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Subprime Crisis, Systematic Risk and Arbitrage

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Abstract

The financial market turmoil of 2007 and 2008 was the most severe recession seen after the Great Depression. The economic momentum, previous to the crisis, created a strong demand for AAA securities that was not available on single bond market. This motivated arrangers to issue high volumes of structured finance securities, collateralized by subprime Residential Mortgage-Backed Securities. Most of the AAA investors based their choices uniquely based on Credit Rating Agencies' assessment that taken into account Probabilities of Default or Expected Losses but mispriced the Systematic Risk. This fact created arbitrage opportunities for Asset-Backed Securities (ABS) and ABS Collateralized Debt Obligations (ABS CDO) arrangers. They exploited it by issuing securities with high levels of Systematic Risk. In this work I run ABS and ABS CDO simulations to exemplify how these instruments were structured to explore these features. To quantify the extent of the mispricing I compare a structured security with a single bond with the same rating. I use the results to exemplify the potential gains obtained by the arranger by taking advantage of the investor's blindness concerning Systematic Risk. I also approach other Structured Finance risks as parameter sensitivity and biased asset pool parameters in rating models.

Keywords: Subprime Crisis, Systematic Risk, Collateralized debt obligations (CDO), Arbitrage CDOs

Jel Classifications: C13, G01, G12, G24

Contents

Nomenclature

ABS – Asset-backed security

Alt-A – Alternative-A

CAPM – Capital Asset Pricing Model

CDO – Collateralized Debt Obligation

CDS - Credit Default Swap

CRA – Credit Rating Agency

EL – Expected Loss

IPD – Implied Probability of Default

LGD – Loss Given Default

MBS – Mortgage-Backed Security

MSE – Minimum Square Root Error

PD – Probability of Default

PIT – Point-in-Time

RMBS – Residential Mortgage-Backed Security

SF – Structured Finance

TTC – Through-the-Cycle

“The crisis was the result of human action and inaction, not of Mother Nature or computer models gone haywire.”

Conclusions of the Financial Crisis Inquiry Commission

1. Introduction

The financial market turmoil of 2007 and 2008 was the most severe recession seen after the Great Depression. Several millions of dollars had to be written-down because of the credit defaults in the U.S. mortgage market, major bank stocks fell down to less than half of their prices, suddenly banks liquidity dried-up and bailouts came into scene.

Although derivatives are not the direct cause of the crisis as debt levels have been building since the 1980's, unregulated derivatives facilitated leverage and thus the potential for contagion to unrelated markets and the "real" economy. Complex OTC derivatives helped to increase the opaqueness of risk in the markets. In particular "securitized instruments like CDO are thought to be not only a driving force behind the housing market boom but also largely responsible for the damage to the banking sector" (Griffin and Tang, 2010).

At the same time, important transformations occurred in the banking system. Traditionally banks adopted a "sell and hold" policy for their loans but in the beginning of this century, this approach has been widely substituted by a "sell, pool, tranche and resell via securitization" policy. By doing this, banks were arguing that the system was being more and more stable because the agents most prepared to take it carried the risk. To discharge the risk, banks created more and more structured products, among which were the Asset-Backed Securities (ABS). Besides the risk allocation motivation, there were clearly an arbitrage motivation, with the purpose of gaining from the difference between the assets and liabilities of the structures. The ABSs were sometimes repackaged to generate CDOs (Collateralized Debt Obligations) in order to potentiate even more arbitrage gains.

The result of this growing activity was a significant contribution for the creation of what is known as "shadow banking system" with a size compared to the traditional system but with no capital requirements (or lender of last resort), which helped to feed the boom in housing prices.

By 2006, the underlying collateral for many CDOs, subprime MBS, began to experience sharp increases in delinquencies and defaults. In the wake of sharp erosion in collateral performance, spreads began to widen markedly in both subprime MBS and CDOs and consequently the issuance of CDOs began to decline. By 2007, the issuance of CDOs backed by subprime MBS had ceased. See Figure 1 (List of Figures).

The rating agencies have come under a huge condemnation since the subprime crisis started in July 2007, because of the rating they attributed to some structured finance products linked to the subprime real estate segment. It is a common place to consider that the agencies behaviour in the process of ratings was both immoral[1], because of the agency risk, and also negligent, as they were not aware of the risk and the dynamics of those products. More than a condemnation of rating agencies, it was created hysteria against the "demons" of quantitative finance in general and credit derivatives related with the subprime in particular. Public press and general public opinion makers were tough to the Wall Street barons and to the Math Finance "geeks". Suddenly all real economy problems had a well-defined origin. But the structured finance industry didn't really appear from one day to another. There was a long maturity process before achieving the complexity of instruments we've seen. Furthermore the arguments against any kind of the structured instruments weren't based on solid arguments. This work aspires to be a contribution to a more serious and rigorous discussion.

Firstly I present the literature over the matters related to the subprime crisis in a sequence that makes clear some motivations behind relevant "players". In this review I start by introducing the ABS (Asset-Backed Securities) and Residential Mortgage-Backed Securities (RMBS) in general and Subprime RMBS in particular as the first layer securitization. Then I focus on ABS CDO, a second layer securitization. In this section, besides describing the structures and motivations behind ABS CDO, I also introduce a demand side factor that could be the origin and motivation for using Subprime RMBS as collateral. Afterwards I summarize the main risks concerning these complex structured finance securities with special focus on Credit Rating Agencies (CRA) and the mispricing of systematic risk. The mispricing of the systematic risk may be one of the causes to support the explosive growth of Arbitrage CDO in the years before the crisis. The theoretical section closes with an overview of credit structural models that will support the second section of this work.

In the second section I describe the theoretical framework to create a simulation model based on VBA and MS Excel, capable of giving conditional and unconditional Probability of Default, Expected Loss and Implied Probability of Default. Furthermore by exploiting the solver add-in I also generate an implied correlation factor of a single-name instrument that matches a multi-name security for the same level of rating.

[1] See, for example, "The Alchemy of CDO Credit Ratings", Benmelech and Dlugosz (2008)

2. Theoretical Framework

Mortgage-Backed Securities (MBS)

In 1970's U.S. Government National Mortgage Association (GNMA) sold the first mortgage-backed securities. In 1980's the so-called ABS market expanded through other assets as credit cards and car loans. There is a much diversified range of justifications attributed to securitization; among them we highlight the most quoted.

