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ON EXCHANGE RATE DYNAMICS(*)

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Introduction

In this paper, a simple macroeconomic model is developed for the study of exchange rate movements. It is based on the model set forth by Dornbusch (1976). The model is consistent with the monetary approach to the balance of payments which assumes instantaneous adjustments in all markets. Dornbusch introduced a modification whereby asset markets adjust instantaneously and goods markets adjust more slowly. This important contribution retains all the long-run equilibrium, or steady-state properties of the monetary approach. However, in the short run, the real exchange rate and the interest rate can diverge from their long-run levels.

Exchange rate dynamics, or «overshooting», can occur in any model in which some markets do not adjust instantaneously. That being the case, the response to a shock to the system, for instance, an increase in the money supply, would be a proportionately larger once-and-for-all depreciation of the exchange rate, followed by exchange rate appreciation throughout the rest of the adjustment process. During the adjustment process, domestic prices are increasing while the exchange rate is appreciating, so purchasing power parity (1) does not hold, except at long-run equilibrium.

This result is a consequence of the assumption that asset markets clear instantaneously, whereas goods markets do not. In the Dornbusch framework, asset markets must bear the entire initial impact burden of adjustment, and this can only occur when the exchange rate overshoots. The overshooting effect will be lower, the higher the interest response of the demand for money and the greater the expectations coefficient associated with divergences between the spot and the long-run exchange rate. The basic Dornbusch model

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(1) For a good survey on purchasing power parity, see, for example, Frenkel (1976) and Krueger (1983).

assumes that output is fixed at the full-employment level, but Dornbusch extended it to the case in which there are short-run adjustments in output. In this case, the extent of overshooting is reduced by income responses to changes in aggregate demand, and the phenomenon may in fact disappear entirely.

The overshooting result is consistent with perfect foresight. Initial shocks are unanticipated.

Recently, Frenkel and Rodriguez (1982) extended the Dornsbusch analysis by considering the case of finite speed of adjustment in asset markets as well as in goods markets. They concluded that exchange rate overshooting occurs with a sufficiently high degree of capital mobility; but with capital immobility, undershooting will occur. As in the case of the Dornbusch model, it is price stickiness that generates overshooting.

This model attempts to study the same issues as the Dornbusch model, but with a different assumption. It is presented in a stochastic way, assuming that all markets clear continuously.

The result seems to be that the overshooting effect is not very likely to occur. On the contrary, undershooting is the phenomenon which is likely to occur, since all markets would share the impact burden of adjustment caused by a shock to the system, in order to maintain equilibrium. To this extent, this paper supports the conclusions of the Dornbusch model. The way in which it differs is that this model allows for the possibility of overshooting. That being the case, it would not support the views of the Dornbusch model where overshooting is a consequence of price stickiness in the goods markets and instantaneous adjustment in asset markets.

I — The model

1 — The foreign exchange market

We shall begin by considering the case of a country that is small in the world capital market in such a way that it faces a given world interest rate, r_t^*. Perfect capital mobility is assumed between countries, so the domestic interest rate for the same period, r_t, must satisfy the interest rate parity condition [2]. Under the assumption of rational expectations, we can write the interest parity condition as

$$r_t = r_t^* + E\ e_{t+1} \mid I_t - e_t \quad \textbf{(1)}$$

where e_t is the logarithm of the spot exchange rate, measured as the domestic price of foreign currency at time t and $E\ e_{t+1} \mid I_t$ is the expected future spot exchange rate at time $t+1$ conditioned on the available information at

[2] On this issue, see, for instance, Krueger (1983).

time t. Equation (1) states that domestic interest rate, r_t, must equal the world interest rate, r_t^*, plus the expected rate of depreciation of the domestic currency, $E\, e_{t+1} \mid I_t - e_t$. This relation can also be identified as the condition for the balance of payments equilibrium, since deviations from (1) lead to infinite capital flows (3).

2 — The money market

We will assume a conventional demand for real balances depending on income and the domestic interest rate, which assumes the following form:

$$m_t^d - p_t = \phi\, y_t - \lambda\, r_t \tag{2}$$

where m_t^d, p_t and y_t are the logarithms of the money demand, the price of output (4) and income, respectively at time t, and, ϕ and λ are parameters representing the income and interest elasticities, respectively.

