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MASTER

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MASTER'S FINAL WORK

DISSERTATION

AN INTERNAL MODEL FOR WORKERS COMPENSATION

CARLOS EDUARDO BARRENHO DA ROSA

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SUPERVISION:

PROFESSOR MARIA DE LOURDES CENTENO

PROFESSOR JOÃO MANUEL DE SOUSA ANDRADE E SILVA

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In special, the author thanks Jorge Garcia. He was a teacher in ISEG for 20 years and he was a pioneer researcher in Portugal in the area of statistics applied to Actuarial Science. He was always available to help anyone who needed him. Few days before his death, he answered me and commented the subject in which I was working:

"Para já, chamo a atenção de que os pensionistas de AT não têm o mesmo perfil de risco dos restantes, até porque a pensão resulta de acidente e não de doença ou velhice."

Jorge Garcia - May, 21st, 2012.

Thank you

Carlos Rosa

ABSTRACT

Workers Compensation is one of the most interesting Property and Casualty branches to study in Portugal. Largely influenced by Annuities management that unlike what is common in most countries is classified as Life and Savings risk.

Solvency II introduces new requirements that should be fulfilled by companies in order to protect consumers. Companies can opt to develop internal models or to adopt the standard model defined by European regulators. The internal model allows a company to model better the risks insured. However the model has to be approved by regulation.

Our goal is to build an internal model for Workers Compensation. The model must cover all specificities of this branch. In one part, the model is based on the Merz and Wüthrich (M&W) model developed for Solvency II purposes. The M&W model aims to measure possible reserves' fluctuations between two successive predictions for the total ultimate claim. In the second part, the model is based on longevity study. Longevity is one of the most important risks discussed nowadays and this has large impact on annuities management and lifetime assistance.

We think that it is important to study the longevity risk in the short-term and the long-term perspectives. The short-term perspective has less impact on capital requirement and it is a consequence of a low mortality scenario in one year development. In long-term view, companies have to evaluate the adequacy of their mortality table and its impacts on reserves and assets.

A global internal model needs not only to model the consequence of occurred accidents but also to project the ones which have not occurred yet. Companies must prevent the risk of premiums being insufficient to cover all assumed liabilities. Extensive use of simulation is made to estimate some extreme scenarios.

KEYWORDS: Workers Compensation, Solvency II, Internal Model, Solvency Capital Requirement, Loss Reserving, Longevity Risk, Monte Carlo Simulation.

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SECTION I - INTRODUCTION

This study presents a possible approach to tackle some of the concerns of insurers with the entry into force of Solvency II (2014).

Insurance is defined as the equitable transfer of the risk of a loss, from one entity to another one, in exchange of a payment. Risk management, the practice of appraising and controlling risk, has evolved as a specific area of study and practice. However, up to now, the methods depend from company to company and from country to country. Solvency II opens a new reality for companies.

The Solvency II Directive has the purpose of harmonizing the EU insurance regulation. Primarily this concerns the amount of capital that insurance companies must hold to reduce the risk of insolvency and also to protect consumers. Solvency Capital Requirement (SCR) is the key solvency control level. The SCR must correspond to the Value-at-Risk of the basic own funds of an insurance undertaking subject to a confidence level of 99.5% over one-year period. This is equivalent to ensure that on average ruin in one year view occurs no more often than once in every 200 years. Companies have two options to calculate the SCR: to adopt the standard model defined by European regulators or to build their own internal models. In this study, we will focus on an internal model for a specific line of business – Workers Compensation.

Fundamentally, an internal model is a mathematical representation of the insurer's business operation. It is based on past empirical data and assumptions regarding the insurer's future experience with respect to a variety of factors including risk drivers. This is a definition of an internal model framed by the International Association of Insurance Supervisors (IAIS) and contained in the Comité Européen des Assurances – Groupe Consultatif Glossary (Brussels, March 2007, p.35):

> "An Internal model is a risk management system of an insurer for the analysis of the overall risk situation of the insurance undertaking, to quantify risks and/or to determine the capital requirement on the basis of the company specific risk profile."

Workers Compensation is a specific branch in Property & Casualty (P&C). Although classified as General Insurance, it has some of the features of Life & Savings Insurance (L&S). These specificities make the Workers Compensation internal model different from other branches' P&C internal models. Broadly speaking we need to design two sub models, one for P&C Risks and another one for L&S Risks.

In section II, we will introduce Workers Compensation Insurance and some components of Legislation about Workers Compensation. The Internal Model will be developed in section III. We will start with the Model Architecture and we will describe its components. In section IV, we will present a case study.

Due to space limitation we will only discuss the main points of the model.

SECTION II – Workers Compensation

In Portugal, Workers Compensation (WC) is mandatory according to Law No. 98/2009 of September 4th (see [9]); all employers have the obligation to insure the risk (all employees) in an insurance company. Also, all self-employees have the obligation to subscribe WC insurance.

This line of business includes obligations to compensate the victim and/or respective beneficiaries in case of accidents at work or occupational diseases occur (also including rehabilitation and reintegration). The accident on the journey between the employee's residence and the workplace and *vice versa*, and others specific situations, for example, when the accident occurred between the workplace and the place where the employee takes meals are also taken into account.

The nature of the disability may be temporary or permanent. The temporary disability may be partial or absolute. The permanent disability may be partial, permanent for usual work or permanent for any work.

To make it easier for the readers to understand this issue, we will divide the losses in two parts: Compensations and Annuities.

Compensations:

The right to reparation includes the following forms: in kind and in cash. In kind, the main benefits are medical, pharmaceutical and hospital assistance needed to restore health and work capacity. Included are also transportation and accommodation, technical help for functional disabilities, thermal treatments and dependent relatives' Psychological assistance. All compensations provided by law, such as, Compensation for temporary disability, death and funeral expenses, subsidies for high disability (above 70%), house adaptation, rehabilitation and social integration are paid in cash.

The compensations for temporary disability intend to compensate the victim for the temporary loss of work capacity while under ambulatory treatment or vocational rehabilitation. In absolute temporary disability, the victim earns a daily compensation equal to 70% of daily salary during the first 12 months and 75% in the following period. For partial temporary disability, the victim has the daily compensation equal to 70% of daily salary times the degree of disability.

Annuities:

Financial compensations for permanent disability are more complex because they include not only the victim but they may include others beneficiaries (Orphans, Husband/Wife, Parents or equivalents that live together and have earnings below the social pension). In Portugal this takes a significantly different character from what is found in most European countries (Belgium, Finland and Denmark are the exceptions that we know similar to Portugal). The management of this risk is maintained in P&C team and it is present on P&C Balance sheet (it is usually transferred for L&S Balance sheet). This requires having a specific sub model according to L&S principles.

Law defines that in absolute permanent disability for any kind of work, the victim has the right to an annual pension equals to 80% of the salary and can add 10% per dependent person until the salary limit is reached. In absolute permanent disability for usual work, the victim has the right to an annual pension between 50% and 70% of his salary, depending on the functional capacity to develop another compatible work. In partial permanent disability, the victim has the right to an annual pension equal to 70% of his salary devaluated by the degree of ability. Providing additional support for third person is assigned to victims without the capacity for basic daily needs.

In case of death, the family or equivalent beneficiaries have the right to compensation:

– Husband/Wife or equivalent beneficiaries: compensation is 30% of the victim's salary until the retirement age of the beneficiary and 40% above retirement age or, 40% when disability or chronic illness is verified;

– Orphans: compensation is 20% of victim's salary if there is only one; it's 40% if there are two orphans; and 50% if there are three or more orphans (may be 80% for orphans of both parents). The orphans have the right for compensation until 25 years old as long as they are students. Orphans are entitled to a pension for life in case of disability or chronic illness;

– Parents or equivalent beneficiaries: compensation is 10% of the victim's salary for each beneficiary, limited to 30% of the salary. When there isn't husband/wife or orphans, the parents or equivalents earn 15% for each until retirement age and 20% above retirement age or, 20% when disability or chronic illness is verified (however limited to 80%);

There are two types of pensions: the compulsorily recoverable and the not compulsorily recoverable. A pension is compulsorily recoverable for a victim when he has less than 30% of disability and his annual pension is less than six times the minimal national salary. For other beneficiaries (except Orphans) only the second condition applies. On the other hand, a pension can be partially recoverable for victims if they have 30% or more of disability. Other beneficiaries can be partially recoverable if their pension leftover is not less than six times the minimal national salary and Capital Redemption cannot be more than the capital that results of 30% of disability. The law defines that the mathematical provisions for compulsory recoverable and partial recoverable are calculated applying the following conditions: mortality table – TD 88/90 and rate of interest – 5.25% (maybe with rate of management).

The victims can require the revaluation of their disability once a year. In outdated law (before 2010), this situation is only possible during 10 years after the pension has been fixed.

The Workers Compensation Fund (known by "FAT") is responsible for pensions' actualization (see [10]). The Fund receives from companies two types of contributions: 0.15% of Sum Insured and 0.85% of Capital Redemption of pensions' stock at December 31 (that includes the Mathematical Provision of third person's assistance). The Capital Redemption amount is calculated applying the following conditions: Mortality table – TD 88/90; Rate of interest – 5.25%; and Rate of management – 0%. The companies have predicted the provision for Future "FAT" in their Balance Sheet.

SECTION III – Model

The specificities of Workers Compensation described in Section II are quite important to understand the Model design. Two questions need to be answered: "What will be the consequences of occurred accidents?" and "What do I need to project concerning what has not occurred yet?".

The first question is easier to answer since the event has occurred and the actuary can monitor. The traditional way to monitor something in insurance is using triangles. The triangles allow us to understand how something develops (for example, compensations or annuities mathematical reserves). Usually, the analysis cross occurrence year and development year, *i.e.*, in practice, we can understand the gap between occurrence claims and payment moments. For annuities, the gap between the occurrence and reserve constitution year.

The second question is more difficult to tackle. The actuary has to know business and its main indicators. In insurance business, companies usually monitor premiums, policies in force, sum insured, frequency, mean cost, loss ratio, *inter alia*. Predicting something is a task far from simple and requires great sensitivity on the subject.

