MASTER
ACTUARIAL SCIENCE

MASTER'S FINAL WORK
INTERNSHIP REPORT

CREDIBILITY MODELS APPLIED TO WORKER'S COMPENSATION INSURANCE

FILIPE ANDRÉ CAROLINO MOURA

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LISBOA SCHOOL OF ECONOMICS & MANAGEMENT
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To all my friends, for all the good moments spent together.

And to my parents, for the investment made in my education and for all the support, through the good and the bad times.

Abstract

Credibility Theory provides a way to rate a risk in insurance, making use of both the individual and collective claim experience to find the pure premium. It has been in development for almost a century now, and is an important tool that every actuary should know.

From the many Credibility Models available to use, we chose the Bühlmann-Straub model, widely regarded as the most important one in insurance, and the Hierarchical Model, that expands upon the Bühlmann-Straub model by implementing an hierarchical structure in the model. We apply these to the Worker's Compensation line of business, to determine what the best approaches to each model are.

Keywords: Credibility Theory, Bühlmann-Straub Model, Hierarchical Credibility, Worker's Compensation.

Resumo

A Teoria da Credibilidade providencia uma maneira de tarifar um risco no ramo segurador, fazendo para isso uso do histórico de sinistros, tanto individual como colectivo, para encontrar o prémio puro. Tem estado em desenvolvimento durante aproximadamente um século e é uma ferramenta importante que qualquer actuário deverá saber utilizar.

Dos vários modelos de Credibilidade disponíveis, escolhemos o Modelo Bühlmann-Straub, amplamente reconhecido como o mais importante do ramo segurador, e o Modelo Hierárquico, que expande o Modelo Bühlmann-Straub através da implementação de uma estrutura hierárquica. Estes são aplicados à linha de negócio Acidentes de Trabalho, para determinar quais as melhores abordagens a cada um dos modelos.

Palavras-chave: Teoria da Credibilidade, Modelo Bühlmann-Straub, Credibilidade Hierárquica, Acidentes de Trabalho.

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1. Introduction

The concept of insurance is to create a way for a group of individuals, exposed to one same risk, to transfer this risk to an insurance company, in exchange of a risk premium. This premium (not counting necessary loadings) will have to be at least equal to the aggregate claim average amount given by this risk. If this group of individuals, or risk class, is homogeneous, then the premium can be distributed equally among them.

However, because of the great amount and variety of factors that influence this risk, it isn't possible to have truly homogeneous risk classes. Therefore, if the individuals aren't equal, they shouldn't pay the same premium. If all individuals pay the same premium, that would pose a problem, as this would attract bad risks (who would be underpaying) and drive away good ones (who would otherwise be overpaying).

The idea, then, is to rate individual risks through both their individual claims experience and the collective's claim experience, to get the fairest individual premiums. This is of major interest to an insurance company, as it allows increasing both the volume of business and competitiveness. For that, we could use Credibility Theory.

Credibility Theory is the branch of actuarial science that studies how much an individual risk's experience inside a portfolio, alongside the collective's risk experience, should contribute to the calculation of the insurance pure premium. It does so through analysis of the individual's claim history.

This report will focus on the application of Credibility Models to a specific line of business in insurance, in this case, Worker's Compensation.

Worker's Compensation is a Property and Casualty line of business in insurance. It entitles the worker to a benefit, should the worker have an accident at work. It's a group insurance, in which the policyholder is the employer, the insured person is the employee, and each policy may have many insured persons.

In Portugal, it has been mandatory for the employer to compensate his employees for work accidents since 1913 (Law no. 83/1913, July 24th). The former is obliged by law to purchase Worker's Compensation insurance, to assure the employee and his family the necessary conditions to repair whatever damage that comes out from a work accident. By the Portuguese legislation (Law no. 98/2009, September 4th), a work accident is one that occurs at the workplace and during working time and that results in any kind of injury, disability or disease that affects the work or earning capacity, or in death. The definition of work place and time also includes, for instance, the lunch break and the trip from home to work and vice-versa.

There are two types of benefit: in kind and in cash. The benefits in kind include all those necessary to restore the health and the work and earning capacity of the beneficiary, for instance, medical assistance, surgical intervention, medication, professional rehabilitation, and so on. The cash benefits contemplate temporary allowances, subsidies and pensions provided by law. In case of death, the benefits (death subsidy and pension) will be paid to the insured's family.

The next chapters will cover the Credibility Models to be applied to Worker's Compensation. These will be the Bühlmann-Straub Model and the Hierarchical Model.

2. Bühlmann-Straub Credibility Model

This section follows closely the book by Bühlmann & Gisler (2005) and it's given without proofs. That is out of the scope of this report. For proofs, one should check the above reference.

2.1. The model

The Bühlmann-Straub model is the most important and used model in Credibility for insurance practice.

When we want to find individual premiums for a certain line of business, this line is distributed into more or less equal risk classes and is then rated. Consider a portfolio composed of I risks. For each risk $i = 1, \dots, I$, there's usually information available about S_{ij}, a random variable representing the aggregate claim amount for risk i in year j, and w_{ij}, an associated known weight for risk i in year j. From these, we get $X_{ij} = S_{ij}/w_{ij}$, which is the average claim cost (loss ratio) for risk i in year j.

The goal is to get a credibility estimator for the average claim cost for each individual policy, which is to say, we're getting a pure rate for each policy in terms of the associated known weight. The Bühlmann-Straub's model assumptions are the following:

1. *Conditionally, given Θ_i (the individual risk profile) , the $\{X_{ij}: j = 1, 2, \dots, n\}$ are independent with $E\left[X_{ij}|\Theta_i\right] = \mu(\Theta_i)$ and $\operatorname{Var}\left[X_{ij}|\Theta_i\right] = \sigma^2(\Theta_i)/w_{ij}$,*

2. *The pairs $(\Theta_1, X_1), (\Theta_2, X_2), \ldots$ are independent, and $\Theta_1, \Theta_2, \ldots$ are independent and identically distributed random variables. This means that the risks are independent and a priori equal.*

The credibility estimator will be given by:

$$\widehat{\widehat{\mu(\Theta_i)}} = \alpha_i X_i + (1 - \alpha_i)\mu_0, \tag{1}$$

with

$$X_i = \sum_j \frac{w_{ij}}{w_{i\bullet}} X_{ij},$$

$$w_{i\bullet} = \sum_j w_{ij},$$

$$\alpha_i = \frac{w_{i\bullet}}{w_{i\bullet} + \kappa} = \frac{w_{i\bullet}}{w_{i\bullet} + \frac{\sigma^2}{\tau^2}}, \tag{2}$$

$$\mu_0 = E[\mu(\Theta_i)], \tag{3}$$

$$\sigma^2 = E[\sigma^2(\Theta_i)], \tag{4}$$

$$\tau^2 = \mathrm{Var}[\mu(\Theta_i)]. \tag{5}$$

The overall expected value μ_0 in (1) is, however, usually unknown. To get the homogeneous credibility estimator, defined as $\widehat{\widehat{\mu(\Theta_i)}}^{hom}$, we replace the former with the collectively unbiased estimator $\widehat{\widehat{\mu_0}}$:

$$\widehat{\widehat{\mu(\Theta_i)}}^{hom} = \alpha_i X_i + (1 - \alpha_i)\widehat{\widehat{\mu_0}}, \tag{6}$$

$$\widehat{\widehat{\mu_0}} = \sum_{i=1}^{I} \frac{\alpha_i}{\alpha_\bullet} X_i, \tag{7}$$

$$\alpha_\bullet = \sum_i \alpha_i.$$

It's also needed to estimate the structural parameters $\sigma^2$and τ^2, since these two parameters are unknown. Reasonable choices are:

$$\hat{\sigma}^2 = \frac{1}{I}\sum_{i=1}^{I}\frac{1}{n-1}\sum_{j=1}^{n} w_{ij}(X_{ij} - X_i)^2, \tag{8}$$

And

$$\hat{\tau}^2 = \max\left(\hat{\hat{\tau}}^2, 0\right), \tag{9}$$

$$\hat{\hat{\tau}}^2 = c \cdot \left\{\frac{I}{I-1}\sum_{i=1}^{I}\frac{w_{i\bullet}}{w_{\bullet\bullet}}(X_i - \bar{X})^2 - \frac{I\hat{\sigma}^2}{w_{\bullet\bullet}}\right\}, \tag{10}$$

$$c = \frac{I-1}{I}\left\{\sum_{i=1}^{I}\frac{w_{i\bullet}}{w_{\bullet\bullet}}\left(1 - \frac{w_{i\bullet}}{w_{\bullet\bullet}}\right)\right\}^{-1}, \tag{11}$$

$$\bar{X} = \sum\nolimits_i \frac{w_{i\bullet}}{w_{\bullet\bullet}} X_i,$$

$$w_{\bullet\bullet} = \sum\nolimits_i w_{i\bullet},$$

Both estimators for the structural parameters are unbiased and consistent. If $\hat{\hat{\tau}}^2$ is negative, we set $\hat{\tau}^2 = 0$. That means that there are no significant differences in the expected value between the risks.

With these, it's now possible to get the empirical credibility estimator, which follows from the homogeneous credibility estimator, but with the above estimators incorporated in the formula:

$$\widehat{\widehat{\mu(\Theta_i)}}^{emp} = \hat{\alpha}_i X_i + (1 - \hat{\alpha}_i)\widehat{\widehat{\mu_0}}, \tag{12}$$

$$\hat{\alpha}_i = \frac{w_{i\bullet}}{w_{i\bullet} + \hat{\kappa}} = \frac{w_{i\bullet}}{w_{i\bullet} + \frac{\hat{\sigma}^2}{\hat{\tau}^2}}, \tag{13}$$

$$\widehat{\widehat{\mu_0}} = \sum_{i=1}^{I}\frac{\hat{\alpha}_i}{\hat{\alpha}_\bullet} X_i. \tag{14}$$

If $\hat{\tau}^2$ takes the value of 0, then $\hat{\alpha}_i$ will also be 0, and $\widehat{\widehat{\mu_0}}$ will simply be represented by $\widehat{\widehat{\mu_0}} = \sum_i (w_{i\bullet}/w_{\bullet\bullet})\, X_i$.

2.2. Model application

The data to be used refers to the time period from the 1st of January of 2005 through the 31st of December of 2013. It consists of all 15.558 Worker's Compensation policies in a particularly relevant type of business for the company (not revealed for confidentiality purposes). The majority of these policies (9504) had no claims during the observed period. The aggregate claim amount S_{ij} of any given policy *i* in any given year *j*, will consist of the sum of both the expenses and pensions paid or provisioned (no distinction is made between the two kinds of payment). The weights w_{ij} are assumed constant for every year, and represent the insured capital of a policy in the end of 2013 (or last observed). The average claim cost in a given year is represented by $X_{ij} = S_{ij}/w_{ij}$.

When applying the Bühlmann-Straub model, the results for the unbiased estimators of the structural parameters and collective were as follows: $\boldsymbol{\hat{\sigma}^2} = 1159{,}11$; $\boldsymbol{\hat{\tau}^2} = 0{,}00299564$; $\boldsymbol{\hat{\kappa}} = \boldsymbol{\hat{\sigma}^2}/\boldsymbol{\hat{\tau}^2} = 386.932{,}00$; $\boldsymbol{\widehat{\mu_0}} = 0{,}03457231$.

The $\hat{\sigma}^2$ is the estimator for the variance within each policy for the claim amounts S_{ij}, and is 1159,11 for this particular 9-year time lapse, while the $\hat{\tau}^2$ is the estimator for the variance among policies, for the average claim cost X_i (average of all years), and is 0,00299564. In this context, the credibility coefficient $\hat{\kappa}$ has a very direct meaning. Basically, a policy with total weights $w_{i\bullet}$ of 386.932,00€ would have a credibility weight $\hat{\alpha}_i$ of exactly 50% (this figure being higher for bigger weights and vice-versa). The estimator for the collective $\widehat{\mu_0}$ has a value of 0,03457231, which is the average of the individual credibility estimators $\widehat{\widehat{\mu(\Theta_i)}}$.

