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# PRODUCTION FUNCTIONS: RESEARCH LACUNAE

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## 1) Why we need to know them: some examples; organization of paper

Information on production functions is needed for a number of problems in economic science, positive as well as normative. As an introduction to this article we shall mention a few of such problems, by way of examples rather than providing an exhaustive list.

Production functions may be the source of information on marginal productivity of production factors, especially labour, by which, under certain conditions, earnings may be explained that firms are prepared to pay. Production functions for goods or services produced by private firms may be compared to production functions for the same goods or services produced by public authorities, in order to choose the cheapest supplier.

Besides this use in positive economic science we may want to use the knowledge of production functions in normative economics. We may have reasons to advocate some form of incomes policy. For a proper implementation of such a policy we may need production functions.

Econometricians have devoted considerable efforts to estimate production functions and an extensive literature is the result. Some of it will be discussed in the light of the example of problems mentioned, again without claiming completeness. The main emphasis will be on a number of lacunae we think econometric research shows. We propose to start with a general characterization of the work done so far. This will be offered in the remainder of this section. Each of the other five sections will deal with a neglected area, constituting a lacuna. The reasons of neglect are not always the same, as a closer discussion will show.

Starting with a description of what has been done — again without claiming to give an exhaustive picture — we will list some of the aspects dealt with.

**1.** A large number of mathematical shapes of production functions has been tried out. The pioneer was Douglas (1934), in close collaboration with Cobb, who advised the well-known linear relationship explaining the logarithm of production $y$ by a linear expression in the logarithms of production factors $x$ considered. In their earliest attempts, two factors called labour and capital

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were used. One generalization consisted of adding quadratic terms (the translog function). Another was the introduction of more than two factors, for instance blue-collar and white-collar workers and equipment and structures (Berndt and Christensen, 1973). The third generalization was one introduced by Diewert (1974), who proposed a function:

$$y=\alpha_o \Pi_i \Pi_j \left(\frac{1}{2}x_i+\frac{1}{2}x_j\right)^{\beta_{ij}}$$

The inclusion of more than two factors was often combined with the introduction of two-level functions, with the aid of the well-known CES (constant elasticity of substitution) production functions introduced by Arrow, Chenery, Minhas and Solow (1961). In its simplest form it may be written as an additive function of powers of the factors explaining the same power of production $y$:

$$y^{-\varrho}=\Sigma a_h x_h^{-\varrho}$$

where $x_h$ is the quantity of production factor $h$, and $a_h$ $(h=1 \dots H)$ and $\varrho$ are constants.

The two-level idea consists of assuming $x_h$ to be a similar function of sub-factors (blue-collar and white-collar labour; or equipment and structures, constituting sub-factors, respectively, of labour and of capital). Such $x_h$ could also be introduced into a Cobb-Douglas function.

The CES function has been generalized in several ways: for instance by the introduction of larger values of $H$ than just 2, or by the introduction of different powers instead of the same $-\varrho$ for all. The latter generalization has also been called the addilog function, used by Mukerji (1963).

Still another aspect of the functions proposed is the change over time of technology. The simplest treatment consisted of the assumption that the product obtained from a given quantitative combination of all production factors rises by a constant percentage per annum. This example also constitutes a case of so-called unembodied (1) technological change: the change was assumed not to be caused by a change in design of capital goods used or a change in the education level of the labour force. Rather a change in organization or quantity of equipment per person was thought of. In contrast, embodied technological change is one of the alternatives just mentioned. Rather sophisticated discussions have taken place around concepts such as labour saving or capital saving changes in technology. These discussions need not be sophisticated if the production process is described in the «recipe form», usual for short-term changes in production. Contrary to what was discussed so far production volume $y$ is then considered to be the

(1) It seems to me that this word expresses the concept better than the usual word «disembodied». Professor James E. Meade agrees with this opinion.

independent variable and the quantities needed of the various production factors are considered to be the dependent variables. The simplest well-known example is the one where all inputs of production factors are proportional to *y:*

$$x_h = a_h y \ (h = 1 \ldots . H)$$

Technological change may then consist of a change in some of the $a_h$ and a pure case of a labour-saving invention consists of a reduction in $a_{h'}$, if *h'* refers to labour. Similarly capital-saving changes can be defined; and savings in any other input. Concepts have to become more sophisticated if substitution between factors is assumed to be possible; we will not pursue this issue.

**2.** Continuing our sketch of the work on production functions done so far, we want to state that in most of this work no intimate link exists between the formulae used and the description of the technology applied. The main exception to this statement is the input-output approach to process industries. But even for non-process industries, say construction, the input-output method tells us all about the supplies needed, but not very much about how to use the bricks, the timber, the nails, the paint, the glass, etc. in order to build a house. A small group of economists dealing with this subjects (e.g. Boon, 1964, 1981) works in some isolation.

**3.** In a large part of the research on production functions only a limited number of production factors has been considered, each of them representing, as a consequence, a group of heterogeneous microfactors. Thus, capital stands for a collection of very different types of equipment, from simple tools to railway stations; and labour for a wide range of workers, from labourers to managers.