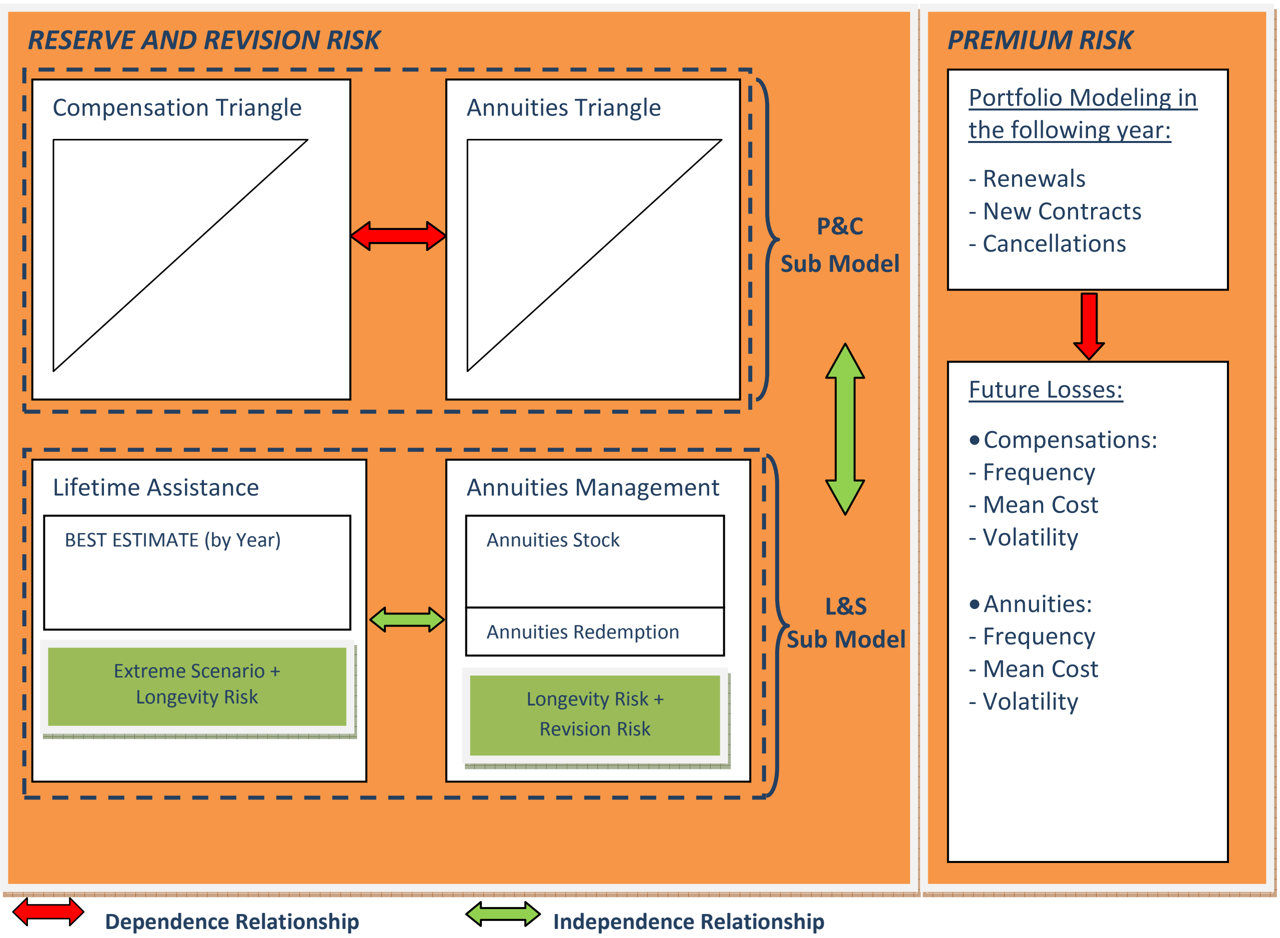


Figure 3.1. Model Architecture.

Figure 3.1 introduces our Model architecture proposal in which all components that make sense to be considered are visible. The objective is to make the bridge between monitoring (currently existing) and modeling (Solvency II requirements). Note that before Solvency II, modeling already exists in a deterministic concept and based on actuary sensibility. Solvency II introduces a new step, based on stochastic concepts (more challenging). Note that, the SCR is the capital that companies have in account at December 31, in addiction to technical provisions to cope with an extreme scenario in the following year.

In the Compensations triangle, companies monitor all payments made, for example, subsidies, hospitalization, surgery, orthopedics and transportation costs. Workers Compensation Insurance is long term management insurance. Companies have to restore the victims' work capacity and this may take a long time in complex cases. It is important for companies to monitor the payments made in order to study their behavior and unpredictability. Sub model P&C capture volatility in order to predict an extreme scenario.

The Annuities triangle is similar to the previous one, however it monitors the reserve for financial compensation (assigned victim and other beneficiaries). Independent on annuities management, the annuities triangle monitors the reserve at date of court decision. Essentially, the triangle has various objectives: to capture the period elapsed between the accident and the assignment of a pension to the beneficiaries (there are two fases in between: the recovery of the injured and the evaluation of permanent disability); to evaluate the severity of impairment and / or temporal prediction of responsibility. Annuities' triangle will have the same treatment as the compensations' triangle (Sub model P&C).

However, the annuities' triangle doesn't capture the revision risk. The revision risk can only occur for victims (not for other beneficiaries) and it depends on disability evolution. It is applied to all victims independently whether the pension is active or not (there are some cases in which pension has been redeemed and the subsequent disability was revised). According to QIS5 [2] (reference Shock risk for revision, page 257), the computation of the risk has to take into account the historical relative change of individual annuities. The aim is to forecast the individual annuities for which a revision process is possible to occur during the next year.

Companies are required to manage pensions which are not compulsorily recoverable. This means that the company will support a series of payments until the death of the pensioner. Companies have to predict the amount of payments discounted reflecting the mortality effect for all pensioners (this task is monitored monthly). However, a decrease on mortality rates leads to an increase in technical provisions. This is one of the Life and Savings (L&S) risks, known as Longevity Risk. In recent years, people have got better health care; science and technology have evolved in cases of cancer or other diseases, and so on. Hence an increase in the life expectancy is expected. Companies have to be prepared for this scenario.

The Lifetime Assistance is a provision that companies create to assist more complicated victims' cases. For example: victims who are in wheelchairs; victims that use advanced prosthesis to address causes of the accident; victims who need regular surgeries to keep and/or not to deteriorate their quality of life; and so on. These are some of the regular needs which follow the victims until their death. In the company under study, this provision is only calculated for compensations that are paid 15 years after the occurrence of the claim. The severity of annual payments and the longevity of these complex cases are the two risks implicit in lifetime assistance.

Annuities Management and Lifetime Assistance are supported by a mortality table reference and using a given interest rate. We will only focus in the first problem because Interest Rate Risk is a market risk (in Solvency II framework) and not addressed in our internal models (internal models only have to address cash-flow projections). The L&S Sub model tackle the longevity risk and will be supported in two steps: the adjustment of the mortality experience and the variability of expected life time. INE Statistics Portugal helped to launch the first pillar when it published on May 30, 2012, *the Complete Mortality Tables for Portugal* - 2009-2011.

Until now, we only focused on one part of the model that answers the question “What will be the consequence of occurred accidents?” (risks that have already occurred). The second step is to answer “What do I need to project concerning what has not occurred yet?” (risks that will probably occur), *i.e.*, we have to model our portfolio and the future losses to calculate the premium risk (the risk of premiums being insufficient).

Portfolio modeling is not a simple task, nobody has a crystal ball. However, the evolution of some principal indicators and the expectancies of companies can help us. Usually, a Workers Compensation tariff is a rate that is multiplied by the sum insured (equivalent to the total of salaries of the insured enterprise). The rate depends on the activity of the enterprise and the risk associated with it. Companies monitor the number of policies, the sum insured, the average rate used in their portfolio (renewals, new contracts and lapse contracts) over several years. This is important to help us to model the portfolio in the following year.

The future losses are correlated with the portfolio insured. Depending on the sum insured, the company expects more or less losses. However, the model has to absorb possible bad scenarios in losses behavior. The future losses involve compensations and annuities. We have chosen to model the losses using two simple indicators: frequency (for example, number of claims per one million Euros insured) and mean cost (per claim). The model captures the trend of these indicators and their standard deviation. Note that, the lifetime assistance is considered as not having impact in future losses in a one year development.

The Risk of Premiums may occur for several reasons. In portfolio perspective, it may occur if the company fails to renew policies, if the policies are revised by lower rates and/or the average capital per policy decreases. In losses perspective, the risk of premiums may occur if the frequency and mean cost increase more than expected.

In this Model, due to size and time limitations, CAT Risk will not be treated.

SUBSECTION III-A – Sub Model for P&C Risks

This Sub Model will be applied to estimate the Compensation and Annuities reserves. Merz and Wüthrich (M&W) developed a method based on claims development result for the Solvency II purposes [8]. Adopted by QIS5 Technical specifications [2, page 255], this method is based on the mean squared error of prediction (MSEP) of the claims development result over one year volatility.

Loss reserving is one of the basic actuarial tasks. Based on observed claims development figures (usually, triangles) actuaries have to predict the total ultimate claim. At time I we predict the total ultimate claim with the information available at time I. Repeating at time $I+1$ we predict the same total ultimate claim with the information available at time $I+1$. The claims development result for accounting year $(I, I+1]$ is the difference between these two successive predictions of the total ultimate claim.

Chain Ladder (CL) method is one of the most well known methods to predict the total ultimate claim. Mack was the first to study the total uncertainty in claim development until the total ultimate claim (see Mack [7]) based on a long-term view (the total uncertainty of the full run-off triangle). M&W took advantage of Mack's approach but focused on the short-term view which is in accordance with the Solvency II purposes (for more details see M&W [8]).

Denote by $C_{i,j}$ the cumulative payments for accident year $i \in \{0,\dots,I\}$ and development year $j \in \{0,\dots,J\}$. The ultimate claim for accident year i is denoted by $C_{i,J}$. For simplicity, assume that $I = J$.

Table 3.1. Triangle of cumulative payments.

	Development Year				
Accident Year	0	1	...	J-1	J
0	$C_{0,0}$	$C_{0,1}$		$C_{0,J-1}$	$C_{0,J}$
1	$C_{1,0}$	$C_{1,1}$		$C_{1,J-1}$	
...					
I-1	$C_{I-1,0}$	$C_{I-1,1}$			
I	$C_{I,0}$				

Model Assumptions (by Mack):

- Cumulative Payments $C_{i,j}$ in different years $i \in \{0,\dots,I\}$ are independent;
- $(C_{i,j})_{j\geq 0}$ are Markov processes and there are constants $f_j > 0$, $\sigma_j > 0$ such that for all $1 \leq j \leq J$ and $1 \leq i \leq I$ we have
    - $E[C_{i,j} \mid C_{i,j-1}] = f_{j-1} \times C_{i,j-1}$ (3.1)
    - $Var[C_{i,j} \mid C_{i,j-1}] = \sigma_{j-1}^2 \times C_{i,j-1}$ (3.2)

Let $D_I = \{C_{i,j} : i + j \leq I \wedge i \leq I\}$ denote the claims data available at time $t = I$ and $D_{I+1} = \{C_{i,j}; i + j \leq I+1 \wedge i \leq I\} = D_I \cup \{C_{i,I-i+1}; i \leq I\}$ denote the claims data available one period later, at time $t = I+1$.

