A Finite Element Analysis of a Moving Load on Slab Track by a High-Speed Train

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List of Abbreviations

HSR - HIGH SPEED RAIL

ERS – EMBEDDED RAIL SYSYTEM

RTS – RHEDA TRACK SYSTEM

HBL – HYDRAULICLY BOUND LAYER

FPL – FROST PROTECTION LAYER

CBL – CONCRETE BEARING LAYER

KPH – KILOMETERS PER HOUR

UIC – INTERNATIONAL UNION OF RAILWAYS

HS2 – HIGH SPEED 2

FEA – FINITE ELEMENT ANALYSIS

ST – SLAB TRACK

TGV – TRAIN à GRANDE VITESSE (HIGH SPEED TRAIN)

ICE – INTERCITY EXPRESS

Abstract

As the challenge for rapid transport and higher frequency rail services around the world increases, greater stresses are induced on the track, thus the necessity for a sustainable and durable railway structure is essential. To meet these demands, it is seen that Slab Track (ST) construction is generally preferred for High-Speed Railways (HSR) as opposed to the conventional ballasted track as the ST can sustain higher dynamic loading thus less maintenance is required, ultimately lowering the life cycle costs of the track. The principal drawback of this design technique is somewhat due to the high construction costs, mainly due to earthworks. In line with this research, a study on the development and application of a 3D structural model using ABAQUS software is carried out to evaluate the dynamic behaviour of HSR ST and to further evaluate the displacements that occur at the subgrade/concrete layer during the high-speed passage of trains. This research aims at evaluating two types of ST systems, namely:

1. The continuously fastened Embedded Track System (ETS) which was first conceived by Charles Penny in the UK. This system offers superior safety, performance, and availability (Penny, C., 2009).
2. The RHEDA (RTS) ST system, which is discreetly fastened. This approach to railway design and build was originally used in Germany by the Deutsche Bahn at Rheda-Wiedenbruck station (Michas, G., 2012).

The two modelled rail tracks consist of a rail fastened onto a slab laid on a suitable foundation. The foundation consists of a subbase layer which railway engineers label as the hydraulically bound layer (HBL) placed on a capping layer, referred to as frost protection layer (FPL) overlaying the subgrade soil. This research is vital as ST construction is commonly used when building a HSR line due to its evident advantages, however we still do not know fully how the behaviour of this type of structure performs and design standards have not yet been fully specified when compared to ballasted track which has been the conventional way for building high speed lines for the past 150 years. The study is imperative as it sheds light on the performance of the ETS (not as frequently used as the RTS) which if effective can be more economical and sustainable as it has far less components.

This thesis considers findings of dynamic analysis of various parameters which are affected during the high-speed passage of trains. Constant speeds are applied to a moving load as part of dynamic analysis. Parametric studies are performed for the concrete bearing layer (CBL) stiffness, soil stiffness and CBL thickness. The motion speeds are also varied to see its effect on the rail track. In this study, the behaviour of the two ST systems with changing parameters are illustrated. It is seen that the influence of the soil stiffness on the rail track behaviour is far more than the concrete stiffness. Moreover, increasing speed from 180kph to 360kph increases the deformation response of the track significantly. It can be said that the speeds affect vertical deformations in the soil layer. The overall deformation does not change much from the CBL to the soil layer, implying that CBL is absorbing much of the load.

The intensity of static load is minimal when compared with moving load analysis. Finally, it is seen that the point of maximum deflection shifts behind the load with increasing speed.

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Chapter-1: Introduction

Due to the worldwide increase in population growth. a substantial amount of overcrowding in towns and cities began to occur. People set out to reside outside of the major cities into smaller towns and suburbs. Subsequently, Railway systems began to become favoured over road and air transportation when travelling intermediate distances to commute in and out of the city. The reason behind this was because railways are efficient, reliable, and cheap as well as the higher speeds that can be achieved with the advances in new design methods and technology. As a result of this, a competitive high-speed rail (HSR) system is one of the biggest challenges any country faces in terms of infrastructure development.

Developments in technology have resulted in a fast-moving expansion of HSR in the last two decades with many lines planned for construction. This movement is because HSR is far more environmentally friendly than road and air travel. As HSR generally runs on electricity it produces less air pollution which is terrific for the environment as it reduces global warming since there is less air pollutants (Krylov, 2001) thus, there will be far less deterioration of human health. HSR reduces the burden on roadways by reducing the traffic density by adding more commuters to railways.

ST concepts have become favourable when constructing a modern HSR as opposed to the traditional ballasted track system. The ST is a system whereby concrete is cast-in-situ or constructed in discrete precast sections that are laid resulting in a continuous path of concrete or sometimes asphalt; thus, replacing the conventional ballasted track method. This technique of railway construction was found to be very much appealing as huge amounts of soil treatment can be avoided, it was revealed that by increasing the width of the concrete slab and by applying reinforcement in the concrete bearing layer (CBL), a remarkable amount of soil treatment can be avoided (Steenbergen, 2007). Asphalt is accepted and is used in the construction of the ST systems; however, a large number of ST systems use concrete.

1.1 Aims and Objectives

This dissertation aims to contribute to the design and development of future ST systems. The objectives include to investigate the ST structure by modelling it as a 3D problem where the solution is then obtained by using the Finite Element Analysis (FEA) software ABAQUS. The designs like the RHEDA (RTS) and Balfour Beatty Embedded (ETS) ST system have been modelled. This consists of three layers; namely the concrete bearing layer (CBL), Subbase (Hydraulically bonded layer, HBL) and Capping layer (Frost protection layer, FPL). FPL is used for freezing temperatures over a prepared subgrade.

1.2 Scope of Study

The dynamic analysis was carried out to see the effect of load motion on the elastic rail track. Parametric studies on stiffness of materials and thicknesses of layers are performed with dynamic load. The results of these studies can be used to design an economically viable rail track based on the speed and load regulations.
The two systems studied are;

a) The German RHEDA 2000 system (RTS), a widely used discretely supported system, where the rail is supported by sleepers encased in a concrete bearing layer (CBL).

b) The continuously supported Embedded Rail Structure (ETS), where the rails are embedded into the concrete slab.

1.3 Historical Background

The RTS system seen in figure 1, is the most frequently used ST system in the world with over 400 kilometres in Germany (Bastin, 2005) and other short sections in Holland, Taiwan, Spain, China, Greece and in Britain. The reason for this is because the RTS system has performed satisfactorily since it was first introduced in Germany and has had longer experience in the developmental and construction stage thus allowing it to be favoured when opting for the ST option. Furthermore, the RTS system is free from any patent rights meaning it has been under continuous development by contractors since it was first introduced (Talampekos, 2000).

There are various versions of ETS system as these have been used in Europe since the 1970s (Tayabji and Bilow 2001). This paper studies the Balfour Beatty ETS, an innovative low noise ST specifically designed for high speed-rail but also ideal for heavy haul, mixed traffic, metros, and light rail (Penny, 2004). In the ETS a block rail (applied to achieve improved acoustic properties and low structure height) together with pad and shell and this assembly is then grouted into the concrete as seen in figure 2. It has been identified in existing literature that the continuous support on the ETS cause wheels to not experience any differences in vertical stiffness, a major source of corrugation development (Esveld, 2003) and as a result can be very expensive in the maintenance of the rails which ultimately impact the substructure due to the dynamic impact loading caused by corrugations and irregularities in the rail head. These knock-on effects cause unanticipated track closures which increase expenses and ultimately increase the rail industry’s carbon footprint.

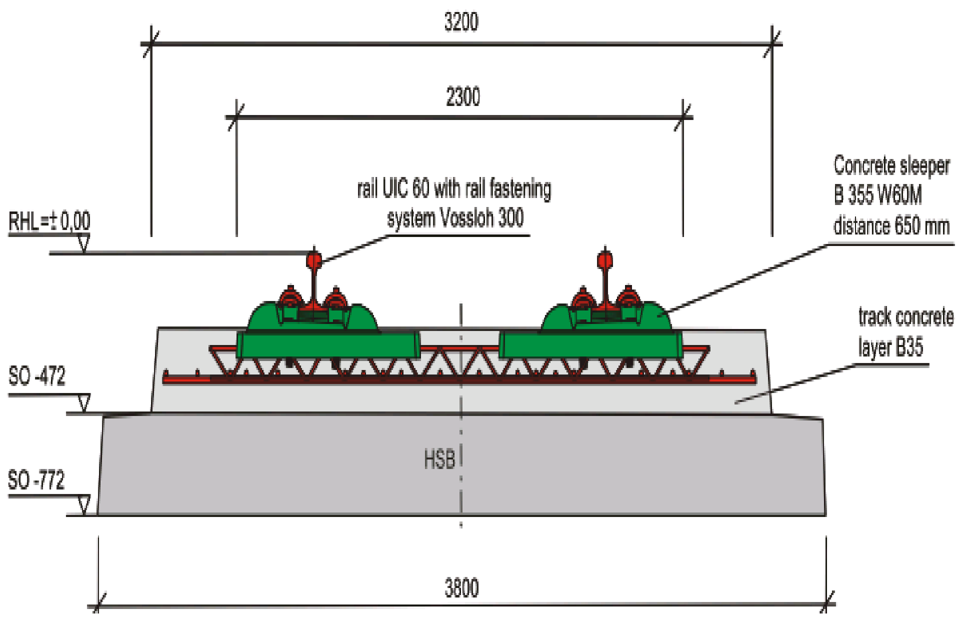

Figure 1. RHEDA system (Esveld, 2003)

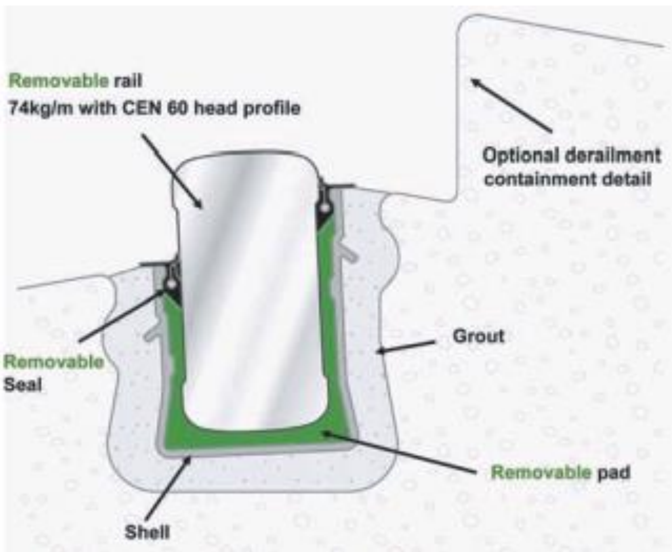

Figure 2. ETS rail (Esveld, 2003)

Chapter-2: Literature review

Currently the most used rail track system is still the ballasted track. Although this track system is outdated it is still favoured amongst all other track concepts as it is the one which railway engineers have known and developed for the past 150 years (Whitmore, 2014). This construction method consists of two parts:

1. Substructure
2. Superstructure

The superstructure of ballasted track system includes the rails, fastening system, sleepers, and ballast bed. The main function of the ballast is to absorb the loads induced by passing trains, provide good drainage and support the weight of the track thus ensuring track stability (Indraratna et al, 2010).

It is of high value to use stiff stone like granite or other equivalent materials for the ballast layer. These stones should be angular in shape so that they interlock with each other thus provide friction between the grains which prevents undue movement of sleepers. The ballast layer ought to be at least 300mm in thickness underneath the sleeper. The connection between the superstructure and substructure is guaranteed by the sleepers which have the functions to receive and distribute loads from the rails, to transfer them to the ballast. Sleepers also ensure that track gauge is maintained and are laid in discrete sections approximately 0.6m to 0.65m (Whitmore, 2014).

Ballasted track is a longstanding concept for the design and build of railways. It was first established in the UK and as demand for rail networks grew, this method of construction developed to be the standard railway construction approach around the world.

The traditional track foundation design comprises three layers, namely:

1. Ballast
2. Sub-ballast
3. Subbase

These materials are all aggregates that consist of different material properties i.e., rock strength, grain size distribution and cohesion. The ballast aggregate in this system plays a crucial role as it not only transfers the various loads induced on the track to the ground but also help drainage which is a crucial issue especially in wet climates (Whitmore, 2014).

