

# Distance-independent individual tree diameter-increment model for *Thuya* [*Tetraclinis articulata* (VAHL.) MAST.] stands in Tunisia

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## Abstract

**Aim of study:** The aim of the work was to develop an individual tree diameter-increment model for *Thuya* (*Tetraclinis articulata*) in Tunisia.

**Area of study:** The natural *Tetraclinis articulata* stands at Jbel Lattrech in north-eastern of Tunisia.

**Material and methods:** Data came from 200 trees located in 50 sample plots. The diameter at age  $t$  and the diameter increment for the last five years obtained from cores taken at breast height were measured for each tree. Four difference equations derived from the base functions of Richards, Lundqvist, Hossfeld IV and Weibull were tested using the age-independent formulations of the growth functions. Both numerical and graphical analyses were used to evaluate the performance of the candidate models.

**Main results:** Based on the analysis, the age-independent difference equation derived from the base function Richards model was selected. Two of the three parameters (growth rate and shape parameter) of the retained model were related to site quality, represented by a Growth Index, stand density and the basal area in larger trees divided by diameter of the subject tree expressing the inter-tree competition.

**Research highlights:** The proposed model can be useful for predicting the diameter growth of *Tetraclinis articulata* in Tunisia when age is not available or for trees growing in uneven-aged stands.

**Key words:** age-independent growth model; difference equations; *Tetraclinis articulata*; Tunisia.

## Introduction

*Thuya* [*Tetraclinis articulata* (Vahl.) Mast.] is endemic to the western Mediterranean region. It is native to North-Western Africa in the Atlas Mountains of Morocco, Algeria and Tunisia, with two small outlying populations on Malta, and near Cartagena in South-East Spain (Blondel and Aronson, 1999). It grows at relatively low altitudes in a hot, dry subtropical Mediterranean climate (Farjon, 2005) and it can be trimmed as a hedge (Rushforth, 1999). *Thuya* grows in dry conditions, so it is suitable for planting to prevent soil loss and desertification. In Tunisia, the geographical distribution of *Thuya* is limited to the areas close to the Cap Bon, Boukornine and a side part of Zaghouan

(Rejeb *et al.*, 1996), occupying approximately 33,000 ha (DGF, 1995) (see Fig. S1).

*Tetraclinis articulata* is a small slow-growing tree that attains at most 6-15 m of height (rarely 20 m), often with two or more trunks from the stump (Farjon, 2005). The two main products of this species have been the resin and the wood. The resin has various industrial uses. It is used to produce varnish and lacquer and it is particularly valued for preserving paintings. The wood is prized for cabinetry and was extensively used in construction by the Romans (Stevens, 2000).

Actually, the only product from *Tetraclinis articulata* forests in Tunisia is fire wood due to its small dimension. However, in order to improve the profitability of these forests, other uses of the wood of *Thuya* are possible (industry, decorative objects, etc.). To develop an artisanal and/or industrial use of the wood of *Thuya* in Tunisia, and develop a possible reforestation plan with this species there is the need to know the potential growth and production of these forests.

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This work has five Supplementary Figures that do not appear in the printed article but that accompany the paper online.

**Table 1.** Descriptive statistics for the measured diameter ( $d$ , in cm) and diameter-increment ( $id5$ , in cm) for the last five years for *Tetraclinis articulata* stands in Tunisia

Sample	Number of trees	Variable	Average	Min	Max	Standard deviation
Trees where cores was extracted	200	$d$	9.70	5.00	21.70	3.051
		$id5$	0.826	0.166	1.990	0.350
All trees	2,000	$d$	9.09	5.00	22.00	2.870

In this case, the diameter-growth dynamics of trees is one of the most important factors to study.