The CL factors f_j can be estimated as follow:

1. At time $t = I$, given information D_I, the f_j are estimated by

$$\hat{f}_j^I = \frac{\sum_{i=0}^{I-j-1} C_{i,j+1}}{S_j^I}, \text{ where } S_j^I = \sum_{i=0}^{I-j-1} C_{i,j}. \tag{3.3}$$

2. At time $t = I + 1$, given information D_{I+1}, the f_j are estimated by

$$\hat{f}_j^{I+1} = \frac{\sum_{i=0}^{I-j} C_{i,j+1}}{S_j^{I+1}}, \text{ where } S_j^{I+1} = \sum_{i=0}^{I-j} C_{i,j} . \tag{3.4}$$

Then, given D_I and $C_{i,I-i}$, $E[C_{i,j} \mid D_I]$, $j \geq I - i$, can be estimated by

$$\hat{C}_{i,j}^{I} = C_{i,I-i} \hat{f}_{I-i}^{I} \ldots \hat{f}_{j-2}^{I} \hat{f}_{j-1}^{I} . \tag{3.5}$$

Given D_{I+1} and $C_{i,I-i+1}$, $E[C_{i,j} \mid D_{I+1}]$, $j \geq I - i + 1$,can be estimated by

$$\hat{C}_{i,j}^{I+1} = C_{i,I-i+1} \hat{f}_{I-i+1}^{I+1} \ldots \hat{f}_{j-2}^{I+1} \hat{f}_{j-1}^{I+1} . \tag{3.6}$$

M&W define the true Claims Development Result (denoted by CDR) as the margin between the expected total ultimate claims at time I and the expected total ultimate claims at time $I+1$. The true CDR for accident year $i \in \{1,\ldots,I\}$ in accounting year $(I, I+1]$ is defined by

$$CDR_i(I+1) = E[C_{i,J} \mid D_I] - E[C_{i,J} \mid D_{I+1}] \tag{3.7}$$

Note that, $E[C_{i,J} \mid D_I]$ can be estimated and it predict $C_{i,j}$ at time I . The true aggregate CDR is given by

$$\sum_{i=1}^{I} CDR_i(I+1) \tag{3.8}$$

M&W proved that

$$E[CDR_i(I+1) \mid D_I] = 0 . \tag{3.9}$$

The prediction uncertainty of this prediction 0 can be calculated by

$$msep_{CDR_i(I+1)|D_I}(0) = Var[CDR_i(I+1) \mid D_I] = E[C_{i,J} \mid D_I]^2 \frac{\sigma_{I-i}^2 / f_{I-i}^2}{C_{i,I-i}} . \tag{3.10}$$

However, the true CDR is not observable and the expected ultimate claims $E[C_{i,J} \mid D_I]$ and $E[C_{i,J} \mid D_{I+1}]$ can be estimated by $\hat{C}_{i,J}^{I}$ and $\hat{C}_{i,J}^{I+1}$. Then, the observable CDR for accident year $i \in \{1,\ldots,I\}$ in accounting year $(I, I+1]$ is defined by

$$C\hat{D}R_i(I+1) = \hat{C}_{i,J}^{I} - \hat{C}_{i,J}^{I+1} \tag{3.11}$$

And the observable aggregate CDR is given by $\sum_{i=1}^{I} C\hat{D}R_i(I+1)$. (3.12)

The M&W goal is to quantify the MSEP of claims development result, as follow,

$$msep_{C\hat{D}R_i(I+1)|D_I}(0) = E\left[\left(C\hat{D}R_i(I+1)-0\right)^2 \mid D_I\right]. \tag{3.13}$$

In order to quantify the conditional MSEP, we need an estimator for variance parameters $\hat{\sigma}_j^2$,

$$\hat{\sigma}_j^2 = \frac{1}{I-j-1}\sum_{i=0}^{I-j-1} C_{i,j}\left(\frac{C_{i,j+1}}{C_{i,j}} - \hat{f}_j\right)^2 . \tag{3.14}$$

Define the estimators for a single accident year,

$$\hat{\Delta}_{i,J}^I = \frac{\hat{\sigma}_{I-i}^2/(\hat{f}_{I-i}^I)^2}{S_{I-i}^I} + \sum_{j=I-i+1}^{J-1}\left(\frac{C_{I-j,j}}{S_j^{I+1}}\right)^2 \frac{\hat{\sigma}_j^2/(\hat{f}_j^I)^2}{S_j^I} \tag{3.15}$$

$$\hat{\Phi}_{i,J}^I = \sum_{j=I-i+1}^{J-1}\left(\frac{C_{I-j,j}}{S_j^{I+1}}\right)^2 \frac{\hat{\sigma}_j^2/(\hat{f}_j^I)^2}{C_{I-j,j}} \tag{3.16}$$

$$\hat{\Psi}_i^I = \frac{\hat{\sigma}_{I-i}^2/(\hat{f}_{I-i}^I)^2}{C_{i,I-i}}. \tag{3.17}$$

The variance of the true CDR is estimated by,

$$V\hat{a}r\left(CDR_i(I+1)\mid D_I\right) = \left(\hat{C}_{i,J}^I\right)^2 \hat{\Psi}_i^I , \tag{3.18}$$

and the estimates for the conditional MSEP's are given by

$$m\hat{s}ep_{C\hat{D}R_i(I+1)|D_I}(0) = (\hat{C}_{i,J}^I)^2(\hat{\Phi}_{i,J}^I + \hat{\Psi}_i^I + \hat{\Delta}_{i,J}^I). \tag{3.19}$$

However, (3.19) has not taken into account the correlations between different accident years. For the conditional aggregate observable CDR around 0, M&W obtain the following estimator

$$m\hat{s}ep_{\sum_{i=1}^I C\hat{D}R_i(I+1)|D_I}(0) = \sum_{i=1}^I m\hat{s}ep_{C\hat{D}R_i(I+1)|D_I}(0) + 2\sum_{k>i>0}\hat{C}_{i,J}^I\hat{C}_{k,J}^I\left(\hat{\Xi}_{i,J}^I + \hat{\Lambda}_{i,J}^I\right) \tag{3.20}$$

where,

$$\hat{\Lambda}_{i,J}^I = \frac{C_{i,I-i}}{S_{I-i}^{I+1}}\frac{\hat{\sigma}_{I-i}^2/(\hat{f}_{I-i}^I)^2}{S_{I-i}^I} + \sum_{j=I-i+1}^{J-1}\left(\frac{C_{I-j,j}}{S_j^{I+1}}\right)^2\frac{\hat{\sigma}_j^2/(\hat{f}_j^I)^2}{S_j^I} \text{ and } \hat{\Xi}_{i,J}^I = \hat{\Phi}_{i,J}^I + \frac{\hat{\sigma}_{I-i}^2/(\hat{f}_{I-i}^I)^2}{S_{I-i}^{I+1}}. \tag{3.21}$$

Note that, the prediction uncertainty relative to the observable CDR is evaluated by the conditional aggregate observable CDR, where $\hat{msep}_{\sum_{i=1}^{I} C\hat{D}R_i(I+1)|D_I}(0)^{1/2}$ is prediction standard deviation of 0 to $C\hat{D}R_i(I+1)$.

Assuming that

$$C=\sum_{i=1}^{I} C_{i,J} \cap Normal\left(\sum_{i=1}^{I} \hat{C}_{i,J}^{I}\ ,\ m\hat{s}ep_{\sum_{i=1}^{I} C\hat{D}R_i(I+1)|D_I}(0)^{1/2}\right), \tag{3.22}$$

the Solvency Capital Requirement is estimated by $S\hat{C}R = m\hat{s}ep_{\sum_{i=1}^{I} C\hat{D}R_i(I+1)|D_I}(0)^{1/2}\times\Phi^{-1}(0.995)$ where Φ is the distribution function of Standard Normal.

SUBSECTION III-B – Sub Model for L&S Risks

The L&S Sub model aims to respond to the longevity risk and will be supported by two pillars: the adjustment of mortality experience and the variability of expected life time.

The first step that supports this sub model is to find a recent mortality table as adequate as possible. Fortunately, INE Statistics Portugal published *the Complete Mortality Tables for Portugal* - 2009-2011 (called PT0911 and presented in annex II).

Another source, the Portuguese Association of Insurers (known as APS) compiles the information about mortality of Workers Compensation pensioners in a bench study. Initially, this information is reported by each company to Portuguese Insurance Institute (ISP). We have used the information reported by companies between 2006 and 2010, and we have built a crude mortality table for Workers Compensation experience (named WC0610 and presented in annex III). Note that, this table alone is not a good choice because, for some ages, it has little or no experience.

Remember that the Longevity Risk has only impact in pensions not compulsorily recoverable and Life Assistance. In general, due to disability, the victims of accident do not have the same mortality behavior of the Portuguese population. For this purpose, INE table is onerous and inadequate but useful to retain the mortality behavior.

Using a methodology that appeared first in Financial Mathematics and later in Wang (see [12]) with respect to premium principles, we distort the survival function

$$S_{PT0911}(x)=1-F_{PT0911}(x) \tag{3.23}$$

by means of the power function $g(w)=w^{\frac{1}{p}}$, with $p < 1$, obtaining the survival function,

$$S_g(x)=g\left(1-F_{PT0911}(x)\right)=\left[S_{PT0911}(x)\right]^{\frac{1}{p}}. \tag{3.24}$$

Note that, our application is inverse than Wang when he calculated the premium by transforming the survival function with $p > 1$. Our aim is to devaluate PT0911 table using the WC0610 experience in order to retain the mortality behavior of Portuguese population and to take the lifetime expectancies of workers compensation victims.

To calibrate the value p, we have minimized the sum of the weighted quadratic differences between the survival functions,

$$Dif = \sum_{x=0}^{\infty} \frac{\left(S_{wc0610}(x) - S_g(x)\right)^2}{S_g(x)} \tag{3.25}$$

Note that, the survival function g will result in a new mortality table. Supported by this new mortality table, we calculate the traditional components of life contingent risks for typical beneficiaries.

Let Y_x be a random variable representing the present value of unit monthly payments that will be made in advance for a pensioner aged *x*. The expected value of Y_x depends on the beneficiary:

Victims: $E[Y_x] = \ddot{a}_x^{(12)} \approx a_x + \frac{13}{24}$ [whole life annuity in advance] (3.26)

Orphans: $E[Y_x] = \ddot{a}_{x:25-x}^{(12)} = \ddot{a}_x^{(12)} - {}_{25-x}E_x \times \ddot{a}_{25}^{(12)}$ [temporarily life annuity in advance] (3.27)

Husband/Wife: $E[Y_x] = \ddot{a}_{x:65-x}^{(12)} + \frac{4}{3} \times {}_{65-x}E_x \times \ddot{a}_{65}^{(12)}$ [whole life annuity in advance] (3.28)

Parents: $E[Y_x] = \ddot{a}_{x:65-x}^{(12)} + \frac{4}{3} \times {}_{65-x}E_x \times \ddot{a}_{65}^{(12)}$ [whole life annuity in advance] (3.29)

where,

$a_x = \sum_{k=1}^{\infty} v^k \, {}_k p_x$ [whole life annuity in arrear] (3.30)

$v = (1+i)^{-1}$ [discounting factor] (3.31)

${}_t p_x = \Pr[T_x > t] = S_x(t)$ [Prob. a life aged *x* survived for at least *t* years] (3.32)

${}_t q_x = \Pr[T_x \le t] = 1 - S_x(t)$ [Prob. a life aged *x* does not survive beyond age *x*+*t*] (3.33)

${}_n E_x = v^n \, {}_n p_x$ [Expected present value of the pure endowment] (3.34)

Note that in (3.28), we don't consider the possibility of the husband or the wife getting married again. In this case, the husband or the wife loses the right to pension but the company has to pay three times the annual pension amount in one time. We observed from the APS benchmark study, that since 2006, this possibility has almost not been used. The worst scenario for companies is to consider the rate of remarriage equal to zero and it represents a cost (see in section IV).

Let R^I be the reserves for a annuities portfolio at time I , then,

$$R^I = \sum_{w=1}^{W} R_w^I \tag{3.35}$$

in which W is the number of pensioners and R_w^I is the reserve for beneficiary *w* at time I (depending of annual amount and the expected present value $E[Y_x]$) .

It is expected that some pensioners die during accounting year $(I, I+1]$ and release reserve at time $I+1$. When it does not occur the reserve will be recalculated at time $I+1$ with the pensioner one year older.

Let P^{I+1} be the payments that will occur during accounting year $(I, I+1]$,

$$P^{I+1} = \sum_{w=1}^{W} \frac{P_w}{12} \times \left[\ddot{a}_{\overline{12}|i'} \times I_w + \ddot{a}_{\overline{6}|i'} \times (1 - I_w)\right], \tag{3.36}$$

where, P_w is the annual amount paid to pensioner *w,*

$$I_w = \begin{cases} 1, \textit{pensioner w doesn't die during accounting year } (I, I+1] \\ 0, \textit{pensioner w dies during accounting year } (I, I+1] \end{cases},$$

and, the factor $\ddot{a}_{\overline{n}|i'}$ corresponds to the present value of the *n* certain monthly payments of one monetary unit in advance (not depending on human life) and *i´* is the nominal annual rate of interest convertible in 12 times per year.