In general, it is extremely difficult to predict the future path of the money supply, so we will assume that the money supply follows a random walk (5)

$$m_t^s = m_{t-1} + \varrho_t \tag{3}$$

where m_t^s is the logarithm of the money supply at time t, m_{t-1} is the logarithm of the money supply in the previous period, and ϱ_t is an independently distributed random variable with zero mean. Under these assumptions, the expected changes in the money supply are simply the changes at the time at which the expection is formed, i. e.,

$$E(m_{t+i} \mid I_t) = m_t$$

and

$$E(\varrho_{t+i} \mid I_t) = 0$$

This assumption makes all monetary changes unanticipated and thus makes the model correspond to the Dornbusch (1976) treatment of a one time unanticipated monetary change. More complex stochastic processes could be used to describe the money supply without altering the qualitative conclusions drawn from the model.

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(3) This view is consistent with the monetary approach to the balance of payments.

(4) In (2) it is assumed, for simplicity, that the deflator for the money demand is the price of domestic output. An alternative of a weighted average of domestic and import prices, for instance, would not alter any qualitative results described below.

(5) The idea supporting this money supply process is that the monetary authority tries to follow a rule, which in this case is the rate of growth of the previous year, but it allows for a certain interval around that target, as it may not always be possible to achieve.

Money market equilibrium requires that the supply of money equals demand, that is, $m^d_t = m_t = m^s_t$. Therefore, the money market equilibrium condition can be obtained as follows:

$$m_{t-1} + \varrho_t - p_t = \phi\, y_t - \lambda\, r_t$$

which, rearranged, gives

$$r_t = (1/\lambda)\ [\phi\, y_t - (m_{t-1} + \varrho_t) + p_t] \qquad \textbf{(4)}$$

where r_t is the dometic interest rate that clears the money market.

3 — The goods market

We will assume a 2×2×2 model, where residents of the home country consume two goods, one of which is produced domestically and one of which is imported. The good which is domestically produced is also exportable.

The demand for domestic output combines domestic residents and foreign residents, since the domestic output is an exportable good. It depends on income, the domestic interest rate and the relative price of the domestic good, defined as $e_t + p^*_t - p_t$, where p^*_t is the logarithm of the foreign currency price of foreign goods at time t. The price p^*_t is given, since the home country is considered to be small in the world market for imports.

The demand for domestic output can be described as

$$y^d_t = \gamma\, y_t - \sigma\, r_t + \delta(e_t - p_t) + u_t \text{ (6)} \qquad \textbf{(5)}$$

where y^d_t is the logarithm of the demand for domestic output, γ and σ are parameters which represent the income elasticity and interest rate elasticity, respectively. The parameter δ captures the substitution of both domestic and foreign residents towards domestic goods as their relative price falls.

In reality, foreign income and interest rates should also influence the aggregate demand for domestic output by foreign residents, but due to the assumption of a small country, they are assumed to be exogeneous.

The supply of domestic output, y^s_t, is assumed to respond positively to unanticipated changes in the price of domestic output. The supply function, or Phillips curve, is assumed to have the form (7)

$$y^s_t = \bar{y}_t + \theta(p_t - E\ p_t \mid I_{t-1}) + \varepsilon_t \qquad \textbf{(6)}$$

where $\bar{y}_t$ is the «natural» level of domestic output, and $E\ p_t \mid I_{t-1}$ is the expected domestic price level at time t, conditioned on information available at time $t-1$. The coefficient θ is a parameter. It should be noted that (6) em-

(6) The complete relative price argument in (5) is $(e_t + p^*_t - p_t)$, but it is assumed that the foreing price level is equal to unity which implies that $p^*_t = 0$.

(7) This is what is often called a Sargent-Wallace supply function.

bodies the «natural rate hypothesis», which amounts merely to asserting that agents' decisions depend only on relative prices. It is apparent that increases in aggregate output occur because of deviations between the price level at time t and what it was expected to be, based on the available information at time $t-1$. The parameter θ measures the impact on aggregate output from that deviation. It is assumed to be positive and less than one.

Under such a hypothesis, if one is to explain why high inflation and high nominal aggregate demand seem to induce high aggregate output, it is necessary to explain (6) within an operational model of «money illusion», compatible with rational, optimizing behaviour.

