Instituto Superior de Economia e Gestão
UNIVERSIDADE TÉCNICA DE LISBOA
DESDE 1911

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THESIS

ANALYSIS OF PORTFOLIO INSURANCE STRATEGIES BASED UPON EMPIRICAL DENSITIES

RICARDO JORGE DA GRAÇA RODRIGUES DE ALMEIDA

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GUIDANCE:

Professor Dr. Raquel M. Gaspar

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Abstract

This study evaluates the performance of the most common Portfolio Insurance Strategies based on a block-moving bootstrap simulation. We consider not only the traditional mean-variance approach, but also some measures of downside risk and stochastic dominance. We find that CPPI 1 should be preferred in terms of stochastic dominance. We also find that SLPI is constantly dominated by all the other strategies and a floor of 100% should be preferred to lower ones. Consistently, and purely in terms of performance analysis, CPPI 3 tends to outperform other strategies.

During this analysis, we try to provide another insight into the controversy over Portfolio Insurance strategies, turning the decision-making process for future investors more efficient.

JEL Classification: C15, C60, D81, G01, G11, G15, G23.

Keywords: Portfolio Insurance, Constant Proportion Portfolio Insurance, Option Based Portfolio Insurance, Stop-loss Portfolio Insurance, Moving Block Bootstrap.

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Resumo

Este estudo avalia a performance das mais comuns estratégias de *Portfolio Insurance*, baseando essa análise em simulações de blocos móveis de *Bootstrap*.

Nesta análise consideramos não apenas as tradicionais medidas associadas à Teoria Média-variância, mas também outras medidas associadas ao *Downside Risk*, bem como classificações de dominância estocástica. Foram identificadas evidências que suportam que a estratégia CPPI 1 deve ser preferida em termos da sua dominância face às restantes. Contrariamente, a estratégia SLPI deverá ser preterida face a outras estratégias de *Portfolio Insurance*. Encontrámos igualmente evidências de que deverão ser escolhidas barreiras mínimas mais elevadas, com o objectivo de maximizar a utilidade da generalidade dos investidores. Consistentemente, e meramente em termos de performance, a estratégia CPPI 3 é aquela que apresenta resultados mais satisfatórios.

Ao longo desta análise, tentamos proporcionar uma nova visão sobre as controversas estratégias de *Portfolio Insurance*, tentando tornar mais eficiente a decisão de futuros investidores.

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1 Introduction

In the years before the 2008 crisis, private arrangements have become more and more important in financial markets. During this period, many investors put their money in financial assets that should provide protection against market falls and guarantee the upside potential. However, after the recovery of the markets in 2009, many of these investors started to ask why their assets didn´t grow with the market – they had the same investment, Portfolio Insurance instruments (PI).

Traditionally, costumers demand a guaranteed minimum performance for their invested money. In order to guarantee this goal, offering-banks and insurance companies designed instruments that could fulfil this purpose, providing the ability to limit downside risk in bearish markets. Portfolio Insurance instruments could be divided into three categories: Constant Proportion Portfolio Insurance, Option Based Portfolio Insurance and Stop-loss Portfolio Insurance.

In this study, we will try to analyse if these types of instruments really do what they should do – protection – and if they can outperform the payoff provided by traditional investment strategies such as the buy-and-hold strategy. To perform this analysis we used both performance measures and stochastic dominance criteria.

This thesis is organised as described ahead. In Chapter 2, we discuss the different PI strategies and the main advances in this area. Chapter 3 describes some data analysis. Chapter 4 presents the methodology. Chapter 5 describes the empirical setup and the simulation results and Chapter 6 presents the main conclusions and some clues for further research.

2 Overview of the Literature

2.1 Portfolio Insurance

The main goal of a risk-averse economic agent is to reduce future uncertainty. Portfolio Insurance Strategies are investment schemes where the total amount (or, at least, the major part) of the initial investment is guaranteed in the future.

The first approach to portfolio insurance strategies was developed in the early eighties by Leland and Rubinstein (1988), who analysed the portfolio insurance techniques based on the options pricing formula of Black and Scholes (1973). The underlying idea of these strategies is to provide protection against potential market losses, while preserving the upward potential (see e.g. Grossman and Villa (1989) and Basak (2002)), allowing participation in market rallies. Thanks to a dynamic allocation strategy, the portfolio is protected against market falls by a guaranteed floor, which preserves a guarantee of a minimum level of wealth at a specific time horizon.

The most preeminent strategies of portfolio insurance are the Constant Proportion Portfolio Insurance (CPPI), the Option-based Portfolio Insurance (OBPI), and the Stop-loss Portfolio Insurance (SLPI). From a micro-economic perspective, such strategies using insurance properties that are thus rationally preferred by individuals that are specially concerned with the potential losses represented on the left side of the returns distribution function.

2.1.1 Constant Proportion Portfolio Insurance (CPPI)

Among the most popular strategies of portfolio insurance, the Constant Proportion Portfolio Insurance (CPPI) allocation strategy is one of the most used in financial markets. The CPPI method was first introduced by Perold (1986) (see also Perold and Sharpe (1988)) for fixed income instruments and Black and Jones (1987) for equity markets. CPPI strategies are very popular, commonly used by hedge funds, and are held essentially for portfolio protection.

This strategy is based on a specific simplified method of dynamic allocation on the risky asset, denoted *S*, and a risk-free asset[2], denoted *B*, over time, in order to guarantee a predetermined floor, which should be equal to the lowest acceptable value of the portfolio at maturity. The properties of the strategy are extensively studied in the literature (see, e.g. Black and Rouhani (1989), Black and Perold (1992) and Bookstabber and Langsam (2000)).

As explained before, the underlying idea of the CPPI strategy consists of managing a dynamic portfolio, to guarantee at maturity that the portfolio terminal value, denoted by V_T^{CPPI}, lies above the guaranteed level defined at t_0, denoted *K*, function of the initial investment, denoted V_0^{CPPI}, as shown below:

$$V_T^{CPPI} \geq K \times V_0^{CPPI} \tag{1}$$

Let $F_{t;0\leq t\leq T}$ denote the present value of the guarantee, the so-called floor. This value should be discounted at a risk-free rate, as shown by:

$$F_T = K \times V_0^{CPPI} \Rightarrow F_t = K \times V_0^{CPPI} \times e^{-r(T-t)} \tag{2}$$

The surplus of the current portfolio value V_t^{CPPI}, denoted C_t, over the floor F_t is called cushion, and its value at time *t* ($t \in [0,T]$) is given by:

[2] This instrument is usually a liquid instrument with residual risk, such as Treasury bills or other liquid money market instruments;

$$C_t = \max\left\lfloor V_t^{CPPI} - F_{t,} 0 \right\rfloor \quad (3)$$

This cushion is the total amount that the investor can use to expose himself to risky assets. This value can be leveraged over the introduction of a *multiplier*, denoted *m*, which is a constant function of the *cushion* $[C_t]$. The portfolio volatility is crucially dependent upon this parameter, which influenced the payoff function, turning it convex (only if *m* satisfies the condition $m \geq 1$). As the *multiple* is higher, larger is the portfolio value in a bullish market tendency. Nevertheless, the higher the multiple, the faster the portfolio will approach the floor when there is a decrease in the value of the risky asset.

Usually, this *multiple* tends to be unconditional (see, e.g. Bertrand and Prigent (2005) and Prigent and Tahar (2005)), which means that the multiple must not depend on market conditions but on investor´s risk tolerance function[3]. Others (see, e.g. Ameur and Prigent (2006)) defend that in a time-varying framework, the multiple must be conditionally determined in order to guarantee that the risk exposure remains constant, but path dependent from market conditions. This exposure $(E_{t:0 \leq t \leq T})$, which is the total amount invested in the risky asset, can be determined by:

$$E_t = m \times C_t \qquad [m \geq 1] \quad (4)$$

The remaining part is invested in the risk-free asset:

$$V_{R_f} = V_0^{CPPI} - E_t \quad (5)$$

As the value of the portfolio approaches the *floor*, the cushion approaches zero, and the exposure presented in (4) tends to approach zero as well. In theory, the value of exposure

[3] This relation is also true for the floor and the initial *cushion*, that is also a function of the investor´s risk tolerance;

should never be negative. However, this relation is only true if we consider a market with no jumps, keeping the portfolio value away from falling below the floor. This contingent question is well known and far presented in the literature (see e.g. Grossman and Villa (1989)), observed especially during financial crisis, when a very sharp drop in the market may occur before the investor has the chance to re-balance his position.

The behaviour of these type of strategies is shown on Figure 1.

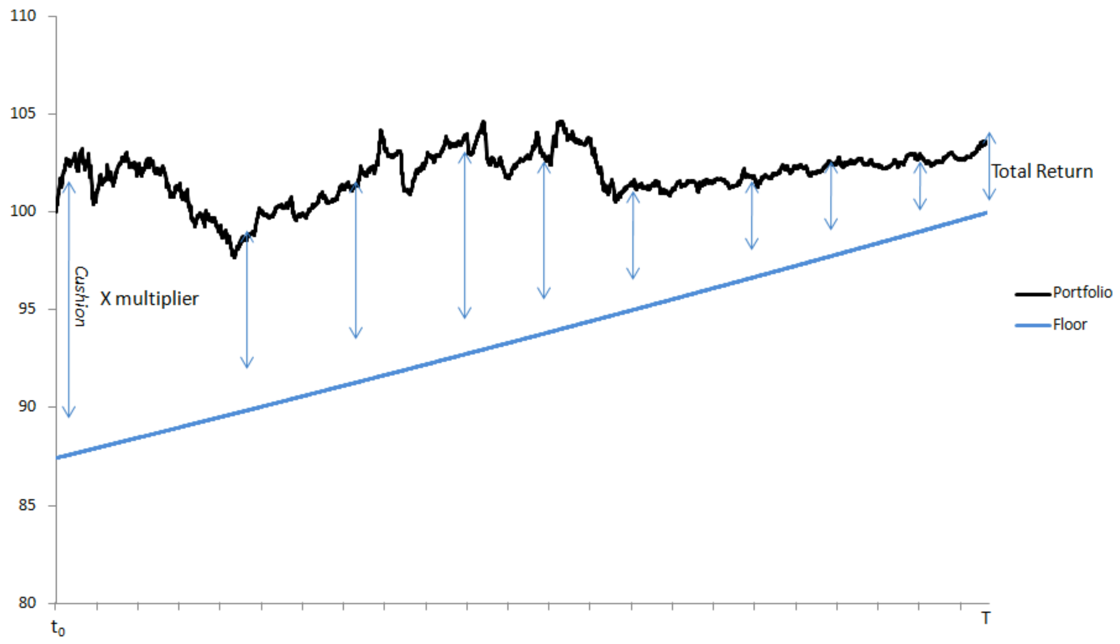


Figure 1: Simulated behaviour for a standard $CPPI_n$ strategy

However, it is important to notice that in this strategy the transaction costs are high, due to the frequent rebalance of the portfolio´s positions, which can be done daily, weekly or monthly.

Note that the portfolio value [V_t^{CPPI}] is always above (or at least equals) the floor [F_t] and the *cushion* [C_t] is always higher (or at least equal) than 0.

2.1.2 Option-based Portfolio Insurance (OBPI)

The Option-based Portfolio Insurance strategy, commonly designated by OBPI, is a dynamic portfolio strategy that guarantees a percentage of the initial investment, denoted by V_0^{OBPI}. Dated from the seventies, this strategy was first developed by Leland and Rubinstein (1976), and consists in maintaining a static position either in options and in the risk-free asset (1) or in the underlying asset (2).