The objective of the present study was to develop a distance-independent individual tree diameter-increment model for *Tetraclinis articulata* stands in Tunisia. Age of *Thuya* stands is usually not known, therefore the model should not include age as a regressor. For this reason we used the methodology proposed by Tomé *et al.* (2006). Four sigmoid functions usually used to model tree and stand growth were used as candidate models using the diameter growth measured for the last five years ( $idu5$ ). In order to make the functions flexible enough to model trees growing in stands of different characteristics, the parameters of the growth functions were expressed as a function of site index, stand variables and a distance-independent competition index.

## Material

The natural *Tetraclinis articulata* stands at Jbel Lattrech in north-eastern of Tunisia (Fig. S1), with a semi-arid and warm climate, are essentially mono specific and homogeneous forests and cover about 3,000 ha. The mean annual precipitation is 393 mm. The mean of the minimum temperatures for the coldest month (January), and the mean of the maximum temperatures for the warmest month (August) amount to 8.4 and 30.5°C, respectively. The surface material in the study zone consists of Oligocene sandstone (Ben Mansoura and Garchi, 2001).

The data used to develop the diameter-increment model were obtained from a total of 50 temporary sample plots, which were installed and measured in 2009. The plots are circular, with plot size ranging from 88 m<sup>2</sup> to 835 m<sup>2</sup>, in order to assure 40 trees per plot. The diameter at breast height ( $d$ ) of trees with  $d > 5$  cm was measured to the nearest 0.1 cm. Radial increment cores were obtained using a Pressler increment borer at breast height from four trees per plot, selecting those

nearest to the plot centre within the four main quadrants. Annual radial growth was measured from the cores with the LINTAB table and TSAP software (Rinntech, 2003) and annual diameter increments were grouped by 5-year intervals from the last year (year of measurement). A total of 200 5-year diameter increment measurements, from 200 trees in 50 plots, were available for the study. Furthermore, one dominant tree per plot was felled and discs were cut at stump height (0.30 m), and from there, every 0.5 m along the stem. At each disc the number of rings was counted in order to determine the age at which the tree reached that disc height level. This information was used to develop a site index model in function of the dominant height. More information about the plots installation and data could be found in Calama *et al.* (2012). Table 1 gives the descriptive statistics for the diameter ( $d$ ) as well as the last five years diameter growth ( $id5$ ).

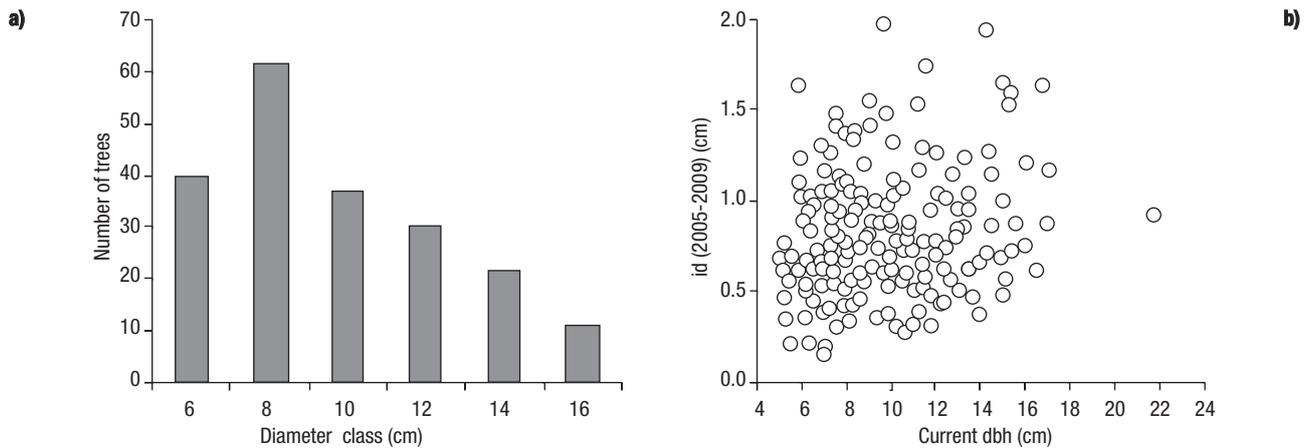
Fig. 1 presents the histogram of frequencies for the measured trees by diameter classes (a) and the relations between five years (2005-2009) diameter-increment versus current diameter at breast height (b).

