MODELLING BANK OPERATING COSTS WITH AN UNDERLYNG CES PRODUCTION FUNCTION (*)

José Martins Barata (**)

1 — Introduction

The objective of this article is to present economic reasons for underlying production functions, in the case of bank operating costs, as well as to show that, if such a function is CES-Dhrymes, the operating costs function has a special formula. Empirical evidence, concerning the Portuguese banking, will be presented, and results will show the difference between the assumption of Cobb-Douglas underlying production function and of a CES-Dhrymes one.

Morever, the study will be carried out under the assumption that pure competition does not exist, which is believed to correspond to the reality of less developed countries.

Banks are assumed to maximize their financial margin (bank net output) instead of profits. Arguments about such a choice may be read in Barata (1985) and will not be repeated here. The main point is that the financial margin maximization avoids the difficulties regarding «side payments», which is not the case for profit maximization, as one concludes following Cyert, March, Marris and Hague arguments.

As J. Johnston (1960, pp. 171-173) showed, if the production function has a Cobb-Douglas form then the cost function has a form of the same kind. His demonstration assumes the existence of pure competition and profit maximization.

Benston (1965; 1972) and many other authors following him assumed an underlying Cobb-Douglas production function in their studies on economies of scale in banking, without presenting any theoretical reason for such a hypothesis. Although that kind of analysis was a great progress at the time, other authors suggested different models for operating costs. For example, J. A. Clark (1984) argues that «there does not appear to be any economic justication for assuming the underlying production process to be Cobb-Douglas» (cf. p. 55)

(*) This paper was presented at the «3rd International Meeting on Monetary Economics and Banking», organised under patronage of the Parlementary Assembly of the Council of Europe, Europe Parliament, Strasbourg, 5 & 6 June 1986.

The author is indebted to Professor E. Apel, Otawa University, for many helpful suggestions which contributed to a more readable paper. The responsability for any errors or imperfections remains, of course, strictly the author's own.

(**) Professor associado at the Instituto Superior de Economia.

and proposes a Box-Cox generalized functional form. But, after estimation, he found that «the hypothesis of an underlying Cobb-Douglas production function cannot be rejected by the data» (cf. p. 64). Barata (1981) proposed economic reasons for the case where the production function is Cobb-Douglas, as well as for another type of CES production function. We continue this analysis in this article, taking the general CES case and testing it with the Box-Cox generalized functional form methodology.

2 — The production function

Considering the bank production defined by the real value added, we can write:

$$X_1 = LW + Kr \tag{1}$$

where:

- X_1 = output;
- L = number of employees;
- W = wages/salaries rate;
- K = productive capital;
- r = rate of return on productive capital.

Given the definition of value added, it is clear that Kr is the profit before depreciation, provisions and taxes.

Given the identity (1) and dividing by X_1 (afterwards simply denoted x), we obtain:

$$1 = \frac{LW}{X} + \frac{Kr}{X} \tag{2}$$

which is the income distribution structure identity.

The behaviour of this structure, as X changes, is the basis for deriving the form the production function. If LW/X remains constant while factor prices change, the function will be Cobb-Douglas; otherwise it will have another mathematical expression.

The equation **(2)** can be expressed in terms of productivity, as follows:

$$1 = \frac{W}{p_L} + \frac{r}{p_K} \tag{3}$$

where:

- $p_L = X/L$ — average productivity of labour;
- $p_K = X/K$ — average productivity of capital;

and:

$$LW/X = W/p_L.$$

Thus, LW/X remains constant when real wages vary proportionally to labour productivity.

How does this structure react to an increase of output, X? All hypotheses are possible, but there are two cases that must be more probable in an economic system like the one described in this study. They are depicted in figure 1, where the X's are cross-section values. The domain of the function depicted is the interval $[X_m, X_M]$, where X_m is the minimal size possible in a cross-section of a given economy in a certain period and X_M is the maximal one (1), in the same period.

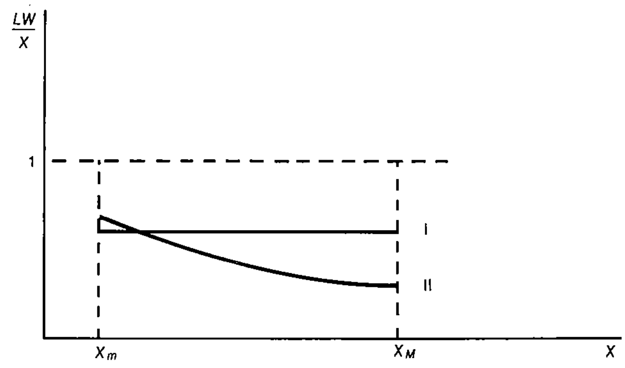

Figure 23.1 — Labour income distribution rate as function of size.

The case I is that in which LW/X remains constant. This is equivalent to say that W/p_L is constant. This case is plausible, since unions are usually ready to accept that wages increase at the same rate of labour productivity.

In case II the ratio LW/X decreases as a function of X, but with a increasing marginal rate.