A very short sample of the vast literature on the theoretical justifications of securitization are the works of Stulz and Johnson (1985), DeMarzo (2005), Parlour and Plantin (2008), and Brennan, Hein, and Poon (2009), just to quote few examples.

Ashcraft and Schuermann (2008) meticulously explain the MBS securitization process and its potential drawbacks.

The prevalence of securitization suggests the existing of long-term motivations for these instruments.

Subprime MBS

To understand the growth of subprime mortgages market we must recall that in the period before the crisis it was observable two different states of these factors according to Coleman, LaCour-Little, and Vandell, (2008). From 2001 to 2003 house prices were increasing and interest rates were decreasing preparing extraordinary benign conditions for home ownership. From late 2003 to 2006 house prices kept increasing but interest rates increased, too, thus reducing housing affordability. See Figures 3 and 4 (List of Figures).

The U.S. governmental agencies, Fannie Mae, Freddie Mac and Ginnie Mae were mandated to purchase mortgage loans to local banks. The purchase of these loans was based on verification of eligibility criteria in terms of quality and size of the portfolio. Loans were then repackaged and placed in the capital markets with U.S. government guarantee.

The mortgage loans that by some reasons were not eligible could be maintained by local banks or be placed in the secondary market with "private-labels" by Wall Street banks. With the mentioned reduction of affordability since 2003, there was a growing share of non-agency mortgages and MBS.

From the total securitizations in 2007 (2Q), the agency mortgages represent about 66%, leaving the rest as non-agency mortgages. Within the category of non-agency are the jumbo prime, too large mortgages portfolios to be eligible for government programs (8%), the alternative-A or alt-A mortgages, more risk layering than those of the standard governmental agencies (13%) and subprime borrowers that have generally lower credit quality than the required by the agencies (13%)[2].

In the years before the crisis there was a significant increase in the issue of securitizations based on the subprime segment. Furthermore, it was observed, as for example in Ashcraft and Schuermann (2008), the deterioration in the credit indicators in the segment.

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[2] *Source: Inside MBS & ABS*, LoanPerformance, and UBS.

ABS CDO and the demand for AAA securities

ABS CDO

ABS CDO is a re-securitization based on ABS. Sometimes these second securitizations can be called as outer CDO (in contrast with inner CDO, first securitization).For the purposes of this work, we will focus primarily on CDO based on subprime segment ABSs.

In recent years, as a consequence of the crisis, an increasing number of authors focused on the structure of CDO. To quote some examples Longstaff and Rajan (2008), Benmelech and Dlugosz (2008), and Sanders (2009) present an overview of CDO structure.

Basically three types of CDO structures can be considered. First, in a Cash-flow CDO, a SPV (Special Purpose Vehicle) invest in assets such as loans, mortgages or bonds and issues notes to investors. The proceeds gained from collateral pool are then distributed to investors according to the "waterfall" – pre-specified interest and principal payment scheme. Second, in a Synthetic CDO, the asset-side is composed by CDS (Credit Default Swaps) and earnings are therefore gained by selling protection. The premium is usually invested in risk-free securities and in case of default of one of the CDS underlings, the principal of the CDO is written down. Third, in a Market-value CDO, that are very similar to Cash-flow CDO, the collateral pool is marked-to-market.

In terms of motivations two main types of CDO must be considered. The Balance-sheet CDO is motivated by the desire to achieve relief from regulatory capital requirements. The Arbitrage CDO aims to generate spread between the asset pool and the funding of CDO. The excess spread is then paid to equity investors.

As indicated in Figure 1, the vast majority of CDO issuance was driven by arbitrage motivations. For sure, the balance-sheet CDOs played an important role, but it is undeniable that the majority of the explosive growth observed is motivated by the potential gains from the CDO structures.

Demand for AAA CDO Securities

At the demand side of our discussion is important to address some issues that could complete the understanding of the process that ended with the subprime crisis. The question of who was buying the CDO tranches has to be seen by the perspective of two different generic types of investors. As Adelino (2009) shows, the investors in AAA tranches were less informed about the quality of the assets collateralizing their investments than the ones investing in tranches with higher risk.

Investors in AAA tranches, less sensitive to collateral information, were those who demanded large amounts of securities. These investors were most of the types constrained by restrictions in terms of ratings, so in order to maximize their return given a specific rating; they could choose these structured investments. The fact that ratings for structured products and traditional bonds were based on similar scale is seen as a main reason to the creation of an illusion of comparability in terms of risk between them. Examples of these groups are conservatively managed institutions that require AAA securities to hedge funds.

Generally risk-based capital requirements take credit rating as an input. Therefore, and since there is no regulatory distinction between single or structured instrument for capital requirement purposes, then there is a strong motivation to try to gain more yield maintaining the levels of capital.

A second group invested in securities with lower ratings. They usually were very specialized in quantitative competencies, with much more sensitivity to the risk of the instruments, and usually had also more ambitious targets for rates of return. Recall that most of the issues had the arbitrage motivation and that the profile of this second group matches the one of the possible arbitrageurs. Indeed, they could take the chance of the high demand of the AAA rated securities to enhance their gains, as it will be illustrated in the practical example of this work. Examples of these groups are private equity funds or investment banks that are looking for high-yield returns.

Plantin (2003) besides presenting a theoretical analysis of the rationale for tranching and securitization activities such as CDO, shows that they arise as a natural profit maximizing strategy of investment banks, and predicts that investors with increasing sophistication acquire tranches with decreasing seniority.

It has to be highlighted that the arguments above raise some doubts about the motivation of some CDO arrangers that maintained the first piece of loss, the equity tranche, on their balance-sheets, "just" to show their confidence on the collateral assets quality. Furthermore, the inclusions of the above mentioned subprime RMBS as collaterals of these structures reinforces suspicious about the motivations. The subprime RMBS figured as an attractive alternative since it had high spreads that could be potentially used as a source of arbitrage. Also, it was seen as an investment able to yield diversification benefits.

Use of RMBS as Collateral

Hu (2007) provides some justifications for the use of RMBS as collateral on CDO issues.

The first is the weak performance of the high yield CBO (Collateralized Bond Obligations) during the crisis of 2000-2001, that created a demand for alternative investments to include in the CDO pools. See Figure 2.

Second, the economic momentum, including low interest rates until 2003 and booming housing market since 2001 were a strong contribution for a massive collateral generation. Finally, the introduction of CDS on RMBS allowed more flexibility for collateral managers to access exposures that were previously unavailable in a cash only market.

Note that at the time and given the circumstances, RMBS were seen as both performing and familiar to the asset managers.

