-5x-2>0$	
$x^2+4x-2 \geq 0$		$x=\frac{5-\sqrt{33}}{2}$ or $x=\frac{5+\sqrt{33}}{2}$
$x=\frac{-4\pm\sqrt{4^2}-4(1)(-2)}{2(1)}$	$-4x-2>0$	
	$-4x>2$	$x=\frac{5-\sqrt{33}}{2}$ or $x=\frac{5+\sqrt{33}}{2}$
$x=-2+\sqrt{6}$ or $-2-\sqrt{6}$		
$x=0.45$ or -4.45	$x>\frac{1}{2}$	$x=\frac{5-\sqrt{33}}{2}$ or $x=\frac{5+\sqrt{33}}{2}$
$-4.45 \geq x \geq 0.45$		$x=-0.37$ or $x=5.37$
		$-0.37>x>5.37$

Typical incorrect procedures and answers observed with this misconception are shown in Table 4. Seventy five percent (75%) of the learners multiplied both sides of a fractional inequality by a variable or expression involving a variable in all the three questions. Most learners applied their knowledge of fractions and procedures for clearing denominators and converting the whole expression into a quadratic inequality. After they obtain quadratic inequality, they apply the quadratic formula or factorisation to find critical values. This misconception could be due to the fact that learners know the rules for manipulating fractions but lack knowledge of the fact that multiplying both sides by a variable depends on whether the variable is a positive or negative number.

Learners indicated that they are not aware of the fact that multiplying both sides by a variable is only possible where there is an extra condition that the variable is positive or if they are multiplying across an equal sign. Thus, learners apply the rules for manipulating fractions without conceptual understanding. These misconceptions can be prevented by exposing the underlying structure of algebraic fractions to learners while working with arithmetic prior to learning formal algebra. The use of number lines to represent magnitude of fractions helps learners to grasp the concept. Misconceptions with fractions are believed to be rooted in learners' belief that the properties of whole numbers can be applied to fractions. Banerjee and Subramaniam (2012) refer to these misconceptions as 'detachment errors' due to lack of understanding of mathematical structures.

Misconceptions with interpretation of word problems

The following question was given to the learners:

> The object distance p, and the image distance q, for a camera of focal length 3 cm is given by $p=\frac{3q}{q-3}$. For what values of q is p > 12 cm?

Table 5. Misconceptions with interpretation of word problem (Mutodi, 2019)

Learner L41	**Learner L29**
$p=\frac{3q}{q-3}$	$p=\frac{3q}{q-3}$
$12=\frac{3q}{q-3}$	$12=\frac{3q}{q-3}$

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$p(q-3) = \frac{3q}{q-3}(q-3)$	$p(q-3) = \frac{3q}{q-3}(q-3)$
$pq - 3p = 3q$	$pq - 3p = 3q$
$12q - 3(12) = 3q$	Let $p = 12$
$12q - 36 = 3q$	$12q - 3(12) = 3q$
$12q - 3q = 36$	$12q - 36 = 3q$
$9q = 36$	$12q - 3q = 36$
$q = 4$	$9q = 36$
	$q = 4$
	When $q = 4$, $p > 12$

Analysis of learners' responses indicated that they lack knowledge of technical language which characterises the mathematics register. Learners struggled to decode the meanings of words and symbols associated with inequalities such as 'smallest', 'value(s)', 'modulus sign', 'derivatives $\frac{dy}{dx}$ sum to infinite (S_n)' and 'sum(S_∞) of n terms'. Many learners (68) skipped question 3 probably due to language barriers, even though the question had nothing to do with the knowledge of the context described in the question. The main misconception emerging from this item was solving for q after substituting p by 12. Learners 54%) incorrectly interpreted the question as an equation instead of an inequality. The two samples in Table 5 show the misconceptions that learners exhibited when attempting this question.

Teachers should explain explicitly the difference between solving an equation and solving an inequality though the two seem to have a common procedure. The same recommendation was made by Lim (2006). The solutions have different meanings and learners' attention should be drawn to these meanings. An equation has a unique solution or unique solutions while an inequality has infinitely many solutions in each interval (Bazzini & Tsamir, 2001). Through analysis of the learners' incorrect or partially correct answers it became apparent that most of the misconceptions are rooted in algebra, especially the misconceptions with equations and inequalities.