It is of high importance to emphasize that the design of ballasted track has been almost untouched since it was first introduced several hundred years ago (Indraratna, 2011). Ballasted rail track systems are widely used all over the world because of the high resilience it provides to the repeated wheel loads of trains. Furthermore, ballasted track is very cheap to install and is easily maintained (Indraratna, 2010). In the early days of track construction, the railways were ballasted with a variety of materials which today would be deemed as completely unsuitable. An example of these materials would be the use of ashes, chalk, burnt clay etc (Cope, 1993). Understanding of the importance of materials later demonstrated that it is very much essential to use a layer of good quality ballast under and around the sleepers, thus the use of gravel, crushed limestone or igneous rock was used. Due to the longstanding use of ballast a surplus of experience has been amassed relating to the requirements of the depth and layers of ballasted track for minimum maintenance and good running. There are also codes of practise which have been produced over the years that are constantly being reviewed and amended. Figure 3 Shows a cross section of a ballasted track structure

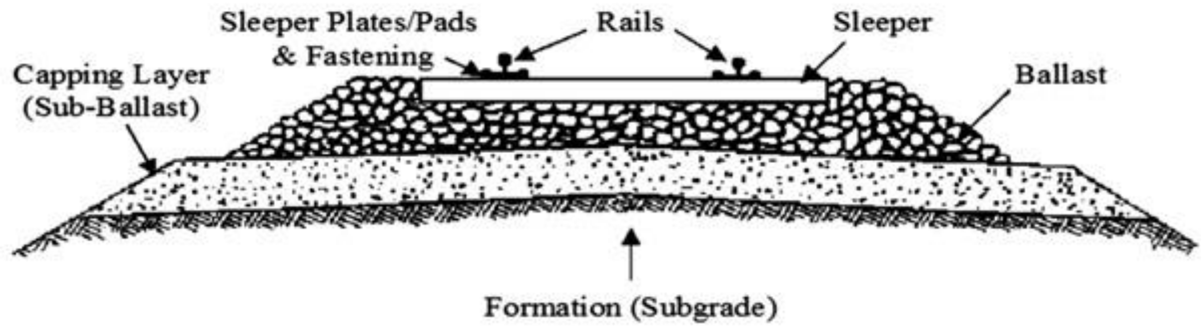

Figure 3 Ballasted Track cross section (Elkhoury 2018)

2.1 Slab Track Technology

An Increase in demand for HSR and higher axle loads necessitated the development of a more rigid and low maintenance track structural solutions. These track solutions are composed of rails fastened to a concrete slab which sits on an appropriate foundation and is typically known as Slab Track (ST). Japan was the first to use ST in the construction of the Shinkansen (bullet train) which later stirred the French in the 1960's to build the Train à Grande Vitesse (TGV) translated to "high speed train" (Arduin et al, 2005). Although the French focused more on the development of the rolling stock for its high speed, the Japanese chose to concentrate more on the geometry and track structure. The ST system consists of prefabricated slabs which are 5m long, this design has practically been unchanged since its birth in 1972 (Tayabji et al 2001). Currently HSR has expanded in Europe due to its demand (Khabbaz, et al, 2014) with networks successfully operated in the Netherlands, Spain, Italy, Germany, Russia, Austria, Sweden, Belgium, and the United Kingdom. Although some of the above-mentioned countries still resort to the conventional solution for building its rail track, many are now shifting from the traditional system and opting for ST solutions for its railways. The reason for this is that ST construction has many advantages when compared to the conventional ballasted track. These advantages include excellent geometrical stability, lower construction depth (great for tunnels), good riding comfort at high speed, allows for steeper route gradients (Gautier, P.E., 2015). Still, it comes with disadvantages like small adaptability to high track displacements, is very expensive and can be even more costly in areas of land with soft clays.

the ST structure is a concrete or sometimes asphalt layer which reinstates the ballast in the conventional track system. As the end results of this form of construction is a rigid superstructure, the required elasticity is achieved by inserting an elastomeric layer underneath the slab or sometimes below the rail or the sleeper (Lichtberger, 2011). With ST construction the structure is usually made up of five layers (Figure 4). ST design is attempted in two different ways, namely discrete rail support or continuous rail support and based on these two methods many ST concepts have been developed and used around the world for its HSR lines.

At Present there are many ST designs which have been established and what identifies which system is best, depends on key factors which according to Esveld, 2001 and Lichtberger, 2011, need to be addressed prior to the construction process; these include but are not limited to the soil conditions, the supporting layers to be used underneath, the location of construction e.g. tunnels, bridges or open sections, the amount of traffic, the axle load, noise restrictions, construction costs, climate/terrain.

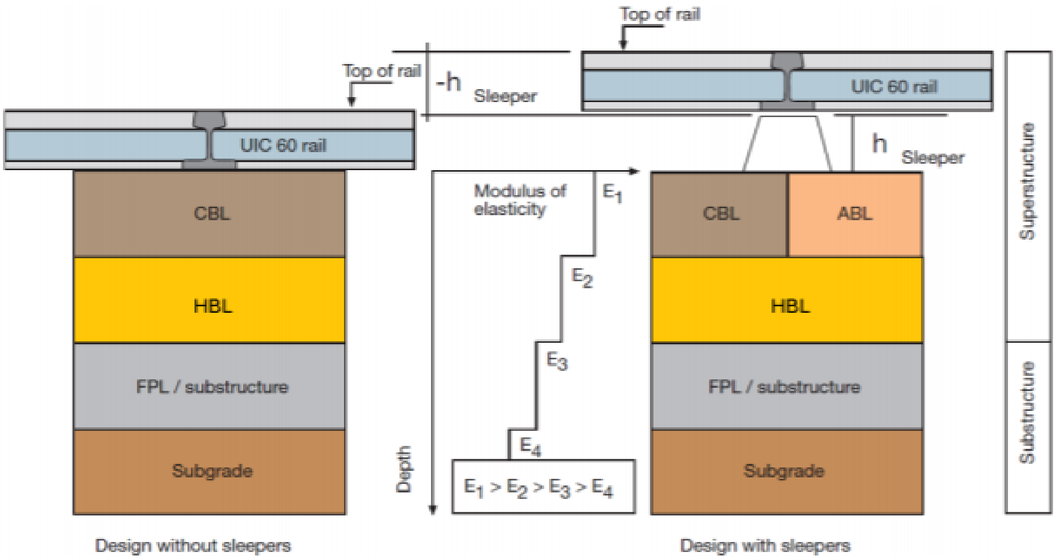

Figure 4 Construction profiles for Slab Track (Darr, 2000)

In the UK, this type of railway construction is mainly limited to tunnels, underground networks, and bridges. This is partly due to the rigid supporting conditions provided (Esveld, 2003). Although the initial costs of ST are high, ST is superior when it comes to passenger comfort (smoothness of ride) and maintenance costs which are exceedingly low when matched with ballasted track construction, thus providing long term benefits like the reduction in life cycle costs (Bilow, and Randich, 2000). There are still however uncertainties when calculating the overall behaviour of ST construction as there is no ST line currently in the world which has completed its design period (Gautier, 2015).

The main differences between the two track types (ballasted and non-ballasted) are that despite the high construction costs, maintenance is far less for ST systems which reduces life cycle costs over time (Esveld, 1999). Furthermore, since the ballast is eliminated, there is less dust released into the surroundings. Additionally, advancements in ST technologies permit higher speeds with less maintenance since the superstructure of the track is far more rigid than the ballasted track, thus allowing for an increase in lateral and longitudinal stability. This facilitates in the reduction of track closures and consequently allows for higher train frequencies (Esveld, 2010).

2.2 Slab Track Systems categorization

There are many types of ST systems which are used around the world and can be divided into two classifications, namely, the discrete rail support and the continuous rail support system as shown in figure 5. These two categories can be further divided into subcategories as shown on Table 1 and Table 2.

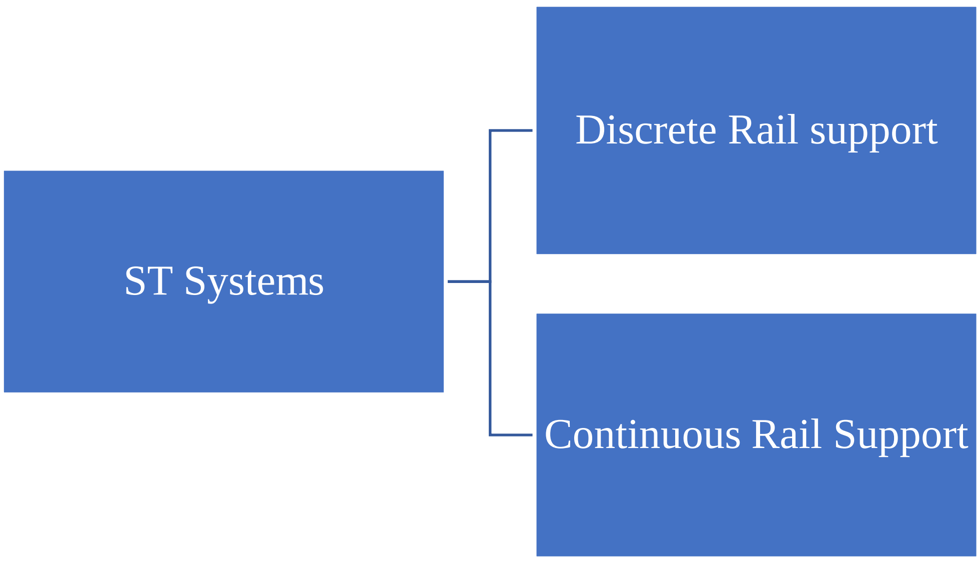

Figure 5 Slab Track system classification

Table 1. Discrete Rail Support

Prefabricated concrete Slab	Sleepers/Blocks encased in concrete	Sleepers on Top of Asphalt Roadbed	Monolithic Designs
SHINKANSEN	RHEDA 2000	GETRAC	PACT
BOGL	SONNEVILLE-LVT	WALTER	LAWN TRACK
OBB-PORR	ZUBLIN	SATO	HOCHTIEF

Table 2. Continuous Rail Support

Embedded Rail Structure	Clamped and Continuously Supported Rail
BALFOUR BEATTY EMBEDDED RAIL SYSTEM	Vanguard
DECK TRACK	SAARGUMMI
ENFUNDO – EDILON	COCON TRACK

2.3 Balfour Beatty Embedded Rail System (ETS))

The ETS emerged since the early 1990’s structuring its ideas on European concepts. The ETS’s main objective was to provide a reduced whole life cost, have greater performance and low maintenance. Its developer Charles Penny explored many solutions merging all the different advances in structural, geotechnical, and material sciences.

In the early 2000’s Balfour Beatty was able to develop the ERS. It can be seen in figure 6 this ground-breaking advancement encompasses a continuous concrete slab that includes two slots at either end, allowing the rail sub-system to be embedded into the slab. After much development and testing at Munich Technical University, which further established the unique performance of the system. It was then decided that trial instillation tests were to be carried out at Beeston, UK which demonstrated its effectiveness. This was then followed by successful installations in a high-speed test track at Medina el Campo, Spain in 2002 followed by Crewe, in the UK in August 2003 to which the system received Network Rail acceptance in February 2006 (Charles Penny 2009).

It was noted that this system had many advantages including quick instillation which therefore means less risk of programme overrunning. Moreover, as there are only two replaceable components there is less risks of failure and considerably fewer checks on foot thus decreasing health and safety consequences and costs for labour. As shown in figure 6, The ETS uses a U-shaped glass reinforced plastic shell and a U-shaped pad made up of micro cellular polyurethane to fit both the shell and rectangular rail.

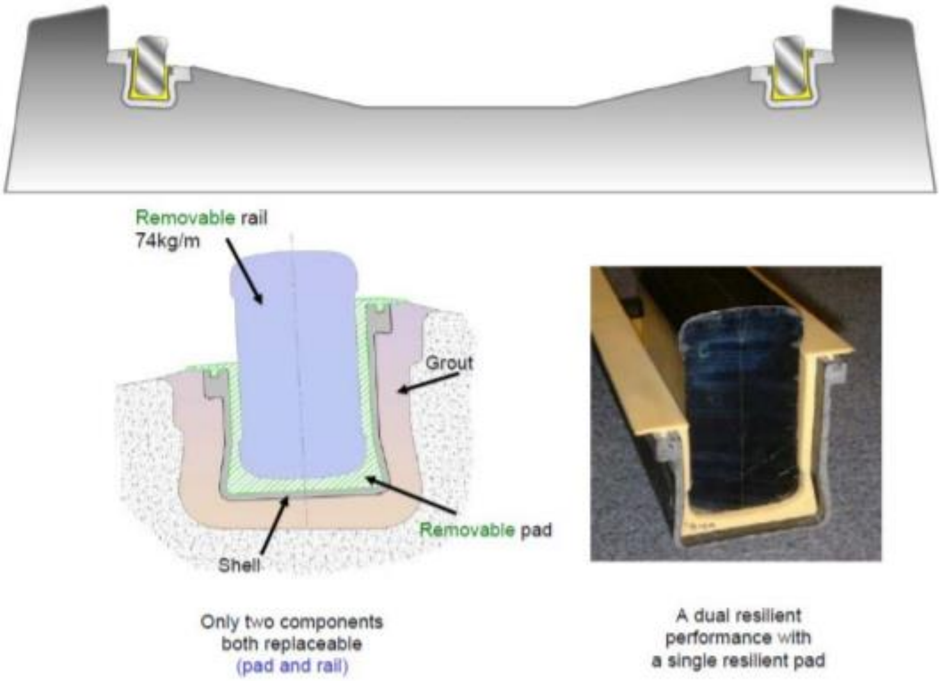

Figure 6 ETS System (Penny, 2009)

The construction process for this system can be done in three different methods which is very ideal in case of any drawbacks/time constraints. One can install this system by means of slip forming. This is done by initially excavating and preparing the formation in order for the lean mix base for the sab to be placed. Reinforcement is then placed followed by slip form. The shell and pad are then aligned and grouted into final position. Finally, the rails are aligned, installed, and welded.

The second method is to use pre-cast concrete slabs. For this you will need to excavate and prepare formation for laying of slabs (depending on the ground conditions). The pre-cast slabs are then laid on the ground and aligned before grouting into final position. The pre-cast slabs are then joined, and the rails are finally installed and welded.

The third and final method for constructing the ERS system is by casting in-situ. Again, it is worth mentioning that it is of high importance to prepare the formation and ensure it is free from any settlement before the slabs are laid on the ground. This is because with slab track construction the tolerances for alignment are marginal once the track is laid and it can be very costly to reinstall the slabs again. A lean mix for the slab is then placed followed by steel reinforcement. Thereafter the shuttering is assembled, and concrete is poured. The sub-system is then aligned, and the shell is grouted to its final position. Finally, the rails are distributed, aligned, and welded into their final position.

2.4 Rheda System (RTS)

The RTS (Figure 7) is highly adaptable thus enabling alterations in design and allowing improvements and hence permitting it to fit the requirements of each individual construction plan. This design has many advantages including years of experience in the design and installation process in comparison to other ST systems. The RTS was first installed in the early 1970s, this system is continually refined as it is free from any patent rights thus allowing development of the system to continue and grow since its very first emergence (Talampekos, 2000).

The RTS was first introduced in 1972 in Rheda (Germany). Although the design has improved throughout the years the original classic Rheda model is still used as a template for design and keeps most of its usual characteristics. The construction method for the RTS includes using horizontal and vertical adjustments to alter track position, the use of encased concrete sleepers which sit on a HBL (Hydraulically bonded layer) that is 30cm in thickness and a frost protection layer (FPL) around 50cm in thickness. Furthermore, an additional common characteristic is the fact that C30/37 with regards to the concrete slab is the minimum quality of the concrete (Esveld, 2003).

