PERSISTENCE IN PORTUGUESE ECONOMIC ACTIVITY (*)

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1 — Introduction

The evaluation of the stochastic memory of macroeconomic time series has recently become an attractive topic in economics. The «unit-root literature» in particular has singled out the theoretical and practical importance of memory evaluation for the study of the long-run implications of political economic measures. Diebold and Nerlove (1990) present a comprehensive and updated survey and Rothman and Crato (1991) discuss the Portuguese case introducing some new procedures.

The claim of long-memory on macroeconomic variables goes further than the simple claim of a stochastic dependence. It states that correlations between changes of the stationarized series die out very slowly, in a sense made precise below; the actual movements are stochastically influenced by the recent to the most remote past.

That claim was suggested by the seminal work of Mandelbrot (1972) on the fractional Brownian motion and on the celebrated R/S analysis. Several researchers used the tools developed by Mandelbrot and by means of R/S analysis arrived at the surprising conclusion that some economic time series, namely economic output, had a long-memory behavior.

Recently, Lo (1991) and Haubrich and Lo (1989) showed that the R/S statistic was biased and unable to distinguish, in finite samples, short- from long-memory characteristics of a time series. Lo developed a modified R/S statistic that overcomes those problems and Haubrich and Lo applied it to US GNP rates. Their conclusions contradicted previous research and imputed the results of preceding R/S analysis to the sole existence of short-memory in GNP rates of increase.

This paper presents a different approach. Instead of using a non-parametric test as before, a class of discrete-time long-memory models is fitted to the data, and the compatibility of the fitted parameters with a short-memory assumption is tested. The class of models used is the autoregressive frac-

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tionally integrated moving average (ARFIMA), a generalization, introduced by Granger and Joyeux (1980) and by Hosking (1981), of the well-known ARIMA models. The estimation procedures used here are the new exact maximum likelihood approach of Sowell (1992) and a spectral regression method developed by Geweke and Porter-Hudak (1983). This way, the problem is readdressed from a new and more pragmatic angle; the question is whether or not the large class of linear long-memory models that generalizes the ARIMA processes needs a «long-memory parameter» to better fit the data.

The plan of the paper is as follows. Section 2 defines the concepts of short- and long-memory of a random process. Section 3 presents a fractional model with long-memory behavior, the ARFIMA model, and describes the available estimation procedures for its parameters. Section 4 presents the data and discusses the results of the estimations. Section 5 concludes.

2 — Time series memory

Let the time series under consideration be represented by (X_t). In what follows it is assumed that the time series are of *second order,* i. e., that their second moments EX_t^2 is finite for every t. It is also assumed that the time series, directly or after some transformation, are *stationary,* that is, their first and second moments are independent of t: EX_t is constant and Cov $[X_t, X_{t+h}] = :\gamma(h)$, is a function of the lag h alone. Also, (ε_t) will denote a zero mean *white noise* process with variance $\sigma^2 < \infty$; in symbols $\varepsilon_t \sim WN(0,\sigma^2)$.

Time series studied in economics reflect a characteristic behavior of economic variables; their autocovariances decay to zero reflecting the fact that the influence of the past values decreases with the lags under consideration. The speed of that decay is a measure of the internal memory of the random event.

If a white noise is the appropriate model, then the random event is said to have *no memory.* It can be said that such an extreme case seldom occurs in practice, but such a model is a «building block» for more complex models.

An example of model with *short memory* is the so-called auto regressive moving average (ARMA). The rationale is that the decay of the autocorrelations of an ARMA is geometrically bounded, i. e., there exist constants $C > 0$ and r in the open interval $]0,1[$ such that $|\gamma(h)| \leq Cr^{|h|}$. Hence, the sequence $(\gamma(h))$ is absolutely summable.