In a cross section of banks, either case I or II may be observed, since:

— usually labour agreements between unions and banks settle wage levels that are generally observed; case I corresponds to such a situation, where enterprises use the same technology; so, W changes proportionally to p_L;

— otherwise, banks of larger size, which adopt more capital-intensive technological systems, have a higher productivity; although they may pay better salaries and wages in order to attract more qualified personnel, the use of more advanced technologies, as well

(1) For a given economy bank size cannot increase indefinitely, due to market and financial constraints, besides practical requirements to implement an investment project, given a certain time period not too long.

as other advantages of larger sizes, afford relatively larger capital incomes and, consequentely, the share LW/X becomes smaller and smaller, as it is dipicted by curve II in the figure. W is not proportional to p_L.

Both cases may be represented by the following general expression:

$$\frac{LW}{X}=A\ X^{\alpha}\ L^{\beta},\ \text{with}\ 0<A\leqslant X_M^{-\alpha}\ L^{-\beta}\leqslant 1;\ -1<\alpha\leqslant 0$$
$$X_m\leqslant X\leqslant X_M\ \text{and}\ 0\leqslant\beta,$$

which gives:

(23.4):

$$W=A\ X^{a}\ L^{b}$$

where:

$a=\alpha+1$ and $b=\beta-1$.

In case I it will be $\alpha=\beta=0$, otherwise $-1<\alpha<0;\ 0<\beta$.

It is possible to show that case I corresponds to a Cobb-Douglas production function, where, it is pointed out again, *the income distribution structure is constant.*

If the banking income distribution structure does not remain constant when the output level changes, then the production function is no longer Cobb-Douglas.

If $-\frac{\delta K}{\delta L}=\frac{W}{r}$ and given a constant elasticity of substitution, it is possible to write:

$$\frac{Kr}{LW}=a\left(\frac{r}{W}\right)^{\sigma-1},\ \text{with}\ \sigma\neq 1$$

If $\frac{Kr}{LW}=a\left(\frac{r}{W}\right)^{\sigma-1}$ is not constant, it should be interesting to find a necessary and sufficient condition about a CES specification of the production function, namely the most general CES-DHRYMES-type.

Following the pioneering paper of Arrow, Chenery, Minhas and Solow (1961), Dhrymes (1965) derived a production function specified by:

$$X=c\ [a_1\ K^{h\delta}+a_2\ L^{h\delta}]^{1/\delta} \qquad (5)$$

The necessary and sufficient condition for such a function specification is

$$X=\hat{A}\ L^{\alpha}\ W^{\beta} \qquad (\alpha\neq 1;\ \beta\neq) \qquad (6)$$

which is equivalent to (4), with $a\neq 1$ and $b\neq -1$, on which Dhrymes (5, p. 360) based the derivation of his CES production function. It is equivalent to $Kr/LW=a\ (r/W)^{\sigma-1}$ (demonstration in appendix 1).

The objective of the enterprise is assumed here to be the maximization of the financial margin, that is, net interest income plus other operating income. Although no systematic study was carried out on bank managers behaviour, we can observe directly in the profession that they adopt that objective, as well as to maximize the amounts of credit and of costless deposits and other objectives of this kind, which are equivalent. As financial margin is equal to costs of intermediate inputs (Ci) plus value added, its maximization is equivalent to the maximization of Ci, which may be assumed, and to maximize value added. This is a long-run objective and it is subject to budgetary constraints, which determine specified cost levels ($\overline{OC}$).

On these bases, the optimal plans of the firm assume the following:

a) Short-run costs are minimized, which determines short-run allocation of productive factor quantities;

b) Under some conditions, namely the financial resources available, the firm will try to attain the highest isoquant possible, in the long-run (this corresponds to the Baumol's size maximization assumption).

Finally, it must be stressed that the production functions used here are long-run functions, with each firm attaining optimal costs and production levels. As they are derived from accounting identities, they are not directely related with physical conditions of production, but they are useful to construct banking profitability models, since data are those provided by accounting statements. If our aims were different, the criticism of Simon (10) would be relevant.

3 — The costs function in the case of a CES production function

The CES production function (5) will be assumed as corresponding to the reality of the universe being studied (case II of figure 1). Thus, taking a cross-section of data, changes in the structure of income distribution, as size increases, must be observed, as a consequence of higher salaries and wages paid by large banks, which use more advanced technologies and need more qualified personnel.

The accounting identity for operating costs may be written:

$$OC = LW + K\ r^* \text{ with } r^* = OOC/K \tag{7}$$

where OC denotes operating costs, LW total wages and salaries, OOC other operating costs and K the productive capital. This provides the optimal short-run solution of factors allocation, as it is usually derived in microeconomics.

For the long-run, let us consider the case of the largest firm, the maximal size being X_M.

Given identity (1) and the assumption that W is an exogenous variable (e. g., it is fixed after agreements with unions), as well as r, the long-run value added maximization is:

$$\begin{gathered} \text{Max } LW + Kr \\ \textit{subject to} \\ X = F\ (K,\ L) \leqslant X_M \\ \overline{OC} = LW + Kr^* \end{gathered}$$

for which the solution given the Kuhn-Tucker conditions (2), implies

$$-\delta K/\delta L = W/r. \qquad (8)$$

From (8) and other equalities presented or derived in appendix 2, we get

$$OC' = \Psi\ (P,\ P_r,\ N_c,\ T_1,\ T_2)\ X'^{1/h} \qquad (18)$$

where OC' and X' stand for nominal values of operating costs and production, respectively.