Theoretically, the relation between reserves at time I (already observed)*, expected* reserves at time $I+1$ *and expected payments* occurring in accounting year $(I, I+1]$ is

$$R^I = E\left[R^{I+1}\right] \times v + E\left[P^{I+1}\right]. \tag{3.37}$$

Note that the *expected* reserves at time $I+1$ *and expected payments that will* occur in accounting year $(I, I+1]$ depend on the volatility inherent to whether pensioners die or not. For solvency purposes, companies should perform by simulation the behavior of the second part of (3.37). For replica *m*, the capital requirement, CR^I , will be given by

$$CR^I(m) = R^{I+1}(m) \times v + P^{I+1}(m) - R^I \tag{3.38}$$

where $R^{I+1}(m)$ are the observed reserves and $P^{I+1}(m)$ are the observed payments. Note that, we simulate at each replica if each pensioner dies or survives (for a pensioner aged *x*, we used the distribution $BERNOULLI({}_0q_x)$).

When $CR^I(m)$ is positive, we can conclude that the reserves are insufficiency for that replica. The solvency capital requirement for longevity risk is given by confidence level 99.5% of distribution of CR^I .

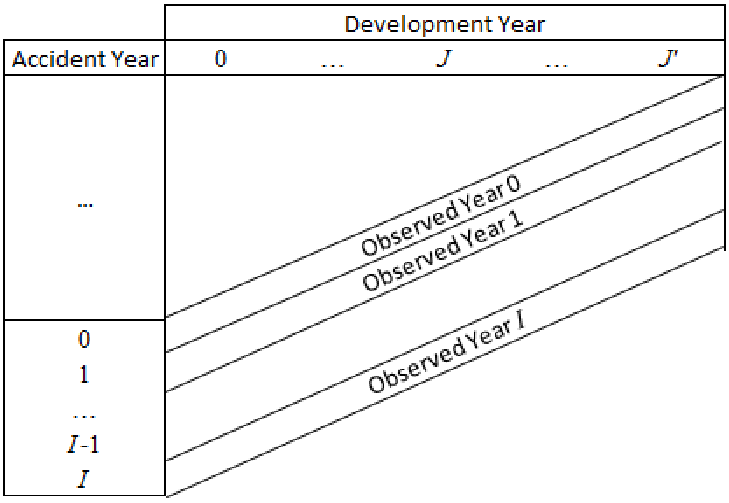
SUBSECTION III-C – Sub Model for Lifetime Assistance

The Lifetime Assistance includes all payments that companies have supported J years after the claim occurs. Until J years, the payments are included in the compensations' triangles. The company under study uses $J = 13$.

For example, the victims with permanent disabilities need actualization of prosthesis and/or chirurgical intervention to maintain their life quality. The frequency of payments after J years is low and we only consider the Amount of Annual Payments to construct the model.

Table 3.2 continues the illustration presented in Table 3.1 and introduces a new perspective in diagonal – the observed year. Each observed year reports for lifetime assistance payments (after J years of development). This information is particularly useful to predict a mean annual payment expected in the future.

Table 3.2. Triangle of cumulative payments for Lifetime Assistance illustration.



Using the same notation presented on Table 3.1, we define,

$$Mean\_Paym_j^I = \begin{cases} \dfrac{\sum_{i=-j}^{I-j} C_{i,j} \times (1+r)^{-i-1}}{I}, & j \in \{J+1, \ldots, J'\} \\ \dfrac{\sum_{k=J+1}^{J'} \sum_{i=-k}^{I-k} C_{i,k} \times (1+r)^{-i-1}}{I \times (J'-J)}, & j \in \{0, \ldots, J\} \end{cases} \tag{3.39}$$

where r is the annual inflation rate. On the other hand, we define the $Mean\_Age_j^I$ as the mean age of victims that originated the payments on development year j at time I.

The Lifetime Assistance Reserve at time I is

$$L^I = \sum_{j=0}^{J} Mean\_Paym_j^I \times \left(\frac{1+r}{1+i}\right)^{J-j+1} \times a_{Mean\_Age_j^I+J-j+1} + \sum_{j=J+1}^{J'} Mean\_Paym_j^I \times a_{Mean\_Age_j^I} \quad (3.40)$$

where and a_x is the present value of a whole life immediate annuity at age *x* for a monetary unit. For application purposes we use table PF6064 and 4.5% of interest rate (*i*).

Note that, in formula (3.40) we use the mean age at time I assuming that the last payments are a representative sample of the victims that will need lifetime assistance.

Using this methodology, we need to take care of two risks: the risk of volatility of payments and the risk of the beneficiaries' longevity.

For the risk of volatility of payments, we considered that the amount of lifetime assistance payments concerning accident year i (actually on development year j), follows a Normal Distribution. Then, at time $I+1$,

$$Amount\_Paym_j^{I+1} \sim N\left(Mean\_Paym_j^I, Var\_Paym_j^I\right),\ j \in \{0,\dots,J,\dots,J'\} \quad (3.41)$$

where

$$Var\_Paym_j^I = \begin{cases} \dfrac{\sum_{i=-j}^{I-j}\left[C_{i,j}\times(1+r)^{-i-1} - Mean\_Paym_j^I\right]^2}{I-1}, & j \in \{J+1,\dots,J'\} \\ \dfrac{\sum_{k=J+1}^{J'}\sum_{i=-k}^{I-k}\left[C_{i,j}\times(1+r)^{-i-1} - Mean\_Paym_k^I\right]^2}{I\times(J'-J)-1}, & j\in\{0,\dots,J\} \end{cases}. \quad (3.42)$$

The extreme scenario for annual payments lifetime assistance at time $I+1$ rises to

$$Amount\_Paym_j^{I+1} = Mean\_Paym_j^I + \sqrt{Var\_Paym_j^I} \times \Phi^{-1}(0.995) \quad (3.43)$$

where Φ is the distribution function of Standard Normal.

Then, the new mean payment at time $I+1$ is

$$Mean\_Paym_j^{I+1} = \begin{cases} \dfrac{\sum_{i=-j}^{I-j} C_{i,j}\times(1+r)^{-i} + Amount\_Paym_j^{I+1}}{I+1}, & j\in\{J+1,\dots,J'\} \\ \dfrac{\sum_{k=J+1}^{J'}\left[\sum_{i=-k}^{I-k} C_{i,k}\times(1+r)^{-i} + Amount\_Paym_j^{I+1}\right]}{(I+1)\times(J'-J)}, & j\in\{0,\dots,J\} \end{cases}. \quad (3.44)$$

Consequently, the Lifetime Assistance Reserves at time $I+1$ become

$$L^{I+1}=\sum_{j=0}^{J-1} Mean\_Paym_j^{I+1}\times\left(\frac{1+r}{1+i}\right)^{(J-j+1)}\times a_{Mean\_Age_j^I+J-j+2}+\sum_{j=J}^{J'} Mean\_Paym_j^{I+1}\times a_{Mean\_Age_j^I+1} \quad (3.45)$$

And the Lifetime Assistance Capital Requirement (LCR) will be given by

$$LCR=L^{I+1}\times v+\sum_{j=J-1}^{J'} Amount\_Paym_j^{I+1}-L^I\,. \quad (3.46)$$

Note that the longevity risk results from comparing the LCR (from (3.46)) using the PF6064 mortality table with the new mortality table g (shock on reserves L^{I+1}).

SUBSECTION III-D – Sub Model for PREMIUM RISK

This sub model proposes to study the risk of premiums being insufficient to cover all implicit liabilities on the following year. Normally, companies tariff their risks taking the past experience and define the theoretical premium according to one's needs. The premium value takes into account expected losses by risk unit, a security margin, taxes, general expenses of the company and commissions. The security margin is an important part of the premium that companies have reserved for the worst years.
In order to measure the premium risk, we need to model the Portfolio and the Future Losses in a coherent and realistic way.

1. PORTFOLIO MODELING

In Workers Compensation, it is common for companies to price risks based on a tariff rate that multiplies the capital insured (the total salaries of an enterprise). The activity of the enterprise is also taking into account. There are activities with low risk, medium risk or high risk. In our approach, we opt to model separately the new contracts, the cancellations and the renewals (described in annex1). We tried to model using the principal activities (agriculture, construction, independent workers, industry, services and transports) but the lack of experience did not allow us to follow this approach.
The principal indicators of business used in the model are: exposure, capital insured, earned premium, tariff rate and the mean capital by policy. Our strategy in portfolio modeling is to model the mean capital, the tariff rate and the rate of renewals. The other indicators are calculated from the mentioned ones.
Portfolio modeling is not a simple task and there are a lot of possible approaches. We have chosen a simple strategy which has given a good practical result.

2. FUTURE LOSSES

Depending on the portfolio insured, the future losses can be higher or lower than expected. Losses are correlated with the capital insured and the type of risk insured. No matter the losses amount, there are two metrics used by companies: mean cost and frequency. Our future losses model is supported by them, separately for compensations and annuities.

We perform some sensitivity analysis with respect to the mean cost and frequency in development year 0. We focus on the best estimate for the following accident year and capture the standard deviation of prediction.
Further details are presented in annex I.

3. PREMIUM RISK

The premium risk model measures the capital requirement for an extreme scenario. The based scenario is known by the best estimate. The extreme scenario is based on possible fluctuations of principal metrics that change the portfolio and the losses.
In the portfolio model, we have three elements that influence the portfolio: rate of renewals, mean capital by policy and tariff rate. The tariff rate depends on the company policy and it is sensitive to external influences. However, the other two suffer external influences and can be correlated. Hence, we have opted by choosing only one - the mean capital insured.
For Future Losses, we use the same methodology to model the indicators. We adjust a regression to the passed values.
In each replica we generate a value according to the regression using the normal assumption. We define the margin as the difference between the earned premiums and the expected losses. By simulation, we get the margin of the extreme scenario (according with Solvency II purposes) and compare it with the margin of the based scenario in order to get the capital requirement for premium risk.

SECTION IV – A case study

In this section we apply our model to a real case. The data has been provided by a company operating in the Portuguese market. Due to confidentiality reasons, they were transformed.

Firstly, we will see the P&C sub model application in which we will measure the impact of an extreme scenario on reserves. Secondly, we will show the L&S sub model in which we will use simulation to the revision risk and to the longevity risk in a short-term view. Thirdly, we will present the lifetime assistance model. Fourthly, we will focus on the premium risk and we will justify all the choices we made. Finally, we will provide the results of all sub models.

A) P&C Sub Model Application

This sub model is applied to the Compensations triangle and Annuities triangle in order to obtain their Solvency Capital Requirement equivalent to Reserve Risk in QIS5 [2]. However, Compensations and Annuities are correlated and will be treaty using Cholesky application (see [11]).

1. COMPENSATIONS TRIANGLE

The dataset for compensations is given in Table 4.1. This Table contains the cumulative payments $C_{i,j}$ for accident year $i \in \{0,...,I\}$ at time $I = 8$, the CL factors estimates $\hat{f}_j^I$ and the variance estimates $\hat{\sigma}_j^2$. To estimate $\hat{\sigma}_7^2$, we use the extrapolation given by Mack [7]:

$$\hat{\sigma}_7^2 = \min\left\{\hat{\sigma}_6^2, \hat{\sigma}_5^2, \hat{\sigma}_6^4 / \hat{\sigma}_5^2\right\}. \quad (4.1)$$

Table 4.1. Compensations and run-off triangle by CL method for time $I = 8$ (in Euros).