The following chart is a visual representation of the individual policies' credibility estimators, $\widehat{\mu(\Theta_i)}$ (represented by μ(Θi)), in comparison to the collective, $\widehat{\mu_0}$ (represented by μ0). Because of the large amount of policies in study, a random sample of 1000 policies was selected, through resampling of the original data:

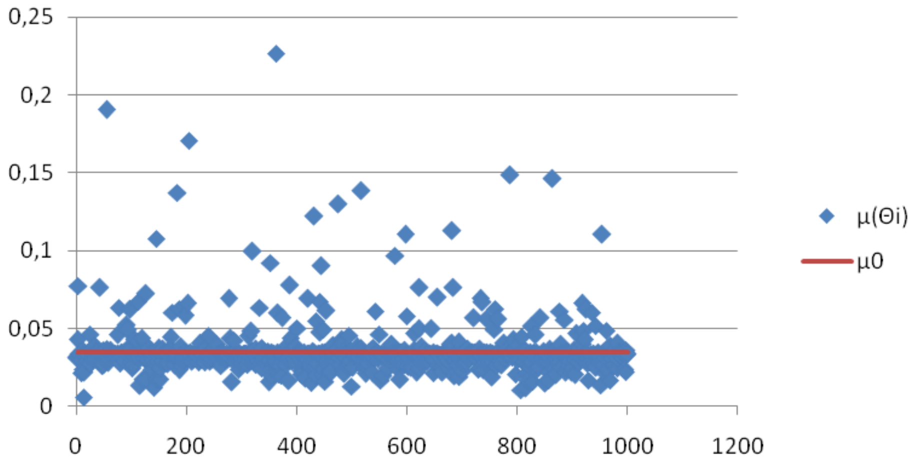

Figure 1: Individual policies, in comparison to the collective.

As it can be seen, the individual results were more or less around the collective (as an observed average credibility weight of 14,2% would suggest), even though some have deviated greatly (the ones with more insured capital and/or higher claim costs). The results showed that 33 policies ended up with a credibility estimator of over 20%. One policy even got an estimator of over 100%.

These results may suggest the presence of outliers in the data, which is to be expected in such a large amount of observations. So, the next step is to strip the data from these outliers to see what the results are. This was done with Chauvenet's criterion, first introduced in Chauvenet (1891). This method identified 58 outliers in the data (all of them in excess, since there are policies with credibility estimators that greatly surpass the vast majority), which were removed before applying the Bühlmann-Straub model again. The new results follow: $\hat{\boldsymbol{\sigma}}^2 = 358{,}39$; $\hat{\boldsymbol{\tau}}^2 = 0{,}00060641$; $\hat{\boldsymbol{\kappa}} = \hat{\boldsymbol{\sigma}}^2/\hat{\boldsymbol{\tau}}^2 = 590.999{,}80$; $\widehat{\boldsymbol{\mu}_0} = 0{,}03015424$.

As expected, the collective's estimator decreases by a fair amount, since the larger observations were removed. Also, the new credibility coefficient is obviously higher, which translates into individual credibility estimators closer to the collective's (the credibility weights fall from an average of 14,2% to 10,8%), and an overall less scattered chart, albeit not by much:

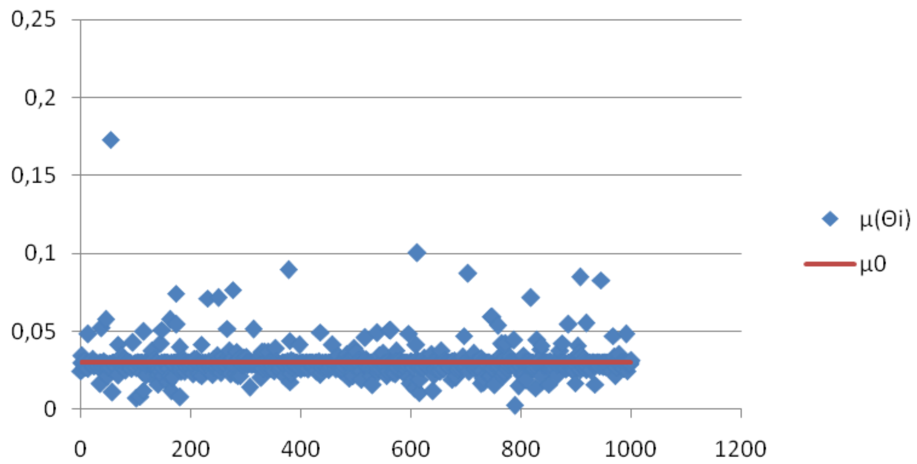

Figure 2: Individual policies, in comparison to the collective (outlier-free).

3. Hierarchical model

Hierarchical Credibility first appeared in Jewell (1975) as a way to make use of collateral data to get a modified credibility formula. Nowadays, it's mostly used to get a fair distribution of premiums in a heterogeneous portfolio with hierarchical structure. As with the previous chapter, this one will also be following the Bühlmann & Gisler (2005) book.

3.1. The order 2 model

Assuming an order 2 model, where the first level consists of the individual risks and the second of another appropriately chosen classification, the hierarchical model assumptions are as follows:

- ***Level 2***

  The random variables $\Phi_h (h = 1, 2, \ldots, |H|)$ *are independent and identically distributed with density* $r_2(\phi)$.

- ***Level 1***

  Given Φ_h *the random variables* $\Theta_i \in \Theta(\Phi_h)$ *are independent and identically distributed with conditional density* $r_1(\theta|\Phi_h)$.

- ***Level 0***

  Given Θ_i *the observations* $X_{ij} \in \mathcal{D}(\Theta_i)$ *are conditionally independent with densities* $r_0\left(x|\ \Theta_i, w_{\text{ij}}\right)$*, for which*

$$E\left[X_{ij}|\Theta_i\right] = \mu(\Theta_i),$$

$$\text{Var}\left[X_{ij}|\Theta_i\right] = \sigma^2(\Theta_i)/w_{ij},$$

where w_{ij} are known weights. New notation includes $|H|, |I|$ and $|I_h|$, which mean, respectively, the total number of classes in level 2, total number of classes in level 1, and total number of classes in level 1 stemming from level 2 class h.

A Hierarchical model is basically an extension of the Bühlmann-Straub model, with a higher amount of levels. Therefore, the calculation of the homogeneous credibility estimator is also similar.

The correct individual premium, $\mu(\Theta_i)$, and its equivalent quantities for the higher levels can be defined as:

$$\mu_0 = E[X_{ij}], \tag{15}$$

$$\mu(\Phi_h) = E[X_{ij}|\Phi_h],$$

$$\mu(\Theta_i) = E[X_{ij}|\Theta_i].$$

With μ_0 being the collective premium.

As for the structural parameters, the variance components are:

$$\sigma^2 = E[\sigma^2(\Theta_i)], \tag{16}$$

$$\tau_1^2 = E[\mathrm{Var}[\mu(\Theta_i)|\Phi_h]], \tag{17}$$

$$\tau_2^2 = \mathrm{Var}[\mu(\Phi_h)]. \tag{18}$$

For levels 0, 1 and 2, respectively.

The best individually unbiased estimator for the collective premium can be calculated from the bottom to the top:

$$B_i^{(1)} = \sum_j \frac{w_{ij}}{w_{i\bullet}} X_{ij}, \qquad w_{i\bullet} = \sum_j w_{ij}, \tag{19}$$

$$\alpha_i^{(1)} = \frac{w_{i\bullet}}{w_{i\bullet} + \frac{\sigma^2}{\tau_1^2}}, \tag{20}$$

$$B_h^{(2)} = \sum_{i \in I_h} \frac{\alpha_i^{(1)}}{w_h^{(2)}} B_i^{(1)}, \qquad I_h = \{i: \Theta_i \in \Theta(\Phi_h)\}, \qquad w_h^{(2)} = \sum_{i \in I_h} \alpha_i^{(1)}, \tag{21}$$

$$\alpha_h^{(2)} = \frac{w_h^{(2)}}{w_h^{(2)} + \frac{\tau_1^2}{\tau_2^2}}, \tag{22}$$

$$\widehat{\widehat{\mu_0}} = \sum_h \frac{\alpha_h^{(2)}}{w^{(3)}} B_h^{(2)}, \qquad w^{(3)} = \sum_h \alpha_h^{(2)}. \tag{23}$$

Afterwards, the homogeneous credibility estimators, defined as $\widehat{\widehat{\mu(\Phi_h)}}^{hom}$ and $\widehat{\widehat{\mu(\Theta_i)}}^{hom}$, can be computed, from the top to the bottom:

$$\widehat{\widehat{\mu(\Phi_h)}}^{hom} = \alpha_h^{(2)} B_h^{(2)} + \left(1 - \alpha_h^{(2)}\right)\widehat{\widehat{\mu_0}}, \tag{24}$$

$$\widehat{\widehat{\mu(\Theta_i)}}^{hom} = \alpha_i^{(1)} B_i^{(1)} + \left(1 - \alpha_i^{(1)}\right)\widehat{\widehat{\mu(\Phi_h)}}^{hom}, \qquad \Theta_i \in \Theta(\Phi_h). \tag{25}$$

For the components of variance, the first two levels, σ^2 and τ_1^2, can be gotten in a way similar to those for the Bühlmann-Straub model. For σ^2, it'll be:

$$S_i = \frac{1}{n_i - 1}\sum_j w_{ij}\left(X_{ij} - B_i^{(1)}\right)^2, \tag{26}$$

$$S = \sum_{i \in I} \frac{n_i - 1}{n_\bullet - |I|} S_i = \frac{1}{n_\bullet - |I|}\sum_{i,j} w_{ij}\left(X_{ij} - B_i^{(1)}\right)^2, \tag{27}$$

$$n_\bullet = \sum_i n_i.$$

Since $E[S_i|\Theta_i] = \sigma^2(\Theta_i)$ and $E[S] = \sigma^2$, then $\widehat{\sigma^2} = S$.

As for τ_1^2, first it's:

$$\widehat{T_h^{(1)}} = c_h \left\{ \frac{|I_h|}{|I_h| - 1} \sum_{i \in I_h} \frac{w_{i\bullet}}{z_h^{(1)}} \left(B_i^{(1)} - \bar{B}_h^{(1)}\right)^2 - \frac{|I_h|\,\widehat{\sigma^2}}{z_h^{(1)}} \right\}, \tag{28}$$

$$z_h^{(1)} = \sum_{i \in I_h} w_{i\bullet},$$

$$\bar{B}_h^{(1)} = \sum_{i \in I_h} \frac{w_{i\bullet}}{z_h^{(1)}} B_i^{(1)},$$

$$c_h = \frac{|I_h| - 1}{|I_h|} \left\{ \sum_{i \in I_h} \frac{w_{i\bullet}}{z_h^{(1)}} \left(1 - \frac{w_{i\bullet}}{z_h^{(1)}}\right) \right\}^{-1}. \tag{29}$$

And given that $E\left[\widehat{T_h^{(1)}}|\Phi_h\right] = \tau_1^2(\Phi_h)$ and $E\left[T_h^{(1)}\right] = \tau_1^2$, then the estimator for τ_1^2 will be:

$$\widehat{\tau_1}^2 = \frac{1}{|H|}\sum_{h \in H} \max\left\{\widehat{T_h^{(1)}}, 0\right\}. \tag{30}$$

For the level 2 variance component, it is first defined the random variables:

$$T^{(2)} = c\left\{\frac{|H|}{|H|-1}\sum_{h\in H}\frac{w_h^{(2)}}{z^{(2)}}\left(B_h^{(2)} - \bar{B}^{(2)}\right)^2 - \frac{|H|\tau_1^2}{z^{(2)}}\right\}, \tag{31}$$

$$z^{(2)} = \sum_{h\in H} w_h^{(2)},$$

$$\bar{B}^{(2)} = \sum_{h\in H}\frac{w_h^{(2)}}{z^{(2)}} B_h^{(2)},$$

$$c = \frac{|H|-1}{|H|}\left\{\sum_{h\in H}\frac{w_h^{(2)}}{z^{(2)}}\left(1 - \frac{w_h^{(2)}}{z^{(2)}}\right)\right\}^{-1}. \tag{32}$$

Then, replacing the structural parameters by their estimates, we get the unbiased random variables $\widehat{T^{(2)}}$. Therefore, the estimator for τ_2^2 will be:

$$\widehat{\tau_2}^2 = \max\left\{\widehat{T^{(2)}}, 0\right\}. \tag{33}$$

3.2. Model application to capital insured

Now, for the application of a hierarchical model, a second level is needed. For this purpose, let the second level be grouped according to the policies' capital insured, in the following ranges (with the percentage of policies within a certain range between brackets):

- Policies with capital insured at or below 125.000€ (94,00%),
- Policies with capital insured in the interval (125.000€, 250.000€] (3,33%),
- Policies with capital insured in the interval (250.000€, 500.000€] (1,59%),
- Policies with capital insured in the interval (500.000€, 1.200.000€] (0,72%),
- Policies with capital insured above 1.200.000€ (0,36%).