Only one sampled tree had a diameter larger than 20 cm (Fig. 1), while the second largest tree had only 17 cm. For this reason we decided to exclude the largest tree from the data set used to construct the diameter growth model.

## Methods

### Models

The diameter growth models were built by assuming that the diameter increment of each tree for the next five years, after the measurements taken during 2009, will be the same as the observed diameter increment of the past five years. This assumption was retained after the comparison of the past 5-year increment with the increment attained in the previous five years (10 to 6 years before plot installation) by applying a paired



**Figure 1.** Histogram of the number of trees per diameter classes (a) and scatter-plot of diameter-increment in the last five years (2005-2009) vs current diameter at breast height (b).

t-test (Dagnelie, 1998). The obtained result showed no significant difference between the two increment periods ( $p = 0.111$ ). This way, the present values of the tree and stand variables (that will be used as explanatory variables in the regression analysis) influencing the future diameter growth are known. This assumption is realistic, namely due to the slow-growing characteristic of this species (Farjon, 2005). Additionally, *Thuja*'s bark, being very thin, was considered negligible. In this way, diameter increment was considered to be equal to the thickness of the last 5 annual growth rings.

To develop the age-independent diameter-increment model for individual tree we used the methodology described by Tomé *et al.* (2006). Four dynamic equations were tested. These equations are derived from the base functions of Richards (Richards, 1959), Lundqvist-Korf (Korf, 1939; Lundqvist, 1957), Hossfeld IV (cited *e.g.* in Zeide, 1993 and Kiviste *et al.*, 2002) which is equivalent to the so-called log-logistic function (*e.g.* Ciesewski, 2000; Dieguéz-Aranda *et al.*, 2006) and Weibull 1951 (cited *e.g.* in Gea-Izquierdo *et al.*, 2008). These four base growth functions (hereafter referred as M1, M2, M3, and M4) are usually used for tree and stand growth modelling. Table 2 gives the base equations and their forms as age-independent difference equations. Omitting the vector of model parameters, dynamic equations expressed as age-independent growth functions have the general form:

$$Y_{i+a} = f(Y_i, a)$$

where  $Y_{i+a}$  is the value of the response variable (breast height diameter) after a period of  $a$  years, and  $Y_i$  is its current value.

### Expressing the parameters of the growth functions as a linear function of stand variables and competition measures

As mentioned by Tomé *et al.* (2006), at least one of the parameters of the age-independent growth functions has to be expressed as a function of site quality and other stand or tree variables that may lead to different growth rates. Otherwise, the model obtained will be an average model that represents growth for average conditions and that is not able to reflect the differences between individuals.

The four candidate models were thereafter adjusted by expressing the parameters  $k$  (growth rate parameter) and  $m$  (shape parameter) as linear functions of variables (or transformations of them) representing the effects of stand density, inter-tree competition and site quality. The stand variables that were screened for this reparameterization were stand basal area ( $G$ ), number of trees per ha ( $N$ ) and stand density index ( $SDI$ ). Stand density index was computed as (Reineke, 1933):

$$SDI = N \left( \frac{25}{d_g} \right)^{-1.605} \quad [1]$$

where  $d_g$  is the quadratic mean diameter. This stand density measure is less influenced by stand age and site quality than  $G$  and  $N$  and has therefore been used with success by several authors (Fabian *et al.*, 2008).