Compensations Triangle									
Accident	Development Year (j)								
Year (i)	0	1	2	3	4	5	6	7	8
0	13 442 040	21 758 105	23 069 075	23 596 880	23 999 776	24 160 536	24 371 445	24 619 060	24 866 711
1	15 146 219	23 740 976	25 145 598	25 894 484	26 100 090	26 401 492	26 855 693	27 058 634	27 330 826
2	16 511 358	25 410 273	27 687 150	28 366 383	28 758 677	29 383 786	29 740 728	30 002 305	30 304 108
3	15 995 429	27 125 347	28 797 782	29 592 273	30 286 762	30 693 684	31 086 081	31 359 492	31 674 947
4	17 766 457	28 578 064	31 567 149	32 904 969	33 330 783	33 787 080	34 219 024	34 519 990	34 867 237
5	20 082 621	32 417 981	35 973 995	37 181 146	37 743 042	38 259 742	38 748 866	39 089 673	39 482 889
6	21 561 689	38 421 434	41 649 820	42 930 307	43 579 086	44 175 681	44 740 436	45 133 941	45 587 958
7	23 500 233	36 777 275	39 839 075	41 063 892	41 684 465	42 255 123	42 795 325	43 171 722	43 606 000
8	19 474 543	31 675 834	34 312 927	35 367 847	35 902 340	36 393 841	36 859 110	37 183 296	37 557 335
estimation of f_j^I	1.6265	1.0833	1.0307	1.0151	1.0137	1.0128	1.0088	1.0101	
estimation of σ_j^2	125 092	12 444	1 473	923	1 052	462	87	16	

For compensations development between the eighth and the thirteenth year, we get a tail factor equal to 1.94%. In Table 4.2 the CL factors are presented and the best estimates for $j = 8,\ldots,13$. For $j \geq 14$, the compensations are included in Lifetime Assistance.

Table 4.2. Compensations CL factors development and Power regression application (section in gray).

j	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Develop Factors CL	162.65%	108.33%	103.07%	101.51%	101.37%	101.28%	100.88%	101.01%	100.49%	100.40%	100.33%	100.28%	100.23%	100.20%
Increment	62.65%	8.33%	3.07%	1.51%	1.37%	1.28%	0.88%	1.01%	0.49%	0.40%	0.33%	0.28%	0.23%	0.20%

Table 4.3 first block presents the ultimate diagonal observed at time $I = 8$, the total ultimate claim for each accident year $i \in \{0,\ldots,8\}$ (including the tail factor effect) and the estimated reserves at time $I = 8$.

In the second block, Table 4.3 provides the estimates for single and aggregated accident years:

- The estimated standard deviation of true CDR using (3.18);
- The squared root of the estimate for MSEP between the true CDR and observed CDR ((3.18) to (3.19));
- The estimated standard deviation of observed CDR as in (3.19) and (3.20) for aggregated Accident Years;

The coefficient of variation of reserves and the Solvency Capital Requirement are presented in the third block of Table 4.3. In conclusion, the company needs approximately 6.136 million Euros for risk of reserves in compensations (it represents 15.1% of estimated reserves). Considering the “extreme case” for each year we would get 9.991 million Euros (note the importance joint calculations). Note also that we only consider the one-year uncertainty of the claims reserves run-off.

Table 4.3. Compensations volatilities of the estimates and Solvency Capital Requirement calculations (in Euros).

COMPENSATIONS Accident Year (i)	Ultimate diagonal observed	Total ultimate claim	Estimated reserves at time I	estimation process variance Var (CDR i (I+1)|DI)	estimation error of msep $_{CDR\ i\ (I+1)\|DI}$ (0)	estimation msep^(1/2) at point 0 msep $_{CDR\ i\ (I+1)\|DI}$ (0)	**Coefficient of Variation**	**Capital Required**	**% Capital Required**
							Normal Aproximation		
0	24 866 711	25 349 296	482 585						
1	27 058 634	27 861 231	802 597	21 354	22 387	30 938	3.85%	79 690	9.93%
2	29 740 728	30 892 216	1 151 488	52 252	43 505	67 992	5.90%	175 137	15.21%
3	30 693 684	32 289 658	1 595 974	123 744	82 626	148 794	9.32%	383 268	24.01%
4	33 330 783	35 543 901	2 213 118	197 017	121 610	231 527	10.46%	596 374	26.95%
5	37 181 146	40 249 128	3 067 982	197 526	130 597	236 795	7.72%	609 944	19.88%
6	41 649 820	46 472 677	4 822 857	268 106	167 662	316 215	6.56%	814 515	16.89%
7	36 777 275	44 452 256	7 674 981	754 841	345 621	830 204	10.82%	2 138 465	27.86%
8	19 474 543	38 286 205	18 811 662	1 886 527	711 588	2 016 269	10.72%	5 193 565	27.61%
Aggregated Acc.Years	**280 773 325**	**321 396 568**	**40 623 244**	**2 072 916**	**1 173 366**	**2 381 968**	**5.86%**	**6 135 542**	**15.10%**
Without Aggregated							9.55%	9 990 957	24.59%

Note: Include Tail factor.

2. ANNUITIES TRIANGLE

The dataset for annuities is given in Table 4.4 where we present the cumulative reserves $C_{i,j}$ for accident year $i \in \{0,...,I\}$ at time $I=8$. The reserve is created after the court decision and it is assigned from development year $j \in \{0,...,J\}$. In Table 4.4 are also present the CL factors estimates $\hat{f}_j^I$ and the variance estimates $\hat{\sigma}_j^2$. For annuities, the CL factors are higher than the CL factors from compensations.

Table 4.4. Annuites and run-off triangle by CL method for time $I=8$ (in Euros).

Annuities Triangle									
Accident	Development Year (j)								
Year (i)	0	1	2	3	4	5	6	7	8
0	9 609 601	15 946 669	18 715 835	20 115 654	20 639 248	21 166 121	21 641 814	21 721 660	21 795 679
1	11 877 741	19 795 357	23 410 664	25 592 110	26 770 851	27 325 051	27 702 204	27 958 812	28 054 085
2	14 858 411	23 209 444	26 920 852	28 819 073	30 317 234	31 259 201	32 029 223	32 247 616	32 357 503
3	11 525 041	20 089 354	23 592 257	25 791 966	27 142 662	28 078 491	28 649 871	28 845 221	28 943 514
4	10 711 443	20 140 610	25 071 509	27 944 596	29 413 713	30 243 610	30 859 048	31 069 462	31 175 334
5	12 660 218	21 250 523	25 371 258	28 232 004	29 557 132	30 391 076	31 009 515	31 220 954	31 327 343
6	10 517 298	19 930 678	24 249 351	26 522 567	27 767 459	28 550 908	29 131 901	29 330 538	29 430 485
7	8 461 797	15 836 719	18 879 565	20 649 400	21 618 622	22 228 584	22 680 921	22 835 572	22 913 387
8	9 637 276	16 684 887	19 890 699	21 755 321	22 776 452	23 419 081	23 895 645	24 058 578	24 140 561
estimation of f_j^I	1.7313	1.1921	1.0937	1.0469	1.0282	1.0203	1.0068	1.0034	
estimation of σ_j^2	178 296	17 612	8 267	2 799	991	920	377	155	

For annuities development, we calculated a tail factor equal to 2.89%. In Table 4.5 the CL factors and the best estimates are presented for $j = 8,...,29$.

Table 4.5. Annuities CL factors development and Power regression application (section in gray).

j	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Develop Factors CL	173.13%	119.21%	109.37%	104.69%	102.82%	102.03%	100.68%	100.34%	100.50%	100.39%	100.31%	100.25%	100.20%	100.17%	100.14%
Increment	73.13%	19.21%	9.37%	4.69%	2.82%	2.03%	0.68%	0.34%	0.50%	0.39%	0.31%	0.25%	0.20%	0.17%	0.14%

j	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
Develop Factors CL	100.12%	100.11%	100.09%	100.08%	100.07%	100.06%	100.06%	100.05%	100.05%	100.04%	100.04%	100.03%	100.03%	100.03%	100.03%
Increment	0.12%	0.11%	0.9%	0.8%	0.7%	0.6%	0.6%	0.5%	0.5%	0.4%	0.4%	0.3%	0.3%	0.3%	0.3%

Table 4.6 contents are similar to those of Table 4.3 for annuities. The company needs approximately 6.464 million Euros according to aim of Solvency II requirements (it represents 16.1% of the estimated reserves). Without aggregation of the accident years, the percentage increases to 28.1%.

Table 4.6. Annuities volatilities of the estimates and Solvency Capital Requirement calculations (in Euros).

ANNUITIES Accident Year (i)	Ultimate diagonal observed	Total ultimate claim	Estimated reserves at time I	estimation process variance Var (CDR i (I+1)|DI)	estimation error of msep $_{CDR\ i\ (I+1)\|DI}$ (0)	estimation msep^(1/2) at point 0 msep $_{CDR\ i\ (I+1)\|DI}$ (0)	Coefficient of Variation	Capital Required	% Capital Required
							Normal Aproximation		
0	21 795 679	22 425 784	630 105						
1	27 958 812	28 865 118	906 306	67 692	76 798	102 373	11.30%	263 696	29.10%
2	32 029 223	33 292 947	1 263 723	113 516	111 906	159 401	12.61%	410 591	32.49%
3	28 078 491	29 780 260	1 701 769	167 088	127 034	209 895	12.33%	540 654	31.77%
4	29 413 713	32 076 602	2 662 889	181 053	138 030	227 668	8.55%	586 433	22.02%
5	28 232 004	32 233 005	4 001 001	306 555	180 660	355 829	8.89%	916 555	22.91%
6	24 249 351	30 281 309	6 031 958	511 201	242 311	565 722	9.38%	1 457 203	24.16%
7	15 836 719	23 575 804	7 739 085	659 495	253 597	706 573	9.13%	1 820 011	23.52%
8	9 637 276	24 838 455	15 201 179	1 951 414	654 436	2 058 228	13.54%	5 301 645	34.88%
Aggregated Acc.Years	**217 231 269**	**257 369 284**	**40 138 015**	**2 162 503**	**1 273 245**	**2 509 497**	**6.25%**	**6 464 035**	**16.10%**
Without Aggregated							10.93%	11 296 788	28.14%

Note: Include Tail factor.

3. CHOLESKY APLICATION

The Solvency Capital Requirement is the congregation of the different risks which may or may not have implicit correlations. The correlation between Compensations and Annuities is 0.6439 and this requires a harmonization between SCR of Compensation and SCR of Annuities.

Cholesky decomposition allows to generate a Gaussian random vector $N_{px1} \cap Normal_p(\mu, \Sigma)$ where Σ is the covariance matrix. The Methodology adapted to our case is the following:

- Find matrix L_{pxp} such as $L \times L^T = \Sigma$, where L is a lower triangular matrix with strictly positive diagonal entries and L^T is the matrix transpose of L;
- Simulate $N_1, ..., N_p \cap Normal(0,1)$ independently and consider $N = (N_1, ..., N_p)^T$;
- Calculate $N_{px1} = \mu + L \times N$; (4.2)
- The sum of the elements of vector N_{px1} represents one replica.

Formulas to obtain L, for $i = 1,...,I$ and $j = i+1,...,I$:

$$l_{ii} = \sqrt{a_{ii} - \sum_{k=1}^{i-1} l_{ik}^2} \quad (4.3)$$

And

$$l_{ji} = \frac{a_{ji} - \sum_{k=1}^{i-1} l_{jk} l_{ik}}{l_{ii}} \quad (4.4)$$

where a_{ij} are the elements of matrix Σ and l_{ij} are the elements of matrix L.

The results are presented in Table 4.7.

Table 4.7. Cholesky Application for Compensations and Annuities reserves.