Since the year 2000, the RTS design has been widely used across the world, it is a ST system that is currently used in more than 10 countries including Britain (J. Kleeberg, 2009). After more than 35 years of operational knowledge of the preliminary system, the RTS can be deemed as technically stable and consequently manageable. The flexibility of the RTS is further highlighted in the fact that it is used worldwide, mainly in the three HSR international train systems, specifically, the Shinkansen for Taiwan, the TGV for France, and the ICE in Germany where its use is seen in a variety of terrains, climates, and subgrade conditions.

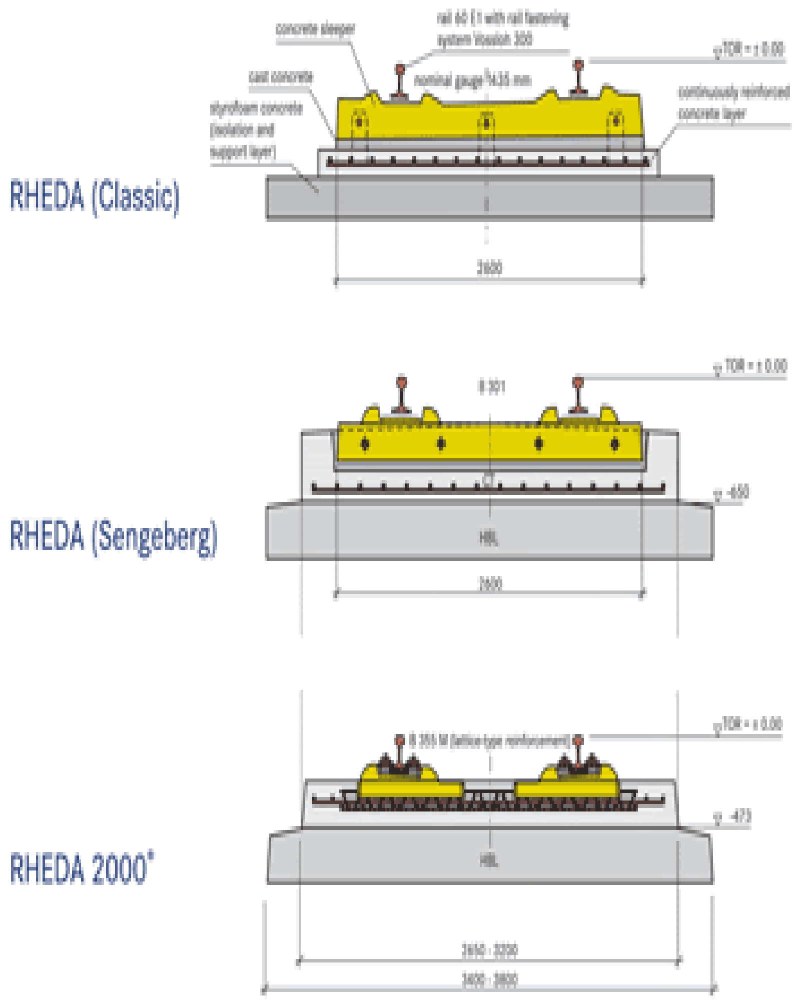

Figure 7 Developmental stages of the RTS design (Giannakos, K., 2010)

2.5 High-Speed Rail Ground Vibrations

Although high speed rail has many positive outcomes ranging from economic, social, and environmental qualities, there are some adverse effects due to the high speed required for speedier travel. It is seen that vibrations from high-speed trains differ from those of low-speed trains which are induced on the track structure and underlying soil. When a high-speed train goes over the ground structure, resonance may occur thus the dynamic response of the track and ground is amplified which causes weakening of the track (Chen 2018).

This amplification of ground-borne vibrations generated at the wheel/rail interface arise from variations in support stiffness, train load and irregularities in the wheel/rail geometry. Additionally, the amplitude of the vibrations is raised radically when the train speed becomes equal with the natural Rayleigh wave speed in the supporting ground (Connolly 2014). In neighbourhoods near to the lines, the vibrations can cause major negative consequences such as personal anxiety if the problem is not addressed and a solution is not found (Persson 2016). Furthermore, these ground vibrations can cause irregularities in the track which as a result can cause major accidents at high-speed travel. This is obvious from the crash reports on high-speed trains; of which is when two sets of Eurostar high-speed train wheels operating at maximum speed (300km/h) derailed due to irregular subgrade in 2000 causing 14 casualties. Train accidents emphasize the fact that railway engineers still require to advance designs and concepts and it is prudent and of high importance to explore the functioning of high-speed rail under dynamic loads. These investigations are compulsory for the train, track, and subgrade to fully grasp the performance of the track system at high speeds (Chen, 2018).

The effect of the subgrade layers and properties of the track system on the critical velocity was studied by Costa et al (2015) via a semi-analytical and 2.5 FEM approach. It was highlighted that the critical velocity was marginally less than the Rayleigh wave velocity in the homogenous subgrade. However, it was observed that in the layered subgrade, the critical velocity depends on the characteristics of the subgrade layers and the mechanical properties of the super/sub - structure of the track system.

Kouroussis et al (2013) emphasized the vibration levels to be higher with train speeds that exceeded the Rayleigh wave velocity. However, it was noticed that layered ground exhibited varied effects for critical velocity in the layered ground which depended on the depth of the ground layers and their material properties. The most well-known case where the critical velocity phenomena was observed (although found on various lines around the world) was at Ledsgard in Sweden. In this track section the subgrade soil layers had a low stiffness which resulted in a low critical velocity of the site. Elevated track deflections were therefore observed, especially as the speed of the train increased (Madshus, 2000).

2.6 Finite Element Analysis

There have been many FEA models established to optimize the design of the railways, these models are primarily for ballasted track for which several non-linear three-dimensional models have been developed (Texeira, 2003).

Feng (2011) studied the dynamic response of track under influence of design parameters. This study used ABAQUS software to carry out the parametric studies to obtain a further comprehensive understanding of the behaviour of the track. The tracks that were simulated include beam on discreet support model. The rails and sleepers were modelled as a Euler-Bernoulli beam element. Spring and dashpot were used for the simulation of the rail pads.

Michas (2012) followed on the research of Feng (2011) and simulated 3D models for the slab (ST) and ballasted track to compare their performance under static loading in ABAQUS. Liu et al. (2011) compared traditional ballasted railway tracks with the structure and mechanics of embedded rail track systems, cementing the better performances of ST systems.

Simulation and field results are in conjunction with one another and can be seen in the works of Kjorling (1995). He has worked on a project to analyse vibrations of track components under moving load in both laboratory and field. Additionally, he performed a comparative study showing similarities between two observations. He took field observations at Algaras, between Laxa and Toreboda in Sweden by using accelerometers. For laboratory setup, he made an 8.5m long model with the same sleeper spacing and mechanical fittings and measured recordings with similar instrumental settings.

Currently the bulk of research on ST looks at the performance and wave propagation and direct effects on ST design with respect to vertical loads that act on the ST structure. However, very little research has been done on the dynamic loads that arise from high-speed ST structures, i.e., moving point loads (producing additional vibration at critical velocity, thus premature ST deterioration). Lei (2016) analysed the effect of moving dynamic loads which happens when wheel/rail profile is asymmetrical, causing wheels to deteriorate and fixed-point dynamic loads that are created when the wheel traverses over irregularities in the ST structure. Indraratna and Nimbalkar (2013) analysed the results of cyclic drained tests and numerical studies carried out on a segment of model railway track supported on geosynthetic reinforced railroad ballast bed. The ability of the geocomposite to provide a significant improvement in the vertical track stiffness through the construction of a resilient pavement was demonstrated using vertical acceleration measurements taken before and after treatment (Woodward et al. 2007).

Recently in Portugal, a continuously supported fastener-less embedded rail system from the CDM range of track solutions (CDM-QTrack, hereinafter referred to as Patrick, C., Louis, V., Paulo, P., Alfredo, R., and Bruno, M. D. CSFERS) was used for the reconstruction of a railway line (Carelsa, 2013). Since The track was projected for 60 km/h speed, trains are now running at 90 km/h, with the chosen track system giving the requested performance in terms of track stability and N&V.

Examples of previous numerical studies of ballastless systems include amongst other: (a) Markine et al (2000) developed a design procedure which includes numerical modelling and dynamic analysis together with laboratory testing and optimisation. They applied the procedure for the design optimization of STs with embedded rails (i.e., ETS) using the model “Rail" developed at Delft University and commercial software ANSYS for 2-D and 3-D finite element models incorporating the track and a moving load. Aggestam et al. (2018) focusing on modelling vehicle–track interaction modelled with a complex-valued modal superposition technique for the linear, time-invariant 2-D track model track and an extended state-space vector approach. The vertical dynamic response can then be calculated by considering a generic initial-value problem initially developed for a ballasted track.

There is a gap in research about the effect of parameters on behaviour of ST system which is addressed in this study. The Embedded Track System (ETS) and the Rheda Track System (RTS) are compared to see which performs better under dynamic loads.

Chapter-3: Methodology

3.1 ABAQUS Overview

ABAQUS is a finite element software, which permits its users to create and explore a 3D model. It is configured in different sections known as 'modules', which are used in the different stages of modelling. These are listed in the following order:

1. Parts module, this allows the user to create different parts as it can be seen in figures 8 and 9.

2. Property module, this allows one to assign material properties to a part.

3. Assembly module, this allows the user to assemble and position the different parts according to the desired position and geometry.

4. Step module, this module allows for the creation of the simulation steps.

5. Interaction module, to assign interactions among parts.

6. Loads module, to assign loads, displacements, and boundary conditions.

7. Mesh module, to mesh the model according to the element type and geometry.

8. Job module, to submit input to processor and providing output definition.

3.2 Geometry, Boundary Conditions and Material Properties

Figure 1 represents schematically the 3D model geometry of the RTS whilst the ETS is represented in Figure 2. The two systems and the respective Finite Element discretisation use 20-noded quadratic brick elements with reduced integration (C3D20R).

Table 3 shows the material properties for the two systems, based on a synthesis of data found in the literature (Batchelor, 1981; Feng, 2011; Michas, 2012). For the subgrade soil, a preliminary parametric study was performed with three different cross section dimensions to represent the theoretical semi-infinite, elastic half space i.e., a 6m x 6m, 8m x 8m and 16m x 16m section, respectively. The layers of both RTS and ETS are hinged together with tied boundary condition causing no relative rotation and translation. Vertical boundaries are applied with roller conditions causing no horizontal movements.

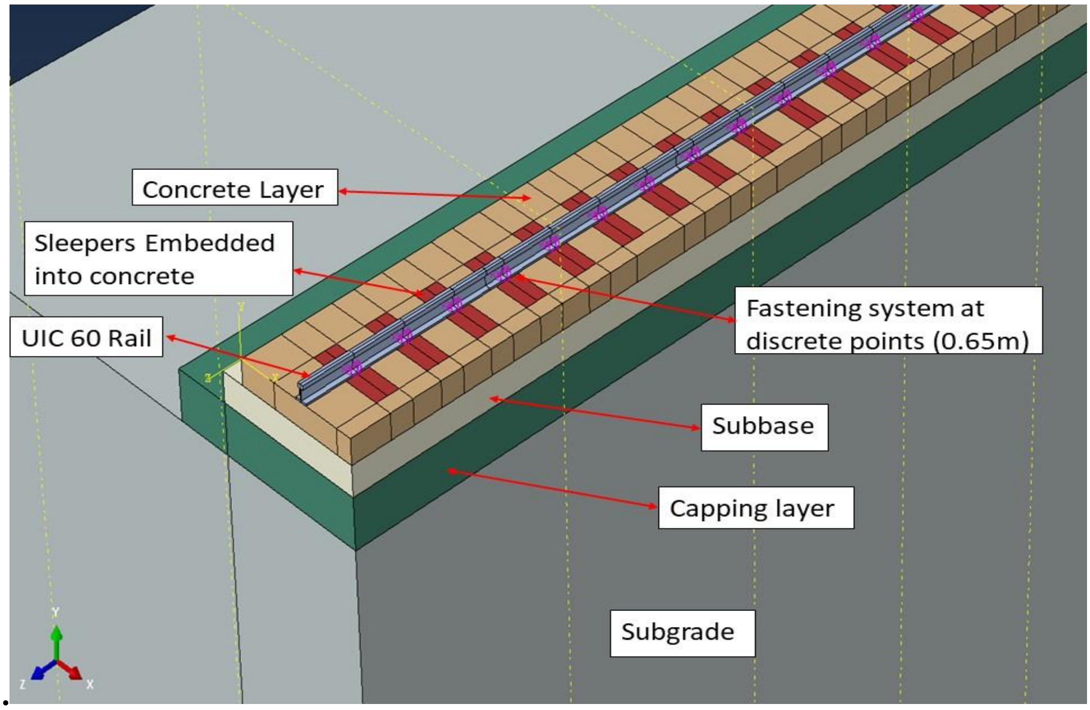

Figure 8 RTS 3D model geometry in ABAQUS (Alkhateeb et al. 2019)