ABS CDO issuers managed to create instruments able to generate a higher weighted average of interest from the collateral pool than the weighted average of interest paid to the holders of the ABS CDO securities. The arbitrage CDO, created to take advantage of this mismatch, was the main type of CDO issued in the years before the crisis[3].

If there is an intention of benefiting from the spreads of the collateral then there is a natural motivation for the inclusion of low quality assets in the pool. The above mentioned RMBS originated from the subprime segment appeared as a natural candidate for the inclusion in the pools since its credit quality was low, generating therefore high yields compared with other assets.

Another way of developing this kind of securities that allows gains from arbitrage is the inclusion of high systematic risk assets. Donhauser, Hamerle and Plank (2010) explain in some detail how to structure this type of securities to improve gains by exploiting the mispricing of systematic risk. Most of this mispricing shall have existed during the boom before the crisis. Below I retake this topic to show a practical example of why there was a mispricing and how the ABS CDO issuers exploited it. For now, I just note that most of the mispricing was induced

[3] Global CDO Market Issuance Data, SIFMA/Thomson Financial

by the rating agencies as I also argue below. The inclusion of MBS, including those from the subprime segment with a relatively high probability of default (low quality) and already diversified (in terms of idiosyncratic risks) fitted in perfection the features sought by arbitrageurs.

The structure of CDO created from MBS is described for example in Gorton (2008). The author presents a perspective from the origination of the subprime mortgages to the ABS structure focusing also on the information complexity between the parties in these deals, the sensitivity of the chain created from subprime mortgages to housing prices, the risk propagation and how the ABX index aggregated and revealed crucial information.

Several authors have been justifying the reduction in the spreads between MBS-yield and treasury bonds by the growing demand of MBS by CDO arrangers. Examples of literature on this issue are for example Brennan, Hein and Poon (2009) and Deng, Gabriel, and Sanders (2009). These latter also highlight that the beginning of the reduction of the referred spread has occurred simultaneously with the issue of the subprime mortgage CDO.

The search for poor quality and high systematic risk assets by arbitrageurs created an incentive for the origination banks to lax the risk assessment of the clients, as long as their intention was to accommodate the demand for more MBS for them. The topic of the adverse selection associated to lending in subprime segment has been widely approached in the recent years by many authors. A broad perspective of the variety of frictions involved in the crisis was given by Ashcraft and Schuermann (2008). An interesting perspective is also presented by Keys, Mukherjee, Seru and Vig (2008), who conclude that securitization affects negatively the incentives for lenders to screen the quality of loans they originated.

Mian and Sufi (2008) shows for example that the expansion in mortgage credit to subprime segment and its dissociation from income growth is closely correlated with the increase in securitization of subprime mortgages. They also note that the period between 2002 and 2005 is the only one in the last eighteen years when income and mortgage credit growth are negatively correlated.

Rajan, Seru, and Vig (2009) noted that as the level of securitization increases, lenders have an incentive to originate loans that rate high based on characteristics that are reported to investors, even if other unreported variables imply a lower borrower quality.

Sanders (2008) demonstrate the sudden increased delinquency of housing markets since 2005. Demyanyk and Van Hemert (2009) argue that subprime loan quality had been decreasing for six years before the crisis.

The Main Risks concerning Structured Finance

The role of Credit Rating Agencies (CRA) through the crisis has been widely debated both in public opinion and in scientific literature. Examples in scientific literature are Bolton, Freixas and Shapiro (2009), Mason and Rosner (2007), and Benmelech and Dlugosz (2009a).

CRA have become specialized over time in the risk assessment of singular investments (traditional corporate bonds), often using qualitative analysis in the course. They have never been called to assess structured instruments, involving the kind of mathematical complexities that cannot be qualitatively judged.

The truth is that despite the vague warning of some supervisory authority as Alan Greenspan who warned in 2005 that investors "should not rely solely on rating-agency assessments of credit risk", almost all parties involved seem to have been caught by surprise by the burst of the crisis. No one had realized the complexity and the danger behind the securities.

Plank (2010) suggests that there were three major issues regarding the subprime crisis of 2007-2008: (i) High systemic risk factor sensitivity of structured debt; (ii) High risk of model choice and parameter sensitivity; (iii) Systematically biased asset pool risk parameters in rating models.

The first risk to be taken into account is that tranching process generates very sensible results given its outputs. Specifically PD, LGD and correlation as inputs can be chosen to match a wide range of Probabilities of default, Expected Losses or other risk measures for tranches. Indeed, this dependence and sensitivity ultimately makes the collateral assets less important. The tranching process and its risks are well described in Brennan, Hein, and Poon (2009). Other authors addressed this issue as Gibson (2004), Fender and Mitchell (2005), Coval, Jurek, and Stafford (2009b), Hull and White (2010), Tarashev (2010) and Heitfield (2009).

Coval, Jurek, and Stafford (2009b) demonstrate that CDO valuation models hinged on a high degree of confidence in the parameter inputs, when a little change in the inputs parameters could lead to a big change in the losses. Besides Coval, Jurek, and Stafford (2009b), also Tarashev and Tarashev and Zhu (2007) and Heitfield (2008) studied this subject. The most quoted as having more influence in the final results is the correlation. Correlation assumptions are a critical component of the CDO rating analysis. Imperfect default correlation of assets is the main reason why a CDO can normally offer tranches with a wide range of risks grades, regardless of the credit quality of assets. Consequently it is no surprise that structured finance is relatively frequently related with lack of transparency, as in Mason (2008).

More generally, the risk of underestimating parameters as the main risk drivers PD, LGD and correlation is called structuring risk in this context. Given the strong usage of models in structured finance it also has an overlap with model risk. As in Plank (2010), the structuring risk is highly dependent on systematic risk.

Secondly, there are evidences that there existed a bias related to input parameters in the years before the crisis or not. Very often CRA were criticized of having underestimated the input parameters (PD, LGD and correlation) and overestimated the house price appreciation.

To a relatively rigorous analysis, there must be a differentiation two basic approaches to the determination of the rating: “point-in-time” (PIT) approach and “through-the-cycle” (TTC) approach.

A TTC rating is based on a stress quantile of an unconditional macro factor distribution:

$$M_q^{TTC} = \Phi^{-1}(q)$$

A PIT rating is based on a stress quantile of a conditional factor distribution:

$$M_q^{PIT} = \mu + \sigma . \Phi^{-1}(q)$$

The conditional factor distribution of PIT rating arises from a forecast of the future mean level of a risk driver M plus a forecast error.