			Covariance Matrix (∑)		Cholesky decomposition(L)	
	μ	σ	Compensation	Annuities	Compensation	Annuities
Compensation	40 623 244	2 381 968	5 673 769 238 603	3 848 964 544 473	2 381 968	0
Annuities	40 138 015	2 509 497	3 848 964 544 473	6 297 572 766 470	1 615 876	1 920 030
TOTAL	80 761 259					

According with the previous P&C sub model application, the company needs more and less 12.600 million Euros (6.136 million Euros from compensations and 6.464 million Euros from annuities). After 10 000 replicas of Cholesky application (following (4.2)), the company needs only 11.422 million Euros for aggregated capital requirement (14.14% of the reserves). The capital requirement reduces by 1.177 million Euros due to the fact of using the joint approach.

B) L&S Sub Model Application

The L&S Sub Model will be applied to pensions not compulsorily recoverable in order to measure the longevity risk. This risk has impact on annuities management and is linked to an unexpected low mortality. However, the annuities' amount can be revised and this fact has impact itself and indirectly through the Longevity Risk simulation. This risk is known as revision risk and it will be primarily simulated.

1. REVISION RISK

Revision risk captures the risk of adverse variation of an annuity's amount, as a result of the disability revision. Revision risk can occur on active pensions and on old pensions that were already redeemed. Revision risk represents a cost for companies and it depends on the average percentage of individual annuities for which a revision process occurred and on the average relative change of individual annuities amount.

Let P be a random variable that takes values 1 or 0 if the revision process has occurred or not, respectively. Let X be a random variable representing the relative change of individual annuities amount.

Taking into account 5 years of observations, the revision risk fits to:

$$P \cap BERNOULLI(\hat{p} = 0.002893)$$

and

$$X \cap GAMMA(\hat{\alpha} = 0.581649, \hat{\theta} = 1.056817)$$

where the parameters have been estimated using the method of moments.

This means that revision occurs only on 0.289% of the annuities for which it is possible and that the average relative change amount is 61.5% (for gamma, $E[X] = \alpha\theta$).

Table 4.8 presents the revision risk impact using 10 000 replicas for the annuities portfolio. Note that the relevant size of the shock is given by the difference between the quantile 99.5% of the distribution and the mean impact. In the present case, the company needs to reserve 344 thousands Euros for Revision Risk.

Table 4.8. Revision risk results for a real annuities portfolio.

REVISION RISK	Mean Impact	Extreme Scenario (99.5%)	Reserve Increase in accounting year (8,9]
Not Compulsorily Recoverable		152 796	
Compulsorily Recoverable		3 826	
Redeemed		452 325	
TOTAL	264 624	608 948	344 324

2. LONGEVITY RISK

By definition, Longevity Risk is associated with insurance obligations (such as annuities) in which a company guarantees to make a series of payments until the death of the beneficiary. A decrease in mortality rates leads to an increase in the technical provisions. This risk will be tackled only for not compulsorily recoverable pensions.

The reserves are based on a recognized mortality table and it represents the base line for longevity risk. In the company under study the base line is supported by mortality table PF6064 and interest rate equal to 4.5%. However, the base line may be outdated due to lifetime expectancies improvement.

Figure 4.1 presents the survival function table PT0911 and WC0610 on which we construct a new mortality table (called g and also present in figure). We obtained 0.8423 as an estimate of p by minimizing the sum of the weighted quadratic differences between the survival functions (reports to (3.24) on page 15).

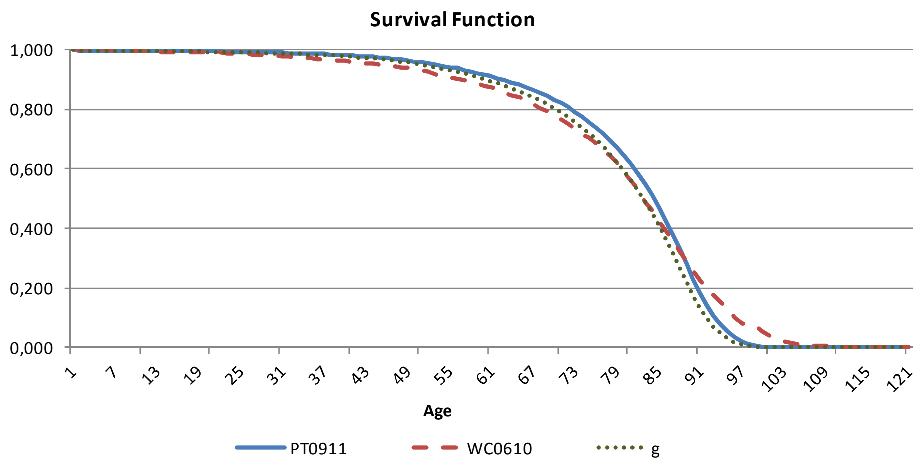


Figure 4.1. Survival Function PT0911, WC0610 and g.

The difference between reserves calculated using the base line and using the new mortality table g is 4.306 million Euros. This amount represents the expected insufficient reserves. However this amount only makes sense in a long term view of longevity risk. Companies should have assets to prevent this scenario occurrence.

Another risk for companies is the non-remarrying risk. In the latest years, the companies realized that the beneficiaries' Husband and Wife don't get married again because they lose the right to pension. The remarried rate observed is almost equal to zero. This represents a concern for companies in the long term because the disappearance of remarriage implies an increase on the reserves. In our company, it represents 909 thousand Euros.

Table 4.9. Long Term risks for a real annuities portfolio.

Long Term	Reserve	Realistic Scenario	Impact
LONGEVITY RISK (c)	101 875 858 (a)	106 181 473 (b)	4 305 615
NON-REMARRYING RISK (a)	24 191 796	25 101 242	909 446
TOTAL			5 215 061

(a) Suported on mortality table PF6064; (b) Suported on new mortality table g.

(c) Including non-remarrying risk impact.

In contrast with long term risk, the longevity risk in a short term perspective represents the risk of having less mortality than it is expected in a one-year period. By simulation (results on Table 4.10), we expected that the company would need 419 thousand Euros on extreme scenario (we made 10 000 replicas). Note that, the reserve increase is not the same for all types of beneficiaries. It is assumed normal due to mutualisation.

Table 4.10. Longevity risk results for a real annuities portfolio (with Revision Risk effect).

LONGEVITY RISK	Realistic Reserve	Extreme Scenario (99.5%)	Reserve Increase in accounting year (8,9]
Victims	73 263 100	73 567 932	304 832
Orphans	5 774 356	5 775 166	810
Husband/ Wife	26 223 878	26 318 429	94 551
Parents	920 139	938 704	18 565
TOTAL	106 181 473	106 600 231	418 758

C) LIFETIME ASSISTANCE Sub Model Application

Remember that lifetime assistance refers only to the payments that the company has to support after development year 14 ($J = 13$). However, the company has reserved a sufficient amount to cover this liability including all accident years.

Table 4.11 shows the lifetime assistance model results. The annual payment for lifetime assistance is inconstant and without trend. The standard deviation of the annual payment is high relative to the mean value. For development year 21 and more, we get a best estimate (600 thousand Euros) because it refers the aggregation of annual payment of various accident years and shows an increasing trend. We assumed the normality assumption to get an extreme annual payment according to the Solvency II purposes. This affects the payments at accounting year (8,9] and the recalculation of reserves at time 9.

Table 4.11. Lifetime Assistance Model Results.

Development Year (*j*)	21 and more	20	19	18	17	16	15	14	13	12	11
Mean Annual Payment (at time 8)	600 000	40 830	53 206	39 596	37 837	42 273	45 539	45 300	44 382	45 270	46 175
Standard Desviation of Annual Payment	146 295	26 284	20 322	18 797	19 431	20 196	22 937	21 729	21 813	22 249	22 694
Extreme Annual Payment at accounting year (8,9]	976 830	108 534	105 553	88 014	87 889	94 293	104 621	101 272	100 569	102 580	104 632
Mean Annual Payment (at time 9)	647 104	48 353	58 441	43 997	42 387	47 002	50 910	50 389	48 783	49 759	50 754
Reserve at time 8 (a)	6 599 315	565 071	688 755	487 723	512 616	608 834	575 353	639 842	599 875	573 714	523 794
Payments at accounting year (8,9]	976 830	108 534	105 553	88 014	87 889	94 293	104 621	101 272	100 569		
Reserve at time 9 (discounted at time 8) (a)	6 599 964	626 870	706 561	505 037	537 435	635 291	600 087	667 327	618 238	590 732	537 714
Reserve with Longevity Risk at time 9 (discounted at time 8) (b)	6 894 268	657 420	748 014	550 842	576 313	673 276	637 390	697 656	646 336	619 520	569 261

Development Year (*j*)	10	9	8	7	6	5	4	3	2	1	0	TOTAL
Mean Annual Payment (at time 8)	47 099	48 041	49 001	49 981	50 981	52 001	53 041	54 101	55 183	56 287	57 413	
Standard Desviation of Annual Payment	23 148	23 611	24 083	24 565	25 056	25 557	26 069	26 590	27 122	27 664	28 217	
Extreme Annual Payment at accounting year (8,9]	106 724	108 859	111 036	113 257	115 522	117 832	120 189	122 593	125 044	127 545	130 096	
Mean Annual Payment (at time 9)	51 769	52 804	53 860	54 937	56 036	57 157	58 300	59 466	60 655	61 868	63 106	
Reserve at time 8 (a)	535 074	499 032	520 752	486 629	474 987	442 090	442 128	451 649	431 139	430 297	410 757	17 499 430
Payments at accounting year (8,9]												1 767 575
Reserve at time 9 (discounted at time 8) (a)	550 416	512 294	536 199	500 081	488 118	453 347	453 877	464 599	443 057	442 635	422 112	17 891 990
Reserve with Longevity Risk at time 9 (discounted at time 8) (b)	579 087	542 350	562 330	527 632	515 010	481 528	480 506	488 800	467 467	465 692	445 368	18 826 065

(a) Suported on mortality table PF6064;

(b) Suported on new mortality table g.

Note that lifetime assistance sub model is easier to present by development year *j*. For example, the development year 0 refers to accident year 8 for which the company reserves 411 thousand Euros (that will start to be used after 13 years). According to Solvency II, an extreme scenario implies additionally 35 thousand Euros (12 thousands for extreme payments and 23 thousands for longevity risk).

The reserve at time 8 is the present value of future annual payments lifetime assistance considering the mean annual payment verified at time 8 and the average age of victims that needed the assistance. The reserve for lifetime assistance amounts 17.499 million Euros considering table PF6064.

The result of an extreme scenario in accounting year (8,9] implies an additional amount near to 1.768 million Euros. Note that the extreme scenario doesn't forecast possible correlations between observed years. It is similar to consider that it will occur an extreme scenario simultaneous in all observed years.

Applying the model at time 9 after an extreme scenario, the reserve undiscounted at time 8 amounts 17.892 million Euros when supported by PF6064 and the victims are one year older. It represents an increase of 12.34%, separately 1.767 million Euros from the payments and 392 thousand Euros from the reverse variation.

Additionally, the longevity risk represents 934 thousand Euros when we apply the new mortality table g. Then, the total capital requirement for lifetime assistance amounts 3.094 million Euros (extreme payment lifetime assistance at accounting year (8,9] and the longevity risk), equivalent to 17.68% of the reserve at time 8.

In all the calculations, we have considered inflation rate equal to 2% and interest rate equal to 4.5%.

D) PREMIUM RISK Sub Model Application

Portfolio and Future Losses are more difficult to model than the other aspects. Depending of the Actuary's sensibility and expectancies, it is common to consider the "BEST ESTIMATE" as the value expected by actuary. Separately, we show portfolio modeling, future losses modeling and Premium Risk as a result of both models.