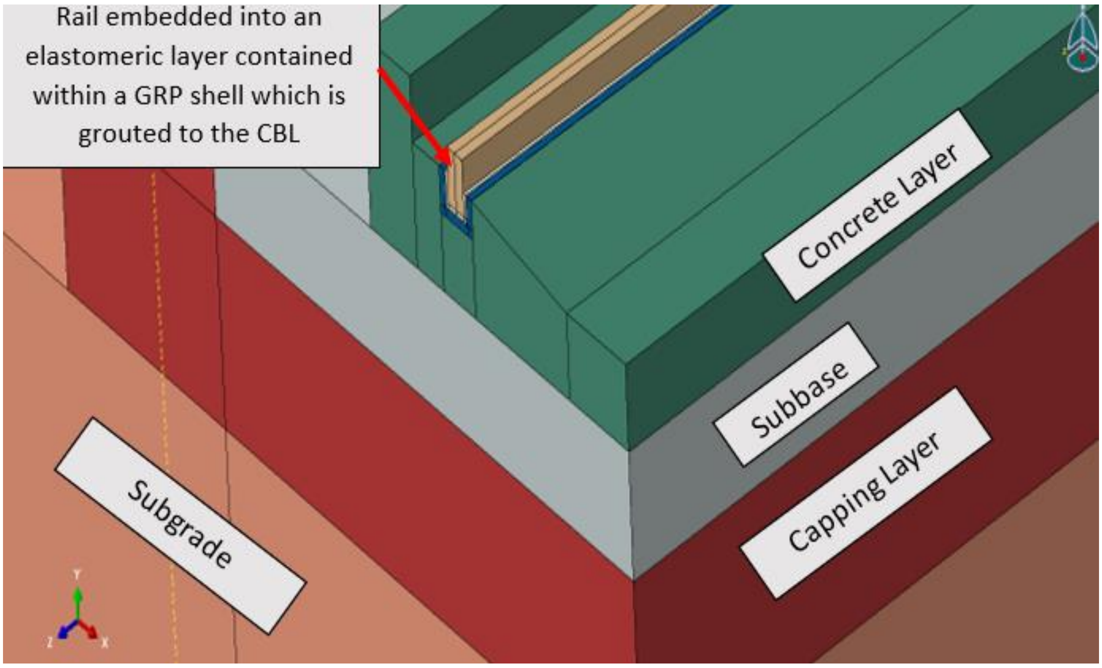

Figure 9 Section view of ETS system in ABAQUS (Alkhateeb et al. 2019)

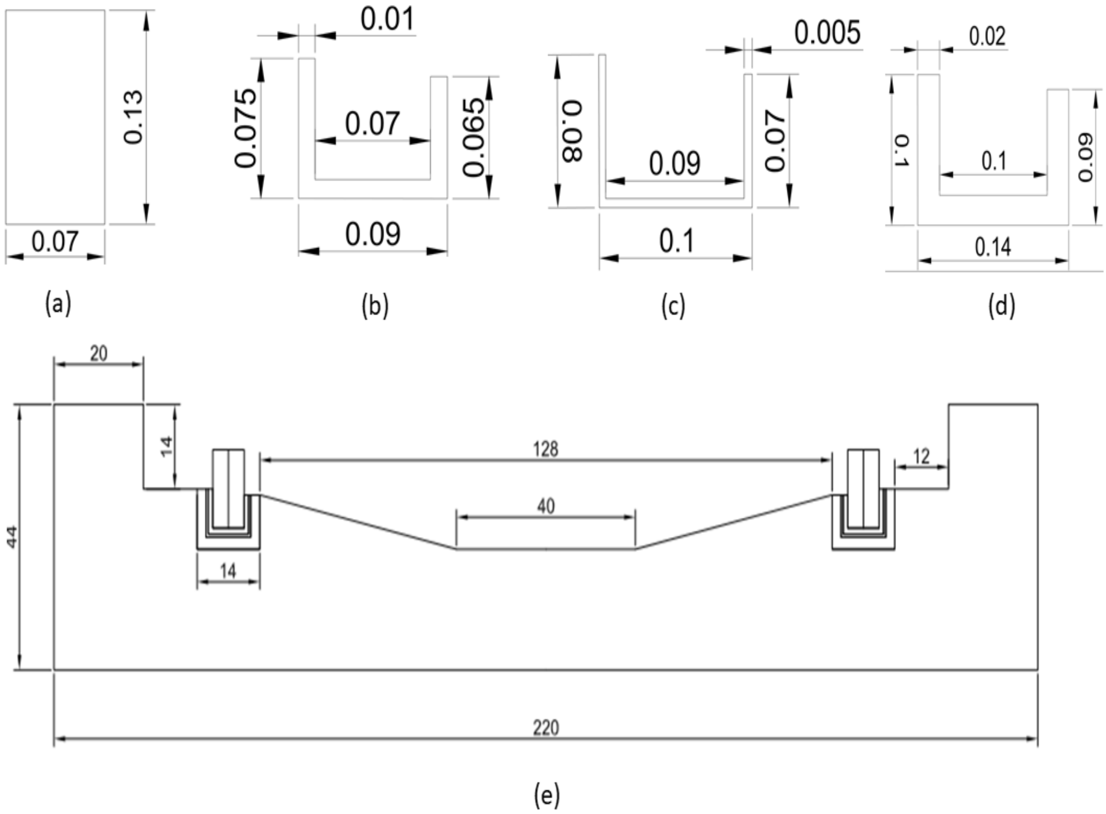

Figure 10 Typical diagrams of ETS track components (a) rail (b) grout (c) shell (d) elastomeric layer (e) slab layer (all dimensions are in mm)

The diagrams shown in figure 10 are the section profiles of ETS track superstructure. The geometry is modelled as half of the cross section to avoid large computation and boundary conditions are applied in such a way that it imitates the complete cross section geometry.

Table 3 shows the dimensions and material properties of components of ETS and RTS. The static load model used here has the rail track length of 21.45 meters and the dynamic load model has the length of 200 meters for both ETS and RTS.

Table 3. Dimensions and material properties of track layers

System	Track component	Dimensions (m^3)	Density (kg/m^3)	Modulus of Elasticity, E (GPa)	Poisson's Ratio, ν
RHEDA2000	Rail	UIC60 approx.	7850	207	0.28
	Sleeper Concrete	0.914 x 0.12 x 0.29	2400	70	0.2
	CBL (concrete)	1.6 x 0.25 x L	2400	34	0.2
	HBL (Subbase)	1.9 x 0.3 x L	2000	5	0.2
	FPL (Capping)	2.6 x 0.5 x L	2000	0.12	0.2
	Subgrade soil	8 x 16 x L	2000	0.01	0.4
Balfour Beatty ESR	Rail	See Figure 10	7850	207	0.28
	Elastomeric pad		500	61	0.3
	GRP shell		2100	17	0.22
	Grout		2400	39	0.45
	Concrete Slab		2400	70	0.2
	HBL (Subbase)	1.9 x 0.3 x L	2000	5	0.2
	FPL (capping)	2.6 x 0.5 x L	2000	0.12	0.2
	Subgrade soil	8 x 16 x L	2000	0.01	0.4

3.3 Static Load Analysis

To simulate the loading pattern of a standard two-coach passenger train, a point load of 83.3 kN is applied either side to replicate a standard 17 tonnes (HS2 Train Technical Specification) axle load at each of the six points on the track as illustrated in figure 11. The diagram also depicts the position of wheels of an approximated standard international union of Railways (UIC) railway passenger wagon.

The centre-to-centre distance between the wheels of same bogie is 2.6m whilst the centre-to-centre distance between two bogies is 3.6m (as derived from the UIC code). Roller boundary is applied to all the soil vertical boundaries and open vertical boundaries of all other layers (including rails). This means that nodes in these vertical planes are constrained to remain in the same plane throughout the analysis. The bottom plane of the model is fixed i.e., no translation is allowed in any of the three co-ordinate directions. At the bottom we consider that the underlying material is bedrock (i.e., a much stiffer material) which undergoes no deformations.

Note that the boundary conditions applied to both the models, i.e., ETS and RTS are the same. The locations of paths for data extraction are shown in Figure 12. The path along CBL is situated at mid-depth of the CBL and for soil layer the path is situated at a depth of 1 meter beneath the soil surface.

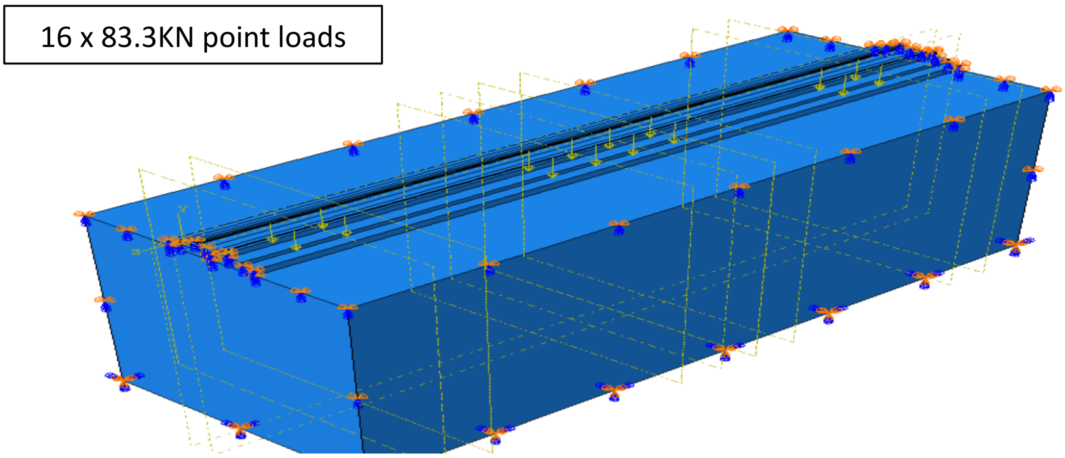

Figure 11 Loading conditions for static load analysis

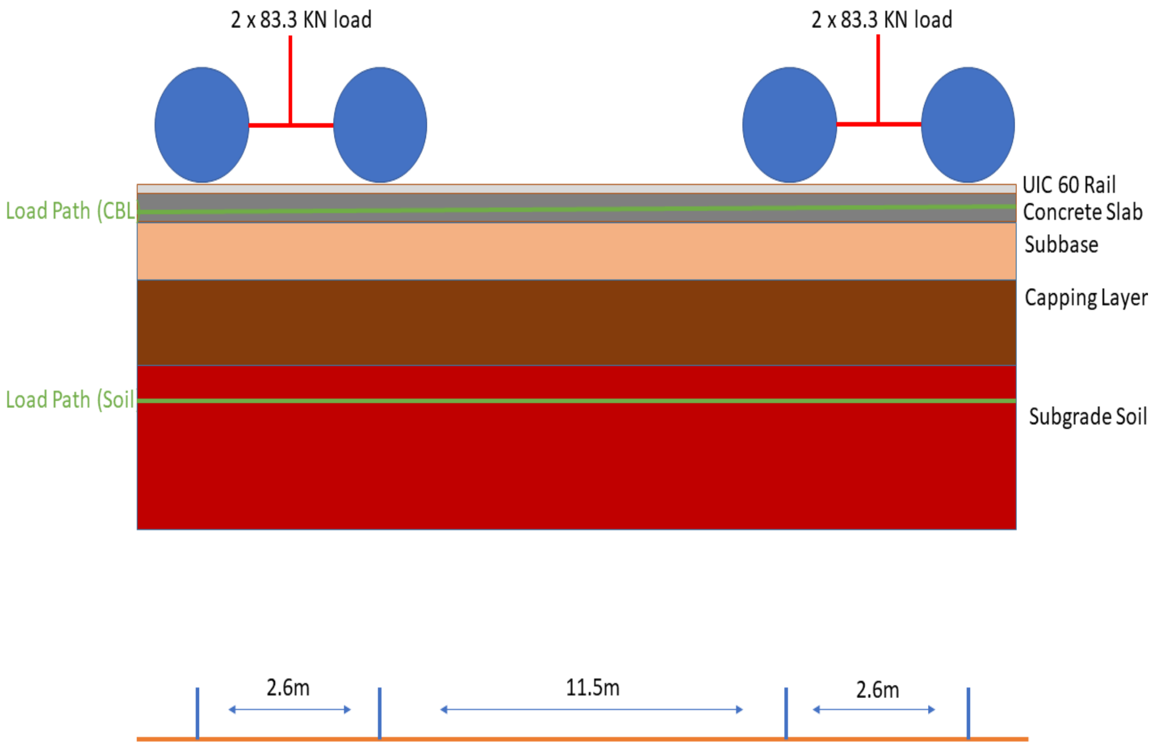

Figure 12 Loading conditions for the models presented in this paper with the modelled half of track outlined

The results of ten finite element analyses are presented and discussed in this section. For each of the ERS and RTS systems analyses were carried out for five different CBL thicknesses (t_e=200mm, t_e =225mm, t_e =250mm, t_e =275mm, and t_e =300mm). In the following results are presented for vertical displacement (U2) and vertical stress (S22) as the most significant results although (of course) results for two other displacement components and five other stress components are available.

Figure 13 shows the vertical displacement contours for the RTS and ERS track bed systems; blue means maximum deformation and red means least deformation. The maximum load intensity is situated near the area where the colour of the plot is blue.

Consider Figures 13 and 14, ABAQUS automatically allocates colours to displacement values in these plots, thus the deep blue colour represents different (although not significantly different) maximum displacements. None-the-less the contour plots demonstrate the essential similarity of the response of the two systems although the structural details of how the rails are supported are quite different. In fact, this is the type of response which can be expected on general principles.