The capital insured can be seen as a measure of the size of the insured company. The ranges presented are the standard in the company, when grouping policies by size.

For the same data set and with the same assumptions, the hierarchical model with 2 levels was then applied, and the results for the unbiased estimators of the structural parameters and collective were: $\hat{\boldsymbol{\sigma}}^2 = 1691{,}88$; $\hat{\boldsymbol{\tau}}_1^2 = 0{,}00042085$; $\hat{\boldsymbol{\tau}}_2^2 = 0{,}00002106$; $\hat{\boldsymbol{\kappa}}_1 = \hat{\boldsymbol{\sigma}}^2/\hat{\boldsymbol{\tau}}_1^2 = 4.020.164{,}76$; $\hat{\boldsymbol{\kappa}}_2 = \hat{\boldsymbol{\tau}}_1^2/\hat{\boldsymbol{\tau}}_2^2 = 19{,}98$; $\widehat{\widehat{\boldsymbol{\mu}_0}} = 0{,}03084340$.

And the second level's best linear individually unbiased estimators, $B_h^{(2)}$**,** credibility weights, $\alpha_h^{(2)}$, and credibility estimators, $\widehat{\widehat{\mu(\Phi_h)}}$, were:

	$\boldsymbol{B_h^{(2)}}$	$\boldsymbol{\alpha_h^{(2)}}$	$\widehat{\widehat{\boldsymbol{\mu(\Phi_h)}}}$
≤125.000€	0,03643604	0,91540783	0,03596295
(125.000€,250.000€]	0,03190409	0,74407114	0,03163263
(250.000€,500.000€]	0,02630550	0,70008868	0,02766647
(500.000€,1.200.000€]	0,03243200	0,64091006	0,03186155
>1.200.000€	0,02497693	0,63922652	0,02709340

Table 1: Second level results (capital insured)**.**

From table 1, a few things can already be said. First, the credibility weights for these second level nodes seem high enough to be relevant. The lowest one (125k) is the one with the most individual credibility, due to the fact that it's there that the bulk of policies reside. 14.620 policies are placed in that range, which is over 93% of the total policies studied. Still, even with a low number of policies, the other second level nodes have a high credibility weight because of the high weights of the individual policies. The credibility estimators also suggest that the smaller policies have a higher average claim cost, while giving somewhat mixed results for the remaining (more or less around the collective's average).

The following charts (figures 3 through 7) show the individual policies' credibility estimators at the first level (blue diamonds), derived from each data range previously defined, along with the corresponding second level (red line) and collective credibility (green line) estimators, the latter for comparison between charts (if only one line is showing, these last two are more or less coincident):

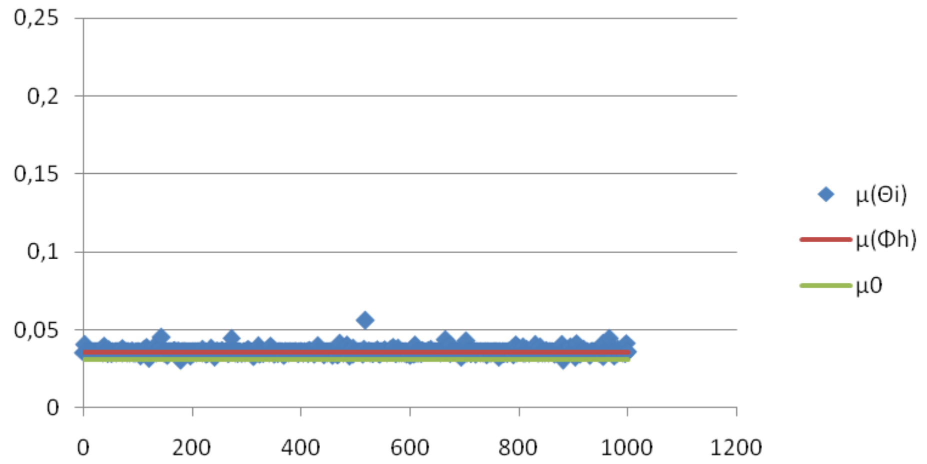

Figure 3: 125k and under policies.

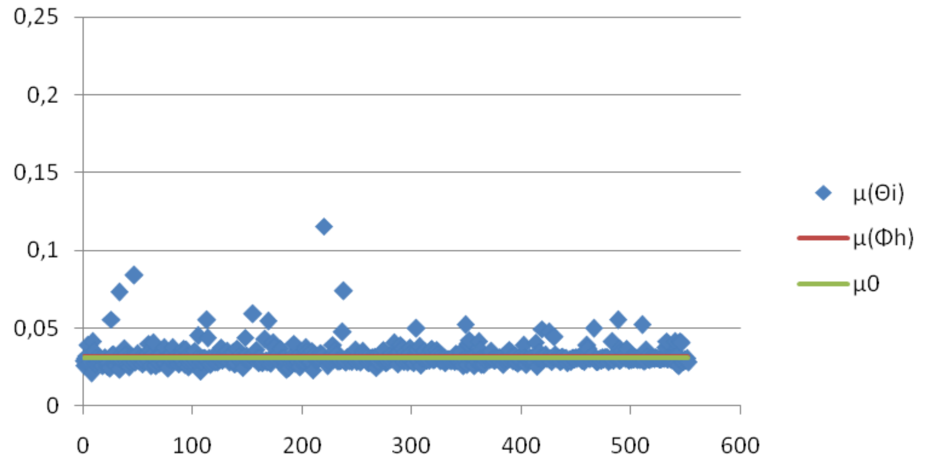

Figure 4: Policies between 125k and 250k.

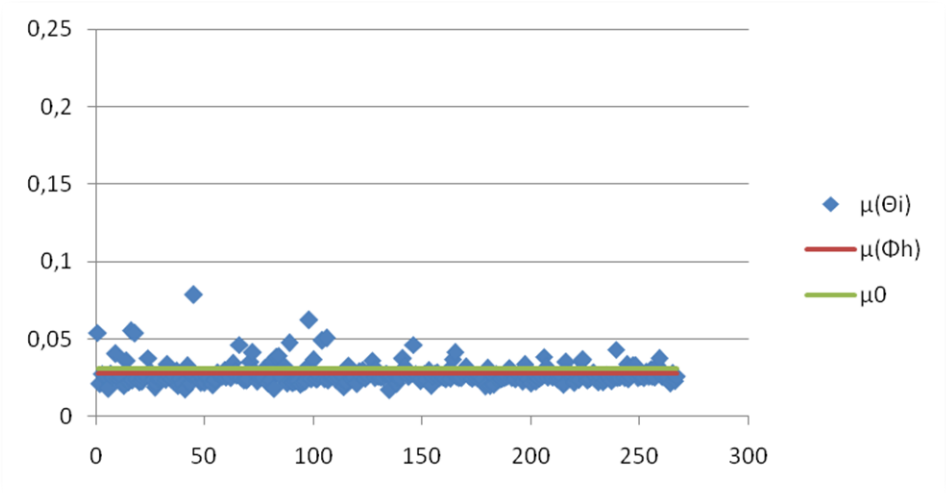

Figure 5: Policies between 250k and 500k.

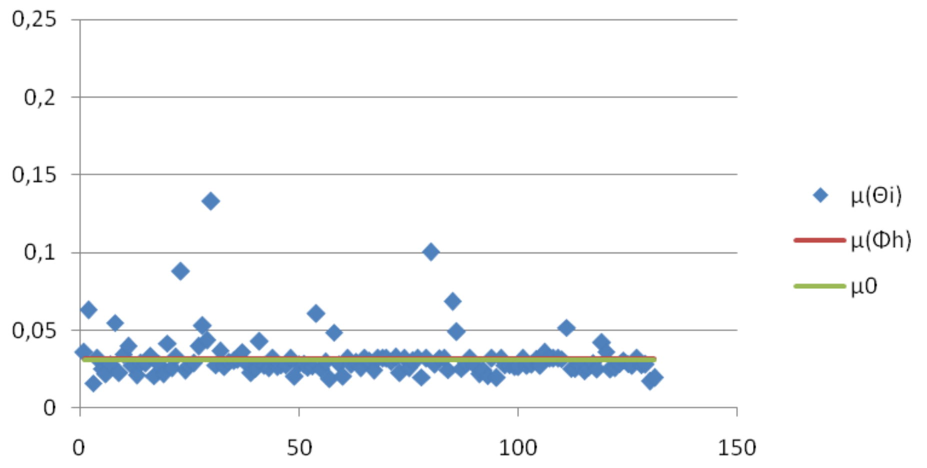

Figure 6: Policies between 500k and 1200k.

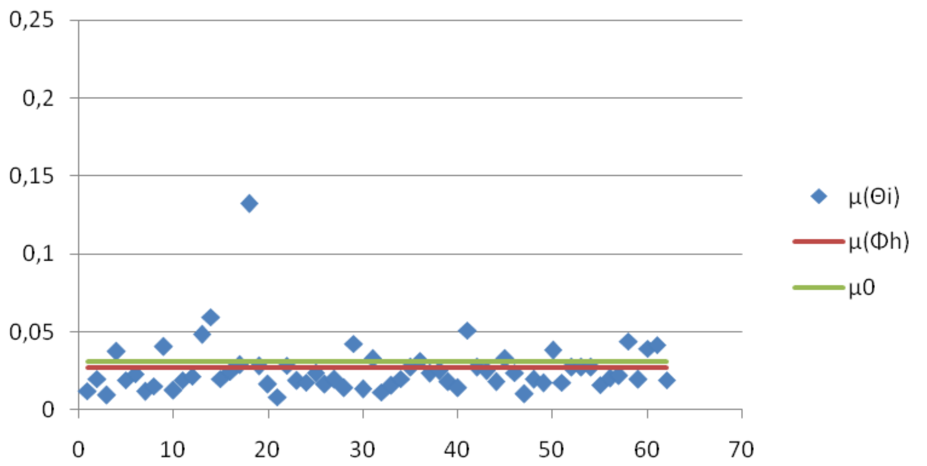

Figure 7: 1200k and over policies.

The individual credibility estimators are, generally, close to the respective second level credibility estimator, more so in the low capital insured groups than in the high. An especially high $\hat{\kappa}_1$ contributes greatly to this, especially in the lowest capital insured range, where there's an almost even distribution of rates among policies, with an average individual credibility weight of close to 1,5%. The remaining ranges have average individual credibility weights of approximately 10,5%, 17,5%, 27,2% and 57,1%, respectively.

Once more, the data was tested for outliers, again with Chauvenet's criterion, but now with each set of capital ranges treated separately. This resulted in a total of 76 outliers this time (56, 9, 8, 2 and 1, respectively). Without these outliers, the model produced the following results: $\boldsymbol{\hat{\sigma}^2} = 652{,}22$; $\boldsymbol{\hat{\tau}_1^2} = 0{,}00006818$; $\boldsymbol{\hat{\tau}_2^2} = 0{,}00001921$; $\boldsymbol{\hat{\kappa}_1} = \boldsymbol{\hat{\sigma}^2}/\boldsymbol{\hat{\tau}_1^2} = 9.566.599{,}03$; $\boldsymbol{\hat{\kappa}_2} = \boldsymbol{\hat{\tau}_1^2}/\boldsymbol{\hat{\tau}_2^2} = 3{,}55$; $\widehat{\boldsymbol{\mu_0}} = 0{,}02632025$.

	$\boldsymbol{B_h^{(2)}}$	$\boldsymbol{\alpha_h^{(2)}}$	$\widehat{\boldsymbol{\mu(\Phi_h)}}$
≤125.000€	0,03130177	0,96342619	0,03111957
(125.000€,250.000€]	0,02801468	0,88229452	0,02781523
(250.000€,500.000€]	0,02203080	0,86342244	0,02261665
(500.000€,1.200.000€]	0,02779468	0,84519764	0,02756643
>1.200.000€	0,02194258	0,87646382	0,02248338

Table 2: Outlier-free second level results (capital insured).

As it can be observed from both $\hat{\kappa}_1$ and $\hat{\kappa}_2$, the credibility weights got smaller at the first level and larger at the second. This means that the individual credibility estimators are much closer to the second level credibility estimators, which are now considerably lower. However, despite the removal of outliers, their structure remained largely the same, with each range dropping around 0,4% or 0,5%.

The average credibility weights are now 0,6%, 4,9%, 8,7%, 15,0% and 41,3%, respectively, an expected drop for all ranges. The visual representations of the individual policies is close to the one presented in figures 3 through 7, but with each policy now closer to the corresponding second level credibility estimator. Therefore, it isn't really important to have them presented here. Regardless, they're available in the appendix.