1. Portfolio Modeling

Cleary, actuary has to define a strategy to model the portfolio. In our model, we have considered (separately) the renewals, the new contracts and the cancellation. We use simple principles and methodologies to estimate the main indicators of business.

Table 4.12 shows past behavior of some indicators for the new contracts. The tariff rate was a little unstable and the estimate was calculated dividing earned premiums by the capital insured (between year 2 and 8). A similar method was applied for exposure rate prediction. The value 8.436% (present in Table 4.16) is obtained dividing the exposure for new contracts between years 3 and 8 by the total exposure between years 2 and 7.

On the other hand we need to estimate the mean capital for year 9. The estimate is obtained using a linear regression on time. Summarizing, we will expect 6 882 policy in exposure and 319 million Euros in capital which represents 5.447 million Euros of Earned Premiums for New contracts.

Table 4.12. Portfolio Modeling for New Contracts.

NEW						
Year (*i*)	Exposure	Capital	Earned Premium	Tariff Rate	Mean Cap	STD DEV Cap
2	6 029	382 209 228	6 636 031	1.736%	63 398	4 382
3	6 445	237 557 621	5 249 165	2.210%	36 861	1 914
4	7 623	360 159 753	6 306 568	1.751%	47 247	2 978
5	7 226	329 407 457	5 601 685	1.701%	45 583	1 812
6	8 218	568 268 115	7 517 400	1.323%	69 150	3 589
7	7 346	266 802 548	4 555 491	1.707%	36 318	1 121
8	6 420	313 273 238	6 115 758	1.952%	48 797	4 596
9	6 882	318 898 136	5 447 423	1.708%	46 338	

The Cancellations show a floating exposure behavior similar to New Contracts (see Table 4.13) and we have chosen the same methodology to estimate year 9. However, the Mean Capital of Cancellations has a significant increase in the last years, while the tariff rate seems to be stabilizing. We opted by the linear regression to estimate the mean capital (65 365 Euros by policy) and the power regression to the tariff rate (1.707%).

Table 4.13. Portfolio Modeling for Cancellation.

CANCELATIONS						
Year (*i*)	Exposure	Capital	Earned Premium	Tariff Rate	Mean Cap	STD DEV Cap
2	5 590	219 003 001	6 009 472	2.744%	39 180	1 432
3	5 478	220 984 797	5 635 432	2.550%	40 342	1 722
4	7 086	255 510 825	6 388 920	2.500%	36 056	2 302
5	7 771	301 956 917	6 645 704	2.201%	38 858	3 231
6	6 285	296 695 914	5 655 308	1.906%	47 204	1 154
7	6 420	377 599 560	6 365 384	1.686%	58 820	2 438
8	6 105	406 779 751	6 962 877	1.712%	66 631	5 990
9	6 225	406 874 770	6 944 627	1.707%	65 365	

Renewals are the greatest part of portfolio and its indicators show more stability over the years (Table 4.14). The trend of renewals is decreasing and it seems to be stabilizing. We used a logarithm trend and we expect 66 thousands of exposure. For tariff rate, we used an AR(2) model and we get a prediction similar to year 8 (1.546%). The mean capital was the more difficult to estimate in year 9. The expectancies are for a further decrease and we have opted to conjugate two scenarios with same weight (50/50): a pessimist scenario (a cubic regression on time) and an optimist scenario (a quadratic regression on time). These regressions are censurable because the time series data don't allow us to adjust so many parameters. This simple method helps actuary to obtain a best estimate.

Table 4.14. Portfolio Modeling for Renewals.

RENEWALS						
Year (i)	Exposure	Capital	Earned Premium	Tariff Rate	Mean Cap	STD DEV Cap
2	73 283	2 746 134 027	69 536 176	2.532%	37 473	1 241
3	72 862	2 948 192 556	68 996 477	2.340%	40 463	1 407
4	72 657	3 059 763 727	67 693 513	2.212%	42 113	1 632
5	69 888	3 477 045 969	66 517 231	1.913%	49 752	2 144
6	72 197	4 015 104 871	63 757 290	1.588%	55 613	2 856
7	70 617	4 260 282 803	65 335 970	1.534%	60 329	2 992
8	69 054	3 851 597 448	59 527 347	1.546%	55 776	2 590
9	66 005	3 498 351 510	54 092 678	1.546%	53 001	

It's evident in Portfolio Modeling that the business is decreasing. Various reasons explain this fact: the unemployment in Portuguese economy leads to a decrease exposure, the competition among players lay down the tariff rate and the resizing of the enterprises cuts down the mean capital.

Table 4.15. Portfolio Modeling Results.

TOTAL						
Year (i)	Exposure	Capital	Earned Premium	Tariff Rate	Mean Cap	STD DEV Cap
2	84 902	3 347 346 256	82 181 679	2.455%	39 426	1 025
3	84 784	3 406 734 974	79 881 074	2.345%	40 181	1 035
4	87 366	3 675 434 305	80 389 001	2.187%	42 069	1 213
5	84 885	4 108 410 343	78 764 620	1.917%	48 400	1 554
6	86 700	4 880 068 900	76 929 998	1.576%	56 287	1 934
7	84 383	4 904 684 910	76 256 845	1.555%	58 124	2 095
8	81 579	4 571 650 437	72 605 982	1.588%	56 040	2 111
9	79 112	4 224 124 415	66 484 728	1.574%	53 394	

The expectancies are to earn 66.485 million Euros considering 79 112 policies in exposure and 4 224 million Euros of capital. It represents a downsizing of 7.6% compared to last observed year.

Table 4.16. Auxiliary to Portfolio Modeling.

Exposure Rate in order to last year			
Year (i)	NEW	CANCELATIONS	RENEWALS
3	7.591%	6.452%	85.819%
4	8.991%	8.358%	85.696%
5	8.271%	8.894%	79.994%
6	9.681%	7.405%	85.052%
7	8.473%	7.404%	81.450%
8	7.608%	7.235%	81.834%
9	8.436%	7.630%	80.909%

2. Future Losses

The purpose of this subsection is to study the past behavior of losses in order to estimate the losses at year 9. As it is usual we will use mean cost and frequency. Normally, frequency is the ratio between the number of claims and exposure. In workers compensation, the principle doesn't adjust perfectly because the risk present in a unit of exposure can be very different. The capital insured is very important and it represents the dimension of the enterprise and consequently the dimension of the risk. Then, we consider the frequency as the number of claims divided by the capital insured (and multiplied by 1 000 000), *i.e.*, the number of claims per million of Euros insured.

Starting with compensations analysis (Table 4.17), the mean cost is increasing almost linearly and the frequency is decreasing almost linearly. We estimate the mean cost and frequency for year 9 using a linear regression on time. The standard deviation of these regressions will be important in following subsection in order to simulate various replicas.

The annuities frequency and the annuities mean cost present a negative trend along last years. However, they show an unstable behavior that introduces some unpredictability. We estimated the mean cost has 7 342 Euros and the frequency as 0.19 claims by one million Euros insured (considering a linear trend). This scenario may be a bit optimistic but coherent with the past. Note that the standard deviation of frequency is relatively high when compares to the standard deviation of compensation frequency.

Table 4.17. Future Losses Results.

	COMPENSATIONS			ANNUITIES		
Year (*i*)	Mean Cost	Frequency	Loss Ratio	Mean Cost	Frequency	Loss Ratio
2	708	6.96	20.09%	10 333	0.43	18.08%
3	727	6.46	20.02%	9 935	0.34	14.43%
4	732	6.60	22.10%	10 077	0.29	13.32%
5	790	6.19	25.50%	9 004	0.34	16.07%
6	883	5.00	28.03%	9 201	0.23	13.67%
7	992	4.83	30.82%	7 916	0.22	11.10%
8	1 010	4.22	26.82%	7 710	0.27	13.27%
9	1 061	3.88	26.13%	7 342	0.19	9.04%
STD DEV	52	0.42		475	0.06	

* Only Development Year 0

Note that, we are only focused to predict the losses in a one-year period, according to our objective.

3. Premium Risk

Essentially, this subsection studies the risk of premiums being insufficient to cover all implicit liabilities. According with the objective we have simulated 10 000 replicas of possible scenarios considering the portfolio model and the future losses expectancies and their variability.

In Portfolio analysis, we only simulate the mean capital risk. The tariff rate and the exposure are more predictable and less unstable. Then, in each replica, we consider a different value for the mean capital by policy and take as constant the tariff rate and the exposure (using the best estimate). As a consequence, the capital and the earned premiums change from replica to replica.

On the other hand, the future losses are influenced by the mean cost and frequency. The two indicators are simultaneously simulated in the two types of losses (compensations and annuities) taking the normality assumption as seen before.

Table 4.18 shows the premium risk results, separately for the best estimate and the extreme scenario. The best estimate predicts 23.417 million Euros of charge at development year 0 and 47.817 million Euros of ultimate charge (considering CL factor discounted assuming the flat interest rate of 3%). The loss ratio becomes 71.92% and the margin is 18.668 million Euros.

Table 4.18. Premium Risk Results.

	Compensations	Annuities	TOTAL
BEST ESTIMATE			
Earned Premiums			66 484 728
Charge at Develop. Year 0	17 398 018	6 018 838	23 416 855
CL factor undiscounted	196.60%	257.73%	
CL factor discounted	190.24%	244.53%	
Ultimate Charge	33 098 554	14 718 049	47 816 603
Loss Ratio	49.78%	22.14%	71.92%
Margin			18 668 125
EXTREME SCENARIO (99.5%)			
Margin			2 351 688
Capital Requirement			16 316 437
Capital Requirement (% of Earned Premiums)			24.54%

The margin is the part of the earned premiums that companies save to pay commissions, general expenses with staff, premises, IT costs, and other implicit costs. The ratio between these expenses and the earned premiums is known as the expense ratio.

In extreme scenario, the margin is only 2.351 million Euros. Assuming that the premiums are adjusted in the best estimate to cover all liabilities, the company needs an additional amount of 16.316 million Euros. This is the capital requirement for Premium Risk and represents 24.54% of Earned Premiums.

E) FULL MODEL RESULTS

The aggregate result of sub models reports 15.35% of capital requirement (see Table 4.19). The two most important are the reserve risk and premium risk that represents 87.72% of 31.416 million Euros expected.

Table 4.19. Full Model Results.

Short Term	Reserve	Extreme Scenario (99.5%)	Reserve Increase in accounting year (8,9]	% Capital Requirement
RESERVE RISK	80 761 259	92 183 494	11 422 235	14.14%
Compensation	40 623 244			-
Annuities	40 138 015			-
REVISION RISK	264 624	608 949	344 324	130.12%
LONGEVITY RISK	106 181 473	106 600 231	418 758	0.39%
LIFETIME ASSISTANCE RISK	17 499 430	20 593 640	3 094 210	17.68%
PREMIUM RISK		16 136 635	16 136 635	-
TOTAL	204 706 786	236 122 948	31 416 162	15.35%

However, we don’t forget in the long term view that the company needs 5.215 million Euros, respectively, 4.305 million Euros for Longevity Risk and 909 thousand Euros for non-remarrying risk.

SECTION V – CONCLUSIONS

We are quite satisfied with our approach and the full model results. As expected on this kind of work, there are some points that allow improvements and reflections.

Positive Points of this model:

- The P&C sub model answers robustly for reserve risk. The Merz and Wüthrich method connected to Cholesky application results on fair capital;
- The solution found for longevity risk seems correct and realistic (L&S sub model). The mortality volatility in short term has direct effect on the capital requirement according to Solvency II. A new mortality table adjusted has impact in a long term perspective and it must require companies' attention.