Misconceptions with applying knowledge from other topics

Table 6. Misconceptions with other algebraic topics (Mutodi, 2019)

Lerner L33	**Learner L23**

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$x^3 - 4x^2 + x + 6 < 0$	$\frac{a}{1-r} - \frac{a(r^n-1)}{r-1} < 1$
$3x^2 - 8x + 1 < 0$	$\frac{45}{1-\frac{9}{10}} - \frac{45\left(\left(\frac{9}{10}\right)^n - 1\right)}{\frac{9}{10}-1} < 1$
$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$	$450 + 450\left(\left(\frac{9}{10}\right)^n - 1\right) < -0.1$
$x = \frac{-8 \pm \sqrt{-8^2-4(3)(1)}}{2(1)}$	$450 + \left(450\left(\frac{9}{10}\right)^n - 450\right) < -0.1$
$x = \frac{-8 \pm \sqrt{64-12}}{2(1)}$	$450\left(\frac{9}{10}\right)^n < -0.1$
$x = -4 + 2\sqrt{3}$ or $-4 - 2\sqrt{3}$	$\left(\frac{9}{10}\right)^n < -0.00022222$
$x = -0.536$ or -7.46	$n\ log\ 0.9 < -0.00022222$
	$n > 4.8$
	$n = 5$

The test items required learners to apply knowledge from other algebraic topics such as exponents, sequences and series, modulus functions, polynomials and differential calculus. The test items also required the learners to understand the contexts in which inequalities are applied and check the meaningfulness of their answers. In Table 6 learner L33 solved the inequality by first differentiating the polynomial while an initial attempt indicates that the learner unsuccessfully tried to factorise the cubic function. In some cases learners managed to find correct expressions for (S_∞ -S_n) and ($\frac{dy}{dx}$) but failed to solve inequalities resulting from these manipulations. Learner L23 seem to have knowledge of sequences and series and successfully handled the difference (S_∞ -S_n) but could not use the resulting function to solve the inequality (S_∞ -S_n) < 1. The learner also failed to reflect on the solution and seem not to understand the meaning of the value of n in the context of the problem. Many learners (44%) had n >1.62 as their final answer. Thus, inequalities should not be taught or dealt with in a purely algorithmic manner that avoids connection with other topics as recommended by De Souza, De Lima and Campos (2015).

Misconceptions with reversing the inequality sign

Table 7. Misconceptions with reversing the inequality sign (Mutodi, 2019)

Learner L47	**Learner L5**
$5x^2 + 4 > 21x$	$2^{2x} - 6.2^x > 16$
$5x^2 - 21x + 4 > 0$	$2^{2x} - 6.2^x - 16 > 0$
$(5x-1)(x-4) > 0$	Let $k = 2^x$
$(5x-1) > 0 \ \ or\ (x-4) > 0$	$k^2 - 6k - 16 > 0$
$5x < 1 \ \ or \ \ x < 4$	$(k-2)(k+8) > 0$
$x < \frac{1}{5} \ \ or \ \ x < 4$	$(k-2) > 0 \ \ or\ (k+8) > 0$
	$k > 2 \ \ or\ k > -8$

Table 7 shows learners' misconceptions with reversing the inequality when moving the numbers to the other side of the inequality symbol or when grouping like terms. Both learners L47 and L5 have partial understanding of the solution

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process but worked out the inequality the way they solve an equation and make mistakes towards the end. It could be reasonably inferred that learners were trying to apply some 'rules' without having the fundamental understanding of how or why the rule works. Learners mistakenly construe that moving a negative number to the other side of the inequality reverses the inequality instead of dividing by a negative number. Teachers often tell learners that, when you move a number to the other side of equation the sign changes to opposite (Powell, 2012), which leads to incorrect answers such as the one shown in Table 7. Teachers normally offer complete explanations such as 'when you divide an inequality by a negative number, the inequality sign changes to opposite' with examples demonstrating why that happens. However, the same rule cannot be extended to inequalities since it results in incorrect answers as shown in learners' work in Table 7. Learners were trying to apply the rule without proper understanding. To overcome this misconception teachers should offer complete and concise explanations such as 'when you divide an inequality by a negative number, the inequality sign is reversed or changes to opposite' with examples demonstrating why that happens. For example, it's a fact that 6 < 8 but upon dividing both sides by -2, we obtain -3 and -4 and comparing the two values shows that -3 > - 4.

Misconceptions with modulus inequality

Table 8. Misconceptions with modulus inequalities (Mutodi, 2019)

Learner L44	**Learner L19**	**Learner L11**
$\|2x-1\|<5$	$\|2x-1\|<5$	$\|2x-1\|<5$
$2x-1<5$	$(2x-1)(2x-1)<5$	$2x+x<5$
$2x<6$	$4x^2-4x+1<5$	$3x<5$
$\frac{2x}{2}<\frac{6}{2}$	$4x^2-4x+4<0$	$\frac{3x}{3}<\frac{5}{3}$
$x<3$	$\frac{4x^2}{4}-\frac{4x}{4}+\frac{4}{4}<\frac{0}{4}$	$x<\frac{5}{3}$
	$x^2-x+1<0$	
	No factors.	