The higher credibility estimator for the small companies is expected. Usually, work accidents decrease if the companies identify the risky situations and invest in safety and preventive measures. This may be difficult for small companies, as these may not have the resources to do so. This is why this result is expected. Conversely, one would expect the credibility estimators to decrease linearly as the capital insured range increased, but this isn't the case, as the credibility estimator goes up in the second-to-last range again. Even after removing the outliers, one gets similar results. Also, no one policy or group of policies showed particular influence to this value, after individual analysis.

3.3. Model application to geographical regions

The second way the hierarchical model is going to be used is to study its influence when grouping the policies by region. The regions will be grouped in the following way (every region's name is presented in Portuguese, to avoid half translations):

- Grande Lisboa (7,32%),
- Extremadura e Ribatejo (except Lisboa) (14,18%),
- Beira Litoral (23,82%),
- Porto (0,87%),
- Minho e Douro Litoral (except Porto) (32,04%),
- Trás-os-Montes e Alto Douro (4,59%),
- Beira Interior (4,70%),
- Alentejo (3,95%),
- Algarve (7,23%),
- Açores e Madeira (1,30%).

These regions were chosen according to the first digit of the postal code numerical structure in Portugal, with a few adjustments to fit better with reality.

Applying the same model as before, the results were now: $\hat{\boldsymbol{\sigma}}^2 = 1692{,}29$; $\hat{\boldsymbol{\tau}}_1^2 = 0{,}00257457$; $\hat{\boldsymbol{\tau}}_2^2 = 0{,}00003545$; $\hat{\boldsymbol{\kappa}}_1 = \hat{\boldsymbol{\sigma}}^2/\hat{\boldsymbol{\tau}}_1^2 = 657.309{,}14$; $\hat{\boldsymbol{\kappa}}_2 = \hat{\boldsymbol{\tau}}_1^2/\hat{\boldsymbol{\tau}}_2^2 = 72{,}63$; $\widehat{\boldsymbol{\mu}_0} = 0{,}03148033$.

	$\boldsymbol{B_h^{(2)}}$	$\boldsymbol{\alpha_h^{(2)}}$	$\widehat{\boldsymbol{\mu(\Phi_h)}}$
Grande Lisboa	0,03191625	0,65188078	0,03176450
Extremadura e Ribatejo	0,03076863	0,74490100	0,03095018
Beira Litoral	0,03320210	0,78320826	0,03282884
Porto	0,03191651	0,25513784	0,03159162
Minho e Douro Litoral	0,04010651	0,89512008	0,03920180
Trás-os-Montes e Alto Douro	0,04177956	0,48260837	0,03645082
Beira Interior	0,03294295	0,42054139	0,03209542
Alentejo	0,02303338	0,37777094	0,02828932
Algarve	0,01913585	0,58061076	0,02431299
Açores e Madeira	0,01821866	0,31387555	0,02731782

Table 3: Second level results (regions).

Again, the credibility weights of the second level, $\alpha_h^{(2)}$, in general seem relevant enough. Naturally, regions with more policies resulted in higher credibility weights, like Minho e Douro Litoral, and vice-versa, like Porto. From the results, both before and after applying credibility, one can say there's a propensity for higher average claim costs in the north of the country (except in Porto), lower in the south and islands, and approximate to the mean in the center of the country. A higher value (than when grouping by capital insured range) for the within group variance $\hat{\tau}_1^2$, and consequently lower $\hat{\kappa}_1$, allows one to conclude that there are more differences in the average claim costs within each group. Therefore, more individual credibility weight must be attributed to each policy, which results in a larger scatter in relation to the collective (the average credibility weights were approximately 11,9%, 9,6%, 7,1%, 18,4%, 12,4%, 9,5%, 7,2%, 7,2%, 8,9% and 16,4%, respectively). Again, we have the charts (figures 8 through 17) with the individual policies' credibility estimators (blue diamonds), along with their respective second level estimator (red line), and collective credibility estimator (green line):

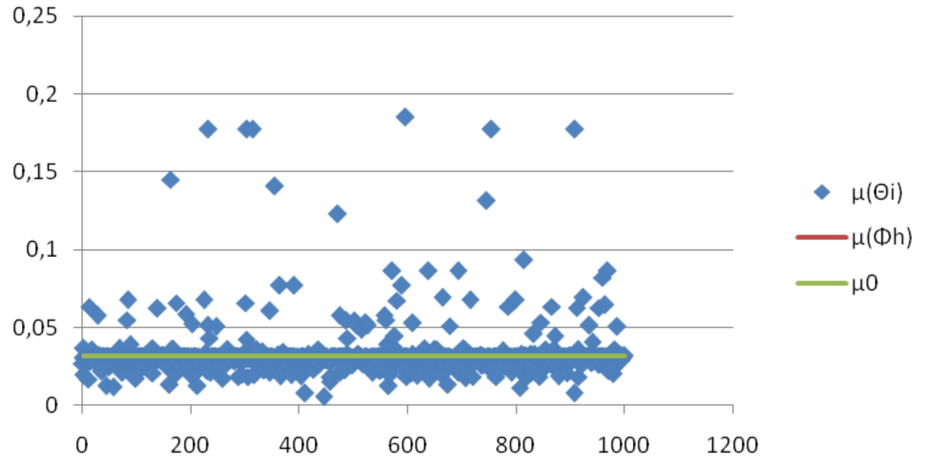

Figure 8: Policies in Grande Lisboa.

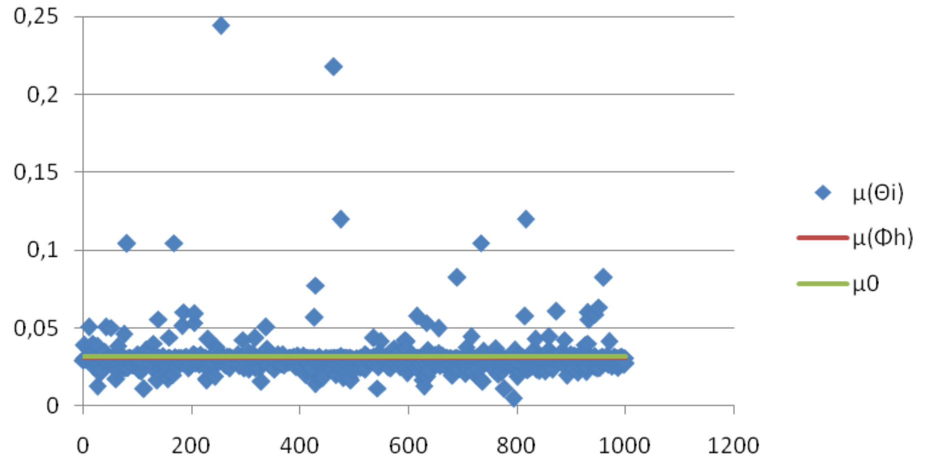

Figure 9: Policies in Extremadura e Ribatejo.

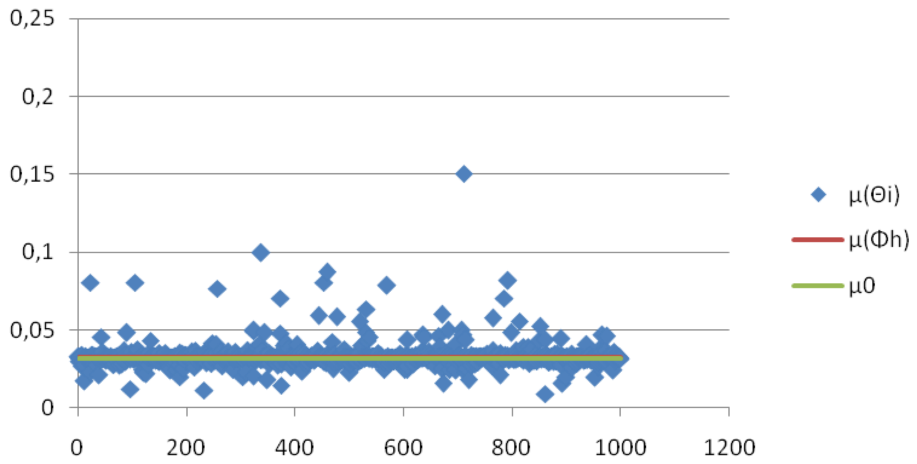

Figure 10: Policies in Beira Litoral.

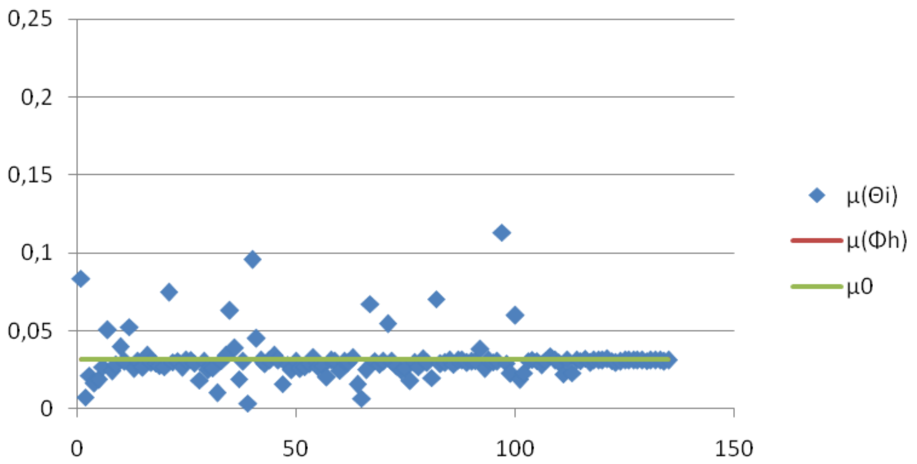

Figure 11: Policies in Porto.

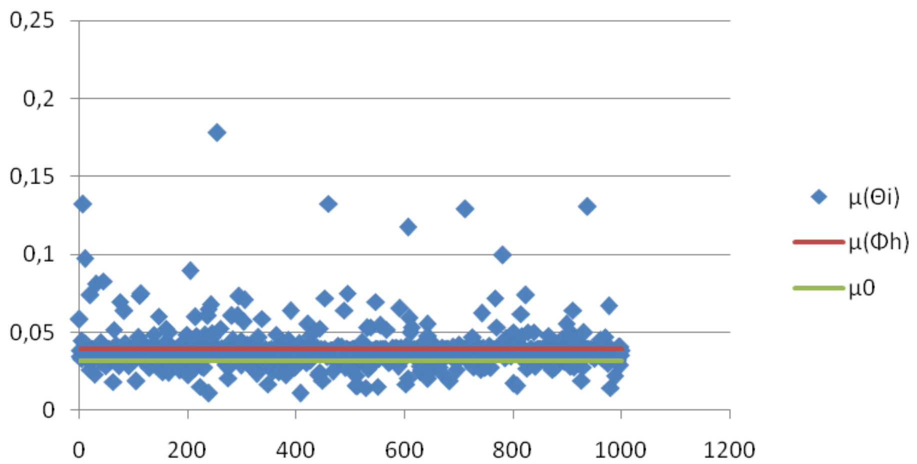

Figure 12: Policies in Minho e Douro Litoral.

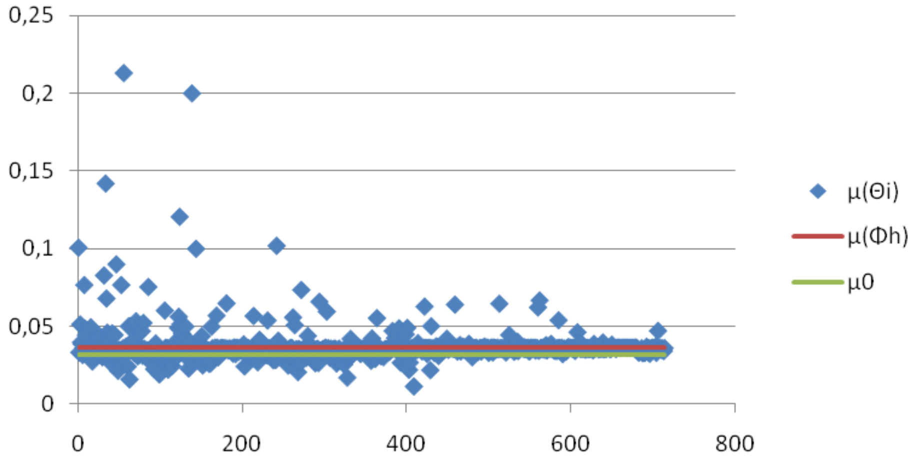

Figure 13: Policies in Trás-os-Montes e Alto Douro.