The first option (1) consists in investing the discounted value of the minimum wealth required at maturity *T* in a risk-free asset and with the remaining part purchasing a call option written on the underlying portfolio, denoted $C_{t;0\leq t\leq T}$, with the *strike* equal to αV_0^{OBPI}, where α is designated by the minimum percentage of the initial amount that the agent requires to invest in this strategy. At maturity, the final value of the payoff would be:

$$\begin{aligned} V_0^{OBPI} + C_{T;Strike=\alpha V_0^{OBPI}} &= V_0^{OBPI} + \max\left(V_T^{OBPI} - V_0^{OBPI}, 0\right) \\ &= \max\left(V_T^{OBPI}; V_0^{OBPI}\right) \end{aligned} \tag{7}$$

Option (2) consists in investing in the underlying portfolio itself in the risky asset and buying a European put option, denoted by P_t, with the *strike* settled equal to the desired floor at *T* (i.e. long positions in the put option and in the underlying risky asset). At maturity, the final payoff is given by (assuming put-call parity):

$$\begin{aligned} V_0^{OBPI} + P_{T;Strike=\alpha V_0^{OBPI}} &= V_0^{OBPI} + \max\left(V_T^{OBPI} - V_0^{OBPI}, 0\right) \\ &= \max\left(V_T^{OBPI}; V_0^{OBPI}\right). \end{aligned} \tag{8}$$

The final value at maturity becomes:

$$V_T^{OBPI} = K + \max\left(0, S_T - K\right), \text{ with } K = \alpha V_0^{OBPI}, \tag{9}$$

where the minimal level required by the investor can be expressed by:

$$V_T^{OBPI} \geq K_T \Rightarrow V_T^{OBPI} \geq \alpha V_0^{OBPI}. \tag{10}$$

The main difference between CPPI and OBPI strategies resides on the mechanism of rebalancing. While CPPI strategy is a investment strategy that requires a continuous reallocation of the corresponding portfolio in order to keep the linear relation between the cushion and the floor, the OBPI perform this rebalancing using the delta of the protecting put option (for analysis of the differences between the two strategies see, e.g. Black and Rouhani (1989) and Bookstaber and Langsam (2000)).

2.1.3 Stop-loss Portfolio Insurance (SL)

Stop-loss portfolio insurance strategy (SL) is a semi-static method of managing insured portfolios, where the total initial capital, denoted V_0^{SL}, is fully invested in the risky asset as long as the value lies above the present value, at t, of the floor [K_t]. Once the portfolio value drops below the discounted floor, the total amount that was previously invested at risky asset is reallocated entirely to the risk-free asset, thereby ensuring that the floor at T [K_T] is reached. This guarantees that, at maturity, if the floor is reached during the time-frame of t $\left(0 < t < T\right)$, the final payoff will be:

$$V_T^{SL} = K \times V_0^{SL}, \tag{11}$$

as presented below:

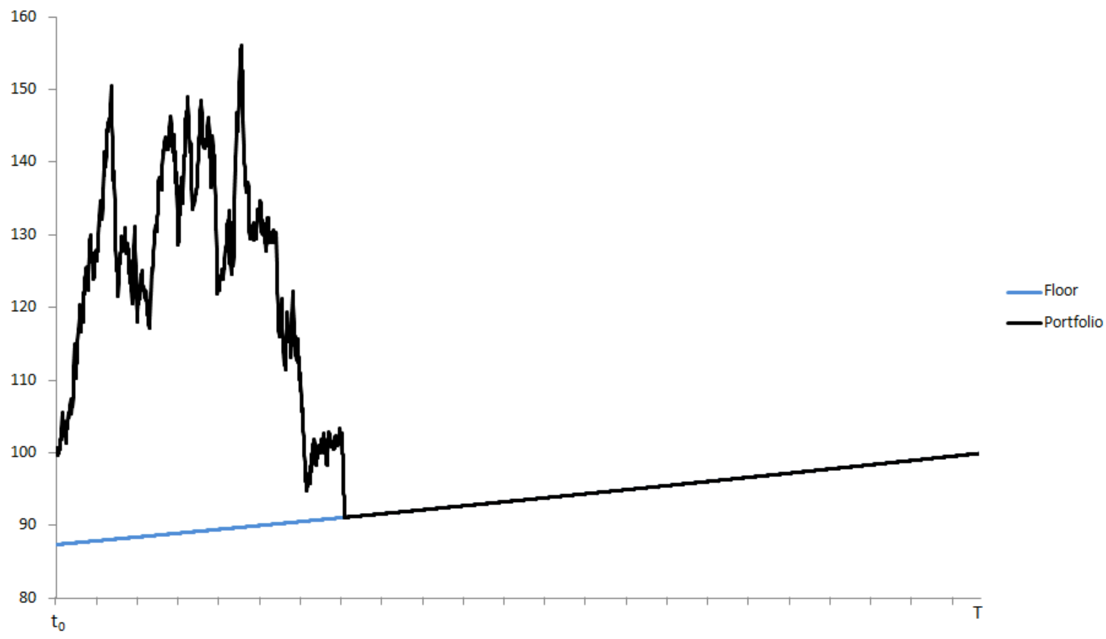


Figure 2: Simulated behaviour for a standard $CPPI_n$ strategy

Otherwise, if the floor was never reached during the investment time horizon, the portfolio's terminal value becomes:

$$V_T^{SL} > K \times V_0^{SL} . \tag{12}$$

This suggests that this type of strategies is most appropriate to implement if the investor expects a bullish market tendency.

2.2 Stochastic Dominance

The common mean-variance approach (Markowit (1987)) is used in finance as a traditional performance analysis that simplifies the decision between investments. However, this approach uses only two criteria: the mean that represents expected outcomes, and the risk, measured as the standard deviations of returns. This allows a simple trade-off analysis but for more complex decisions, the mean-risk approach may lead to inefficient conclusions. To address this problem, a good alternative to improve the investment decision is introduced by stochastic dominance (SD) relations.

The stochastic dominance approach was first developed by Whitmore and Findlay (1978) based on the majorization theory (Hardly et al. (1934)) for discrete distributions, and was later extended to the main generic distributions by Hanoch and Levy (1969) and Royhschild and Stiglitz (1970). Since then, it has been widely used in finance.

The basic approach that tries to rank two or more variables according to special classes of utility functions using an axiomatic model of risk-averse preferences (Fishburn (1964)). To rank them, investors should follow von-Neumann-Morgenstern utility functions assuming that they want to maximise their expected utility.

In the stochastic dominance approach random variables are evaluated by point-wise comparison of some performance functions, which are constructed from their distribution functions. The stochastic dominance criteria can be found by comparing the orders of stochastic dominance.

5.2.1 First order stochastic dominance

The first order stochastic dominance rule was first developed by Quirk and Saposnik (1962), who established the relationship between returns and investor´s preferences. To ensure that one or more investments dominate others in terms of stochastic dominance, the cumulative distribution function of an investment A, denoted $F_A(X)$, is always below the cumulative distribution function of a certain distribution function, denoted as $F_B(X)$.

The first performance function, denoted by $F_x^{(1)}(X)$ is defined as the right-continuous cumulative distribution (also known as *first order stochastic dominance*). This function can be defined as:

$$F_x^{(1)}(X) = F_x(X) = P\left[x \le X\right], \text{ for x Є IR} \quad (13)$$

And ranked by:

$$x \underset{\succ}{\overset{FSD}{}} y \Leftrightarrow F_x^{(1)}(X) \le F_y^{(1)}(X), \text{ for x Є IR} \quad (14)$$

First-order SD implies that investors prefer higher returns to lower ones, which results in a utility function with a non-negative first derivative.

5.2.2 Second-Order stochastic Dominance

The Second-order stochastic dominance is the best way to rank the different investments in terms of risk aversion. The second order dominance is given by the following equation:

$$F_x^{(2)}(X) = \int_{-\infty}^{x} F_x(X)dx, \text{ for X Є IR} \quad (15)$$

The weak relation of the second order Stochastic Dominance is defined as:

$$x \underset{\succ}{\overset{SSD}{}} y \Leftrightarrow F_x^{(2)}(X) \le F_y^{(2)}(X), \text{ for X Є IR} \quad (16)$$

For decision making process under risk, when X is preferred to Y under SSD rules, all risk-averse preference scents prefer the investment X instead of the investment Y, assuring larger outcomes for the same amount of risk.

5.2.3 Third-order Stochastic Dominance

The third order SD was introduced by Whitmore (1970), and consists of adding the condition that utility functions have a positive (or null) third derivative.

The third order dominance also follows the previous relation and can be defined as:

$$F_x^{(3)}(X) = \int \int_{-\infty}^{x} F_x(X)\,dx \text{, for X Є IR} \qquad (17)$$

And the third order relation is represented by:

$$x \underset{\succ}{\overset{TSD}{}} y \Leftrightarrow F_x^{(3)}(X) \leq F_y^{(3)}(X) \text{, for X Є IR} \qquad (18)$$

This relation implies a decreased risk aversion function with the increasing of the level of wealth.

Thus, if X dominates Y under FSD rules, we can assure that X also dominates Y on the following orders. If X dominates Y under SSD rules, we also can guarantee that this relation can be maintained on the following order. On the contrary, if an investment dominates the other under SSD or TSD, we cannot assure any relationship between the previous orders.

3 Data Analysis

The possible benefits of Portfolio Insurance strategies have been recently studied in academia (see, e.g. Bouyé (2009), for a global overview). Unfortunately, the major part of these studies dealt with stochastic distributions based on normally distributed functions (see, e.g. Brooks and Levy (1993) and Costa and Gaspar (2011)). However, it is well presented in the literature that financial returns are not normally distributed. This tends to bias the approach influenced by the non-consideration of relative factors i.e. variance clusters, path-dependency between returns, and fatter tails in the empirical distributions. In order to overcome these constraints, we focused our study on empirical distributions. To set these distributions, we performed a statistical analysis to observe their real distribution shape. For our empirical distributions we used data on historical returns from July 1 of 2002 and June 30 of 2012. Figures 1 to 3 show the obtained empirical distributions.

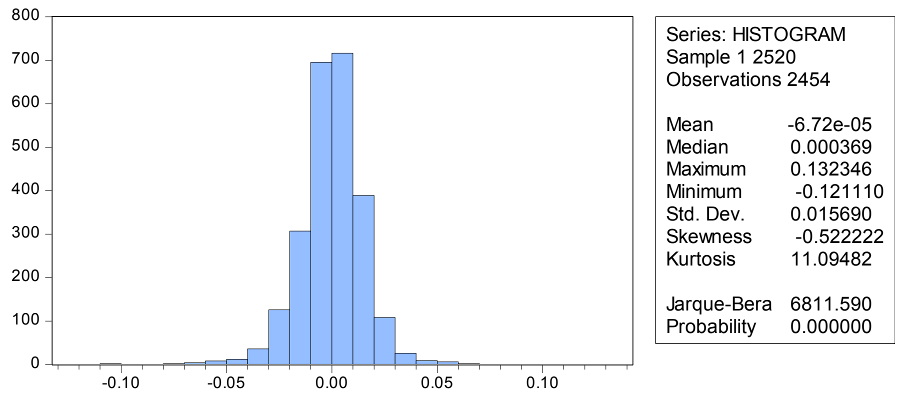


Figure 3: Returns distribution for NIKKEI225.