The intention behind TTC is to maintain rating transition rates as low as possible. Therefore, ratings are determined based on stress PD and LGD for both corporate bonds and structured finance securities. So for the subprime segment, conditional on using this approach, CRA should have used undemanding scenarios for the stress tests.

By using the PIT approach another source of uncertainty arises: the accuracy of mean and error of the factor distribution. It is a common place to point weaknesses of CRA as that they were using used too short time series, ignoring some risks intrinsic to the subprime segment and also underestimated systemic risks. Also, the overestimation of house prices increase was almost taken as granted. If any of the weaknesses take place, then there we have a biased rating methodology and the perception of the risk in structured products in particular would be also biased.

All these issues are related with the information deficits along the securitization value chain (Mason, 2008) or with the information that hard facts about borrower creditworthiness lost explanatory power in the course of time (Rajan, Seru, and Vig, 2008).

Finally, one of the frequent critics that I've already mentioned above, is the use of the same scale to rate both singular and structured investments, and the consequent illusion of comparability. From the asset pricing theory[4], an asset price in a risk-neutral world is given by

$$p = \frac{E(x)}{R_f} + cov(x, m)$$

where p is the asset price, E(x) is the expected value of the asset x, R_f is the risk-free rate and cov(x,m) is the covariance between the asset x return and the stochastic discount factor m.

As in Coval, Jurek and Stafford (2009a), an asset whose payoff covariates positively with the discount factor have its price raised, so that securities that fail to deliver their expected payments in the worst economic states will have low values, because these are precisely the states where the money is most valuable (high marginal utility).

Asset prices should, therefore, reflect the premium for the covariation of its payoffs with priced states of nature. If the assessment of a security is just based on its ratings, and this rating is a function of an unconditional PD, then securities with same ratings would trade at different prices. In order to maximize the return, market arbitrageurs have a motivation to sell digital call options on market because these are the cheapest to supply bonds for a given rating. The digital call option on the market has the largest covariation possible with the market, so if the seller of the call "hides" the second part of the pricing equation then he can sell it receiving more than the fair amount. This is indeed considered as an optimal mechanism to exploit naïve investors that base their decisions solely on securities credit ratings.

Note also that in terms of asset pricing theory, the idiosyncratic risks, uncorrelated with the discount factor, generates no premium. This is well discussed in Cochrane (2001) for example. The diversification effect results in the reduction of the idiosyncratic risk and consequently on the larger share of the systematic risk. Different products with identical exposure to systematic risk must have the same price, because the idiosyncratic risk can be eliminated through diversification. This is a fundamental insight from the CAPM model.

Many authors have been calling attention that by Modigliani and Miller (1958) these gains should not exist, which leads us to consider the complex structure of these instruments in more depth. Three reasons to justify these gains are transaction costs, market incompleteness and asymmetric information (Benmelech and Dlugosz, 2008).

The question is that if CRAs were also fooled or they were informed about the riskiness of Structured Finance products? As it was already shown by Gibson (2004) for example, the mezzanine tranches, typically rated investment-grade, carries risk that can be many times that of an investment-grade corporate bond. Furthermore, the paper shows how the dependence of CDO tranches on default correlation can also be characterized and measured as an exposure to the business cycle.

Structural Models

Before proceeding to the empirical part of the work, I review some of the literature in the field of credit risk structural models, since it still lacks a theoretical framework on the modelling of individual credit risk. A more analytical approach to modelling these risks will be presented below.

Traditionally, the explanation of default processes relied on two main types of models: the structural and the reduced form. Reduced form models do not consider the relation between default and firm value in an explicit manner, since it uses market prices of instruments such as bonds or CDS to extract both their default probabilities and their credit risk dependencies, (Elizalde, 2005b).

Concerning the structural models, modelling default and firm value in an explicit manner involves control when a firm's assets are below a threshold – its debt. The first one attempting to do it was Merton (1974). In Merton Model it is of crucial importance the use of Black-Scholes (1973) options pricing model in the valuation of corporate debts.

Assumptions of the Merton Model are that the capital structure of the firm is composed by equity (E) and by a zero-coupon bond with maturity T and face value of D, whose values at time t are denoted by E_t and z (t, T) respectively, for $0 \leq t \leq T$.

Equity represents a call option on the firm's assets with maturity T (V_T) and strike price of its debt (D). The payoffs to equity holders and bondholders at time T under the assumptions of this model are

$$E_T = Max(V_T - D, 0)$$

$$Z(T,T) = V_T - E_T$$

The rest of assumptions Merton (1974) adopts are the inexistence of transaction costs, bankruptcy costs, taxes or problems with indivisibilities of assets; continuous time trading; unrestricted borrowing and lending at a constant interest rate r; no restrictions on the short selling of the assets; the value of the firm is invariant under changes in its capital structure (Modigliani-Miller Theorem) and that the firm's asset value follows a diffusion process. For a more detailed explanation see Elizalde (2005b). There are several extensions to the model aimed to assume more realistic and relaxed assumptions, but never discarding the easiness of use.

A second approached was introduced by Black and Cox (1976). The main difference in comparison with the previous model was that default could occur at any time. This paper was the first of the First-passage models (FPM). Geske (1977, 1979) extends the model to consider characteristics such as sinking funds, safety covenants, debt subordination, and payout restrictions. These two along with Leland (1994;1998) and Leland-Toft (1996) contributed to extensions introducing endogenous default boundary.

Stochastic interest rates allow the introduction of correlation between the firm's asset value and the short rate, and have been considered, among others, by Ronn and Verma (1986), Kim, Ramaswamy and Sundaresan (1993), Nielsen et al. (1993), Longstaff and Schwartz (1995), Hsu, Saá-Requejo and Santa-Clara (2004), Kim, Ramaswamy and Sundaresan (1993).

Inclusion of jumps had contributions of Zhou (2001a) and Huang and Huang (2003).

In addiction there are new models that are trying to be more consistent with empirical data as Liquidation Process Models (LPM) and State Dependent Models (SDM). SDM assumes that some of the parameters governing the firm's ability to generate cash flows or its funding costs are state dependent, where states can represent the business cycle or the firm's rating. This branch of structural models is able to reduce the problems of predictability of defaults and recovery suffered by standard models because the firm is subject to exogenous changes of parameters which are the main drivers of default probabilities. Hackbarth, Miao and Morellec (2004) and Elizalde (2005b) present two different models illustrating the previous ideas. Like LPM, State Dependent Models (SDM) have only been developed theoretically and their future success in credit risk modelling (if any) lies in their empirical applicability and their ability to replicate and predict credit spreads and default probabilities.