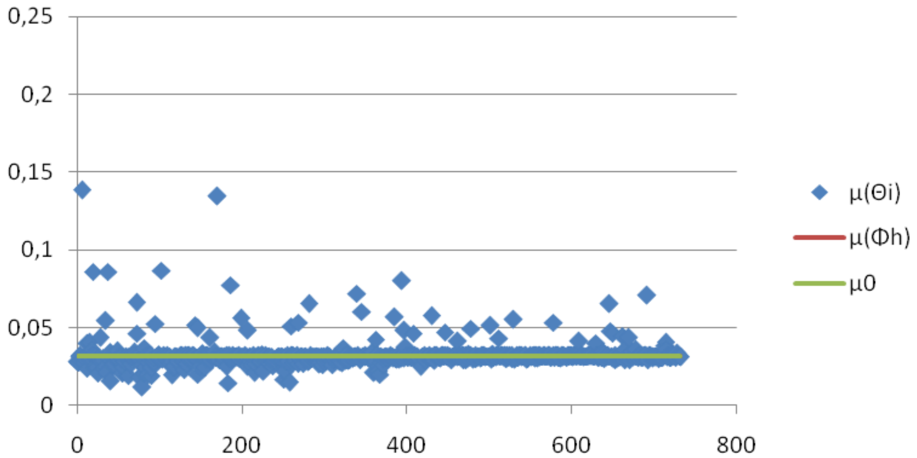

Figure 14: Policies in Beira Interior.

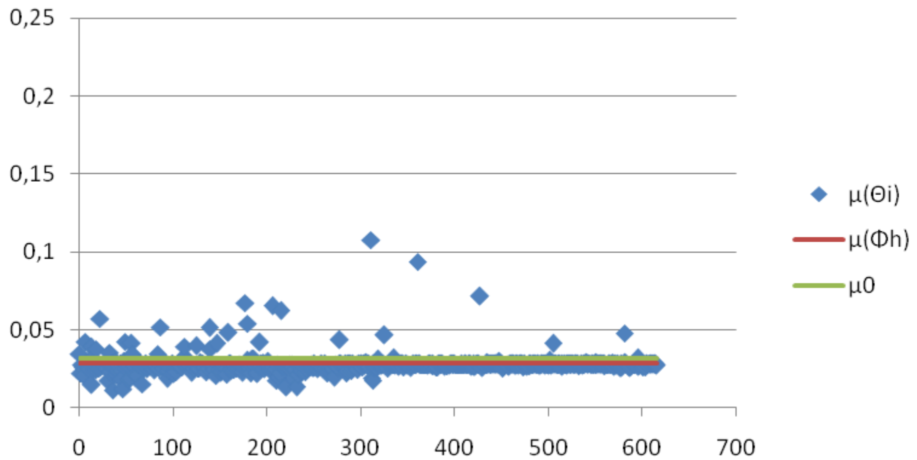

Figure 15: Policies in Alentejo.

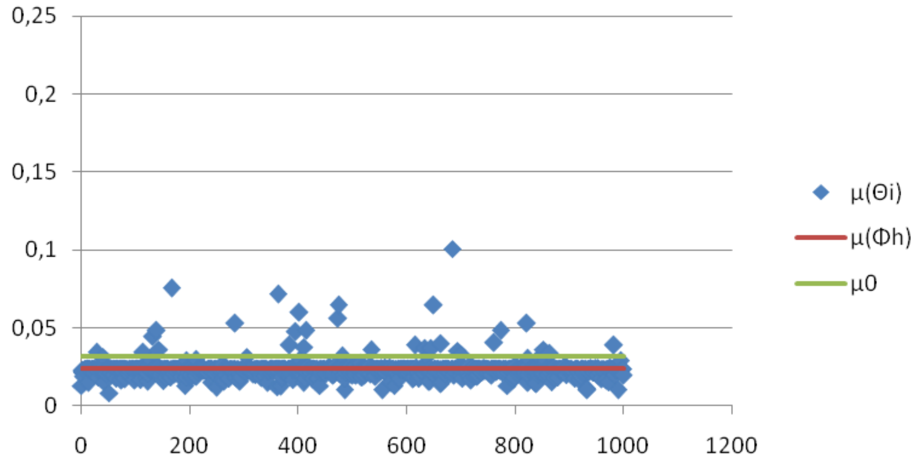

Figure 16: Policies in Algarve.

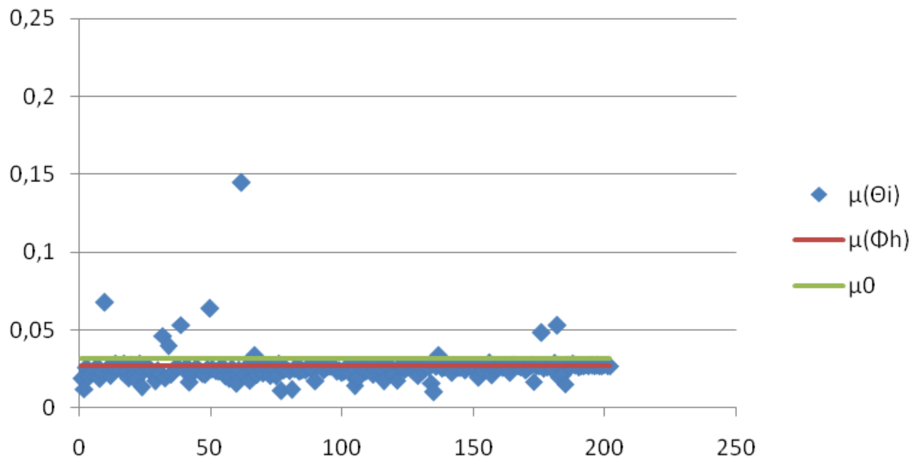

Figure 17: Policies in Açores e Madeira.

With the data grouped in this way, the test for outliers now identified 113 of them. The regions had 14, 16, 20, 2, 28, 11, 15, 2, 1 and 4, respectively. The new outlier-free results were: $\hat{\boldsymbol{\sigma}}^2 = 576{,}73$; $\hat{\boldsymbol{\tau}}_1^2 = 0{,}00001894$; $\hat{\boldsymbol{\tau}}_2^2 = 0{,}00003449$; $\hat{\boldsymbol{\kappa}}_1 = \hat{\boldsymbol{\sigma}}^2/\hat{\boldsymbol{\tau}}_1^2 = 3.0455.983{,}81$; $\hat{\boldsymbol{\kappa}}_2 = \hat{\boldsymbol{\tau}}_1^2/\hat{\boldsymbol{\tau}}_2^2 = 0{,}55$; $\widehat{\boldsymbol{\mu}_0} = 0{,}02504797$.

	$B_h^{(2)}$	$\alpha_h^{(2)}$	$\widehat{\mu(\Phi_h)}$
Grande Lisboa	0,02552047	0,93079928	0,02548777
Extremadura e Ribatejo	0,02545363	0,93634161	0,02542780
Beira Litoral	0,02769203	0,94100617	0,02753605
Porto	0,02179449	0,73419520	0,02265928
Minho e Douro Litoral	0,03296430	0,98282171	0,03282831
Trás-os-Montes e Alto Douro	0,03564826	0,80549422	0,03358644
Beira Interior	0,02644740	0,74406042	0,02608923
Alentejo	0,01907272	0,70642524	0,02082690
Algarve	0,01568322	0,85795973	0,01701339
Açores e Madeira	0,01759849	0,80857492	0,01902450

Table 4: Outlier-free second level results (regions).

Once again, $\hat{\kappa}_2$ dropped and $\hat{\kappa}_1$ went up immensely. This means that credibility shifted mostly to the second level. With a few exceptions, the individual policies' credibility estimators are almost equal to the collective's. The new average credibility weights were 0,7%, 0,4%, 0,2%, 1,1%, 0,6%, 0,3%, 0,2%, 0,2%, 0,3% and 1,2%, respectively.

The structure seems to maintain itself (even though Porto seems to have benefited from the removal of outliers more than the rest). Again, the visual representations of the individual policies aren't really important this time, since all policies are very close to the respective second level credibility estimator. But they are available in the appendix nevertheless.

Since there's evidence of higher claim costs in the north and lower in the south and islands, one may ask what could be the reason. Perhaps an order 3 hierarchical model that includes both the size and the region of the companies will help with this question.

3.4. The order 3 model

For an order 3 hierarchical model, in which level 1 consists of the individual contracts, both the level 2 and 3 will be classifications, and level 4 is the overall portfolio, the assumptions are similar as before, adding a new level:

- ***Level 3***

  The random variables $\Psi_g (g = 1, 2, \dots, |G|)$ are independent and identically distributed with density $r_3(\psi)$.

- ***Level 2***

  Given Ψ_g the random variables $\Phi_h \in \Phi(\Psi_g)$ are independent and identically distributed with conditional density $r_2(\phi|\Psi_g)$.

- ***Level 1***

  Given Φ_h the random variables $\Theta_i \in \Theta(\Phi_h)$ are independent and identically distributed with conditional density $r_1(\theta|\Phi_h)$.

- ***Level 0***

  Given Θ_i the observations $X_{ij} \in \mathcal{D}(\Theta_i)$ are conditionally independent with densities $r_0(x|\ \Theta_i, w_{\mathrm{ij}})$, for which

$$E[X_{ij}|\Theta_i] = \mu(\Theta_i),$$

$$\mathrm{Var}[X_{ij}|\Theta_i] = \sigma^2(\Theta_i)/w_{ij},$$

where w_{ij} are known weights. New notation from the previous order 2 model includes $|G|$ and $|H_g|$, which mean, respectively, the total number of classes in level 3 and the total number of classes in level 2, stemming from level 3 class g. Naturally, we also have one new level of intermediate premiums:

$$\mu_0 = E\left[X_{ij}\right], \tag{34}$$

$$\mu(\Psi_g) = E\left[X_{ij}|\Psi_g\right],$$

$$\mu(\Phi_h) = E\left[X_{ij}|\Phi_h\right],$$

$$\mu(\Theta_i) = E\left[X_{ij}|\Theta_i\right].$$

And a new variance parameter:

$$\sigma^2 = E[\sigma^2(\Theta_i)], \tag{35}$$

$$\tau_1^2 = E\left[\mathrm{Var}[\mu(\Theta_i)|\Phi_h]\right], \tag{36}$$

$$\tau_2^2 = E\left[\mathrm{Var}\left[\mu(\Phi_h)|\Psi_g\right]\right], \tag{37}$$

$$\tau_3^2 = \mathrm{Var}\left[\mu(\Psi_g)\right]. \tag{38}$$

The best individually unbiased estimator for the collective is now computed replacing the last step in the order 2 model (23) by:

$$B_g^{(3)} = \sum_{i \in H_g} \frac{\alpha_h^{(2)}}{w_g^{(3)}} B_h^{(2)}, \qquad H_g = \{h: \Phi_h \in \Phi(\Psi_g)\}, \qquad w_g^{(3)} = \sum\nolimits_{h \in H_g} \alpha_h^{(2)}, \tag{39}$$

$$\alpha_g^{(3)} = \frac{w_g^{(3)}}{w_g^{(3)} + \frac{\tau_2^2}{\tau_3^2}}, \tag{40}$$

$$\widehat{\widehat{\mu_0}} = \sum_g \frac{\alpha_g^{(3)}}{w^{(4)}} B_g^{(3)}, \qquad w^{(4)} = \sum\nolimits_g \alpha_g^{(3)}. \tag{41}$$

And the homogeneous credibility estimators are:

$$\widehat{\widehat{\mu(\Psi_g)}}^{hom} = \alpha_g^{(3)} B_g^{(3)} + \left(1 - \alpha_g^{(3)}\right)\widehat{\widehat{\mu_0}}, \tag{42}$$

$$\widehat{\widehat{\mu(\Phi_h)}}^{hom} = \alpha_h^{(2)} B_h^{(2)} + \left(1 - \alpha_h^{(2)}\right)\widehat{\widehat{\mu(\Psi_g)}}^{hom}, \qquad \Phi_h \in \Phi(\Psi_g), \tag{43}$$

$$\widehat{\widehat{\mu(\Theta_i)}}^{hom} = \alpha_i^{(1)} B_i^{(1)} + \left(1 - \alpha_i^{(1)}\right) \widehat{\widehat{\mu(\Phi_h)}}^{hom}, \qquad \Theta_i \in \Theta(\Phi_h). \tag{44}$$

As for the variance components, the first two previous ones ($\hat{\sigma}^2$and $\hat{\tau_1}^2$) are the same and the level 2 and 3 ones are computed in a similar way. For the level 2:

$$T_g^{(2)} = c_g \left\{ \frac{|H_g|}{|H_g| - 1} \sum_{h \in H_g} \frac{w_h^{(2)}}{z_g^{(2)}} \left(B_h^{(2)} - \bar{B}_g^{(2)}\right)^2 - \frac{|H_g| \tau_1^2}{z_g^{(2)}} \right\}, \tag{45}$$

$$z^{(2)} = \sum_{h \in H_g} w_h^{(2)},$$

$$\bar{B}_g^{(2)} = \sum_{h \in H_g} \frac{w_h^{(2)}}{z_g^{(2)}} B_h^{(2)},$$

$$c_g = \frac{|H_g| - 1}{|H_g|} \left\{ \sum_{h \in H_g} \frac{w_h^{(2)}}{z_g^{(2)}} \left(1 - \frac{w_h^{(2)}}{z_g^{(2)}}\right) \right\}^{-1}. \tag{46}$$

Once more, replacing the structural parameters by their estimates we'll get us to $\widehat{T_g^{(2)}}$.