If a Box-Cox generalized functional form is assumed for Ψ, then we may write (18) as:

$$OC' = A\ P^a P_r^b\ N_c^c\ T_1^d\ T_2^e\ X'^{1/h},\ \text{when}\ \lambda = 0, \qquad (19)$$

or

$$OC' = (\beta_0 + \beta_1 P + \beta_2 P_r + \beta_3 N_c + \beta_4 T_1 + \beta_5 T_2)\ X'^{1/h},\ \text{if}\ \lambda = 1, \qquad (20)$$

as well as other specifications omitted here (for $\lambda = 2$, $\lambda = 3$, etc.).

Thus if the model (19) fits well with a sample of data and (20) was not estimated for comparison, one may present incorrect statements concerning «the underlying Cobb-Douglas production function», as a consequence of a specification error. Indeed, if estimations with (19) are acceptable but those obtained through (20) are better, it seems that the production function will not be Cobb-Douglas.

4 — Measuring output and technology in banking

Two problems have arisen when output must be measured:

1) Banks are multiproduct firms;
2) Their product consists of services.

Even if bank output was of a physical nature, the sole multiproduct feature of the firm would be sufficient to encounter difficulties in measuring production. It would be necessary to find a common measure of the physical output, which would not be practicable in most cases. Hence, in such a case, a measure expressed in money units is usually adopted, such as sales value, as well as the value added. For example, value added is the measure for output in the literature for production functions (e. g. ACMS as well as Dhrymes' articles).

---

(2) Considering $L = \overline{OC} - Kr^* + Kr + \lambda\ [X_M - F\ (K,\ L)]$, we have:

I) $\lambda = 0 \Longrightarrow -\ \partial K/\partial L = W/r;$

II) $\lambda \neq 0 \Longrightarrow \lambda = (W + r\ \partial K/\partial L)/\partial F/\partial L;\ W \partial L/\partial K + r - \lambda \partial F/\partial K = 0 \Longrightarrow$
$\Longrightarrow W \partial L/\partial K + r + (W + r \partial K/\partial L)\ \partial L/\partial K \Longrightarrow -\ \partial L/\partial K = r/W.$

For a bank, the solution might be similar, since it produces no physical goods and, consequently, a physical measure of bank output should seem inconceivable. However, some authors, like Benston (1965; 1972), Bell & Murphy (1968) measured bank output in terms of the number of deposit accounts and loans allowed, because (Benston, 1972, p. 320) output was defined «in terms of what banks or savings and loan associations do that cause them to incur operating costs» and «operating costs are related primarily to the number of documents handled and customers served rather than to the dollars deposited or loaned».

As this measure, obviously, «ignores the jointness of the various bank activities» (J. A. Clark, 1984, p. 54), Benston et al. (1982, pp. 439-442) — to respond to such a requirement — adopted a new measure of bank output, which is a rather complex «Divisia index», calculated through geometric averages depending, namely, on the annual average number of accounts serviced. They used also, as alternative measures, for illustrative purposes, the simple number of accounts as well as the total dollars of deposits and loans, having found that the results, concerning economies of scale measures are not very different.

Most authors used different non-physical measures of bank output (see Barata, 1981, pp. 102-122), such as total value of deposits, total assets, earning assets, sum of weightted lending output with non-lending output (this was done by Greenbaum, 1967, in order to take into consideration the multiproduct nature of lending output), value added and, finally, bank net product (which is bank incomes minus interest paid).

Barata (1981) used bank net product to estimate operating cost regressions, because this measure is compatible with a diagonally recursive model, for which, as it is known, OLS are BLUE (Barata, 1981, p. 248). But he tested also the production specification for the following output measures:

X_1 = bank value added;
X_2 = earning assets;
X_3 = total assets;
X_4 = bank net product;

It seems that no relevant theoretical reason exists, requiring the sole use of «physical measures» of bank output, since banks are service enterprises. Moreover, nowadays, no bank aims to maximize the number of deposit accounts, on the contrary, the goal is to reduce that number and maximize the money of deposits.

A solution like that of S. I. Greenbaum (1967), recently reemployed by J. A. Clark (1984), seems preferable. Morever, the last author concluded «that the estimated output elasticity of cost is rather insensitive to the choice of the measure of output» (*op. cit.*, p. 66).

The theory should recommend the value added as measure of the bank output. However, given the results of Barata (1981) and Clark (1984), earning assets may be used as a good proxy, with the advantage of being a concept well known in banks, since it is an adjusted total assets value.

It must be remembered that maximization of X_4 is the bank's objective and it should be a better proxy of X_1. Only for practical reason X_2 is adopted.

It is obvious that what the bank employees do is mainly to process information, which results in the bank's output. Their work can be done in different ways: entirely manually or with support of several kinds of machines. A system of using machines in the bank activity process will be called *banking techonology.* Several systems and machines can be conceived and used. Barata (1981, pp. 125-132) identified six banking technologies:

T_1 = processing completely manual, non-existence of any machines;
T_2 = use of electro-mechanical accounting machines, even partially;
T_3 = processing with electronic machines not classifiable as computers (microcomputers are included here);
T_4 = hiring of computer time outside the bank;
T_5 = use of a computer in own information processing center, without teleprocessing on-line;
T_6 = teleprocessing on-line.