Several distance-independent competition indices were tested to express inter-tree competition: tree relative dimension ( $d_i/d_g$ ), basal area in larger trees ( $G_{>di}$ : basal area of trees greater than the subject tree)

**Table 2.** Candidate functions

Base equation	Solution for age $t_i$	Dynamic equation	Model
Richards $Y_i = A \left(1 - e^{-kt_i}\right)^{\frac{1}{1-m}}$	$t_i = -\frac{1}{k} \ln \left(1 - (Y_i / A)^{1-m}\right)$	$Y_{i+a} = A \left(1 - e^{-ka} \left(1 - (Y_i / A)^{1-m}\right)\right)^{\frac{1}{1-m}}$	M1
Lundqvist $Y_i = Ae^{\frac{-k}{t_i^m}}$	$t_i = \left(\frac{-k}{\ln(Y_i / A)}\right)^{\frac{1}{m}}$	$Y_{i+a} = Ae^{\frac{-k}{\left(\left(\frac{-k}{\ln(Y_i / A)}\right)^{\frac{1}{m}} + a\right)^m}}$	M2
Hosfeld IV $Y_i = \frac{At_i^m}{k + t_i^m}$	$t_i = \left(\frac{Y_i k}{A - Y_i}\right)^{\frac{1}{m}}$	$Y_{i+a} = A \times \frac{\left(\left(\frac{Y_i k}{A - Y_i}\right)^{\frac{1}{m}} + a\right)^m}{k + \left(\left(\frac{Y_i k}{A - Y_i}\right)^{\frac{1}{m}} + a\right)^m}$	M3
Weibull 1951; Yang <i>et al.</i> , 1978: $Y_i = A \left(1 - e^{-kt_i^m}\right)$	$t_i = \left(-\frac{1}{k} \ln(1 - Y_i / A)\right)^{\frac{1}{m}}$	$Y_{i+a} = A \left(1 - e^{-k \left(\left(-\frac{1}{k} \ln(1 - Y_i / A)\right)^{\frac{1}{m}} + a\right)^m}\right)$	M4

$Y_i$ : variable Y at time  $t_i$ .  $a$ : projection length (years). In: natural logarithm.

and basal area in larger trees divided by diameter of the subject tree ( $G_{>di}/d_i$ ).

To express the parameters  $k$  (growth rate parameter) and  $m$  (shape parameter) as linear functions of site quality, we tested two site quality indices, the site index ( $S$ ) and the growth index ( $GI$ ). The site index is usually used as site quality effect for even-aged stands and was computed with the site index curves (Sghaier *et al.*, in prep) developed by using the Generalized Algebraic Difference Approach (GADA):

$$Y = \exp(X_0)(1 - \exp(-0,0079t))^{(0,539+1/X_0)} \quad [2]$$

with

$$X_0 = -\frac{1}{2} \left( \ln Y_0 - 0,539F_0 \pm \sqrt{(\ln Y_0 - 0,539F_0)^2 - 4F_0} \right)$$

and

$$F_0 = \ln(1 - \exp(-0,0079t_0))$$

where  $Y$  is the predicted height (m) at age  $t$  (years), and  $Y_0$  and  $t_0$  represent the predictor height and age.

The growth index ( $GI$ ) was identical to the one used by Trasobares and Pukkala (2004) and Trasobares *et al.* (2004) in uneven aged stands:

$$GI = \frac{1}{n} \sum_{i=1}^n \left( \frac{id5_i}{\hat{id5}_i} \right) \quad [3]$$

where  $n=4$  (number of trees in which diameter increment in the last 5 years has been measured);  $id5_i$  and  $\hat{id5}_i$  are the observed and estimated average increments for the last 5 years. This last variable represents tree growth for an average site. In order to estimate the diameter average increment in the last 5 years, several variables were tested as regressors such as tree diameter, stand density variables and several distance-independent indices. The retained model to predict the average diameter increment of *Tetraclinis articulata* trees was as follow (Trasobares *et al.*, 2004):

$$id5_i = a_1 + a_2 \times \ln \left( \left( \frac{G_{>di}}{\ln(d_i + 1)} \right) + 1 \right) + e \quad [4]$$

For each function (M1, M2, M3 and M4) the reparameterization with the smallest root mean square error (RMSE) was selected for further comparison with the other candidate models in terms of goodness-of-

fit and predictive ability. All the selected reparameterized models had all the parameters significantly different from zero (with  $\alpha = 0.05$ ), as evaluated by asymptotic t-tests. Violation of the homocedasticity and normality of the errors of the fitted models were checked by inspecting the plots of the studentized residuals *versus* predicted values and the QQ-plots of the studentized residuals, respectively.