Therefore:

$$\hat{\tau_2}^2 = \frac{1}{|G|} \sum_{g \in G} \max\left\{\widehat{T_g^{(2)}}, 0\right\}. \tag{47}$$

And for the level 3:

$$T^{(3)} = c \left\{ \frac{|G|}{|G| - 1} \sum_{g \in G} \frac{w_g^{(3)}}{z^{(3)}} \left(B_g^{(3)} - \bar{B}^{(3)}\right)^2 - \frac{|G| \tau_2^2}{z^{(3)}} \right\}, \tag{48}$$

$$z^{(3)} = \sum_{g \in G} w_g^{(3)},$$

$$\bar{B}^{(3)} = \sum_{g \in G} \frac{w_g^{(3)}}{z^{(3)}} B_g^{(3)},$$

$$c = \frac{|G| - 1}{|G|} \left\{ \sum_{g \in G} \frac{w_g^{(3)}}{z^{(3)}} \left(1 - \frac{w_g^{(3)}}{z^{(3)}}\right) \right\}^{-1}. \tag{49}$$

Again, replacing by the estimators and getting $\widehat{T^{(3)}}$:

$$\hat{\tau}_3{}^2 = \max\left\{\widehat{T^{(3)}}, 0\right\}. \tag{50}$$

It remains to be said that this model can naturally be extended to a higher order. However, the number of parameters to be estimated increases with the order of the model, so one should have that in mind.

3.5. Model application to both capital insured and regions

For the application of an order 3 hierarchical model, it's again going to be considered the same groups as before, now used in the following way:

Level 3:

- Grande Lisboa,
- Extremadura e Ribatejo (except Lisboa),
- Beira Litoral,
- Porto,
- Minho e Douro Litoral (except Porto),
- Trás-os-Montes e Alto Douro,
- Beira Interior,
- Alentejo,
- Algarve,
- Açores e Madeira.

Level 2:

- Policies with capital insured at or below 125.000€,
- Policies with capital insured in the interval (125.000€, 250.000€],

- Policies with capital insured in the interval (250.000€, 500.000€],
- Policies with capital insured in the interval (500.000€, 1.200.000€],
- Policies with capital insured above 1.200.000€.

This is to say that this time, data is grouped by region, and then grouped by capital range, within each region. The results for the estimators of the structural parameters and collective now were: $\hat{\boldsymbol{\sigma}}^2 = 1687{,}03$; $\hat{\boldsymbol{\tau}}_1^2 = 0{,}00122066$; $\hat{\boldsymbol{\tau}}_2^2 = 0{,}00001257$; $\hat{\boldsymbol{\tau}}_3^2 = 0{,}00002806$; $\hat{\boldsymbol{\kappa}}_1 = \hat{\boldsymbol{\sigma}}^2/\hat{\boldsymbol{\tau}}_1^2 = 1.382.060{,}55$; $\hat{\boldsymbol{\kappa}}_2 = \hat{\boldsymbol{\tau}}_1^2/\hat{\boldsymbol{\tau}}_2^2 = 97{,}07$; $\hat{\boldsymbol{\kappa}}_3 = \hat{\boldsymbol{\tau}}_2^2/\hat{\boldsymbol{\tau}}_3^2 = 0{,}44818600$; $\widehat{\boldsymbol{\mu}_0} = 0{,}03005517$.

As for the third level, table 5 contains the data about the third level's best linear individually unbiased estimators, $B_g^{(3)}$, credibility weights, $\alpha_g^{(3)}$, and credibility estimators, $\widehat{\mu(\Psi_g)}$:

	$\boldsymbol{B_g^{(3)}}$	$\boldsymbol{\alpha_g^{(3)}}$	$\widehat{\boldsymbol{\mu(\Psi_g)}}$
Grande Lisboa	0,03165478	0,59098062	0,03100051
Extremadura e Ribatejo	0,02886300	0,62307457	0,02931236
Beira Litoral	0,02956738	0,65558155	0,02973539
Porto	0,03040728	0,25858196	0,03014622
Minho e Douro Litoral	0,03778380	0,81052939	0,03631945
Trás-os-Montes e Alto Douro	0,04158947	0,41589901	0,03485228
Beira Interior	0,03242560	0,35646300	0,03090014
Alentejo	0,02236452	0,31905142	0,02760146
Algarve	0,01835496	0,49967938	0,02420882
Açores e Madeira	0,01836781	0,30631960	0,02647511

Table 5: Third level results (regions).

As expected, the credibility weights are a bit smaller, now that a new level was added, but not by a large amount. The result is that, in relation to the order 2 model, the credibility estimators aren't really all that different, differing the most in Beira Litoral and Minho e Douro Litoral, mostly because of the difference in the compressed data $B_g^{(3)}$, and Trás-os-Montes e Alto Douro, because of a lower credibility weight.

Now onto the second level. The following are the tables relative to each of the regions in table 5:

	$B_h^{(2)}$	$\alpha_h^{(2)}$	$\widehat{\mu(\Phi_h)}$
≤125.000€	0,03309711	0,32899702	0,03169028
(125.000€,250.000€]	0,03315631	0,12279906	0,03126524
(250.000€,500.000€]	0,02544051	0,09560863	0,03046893
(500.000€,1.200.000€]	0,04732977	0,04809859	0,03178593
>1.200.000€	0,01593088	0,05206807	0,03021586

Table 6: Grande Lisboa: second level results (capital insured).

	$B_h^{(2)}$	$\alpha_h^{(2)}$	$\widehat{\mu(\Phi_h)}$
≤125.000€	0,03045850	0,46551985	0,02984591
(125.000€,250.000€]	0,02960517	0,11239339	0,02934527
(250.000€,500.000€]	0,02987951	0,06890465	0,02935144
(500.000€,1.200.000€]	0,01655110	0,03926567	0,02881128
>1.200.000€	0,02132918	0,05478793	0,02887498

Table 7: Extremadura e Ribatejo: second level results (capital insured).

	$B_h^{(2)}$	$\alpha_h^{(2)}$	$\widehat{\mu(\Phi_h)}$
≤125.000€	0,03708524	0,53367886	0,03365785
(125.000€,250.000€]	0,01605508	0,12864096	0,02797554
(250.000€,500.000€]	0,01670095	0,13207908	0,02801381
(500.000€,1.200.000€]	0,02106284	0,04538641	0,02934177
>1.200.000€	0,01540640	0,01331206	0,02954464

Table 8: Beira Litoral: second level results (capital insured).

	$B_h^{(2)}$	$\alpha_h^{(2)}$	$\widehat{\mu(\Phi_h)}$
≤125.000€	0,04152833	0,05527864	0,03077541
(125.000€,250.000€]	0,03205951	0,04610753	0,03023444
(250.000€,500.000€]	0,02960810	0,02127896	0,03013477
(500.000€,1.200.000€]	0,02102538	0,00793850	0,03007382
>1.200.000€	0,00709020	0,02570874	0,02955348

Table 9: Porto: second level results (capital insured).

	$B_h^{(2)}$	$\alpha_h^{(2)}$	$\widehat{\widehat{\mu(\Phi_h)}}$
≤125.000€	0,04309479	0,66894321	0,04085177
(125.000€,250.000€]	0,03856083	0,39699191	0,03720926
(250.000€,500.000€]	0,03270415	0,33105048	0,03512260
(500.000€,1.200.000€]	0,03632626	0,30239692	0,03632151
>1.200.000€	0,02980359	0,21789597	0,03489967

Table 10: Minho e Douro Litoral: second level results (capital insured).

	$B_h^{(2)}$	$\alpha_h^{(2)}$	$\widehat{\widehat{\mu(\Phi_h)}}$
≤125.000€	0,04618001	0,23152731	0,03747496
(125.000€,250.000€]	0,03362234	0,05447677	0,03478527
(250.000€,500.000€]	0,00473806	0,00902764	0,03458042
(500.000€,1.200.000€]	0,05605872	0,01068831	0,03507894
>1.200.000€	0,00795669	0,01340307	0,03449179

Table 11: Trás-os-Montes e Alto Douro: second level results (capital insured).

	$B_h^{(2)}$	$\alpha_h^{(2)}$	$\widehat{\widehat{\mu(\Phi_h)}}$
≤125.000€	0,03468513	0,19547522	0,03164002
(125.000€,250.000€]	0,01499507	0,02382680	0,03052118
(250.000€,500.000€]	0,01021807	0,01874845	0,03051239
(500.000€,1.200.000€]	0,00358855	0,00290379	0,03082084
>1.200.000€	0,09730684	0,00730143	0,03138501

Table 12: Beira Interior: second level results (capital insured).

	$B_h^{(2)}$	$\alpha_h^{(2)}$	$\widehat{\widehat{\mu(\Phi_h)}}$
≤125.000€	0,02540170	0,17471946	0,02721712
(125.000€,250.000€]	0,00752493	0,02573387	0,02708481
(250.000€,500.000€]	0,00642203	0,00285419	0,02754101
(500.000€,1.200.000€]	0,00691711	0,00668540	0,02746318
>1.200.000€	0,00000000	0,00000000	0,02760146

Table 13: Alentejo**:** second level results (capital insured).

	$B_h^{(2)}$	$\alpha_h^{(2)}$	$\widehat{\widehat{\mu(\Phi_h)}}$
≤125.000€	0,02032122	0,30080039	0,02303943
(125.000€,250.000€]	0,02034678	0,07797877	0,02390766
(250.000€,500.000€]	0,00820625	0,05465009	0,02333428
(500.000€,1.200.000€]	0,00480695	0,01418233	0,02393366
>1.200.000€	0,00000000	0,00000000	0,02420882

Table 14: Algarve: second level results (capital insured).

	$B_h^{(2)}$	$\alpha_h^{(2)}$	$\widehat{\mu(\Phi_h)}$
≤125.000€	0,02287301	0,09502832	0,02613280
(125.000€,250.000€]	0,01404532	0,03886932	0,02599197
(250.000€,500.000€]	0,01103726	0,02869449	0,02603212
(500.000€,1.200.000€]	0,01283860	0,01640188	0,02625144
>1.200.000€	0,02053113	0,01891868	0,02636265

Table 15: Açores e Madeira: second level results (capital insured).

From the second level tables, apart from the ones that correspond to the regions of Beira Litoral and Minho e Douro Litoral (from which hail more than half of the total policies studied), the credibility weights are rather small, except maybe in the lowest range. In practice, this translates into credibility estimators that don't really differ all that much from the third level one (being mostly within the same tenth of percentile).

It makes sense to consider if it really worth the effort to produce a hierarchical model with 3 levels with this specific data and levels. Despite there being detectable differences between the risks (as all the variance estimators were positive), it doesn't seem to add much relevancy in comparison to the order 2 models presented earlier. Also, a lot more parameters must be estimated in model of order 3. Therefore, the order 2 models might be better suited in this case, according to the parsimony principle.

One question that was raised before was the reason for higher claim costs up in the north. The reason remains unclear. But, when taking into consideration the order 3 model, the ≤ 125.000€ ranges are the only ones somewhat relevant in the second level. Comparing these with each other:

	Bg(3)	αg(3)	μ(Ψg)
Grande Lisboa	0,03309711	0,32899702	0,03169028
Extremadura e Ribatejo	0,03045850	0,46551985	0,02984591
Beira Litoral	0,03708524	0,53367886	0,03365785
Porto	0,04152833	0,05527864	0,03077541
Minho e Douro Litoral	0,04309479	0,66894321	0,04085177
Trás-os-Montes e Alto Douro	0,04618001	0,23152731	0,03747496
Beira Interior	0,03468513	0,19547522	0,03164002
Alentejo	0,02540170	0,17471946	0,02721712
Algarve	0,02032122	0,30080039	0,02303943
Açores e Madeira	0,02287301	0,09502832	0,02613280

Table 16: Second level results for under 125k policies, by region.