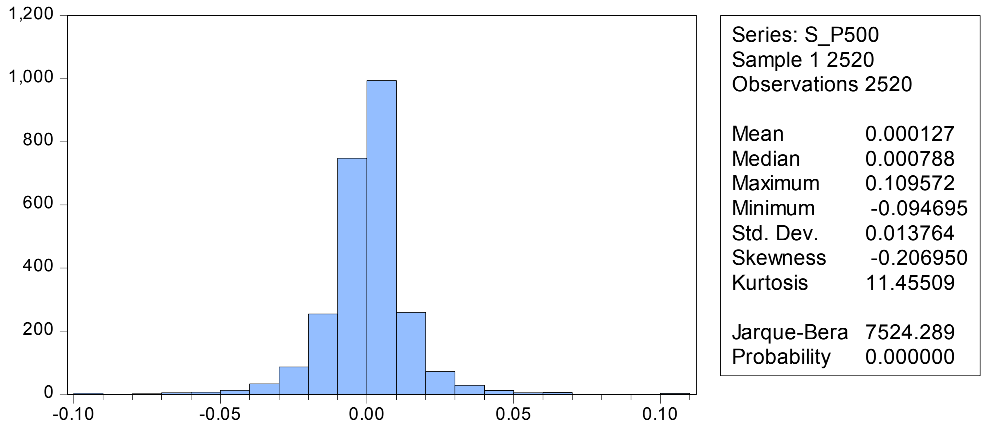


Figure 4: Returns distribution for S&P500.

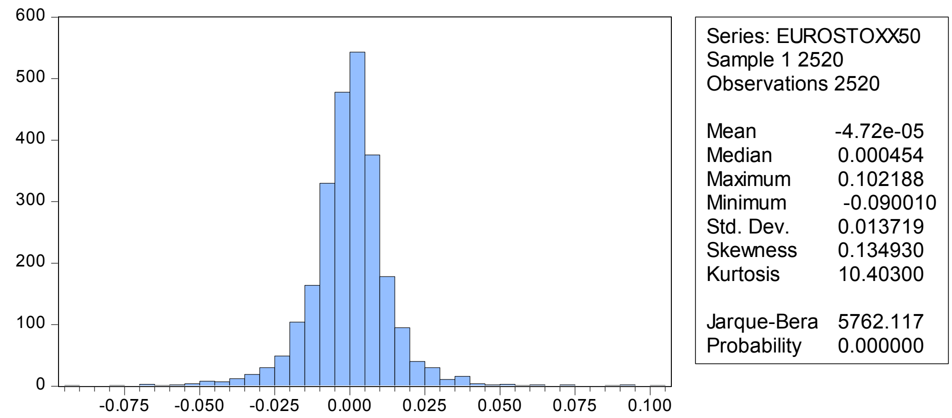


Figure 5: Returns distribution for EuroStoxx 50.

As shown in the previous distributions, and as suggested in the literature, none of the financial series associated with our indices of reference follows a normal distribution. The normal distributions are characterized by the absence of bias (i.e. skewness equals zero) and the stability of kurtosis (i.e. Kurtosis equals three). However, all these financial series exhibit excess kurtosis and a residual probability of normality (Jarque-Bera´s Test). These results support our analysis and are consistent with the findings presented in the literature, refusing the hypothesis of using a normal distribution to study Portfolio Insurance Strategies.

4 Methodology

4.1 Moving Block Bootstrap

The bootstrap method is a computer intensive method for estimation of parameters and future distributions by re-sampling the original time series. It consists of randomly re-sampling series into a so-called bootstrap sample. The classical bootstrap method was introduced by Efron (1979), who developed a technique for application in independent data samples. This approach had a great contribution to academic research since it can overcome the two main strands commonly indicated for prevision with financial series: lack of sufficient data and uncertainty in the nature of the data generating process.

However, as believed by most of the academic community, stock returns tend to be dependent. Hence, the standard bootstrap as some limitations that may destroy the existent dependence across data. Other approaches suggested in the literature (see, e.g. Summers (1986), Fama and French (1988) and Campbell and Shiller (2001)) support this theory, suggesting problems such as: particular excess volatility presented in clusters, short-term momentum, long term reversal in stock prices, and long-run predictability of stock returns. In order to preserve this inter-dependency while performing a bootstrap method, many methods of re-sampling data were suggested during the last few years, such as Block Bootstrap (see e.g. Hall (1995), Carlstein (1986), Künsch (1989)) and Moving Block Bootstrap (see e.g. Graflund (2001), Sanfilippo (2003) and Beach (2007)).

The main differences between basic bootstrap and block bootstrap reside in the preservation of the dependence structure of the original data and not corrupting it by supposing that the data is independent. Blocks of data are randomly re-sampled on consecutive values, with replacement, and then we simulate consecutive versions of the original data, aligning those blocks into a bootstrap sample. Data is processed by joining the blocks together in random order, using overlapping blocks of data instead of individual observations to estimate parameters and distributions. This is important attending that in the traditional bootstrapping method, random observations tend to be independent between themselves, contrarily to block bootstrap where blocks of data are dependent such as in the original time series. The assumption of independence between observations tends to create bias in the bootstrap variance, which can be large if we have a strong dependence between data observations.

4.2 Simulation setup

The simulation of the Moving Bootstrap Methodology presented in Section 3.1 was implemented using R software. The estimation of further information such as the probability distribution functions and the evolution of portfolio insurance strategies were implemented in MATLAB software.

In order to compare the different performances between the portfolio insurance strategies, we used three indexes that are traded continuously during the investment time horizon [0,T] – Eurostoxx50, Nikkei225 and S&P500.

To define the portfolio insurance strategies in terms of performance, we first recovered the daily data of the three major stock indexes, denoted by $S_{t;0\leq t\leq T}$. We applied logarithms to these returns, assuming that the changes in asset prices are supposed to occur at continuous time along the investment period [0,T].

$$\hat{x}_k = \log\left(\frac{S_t}{S_{t-1}}\right),\ 1 \le t \le T \text{ with } T \text{ equals 10 years} \tag{19}$$

To analyse the performance of the three strategies, we conducted a bootstrap simulation based on these continuous daily returns for the three markets[4] on a set of 10.000 paths for each market and each strategy, assuming each year contains 252 observations. These simulations were done creating a Moving Block Bootstrapping sample that offers an effective way of generating return series without making any assumption regarding the real distribution, maintaining the real skewness, fat tails, autocorrelation and heteroscedasticity from the original data sets (see e.g. Sanfilippo (2003)). However, by performing different bootstrapping procedures for each market, dependence between these markets was not considered.

As Portfolio Insurance strategies necessarily need a floor, we used a risk-free asset[5], denoted by B_t, to set it. We assume that lending is possible on a rate that is equal to the risk-free rate of return. As we assumed a risk-free asset, default risk is excluded.

To set the daily floor, and as it could vary according to the risk profile of the investor, we assumed an interval between 80% and 100%, that is the same used by many authors (see, e.g. Annaert et al. (2009) and Bertrand and Prigent (2005)).

After defining these parameters, we applied the Portfolio Insurance properties explained in the previous chapter at the following strategies: CPPI 1, CPPI 3, CPPI 5, OBPI and SLPI. After this, we compared these different strategies among themselves and with a

[4] Data was downloaded from DataStream, and set a time period between 2002 and 2012;
[5] We used the average interest rate that was observed in the last 10 years for German Bunds, Japanese bonds and Treasury bonds;

standard buy-and-hold strategy in terms of performance, risk, stochastic dominance criteria up to third order, and probability functions.

To perform the study, we assumed that borrowing and lending are equally possible, as well as short selling and division of shares are allowed without any restriction.

To compute the options associated with the underlying asset´s performance, we assumed that markets do not provide any arbitrage opportunities, that are nor transactions costs, nor taxes or any margins requirements. The same assumptions were used to determine the floor, referring to the risk-free asset. We also assumed that the strategy is constructed with European options that only can be exercised in the final of the investment time horizon [T], and that the stocks included on the underlying indexes do not pay dividends during the investment time horizon.

5 Results

5.1 Performance Analysis

In this chapter, we analyse the first four moments based upon the mean-variance theory introduced by Markowitz (1952). This is a common procedure to the study of Portfolio Insurance Strategies. However, as we will observe, its analysis is not completely linear when we are studying such different strategies.

The use of risk models is quite straightforward. Usually, performance analysis is based on specific moments of the distribution. Facing the difficulty of preventing the future empirical distributions, investors tend to simulate these distributions based on the past and concentrate their focus on the specific moments such as the expected return or the Sharpe Ratio. This allows a simple trade-off analysis, comparing two scalar characteristics of the distribution – the expected return, which represents the expected outcome and the volatility (risk). To complete this analysis, we also use downside risk metrics.

5.1.1. Mean-Variance

In this first analysis, we try to compare the different strategies, only using traditional ways to measure the return and risk associated with the different strategies. The most common factor, the expected return, is the simplest way to achieve comparisons between different outcomes derived from different distributions. To determine the final profitability of each strategy, we looked at the terminal values of each payoff functions computing it using the discrete return of each path during the 10 years time horizon, using Equation 20.

$$\overline{R_e} = \frac{1}{n}\sum_{t=1}^{n}\left(\frac{\frac{P_T - P_0}{P_0}}{T}\right), \qquad (20)$$

where P_T is the value of the portfolio at maturity, P_0 is the initial investment, n the number of simulations and T the time horizon.

To complete this analysis and take into account the well known fact that bigger payoffs require higher risk, we compared the five standard portfolio strategies using the common risk factor, volatility (σ). This scalar is given by Equation 21.

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}\left(x_i - \overline{x}\right)^2} \qquad (21)$$

Table 1 compares the latter of the five proposed strategies – CPPI 1, CPPI 3, CPPI 5, OBPI and SLPI – with both 80% and 100% floor value.

Table A: Annualized Expected Returns (A) and Volatilities (B)

	EuroStoxx50			NIKKEI 225			S&P500		
	K=80	*K=100*	*Equity*	*K=80*	*K=100*	*Equity*	*K=80*	*K=100*	*Equity*
A									
CPPI 1	3,17%	3,50%	-0,5%	1,38%	1,59%	-1,5%	5,88%	5,36%	3,4%
CPPI 3	2,72%	3,69%	-0,5%	2,33%	2,59%	-1,5%	7,83%	7,53%	3,4%
CPPI 5	2,63%	3,22%	-0,5%	2,11%	2,43%	-1,5%	7,78%	7,48%	3,4%
OBPI	2,96%	3,03%	-0,5%	2,44%	2,61%	-1,5%	7,69%	6,98%	3,4%
SLPI	2,96%	3,09%	-0,5%	1,15%	1,42%	-1,5%	6,68%	6,32%	3,4%
B									
CPPI 1	8,64%	5,89%	21,73%	7,26%	3,23%	24,82%	9,65%	6,71%	21,82%
CPPI 3	15,74%	12,25%	21,73%	14,54%	8,44%	24,82%	17,47%	14,34%	21,82%
CPPI 5	16,37%	13,14%	21,73%	15,34%	9,73%	24,82%	18,01%	15,03%	21,82%
OBPI	15,16%	13,12%	21,73%	15,09%	10,01%	24,82%	15,67%	13,19%	21,82%
SLPI	17,26%	13,30%	21,73%	18,13%	15,04%	24,82%	19,67%	17,10%	21,82%

As presented above, all the strategies can overcome the return associated with the respective benchmark. For markets that exhibit high negative trend (i.e. NIKKEI 225), the SLPI strategy is clearly the worst one. This relationship is even more pressing considering its volatility, which in 83% of cases (on average) makes the strategy reaches the floor, giving returns equal to 0% and -20% to floors of 100%, and 80%, respectively. This relationship tends to dissipate when the market returns are less negative (i.e. EuroStoxx 50), where the CPPI 5 strategy already shows some tendency for returns below their peers. This is not at all extraneous to their high volatility, making them the most risky strategy.