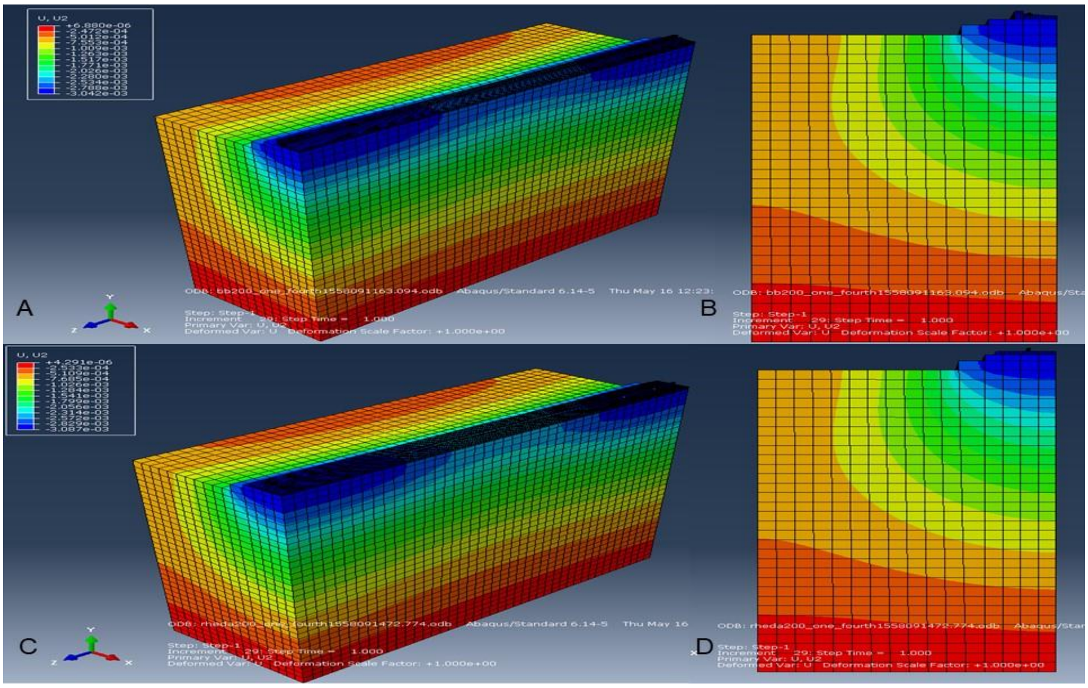

Figure 13 Displacement contours (A) ERS, (B) ERS section view midpoint, (C) RTS, (D) RTS section view midpoint

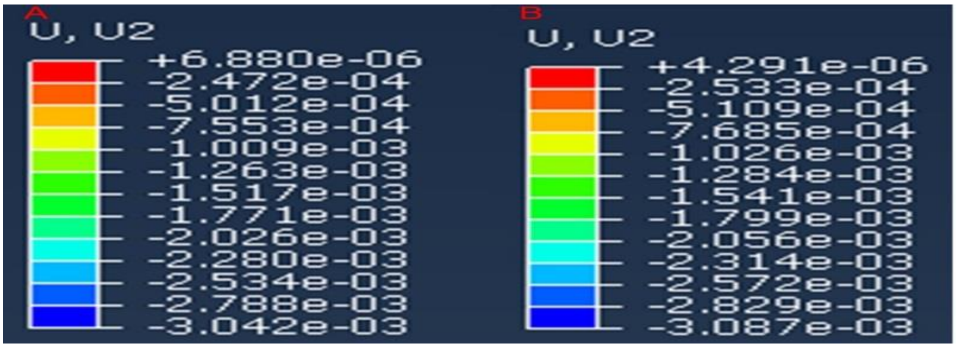

Figure 14 Displacement values for contour plots, (A) ERS, (B) RTS

A major part of the vertical displacements is a direct result of the compression of the most flexible part of the system (the underlying soil) and the loads applied to both systems are identical. According to Saint-Venant's principle the overall results (particularly some distance from where the loads are applied) will be almost independent of the way the loads are transmitted to the rest of the system by the details of the rail support (i.e., slab track system).

The effects of the variation in the thickness of the main concrete slab on the maximum displacement at the bottom of the slab and on the maximum displacement at the top of the subgrade are shown in Table 4 and Figure 14. Generally, the two systems seem to produce comparatively similar values of displacement.

Table 4 Displacements for the RTS and the ERS at the bottom of the CBL (concrete slab) and at the top of the subgrade soil

	Quarter models (finely meshed)			
Slab Thickness (mm)	Maximum displacement in RTS (m)		Maximum displacement in ERS (m)	
	@ Bottom of concrete slab	@Top of Subgrade	@ Bottom of concrete slab	@Top of Subgrade
200	0.003120	0.003001	0.003024	0.002999
225	0.002997	0.002969	0.002998	0.002974
250	0.002967	0.002941	0.002976	0.002953
275	0.002943	0.002917	0.002959	0.002936
300	0.002921	0.002897	0.002943	0.002921

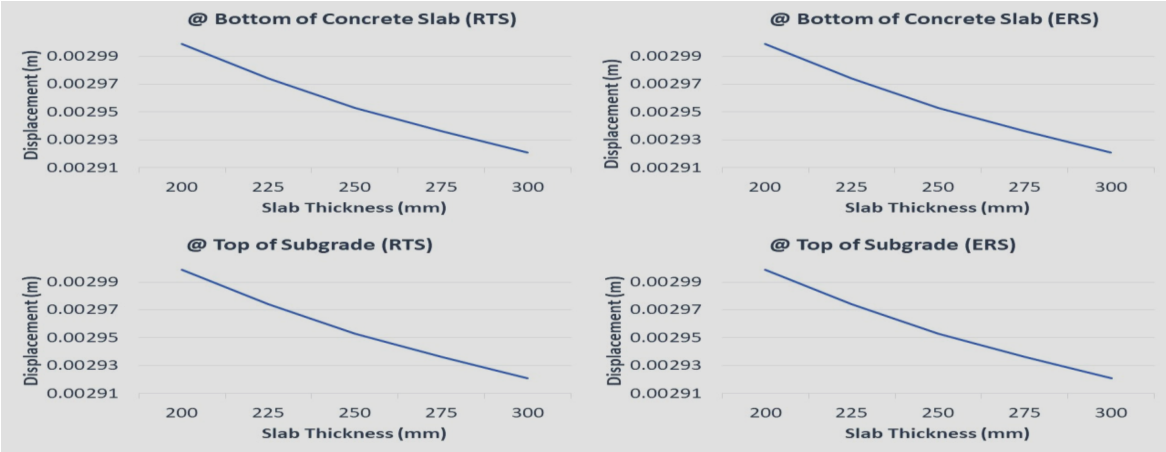

Figure 15 Comparison of thickness versus vertical displacement for RTS and ERS

The effects of the variation in the thickness of the main concrete slab on the maximum stress at the bottom of the slab and on the maximum stress at the top of the subgrade are shown in Table 5 and Figure 15. The results show that whilst the stress at the top of the subgrade is comparable between the two systems, the maximum stress at the bottom of the concrete slab of the RTS is higher than that of the ERS.

Table 5. Maximum stresses for the RTS and the ERS at the bottom of the CBL (concrete slab) and at the top of the subgrade soil

	Quarter models (finely meshed)			
Slab Thickness (mm)	Maximum stress in RTS (Pa)		Maximum stress in ERS (Pa)	
	@ Bottom of concrete slab	@Top of Subgrade	@ Bottom of concrete slab	@Top of Subgrade
200	77314.690	5993.780	50107.650	5977.494
225	63365.880	5794.109	44223.120	5936.263
250	61112.030	5624.439	38065.590	5893.823
275	51103.910	5545.544	32299.300	5868.266
300	43332.440	5500.062	28575.850	5825.750

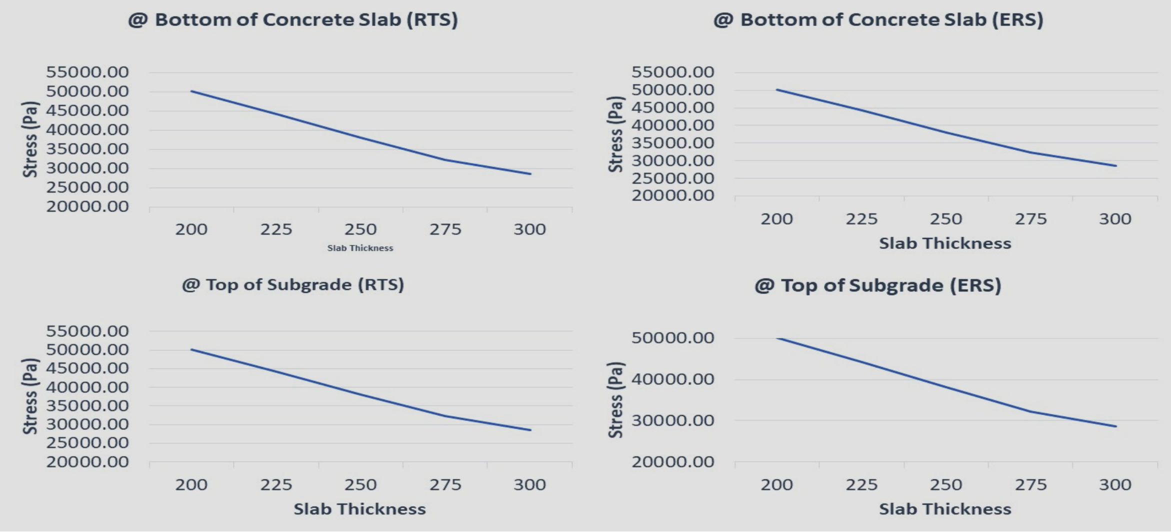

Figure 16 Comparison of thickness versus vertical stress for ERS and RTS

Figure 16 compares ERS and RTS with regards to thickness versus vertical stress. It can be seen that as the slab thickness increases the stresses decrease for both systems. The adjusted values of R-square of the RTS and the ERS for the maximum displacement and maximum stresses at the bottom of the concrete slab and at the top of the subgrade are shown in Table 6. The R values of the ERS system are similar whilst the corresponding R values of the RTS are less similar.

Table 6 Adjusted R-squared values for the linear fit of thickness versus stress and displacement

	RTS		ERS	
Location	@ Bottom of concrete slab	@Top of Subgrade	@ Bottom of concrete slab	@Top of Subgrade
Stress	0.956	0.905	0.991	0. 993
Displacement	0.781	0.989	0.985	0.985

3.4 Dynamic Load Analysis

The dynamic analysis of ETS and RTS is performed for changing motion speeds of loads. The loads are applied in the form of pressure. A block part is created to apply motion and load on its top surface for both RTS and ETS systems. For RTS, the block part has the area with width equal to RTS rail thickness (0.066 meter) and length equal to 0.2 meter. For ETS, the length is the same (0.2 meters) but the width changes to the rail thickness i.e., 0.072 meters. Based on the areas of these block and the static load of 83.35 kN, the pressures are calculated. Pressures at the top of block for ETS and RTS are 6314.88 kPa and 5788.64 kPa, respectively. It should be noted that the pressure values are different for both rail track system This is happening as the area of application of load i.e., 83.35 kN is different. In an overall, the applied loads are same in value.

The load motion is performed using the displacement-controlled method. The displacement and step time are changed to attain a constant velocity with the help of a gradually varied amplitude. For example, a speed of 180 km/h is applied with moving the load to 150 meter in 1.5 second this will create a 100 m/sec (180 km/h) motion speed. The amplitude is applied to gradually vary the displacement and thus making the motion at a constant velocity. The speeds are varied in the rage of 180 to 540 km/h.

For parametric studies, parameters such as thicknesses and stiffnesses of different layer of both RTS and ETS are changed under dynamic loading.

3.5 Mesh Convergence Study

A mesh convergence study is performed to find the optimum size of mesh for ST models. The mesh sizes are varied with following a rule. According to which, as the distance from the static load or pressure (dynamic analysis) increases the mesh size can be coarsened. This is done because the effect of load will decrease with proximity; thus, mesh sizes can be larger to counter computation time.

For mesh convergence study, the initial mesh sizes for soil layer, subbase and CBL are taken as 0.5, 0.3 and 0.2 meter (mesh 1), respectively. These mesh sizes are then changed to 0.4-, 0.2- and 0.1-meter (mesh 2) sizes for soil layer, subbase and CBL, respectively. Final iteration was done with sizes of 0.3, 0.1 and 0.09 meter (mesh 3) for soil layer, subbase and CBL, respectively. No further mesh fineness is applied as the number of elements would exceed the license limit provided by LSBU. It was seen that at global mesh sizes of 0.3, 0.1 and 0.09 meter for soil layer, subbase and CBL, respectively, the convergence of results occurs with minimum computation time. This mesh sizes were adopted for static load analysis.

The length of rail track was increased about ten times the track length for static analysis for dynamic analysis for providing motion space for the load. This required the coarsen the mesh sizes of different layers. The global mesh sizes adopted for dynamic study was 1 meter for substructure (soil layer, FPL, and subbase) parts. However, some geometric divisions are introduced manually to ease the formation of mapped (structured) mesh in these parts. The mesh sizes for rail, and other layers around it was taken as 0.2 meter with geometric divisions for keeping structured mesh. The mesh size for CBL is 0.3 meter with similar geometric divisions.

3.6 Results for Dynamic Analysis

In Figure 17 it can be seen in the plots that as the speed increases the vertical deflection increases. This shows that dynamic effects are dominant with increasing speeds and causing higher deformations.