The conclusion is almost the same, there are higher claim costs in the north and lower in the south, even when taking into account just the small policies. Considering that these make up the bulk of the policies studied, it makes sense to ask if the credibility estimators are higher in the north because the north's smaller policies have higher claim costs (and the converse for the south). This again raises the question of why there are higher claim costs for the small policies in the north and lower in the south. It doesn't seem to have a clear answer.

The order 3 model was also applied with levels 2 and 3 in reverse, which is to say, with the data split first accordingly to the amount of capital insured (third level) and then by region (second level). But, when done this way, the variance estimator $\hat{\tau}_3^2$ for the third level was 0, and so were the credibility weights. This means that there is no detectable difference between the risks and, therefore, this level should be omitted from this classification structure.

4. Conclusions

The problem of charging the right premium to each individual risk is one that isn't possible to solve. However, there are a great variety of methods that one can use to try to get the fairest premiums. This report focused on some of these methods, and the approaches that one can take when using them.

The Bühlmann-Straub model is the most used model in Credibility for a reason. It's quite simple to put in practice and produces good results. From the models applied, it was the one that ended up attributing more credibility to each individual risk, which resulted in a great variation of premiums between policies. The results were acceptable, although one should be careful with the policies with higher claim costs (the outliers), and maybe deal with those individually, applying the model to the outlier-free data only.

The other model applied was the Hierarchical Credibility model. This model allows for a more generic way to get the individual premiums, by first taking into consideration the hierarchical structure of the data, and only afterwards analyzing the individual credibility of each policy, which results in a more leveled load of claims within the collective. The number of hierarchy levels used is up to the user.

For the application of this model (2 levels), the policies were first classified according to their size, and this produced much more leveled results than the Bühlmann-Straub model did. Afterwards, they were classified according to their geographic region, with similar results. In both cases, the removal of outliers didn't seem to really fit with the purpose of Credibility Theory, as most policies ended up with premiums very close to their size range/geographical region's average.

The same can be said when using both classifications in a 3 level structure. While the model works and gives results, they do not seem very relevant, as the 2 level models produced better results. But that isn't to say that one should never use a 3 level model. There may be other classifications, or other sets of data, that would be better suited for such a model.

Therefore, from the models applied, the ones that seemed the most useful were the Bühlmann-Straub model (before or after the removal of outliers) and both 2 level Hierarchical Credibility models (without the removal of outliers).

References

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Chauvenet, W. (1891). *A Manual of Spherical and Practical Astronomy,* Vol. II, 5th Ed. New York: Dover.

Appendix

Appendix I – Chapter 3.2. charts for outlier-free data

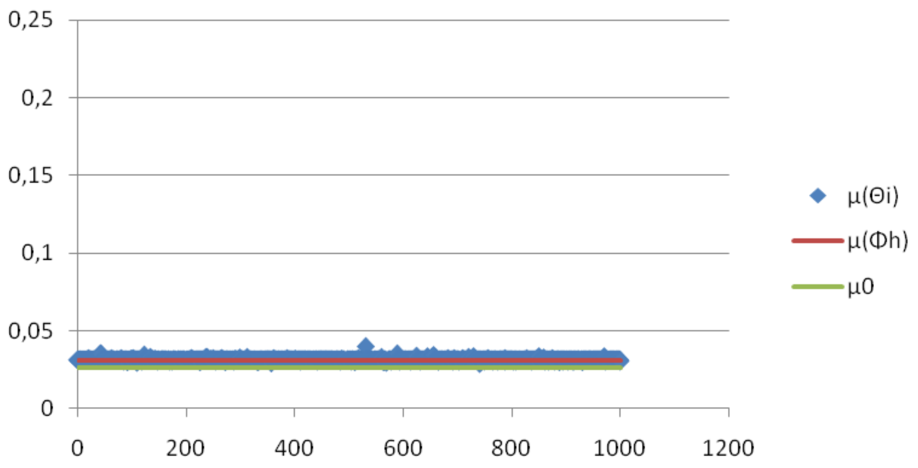

Figure 18: 125k and under policies (outlier-free).

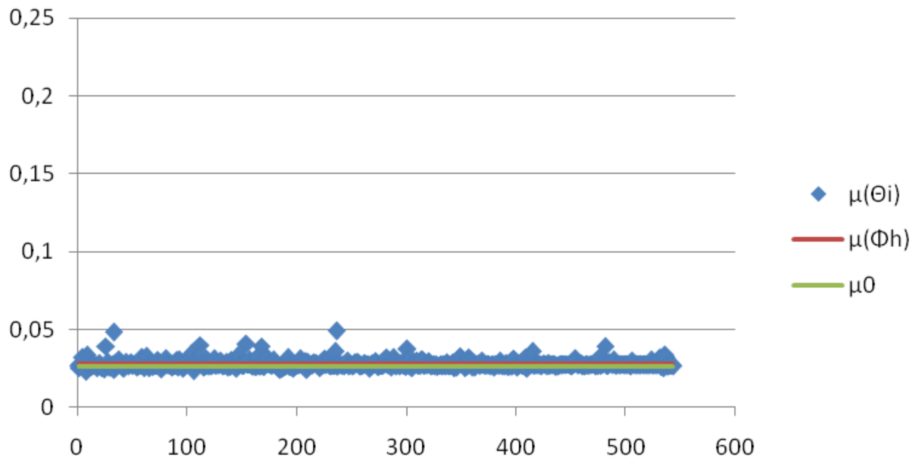

Figure 19: Policies between 125k and 250k (outlier-free).

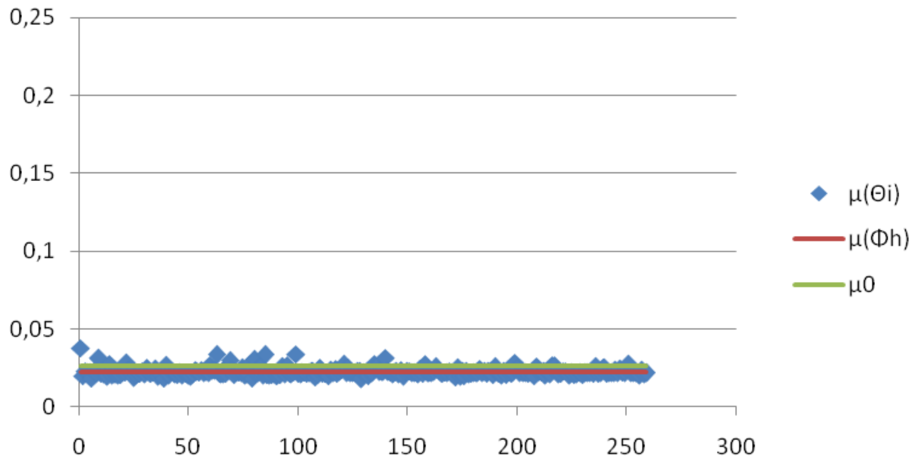

Figure 20: Policies between 250k and 500k (outlier-free).

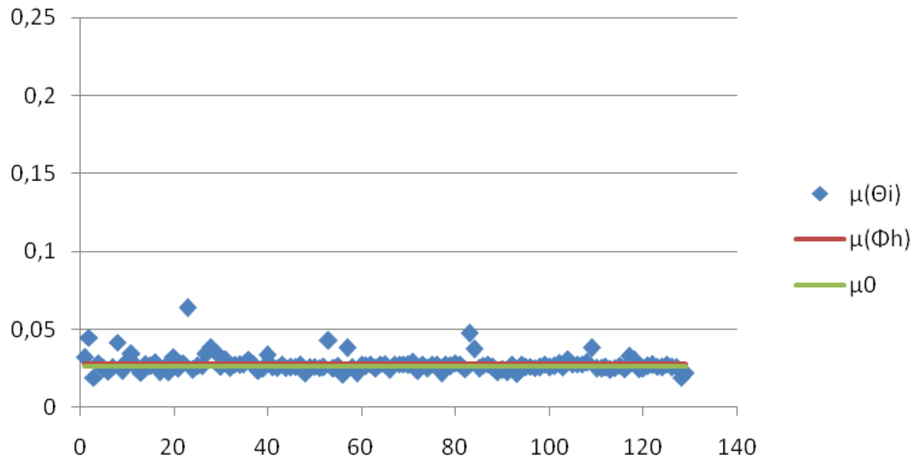

Figure 21: Policies between 500k and 1200k (outlier-free).

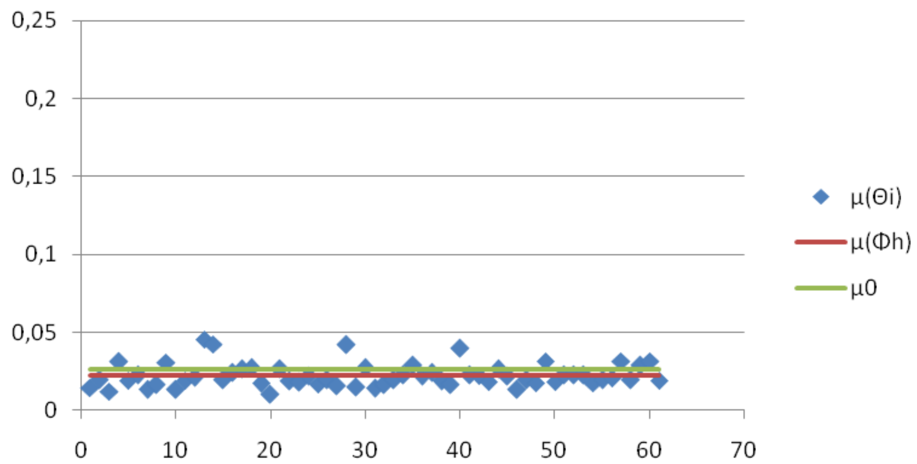

Figure 22: 1200k and over policies (outlier-free).

Appendix II – Chapter 3.3. charts for outlier-free data

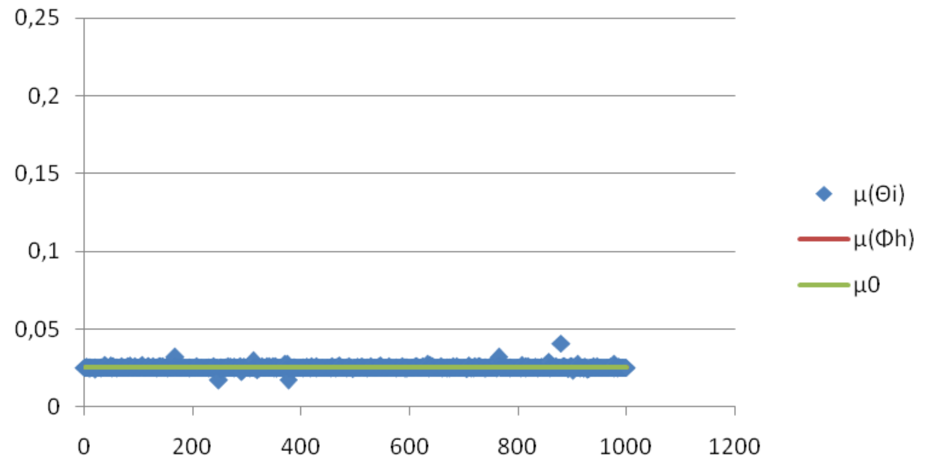

Figure 23: Policies in Grande Lisboa (outlier-free).

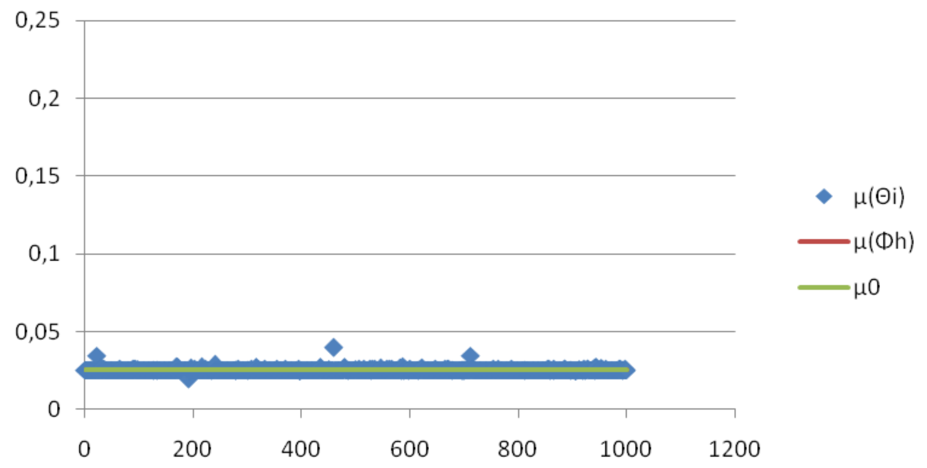

Figure 19: Policies in Extremadura e Ribatejo (outlier-free).