Concerning the existent relation between the floor and return, when the markets have a negative performance and the floor is higher, the strategy also has a higher average return, demonstrating that in this kind of scenarios, and purely in terms of mean-variance theory, investors who are focused exclusively in the highest payoffs, must choose higher floors in order to maximize their profitability. Attending the fact that with lower floors we have higher exposure to the underlying asset, when the corresponding index has negative performance *ceteris paribus*, obviously the strategy with lower floor will have a more negative expected return. For the U.S. market (i.e. S&P500), with higher positive returns, all strategies which have lower floors show higher profitability. In these cases, when the market rises, the lower floors enhance gains in the stock market, since they allow higher exposure to market bullish tendency. However, and contrary to the expectations, the most profitable strategy is the CPPI 3, in favour of CPPI 5. From our analysis, this is due to the high probability of a 5x leverage exposure to the market, which potentiates higher probabilities of reaching the floor when the market goes down in the beginning of the investment time horizon. This prevents its recovery during the market rising, since in these cases the *cushion* is residual, getting the strategy closer to the barrier along the investment horizon. However, this evidence lacks scientific confirmation.

5.1.2. Ratio Analysis

As mentioned above, the performance analysis is traditionally based on the mean-variance analysis. We complete this analysis with some performance measures, such as the traditional Sharpe Ratio and other measures consistent with Downside Risk: the Sortino Ratio and the Omega Ratio.

The first one (1), Sharpe Ratio (Sharpe (1994)) can be described as the cost of each unit of risk, and on the other hand, how much return costs one additional unit of risk. Its value is given by:

$$Sharpe\ Ratio = \frac{\overline{R_p} - R_f}{\sigma_p} \qquad (22)$$

where, $\overline{R_p}$ is the portfolio return, R_f is the risk-free asset and σ_p the total risk of each PI strategy. Despite some disadvantages in terms of consistency with PI strategies (see e.g. Annaert et al. (2009)) this is a commonly used ratio in finance to rank common investments between themselves.

The Sortino Ratio measures the excess return over a Minimum Acceptable Return (*MAR*) defined by each investor (for theoretical explanation, see e.g. Sortino and Price (1994)). Although derived from the Sharpe, the Sortino Ratio uses other type of volatility – the one below *MAR* – giving us the cost of each unit of “negative” volatility. Its value can be reached using the following expression.

$$Sortino\ Ratio = \frac{(r_p - MAR)}{\sigma_d};\ \sigma_d = \sqrt{\sum_{t=1}^{n} \frac{\min[(r_i - MAR),0]^2}{n}} \quad (23)$$

The Omega Ratio was developed by Shadwick and Keating (2002), and is a measure of risk that involves both the likelihood of having a gain associated with an investment in risky assets, such as the probability of having a loss. The higher the ratio, the better is the result since the probability of having a loss rather than a gain will be smaller.

$$\Omega(r) = \frac{\int_r^{\infty}(1 - F(x))\,dx}{\int_{-\infty}^{r} F(x)dx} \quad (24)$$

All the results for our strategies are presented in Table 2.

Table B: Average Sharpe Ratio (A), Sortino Ratios (B) and Omega Ratios (C).

	EuroStoxx50			**NIKKEI 225**			**S&P500**		
	K=80	*K=100*	*Equity*	*K=80*	*K=100*	*Equity*	*K=80*	*K=100*	*Equity*
A									
CPPI 1	-0,043	-0,007	-0,19	0,004	0,074	-0,12	0,216	0,232	-0,02
CPPI 3	-0,052	0,012	-0,19	0,067	0,147	-0,12	0,231	0,260	-0,02
CPPI 5	-0,056	-0,024	-0,19	0,050	0,111	-0,12	0,221	0,245	-0,02
OBPI	-0,038	-0,039	-0,19	0,072	0,126	-0,12	0,248	0,241	-0,02
SLPI	-0,034	-0,034	-0,19	-0,011	0,005	-0,12	0,146	0,147	-0,02
B									
CPPI 1	-0,302	-0,048	-1,95	0,413	0,508	-4,67	1,524	1,651	-1,26
CPPI 3	-0,365	0,079	-1,95	0,730	1,000	-4,67	1,603	1,810	-1,26
CPPI 5	-0,381	-0,175	-1,95	0,635	0,952	-4,67	1,524	1,683	-1,26
OBPI	-0,373	0,063	-1,95	0,608	0,179	-4,67	1,592	1,768	-1,26
SLPI	-0,338	-0,238	-1,95	-1,21	0,023	-4,67	0,324	0,237	-1,26
C									
CPPI 1	0,82	0,88	-	0,71	0,77	-	2,01	1,90	-
CPPI 3	0,77	0,79	-	0,66	0,69	-	2,60	2,38	-
CPPI 5	0,67	0,71	-	0,59	0,61	-	2,62	2,35	-
OBPI	0,80	0,86	-	0,66	0,68	-	2,42	2,25	-
SLPI	0,96	0,97	-	0,88	0,91	-	1,93	1,82	-

In terms of ratio analysis, and considering the basic Sharpe Ratio, the best strategies are clearly OBPI and CPPI, for the two floor levels. In practice, this means that these are the strategies that pay better the assumed levels of risk. On the other hand, if we take into account only the negative returns, the scenario is quite similar. In this case, despite the relative proximity of the OBPI strategy, CPPI 3 is the one which transversely presents best positive results. In terms of negative results, SLPI presents the worst ones. This is due to the high volatility of these instruments, which is not paid for their return.

5.3 Statistical Analysis

Relative Kurtosis and Relative Skewness are two particular ways to define the probability distribution functions expressed by portfolio insurance terminal values. The Skewness coefficient is a measure of asymmetry which studies the probability concentrated in the distribution tails. This allows determining if our probability distribution function has higher probabilities concentrated far from the mean, consistent with the existence of outliers – normal distributions have skewness closer to 0. On the other hand, many researchers suggests that larger skewness makes a protection strategy more appealing (see Harvey and Siddique (2000) and Port et al. (2008)).

The Kurtosis has the same properties of skewness, but it can tell us more particular things about the real distribution. A leading example originated from finance can be seen when performing a distribution function of an index where distribution tends to be leptokurtic – really peaked with fat tails, which concentrates higher probabilities around the mean but also presents a higher number of outliers. For normal distributions, its value should be close to 3. For the studied strategies, the results are presented in Tables C and D.

Table C: Relative Skewness

	EuroStoxx50			NIKKEI 225			S&P500		
	K=80	*K=100*	*Equity*	*K=80*	*K=100*	*Equity*	*K=80*	*K=100*	*Equity*
CPPI 1	0,135	0,137	0,13	-0,557	-0,575	-0,52	-0,218	-0,215	-0,20
CPPI 3	-0,169	-0,331	0,13	-1,340	-1,595	-0,52	-0,506	-0,673	-0,20
CPPI 5	-0,230	-0,605	0,13	-1,737	-2,613	-0,52	-0,592	-0,946	-0,20
OBPI	-0,174	-0,352	0,13	-1,193	-1,023	-0,52	-0,513	-0,692	-0,20
SLPI	2,966	0,264	0,13	3,062	2,123	-0,52	-0,303	-0,292	-0,20

Table D: Relative Kurtosis

	EuroStoxx50			NIKKEI 225			S&P500		
	K=80	*K=100*	*Equity*	*K=80*	*K=100*	*Equity*	*K=80*	*K=100*	*Equity*
CPPI 1	8,723	9,399	7,43	10,341	11,695	7,95	9,222	10,028	8,46
CPPI 3	16,847	22,926	7,43	28,452	38,147	7,95	15,588	21,615	8,46
CPPI 5	19,103	30,835	7,43	40,602	73,648	7,95	17,906	29,684	8,46
OBPI	89,674	96,130	7,43	81,282	74,590	7,95	49,476	34,672	8,46
SLPI	130,507	97,480	7,43	89,154	78,798	7,95	12,262	24,561	8,46

Concerning relative skewness, it is possible to observe that all strategies present a behaviour worse that the market. However, the SLPI strategy presents results clearly far from the market behaviour, for both Kurtosis and Skewness. These results, while unexpected, are easily explained by the tendency to reach the barrier too soon. From the moment the portfolio value reaches the lowest barrier strategy, all returns will be positive, skewing the distribution to the right. On the other hand, if the portfolio´s value reaches the barrier, all the remaining results will be close to the free-risk interest rate, creating large fat tails on the distribution, thereby explaining the results for kurtosis.

5.4 Other Measures

To describe the outcomes for each strategy, we performed a descriptive analysis based on the comparison between equity performance and the portfolio insurance strategies outcomes. These results are presented in Figures 4 through 6.

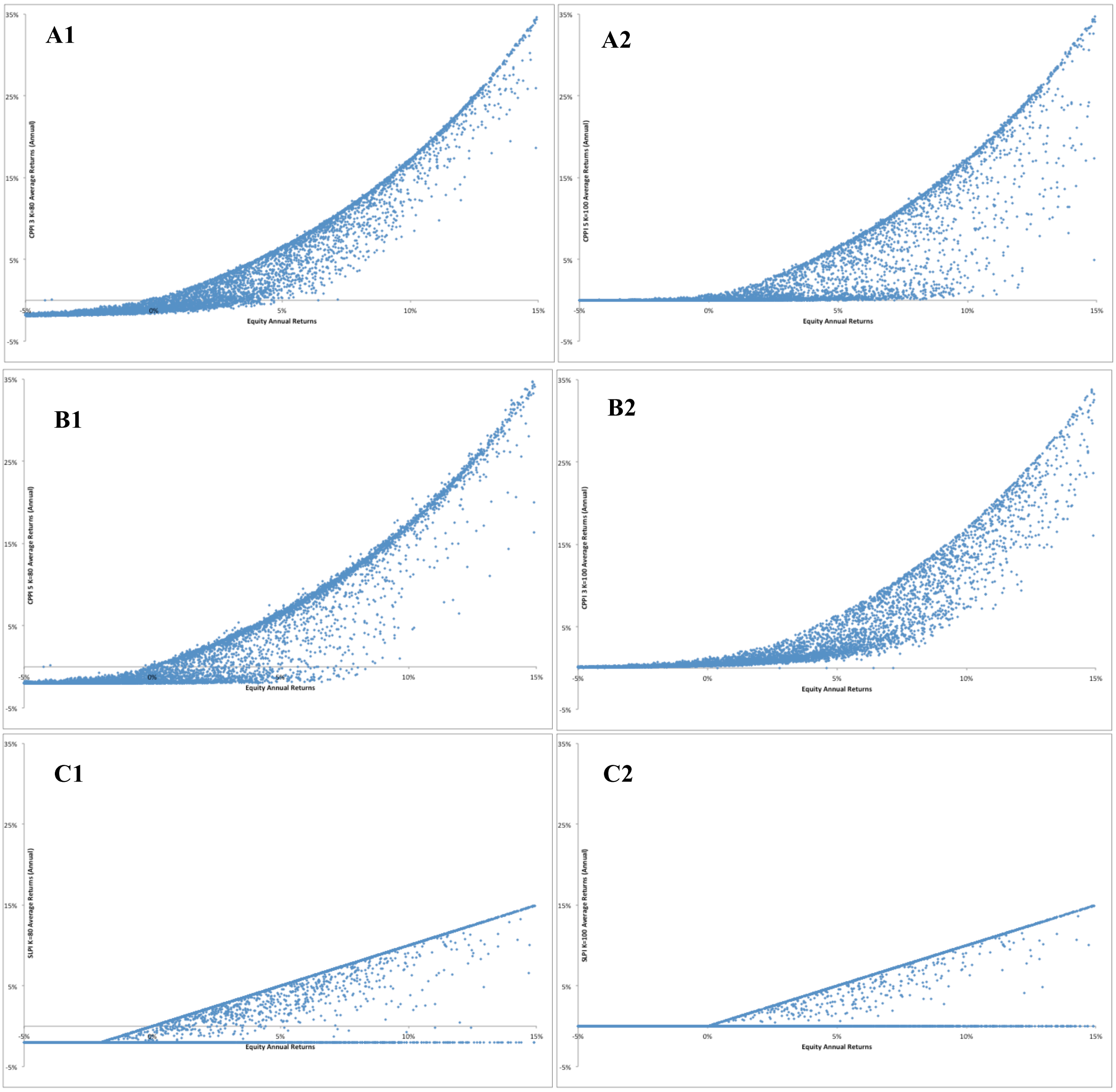
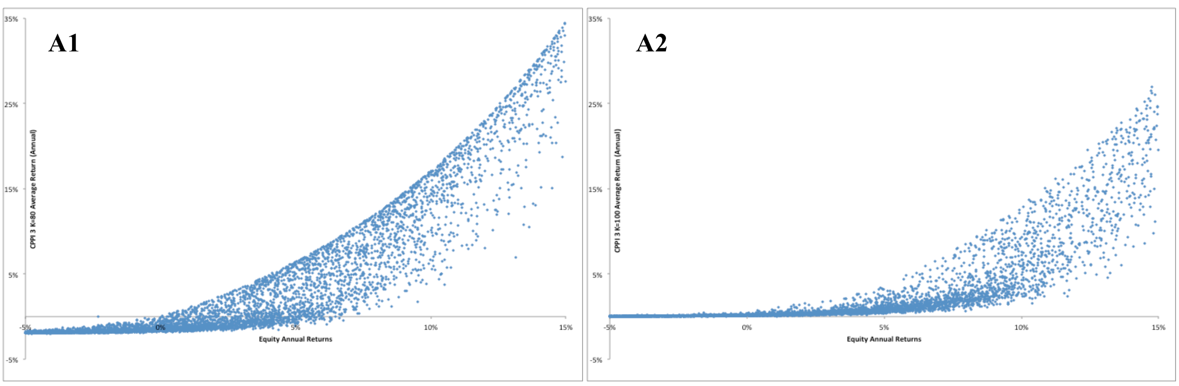