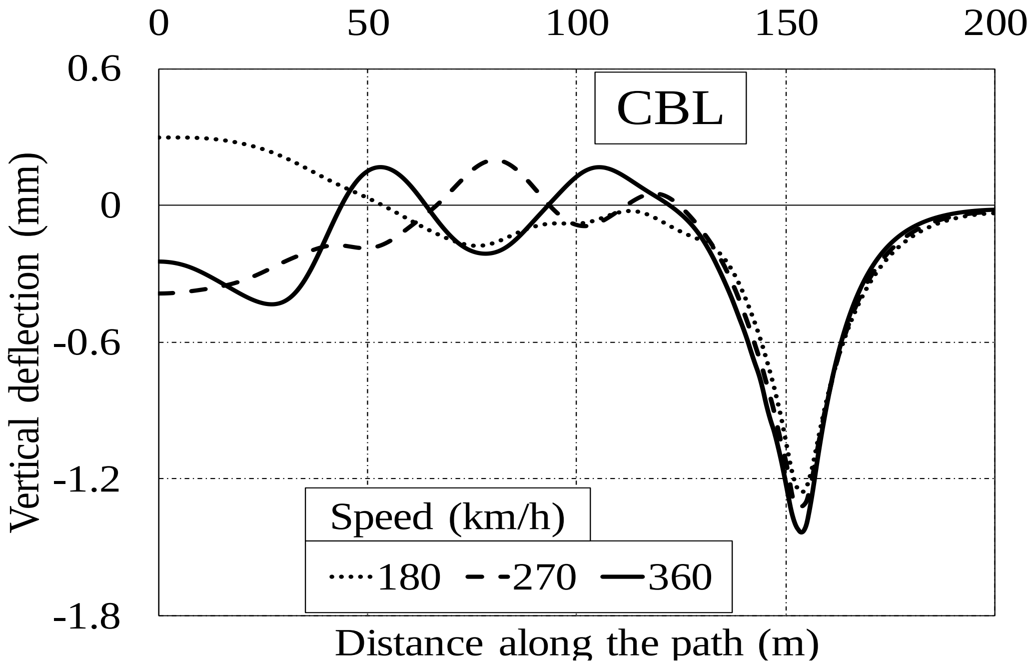

Figure 17 Vertical displacement with motion speed in ETS for CBL

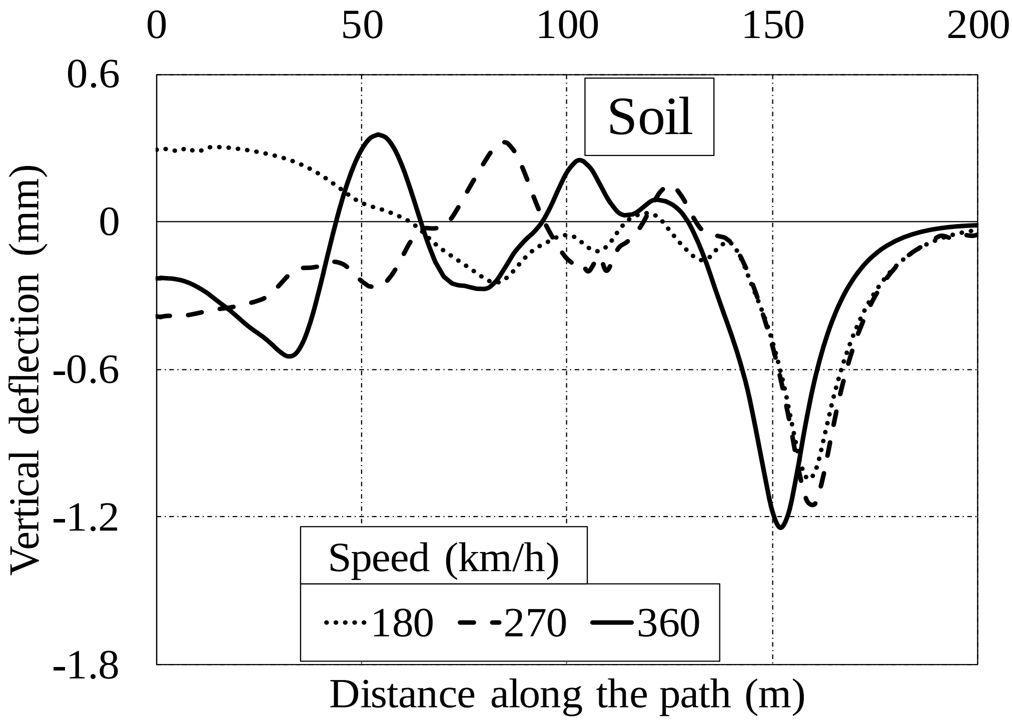

Figure 18 Vertical displacement with motion speed in ETS for Soil layer

Figure 18 shows the results for ETS at depth of 2 meter in soil layer from the top of soil layer. These results comply with the ones shown in Figure 17. It can be said that the speeds affect vertical deformations in soil layer as well. The overall deformation does not change much from CBL to soil layer, implying that CBL is absorbing much of the load.

Figure 19 and 20 show the results of verticla deformations with incresing motion speed for RTS system. Plots for CBL and soil layers are shown and the data is taken at the point same as mentioned earlier. These curves shows the maximum downward deflections for the highest speeds. It can also be seen that CBL absorbs much of the load in RTS also. However, it can be seen with the peak vertical deformations of ETS and RTS that load-deformation behaviour of ETS is slightly better than RTS.

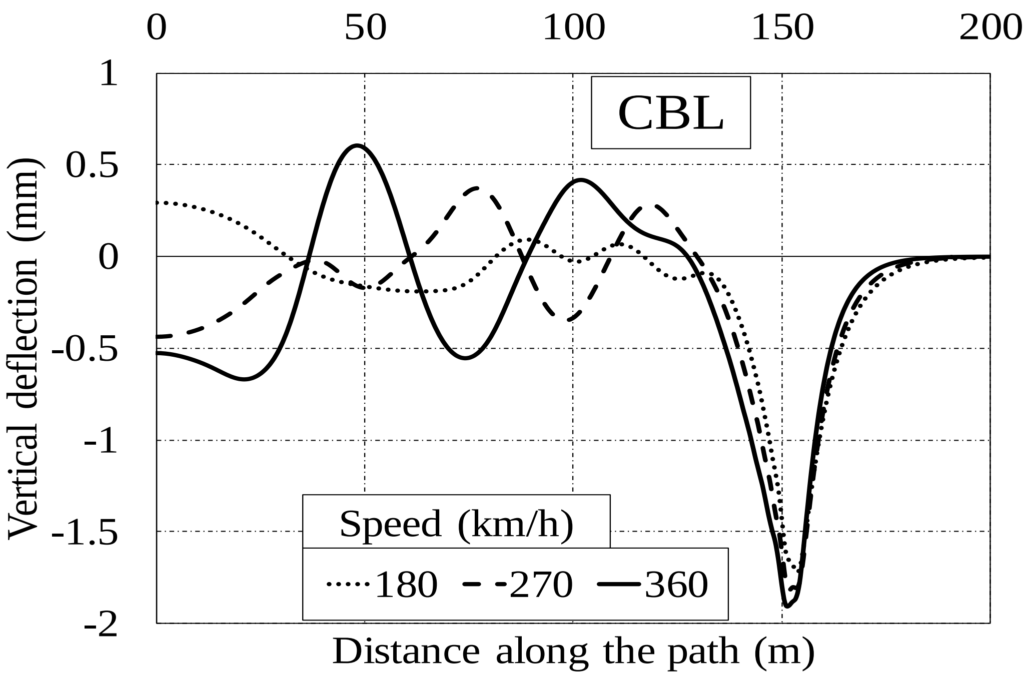

Figure 19 Vertical displacement with motion speed in RTS for CBL

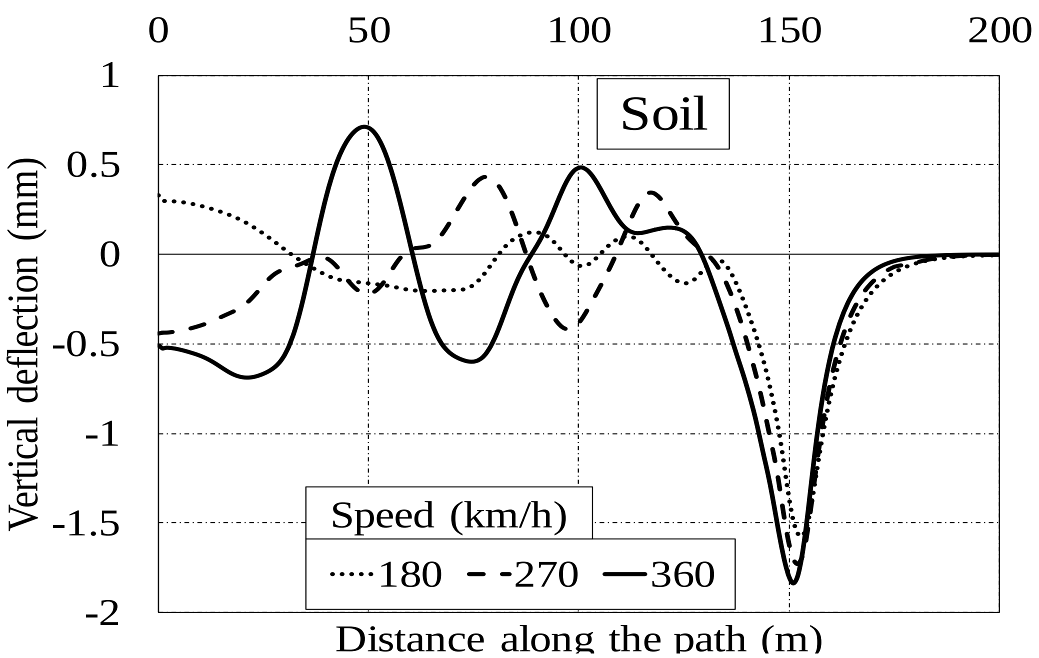

Figure 20 Vertical displacement with motion speed in RTS for Soil layer

A parametric study on CBL (slab) thickness is carried out to see its effect on vertical deformation. Figure 21 and Figure 22 show that both ETS and RTS, respectively, follow same pattern with slab thickness. The vertical deformation is decreasing with the slab thickness. This basically says that a thicker layer will be a better load carrier in a ST.

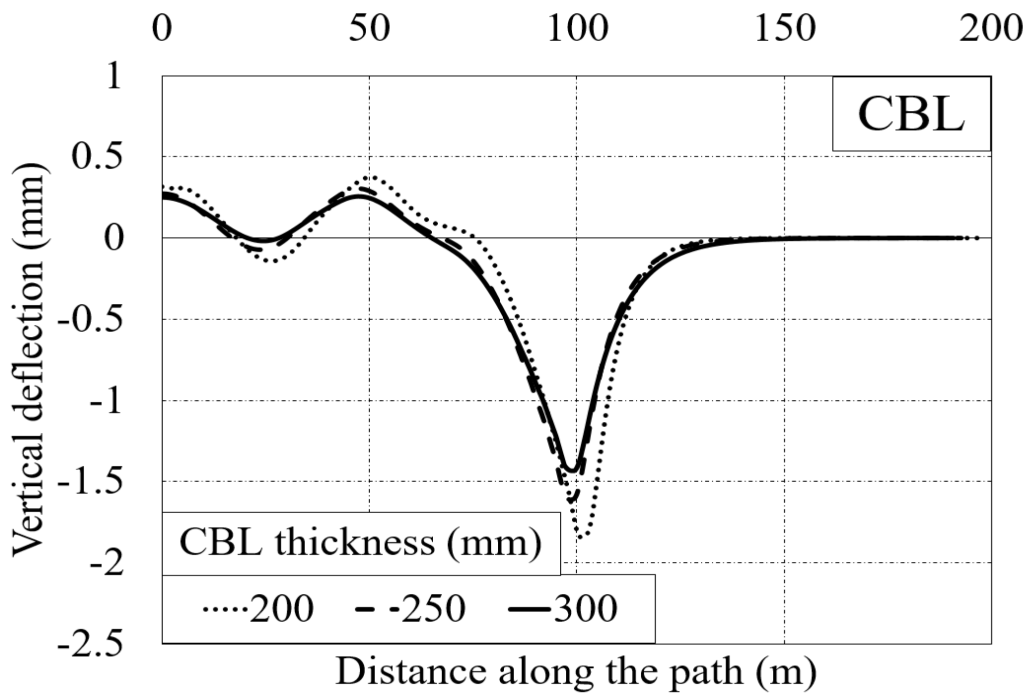

Figure 21 Vertical displacement with CBL thickness in ETS for CBL

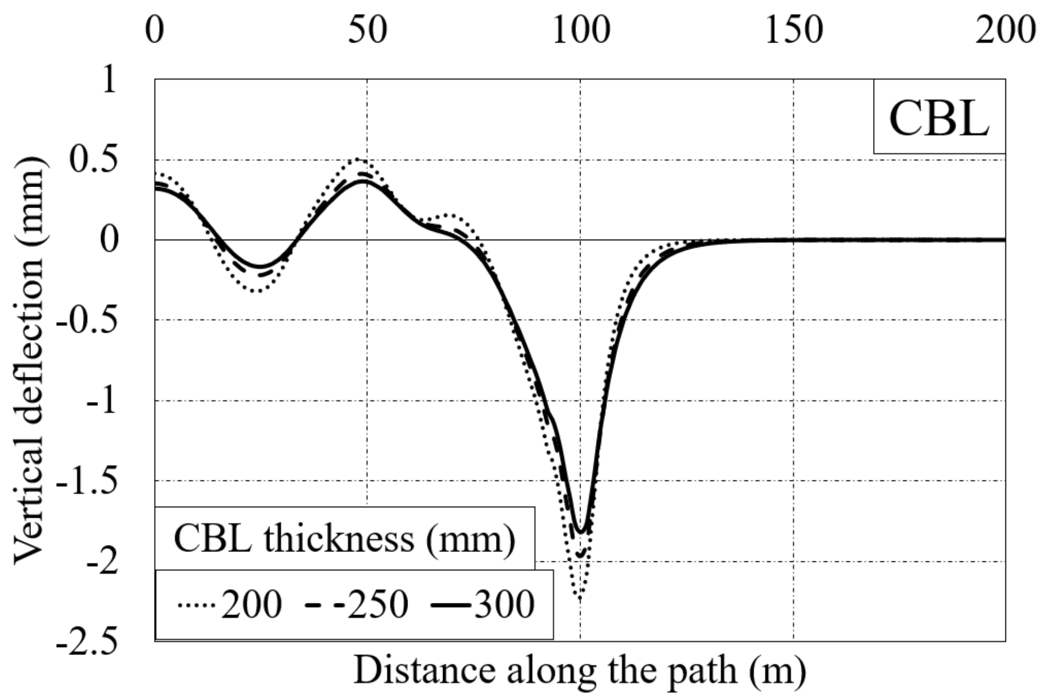

Figure 22 Vertical displacement with CBL thickness in RTS for CBL

Parametric study on soil stiffness is also performed on both ETS and RTS sections. Figure 23 and Figure 24 show the effect of soil stiffness on vertical deformations in ETS for CBL layer and soil layer. These results show that increasing soil stiffness decreases the vertical deflection and the local peaks in the curves.