In contrast, long-memory models have autocorrelations that decay much more slowly, following asymptotically an hyperbolic decay. More precisely, a second-order stationary process (X_t) would be termed *long-memory* if, for some

$C > 0$, and $\alpha < 0$, its autocovariance function has the following asymptotic behavior [1].

$$|\gamma(h)| \sim C|h|^{\alpha} \quad \text{as} \quad h \to \infty \tag{1}$$

If, in addition, $\alpha > -1$ and so $\Sigma|\gamma(h)| = \infty$, it will be said that (X_t) is *persistent.*

The first economic studies on long-range dependence used R/S analysis and tried to detect the existence of long-memory in financial time series, but a number of objections have subsequently been raised. First, the distribution of the R/S statistic is not well known. Second, as Lo (1991) proved, in finite samples the R/S analysis is not capable of discriminating between long-range dependence and sort-range autoregressive models.

A modified R/S statistic was proposed by Lo (1991) in order to overcome these problems. The basic idea is to construct a statistic that is invariant over a general class of short-memory processes but that is sensitive to the presence of long-memory. But this modified R/S statistic also relies solely on the asymptotic theory, and we remain in a fog when using real data.

An alternative approach is a parametric analysis with a class of long--memory models.

3 — Fractional ARIMA models for economic time series

A very general class of long-memory and persistent processes is given by the fractionally differenced noise and by the fractionally differenced ARIMA, models that were introduced independently by Granger and Joyeux (1980) and by Hosking (1981). These models have proved valuable tools in various areas of economic theory, see, e. g., Diebold and Rudebush (1989), and in economic forecasting, see, e. g.,+ Geweke and Porter Hudak (1983) and Crato (1991b).

In order to introduce the models, the notation will follow closely Crato (1991a), where a more detailed development can be found; a basic reference for these models is Brockwell and Davis (1991), section 13.2.

The symbols B and ∇ will represent the backwards and the differencing operators, respectively, i. e., $BX_t := X_{t-1}$ and $\nabla X_t := (1 - B)X_t = X_t - X_{t-1}$. Integer powers of these operators are defined in the usual way, i. e., $B^n = B.B^{n-1}$ and $\nabla^n = \nabla.\nabla^{n-1}$.

(1) The symbol $\sim$ is used in the standard way: $a_n \sim cb_n$ where $c \neq 0$ iff $a_n/b_n \to c$.

If d is any real number $d > -1$ the *fractional difference operator* ∇^d is defined as

$$\nabla^d = (1 - B)^d := \sum_{k=0}^{\infty} \binom{d}{k} (-B)^k, \tag{2}$$

where

$$\binom{d}{k} = \frac{d}{k} \frac{d-1}{k-1} \cdots \frac{d-k+1}{1}. \tag{3}$$

When d is a positive integer the sum is reduced to the terms $k = 1, \ldots, d$, since then, for $k > d$, $\binom{d}{k} = 0$. When the operator is applied to a random process, the sum of the series is to be understood as a mean-square limit.

A process (X_t) is called *fractionally differenced noise* or *fractional noise,* with $d \in]-.5, .5[$ if

$$\nabla^d X_t = \varepsilon_t, \quad \text{with } \varepsilon_t \sim WN(0, \sigma^2) \tag{4}$$

Equation (4) provides an autoregressive representation of the process

$$\sum_{k=0}^{\infty} \pi_k X_{t-k} = \varepsilon_t. \tag{5}$$

with $\pi_k = \binom{d}{k}(-1)^k$. Explicitly

$$X_t - dX_{t-1} + \frac{d(d-1)}{2} X_{t-2} - \frac{d(d-1)(d-2)}{6} X_{t-3} + \ldots = \varepsilon_t.$$

By the laws of exponents,

$$Xt = \nabla^{-d} \varepsilon_t. \tag{6}$$

This expression can be rewritten as

$$X_t - \varepsilon_t + d\varepsilon_{t-1} + \frac{d(d-1)}{2} \varepsilon_{t-2} + \frac{d(d-1)(d-2)}{6} \varepsilon_{t-3} + \ldots$$

The natural generalization of the fractional noise is the fractional integrated ARMA model, i. e., an ARIMA where the order of integration is the non-integer $d \in]-.5, .5[$. In this way, the relatively small flexibility existent in fractional noise, controlled only by the two parameters σ^2 and d, is increased by the $p + q$ parameters of the autoregressive and moving average polynomials.