Figure 6: Payoff function for CPPI 3 (A), CPPI 5 (B) and SLPI (C) with floor equals to (1) 80% and (2) 100% (DJ EuroStoxx50).



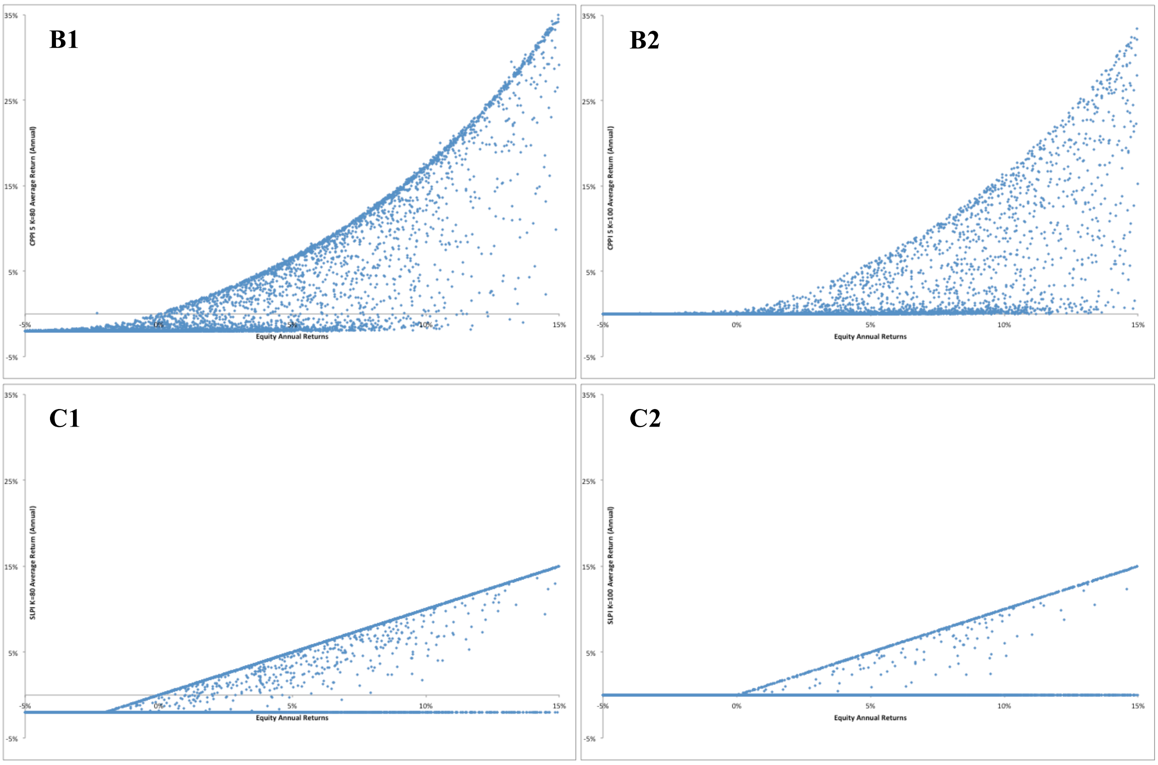
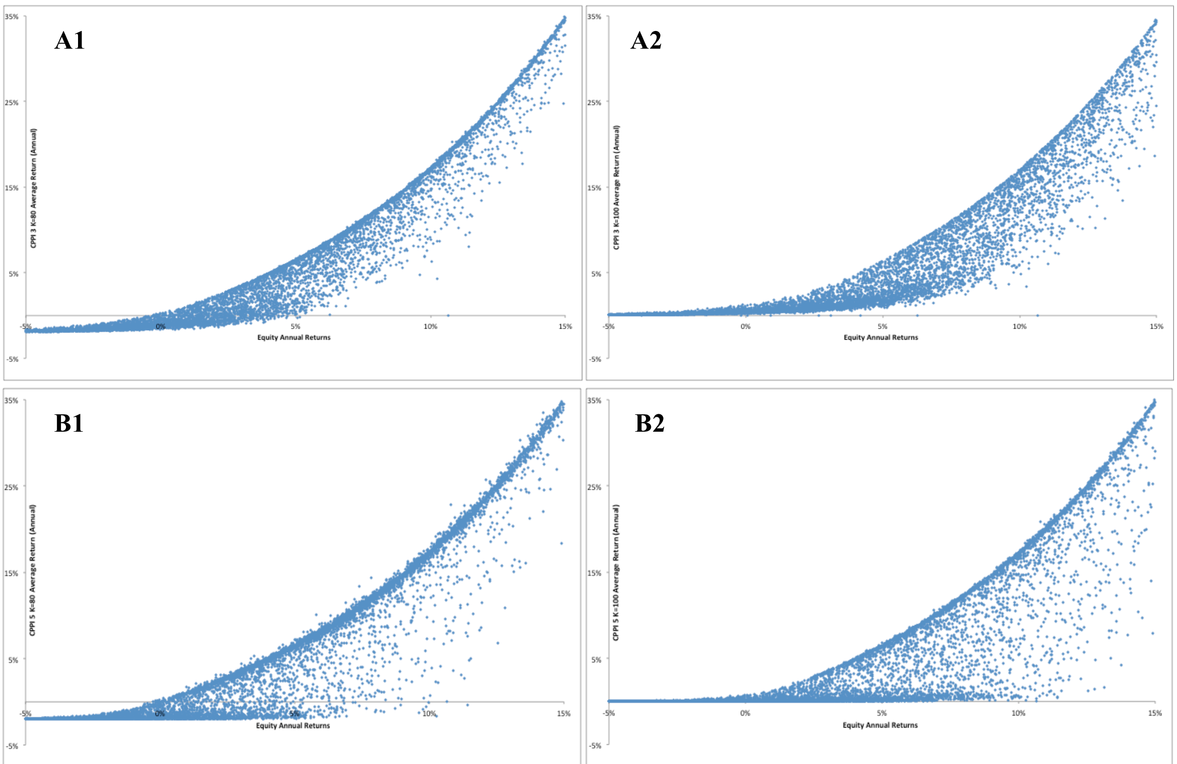


Figure 7: Payoff function for CPPI 3 (A), CPPI 5 (B) and SLPI (C) with floor equals to (1) 80% and (2) 100% (NIKKEI 225).



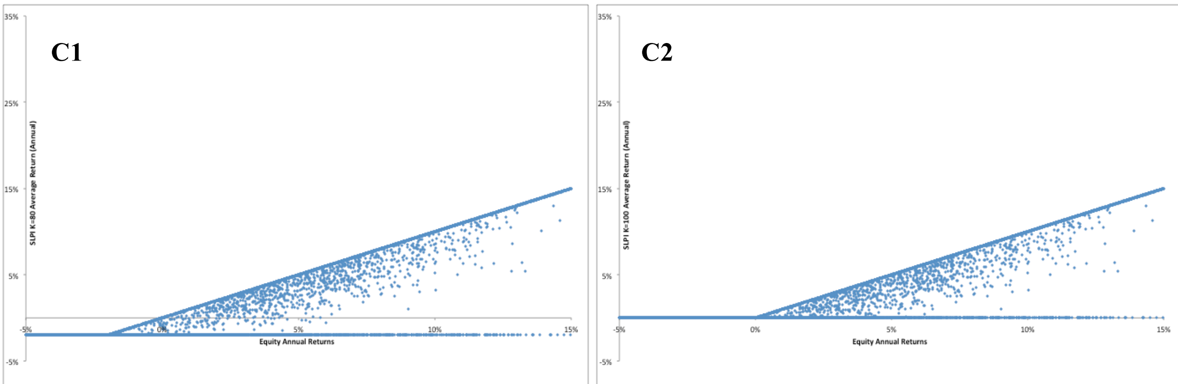


Figure 8: Payoff function for CPPI 3 (A), CPPI 5 (B) and SLPI (C) with floor equals to (1) 80% and (2) 100% (NIKKEI 225).

Opposed to many investors opinion, these strategies cannot be described as linear. This means that it is not only the entry point and the terminal value that matter for the portfolio performance, but also the portfolio's movements during the time horizon. This relation is particularly explicit if we look to the previous diagrams. Other important aspect is the larger dispersion that tends to exist for CPPI 3 and CPPI 5 (Subfigure A and B of each Figure) comparing to SLPI (Subfigure C). These results support our previous analysis in terms of volatility, skewness and kurtosis.

5.5 Stochastic Dominance Analysis

5.5.1 Testing stochastic dominance

For the proposed analysis, we studied the three orders of stochastic dominance (SD) for all the indexes followed in the previous chapter – EuroStoxx50, NIKKEI225 and S&P500. To find the stochastic relation between portfolio insurance strategies, we also define also the two floors suggested before – 80% and 100% - which are commonly used to study these investments. The set of strategies considered are presented in Tables 9 through 11[6].