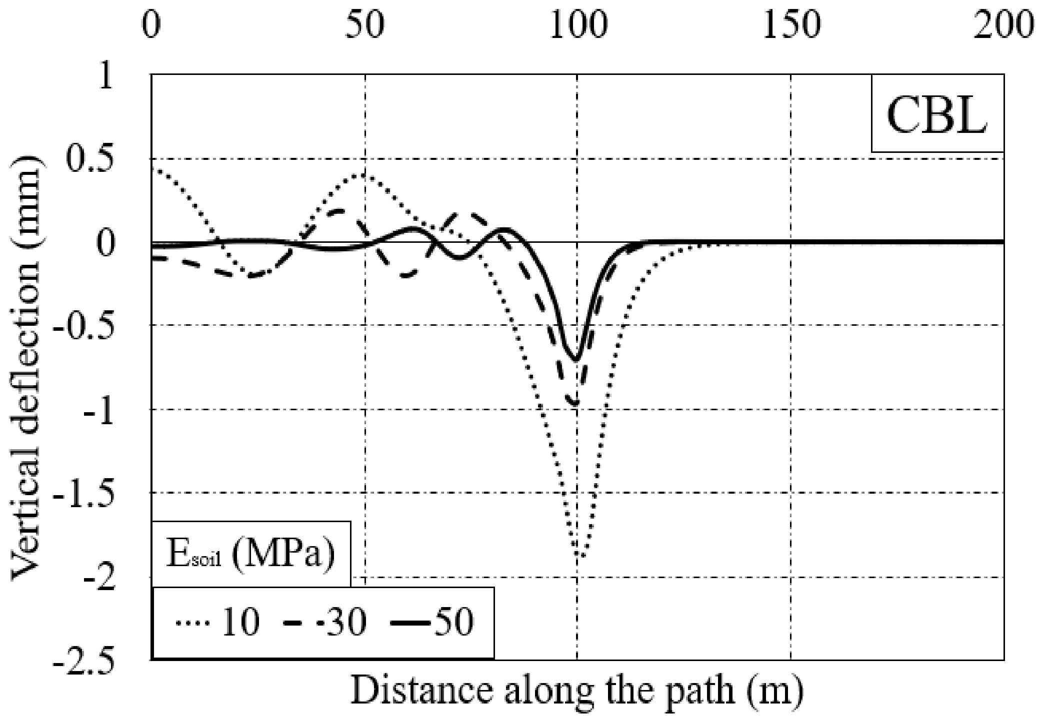

Figure 23 Vertical displacement with Soil stiffness in ETS for CBL

This can be further said that soil stiffness effects the vertical deformation more as compared to CBL stiffness. A soil stiffness corresponding to CBR of 5 (Soil stiffness of 50 MPa) can be said to encompass the best quality subgrade for slab foundation. Further improving the subgrade quality will positively increase the load-deformation behaviour of rail track but the change will not be significant.

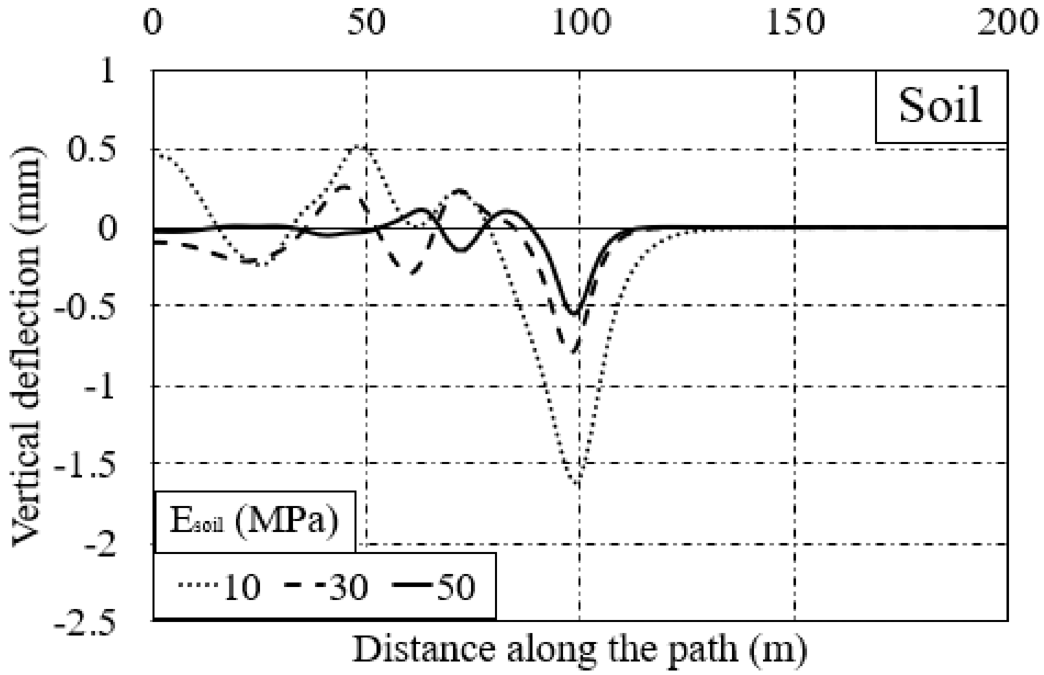

Figure 24 Vertical displacement with Soil stiffness in ETS for Soil layer

Figure 25 and Figure 26 shows the effect soil stiffness on RTS track section at CBL and soil layer, respectively. The behaviour of RTS is similar to ETS section. It can be said that though the load carrying capacity of both track systems may vary but the relative behaviour can be similar. Thus, a common code for construction practice can be followed for all ST models.

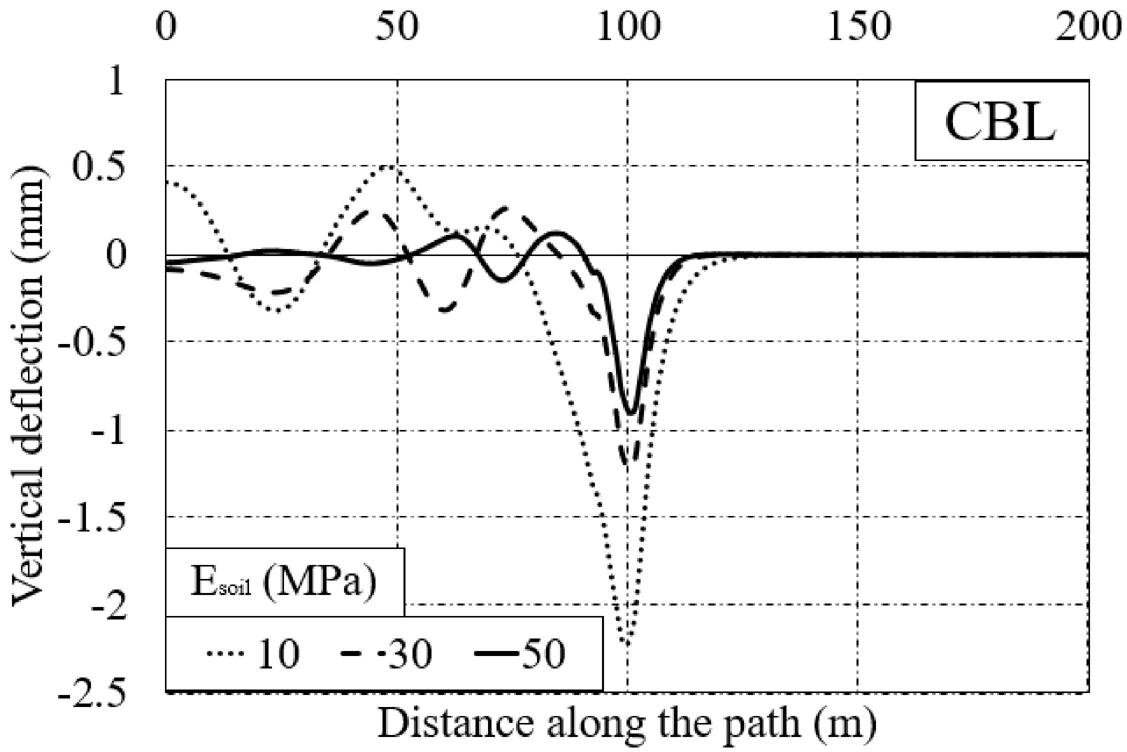

Figure 25 Vertical displacement with Soil stiffness in RTS for CBL

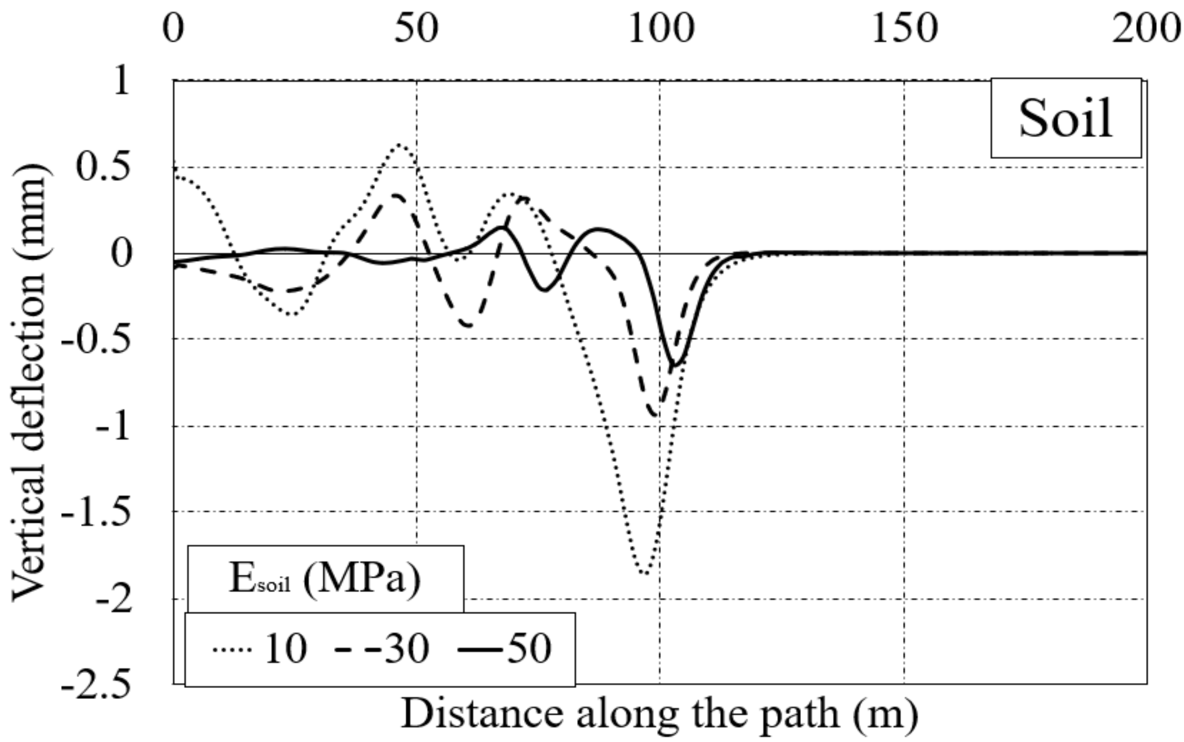

Figure 26 Vertical displacement with Soil stiffness in RTS for Soil layer

Figure 27 and Figure 28 shows the effect of CBL stiffness on the vertical deformation for ETS system at CBL and soil layer. It can be seen in these plots that the increasing CBL stiffness decreases the vertical deformation. However, after a stiffness of 50 GPa for CBL the deformation change is very minute and insignificant.