3. Empirical Analysis

As already mentioned previously, potential arbitrage gains should be created by mispricing systematic risk contained in SF securities. To illustrate this effect of mispricing I setup a model based on Merton model and on Vasicek assumptions to demonstrate the sensitivity of SF securities to systematic risk. The resulting implied correlation can then be used to evaluate prices of created tranches. The setup phase intended to be as simple as possible, but without relaxing the theoretical accuracy. Whenever possible it is used some data from Standard & Poor's to calibrate the model giving some real world flavour to the simulations conducted. Recall that the arbitrage opportunities illustrated in this section had a crucial importance in the growth of CDOs. The model setup is strongly in line with Donhauser, Hamerle and Plank (2010).

CDO Modelling

I start this section by making a basic distinction between the asset and the liability side of the CDO structure. Randomness typically occurs at the asset side of the structure. Consequently, it is assigned a probability space (Ω, F, P) to this side representing the probabilistic mechanism steering all random effects including payment defaults and prepayments. The liability side of the structure consists of tranched securities and is linked to the asset side by a random variable or vector translating asset scenarios into liability scenarios, hereby inducing a probability space $(E, \xi, P_{\vec{X}})$.

$$(\Omega, F, P) \xrightarrow[\vec{X}]{} (E, \xi, P_{\vec{X}})$$

Any CDO can be modelled by the following:

Step 1: Monte Carlo simulation of the underlying reference portfolio at the asset side of the transaction. In our particular case at this side we will start with 100 bonds.

The mathematical part of this first step in the model is the construction of the probability space (Ω, F, P).

Step 2: Modelling of the random vector $\vec{X}$ that translates asset scenarios into liability side scenarios. The mathematical result of this is the $\vec{X}$ -induced probability space $(E, \xi, P_{\vec{X}})$. For instance, this part includes cash flow waterfall modelling.

Step 3: Evaluation of simulation results, e.g., hitting probabilities and expected losses for tranches, distributions of the internal rate of return for tranches, expected interest and principal streams, risk adjusted duration of tranches, and so on.

In this work I use some MS Excel functions along with VBA to setup the simulations exercises. These instruments guarantee some advantages as they are very easy to implement, nevertheless also holds some drawbacks as the lack of efficiency. For this particular thesis it fits very well since these instruments are just demonstrative of the concepts behind it.

In terms of CDO modelling there are analytic, semi-analytic, and comonotonic simulation techniques. These techniques can speed-up CDO evaluations. In comparison with other credit baskets modelling, with CDO one typically has to spend much time in cash flow modelling in the liabilities side. Monte Carlo simulation is the preferred tool for CDO evaluation due to its intrinsic scenario orientation.

The basic inputs in the Monte Carlo approach includes the individual credit spread or default probability for each asset and the correlation matrix of the portfolio. Leaving aside the problematic interpretation of the second input (asset, equity or spread correlation as a proxy for the "desired" default correlation matrix), the basic Monte Carlo setting suffers from two main drawbacks. First, for an accurate estimate of spreads, this technique requires a large number of simulations. Given the large size of a typical underlying portfolio (usually over 100 credits) this procedure can, therefore, be excessively time consuming. Secondly, there is an issue with the estimation of the default correlation matrix. With N obligors, the NxN pairwise correlation matrix requires $\frac{N \cdot (N-1)}{2}$ estimates.

A common trick to reduce the number of estimates is to introduce "factors" that can capture and describe the dependency structure among credits. Thus, instead of analyzing the dependency of each pair of credits, factor models replace this credit-vs.-credit approach with a credit-vs.-common factors approach which is absolutely consistent with the assumption of the Vasicek Model.

Valuation of Single Name Securities

Starting by definition of the value of a single firm, we assume that it follows a Geometric Brownian motion. The respective dynamics is given by:

$$dV_{i,t} = \mu_i V_{i,t} dt + \sigma_i V_{i,t} dW_{i,t}$$

Where $V_{i,t}$ is the single-name security value, μ_i is its drift and σ_i its diffusion. dW represents the dynamics of a Geometric Brownian motion.

It is a very common stochastic equation and it can be solved by applying the Itô formula[5] to $Z = LN(V)$:

$$\begin{aligned} dZ_{i,t} &= \frac{1}{V_{i,t}} dV_{i,t} + \frac{1}{2}(-\frac{1}{V_{i,t}{}^2})(dV_{i,t})^2 = \\ &= \mu_i dt + \sigma_i dW_{i,t} - \frac{1}{2}\sigma_i{}^2 dt = \\ &= (\mu_i - \frac{1}{2}\sigma_i{}^2)dt + \sigma_i dW_{i,t} \end{aligned}$$

With $z_0 = LN(v_0)$

Integrating we obtain

$$Z_t = z_0 + (\mu_i - \frac{1}{2}\sigma_i{}^2)t + \sigma_i W_{i,t}$$

And

$$V_t = v_0 . \exp((\mu_i - \frac{1}{2}\sigma_i{}^2)t + \sigma_i W_{i,t})$$

So at maturity we have

$$V_{i,T} = V_{i,0} \exp\{(\mu_i - \sigma_i{}^2 / 2).T + \sigma_i W_{i,T}\}$$

[5] See Bjork (2010), for example.

Log-return is normally distributed and can be extracted from:

$$S_{i,t} = Ln(V_{i,T} / V_{i,0}) = (\mu_i - \sigma_i^{\ 2} / 2).T + \sigma_i \sqrt{T} W_{i,T}$$

As I mentioned before we simplify the dependence structure by replacing credit-vs.-credit approach with a credit-vs.-common factors. Therefore I split the expression between market (systematic) and idiosyncratic factor:

$$\sigma_i W_{i,T} = \beta_i \sigma_m M_T + \sigma_{u,i} U_{i,T}$$

This gives us:

$$S_{i,t} = (\mu_i - \sigma_i^{\ 2} / 2).T + \beta_i \sigma_m \sqrt{T} M_T + \sigma_{u,i} \sqrt{T} U_{i,T}$$

Following the logic of structural models, firm defaults when it falls below a certain threshold. The objective[6] probability of default of a company is given by:

$$\begin{aligned} p_{i,t} &= P(V_{i,T} \leq K_i \mid V_{i,0}) = \\ &= P(S_{i,T} \leq \ln K_i - \ln V_{i,0}) = \\ &= P(R_{i,T} \leq c^P{}_{i,T}) = \\ &= \Phi(c^P{}_{i,T}) \end{aligned}$$

Where

$$c^P{}_{i,T} = \frac{\ln \frac{K_i}{V_{i,0}} - E(S_{i,T})}{\sqrt{Var(S_{i,T})}} = \frac{\ln \frac{K_i}{V_{i,0}} - (\mu_i - \sigma_i^{\ 2} / 2).T}{\sigma_i \sqrt{T}}$$

This is called the standardized default threshold. The abovementioned probability is exactly the probability of default of each firm *i* at time *t* by Vasicek (1987, 1991 and 2002).