An autoregressive fractionally integrated moving-average process, ARFIMA (p, d, q) with p, q nonnegative integers and d in the open interval $]-.5, .5[$, is a stationary ARMA driven by fractional noise, i. e, (X_t) is an ARFIMA if it is a stationary solution of the difference equation

$$\phi(B)X_t = \theta(B)\xi_t, \quad \text{with } \xi_t := \nabla^{-d}\varepsilon_t, \ \varepsilon_t \sim WN(0, \sigma^2) \tag{7}$$

where $\phi(B)$ and $\theta(B)$ are lag polynomials of order p and q, respectively. Explicitly,

$$X_t - \phi_1 X_{t-1} - \ldots - \phi_p X_{t-p} = \xi_t + \theta_1 \xi_{t-1} + \ldots + \theta_q \xi_{t-q} \tag{8}$$

with

$$\xi_t = \varepsilon_t + d\varepsilon_{t-1} + \frac{d(d-1)}{2}\varepsilon_{t-2} + \frac{d(d-1)(d-2)}{6}\varepsilon_{t-3} + \ldots$$

It can be proved that if (X_t) is a fractional noise or, more generally, an ARFIMA process, then its autocorrelation function has the following asymptotic expression.

$$\gamma(h) \sim Ch^{2d-1}, \quad \text{as} \quad h \to \infty. \tag{9}$$

Hence, for $d \neq 0$, it is a long-memory process, for $-.5 < d < 0$ it is antipersistent, and for $0 < d < .5$ it is persistent. For details, again, see Brockwell and Davis (1991, pp. 520-534).

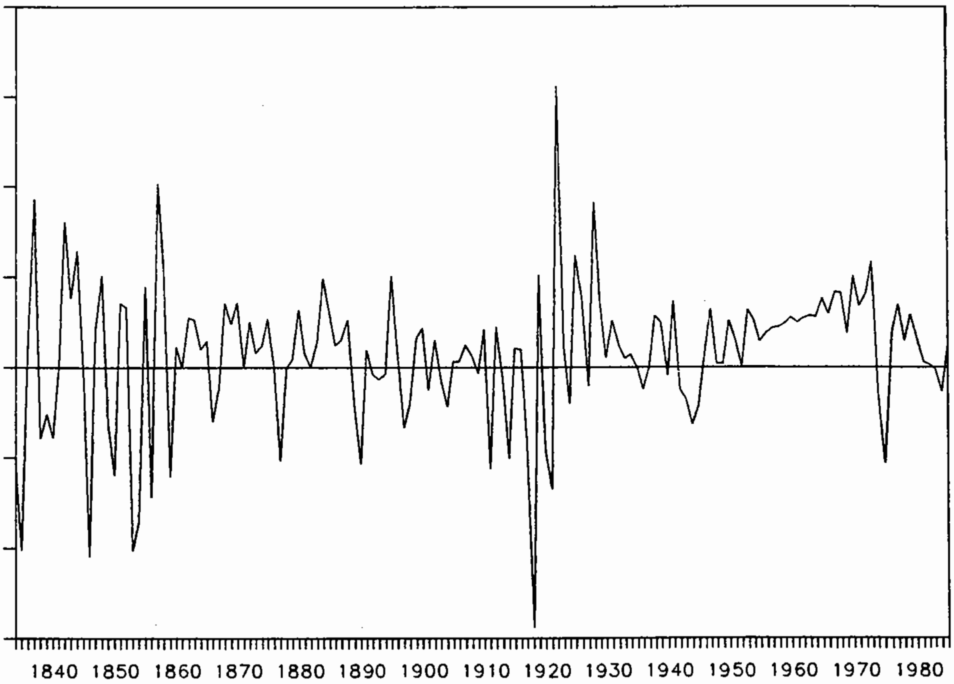
4 — Portuguese data

The data that will be considered represents various aspects of the Portuguese economy: domestic output, financial market returns, and inflation. Other series could be chosen; this selection was restricted not only by the relevance of the figures but also by the availability of long data sets.