### Model comparison

The studied models were compared by taking into account their fitting and predictive abilities. The fitting performance of the selected models was evaluated by examining values of the root mean square error (*RMSE*) and the Akaike's information criterion differences ( $\Delta AIC$ ) (Burnham and Anderson, 2002).  $\Delta AIC < 2$  suggests substantial evidence for the model; values between 3 and 7 indicate that the model has considerably less support, whereas  $\Delta AIC > 10$  indicates that the model is very unlikely.

The predictive ability of the models was evaluated using prediction errors or *PRESS* residuals (Prediction Sum of Squares):

$$PRESS = \sum_{i=1}^n (y_i - \hat{y}_{i,-i})^2 \quad [5]$$

where  $y_i$  is the observed value,  $\hat{y}_{i,-i}$  is the estimated value for observation  $i$  (where the latter is absent from the model fitting data set) and  $n$  is the number of observations.

*PRESS* residuals were also used to compute statistics to evaluate the Prediction Mean of Absolute Deviations (*PREMAD*), bias ( $Bias_p$ ) of prediction corresponding to the mean deviations and modeling efficiency ( $M_{eff}$ ), similar to the coefficient of determination for linear regression, as well as a statistic equivalent to the *RMSE*.

Another important step in evaluating the models was the graphical analysis of the residuals. Visual or graphical inspection is an essential point in selecting the most appropriate model because curve profiles may differ drastically, even though the statistics and residuals of the fit are similar (e.g. Huang *et al.*, 2003). Plots of the  $Bias_p$  and  $RMSE_p$  calculated by diameter classes, site index ( $S$ ) classes, stand density classes and growth index ( $GI$ ) classes were analyzed in order to detect eventual tendencies of the residuals with these variables.

The fitting and analysis of the performance of the models has been accomplished by non-linear least squares using the SAS/ETS<sup>®</sup> MODEL procedure (SAS Institute Inc., 2004).

### Results

For the adjustment of the diameter average increment model in the last 5 years (equation [4]), which is necessary to estimate the growth index ( $GI$ ) for each plot, the following results were obtained:

$$id5_i = 1.051 - 0.177 \times \ln \left( \left( \frac{G_{>di}}{\ln(d_i + 1)} \right) + 1 \right) \quad [6]$$

where the two estimated parameters were significantly different from zero ( $p < 0.0001$ ) with  $R^2 = 0.09$  and  $RMSE = 0.336$ . The  $R^2$  value is low but the fitted function expresses the average growth expected for a tree without taking into account the variability of the site, therefore with a large proportion of the variability not explained by the model. Additionally, it is important to emphasize that the variability in  $id5$  is not very large and that the  $R^2$  is not a good measure of fit when the variability of the dependent variable is low. Anyhow, the value is similar to the value 0.02 obtained by Trasobares *et al.* (2004).

To measure the relationship between the two estimated variables ( $S$  and  $GI$ ) used to describe the site quality of the measured plots, Pearson correlation was calculated. The obtained low value of this relation,  $r_{S \times GI} = 0.367$  ( $p = 0.0088$ ) indicates that the productivity of the measured plots is classified differently according to the index used. For this reason and with the objective to choose the most adequate variable representing the site quality to express the parameters  $k$  and  $m$  for the adjustment of the four tested growth functions, we used each of them separately with the other stand variables to define alternative models. According to the obtained results (Tables 3 and 4) concerning the values of the various statistical criteria used to compare the models, it appeared that models using growth index ( $GI$ ) as site quality measure give more precise results than those obtained by models using site index ( $S$ ). In addition and according to the obtained values of the  $\Delta AIC$  criteria for each case, those obtained for models M2, M3 and M4 by using site index (Table 3) exceed largely the value 10 fixed by Burnham and Anderson (2002) as a critical value to indicate that the models are very unlikely. However, when we used