Table E: Stochastic Dominance Test for EuroStoxx50 with a floor at maturity of 80% (FSD: First Order SD; SSD: Second Order SD; TSD: Third Order SD; NSD: No SD)

	K=80					K=100					
	CPPI 1	**CPPI 3**	**CPPI 5**	**OBPI**	**SLPI**	**CPPI 1**	**CPPI 3**	**CPPI 5**	**OBPI**	**SLPI**	Dominan t
CPPI 1	-	SSD	SSD	SSD	SSD	-	SSD	SSD	NSD	SSD	**CPPI 1**
CPPI 3	NSD	-	SSD	NSD	SSD	NSD	-	SSD	NSD	SSD	**CPPI 3**
CPPI 5	NSD	NSD	-	NSD	NSD	NSD	NSD	-	NSD	SSD	**CPPI 5**
OBPI	NSD	NSD	NSD	-	SSD	NSD	NSD	NSD	-	SSD	**OBPI**
SLPI	NSD	NSD	NSD	NSD	-	NSD	NSD	NSD	NSD	-	**SLPI**

Table F: Stochastic Dominance Test for NIKKEI225 with a floor at maturity of 80% (FSD: First Order SD; SSD: Second Order SD; TSD: Third Order SD; NSD: No SD)

	K=80					K=100					
	CPPI 1	**CPPI 3**	**CPPI 5**	**OBPI**	**SLPI**	**CPPI 1**	**CPPI 3**	**CPPI 5**	**OBPI**	**SLPI**	Dominant
CPPI 1	-	TSD	TSD	TSD	TSD	-	NSD	NSD	TSD	SSD	**CPPI 1**
CPPI 3	NSD	-	SSD	NSD	SSD	NSD	-	SSD	NSD	SSD	**CPPI 3**
CPPI 5	NSD	NSD	-	NSD	SSD	NSD	NSD	-	NSD	SSD	**CPPI 5**
OBPI	NSD	NSD	NSD	-	FSD	NSD	NSD	SSD	-	FSD	**OBPI**
SLPI	NSD	NSD	NSD	NSD	-	NSD	NSD	NSD	NSD	-	**SLPI**

[6] Stochastic dominance is also presented in the appendix in Figures A1 through A6;

Table G: Stochastic Dominance Test for S&P500 with a floor at maturity of 80% (FSD: First Order SD; SSD: Second Order SD; TSD: Third Order SD; NSD: No SD)

	K=80					K=100					
	CPPI 1	**CPPI 3**	**CPPI 5**	**OBPI**	**SLPI**	**CPPI 1**	**CPPI 3**	**CPPI 5**	**OBPI**	**SLPI**	Dominant
CPPI 1	-	NSD	NSD	NSD	NSD	-	NSD	NSD	NSD	TSD	**CPPI 1**
CPPI 3	NSD	-	TSD	NSD	SSD	NSD	-	SSD	NSD	FSD	**CPPI 3**
CPPI 5	NSD	NSD	-	NSD	FSD	NSD	NSD	-	NSD	FSD	**CPPI 5**
OBPI	NSD	NSD		-	SSD	NSD	NSD	NSD	-	FSD	**OBPI**
SLPI	NSD	NSD	NSD	NSD	-	NSD	NSD	NSD	NSD	-	**SLPI**

Concerning the first SD order, we found six cases where the pattern of dominance exists. In such cases, where investors prefer more to less (condition of dominance), the strategy SLPI is clearly the most penalized. However, this strategy is possibly the one that most suffers from the absence of transaction costs considered in this analysis. Nevertheless, high levels of volatility tend to make many of the simulations quickly approach the floor of the strategy continuing on the risk-free rate throughout the time horizon. This relationship is particularly strong in cases where the floor was set at 100% because the floor tends to be on a much higher value than for 80%.

On the second and third order of dominance we found similar results. Taking into account that investors with second order relations of dominance are risk-averse, we concluded that investors who position themselves in European markets (i.e. EuroStoxx 50) should choose less aggressive strategies, such as Buy-and-hold strategies (e.g. CPPI 1), which have higher levels of dominance over the other ones (in this case). The relationship between dominance and multiple associated with CPPI strategies is also present in the dominance relationship between the CPPI 3 and 5 for both floors. In these cases, CPPI 3 dominates CPPI 5 in both markets where distributions are clearly skewed to the left (i.e. EuroStoxx50 and Nikkei 225). This relationship is also true for S&P500, despite its a clearly upward trend, the CPPI 3 continues with second and third order levels of stochastic dominance for a floor of 100% and 80% respectively over CPPI 5.

6 Conclusions and further research

The goal of the present study was to provide another insight into the controversy over Portfolio Insurance strategies and intends to contribute to the decision-making process for future investors in this type of strategies. The main novelty resides in the use of empirical distributions and singular methodologies, and the extensive comparison between such different types of instruments.

First, we found that CPPI 1 outperforms other CPPI strategies with higher multiplier in terms of stochastic dominance. On the other hand, for all the studied indexes with negative performance (i.e. NIKKEI 225 and EUROSTOXX 50) CPPI 5 cannot dominate any of the other strategies (except SLPI). In terms of floor analysis, we found that the highest floor value implies best downside protection.

These results contradicted some other findings such as those presented by Annaert et al. (2009) and Zagst and Klaus (2011) that reject any stochastic dominance between strategies. In our analysis, we found some first order stochastic dominance for all the strategies over SLPI in a market with bullish tendency. This relation also holds for CPPI 1 and 3 over OBPI (with K=100%). In these cases, ambitious investors will choose CPPI and OBPI strategies over SLPI (for K=80%) and CPPI 3 and 5 over SLPI (for K=100%). Our results for CPPI 1 are mainly supported by the research done by Costa and Gaspar (2012).

However, our analysis has some limitations. For future research we suggest that a better way to analyze these strategies in a more realistic scenario is to introduce transaction costs into the analysis and study the inflection point of the multiplier. Some recent studies also suggest that a conditional multiplier could improve the results of this analysis, and this could be a good way to continue this research. We also suggest that in order to make a comparison between strategies, their rebalancing frequency should be studied.

7 References

Ameur, H. and J. Prigent (2006). Portfolio Insurance: determination of a dynamic CPPI multiple as function of state variables. *Working Paper*, University of Cergy.

Annaert, J., S. Osselaer and B. Verstraete (2009). Performance evaluation of Portfolio Insurance strategies using Stochastic Dominance Criteria. *Jornal of Banking and Finance* 33, 272-280.

Basak, S. (2002). A Comparative Study of Portfolio Insurance. *Journal of Economic Dynamics and Control* 26, 1217-1241.

Beach, S. (2007). Semivariance in Asset Allocations: Longer Horizons can Handle Riskier Holdings. *Journal of Financial Planning* 20(1), 60-69.

Benninga, S. and M. Blume (1985). On the Optimality of Portfolio Insurance. *Journal of Finance* 40, 1341-1352.

Bertrand, P. and J. Prigent (2003). Portfolio Insurance Strategies: a comparison of standards methods when the volatility of the stock is stochastic. *International Journal of Business* 8, 15-31.

Bertrand, P. and J. Prigent (2005). Portfolio Insurance Strategies: OBPI versus CPPI. *Finance* 26, 5-32.

Black, F. and R. Jones (1987). Simplifying Portfolio Insurance. *The Journal of Portfolio Management* 14, 48-51.

Black, F. And R. Rouhani (1989). Constant Proportion Portfolio Insurance and the Synthetic Put Option: a comparison. In F. J. Fabozzi (Ed.), *Institutional investor focus on investment management*, Ballinger: Cambridge, 695-708.

Black, F. and A. Perold (1992). Theory of Constant Proportion Portfolio Insurance, *Journal of Portfolio Management*, Fall, 48-51.

Black, F. and M. Sholes (1973). The Pricing of Options and Corporate Liabilities..*Journal of Political Economy* 81, 637-654.

Bookstabber, R. and J. Langsam (2000). Portfolio Insurance Trading Rules. *The Journal of Futures Markets* 8, 15-31.

Brooks, R. and H. Levy (1993). Portfolio Insurance: Does it pay? In: *Advancesin Futures and Options Research*, JAI Press.

Campbell, J. and R. Shiller (1998). Valuation Ratios and the Long-Run stock Market Outlook. *Journal of Portfolio Management* 20(4), 11-26.

Carlstein, E. (1986). The use of subseries values for estimating the variance of a general statistics from a stationary sequence. *Ann. Statistics* 14, 1171-1179.

Cesari, R. and D. Cremonini (2003). Benchmarking, Portfolio Insurance and Technical Analysis: a Monte Carlo Comparison of dynamic strategies of asset allocation. *Journal of Economic Dynamics and Control* 27, 987-1011.

Costa, J. and R. Gaspar (2011). Portfolio Insurance: a comparison of alternative strategies. *Working Paper*, Advance Series.

Efron, B. (1979). Bootstrap methods: another look at the jackknife. *Annals of Statistics* 7, 1-26.

Fama, E. and K. French (1988). Permanent and Temporary Components of Stock Prices. *Journal of Political Economy* 96(2), 246-273.

Garcia, C. and F. Gould (1987). An empirical study of portfolio insurance. *Financial Analysts Journal* 43, 44-54.

Graflund, A. (2001). Expected Real Return from Swedish Stocks and Bonds. *Working Paper*, University of Lund.

Grossman, S. and J. Villa (1989). Portfolio Insurance in Complete Markets: a note. *Journal of Business* 62, 473-76.

Hall, P. (1985). Resampling a Coverage Process. *Stochastic Processes and Their Aplications* 20, 231-246.

Hansson, B. and M. Persson (2000). Time Diversification and Estimation Risk. *Financial Analysts Journal* 56(5), 55-62.

Künsch, H. (1989). The Jackknife and bootstrap for general stationary observations. *Annals of Statistics* 17, 1217-41.

Leland, H. and M. Rubinstein (1976). The Evolution of Portfolio Insurance. In: *D.L. Luskin* (Ed.), Portfolio Insurance: a guide of dynamic hedging, New York: Wiley.

Leland, H. and M. Rubinstein (1988). The evolution of portfolio insurance. In *Dynamic Hedging: a guide to portfolio insurance*: a guide of dynamic hedging, New York: Wiley.

Perold, A. and W. Sharpe (1988). Dynamic Strategies for Asset Allocation. *Financial Analysts Journal* 44, 16-27.

Perold, A. (1986). Constant Proportion Portfolio Insurance. *Working Paper*, Harvard Business School.

Poncet, P. and R. Portait (1997). *Assurance de Portefeuille*. In: *Y. Simon* (ed.), Encyclopédiedês Marchés Financiers, Economica, 140-141.

Prigent, J. and F. Tahar (2005). CPPI with Cushion Insurance. *Working Paper*, University of Cergy-Pontoise.

Sanfilippo, G. (2003). Stocks, bonds and investment horizon: a test of time diversification French market. *Quantitative Finance* 3, 345-351.

Shadwick W. and C. Keating (2002). A Universal Performance Measure. *Journal of Performance Measure* 6(3).

Summers, L. (1986). Does the Stock Market Rationally Reflect Fundamental Values? *Journal of Finance* 41(3), 591-601.

Zagst, R. and J. Kraus (2001). Stochastic Dominance of Portfolio Insurance Strategies. *Annals of Operations Research* 185, 75-103.