[6] The difference between Objective and Risk-neutral probability will be made clear below.

The standardized rate of return is

$$R_{i,T} = \frac{S_{i,T} - E(S_{i,T})}{\sqrt{Var(S_{i,T})}} =$$
$$= \frac{S_{i,T} - (\mu_i - \sigma_i{}^2 / 2).T}{\sigma_i \sqrt{T}} =$$
$$= \frac{\beta_i \sigma_m \sqrt{T} M_T + \sigma_{u,i} \sqrt{T} U_{i,T}}{\sqrt{\beta^2{}_i \sigma^2{}_m + \sigma^2{}_{u,i}} . \sqrt{T}} =$$
$$= \sqrt{\rho_i} M_T + \sqrt{1-\rho_i} U_{i,T}$$

Where

$$\rho_i = \frac{\beta^2{}_i \sigma^2{}_m}{\beta^2{}_i \sigma^2{}_m + \sigma^2{}_{u,i}}$$

By Vasicek, the correlation coefficient $\rho_{i,j,t}$ between each pair of random variables, like rates of return R_i and R_j in a portfolio setting, is the same for any two firms:

$$Corr(R_{i,t}, R_{j,t}) = \rho_{i,j,t} = \rho_t$$

For any $i \neq j$

And since this is true we can write random variables as a function of a common random source:

$$R_{i,t} = \sqrt{\rho_i} M_T + \sqrt{1-\rho_i} U_{i,T}$$

We arrived at a well-known one-factor model. Here we can make clear that the value of a firm is driven by a systematic part (market factor M), interpreted as an indicator of the general state of the business cycle (e.g., some stock, bond index or GDP) and the idiosyncratic factor (firm specific, U) which is the indicator of events strictly linked to the credit itself. The market factor M is a common source of uncertainty. Both M and U are assumed to be (standard) normally distributed and independent.

Resuming, the default occurs whenever

$$\sqrt{\rho_i} M_T + \sqrt{1-\rho_i} U_{i,T} < \Phi(c^P_{c_{i,T}})$$

Risk-neutral Valuation of Systematic Risk

The pricing of financial instruments is based on the risk-neutral probabilities (Q), instead of being based on the objective probability (P). In general, the price of a financial derivative instrument is the present value of the expected cash flows under the risk-neutral measure Q. By Vasicek Model, it is assumed market completeness and the absence of arbitrage opportunities (Bjork, 2010).

For the purposes of this work it is used the class of equivalent probability measures Q called martingales, where non-dividend paying asset processes are discounted using a default-free short rate (r). Absence of arbitrage is a necessary requirement for the existence of one equivalent probability measure (at least), and the assumption of market completeness guarantees its uniqueness. Such an equivalent measure is called a risk neutral measure and will be used to derive bond pricing formulas. In this line of thought it is assumed the independence of the firms' credit risk and the default-free interest rates under the risk neutral probability measure. Now it is clear that we need a risk-neutral default probability formula:

$$\begin{aligned} q_{i,t} &= Q(V_{i,T} \le K_i) = \\ &= Q(S_{i,T} \le \ln K_i - \ln V_{i,0}) = \\ &= Q(R_{i,T} \le c^Q{}_{i,T}) = \\ &= \Phi(c^Q{}_{i,T}) \end{aligned}$$

Note that the logic is almost the same as presented earlier when calculating the objective default probability. The issue here is that instead of having $\mu_{i\,i}$, there is a r as drift of our firm value because it is assumed risk-neutral world (no-arbitrage). This allows to isolate $c^P_{i,T}$:

$$c^Q{}_{i,T} = \frac{\ln \frac{K_i}{V_{i,0}} - (r - \sigma_i{}^2/2).T}{\sigma_i \sqrt{T}} = c^P{}_{i,T} + \frac{\mu_i - r}{\sigma_i}.\sqrt{T}$$

Denoting the Sharpe-ratio by δ and adding the relationship $\mu_i = r_i + \beta_i(\mu_m - r)$ we can conclude that the difference between objective and risk neutral default probability represents a risk premium for systematic risk:

$$q_{i,T} = \Phi(\Phi^{-1}(p_{i,T}) + \frac{\mu_i - r}{\sigma_i}.\sqrt{T}) =$$
$$= \Phi(\Phi^{-1}(p_{i,T}) + \sqrt{\rho_i}.\delta.\sqrt{T})$$

Conditional Valuation on Market Factor

For the purposes of this work the systematic risk represented by the market factor is a key variable. Therefore I show some manipulations of the default probability to find the conditional probability of default given a market realization:

$$p_{i,T}(M_T) = P(V_{i,T} \leq K_i \mid M_T) =$$
$$= P(\sqrt{\rho_i}M_T + \sqrt{1-\rho_i}U_{i,T} \leq K_i \mid M_T) =$$
$$= P(U_{i,T} \leq \tfrac{K_i - \sqrt{\rho_i}M_T}{\sqrt{1-\rho_i}} \mid M_T) =$$
$$= \Phi(\tfrac{c^P{}_{i,T} - \sqrt{\rho_i}M_T}{\sqrt{1-\rho_i}})$$

This is the individual default probability of each firm defaulting at time t, and it is known from Vasicek. We can obtain the default threshold by computing $c^P{}_{i,T} = \Phi^{-1}(p_{i,T})$, since it is considered that the default probability is given externally by rating agencies. This conditional probability on a market realization will be very useful to obtain the Expected Loss-profile of a security (Expected Loss given a determined market realization).

It is an assumption of the Vasicek model that this default probability is the same for each of the securities in the portfolio. It is also assumed that the number of securities in the portfolio is very large. Furthermore the size of each credit in the portfolio is similar.

The Recovery Rate (RR) is the percentage of a credit exposure that is recovered in case of a default event. Therefore the Loss Given Default (LGD) is obviously given by 1-RR. By Vasicek model, the LGD on each credit is deterministic and the same for all firms.