The first two systems have nowadays a mere historical interest in most countries. For example, T_1 still existed in Portugal in 1973, in some small banks. In 1978-1979, it did not exist any more, but T_2 was still used. After 1980 both were eliminated (Barata, 1981; 1984).

Concerning banking technology, it is conceivable to adopt physical measures: for example, total quantity of kilobytes of computers' central memory, as well of microcomputers, terminals and so on. However, this solution would meet serious difficulties, because it would be impossible to consider simultaneously, in a cross-section, different technological systems, for example T_1, T_2 and T_4 could not be taken into consideration with T_5 and or T_6. Moreover, the quantity of machines, or some proxy, would be highly correlated with the variable measuring the size of the bank.

More important than the quantity of machines, concerning banking technology, is information processing system adopted by the bank. Thus, the problem concerns rather the field of «quality» and the statistical way of measuring such a «state» is with dummy variables. Hence, the solution adopted in this study for measuring technology will be the use of dummies.

4 — Estimation

4.1 — The data

The data used in the subsequent econometric tests use 17 Portuguese banks, for the period 1978-1982, with which a pool of cross-section and time series with 85 observations was arranged. It was assumed stable coefficients subject to further tests.

The Portuguese banking system is composed of a central bank, 13 commercial banks — the larger ones nationalized (11) and 3 small private foreign banks — savings and loan associations, as well savings banks. The savings banks operate mainly in the Azores and they are allowed to do the commercial banks operations plus morgage loans.

There are also two investment banks (one of them very recent, which may hold demand deposits but do not discount commercial letters).

For the purpose of estimation only the financial institutions which can hold demand deposits were considered. Balance and income sheets published as well enquiries sent to the banks were the sources of data. Because some institutions did not answer to the enquiry, our sample includes 12 commercial banks, 1 investment bank and 4 savings banks, which represents 85 % of the statistical universe. Only 3 small institutions are not in the sample. Data about monetary policy were colected from the central bank annual reports. The data concerning provision allowance for effective risk were obtained from banks' profit and loss accounts, where provisions for verified losses is recorded.

4.2 — The results

Equation (18) was estimated using a maximum likelihood procedure, with an algorithm provided by J. J. Lacrampe (1982). Different specifications were used:

I) Box-Cox generalized functional forms:

I. *a*):

$$OC'^{(\lambda_0)} = B_0 + B_1 X_2^{(\lambda_1)} + B_2 DF_r^{(\lambda_2)} + B_3 G^{(\lambda_3)} + B_4 P^{(\lambda_4)} + B_i T_j^{(\lambda_i)} + u$$

I. *b*):

$$\Psi^{(\lambda_0)} = b_0 + b_1 DP_r^{(\lambda_1)} + b_2 G^{(\lambda_2)} + b_3 P^{(\lambda_3)} + b_i T_j^{(\lambda_i)} + u$$

where each variable, let us say V_j, has the following power transformation:

$$V_j^{(\lambda_j)} = (V_j^{\lambda_1} - 1)/\lambda_j \quad if \ \lambda_j \neq 0;$$

$$V_j^{(\lambda_j)} = 1n V_j \qquad if \ \lambda_j = 0;$$

II):

$$OC' = \beta_1 X_2'^{1/h} + \beta_2 DP_r X_2'^{1/h} + \beta_3 G \ X_2'^{1/h} + \beta_4 P \ X_2'^{1/h} + \beta_i T_j X_2'^{1/h} + u$$

with the following notation:

OC' = operating costs (interest paid excluded) at current prices;

X'_2 = bank output measured by earning assets (balance sheet total corrected from inter-bank liabilities) at current prices;

DP_r = provisions allowance due to actual risk (balance sheet value, P_r, was corrected from estimated hidden reserves) at constant prices;

G = number of agencies; it is used as a proxy for N_c, since data for this variable are not available in Portugal;

$P = GDP$ deflator;

T_i = dummies for technological states;

h = homogeneity degree of the production function.

The results obtained with specification I. *a*) are presented in table 1 of appendix 2. They show that the Cobb-Douglas form (that is, with $\lambda_i = O$) is preferable to any other.

Specification I. *b*) was estimated following two ways. The first consisted of regressing $1nOC'$ on $1nX_2'$ (regression 1 of table 2, appendix 3) and with the residuals *(ROC)*, to compute regressions following the Box-Cox forms (see regressions 2, 3 and 4, same table), on the variables supposed to explain Ψ. This stage showed that the multiplicative model provided a higher determination coefficiente (see regression 2), but the linear one provided more efficient estimators (regression 3). In the second stage the maximum likelihood of $\frac{1}{h}$ was determined by searching over a grid of different values, from unity to lower ones, first with linear regressions and, after, with logarithms. Some of the results are presented in table 2 of appendix 3. The conclusion was that the best results were provided by a linear regression of $OC'/X_2'^{.55}$ on the variables that explain Ψ.

Estimation of Ψ was carried out in order to determine whether its best specification is a multiplicative model, which would be consistent with a Cobb-Douglas form for operating costs. In the light of the results that have just been presented, we may state that it is the case and, consequentely, a model like equation II) is expected to be better.