Appendix

I - Stochastic Dominance

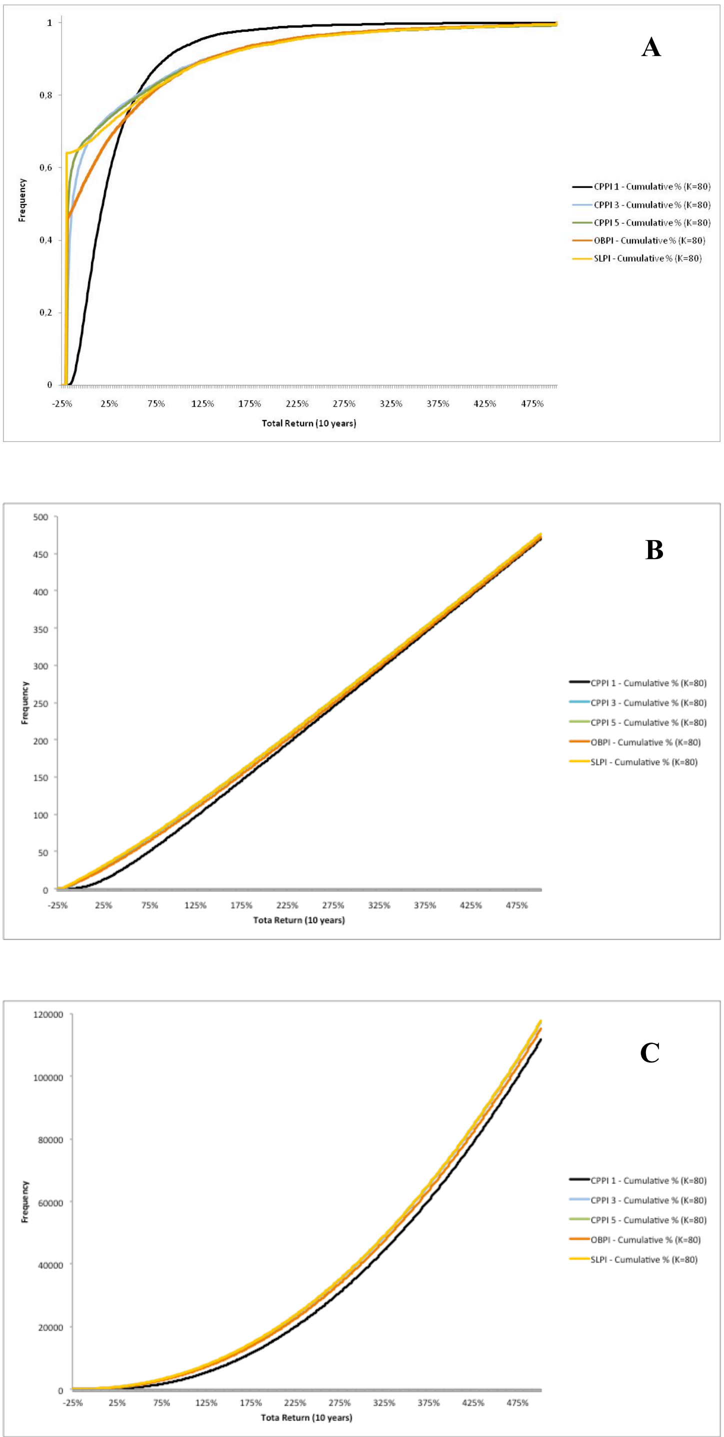


Figure A1: Cumulative distribution function for Eurostoxx 50 with K=80. (A) First order stochastic dominance; (B) Second order stochastic dominance and (C) Third order stochastic dominance.

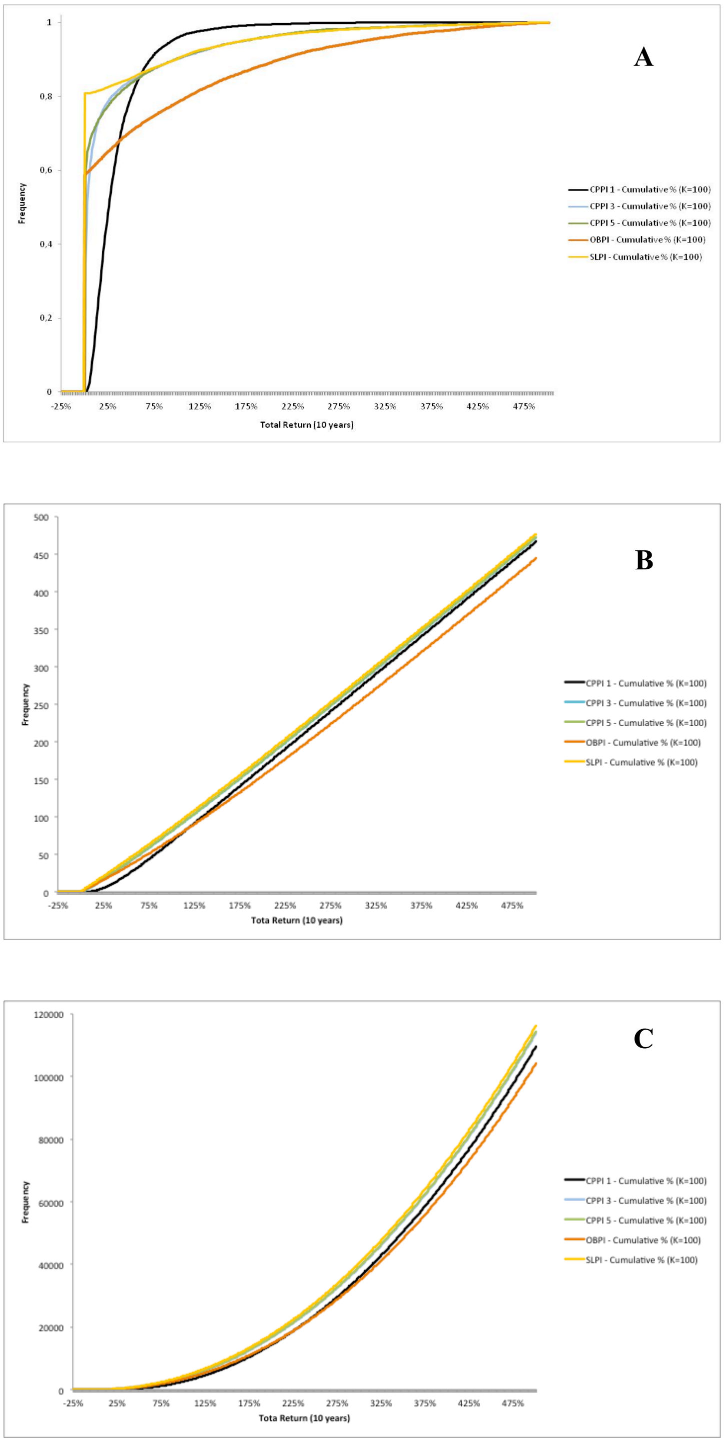


Figure A2: Cumulative distribution function for Eurostoxx 50 with K=100. (A) First order stochastic dominance; (B) Second order stochastic dominance and (C) Third order stochastic dominance.

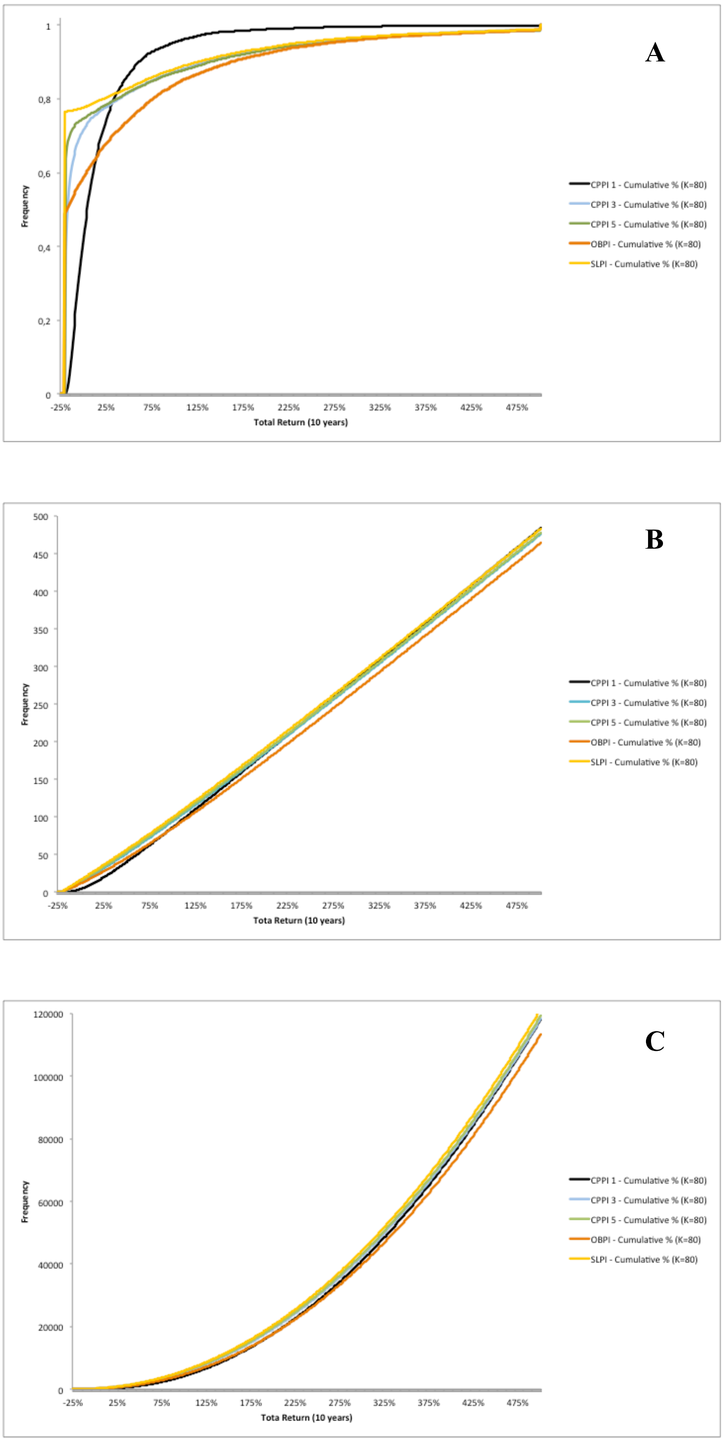


Figure A3: Cumulative distribution function for NIKKEI225 with K=80. (A) First order stochastic dominance; (B) Second order stochastic dominance and (C) Third order stochastic dominance.

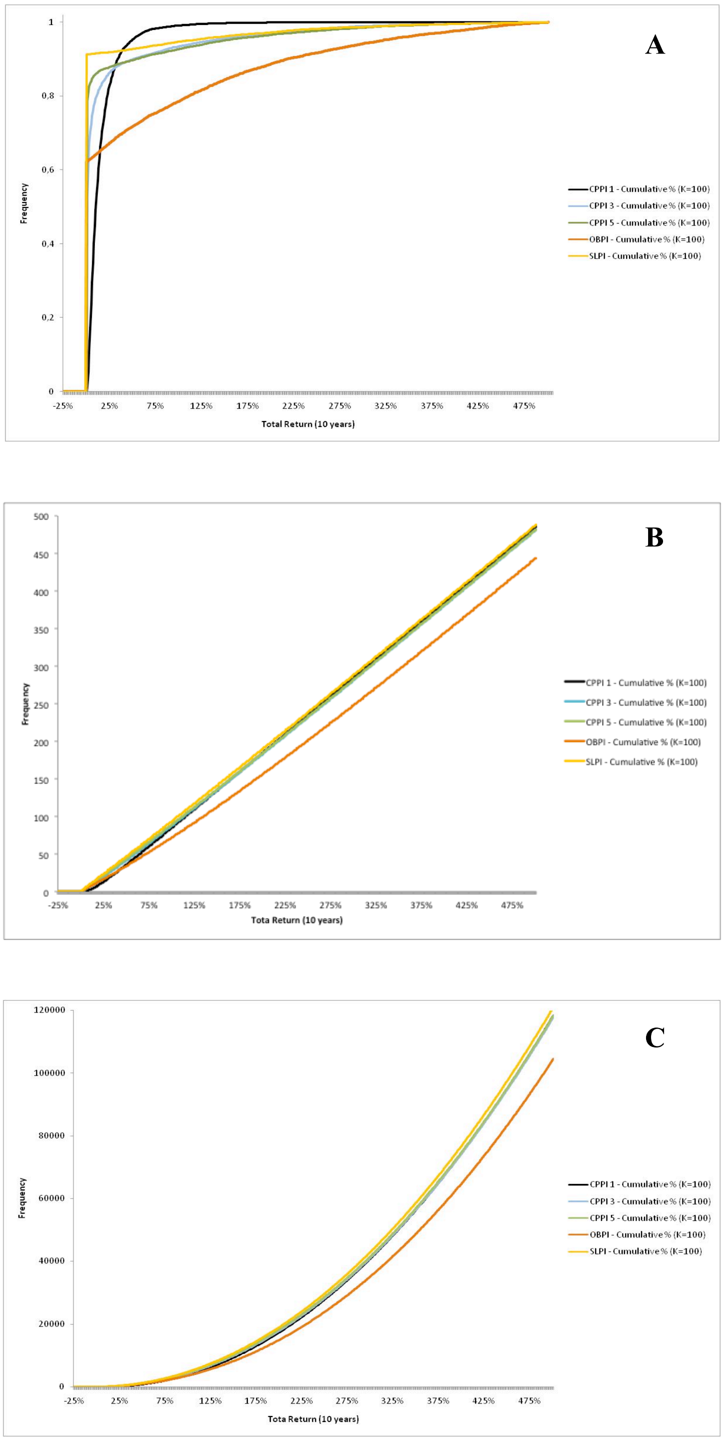


Figure A4: Cumulative distribution function for NIKKEI225 with K=100. (A) First order stochastic dominance; (B) Second order stochastic dominance and (C) Third order stochastic dominance.