The first series, hereafter called *GDP/Capita,* is computed from Nunes, Mata and Valério (1989), and consists of the annual rates of growth of real Gross Domestic Product, per capita, from 1834 to 1985 (152 observations). As far as we know it is the longest Portuguese output series available. Its study with ARFIMA models was done elsewhere (Rothman and Crato 1991), but with a different purpose.

FIGURE 1

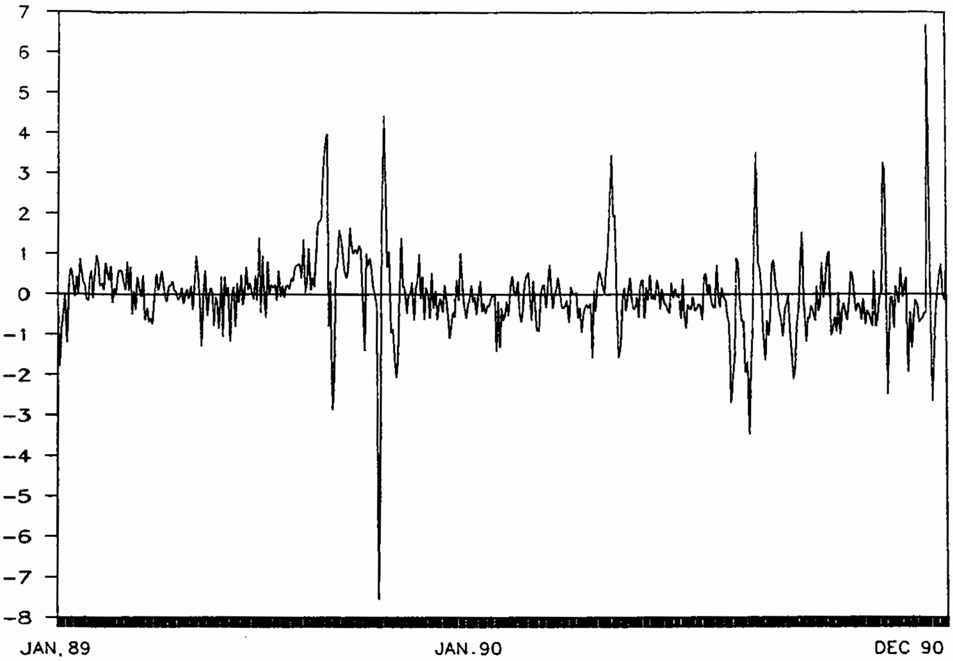
GDP/Capita growth rates



The second series, here called *BTA Returns,* consists of the daily returns given by the aggregated stock price index of Lisbon Stock Exchange, as published by the Banco Totta & Açores. The returns were computed as the first difference of the logarithm of the series. Data ranges from the first transaction day of 1989 (January 3) to the last of 1990 (December 31). It includes 475 observations from a period where the financial market was relatively stabilized, after the turbulent years of 1986 to 1988 where it is well known that price movements were not behaving according to the predictions of the efficient market hypothesis (see, e. g., Crato and Lopes 1989).