The next stage of our analysis was to select the optimal value of $\frac{1}{h}$, through estimation of equation II), using a grid search (3). We started with values around .55, the estimations having shown that R^2 and efficiency of estimators increased as $\frac{1}{h}$ increased until .59 and began to decrease above this value. The best estimation is presented below, in comparison with a Cobb-Douglas estimation.

Specification II):

$$OC' = .492X_2'^{.59} + .0008DP_rX_2'^{.59} + .027G\,X_2'^{.59} + .006P\,X_2'^{.59} - .973T_3X_2'^{.59} - .513T_5X_2'^{.59} + .18T_6X_2'^{.59}$$

	(.259)	(0)	(0)	(0)	(.148)	(.119)	(.104)
t	1.98				6.56	4.3	1.81

---

(3) The values of $\frac{1}{h}$ used to estimate Ψ are not optimal because $\hat{\Psi}X'^{r} = \hat{O}C' - \varepsilon X'^{r} = \hat{O}C'$, with $\varepsilon = \Psi - \hat{\Psi}$.

Significance levels (%)	4.88	0	0	0	0.000	0.17	7.03

$R^2 = 99.02\,\%$

Cobb-Douglas specification:

$$1n\hat{\ }OC' = 3.156 + 0.00978\ 1nDP_r + 0.229\ 1nP - 0.178\ T_3 - 0.253\ T_6 + 0.853\ 1nX'_2$$

	(.775)	(0)	(.153)	(.099)	(.099)	(.025)
t	4.1		1.95	1.78	2.57	33.6
Significance levels (%)	0.03	0	5.15	7.46	1.12	0.000

$R^2 = 97.7\,\%$

Obviously, the Cobb-Douglas regression is the best one for this specification, after elimination of irrelevant variables.

The same kind of estimations above was carried out with respect to X'_4, the bank net output. The best results were provided by specification II), with $\frac{1}{h} = .75$. For this case, R^2 was 98.6 % with regression II) and 87.7 % with the Cobb-Douglas form. Significance of coefficents was much higher with specification II) than with the Cobb-Douglas one.

In any case, estimations carried out show that economies of scale exist in the case of Portuguese banks, but they appear more evident with the CES-Dhrymes formulation of the underlying production function than with the Cobb-Douglas one. The first one is more reliable, because it provides more efficient estimators, as well as higher determination coefficients.

The results obtained with specification II) show that the risk of credit and other risks (DP_r) contribute positively to the bank operating costs, but in a much lower degree than the more advanced stage of data processing installed (T_6) or the number of agencies (G). The use of own computers without teleprocessing on-line reduces operating costs and microcomputers have a stronger effect in the same sense.

These results were not affected by the fact that one investment bank and three savings banks were included in the sample, since the same kind of estimations was run with dummies concerning those institutions, the coefficients of which were not significant.

Morever, since a data pool of 5 cross-sections was used, with assumption of coefficients stability, the Chow test was applied to check that hypothesis. The F statistics were allways very low (between zero and 1.7), which confirms the hypothesis.

5 — Conclusions

One of the aims of this article was to derive the costs function in an imperfect market, when the production function is CES, assuming financial margin maximization as the enterprise objective, rather than profit maximization.

Another objective of this study was to test empirically such a model with the Portuguese banking system.

The tests carried out showed that although a Cobb-Douglas specification could not be rejected by the data the CES-Dhrymes form was found to fit better and provided more efficient estimators.

When the production measure was earning assets, the estimated degree of homogeneity was 1.695 and when measured by the bank net output it was 1.333. Thus, economies of scale were found in the Portuguese banking system.

The credit risk, the number of branches, as well as inflation, contribute positively to operating costs, while technology consisting of microcomputers and computers without teleprocessing on-line revealed to have a negative influence, on operating costs.

APPENDIX 1

The demonstration is as follows. It is assumed here that the variable K may be expressed as a decreasing straight line, in fuction of L, because for each bank size exists an isocost line like

$$K = c - a^* L$$

As $-\frac{\delta K}{\delta L} = a^*$,

$$\left(-\frac{\delta K}{\delta L}\right)^{1-\sigma} = a^{*1-\sigma}, \text{ with } \sigma = -\frac{\delta \ln \frac{K}{L}}{\delta \ln M} \text{ constant.}$$

$$LW \left(-\frac{\delta K}{\delta L}\right)^{1-\sigma} = a^{*1-\sigma} LW$$

$$LW\, a^{*\sigma-1} \left(-\frac{\delta K}{\delta L}\right)^{1-\sigma} = LW$$

The definition of σ conducts to

$$\ln \frac{K}{L} = b + \sigma \ln \left(-\frac{\delta K}{\delta L}\right)$$

This means that

$$\frac{K}{L} = b^* \left(-\frac{\delta K}{\delta L}\right)^{\sigma}$$

with $\ln b^* = b$.