**Table 3.** Parameter estimates and values of statistics appraising goodness-of-fit and predictive ability of the selected models using site index (*S*) as site quality effect

Model	k, m = f(stand variables)	Par.	Estimate	SE	p-value	Fitting ability		Prediction ability			
						RMSE	ΔAIC	Biasp	RMSEp	M <sub>efp</sub>	PREMAD
M1	$k = a_1 \times \sqrt{S}$ $m = a_2 \times \ln(SDI) + a_3 \times \sqrt{\frac{d_i}{d_g}}$	A	190	—	—						
		a <sub>1</sub>	4.98E-4	2.19E-4	0.0243						
		a <sub>2</sub>	8.852E-2	1.87E-2	<0.0001	0.3236	0	-0.0012	0.3287	0.9883	0.2657
		a <sub>3</sub>	-0.419	0.173	0.0164						
M2	$k = a_1 \times \ln(S)$ $m = a_2 \times SDI + a_3 \times \frac{d_i}{d_g}$	A	220	—	—						
		a <sub>1</sub>	5.347	0.4647	<0.0001						
		a <sub>2</sub>	1.976E-4	2.3E-5	<0.0001	0.6882	300.297	-0.3769	0.6959	0.9477	0.5727
		a <sub>3</sub>	0.132	1.73E-2	<0.0001						
M3	$k = a_1 \times \ln(S)$ $m = a_2 \times SDI + a_3 \times \frac{d_i}{d_g}$	A	120	—	—						
		a <sub>1</sub>	90.247	19.288	<0.0001						
		a <sub>2</sub>	8.2E-5	3.8E-5	0.0324	0.4827	159.105	-0.1126	0.4911	0.9749	0.3827
		a <sub>3</sub>	0.628	4.77E-2	<0.0001						
M4	$k = a_1 \times \frac{1}{S}$ $m = a_2 \times SDI + a_3 \times \frac{d_i}{d_g}$	A	120	—	—						
		a <sub>1</sub>	7.827E-2	9.01E-3	<0.0001						
		a <sub>2</sub>	2.91E-4	4.1E-5	<0.0001	0.6257	262.425	-0.2706	0.6335	0.9567	0.5170
		a <sub>3</sub>	0.310	2.65E-2	<0.0001						

*S*: Site Index. *d<sub>i</sub>*: current diameter of the subject tree at time *t<sub>i</sub>*. *d<sub>g</sub>*: quadratic mean diameter. *SDI*: stand density index. *SE*: standard error. *RMSE*: root mean square error. *ΔAIC*: AIC difference from the best model. *Biasp*: bias (computed from *PRESS* residuals). *RMSEp*: root mean square error (computed from *PRESS* residuals). *M<sub>efp</sub>*: model efficiency (computed from *PRESS* residuals). *PREMAD*: prediction mean of absolute deviations.

growth index (*GI*) to fit models, only Model M4 (Weibull) present a value of *ΔAIC* criteria (16.44) greater than the critical value of *ΔAIC* criteria (Table 4). So, on the basis of these results, we concluded that the growth index (*GI*) is a better measure to express the site quality of the studied stands and was therefore used to fit models of diameter increment for Thuya in Tunisia.

Table 4 shows the results obtained for the best models found for each growth function by using the growth index (*GI*) as a site quality measure. The four compared models showed a good fitting and predictive ability with a slight superiority of model M1 (Richards) followed by model M3 (Hossfeld IV). No heteroecasticity problems were found (Fig. S2) and the QQ-plots do not give evidence of non-normality of the model errors except for model M4 (Fig. S3).