Let us assume that there is an increase in aggregate demand which pushes up the price level. As information is not perfect, agents receive additional information about certain prices more than they do about others. As they do not have all the necessary information to compute perfectly all the relative prices they are concerned with, they make mistakes. For example, suppliers tend to misinterpret a general increase in all absolute prices as an increase in the relative price of the good they sell, and this in turn leads to an increase in their supply above their previously planned levels. Since, on average, everyone is making the same mistake, aggregate output rises above what it would have been. Workers also make mistakes, because of limited information misinterpreting the increase in the general price level —which is above what they had previously expected— as an increase of the relative price of the good they sell, i. e., labour. Since, on average, all workers make the same mistake, they make errors in computing the general price level which they have to know to determine their real wage and their supply of labour when they bargain and agree with firms for a nominal wage contract.

Firms will increase their demand for labour in the belief that they are paying a lower real wage and selling their output for a higher relative price, whereas workers wil increase their supply of labour believing that they will get a higher real wage, but as they are only guessing, they may end up working for a lower real wage. Agents are still rational, making the best use of the information they possess. The stochastic element of (6), ε_t, accounts for unpredictable movements in variables, such as the money supply, which make their decisions even more uncertain.

One important point to be noted is the fact that the information set for expenditure and portfolio decisions includes information up to time t. On the other hand, the information set for output (labour) supply decisions includes information only up to the period $t-1$. The first case is shown in (1), which assumes that the expected future spot exchange rate is based on information available at time t. The second case is shown in (6), where the expected value of the current general price level is based on the available information at time $t-1$ [8].

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[8] One can consider the aggregate supply function as being derived along the lines of Fischer (1977).

The idea is that at the start of each period workers and firms negotiate and sign contracts in which workers agree to supply a certain amount of labour at each nominal wage. Both workers and firms are concerned with the real wage, so the labour supply function facing firms will depend on the expected price at period *t*, but at the beginning of each period, when contracts are signed, agents only have information up to period $t-1$. Through period *t* firms know the current price of domestic goods and therefore they can choose the point on the labour supply function that maximizes profits. It is also during period *t* that agents make their expenditure and portfolio decisions. That being the case, the relevant expected future spot exchange rate for those decisions is based on information available at time *t*.

In other words, agents are assumed to have full current information when making asset market decisions, but not when making output (labour) supply decisions.

In this model, it is assumed that all markets clear continuously.

II — Solution of the model

The market clearing condition for the goods market is

$$y_t^d = y_t = y_t^s \quad \textbf{(7)}$$

Combining the two equations from the asset markets (1) and (4), we obtain

$$e_t = E\, e_{t+1} \mid I_t + r_t^* - (1/\lambda)\, [\phi\, y_t - (m_{t-1} + \varrho_t) + \varrho_t] \quad \textbf{(8)}$$

Substituting (8) into (5) and eliminating the spot exchange rate and the domestic interest rate, we have

$$y_t = (1/A)\, [C\, (m_{t-1} + \varrho_t) + \delta\, (E\, e_{t+1} \mid I_t + r_t^*) - B\, p_t + u_t] \quad \textbf{(9)}$$

where:

$A = 1 - \gamma + (\phi/\lambda)\, (\sigma + \delta)$;
$B = \delta + (1/\lambda)\, (\sigma + \delta)$; and
$C = (1/\lambda)\, (\sigma + \delta)$.

From the market clearing condition, equating (9) to the aggregate supply, (6), we have

$$p_t = [1/(A\theta + B)]\, [C\, (m_{t-1} + \varrho_t) - A\, \bar{y}_t + A\theta\, E\, p_t \mid I_{t-1} + \\ + \delta\, (E\, e_{t+1} \mid I_t + r_t^*) + u_t - A\, \varepsilon_t] \quad \textbf{(10)}$$

which is the equilibrium domestic price level at time *t*.