Points that need a second approach:

- The premium risk sub model may allow more improvements. We have adopted a simple sensitive perspective. However, an exhaustive study across the correlations between and within the portfolio and the losses would be welcomed. As another improvement, we could have modeled the general expenses, taxes and commissions.
- The capital requirement for Lifetime Assistance is overestimated because the sub model considers an extreme aggregate scenario simultaneously in all observed years. The same question appears in the full model when we model separately some risks (Reserve risk, Premium risk, Longevity risk and so on). The worst scenario is not expected for all risk at the same time.

In the description of Workers Compensation, we focused in what we think the most important points of the law. Only Workers Compensation Fund (known as FAT) wasn't considered in the model architecture. The contribution for this fund is made once a year if the pension is active. The Fund risks are the same as described for annuities management (the revision risk and the longevity risk).

Finally, note that the assumptions for the interest rate and the inflation rate have impact on the capital requirement. Solvency II defines these risks in a specific module - market risks. The Workers Compensation model has to provide all cash-flow projected without assumptions effect in order to allow their specific analysis.

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ANNEXES

ANNEX I - Sub Model for Portfolio and Future Losses (detailed)

1. PORTFOLIO MODELING

In Portfolio Modeling, the objective is to forecast the behavior of portfolio in the following year. Our approach has considered the three principal partitions of portfolio: the new contracts, the cancelation contracts and the renewal contracts.

The model components are:

Exposure – correspond to number of policies in force;

Capital – corresponds to the sum insured;

Earned Premiums – the premiums that policyholders pay to company;

All components are calculated in a *pro-rata temporis*.

The evolution of these components depends of the evolution of main principal indicators of business. There are many indicators of business that companies report and monitor. We intended to model only three: the mean capital by policy, the tariff rate and the rate of renewals.

Notation:

PAR is a partition of portfolio. The elements of PAR are *NEW* (for new), *CAN* (for cancelations) and *REW* (for renewals)

i is the exercise year and $i \in \{0,...,I\}$

Exp_i^p - Exposure at year *i* and $p \in PAR$

Cap_i^p - Capital at year *i* and $p \in PAR$

EP_i^p - Earned premiums at year *i* and $p \in PAR$

$Rate\_Exp_i^p$ - Exposure rate at year *i* and $p \in PAR$

$Rate\_Tar_i^p$ - Tariff Rate at year *i* and $p \in PAR$

$Mean\_Cap_i^p$ - Mean Capital at year *i* and $p \in PAR$

Principal Indicators for each partition ($p \in PAR$):

$$Rate\_Exp_i^p = \frac{Exp_i^p}{Exp_{i-1}^{NEW} + Exp_{i-1}^{CAN} + Exp_{i-1}^{REW}} \quad (A.1)$$

$$Rate\_Tar_i^p = \frac{EP_i^p}{Cap_i^p} \quad (A.2)$$

$$Mean\_Cap_i^p = \frac{Cap_i^p}{Exp_i^p} \quad (A.3)$$

Additionally, we analyze the variance of the capital at year *i* and $p \in PAR$ (named $Var\_Cap_i^p$).

For all members of the partition, the objective is to predict the value of the indicators at year $I+1$. In the case study, the lack of a historical time series didn't allow us to apply known models as Auto regressive and/or Mean Average (we had only 8 historical years observed). We have adjusted regressions (Linear, quadratic or polynomial) to get the trend of indicator.
Predictions for year $I+1$:

$$Exp_{I+1}^p = \left(Exp_I^{NEW} + Exp_I^{CAN} + Exp_I^{REW}\right) * Rate\_Exp_{I+1}^p \quad (A.4)$$

$$Cap_{I+1}^p = Exp_{I+1}^p * Mean\_CAP_{I+1}^p \quad (A.5)$$

$$EP_{I+1}^p = CAP_{I+1}^p * Rate\_Tar_{I+1}^p \quad (A.6)$$

The risk of premiums decreases or increases more than it is expected depending on the uncertainty of some indicators. As we mentioned, we have kept the Mean Capital as the most unpredictable indicator. Then, by the Central Limit Theorem,

$$Mean\_CAP_{I+1}^p \sim N\left(Me\hat{a}n\_CAP_{I+1}^p, V\hat{a}r\_CAP_{I+1}^p\right) \quad (A.7)$$

Where, $V\hat{a}r\_CAP_{I+1}^p = E\left[Var\_CAP_I^p\right] + Var\left[Mean\_CAP_I^p\right]$. (A.8)

2. FUTURE LOSSES

There are two types of losses that are considered in this subsection: Compensations and Annuities. In both, the principals' indicators to monitoring are frequency and mean cost.

The notation is:

$Nb_{i,j}^{COMP}$ - Number of claims for compensations with reference accident year *i* and development year *j*;

$Nb_{i,j}^{PENS}$ - Number of pensioners with reference accident year *i* and development year *j*;

$Paym_{i,j}^{COMP}$ - Compensations payments with reference accident year *i* and development year *j*;

$PMs_{i,j}^{PENS}$ - Present value at the court decision in respect of all monthly payments that will be made in advance for all pensioners reference accident year i and development year j;

$Freq_{i,j}^{COMP}$ - Frequency for compensations with reference accident year i and development year j;

$Freq_{i,j}^{PENS}$ - Frequency for annuities with reference accident year i and development year j;

$Mean\_Cost_{i,j}^{COMP}$ - Mean cost for compensations with reference accident year i and development year j;

$Mean\_Cost_{i,j}^{PENS}$ - Mean cost for annuities with reference accident year i and development year j.

We defined the usual indicators as

$$Freq_{i,j}^{COMP} = \frac{Nb_{i,j}^{COMP} * 1\,000\,000}{Cap_i^{NEW} + Cap_i^{CAN} + Cap_i^{REW}} \tag{A.9}$$

$$Freq_{i,j}^{PENS} = \frac{Nb_{i,j}^{PENS} * 1\,000\,000}{Cap_i^{NEW} + Cap_i^{CAN} + Cap_i^{REW}} \tag{A.10}$$

$$Mean\_Cost_{i,j}^{COMP} = \frac{Paym_{i,j}^{COMP}}{Nb_{i,j}^{COMP}} \tag{A.11}$$

$$Mean\_Cost_{i,j}^{PENS} = \frac{PMs_{i,j}^{PENS}}{Nb_{i,j}^{PENS}} \tag{A.12}$$

In future losses, the our objective is to predict $Freq_{I+1,0}^{COMP}$, $Freq_{I+1,0}^{PENS}$, $Mean\_Cost_{I+1,0}^{COMP}$ and $Mean\_Cost_{I+1,0}^{PENS}$. In addition to portfolio modeling, we have estimated the Loss ratio for one year.

Then,

$$Nb_{I+1,0}^{COMP} = \frac{Freq_{I+1,0}^{COMP} * \left(Cap_{I+1}^{NEW} + Cap_{I+1}^{CAN} + Cap_{I+1}^{REW}\right)}{1\,000\,000} \tag{A.13}$$

$$Nb_{I+1,0}^{PENS} = \frac{Freq_{I+1,0}^{PENS} * \left(Cap_{I+1}^{NEW} + Cap_{I+1}^{CAN} + Cap_{I+1}^{REW}\right)}{1\,000\,000} \tag{A.14}$$

$$Paym_{I+1,0}^{COMP} = Mean\_Cost_{I+1,0}^{COMP} * Nb_{I+1,0}^{COMP} \tag{A.15}$$

$$PMs_{I+1,0}^{PENS} = Mean\_Cost_{I+1,0}^{PENS} * Nb_{I+1,0}^{PENS} \tag{A.16}$$

and consequently,

$$Loss\_Ratio_{I+1,0}^{COMP} = \frac{Paym_{I+1,0}^{COMP}}{EP_{I+1}^{NEW} + EP_{I+1}^{CAN} + EP_{I+1}^{REW}} \tag{A.17}$$

$$Loss\_Ratio_{I+1,0}^{PENS} = \frac{PMs_{I+1,0}^{PENS}}{EP_{I+1}^{NEW} + EP_{I+1}^{CAN} + EP_{I+1}^{REW}} \tag{A.18}$$

$$Loss\_Ratio_{I+1,0} = Loss\_Ratio_{I+1,0}^{COMP} + Loss\_Ratio_{I+1,0}^{PENS} \; . \tag{A.19}$$

ANNEX II – *The Complete Mortality Tables for Portugal* – 2009-2011 by INE Statistics Portugal (PT0911)

x	*lx*	*x*	*lx*	*x*	*lx*
0	100 000	35	98 535	70	82 025
1	99 704	36	98 437	71	80 650
2	99 680	37	98 338	72	79 134
3	99 665	38	98 235	73	77 547
4	99 649	39	98 113	74	75 791
5	99 637	40	97 977	75	73 940
6	99 626	41	97 814	76	71 861
7	99 614	42	97 647	77	69 563
8	99 605	43	97 464	78	67 081
9	99 592	44	97 262	79	64 340
10	99 580	45	97 058	80	61 500
11	99 571	46	96 842	81	58 281
12	99 559	47	96 590	82	54 809
13	99 548	48	96 306	83	51 089
14	99 535	49	95 996	84	47 028
15	99 521	50	95 701	85	42 588
16	99 503	51	95 367	86	37 758
17	99 479	52	94 991	87	32 772
18	99 449	53	94 593	88	27 767
19	99 412	54	94 153	89	22 913
20	99 376	55	93 691	90	18 357
21	99 333	56	93 203	91	14 191
22	99 284	57	92 715	92	10 573
23	99 237	58	92 157	93	7 560
24	99 186	59	91 573	94	5 162
25	99 137	60	91 010	95	3 349
26	99 091	61	90 376	96	2 053
27	99 043	62	89 667	97	1 181
28	98 987	63	88 914	98	634
29	98 939	64	88 137	99	314
30	98 890	65	87 276	100	143
31	98 832	66	86 384		
32	98 766	67	85 406		
33	98 704	68	84 362		
34	98 625	69	83 240		

ANNEX III – The Mortality Table for Workers Compensation experience – 2006-2010 (WC0610)

x	lx	x	lx	x	lx
0	100 000	35	96 855	70	76 771
1	100 000	36	96 685	71	75 214
2	100 000	37	96 484	72	73 367
3	100 000	38	96 253	73	71 654
4	100 000	39	96 075	74	70 472
5	100 000	40	95 869	75	68 582
6	99 590	41	95 770	76	66 404
7	99 590	42	95 519	77	64 176
8	99 590	43	95 231	78	61 956
9	99 590	44	95 028	79	58 974
10	99 590	45	94 692	80	56 098
11	99 311	46	94 481	81	53 347
12	99 311	47	94 025	82	49 123
13	99 202	48	93 738	83	45 966
14	99 202	49	93 342	84	42 459
15	99 115	50	92 812	85	38 923
16	99 115	51	92 459	86	35 577
17	99 115	52	91 813	87	32 068
18	99 052	53	91 345	88	28 816
19	98 990	54	90 856	89	25 942
20	98 990	55	90 391	90	22 699
21	98 926	56	89 696	91	20 063
22	98 795	57	89 165	92	17 433
23	98 662	58	88 656	93	15 043
24	98 662	59	88 063	94	13 307
25	98 380	60	87 429	95	9 947
26	98 297	61	86 710	96	8 310
27	98 118	62	85 662	97	7 544
28	97 953	63	84 704	98	6 995
29	97 953	64	83 937	99	5 261
30	97 745	65	82 913	100	3 832
31	97 622	66	81 801		
32	97 286	67	80 467		
33	97 185	68	79 352		
34	96 992	69	78 042		