When plotting the bias and RMSE (Fig. S4 and S5), computed with *PRESS* residuals vs diameter class, site index class, growth index class and stand density class, the superiority of models M1 and M3 is apparent especially for the bias (Fig. S4). On another side, M3 had the smallest value of *ΔAIC* criteria (1.54) after M1 which had zero as obtained value for the same criteria when the site index (*S*) and the growth index (*GI*) were

used separately as a site quality measure to express the parameters *k* (growth rate parameter) and *m* (shape parameter) as linear functions of site quality (Tables 3 and 4). According to Burnham and Anderson (2002), this value of *ΔAIC* criteria less than 2 suggests that models M1 and M3 performance is similar. However, model M1 is the one that provides the smaller values of RMSE for all used stand variables classes (Fig. S5).

The selected age-independent diameter-increment model, derived from the Richards function, is as follow:

$$d_{i+a} = A \left[ 1 - e^{-k \cdot a} \left( 1 - \left( \frac{d_i}{A} \right)^{1-m} \right) \right]^{\frac{1}{1-m}} \quad [7]$$

with:

$$A = 170$$

$$k = 0.002146 \times \sqrt{GI} - 0.00013 \times \ln(N)$$

$$m = 0.08657 \times \sqrt{\frac{G_{>d_i}}{d_i}}$$

where *A* indicates the upper fixed asymptote (cm), *GI* is the growth index (equation [3]), *d<sub>i</sub>* is the current diameter of tree at time *t<sub>i</sub>* (cm), *G<sub>>d<sub>i</sub></sub>* is the basal area in larger trees or basal area of trees greater than the

**Table 4.** Parameter estimates and values of statistics appraising goodness-of-fit and predictive ability of the selected models using growth index (*GI*) as site quality effect

Model	k, m = f(stand variables)	Par.	Estimate	SE	p-value	Fitting ability		Prediction ability			
						RMSE	ΔAIC	Biasp	RMSEp	M <sub>efp</sub>	PREMAD
M1	$k = a_1 \times \sqrt{GI} + a_2 \times \ln(N)$ $m = a_3 \times \sqrt{\frac{G_{>di}}{d_i}}$	A	170	—							
		a <sub>1</sub>	2.146E-3	1.88E-4	<0.0001						
		a <sub>2</sub>	-1.3E-4	2.5E-5	<0.0001	0.2752	0	0.0010	0.2798	0.9916	0.2217
M2	$k = a_1 \times \frac{1}{GI} + a_2 \times \ln(N)$ $m = a_3 + a_4 \times \frac{G_{>di}}{d_i}$	A	150	—							
		a <sub>1</sub>	32.299	16.261	0.0484						
		a <sub>2</sub>	2.513	0.819	0.0025	0.2765	2.78	-0.0030	0.2836	0.9914	0.2250
		a <sub>3</sub>	0.643	0.074	<0.0001						
M3	$k = a_1 \times \frac{1}{GI} + a_2 \times \sqrt{N}$ $m = a_3 + a_4 \times \sqrt{\frac{d_i}{d_g}}$	A	120	—							
		a <sub>1</sub>	250.821	97.924	0.0112						
		a <sub>2</sub>	1.171	0.574	0.0428	0.2756	1.54	0.0021	0.2828	0.9914	0.2228
		a <sub>3</sub>	0.636	0.107	<0.0001						
		a <sub>4</sub>	0.217	0.042	<0.0001						
M4	$k = a_1 \times \sqrt{GI} + a_2 \times SDI$ $m = a_3 + a_4 \times \sqrt{\frac{d_i}{d_g}}$	A	120	—							
		a <sub>1</sub>	4.273E-3	2.03E-3	0.0363						
		a <sub>2</sub>	-1.27E-6	4.84E-7	0.0094	0.2861	16.44	0.0072	0.2928	0.9908	0.2314
		a <sub>3</sub>	0.601	0.141	<0.0001						
		a <sub>4</sub>	0.211	0.046	<0.0001						

*GI*: growth index. *SDI*: stand density index. *G<sub>>di</sub>*: basal area in larger trees (basal area of trees greater than the subject tree). *d<sub>i</sub>*: current diameter of the subject tree at time *t<sub>i</sub>*. *d<sub>g</sub>*: quadratic mean diameter. *N*: stand density. *SE*: standard error. *RMSE*: root mean square error. *ΔAIC*: AIC difference from the best model. *Biasp*: bias (computed from *PRESS* residuals). *RMSEp*: root mean square error (computed from *PRESS* residuals). *M<sub>efp</sub>*: model efficiency (computed from *PRESS* residuals); *PREMAD*: prediction mean of absolute deviations.

subject tree (m<sup>2</sup>), *N* is the stand density (trees/ha) and *a* indicates the projection length (years).