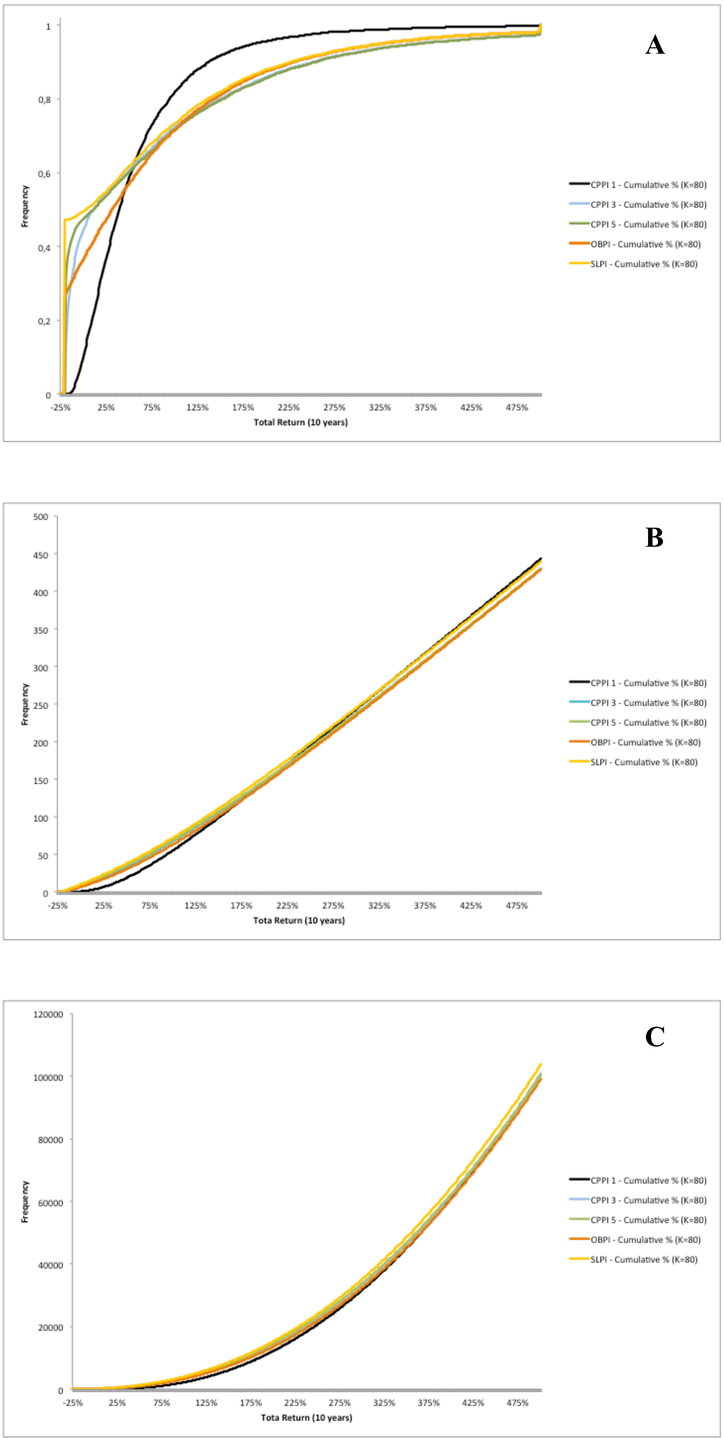


Figure A5: Cumulative distribution function for S&P500 with K=80. (A) First order stochastic dominance; (B) Second order stochastic dominance and (C) Third order stochastic dominance.

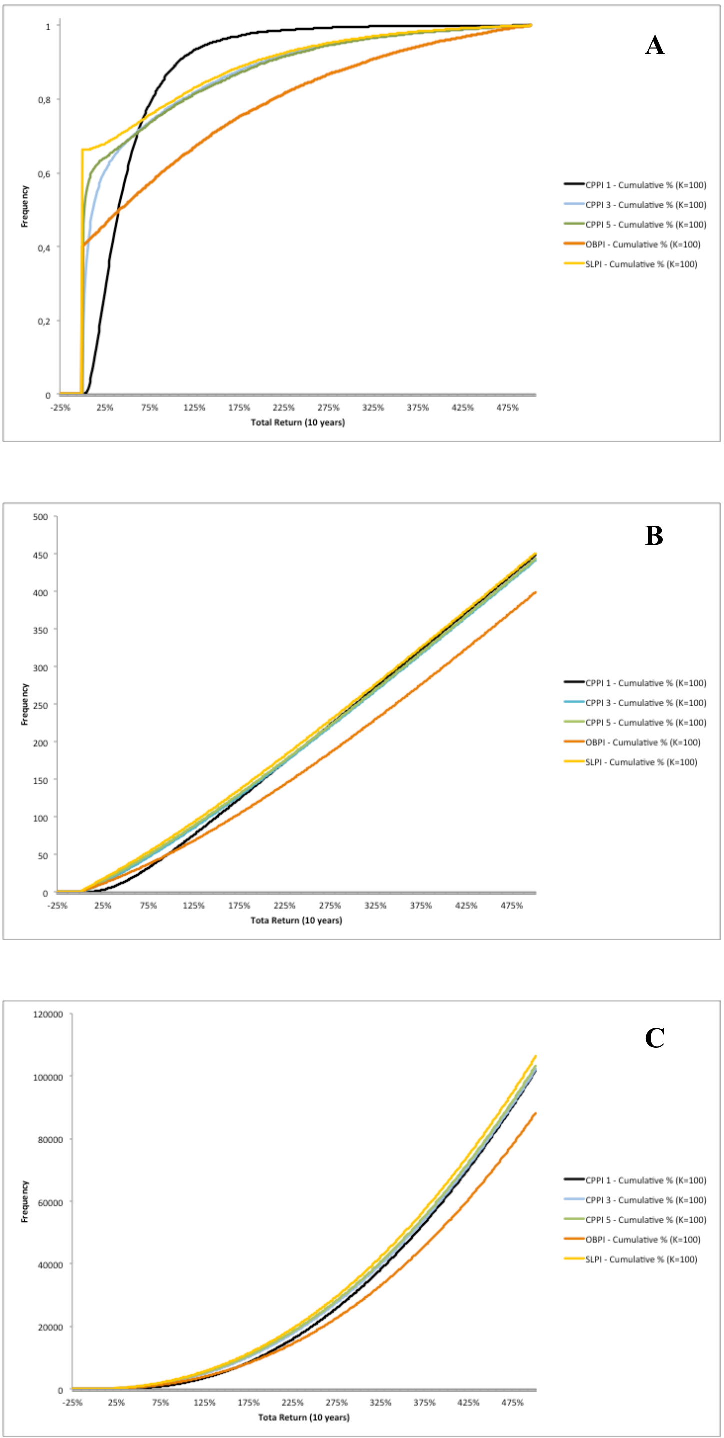


Figure A6: Cumulative distribution function for S&P500 with K=100. (A) First order stochastic dominance; (B) Second order stochastic dominance and (C) Third order stochastic dominance.

III - Other Statistics

Table A1: Probabilities associated with the returns provided by Portfolio Insurance Strategies (EUROSTOXX 50)

	$E(R)$	P[$E(R)$]>0	P[$E(R)$]>r_f	$[E(R)]^A - r_f$	P{$[E(R)]^A$}>0	P{$[E(R)]^A$}>r_f
CPPI 1 (K=80)	3,17%	78,47%	31,58%	2,38%	81,63%	29,53%
CPPI 1 (K=100)	3,50%	100%	34,24%	2,95%	100%	32,49%
CPPI 3 (K=80)	2,72%	34,39%	23,38%	1,01%	32,66%	20,42%
CPPI 3 (K=100)	3,69%	100%	19,02%	1,86%	100%	15,58%
CPPI 5 (K=80)	2,63%	32,29%	24,05%	0,88%	30,32%	21,17%
CPPI 5 (K=100)	3,22%	100%	20,07%	1,52%	100%	16,74%
OBPI (K=80)	2,96%	43,31%	28,46%	2,12%	42,57%	26,07%
OBPI (K=100)	3,03%	100%	34,83%	2,51%	100%	33,14%
SLPI (K=80)	2,96%	33,47%	25,67%	1,12%	31,63%	22,97%
SLPI (K=100)	3,09%	100%	16,82%	1,33%	100%	13,13%

Table A2: Probabilities associated with the returns provided by Portfolio Insurance Strategies (NIKKEI 225)

	$E(R) - r_f$	P[$E(R)$]>0	P[$E(R)$]>r_f	$[E(R)]^A - r_f$	P{$[E(R)]^A$}>0	P{$[E(R)]^A$}>r_f
CPPI 1 (K=80)	1,38%	57,97%	36,33%	1,05%	58,86%	34,81%
CPPI 1 (K=100)	1,59%	100%	37,93%	1,11%	100%	36,59%
CPPI 3 (K=80)	2,33%	35,52%	23,22%	0,79%	33,91%	20,24%
CPPI 3 (K=100)	2,59%	100%	18,93%	0,99%	100%	15,48%
CPPI 5 (K=80)	2,11%	25,30%	23,27%	0,56%	22,56%	20,30%
CPPI 5 (K=100)	2,43%	100%	14,17%	0,86%	100%	10,19%
OBPI (K=80)	2,44%	41,25%	36,03%	0,54%	40,28%	34,48%
OBPI (K=100)	2,61%	100%	38,59%	0,96%	100%	37,32%
SLPI (K=80)	1,15%	22,31%	20,81%	0,18%	19,23%	17,57%
SLPI (K=100)	1,42%	100%	8,98%	0,23%	100%	4,42%

Table A3: Probabilities associated with the returns provided by Portfolio Insurance Strategies (S&P500)

	$E(R) - r_f$	$P[E(R)]>0$	$P[E(R)]>r_f$	$[E(R)]^A - r_f$	$P\{[E(R)]^A\}>0$	$P\{[E(R)]^A\}>r_f$
CPPI 1 (K=80)	5,88%	89,75%	53,42%	5,02%	94,17%	53,80%
CPPI 1 (K=100)	5,36%	100%	56,25%	4,77%	100%	56,94%
CPPI 3 (K=80)	7,83%	55,44%	42,92%	5,57%	56,04%	42,13%
CPPI 3 (K=100)	7,53%	100%	36,81%	5,28%	100%	35,34%
CPPI 5 (K=80)	7,78%	52,54%	43,54%	5,50%	52,82%	42,82%
CPPI 5 (K=100)	7,48%	100%	35,59%	5,14%	100%	33,99%
OBPI (K=80)	7,69%	63,52%	48,55%	5,77%	65,02%	48,39%
OBPI (K=100)	6,98%	100%	55,53%	5,54%	100%	56,14%
SLPI (K=80)	6,68%	50,52%	42,36%	4,67%	50,58%	41,51%
SLPI (K=100)	6,32%	100%	31,88%	4,20%	100%	29,87%