The equilibrium domestic output at time t can be obtained by substituting (10) into (6), yielding

$$y_t = [B/(A\theta + B)]\ [\bar{y}_t + \varepsilon_t - \theta E\ p_t \mid I_{t-1}] + [\theta/(A\theta + B)]\ [C\ (m_{t-1} + \varrho_t) + \\ + \delta\ (E\ e_{t+1} \mid I_t + r_t^*) + u_t] \qquad \textbf{(11)}$$

Substituting (10) and (11) into (4), we obtain

$$r_t = \frac{1}{\lambda\ (A\theta \cdot B)} \Big\{ (\phi B - A)\ (\bar{\xi}_\tau + \varepsilon_\tau - \theta E\ \pi_\tau \ /\ I_{t-1}) - [\theta\ (1-\gamma) + \delta]\ (m_{t-1} + \varrho_t) + \\ + (1 + \phi\theta)\ [\delta(E\ e_{t+1} \mid I_t + r_t^*) + u_t] + u_t \Big\} \qquad \textbf{(12)}$$

which represents the equilibrium domestic interest rate at time t.

Now using (12) in (1), we can obtain the equilibrium spot exchange rate at time t

$$e_t = D(E\ e_{t+1} \mid I_t + r_t^*) - \frac{1}{\lambda\ (A\theta + B)} \Big\{ (\phi B - A)\ (\bar{y}_t + \varepsilon_t - \theta E\ \varrho_t \ /\ I_{t-1}) - \\ - [\theta\ (1-\gamma) + \delta]\ (m_{t-1} + \varrho_\tau) + (1 + \phi\theta)\ u_t \Big\} \qquad \textbf{(13)}$$

where:

$$D = 1 - \frac{\delta(1 + \phi\theta)}{\lambda\,(A\theta + B)}$$

$$= \frac{\lambda[\theta(1-\gamma) + \delta] + (1 + \phi\theta)\sigma}{\lambda[\theta(1-\gamma) + \delta] + (1 + \phi\theta)\ (\sigma + \delta)}$$

It should be noted that $0 < D < 1$.

We have obtained all the equilibrium values for the endogeneous variables p_t, y_t, r_t and e_t, for the current period. Looking at equations (10), (11), (12) and (13), we note that the endogeneous variables are as functions of a set of exogeneous variables, including $E\ p_t \mid I_{t-1}$ as it is predetermined at time t. So, in equation (13), at time t, all the variables on the right hand side are predetermined.

We shall now need to obtain an expression for the future spot exchange rate. Taking expectations of (10) conditioned on information available at time $t-1$, we have

$$E(p_t \mid I_{t-1}) = [1/(A\theta + B)]\ E[C(m_{t-1} + \varrho_t) - A\ \bar{y}_t + A\theta\ E\ p_t \mid I_{t-1} + \qquad \textbf{(14)}$$

$$+ \delta(E\ e_{t+1} \mid I_t + r_t^*) + u_t - A\ \varepsilon_t] \mid I_{t-1}$$

As we know, by the law of iterated projections

$$E(E\ e_{t+1} \mid I_t) \mid I_{t-1} = E\ e_{t+1} \mid I_{t-1}$$

We also know that

$$E(u_t \mid I_{t-1}) = 0$$
$$E(\varepsilon_t \mid I_{t-1}) = 0$$

and

$$E(\varrho_t \mid I_{t-1} = 0$$

Expression (14) can be rearranged, yielding

$$E(p_t \mid I_{t-1}) = (1/B)\ E(C\ m_{t-1} - A\ \bar{y}_t + \delta\ r_t^*) \mid I_{t-1} + + (\delta/B)\ E\ e_{t+1} \mid I_{t-1} \tag{15}$$

The substitution of expression (15) into (13) yields

$$e_t = DE\ e_{t+1} \mid I_t + \frac{\delta\theta(\phi B - A)}{B\lambda(A\theta + B)} E\ e_{t+1} \mid I_{t-1} + v_t + E\ w_t \mid I_{t-1} \tag{16}$$

where:

$$v_t = D\ r_t^* - \frac{1}{\lambda(A\theta + B)} \Big\{ (\phi B - A)\ (\bar{y}_t + \varepsilon_t) - - [\theta(1 - \gamma) + \delta]\ (m_{t-1} + \varrho_t) + (1 + \phi\theta)\ u_t \Big\}$$

and

$$w_t = \frac{\theta\ (\phi B - A)}{B\lambda\ (A\theta + B)}\ [C m_{t-1} - A\ \bar{y}_t + \delta\ r_t^*]$$