## Discussion and conclusions

The technique of modeling used in this study was based on the development of the growth models of the trees independently of their age. This approach of modeling which can be used when age is not available (or difficult to obtain) or for trees growing in uneven-aged stands, was used successfully for the first time by Tomé *et al.* (2006) to establish models for dominant height growth of eucalyptus (*Euclayptus globulus* Labill.) plantations and the individual tree growth in diameter at breast height in sparse cork oak (*Quercus suber* L.) stands in Portugal. Thereafter, the same technique was used in Spain to model growth in diameter of *Quercus ilex* L. (Gea-Izquierdo *et al.*, 2008). The present study represents another application of this new approach of modeling growth in diameter of another Mediterranean

species (*Tetraclinis articulata*) which is not well known and less studied.

The Tunisian *Tetraclinis* forests are a natural forests which assumes until now a protective role. So, few forestry interventions are made in them. Moreover, as the studied *Tetraclinis articulata* area was cut in the beginning of the last century, it was originally regenerated as even-aged but later on the abundance of the natural regeneration lead to a more or less uneven-aged structure which can be seen in Fig. 1 by the typical inverse j or negative exponential distribution of number of trees over diameter classes. So, to localize the models for the different sites, stands and trees and to express the parameters of the growth functions as a linear function of stand variables and competition measures, we compared the use of the site index in function of the dominant height (*S*) and of the growth index (*GI*) presented by Trasobares and Pukkala (2004) and Trasobares *et al.* (2004) to describe the site quality, with the other stand variables. The site index variable is used for the even-aged forest; however the growth index can be used for

uneven-aged forest. The obtained results showed the superiority of the growth index to explain the quality site of the studied stands which originated smaller values of the residual mean square error that, according to the *AIC* criterium, were considered important. In addition, contrary to the site index that has significance for the even-aged stands, the growth index can be considered a good stand parameter in predicting diameter-increment model for both even-aged and uneven-aged stands. In this case, the potential application of the selected model is wide as it could be used with all stand structures.

The model selection procedure indicates that the Richards function (7) explained 99.2% of the variation, with the parameters expressed as linear functions of site quality, expressed by the growth index, of stand density and of inter-tree competition, resulted in the best compromise between biological and statistical aspects, producing the most adequate equation to predict the individual tree diameter-increment for *Thuya* in Tunisia. To give a large use of this model, two of the three parameters of the model were related to the stand characteristics. The parameter  $k$  (growth rate parameter) was related to the site quality indicator represented by the Growth Index (*GI*) and the stand density ( $N$ ). The parameter  $m$  (shape parameter) was related to the basal area in larger trees divided by the diameter of the subject tree ( $G_{>d_i}/d_i$ ) expressing the inter-tree competition.

Today is accepted that to apply a sustainable management of the forest systems and especially in fragile systems such as the Mediterranean forest we need to have knowledge of the dynamics and growth of these forest systems. In this sense this work provides an individual tree diameter increment model in function of growth index, stand variables and competition index for *Thuya* in Tunisia. The constructed model, together with the already existing single tree models for breast height diameter- total height relation, crown dimensions and stem curve (Calama *et al.*, 2012), The diameter distribution model (Sghaier *et al.*, *in review*) and the dynamic and static models for stand growth and yield presented in Sghaier *et al.* (in prep), constitute the basis for developing an integrated distance independent single tree-level model for the sustainable management of the species.

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