Risk Measures for CDO Tranches

Hitting probability measures the probability of a tranche being hit, independently of the width of the loss.

$$p_T^{tr} = P(L(T) > a)$$

Standard and Poor's and Fitch determine ratings based on this measure.

Another risk measure for CDO is the Expected Loss. This, differently from the previous measure, takes into account the extension of the loss and not just the hitting probability.

$$L^{tr} = \begin{cases} 0, L \leq a \\ \dfrac{L-a}{b-a}, a < L \leq b \\ 1, L > b \end{cases}$$

Moody's uses this measure.

Loan equivalent "Bond Representation"

In order to quantify the systematic risk in the tranches we can follow different approaches presented in recent literature.

A first approach is used by Donhauser, Hamerle and Plank (2010), that presents the virtual asset correlation (as a quantification of systematic risk) using the methodology followed also by Hamerle, Jobst, and Schropp (2008).

Another possible approach is the one used by Yahalom, Levy, and Kaplin (2008). These authors provide an overview of how to calibrate loan-equivalent correlation parameters. They assume two identical CDO tranches with collateral pools containing different assets with the same characteristics concerning number and risks. Both are driven by a single factor model with identical risk parameters. They can then obtain the virtual asset correlation by using the formula

$$JPD(PD_1, PD_2, \rho_{1,2}^A) = JPD_{sim}$$

The conclusions suggest that structured instruments have far higher correlation parameters than single-name instruments.

I followed Hamerle, Jobst, and Schropp (2008) to find the $\widehat{\rho}^{tr}$ (virtual asset correlation) implicit in a virtual CDO tranche, so that it has the same Hitting Probability profile that was found in a specific simulation run. This simulation was conducted to result in a HP-profile for each tranche.

A CDO tranche is approximated by a single-tranche bond in a single factor model with probability of default $p^{tr}_{virtual}$, asset correlation $\widehat{\rho}^{tr}$ and $L\widehat{G}D^{tr}$ as loss given default of the virtual bond. The approach ensures that the EL-profile of the single tranche bond resembles the simulated CDO tranche.

This is achieved by finding a $\widehat{\rho}^{tr}$ such that

$$\arg\min_{\widehat{\rho}^{tr}_{virtual}} \left\{ \sum_{k=1}^{K} \left[p^{tr}_{sim}(m_k) - p^{tr}_{virtual}(m_k) \right]^2 \mid \rho \in [0,1] \right\}$$

With

$$p^{tr}_{virtual}(m_k) = \Phi\left(\frac{\Phi^{-1}(\widehat{p}^{tr}) - \sqrt{\widehat{\rho}^{tr}_{virtual}}.m_k}{\sqrt{1-\widehat{\rho}^{tr}_{virtual}}} \right)$$

The parameter $\widehat{p}^{tr}$ is the hitting probability of the virtual bond, obtained by the following formulas

$$\widehat{p}^{tr} = \frac{E(L^{tr})}{L\widehat{G}D^{tr}}$$

$$L\widehat{G}D^{tr} = \begin{cases} \dfrac{\min(L_{\max}, b) - a}{b-a}, if \quad L_{\max} > a \\ 0, else \end{cases}$$

$L_{\max}$ is the maximum LGD for a tranche. a and b are the attachment and detachment points respectively. The approach used for the LGD, in the words of the authors, “aims to be as basic as possible and comparable to approaches that are used for modeling traditional single-name products”. To examine the goodness of the fit I also introduced the mean squared error as a measure:

$$MSE = \tfrac{1}{K} \sum_{k=1}^{K} (p^{tr}_{sim}(m_k) - p^{tr}_{virtual}(m_k))^2$$

Valuation of CDO Tranches

The pricing of a defaultable bond $B^d(0,T)$ can be priced with the following equations:

$$B^d(0,T) = B(0,T).\left(Q\left(\tau > T\right) + RR.(1 - Q\left(\tau > T\right)\right)$$

$$1 - Q\left(\tau > T\right) = q_{i,T} = \Phi(\Phi^{-1}(p_{i,T}) + \sqrt{\rho_i}.\delta.\sqrt{T})$$

Sample Collateral Pool and Input Parameters

Our base case portfolio will be composed by 100 equally weighted BB+ rating bonds with five years to maturity that is the average mortgage securities maturity. Each of our portfolio bonds has a default probability of 7.0% consistent with its rating.

%	Default Rate	
	From	To
BB+	5.18	7.02

The complete curve values for five year maturity is presented in annex I.

We will also assume a deterministic recovery rate of 40% for the assets in the collateral pool. S&P was applying a 50% mean and a 20% standard deviation for secured US corporate bonds, and a 38% mean and a 20% standard deviation for unsecured US corporate. This work intends to be as simplistic as possible in terms of modelling the LGD parameter, but for reference I mention that almost all modelling of this parameter is done via a Beta Distribution with alpha parameter referring the mean and beta referring to the standard deviation.

It is also assumed that $\rho = 10\%$ for all bonds in the portfolio. S&P applied a correlation parameter according with the table below:

Table 3.1: Correlations assumptions of S&P

		Between Sectors		**Within Sectors**	
		Corp	ABS	Corp	ABS
Within Country		5%	10%	15%	30%
Within Region	Local	5%	10%	0%	20%
	Regional			15%	
	Global			15%	
Between Regions	Local	0%	0%	0%	0%
	Regional			0%	
	Global			15%	

In order to give a real world flavour to the work I'm going to base the CDO liability side on "Octagon Investment Partners V". The structure is divided into five tranches with different seniority levels: Equity, Junior, Senior, Mezzanine, Senior and Super-Senior.

Table 3.2: Octagon Investment Partners V CDO Structure

Tranche	Attachment Point (a)	Detachment Point (b)
Equity	0.0%	7.8%
Junior	7.8%	9.3%
Mezzanine	9.3%	14.3%
Senior	14.3%	21.3%
Super-Senior	21.3%	100.0%
Pool	0.0%	100.0%

Simulation Results

Unconditional Results

Using a simulation[7] with 1.000.000 trials, we got the results for unconditional Probability of Default and unconditional Expected Loss. Based on the table in annex I it was assigned a rating to each tranche of the liabilities side of the structure. The exception is the equity tranche that usually is not rated.