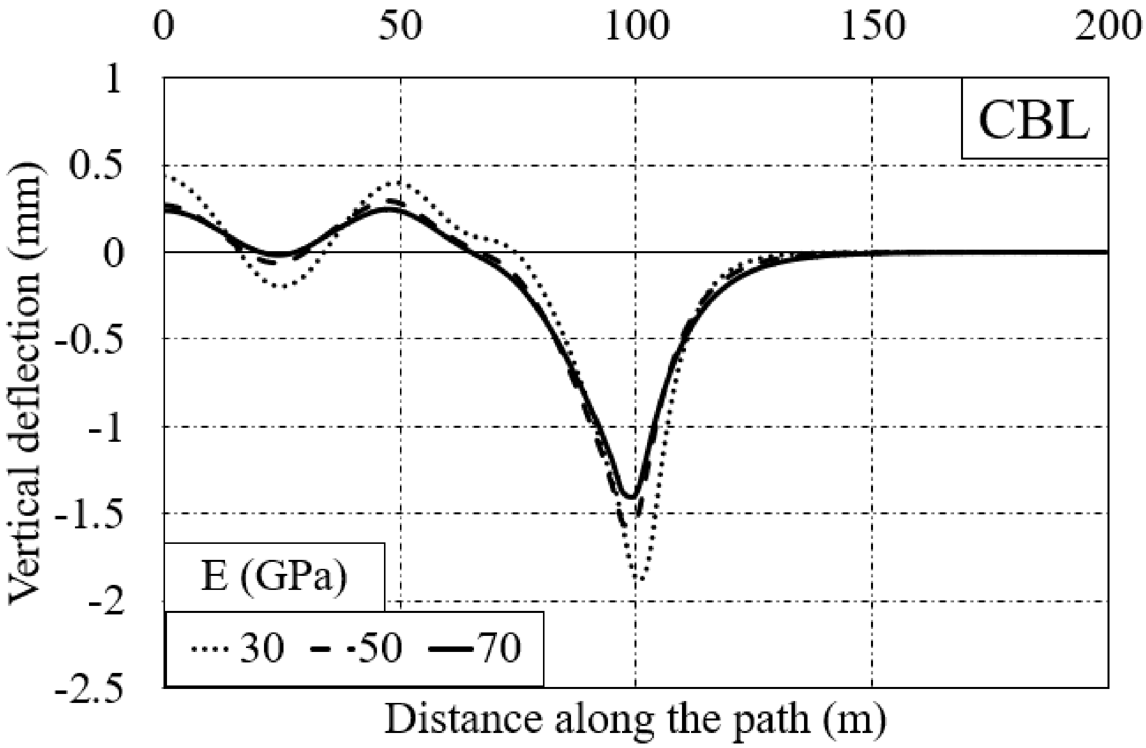

Figure 27 Vertical displacement with CBL stiffness in ETS for CBL

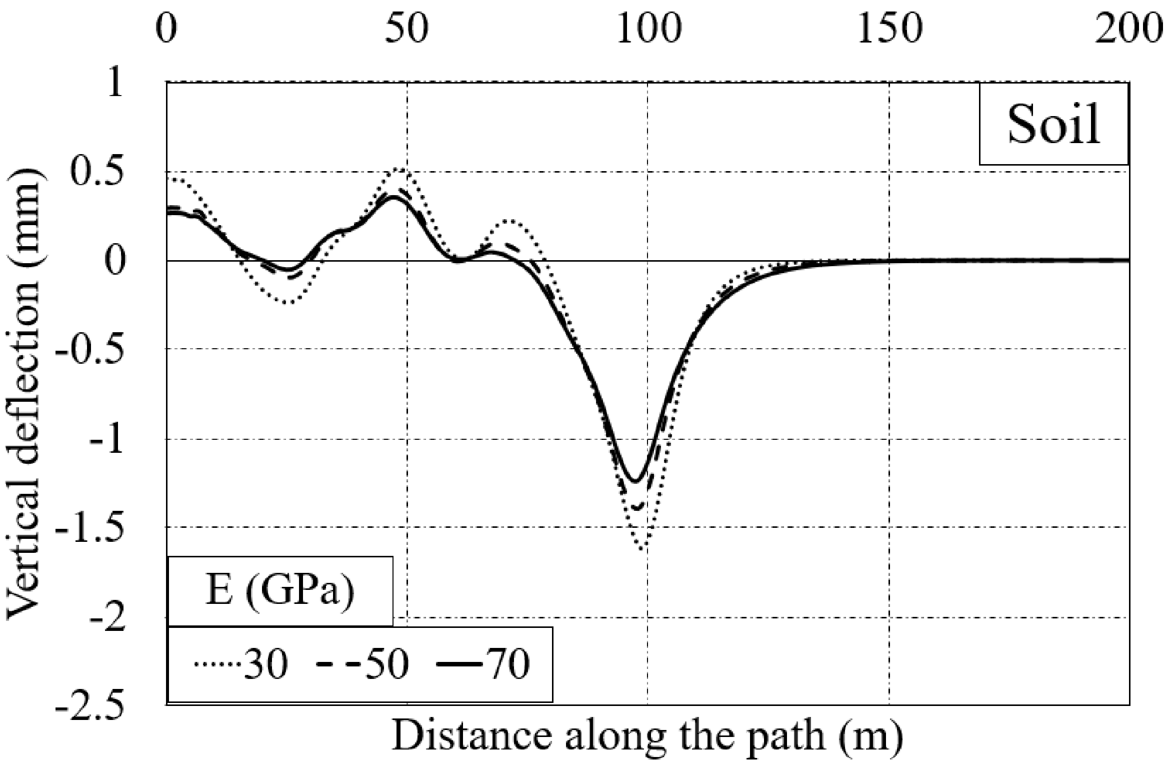

Figure 28 Vertical displacement with CBL stiffness in ETS for Soil layer

Figure 29 and Figure 30 show the effect of CBL stiffness on RTS system for CBL and soil layers. The behaviour is same as ETS system and it can be said that the stiffness of 50 GPa is good enough for the construction of ST for a speed of 360 km/h in both cases. The vertical deformation decreases with increasing CBL stiffness. However, the soil stiffness affects the load-deformation behaviour of slab more than the CBL stiffness.

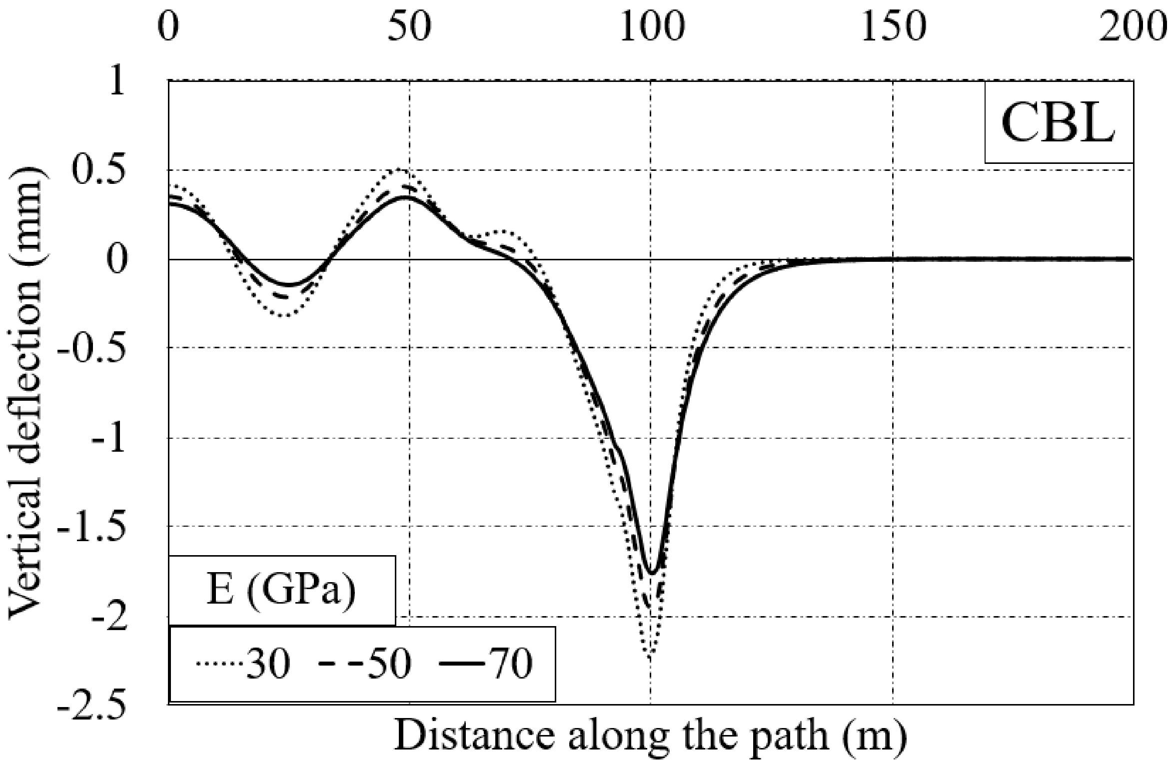

Figure 29 Vertical displacement with CBL stiffness in RTS for CBL

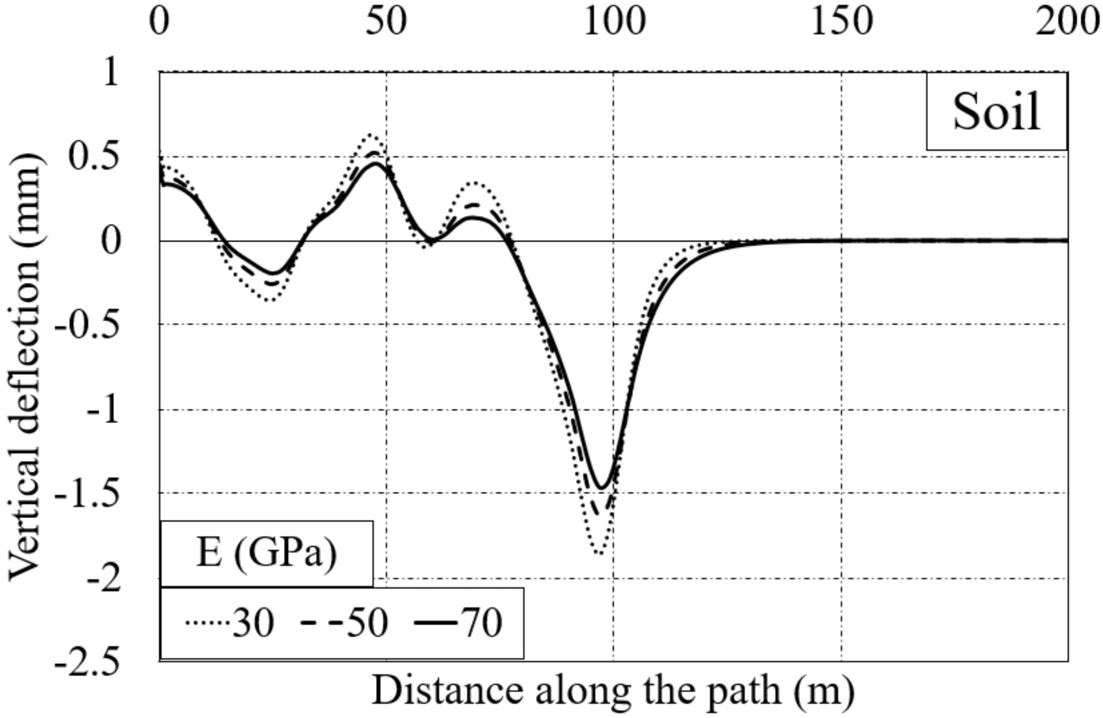

Figure 30 Vertical displacement with CBL stiffness in RTS for Soil layer

The results shown in plots above are analysed and the peak values of vertical deflection and vertical velocity are shown in Tables 7 and 8. The parametric study results for soil stiffness, CBL stiffness and motion speeds are exhibited, the R-squared values for soil stiffness versus different parameters are given. These values signify good quality of linear fit for the data. Based on these values, it can be said that Peak vertical velocity is better correlated with increasing soil stiffness as compared to maximum displacement. It can also be stated that the soil layer data is better correlated than CBL data based on R-squared values.

Table 7 Peak values of parameter with soil stiffness (ETS)

Soil Stiffness (MPa)	Maximum Displacement (CBL, mm)	Maximum Displacement (Soil layer, mm)	Peak Vertical Velocity (CBL, mm/sec)	Peak Vertical Velocity (Soil layer, mm/sec)
35	0.855114	0.547072	14.33725	10.46167
50	0.669569	0.442853	12.31112	8.534629
65	0.41999	0.369951	10.74323	6.850802
80	0.414895	0.310648	9.532265	5.387248
100	0.409351	0.275830	8.637782	4.315284
R-sq	0.905	0.983	0.987	0.996

Table 8 Peak values of parameter with soil stiffness (RTS)

Soil Stiffness (MPa)	Maximum Displacement (CBL, mm)	Maximum Displacement (Soil layer, mm)	Peak Vertical Velocity (CBL, mm/sec)	Peak Vertical Velocity (Soil layer, mm/sec)
35	1.117236	0.843681	23.69441	14.02039
50	0.903255	0.646748	20.2840	11.52090
65	0.757838	0.500196	17.63666	9.538648
80	0.569649	0.401509	15.00007	7.737971
100	0.561117	0.33346	14.22473	6.435998
R-sq	0.995	0.978	0.996	0.994

Tables 9 and 10 show the peak values of changing speeds for ETS and RTS. One can see that as the speed increases the displacement (mm) also increases. Moreover the peak vertical velocity also increases as the speed increases.

Table 9 Peak values of parameters with motion speed (ETS)

Speed (km/h)	Maximum Displacement (CBL, mm)	Maximum Displacement (Soil layer, mm)	Peak Vertical Velocity (CBL, mm/sec)	Peak Vertical Velocity (Soil layer, mm/sec)
180	0.581515	0.407215	5.624759	2.85987
270	0.675218	0.470178	9.132244	5.170173
360	0.704119	0.546277	12.31112	8.534629
450	0.746577	0.545887	20.04624	11.55012
540	0.800145	0.576895	25.58180	12.88448
R-sq	0.963	0.896	0.971	0.984

Table 10 Peak values of parameters with motion speed (RTS)

Speed (km/h)	Maximum Displacement (CBL, mm)	Maximum Displacement (Soil layer, mm)	Peak Vertical Velocity (CBL, mm/sec)	Peak Vertical Velocity (Soil layer, mm/sec)
180	0.66375	0.435336	7.520939	3.121326
270	0.720042	0.506078	10.63859	5.865894
360	0.903255	0.646748	20.284	11.5209
450	0.872217	0.682554	28.27748	15.924
540	0.928719	0.695052	29.32568	17.19641
R-sq	0.833	0.901	0.948	0.967

Chapter-5: Discussion and Conclusion

The results shown from the study sheds light on how the different parameters affect the load-deformation behaviour of the ETS and RTS. At a glance it can be said that increasing material stiffness and thickness of the load carrying layer will increase the resistance by the overall rail track structure. This study shows that the influence of the soil stiffness on the rail track behaviour is more than the concrete stiffness. Increasing thickness changes the deflection and vertical velocity responses but its effect is also not more than what soil stiffness can achieve. Increasing speeds also increase the deformations response and vertical velocity response of the track. The effect of increasing motion velocity is better encapsulated by vertical velocity as compared to vertical displacement based on statistical analysis. It is also inferred that the data from the soil layer is better correlated with various parameters than the data from CBL layer. It is known that larger wavelength gets attenuated faster than smaller ones. When the depth of a material is high it experiences a uniform set of wavelengths from the dynamic effects as compared to the part near the application of the load.

The intensity of static load is minimal as compared to moving load analysis. Moving load produces deflection waves i.e., its effect remains at a point even though it might have moved away from the point of action, this is the reason for its higher intensity then stationary load, finally it is seen that the point of maximum deflection shifts behind the load with increasing speed.

On review of the results obtained, it can be seen that there is a general pattern whereby the ETS tends to show less displacement compared with RTS with changing parameters in general, such as increased speed, increased slab thickness, or with increasing soil stiffness. Many factors play a part in this, it could be due to the layers, and possibly the material properties used. However, the fact that it is a continuous embedded system could be playing a major role in obtaining these results as the load is distributed more evenly across the track and thus the wheels do not experience any differences in vertical stiffness. This is further evident through existing literature by Esveld, 2003, where it was seen that the continuous support on the ETS cause wheels to not experience any differences in vertical stiffness, a major source of corrugation development. This subsequently results in higher costs of maintenance and ultimately increases the rail industry’s carbon footprint.

Scope for further studies:

1. The Length of the model can be increased to analyse it for higher speeds.

2. Failure analysis can also be done for concrete if plasticity is introduced in the model.

3. Failure analysis of the soil layer with inclusion of any critical state mechanism of soil can be done. A junction of changing subsoil property can be introduced and the effect of change in the subsoil strata can also be analysed.

4. Addition or replacement of different layers can also be used e.g., adding an asphalt layer underneath or on top of the concrete layer.

5. Use of multiple moving loads can be done to see combined effect of motion on the rail track.

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