Table 8 shows samples of learners' responses when solving modulus inequalities. The three samples of learners' work show three different interpretations of the modulus inequality. Learner L44's written response indicates that the learner perceived $|2x-1|=2x-1$ while learner L19 construed $|2x-1|=(2x-1)(2x-1)$ and learner C interpreted $|2x-1|=2x+1$. Learner L44 interpreted |2x-1| as a constant, hence, |2x-1|=2x-1, and learner L19 interpreted modulus as squaring, that is, $|2x-1|=(2x-1)^2$. Learner L11 interpreted modulus as making everything positive, that is,$|-3|=3$. The majority (80%) of the learners had no clue of the meaning of the modulus function while 20% had a partial understanding of the modulus function. The results indicate that learners have considerable difficulties solving such inequalities. The same observation was made by El-khateeb (2016). Based on these varieties of learners' interpretations of the modulus inequalities, teachers should understand learners' thought processes and use this understanding to conduct remediation and enhance proper conceptual understanding of the modulus inequality.

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Discussion

The analysis of the results shows that learners have difficulties in solving inequalities. The study found that the errors do not arise by chance as learners work on the problems, but rather learners have conceptual misconceptions based on mathematical concepts in the earlier grades. Results for this study are consistent with the finding of Blanco and Garrotte (2007) who confirmed that the basis of learners' errors lies in their prior experiences. Learners apply knowledge of solving linear and quadratic equations to inequalities. Most (85%) of the participants changed inequality symbols to an equal-sign in most non-linear inequalities, and solved the problems as equations instead of inequalities.

This is consistent with the findings of El-khateeb (2016)- who found that learners lack the basic concepts and skills associated with the concept of equations and inequalities and ways of solving them. The results concur with Switzer(2014) who argued that neither in the classroom nor in the textbook are the properties of inequalities and of equations made explicitly visible for the learners. The results of the study are consonant with Balomenou et al (2017) who reported that learners' solutions to inequalities are incorrect because of the incorrect application of the 'balance method' that is used when solving equations. Thus, learners' errors in solving inequalities have their origins in quadratic equations and linear equations. They recommended that in order to promote conceptual learning, these properties should have been introduced and exemplified. In this study we strongly argue this is actually based on their prior knowledge of concepts covered in earlier grades. Learners who use algebraic methods tend to get correct critical values but they fail to use these values to find intervals in which the inequality holds. The meanings of the symbols '<', '>', '=', '≤', and '≥' were not yet grasped by learners. Such misconceptions were common in learners' final answers such as $\frac{1}{5} < x > 4$.

Serious misconceptions were also observed when solving inequalities involving fractions. Many learners multiplied both sides of inequalities by a variable or by an algebraic expression without changing the inequality sign. Thus, learners treat variables as if they belong to a set of natural numbers. Learners perceive the unknown and endow it with the need to find specific values for the letter deriving from its use in equations represents a major barrier to their interpretation of the solution of an inequality.

CONCLUSION

The findings have implications for teaching inequalities, particularly in raising teachers' awareness of learners' errors and misconceptions. The results raise questions about the methods of teaching this section in mathematics, such as how to teach various approaches to solving specific inequalities such as the graphical approach and functional approach. Error analysis may be incorporated in the teacher training curriculum as it will assist in reducing or eliminating learner errors. It will assist educators to be able to identify learner errors, assist learners in eliminating those errors and encourage learners to review the work before submission. Understanding learners' rationale when going through their work can also assist teachers to institute remedial lessons. Educators need to incorporate error

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analysis in their lesson designs, as knowledge of why learners commit errors is valuable to the educators as it will help in selecting the relevant strategies.

Based on the errors and misconceptions emerging from this research, teachers should take note of following when teaching inequalities:

a. Ensure that learners are familiar with inequality symbols in terms of shape, meaning and semantic.
b. Ensure that learners clearly acquire the procedural and conceptual differences between an equation and an inequality, with clear implications of what it entails especially when interpreting their solutions. Equations have unique solutions while inequalities have infinitely many values as solutions.
c. Promote the use of multiple strategies such as graphs and number lines when solving inequalities in order to enrich learners' acquisition of the inequality concept.

The findings of this study revealed that some of the errors and misconceptions exhibited by learners were due to lack of conceptual understanding and procedural fluency as well as limited knowledge of applying inequalities in other algebraic topics. It is therefore essential that teachers devote equal importance on both conceptual understanding and procedural fluency. Knowledge of common errors and misconceptions, how they are caused, and how they can be prevented and remedied should be known by teachers to ensure they plan classroom instruction and activities that reduces the occurrences of these errors and misconceptions.

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