If we now lead expression (16) one period and take expectations conditioned on the information availabe at time t, we have

$$E\ (e_{t+1} \mid I_t) = \pi\ E\ e_{t+2} \mid I_t + E\ \Omega_{t+1} \mid I_t \tag{17}$$

where:

$$E\ (E\ e_{t+2} \mid I_{t-1}) \mid I_t = E\ e_{t+2} \mid I_t$$

$$E\ (\Omega_{t+1} \mid I_t) = \left[D + \frac{\delta\theta\ (\phi B - A)}{B\lambda\ (A\theta + B)}\right] E\ r_{t+1}^* \mid I_t - \left[\frac{(\phi B - A)}{\lambda\ (A\theta + B)} + \frac{A\theta\ (\phi B - A)}{B\lambda\ (A\theta + B)}\right] E\ \bar{y}_t \mid I_t - - \frac{(\phi B - A)}{\lambda\ (A\theta + B)}\ E\ \varepsilon_{t+1} \mid I_t + \left[\frac{\theta\ (1 - \gamma) + \delta}{\lambda\ (A\theta + B)} + \frac{C\theta\ (\phi B - A)}{B\lambda\ (A\theta + B)}\right] E\, m_t \mid I_t + + \frac{\theta\ (1 - \gamma) + \delta}{\lambda\ (A\theta + B)}\ E\ \varrho_{t+1} \mid I_t - \frac{(1 + \phi\theta)}{\lambda\ (A\theta + B)} E\, u_{t+1} \mid I_t$$

$$E\ (E\Omega_{t+1} \mid I_{t-1}) \mid I_t = E\ \Omega_{t+1} \mid I_t$$

and

$$\pi = D + \frac{\delta\theta\ (\phi B - A)}{B\lambda\ (A\theta + B)}$$

$$= 1 - \frac{\delta}{(1 + \lambda)\ \delta + \sigma} \qquad (0 < \pi < 1)$$

Expression (17) is a first order difference equation that can be solved to yield the optimal path for the expected future spot exchange rate. The general solution of this equation (optimal path) is

$$E\,(e_{t+1} \mid I_t) = \sum_{i=0}^{\infty} \pi^i \, E\,\Omega_{t+i+1} \mid I_t \tag{18}$$

From (18), it can be seen that it is a stable path as shown in the appendix.

The interpretation that follows is that the spot exchange rate adjusts exponentially to its long-run value.

III — The efects of an (unanticipated) monetary expansion

We shall start by defining the unanticipated component of a variable as the difference between the actual value of a variable at time t, and what it was expected to be based on the available information at time $t-1$. The notation used will be:

$$x_t^u = x_t - E\,x_t \mid I_{t-1} \tag{19}$$

where x_t is any variable at time t and u stands for unanticipated. In this case, from expression (16) we have

$$\begin{aligned} e_t - E\,e_t \mid I_{t-1} = {} & D\,(E\,e_{t+1} \mid I_t - E\,e_{t+1} \mid I_{t-1}) + \frac{\delta\theta(\phi B - A)}{B\lambda(A\theta + B)}\,(E\,e_{t+1} \mid I_{t-1} - \\ & - E\,e_{t+1} \mid I_{t-1}) + D\,(r_t^* - E\,r_t^* \mid I_{t-1}) - \frac{1}{\lambda(A\theta + B)}\Big\{(\phi B - A)\,[(\bar{y}_t - E\,\bar{y}_t \mid I_{t-1}) + \\ & + (\varepsilon_t - E\,\varepsilon_t \mid I_{t-1})] - [\theta(1 - y) + \delta][(m_{t-1} + \varrho_t) - E\,(m_{t-1} + \varrho_t) \mid I_{t-1}] + \\ & + (1 + \phi\theta)\,(u_t - E\,u_t \mid I_{t-1})\Big\} + E\,w_t \mid I_{t-1} - E\,w_t \mid I_{t-1} \end{aligned} \tag{20}$$

As already explained, the money supply process follows a random walk, so from (20) we can see that

$$E\,(m_{t-1} + \varrho_t) \mid I_{t-1} = E\,m_{t-1} \mid I_{t-1} + E\,\varrho_t \mid I_{t-1} = m_{t-1},$$

and

$$m_{t-1} + \varrho_t - E\,(m_{t-1} + \varrho_t) \mid I_{t-1} = m_{t-1} + \varrho_t - m_{t-1} = \varrho_t \tag{21}$$

which is the unanticipated monetary component of the money supply function, defined as $\varrho_t = m_t^u$.