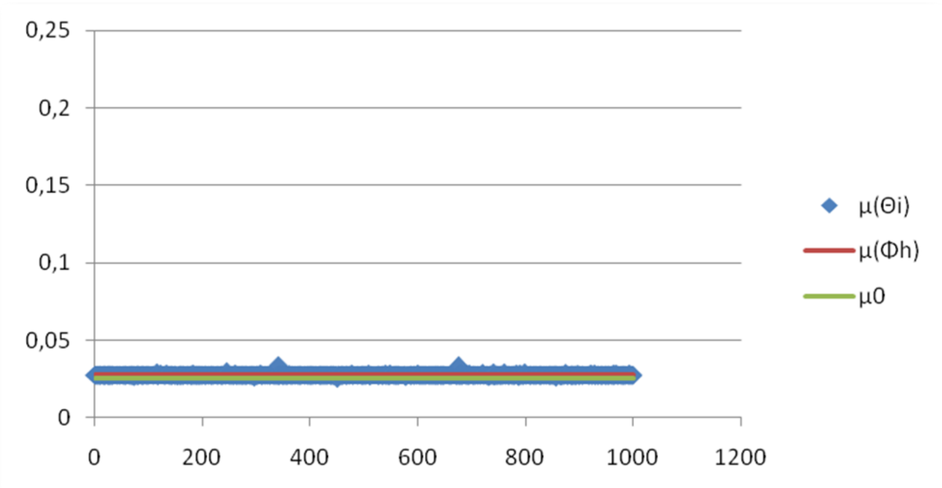

Figure 25: Policies in Beira Litoral (outlier-free).

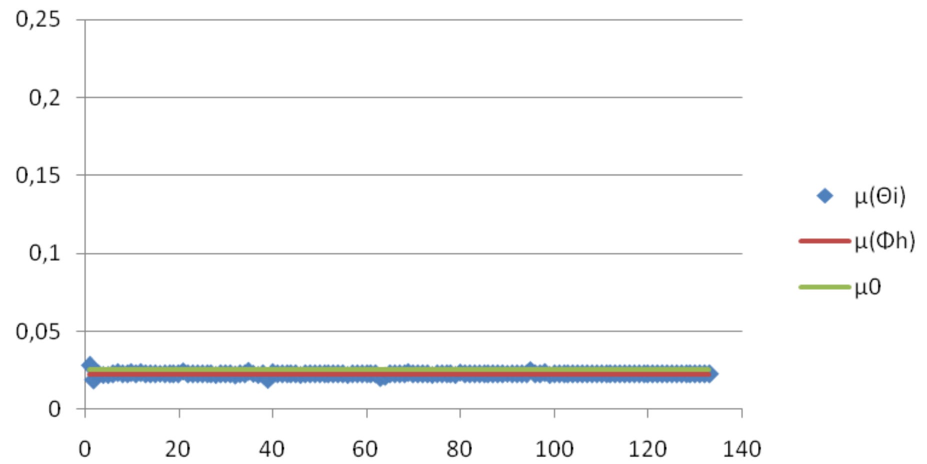

Figure 20: Policies in Porto (outlier-free)**.**

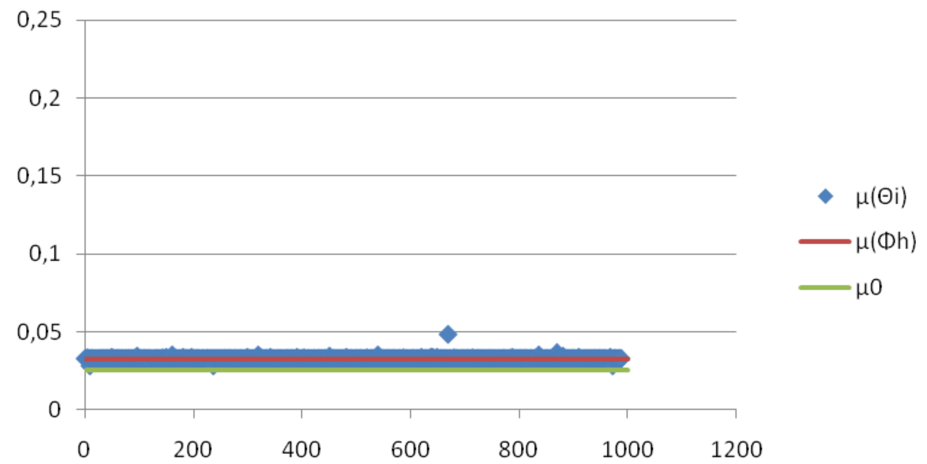

Figure 21: Policies in Minho e Douro Litoral (outlier-free).

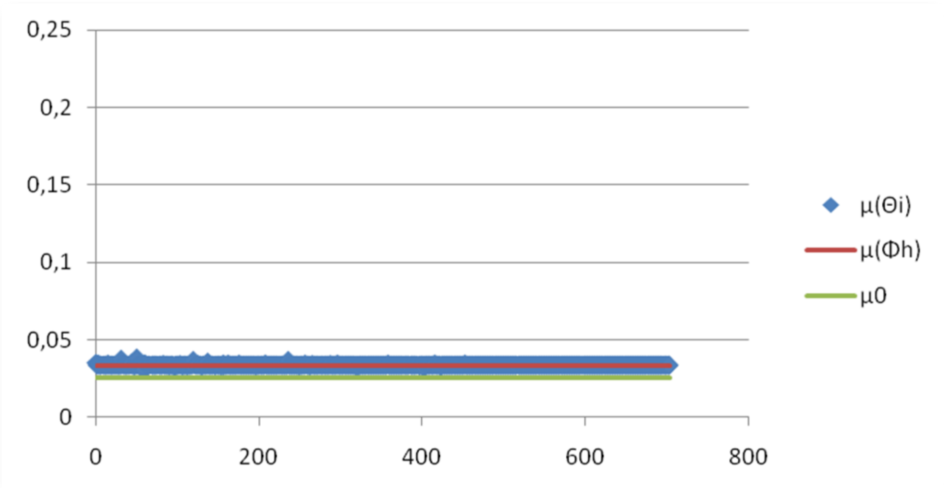

Figure 22: Policies in Trás-os-Montes and Alto Douro (outlier-free).

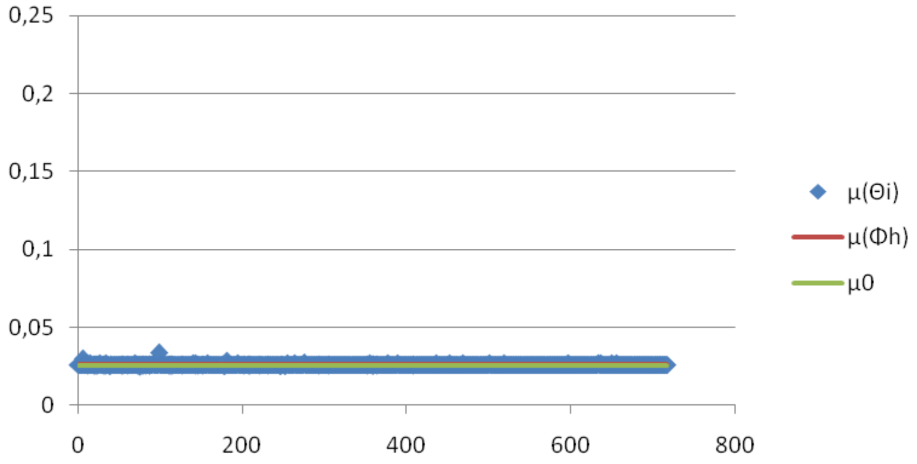

Figure 23: Policies in Beira Interior (outlier-free).

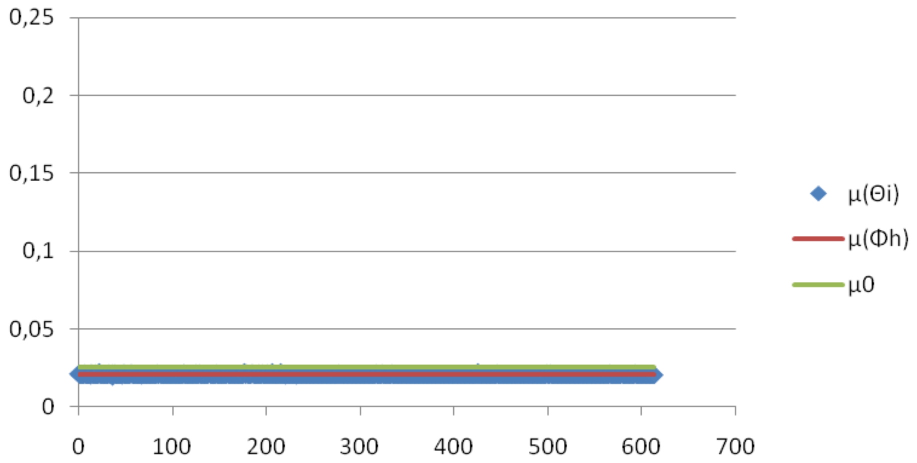

Figure 30: Policies in Alentejo (outlier-free).

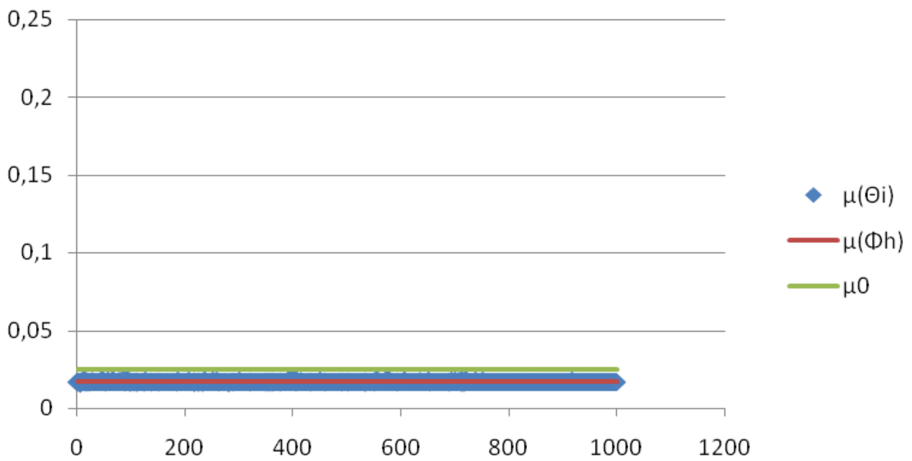

Figure 31: Policies in Algarve (outlier-free).

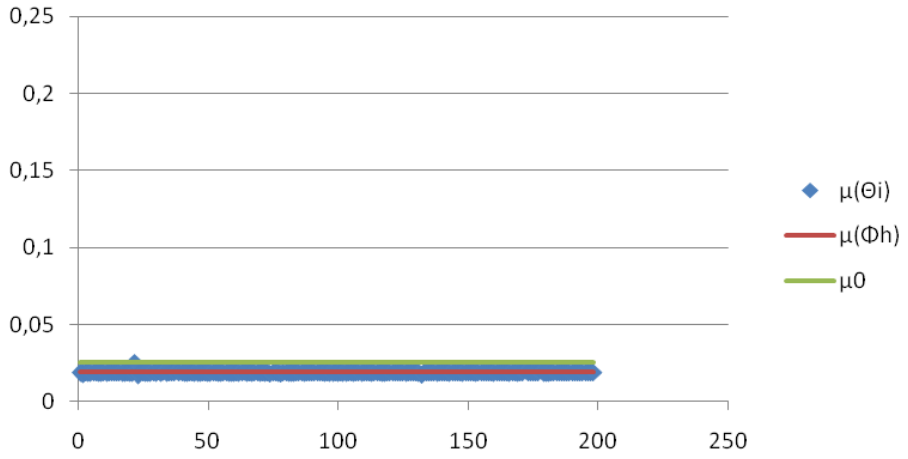

Figure 32: Policies in Açores e Madeira (outlier-free).