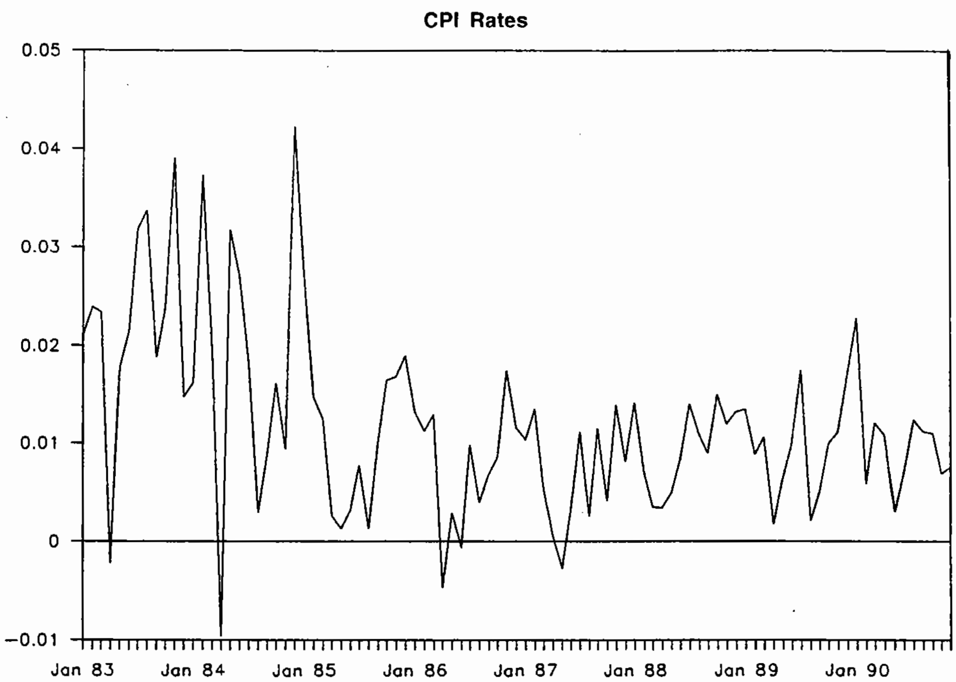
FIGURE 2

BTA Returns



The third series, called *CPI Rates,* includes the monthly inflation computed as the first difference of the logarithms of the Portuguese consumer price index (total mainland, excluding housing) published by the Instituto Nacional de Estatística. The original series had, as basis, average 1983 prices and was built by merging the two series available for the period January 1983 to December 1990. Details about the construction of the series can be found in Crato (1990).

FIGURE 3



The graphs of all series show a type of nonperiodic cycle that can be interpreted as a sign of long-memory (see Mandelbrot 1972). Relatively long periods with high values of the rates are followed by periods of low values. In some phases the assumption of stationarity can even be questioned. The whole point of the long-memory models is that they provide an explanation for the nonperiodic cycles without relaxing the stationarity assumption, so further differencing is not required.

Two estimation procedures were used: a maximum likelihood approach and a spectral regression method.

The maximum likelihood Gaussian estimation used the program GQSTFRAC of Fallaw Sowell and estimated 16 alternative ARFIMA models, with $p = 0, 1, 2, 3$ and $q = 0,1, 2, 3$. In order to select what could be reasonable ARFIMA models, appropriate to each series, AIC and SIC criteria were used. These are common selection criteria used in today's time series analysis. They measure the goodness of fit of the model to the data and subtract a penalization that increases with the number of parameters of the model. AIC stands for Akaike Information Criteria, and SIC is the Bayesian criterion of Schwarz. For details the reader is referred to de Gooijer *et al.* (1985).

Next, all selected models were inspected and the estimated values for the differenced parameter d were tested. The standard t-test for the nullity of the parameter was performed using the estimated standard deviation of the estimate for d. The non-rejection of the null hypothesis $d = 0$ leads to the conclusion of short-memory in the series and the rejection leads to the existence of long-memory. The results are presented on table I. Estimations were also performed on a broader set of ARFIMA models, using the approximate maximum likelihood estimation program LONGMEM, from the new version of the package ITSM of Brockwell and Davis. The results obtained confirmed the selected models on table I.

The second procedure was the spectral estimation technique of Geweke and Porter-Hudak (1983). These authors suggested a regression over the logarithm of the periodogram at low frequencies, where its slope is essentially dependent on the long-memory parameter d. The rationale lies in the form of the spectral density $f(\lambda)$ of an ARFIMA process near zero,

$$f(\lambda) \sim C|1 - e^{-i\lambda}|^{-2d} \quad \text{as} \quad \lambda \to 0 \qquad \textbf{(10)}$$

Geweke and Porter-Hudak argued that their regression estimator could capture the long-memory characteristic of the process, without being «contaminated» in the estimation by the other $p + q$ parameters of the ARFIMA model. Using the periodogram as an estimator of the spectral density, d can be estimated by regression. As a truncation parameter m that would choose the low Fourier frequencies to be considered, simulations by Geweke and Porter-Hudak suggest the use of m around $n^{.55}$, where n is the number of observations. It has recently been proved that this estimate is consistent for a range of values larger than the inicially considered (Crato 1992).