We also know that

$$E\,(\bar{y}_t \mid I_{t-1}) = \bar{y}_t$$
$$E\,(\varepsilon_t \mid I_{t-1}) = 0$$
$$E\,(u_t \mid I_{t-1}) = 0$$
$$E\,(r_t^* \mid I_{t-1}) = r_t^*$$

because r_t^* is an exogenenous variable determined outside the home country and assumed to be constant.

We can now rearrange expression (20) as

$$e_t - E\, e_t \mid I_{t-1} = D\,(E\, e_{t+1} \mid I_t - E\, e_{t+1} \mid I_{t-1}) - \\ - \frac{1}{\lambda\,(A\theta + B)} \{(\phi B - A)\, \varepsilon_t - [\theta\,(1-\gamma) + \delta]\, m_t^u + (1 + \phi\theta)\, u_t\} \qquad \textbf{(22)}$$

Next, we will determine the change in the expected future spot exchange rate between the two periods t and $t-1$, i. e.,

$$E\, e_{t+1} \mid I_t - E\, e_{t+1} \mid I_{t-1} \qquad \textbf{(23)}$$

To obtain expression (23), we shall make use of expression (18) above. That expression should also hold true if we change the information set which conditions expectations from time t to $t-1$, becoming

$$E\,(e_{t+1} \mid I_{t-1}) = \sum_{i=0}^{\infty} \pi^i\, E\, \Omega_{t+i+1} \mid I_{t-1} \qquad \textbf{(24)}$$

Another way to verify this would be to follow the same procedure as shown in the appendix for finding expression (18), the difference being that expectations would be conditioned on information available at time $t-1$. Therefore, the change in the expected future spot rate is given by the difference between expressions (18) and (24). Setting $i = 0$ in both of them, we have

$$E\,(e_{t+1} \mid I_t) = \left[D + \frac{\delta\theta\,(\phi B - A)}{B\lambda\,(A\theta + B)}\right] E\, r_{t+1}^* \mid I_t - \left[\frac{(\phi B - A)}{\lambda\,(A\theta + B)} + \frac{A\theta\,(\phi B - A)}{B\lambda\,(A\theta + B)}\right] E\, \bar{y}_t \mid I_t + \\ + \left[\frac{\theta\,(1-\gamma) + \delta}{\lambda\,(A\theta + B)} + \frac{C\theta\,(\phi B - A)}{B\lambda\,(A\theta + B)}\right] E\, m_t \mid I_t + \frac{\theta\,(1-\gamma) + \delta}{\lambda\,(A\theta + B)}\, E\, \varrho_{t+1} \mid I_t - \\ - \frac{(\phi B - A)}{\lambda\,(A\theta + B)}\, E\, \varepsilon_{t+1} \mid I_t - \frac{(1 + \phi\theta)}{\lambda\,(A\theta + B)}\, E\, u_{t+1} \mid I_t \qquad \textbf{(25)}$$

and

$$E\,(e_{t+1} \mid I_{t-1}) = \left[D + \frac{\delta\theta\,(\phi B - A)}{B\lambda\,(A\theta + B)}\right] E\, r_{t+1}^* \mid I_{t-1} - \left[\frac{(\phi B - A)}{\lambda\,(A\theta + B)} + \frac{A\theta\,(\phi B - A)}{B\lambda\,(A\theta + B)}\right] \times E\, \bar{y}_t \mid I_{t-1} + \\ + \left[\frac{\theta\,(1-\gamma) + \delta}{\lambda\,(A\theta + B)} + \frac{C\theta\,(\phi B - A)}{B\lambda\,(A\theta + B)}\right] E\, m_t \mid I_{t-1} + \frac{\theta(1-\gamma) + \delta}{\lambda\,(A\theta + B)}\, E\, \varrho_{t+1} \mid I_{t-1} - \\ - \frac{(\phi B - A)}{\lambda\,(A\theta + B)}\, E\, \varepsilon_{t+1} \mid I_{t-1} - \frac{(1 + \phi\theta)}{\lambda\,(A\theta + B)}\, E\, u_{t+1} \mid I_{t-1} \qquad \textbf{(26)}$$

respectively.