Therefore

$$\frac{K}{L} = b^* \left(\frac{W}{r}\right)^{\sigma}$$

or, then,

$$\frac{Kr}{LW} = b^* \left(\frac{W}{r}\right)^{\sigma-1},$$

which implies

$$Kr = b^* LW \left(-\frac{\delta K}{\delta L}\right)^{\sigma-1}$$

Since

$$X = LW + Kr,$$

$$X = LW\ a^{*1-\sigma} \left(-\frac{\delta K}{\delta L}\right)^{\sigma - 1} + b^*\ LW \left(-\frac{\delta K}{\delta L}\right)^{\sigma - 1};$$

(A.1) $$X = LW \left(-\frac{\delta K}{\delta L}\right)^{\sigma - 1} (a^{*1-\sigma} + b^*)$$

As it was written above, it is possible to accept that the ratio $\frac{W}{r} = -\frac{\delta K}{\delta L}$, may change with $\frac{X}{L}$, that is, in function of the productivity of labour, which will be as much higher as more automation is used in the bank (and this is true mostly for higher sizes). But it can change also in function of the number of personnel (which is correlated with size). Morever, both hypotheses may occur simultaneously.

Therefore, we can write:

$$-\frac{\delta K}{\delta L} = \theta \left(\frac{X}{L}\right)^{\psi} L^{\gamma}, \qquad \text{with } \theta > 0;\ \psi \text{ and } \gamma \text{ any values} \neq 0.$$

Then, deducing from (A.1), we shall have:

$$X = LW\ \theta^{\sigma - 1} \left(\frac{X}{L}\right)^{\psi\ (\sigma - 1)} L^{\gamma\ (\sigma - 1} \left(a^{*1-\sigma} + b^*\right);$$

$$X^{1 - \psi\sigma + \psi} = \theta^{\sigma - 1} \left(a^{*1 - \sigma} + b^*\right)\ W\ L^{1 - \psi\sigma + \psi + \gamma\sigma - \gamma};$$

(A.2) $$X = A\ W^{\alpha} L^{\beta}$$

with $$A = [\theta^{\sigma - 1}(a^{*1-\sigma} + b^*)]^{\frac{1}{1 - \psi\sigma + \psi}}$$

$$\alpha = \frac{1}{1 - \psi\lambda + \psi}$$

$$\beta = 1 + \frac{\gamma\ (\sigma - 1)}{1 - \psi\sigma + \psi}.$$

This demonstration is reversible, since from $X = A\ W^{\alpha}\ L^{\beta}$ and the definition of σ it is always possible to meet two real numbers ψ and γ permiting to establish a relation between α and β with γ, respectively. Thus (6) is a necessary and sufficient condition of a CES function.

As a particular case, if $\sigma = 1$, then $\alpha = \beta = 1$ in (A.2) and the function is Cobb-Douglas.

APPENDIX 2

Consider the elasticity of substitution, σ, and the marginal rate of substitution, M, defined by

$$\sigma = \frac{d\ \ell n\left(\frac{K}{L}\right)}{d\ \ell n\ M} \qquad (9)$$

$$M = \frac{\delta X / \delta K}{\delta X / \delta L}. \qquad (10)$$

The identity (10) is equivalent to:

$$M = -\frac{\delta L}{\delta K}.$$

In this case

$$\frac{\delta X/\delta L}{\delta X/\delta K}=\frac{W}{r}$$

becomes

$$\frac{\frac{1}{\delta}\,c\,[a_1\,K^{h\delta}+a_2\,L^{h\delta}]^{\frac{1}{\delta}-1}\,h\delta a_2\,L^{h\delta-1}}{\frac{1}{\delta}\,c\,[a_1\,K^{h\delta}+a_2\,L^{h\delta}]^{\frac{1}{\delta}-1}\,h\delta a_1\,K^{h\delta-1}}=\frac{W}{r},$$

which, after simplifying, gives:

$$\frac{a_2}{a_1}\left(\frac{L}{K}\right)^{h\delta-1}=\frac{W}{r},$$

hence

$$\frac{K}{L}=\left(\frac{a_1}{a_2}\,\frac{r}{W}\right)^{\frac{1}{h\delta-1}} \tag{11}$$

Therefore, from (11), we get:

$$L=\left(\frac{a_1}{a_2}\right)^{\frac{1}{h\delta-1}}\left(\frac{W}{r}\right)^{\frac{1}{h\delta-1}}K; \tag{12}$$

$$K=\left(\frac{a_2}{a_1}\right)^{\frac{1}{h\delta-1}}\left(\frac{r}{W}\right)^{\frac{1}{h\delta-1}}L. \tag{13}$$

Replacing L by (12) into the production function, it is obtained:

$$X^{\delta}=\hat{C}\;a_1\;K^{h\delta}+\hat{C}\;a_2\left(\frac{a_1}{a_2}\right)^{\frac{h\delta}{h\delta-1}}\left(\frac{W}{r}\right)^{\frac{h\delta}{h\delta-1}}K^{h\delta},$$

where $\hat{C}=C^{1/\delta}$.