The results of the spectral method are presented on table II. Again, the test on the nullity of the parameter d is performed as an usual t-test, using the standard deviation given by the regression.

An inspection of the two tables shows immediately that the null hypothesis of short-memory is difficult to reject in all series.

For the GDP/Capita the maximum likelihood estimation gives two competitive models with small $\hat{d}$ and very small corresponding t-values. But the spectral approach leads to significant estimates of d in two out the of the three values for α.

The results are much more clear in the series of BTA Returns. The model chosen by AIC gives a value of $\hat{d}$ that is different from zero at a significance of 5% or higher. But the model chosen by SIC gives a similar estimate for d that is significant at a level of 5%, and all estimates from the spectral method give similar estimates and all highly significant.

For CPI Rates the significance of $\hat{d}$ is even higher. Both AIC and SIC chose the fractional noise model and give estimates for d that are statistically significant at any reasonable significance level. The spectral regression leads to similar values of the differencing parameter and the p-values of the corresponding nullity tests are between .10 and .01.

These results are very clear and support the claim of the existence of long-memory in the series under consideration.

5 — Conclusions

A broad class of time series models capable of reproducing long-memory behavior was fitted to some relatively long series fo relevant Portuguese economic variables. The time series models that were used are the autoregressive fractionally differenced moving average models, ARFIMA.

On these ARFIMA models applied to the growth rate series, the hypothesis of nullity of the differencing paremeter d corresponds to the hypothesis of nonexistence of long-memory. That hypothesis was tested in the series and, in general, the tests supported the existence of long-memory in the rates of change. Further, the estimates for d were all positive, thus corresponding to persistent long-memory models.

These results suggest that long-memory models can provide competitive tools to explain the evolution of some Portuguese macroeconomic variables and to test important hypothesis about economic growth. Some partial results with aggregate output have already been presented out in Rothman and Crato (1991), but it is clear that much more research is needed. In particular we would suggest research in different periods and with longer series, and the test of usefulness of the models with out-of-sample forecasting. A comparison of in- and out-of-sample fit with short-memory ARIMA models would also be relevant. The main contribution of this paper is that these new lines of research may be fruitful.

TABLE I

Results of the maximum likelihood estimation

(Models chosen by AIC and SIC criteria, estimated *d* and corresponding *t*-values

Series	AIC	SIC
GDP/Capita	(2,*d*,1) $\hat{d}$ = .12 *t* = .11	(1,*d*,1) $\hat{d}$ = .03 *t* = .07
BTA Returns	(2,*d*,3) $\hat{d}$ = .22 *t* = 1.81	(2,*d*,1) $\hat{d}$ = .25 *t* = 2.18
CPI Rates	(0,*d*,0) $\hat{d}$ = .28 *t* = 3.70	(0,*d*,0) $\hat{d}$ = .28 *t* = 3.70

TABLE II

Results of the estimation by regression over the log-periodogram

(Estimated *d* and *t*-values for different *m* low-frequency ordinates used, with $m = [n^{\alpha}]$)

Series	α = .50	α = .55	α = .60
GDP/Capita	$\hat{d}$ = —.30 *t* = 1.30	$\hat{d}$ = .48 *t* = 2.09	$\hat{d}$ = .37 *t* = 2.05
BTA Returns	$\hat{d}$ = .33 *t* = 3.14	$\hat{d}$ = .29 *t* = 2.81	$\hat{d}$ = .23 *t* = 2.51
CPI Rates	$\hat{d}$ = .40 *t* = 1.64	$\hat{d}$ = .48 *t* = 1.94	$\hat{d}$ = .56 *t* = 2.98

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