**4.** Empirical testing very often has been limited to macrodata on production, such as all national production, all manufacturing and a restricted number of industries.

**5.** Sometimes competitive markets for productions factors have been assumed to exist as a justification to use price data as a measure of marginal product, as in Arrow *et al.* (1961).

**6.** A special role has been played by the concept of human capital, generalizing the concepts of capital as well as education. Through it the heterogeneity of labour could be expressed in a macroconcept.

## 2) The need to introduce many types of labour

After the education explosion of the nineteenfifties and -sixties labour has become still more heterogeneous. Moreover, income distribution has become a more important component of policy discussions and the spreading unionization of all types of labour may make an incomes policy — whether indicative or compulsory — unavoidable. Among the knowledge needed is the marginal productivity of a larger number of worker types from the lowest-paid to the highest. Here we hit on an important lacuna. The number of studies devoted to estimating production functions in which between five and ten types of labour appear as arguments is extremely limited. Besides, the innovative research done by Gottschalk (1978) was barely discussed. The same is true for some of the new problems his results pose. A brief overview seems useful.

Gottschalk estimated the marginal value product for six categories of workers outside agriculture, namely managers, sales workers, professionals, craftsmen, operatives and «supporting» workers (clerical, labourers and service workers). The terms are abbrevations of the main categories shown in the US Censuses. The ratios of earnings (median) to marginal revenue product he found are shown in table I.

TABLE I

**Median Earnings *E* and Ratio of earnings/ product *R* for six types of labour**

(USA, 1959)

| Type | *E* ($) | *R* |
|---|---|---|
| Managers | 8 189 | 2.03 |
| Sales Workers | 6 136 | 2.64 |
| Professionals | 6 007 | 1.12 |
| Craftsmen | 4 875 | 0.35 |
| Operatives | 3 797 | 0.44 |
| Supporting (*) | 3 222 | 0.55 |

(*) Unweighted average of clerical, labourers and service workers.

If free competition had prevailed on the six labour market compartments, all ratios should have been equal to unity, at least if the production function used (which was the Cobb-Douglas function) is correct. It remains a subject for research whether other production functions may lead to different results; and there are many to be tried out, as we have seen.

On the other hand, if we accept the Cobb-Douglas function as a correct approximation, Gottschalk's results pose a number of interesting questions to be taken up later (cf. Section 4).

Gottschalk's results find some support by another attempt to estimate production functions for the United States (Tinbergen and Kol, 1980), accompanied by a similar attempt for Japan. In this study apart from capital, five types of labour were included, and the coefficients of the linear formula for the logarithms of the factor quantities (hence, again the Cobb-Douglas function) estimated. Among the results the following seems noteworthy: (*a*) significantly negative coefficients were found for farm workers (including farm managers); (*b*) the coefficients for managers were found to be non-significantly different from zero; (*c*) significantly positive coefficients were found for manual workers in Japan, white-collar workers in the USA and more or less significantly positive coefficients for professional workers and technicians in the USA; (*d*) negative coefficients were found also for blue-collar workers in the USA, and for small entrepreneurs in Japan.

Theoretical explanations can be given of (*a*) (cf. Mahmood and Nadeem--ul-Haque, 1981) and (*d*) (cf. Miller, 1971). These explanations may be briefly indicated as the crop maximizing argument for (*a*) and the labour reserve argument for (*d*).

What we want to stress here, however, is the low level of reliability (as expressed by *t* values) of the Tinbergen-Kol results and the much higher level of reliability of Gottschalk's method, which justifies a brief characterization of it.

Gottschalk assumes that the production process consists of a combination of two processes, for instance technical production and administration. (More processes would also be possible.) The two processes use different production factors (types of work), but one factor is used by both processes. The production functions of each process can be estimated and contain a smaller number of factors. This makes the regression coefficients more reliable: his *t* values vary from 1.37 to 14.83 with a median of 3.46. Also the adjusted $R^2$ are 0.89 and 0.98. Although in my opinion his method contains one technical error, this error can be easily avoided an does not change the essence of the results (cf. Tinbergen 1982 *a*). So this method constitutes a considerable contribution to the possibility of including a larger number of types of labour.

### 3) The phenomenon of counterproduction

Some aspects of Gottschalk's findings and some features of reality can be understood with the aid of counterproduction, a term which I proposed (Tinbergen, 1981) for a phenomenon identified by others quite some time ago, for instance Mishan (1967), who simply spoke of the «costs of economic growth». What I have in mind need not be linked with growth; it also exists in a stationary economy. In its more extreme forms it is the visible annihilation of somebody else's production, as with an act of vandalism. Counterproduction also takes more hidden shapes: a driver's error may cause an accident which then requires work by a doctor and hospital personnel. Still more hidden is the counterproductive character of two competing sales workers or sales managers or just managers. Sales manager A of firm I tries to enlarge firm I's market share, but sales manager B of firm II in the same industry branch takes action to prevent this from happening. The production of the surplus consumption of many Western citizens which makes them suffer from a heart attack is counterproducing the efforts of the medical profession to treat the heart patient.