Hence

$$K=\left[\hat{C}\;a_1+\hat{C}\;a_2\left(\frac{a_1}{a_2}\,\frac{W}{r}\right)^{\theta}\right]^{-1}X^{1/h} \tag{14}$$

with

$$\theta=\frac{h\delta}{h\delta-1}.$$

Now, substituting (13) into the production function,

$$X^{\delta}=\left[\hat{C}\;a_1\left(\frac{a_2}{a_1}\,\frac{r}{W}\right)^{\theta}+\hat{C}\;a_2\right]L^{h\delta}$$

hence

$$L=\left[\hat{C}\;a_1\left(\frac{a_2}{a_1}\,\frac{r}{W}\right)^{\theta}+\hat{C}\;a_2\right]^{-1}X^{1/h}. \tag{15}$$

Taking (7) again and substituting into it L and K for (15) and (14) respectively, we obtain:

$$OC=\underbrace{\left\{\left[\hat{C}\;a_1\left(\frac{a_2}{a_1}\,\frac{r}{W}\right)^{\theta}+\hat{C}\;a_2\right]^{-1}W+\left[\hat{C}\;a_1+\hat{C}\;a_2\left(\frac{a_1}{a_2}\,\frac{W}{r}\right)^{\theta}\right]^{-1}r^*\right\}}_{A(W,r,\;r^*)}X^{1/h} \tag{16}$$

or

$$OC'=A\;(W,\;R,\;R^*)\;X'^{1/h}\;P^{(h-1)/h} \tag{17}$$

with $OC'=OC\;P$; $X'=X\;P$, where P is the general index of prices, and

$$A\;(W,\;r,\;r^*)=\frac{W}{\hat{C}\;a_1\left(\frac{a_2}{a_1}\,\frac{r}{W}\right)^{\theta}+\hat{C}\;a_2}+\frac{r^*}{\hat{C}\;a_1+\hat{C}a_2\left(\frac{a_1}{a_2}\,\frac{W}{r}\right)^{\theta}}$$

As in the Cobb-Douglas case, A (W, r, r^*) would be a constant if prices were given, under pure competition. In such a situation the cost functions would have identical formulae.

It is useful to change the arguments of $A(\ )$ for other on which depend W, r, and r^*, since data for r and r^* are usually not available. Real wages will be assumed to depend exclusively on P and the technology used by the bank (for which two dummies, T_1 and T_2, are assumed below to correspond to the existing technological states). Technology influences labour productivity and may imply higher wages (skilled employees). The rate r depends on P (since profit is here net bank output less LW) and r^* depends on P, the provisions to face several risks (P_r), the number of current accounts processed (N_c) as well as T_1 and T_2.

Thus, we may state

$$A(\mathrm{W},\ \mathrm{r},\ \mathrm{r}^*)\ \mathrm{P}^{(h-1)/h} = \psi(P,\ P_r,\ N_c,\ T_1,\ T_2)$$

and (17) becomes

$$OC' = \psi(P,\ P_r,\ N_c,\ T_1,\ T_2)\ X'^{1/h} \tag{19}$$

APPENDIX 3

TABLE 1

Box-Cox estimations of operating costs (85 observations)

Explained variable	Lambda	Intercept	X2	DPr*	P	T3	T5	T6
OC	0	−3.156 (.775) −4.07 *****	.853 (.025) 33.6 ***** R^2=97.736 %	.00978 (0) *****	.299 (.153) 1.952 *	−.178 (.099) −1.784 *	-	−.254 (.099) −2.567 ***
OC	1	−169.007 (−313.503) .539	.024 (0) ***** R^2=91.170 %	1.449 (.631) 2.289 ****	.282 (1.879) .15	−141.188 (−189.23) .746	-	−370.166 (187.18) 1.978 **
OC	2	1.91319 e 15 (5.689E+15)	5920000 (0) ***** R^2=88.119 %	1.2887 e 11 (6.001E+10) 2.147 **	3.6719 e 11 (4.278E+11)	−9.7099 e 14 (−1.2E+06) .078	5.85666 e 15 (1.01E+16) .575	-

*; **; ***; ****; ***** → significance levels of 10 %, 5 %, 2.5 %, 1 % and .5 %, respectively.

Variables:

OC = operating costs at current prices;
X2 = earning assets at current prices;
DPr* = provision allowances for actual risks at constant prices;
P = GDP deflator;
T3 = dummy for using of microcomputers;
T5 = dummy for own computer;
T6 = dummy for own computer with teleprocessing on-line.

TABLE 2

Box-Cox estimations with residuals of OC (85 observations)

Explained variable	Lambda	Intercept	X2	DPr*	G	P	T3	T4	T5	T6
(1) OC	0	−2.86 (.212) −13.44 *****	.93 (.021) 33.6 ***** R^2=95.773%	-	-	-	-	-	-	-
(2) ROC	0	-	- R^2=41.133%	.00767 (0)	−.052 (.0349) −1.5	.101 (.032) 3.14	−.175 (.111) −1.566	-	.1347 (.128) 1.053	−.391 (.106) −3.675
(3) ROC	1	−.673 (.229) 2.938 *****	- R^2=32.233%	.000288 (0) *****	.00139 (0) *****	.00277 (0) *****	−.263 (.125) −2.106 **	-	.299 (.126) 2.366 ****	−.518 (.114) −4.554 *****
(4) ROC	2	.407 (.311) 1.307	- R^2=17.869%	−.000000197 (0) *****	−.000007 (0) *****	.0000191 (0) *****	−.926 (.389) −2.375 ****	.932 (.408) 2.282 ****	.479 (.384) 1.251	-

*; **; ***; ****; ***** → significance levels of 10%, 5%, 2.5%, 1% and .5%, respectively.