Since we know that ε, u and ϱ are random components, independently distributed with zero mean, we have

$$E(\varepsilon_{t+1} \mid I_t) = E(\varepsilon_{t+1} \mid I_{t-1}) = 0$$

$$E(u_{t+1} \mid I_t) = E(u_{t+1} \mid I_{t-1}) = 0$$

and

$$E(\varrho_{t+1} \mid I_t) = E(\varrho_{t+1} \mid I_{t-1}) = 0$$

We will also assume that there is no change in the «natural» level of output, so that $E(\bar{y}_t \mid I_t) = E(\bar{y}_t \mid (I_{t-1}) = \bar{y}_t$, and that, apart from the world interest rate being constant between the two periods, information from the outside travels very slowly to the home country. That is to say, there is a lack of updated information concerning variables determined outside the home country, in such a way that,

$$E(r^{*}_{t+1} \mid I_t) = E(r^{*}_{t+1} \mid I_{t-1})$$

We also know that substituting m_t by (3) in (25) and (26) yields

$$E(m_{t-1} + \varrho_t) \mid I_t — E(m_{t-1} + \varrho_t) \mid I_{t-1}) = m_{t-1} + \varrho_t — m_{t-1} = \varrho_t = m^u_t$$

We can now rewrite (23) as

$$E\ e_{t+1} \mid I_t — E\ e_{t+1} \mid I_{t-1} = \left[\frac{\theta(1-\gamma)+\delta}{\lambda(A\theta+B)} + \frac{C\theta(\phi B - A)}{B\lambda(A\theta+B)}\right] m^u_t \qquad \textbf{(27)}$$

The substitution of (27) into (22) yields.

$$e_t — E\ e_t \mid I_{t-1} = D\left[\frac{\theta(1-\gamma)+\delta}{\lambda(A\theta+B)} + \frac{C\theta(\phi B - A)}{B\lambda(A\theta+B)}\right] m^u_t — \frac{1}{\lambda(A\theta+B)}\Big\{ (\theta B - A)\varepsilon_t — \qquad \textbf{(28)}$$

$$—[\theta(1-\gamma)+\delta]\ m^u_t + (1+\phi\theta)\ u_t \Big\}$$

We can now analyse how unanticipated monetary changes will cause unanticipated changes in the spot exchange rate, by differentiating expression (28) with respect to m^u_t, which yields

$$\frac{\Delta e^u_t}{\Delta m^u_t} = D\left[\frac{\theta(1-\gamma)+\delta}{\lambda(A\theta+B)} + \frac{C\theta(\phi B - A)}{B\lambda(A\theta+B)}\right] + \frac{\theta(1-\gamma)+\delta}{\lambda(A\theta+B)} \qquad \textbf{(29)}$$

The interpretation of expression (29) is as follows: the unanticipated change in the exchange rate results from two sources. Firstly, from the change in expectations concerning the future spot exchange rate, reflected by the information received by agents and the way in which unanticipated changes in the money supply affect those expectations, that is

$$D\left[\frac{\theta(1-\gamma)+\delta}{\lambda(A\theta+B)}+\frac{C\theta(\phi B-A)}{B\lambda(A\theta+B)}\right] \tag{30}$$

and secondly, from the unanticipated change in the domestic money supply itself because of its effects on the level of economic activity at time t, given by

$$\frac{\theta(1-\gamma)+\delta}{\lambda(A\theta+B)} \tag{31}$$

Both effects reflect changes in information that agents receive between the period t and the period $t-1$ ([9]).

From expression (29), we are able to see that the exchange rate can overshoot or undershoot depending on whether

$$D\left[\frac{\theta(1-\gamma)+\delta}{\lambda(A\theta+B)}+\frac{C\theta(\phi B-A)}{B\lambda(A\theta+B)}\right]+\frac{\theta(1-\gamma)+\delta}{\lambda(A\theta+B)} \gtreqless 1 \tag{32}$$

It should be noted that the criteria for over-or undershooting used here is very much the same as Dornbusch (1976) obtains in his appendix where he introduces a Phillips curve. The difference between the Dornbusch model and this one is that the Dornbusch framework maintains the assumption of sticky prices, whereas here, flexible prices and market clearing are assumed.