In an attempt to penetrate to the deeper causes of counterproduction one is confronted with the impossibility to plan every detail of the operation of an economy (production, distribution, consumption, etc.), because our economies are too complicated. But also human behaviour contains elements causing counterproduction: we mentioned vandalism as an example. In addition, recent development of chemical and radio-active processes has introduced environmental pollution requiring counteractivities.

Production functions should reflect the phenomenon of counterproduction and considerable work in this field has been done (cf. Leontief *et al.* 1977), but much remains to be done.

### 4) Degree of aggregation in production function estimation

Our example of Sales managers A and B in Section 3 partly explains Gottschalk's findings, since his data are data for entire manufacturing branches in each state of the United States. Gottschalk gives this explanation in footnote 29 on p. 374. This implies that data for single firms are needed if changes in market share as the cause of a sales worker's earnings have to be included in a model used for such an explanation. Branch data are already too aggregated. Data for single firms are not available from official statistics and can only be obtained from voluntary contributions of the firms concerned. Presumably the best setup would be to try to explain company sales with the aid of the numbers of the types of workers employed, including the sales workers. A desirable cross check would be to run the same regression for branch data. In it the coefficients for the other types of labour should not be very different from the ones found with the aid of the firm data, but those for sales workers should be considerably lower and those for the managers should also be lower, but not necessarily as much as those for the sales workers.

The deviations between coefficients found for the numbers of sales workers with the aid of company data and with the aid of branch data may explain the deviation Gottschalk found between marginal productivity and earnings. The value 2.64 shown in Table I for the sales workers' $R$ may come closer to 1. The same may happen with the $R=2.03$ found for managers. It is also possible that the ratios would not become unity, but remain above 1. The remaining ratio $R'$ may then point to a not completely competitive market.

Together with the desirable estimations based on other production functions an extensive programme of data collection and processing appears to be waiting for implementation.

### 5) Essential and non-essential production factors; required versus available capabilities

Another lacuna in the work devoted to the estimation of production functions may be traced around the distinction between essential and non-essential production factors (cf. De Boer, 1981). An essential production factor is one which must be used in order to obtain the product; if the input of an essential production factor is zero also the quantity of product vanishes. Production factors whose absence does not make a positive production volume impossible are called non-essential. This characterization should not lead us astray. A non-essential production factor may be useful nevertheless, in the sense of showing a positive marginal product. It may be substituted for another production factor which happens to be unusually scarce.

A range of concrete examples exists whose production factors can conveniently be characterized by an even number of indexes. The simplest example evidently is one where two indexes are appropriate. These may refer to a feature required and the actual feature of the type of factor (say, labour) considered. The feature may be level of education; indicating the required level by an index $h$ and the actual level by $h'$, a quantity $\varphi_{hh'}$ of such a «double indexed» type of labour may participate in an economy's production process. A concrete example will be found in Tinbergen 1982, where three levels only of education ($h$, $h'=1, 2, 3$) are considered: primary, secondary and third-level education. Data on required level of some capability are relatively rare; they are available for schooling, in the USA thanks to Rumberger (1981); also in the Netherlands, thanks to Zanders *et al.* (1977), and analyzed and processed by Hartog (1983).

It is here that the lacuna now to be discussed shows up. Very few data are available when it comes to required levels and levels attained of other capabilities. Job evaluation provides us with required levels of some other features, but the levels attained by those on the job are available only in small samples of special inquiries. For the estimation of production functions such data could be very helpful.

For one category of production functions, namely those describing education processes, a distinction between innate and learnable capabilities deserves particular attention. This distinction itself may be possible only after research on learning processes is directed at it.

### 6) Public and private production

The last example of lacunae we want to discuss refers to the estimation of production functions for the same good or service, produced in publicly owned enterprises and privately owned. The issue in a way constitutes the main issue of the East-West controversy; that is its macro aspect. But it has a micro aspect also, since in both social orders the choice is under discussion for narrowly defined goods or services. There is a vast literature on the relative productivity in especially the United States of America and the Soviet Union. The evidence we have for individual goods or services in the same country produced publicly or privately is much less organized. Incidental news items seem to indicate that more evidence exists than scientific journals so far published. In addition, incidental political decisions refer to the problem, for instance the nationalization and reprivatization of steel production in the United Kingdom.

There are some interesting historical examples. In 1902 the Dutch State Mines (DSM) were established in a period where socialist influence on government was minimal. The reason was that no private initiative and capital were available to expand coal mining, considered of vital interest by the government. In 1923 the Turkish government under Atatürk which wanted to modernize the country, created a number of «state economic enterprises», for exactly the same reason. Today; both DSM and the Turkish state economic enterprises exist. DSM is considered to be a successful corporation which in the meantime abandoned coal mining and produces a variety of chemical products. In the Netherlands also state farms exist. They are located in newly reclaimed land. The reason for their being state owned is that after previous reclamations the first generation of farmers went bankrupt because of some particular risks of farming on newly reclaimed soil. After some years the state farms are sold to private farmers.

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