Variables:

OC = operating costs at current prices;
X2 = earning assets at current prices;
G = agencies;
DPr* = provision allowances for actual risks at constant prices;
P = GDP deflator;
T3 = dummy for using of microcomputers;
T5 = dummy for own computer;
T6 = dummy for own computer with teleprocessing on-line.
ROC = residuals of regression (1)

TABLE 3

Estimation of Ψ = OC/(X2^1/h) (85 observations)

Explained variable	Intercept	DPr*	G	P	T3	T5	T6
OC/(X2^.853)	.072 (.022) 3.279 *****	–.0000203 (0) ***** R^2-52.912 %	.000502 (0) *****	.000359 (0) *****	–.033 (.013) – 2.614 ****	.038 (.013) 3.007 *****	–.054 (.012) – 4.699 *****
OC/(X2^.8)	.097 (.034) 2.87 *****	–.0000122 (0) ***** R^2-68.250 %	.00121 (0) *****	.000633 (0) *****	–.058 (.019) – 2.988 *****	.064 (.019) 3.28 *****	–.082 (.018) – 4.367 *****
OC/(X2^.6)	.096 (.206) .46	.000855 (0) ***** R^2-92.011 %	.019 (0) *****	.00618 (0) *****	–.484 (.112) – 4.068 *****	.322 (.121) 2.67 ****	–.319 (.108) – 2.946 *****
OC/(X2^.55)	–.092 (.345) .265	.00191 (0) ***** R^2-93.550 %	.036 (0) *****	.011 (0) *****	–.826 (.199) – 4.142 *****	.431 (.202) 2.134 **	–.41 (.181) – 2.259 ***
		ln DPr*	ln G	ln P			
OC/(X2^.55)	– 3.341 (.692) 4.825 *****	.00715 (0) ***** R^2-91.241 %	.369 (.030) 12.214 *****	.625 (.136) 4.588 *****	–.012 (.095) –.121	.296 (.109) 2.718 *****	–.266 (.092) – 2.901 *****

*; **; ***; ****; ***** → significance levels of 10 %, 5 %, 2.5 %, 1 % and .5 %, respectively.

Variables:

OC = operating costs at current prices;
X2 = earning assets at current prices;
G = agencies;
DPr* = provision allowances for actual risks at constant prices;
P = GDP deflator;
T3 = dummy for using of microcomputers;
T5 = dummy for own computer;
T6 = dummy for own computer with teleprocessing on-line.

REFERENCES

(1) BARATA, José Martins, *Analyse de la rentabilité bancaire au Portugal,* Lisbon, 1981; «A global model of bank profitability; a non-competitive market case, with restrictive monetary policy», in *Economic Notes,* vol. 14, no. 2, 1985, Siena, pp. 78-96.

(2) BELL, F. W., & MURPHY, N. B., *Costs in commercial banking: a quantitative analysis of bank behavior and its relation to bank regulation,* 1968, Federal Reserve Bank of Boston.

(3) BENSTON, G. J., «Economies of scale and marginal costs in banking operations», in *The National Economic Review,* June 1965, pp. 507-549; «Economies of scale of financial institutions», in *Journal of Money, Credit and Banking,* no. 2, May 1972, pp. 312-341.
BENSTON, G. J., HANWECK, C. A., & HUMPHREY, D. B., «Scale economies in banking», in *Journal of Money, Credit and Banking,* November 1982, pp. 435-456.

(4) CLARK, J. A., «Estimation of economies of scale in banking using a generalized functional form», in *Journal of Money, Credit and Banking,* February 1984, pp. 53-68.

(5) DHRYMES, P. J., «Some extensions and tests for the CES class of production functions», in *Review of Economics and Statistics,* November 1965, pp. 357-366.

(6) GREENBAUM, S. I., «A study of bank costs», in *The National Banking Review,* June 1967, pp. 415-434.

(7) JOHNSTON, J., *Statistical cost analysis,* McGraw Hill, 1960.

(8) LACRAMPE, J. J., *Modèles à une équation en polinômes de retard,* 1982, Institut Orléanais de Finance, Orléans.

(9) MURRAY, J. D., & WHITE, R. W., «Economies of scale and deposit-taking financial institutions in Canada», in *Journal of Money, Credit and Banking,* February, 1980, pp. 58-70.

(10) SIMON, H. A., «On parsimonious explanations of production relations», *Scandinavian Journal of Economics,* 1979, pp. 459-474.

BARATA, José Martins — **Modelo para os custos operacionais bancários no caso de uma função CES subjacente.**

Neste artigo são apresentadas justificações de natureza económica relativamente ao tipo de função de produção subjacente a determinada formulação da expressão dos custos operativos na banca e demonstra-se que, no caso de tal função ser CES-DHRYMES, a função dos custos operativos tem uma forma especial.

Após especificação da função de custos operativos e sua estimação baseada em dados relativos a instituições de crédito portuguesas, conclui-se que a fórmula derivada da função CES--DHRYMES proporciona melhores resultados do que a de tipo COBB-DOUGLAS, verificando-se, com maior evidência, a existência de economias de escala.

BARATA, José Martins — **Modelling bank operating costs with an underlying CES production function.**

In this article the author present economic reasons for underlying production functions, in the case of bank operating costs and shows that, if such a function is CES-DHRYMES, the operating costs function has a special formula.

After the model specification, empirical evidence concerning Portuguese banking was presented. Results show that economies of scale are more evident with the CES-DHRYMES formulation of the underlying production function than with the COBB-DOUGLAS one and that first is more reliable, because it provides more efficient estimators, as well as higher determination coefficients.