From expression (32), it is very difficult to analyse the conditions under which over-or undershooting will occur and what determines either effect. We believe that undershooting is more likely to occur because all markets will bear the initial impact burden of adjustment.

Nevertheless, one cannot exclude the possibility of overshooting and that being the case, it would show that the overshooting effect is not necessarily a consequence of a disequilibrium caused by price stickiness. In this case, it would mean that the exchange rate might have to overshoot, or for that matter, undershoot in order to clear all the markets. It is not a consequence of the failure of the goods market to clear, but a necessity for the maintenance of equilibrium.

APPENDIX

Stability of the optimal path

$$E(\Omega_{t+1} \mid I_t)=\left[D+\frac{\delta\theta(\phi B-A)}{B\lambda(A\theta+B)}\right]E\, r^*_{t+1} \mid I_t-\left[\frac{(\phi B-A)}{\lambda(A\theta+B)}+\frac{A\theta(\phi B-A)}{B\lambda(A\theta+B)}\right]E\,\bar{y}_t \mid I_t+ \tag{A21}$$

$$+\left[\frac{\theta[(1-\gamma)+\delta}{\lambda(A\theta+B)}+\frac{C\theta(\phi B-A)}{B\lambda(A\theta+B)}\right]E(m_{t-1}+\varrho_t) \mid I_t+\frac{\theta(1-\gamma)+\delta}{\lambda(A\theta+B)}E\,\varrho_{t+1} \mid I_t-$$

$$-\frac{(\phi B-A)}{\lambda(A\theta+B)}E\,\varepsilon_{t+1} \mid I_t-\frac{(1+\phi\theta)}{\lambda(A\theta+B)}E\,u_{t+1} \mid I_t$$

([9]) See Frenkel (1981-b) for a discussion on the role of new information in determining unexpected changes in the exchange rate.

It should be noted that $m_{t-1} + \varrho_t = m_t$, but it was substituted by the money supply process as described by expression (3).

We also know that

$$E(m_{t-1} \mid I_t) = m_{t-1}$$

and

$$E(\varrho_{t+1} \mid I_t) = 0$$

Substituting successively forwards for $E\ e_{t+2} \mid I_t,\ E\ e_{t+3} \mid I_t$ into (A20), we obtain

$$E(e_{t+2} \mid I_t) = E\ \Omega_{t+2} \mid I_t + \pi\ E\ e_{t+3} \mid I_t \qquad \textbf{(A22)}$$

$$E(e_{t+1} \mid I_t) = E\ \Omega_{t+1} \mid I_t + \pi\ E\ \Omega_{t+2} \mid I_t + \pi^2\ E\ e_{t+3} \mid I_t$$

$$E(e_{t+3} \mid I_t) = E\ \Omega_{t+3} \mid I_t + \pi\ E\ e_{t+4} \mid I_t$$

$$E(e_{t+1} \mid I_t) = E\ \Omega_{t+1} \mid I_t + \pi\ E\ \Omega_{t+2} \mid I_t + \pi^2\ E\ \Omega_{t+3} \mid I_t + \pi^3 E\ e_{t+4} \mid I_t$$

Expression (A22) can be generalised in the following way:

$$E(e_{t+1} \mid I_t) = \pi^N\ E\ e_{t+N+1} \mid I_t + \sum_{i=0}^{\infty} \pi^i\ E\ \Omega_{t+i+1} \mid I_t \qquad \textbf{(A23)}$$

It is now important to find out whether the optimal path is stable or not.

Let $N \rightarrow \infty$, so that $E e_{t+N+1} \mid I_t) \rightarrow 0$. Since $0 < \pi < 1$, as $N \rightarrow \infty$, we see that $\pi \rightarrow$ $\rightarrow 0$, which implies

$$\pi^N\ E\ e_{t+N+1} \mid I_t \rightarrow 0$$

As this is the case, expression (A23) becomes:

$$E(e_{t+1} \mid I_t = \sum_{i=0}^{\infty} \pi^i\ E\ \Omega_{t+i+1} \mid I_t \qquad \textbf{(A24)}$$

which is a stable path

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