Table 3.3: Results of a simulation of Unconditional PD and EL given 100 BB+ bonds as collateral

Tranche	Attachment Point (a)	Detachment Point (b)	PD	EL	Rating S&P
Equity	0,0%	7,8%	96,99%	49,82%	-
Junior	7,8%	9,3%	10,83%	9,18%	BB-
Mezzanine	9,3%	14,3%	6,92%	3,04%	BB+
Senior	14,3%	21,3%	1,02%	0,30%	BBB+
Super-Senior	21,3%	100,0%	0,04%	0,00%	AAA
Pool	0,0%	100,0%	96,99%	4,20%	BB+(*)

Note that the collateral pool is composed by BB+ bonds and that 90.7% of the tranches are at least BB+.

Expected Loss Profile

I now analyse the expected tranche loss conditional on market factor realization by applying the already mentioned formulas:

$$EL_i(T \mid M) = (1 - RR_i).p_{i,T}(M_T)$$
$$p_{i,T}(M_T) = \Phi\left(\frac{c^{P}{}_{i,T} - \sqrt{\rho_i}M_T}{\sqrt{1-\rho_i}}\right)$$

By doing so, the intention is to call the attention to the risk profile of a portfolio of defaultable securities in terms of systematic risk in opposition to a single-name security.

[7] See annex III for VBA code used
(*)Average of collateral pool

Comparing the EL-profile, obtained via simulation, of a BB+ CDO Tranche (Mezzanine) and a BB+ bond we have a clear image of the differences between both.

As seen in the Figure 5, same hitting probabilities (same ratings) don't reflect the real EL of tranches, mainly in worst phases of the economic cycle, as already discussed in the theoretical framework. It is also true that a market factor of -4, when most of the tranche loss has already been materialized, is a very rare event as $\Phi\ (-4) = 0.0032\%$ but empirical evidence shows that these rare events occur to often against what is expected. So a potential danger resides in assuming these events probability based on normal distributions.

We can try to demonstrate that creating an illusion of comparability between single and multi-name securities can drive to a mispricing of risk creating arbitrage opportunities that explain the explosive growth of CDOs before 2007. So the next step will be to quantify this systematic risk of the tranches of our sample CDO by applying the "bond representation" methodology.

Correlation of CDO Tranches

The table below presents the results following the "bond representation" methodology. In practice I used "Solver add-in" of MS Excel to find the results.

Table 3.4: Asset Correlation and Minimum Squared Root Error results

Tranche	PD	EL	Asset Correlation	MSE
Equity	96.99%	49.82%	0.3924	0.000352
Junior	10.83%	9.18%	0.8142	0.000001
Mezzanine	6.92%	3.04%	0.7914	0.000002
Senior	1.02%	0.30%	0.8023	0.000001
Super-Senior	0.04%	0.00%	0.4167	0.003058
Pool	96.99%	4.20%	0.1000	0.000000

As seen in the figure 6, the adjustment we made following the adopted methodology has a very good fit in terms of Expected Loss-profile (or implied hitting probability). This is a perfectly consistent conclusion with the MSE value.

The better approximation of the mezzanine and junior tranches is generally due to the lack of flexibility of the Gaussian copula model. Because of their position at the ends of the capital structure the equity as well as the senior tranche profile does not fulfil the symmetry criterion that is fulfilled by the middle tranches. Therefore, the goodness of fit of these tranches is less satisfactory.

Hull and White (2004) show that any zero mean unit variance distributions can be chosen for market and idiosyncratic factors. They find that the “double t” copula model where both have t-distributions with 4 degrees of freedom (scaled so that the variance is one) fits market data on synthetic CDOs well. It has considerably more tail default correlation (i.e., it has a higher probability of extreme clustering of defaults) than the Gaussian copula model.

The graphs from the other four tranches are presented in the annex II.

Valuation of CDO tranches

The valuation table of CDO tranches are presented below.

Table 3.5: CDO tranches pricing parameters and prices by “Bond Representation”

Tranche	IPD	Corr (p)	LGD	Tranche Price
Equity	49.82%	0.3924	100.0%	23.67
Junior	9.18%	0.8142	100.0%	57.27
Mezzanine	3.04%	0.7914	100.0%	70.39
Senior	0.30%	0.8023	100.0%	79.74
Super-Senior	0.00%	0.4167	49.2%	81.86
Pool	7.00%	0.1000	60.0%	76.15

These are considered as “fair” arbitrage-free prices. The argument for the explosive growth of CDOs has been the mispricing of the systematic risk so I compare the results above with a valuation that assumes a correlation of 0.1. Furthermore as the arbitrageurs’ earnings are residual I calculate the price of the equity tranche (retained by those arbitrageurs) as the difference between the collateral pool price and the weighted average of all the other tranches calculated with the same methodology as previously.

Table 3.6: CDO tranches pricing parameters and prices assuming 10% sensitivity to the market factor

Tranche	IPD	Corr (p)	LGD	Tranche Price
Equity	49.82%	0.1000	100.0%	14.35
Junior	9.18%	0.1000	100.0%	69.79
Mezzanine	3.04%	0.1000	100.0%	77.31
Senior	0.30%	0.1000	100.0%	81.31
Super-Senior	0.00%	0.1000	49.2%	81.87
Pool	7.00%	0.1000	60.0%	76.15

Comparing the tables above the conclusion is that the arranger (the alleged arbitrageur) of CDO that retains the equity tranche can exploit the situation in his advantage by selling tranches of non-equity CDO tranches at prices assuming low correlation parameter. The non-equity tranches that the arranger usually does not maintain in his balance sheet has a "fair" value price below what is obtained assuming a market standard methodology of cash-flow waterfall.

If the investors rely on ratings then there is a ground for mispricing and therefore to arbitrage. This can be an explanation to the explosion of arbitrage during 2006 and 2007, prior to the crisis.

Table 3.7: Comparison between prices obtained by "Bond Representation" and assuming 10% sensitivity to common market factor

Tranche	Bond Representation	Correlation=10%	Difference
Equity	23.67	14.35	9.32
Junior	57.27	69.79	-12.53
Mezzanine	70.39	77.31	-6.92
Senior	79.74	81.31	-1.56
Super-Senior	81.86	81.87	-0.01
Pool	76.15	76.15	0.00

CDO Squared: Second Layer Securitization

As already mentioned, much of the CDO issuances were created with other Structured Finance securities as collateral (recall Figure II). This is, allegedly, a reason for the increasing systematic risk. Imagine, in a purely theoretical exercise, that I incorporate some of abovementioned BB+ tranches (Mezzanine tranches) in the structure presented earlier. The key point is that while PD's are comparable (7% versus 6.92%) the systematic risk of the pool is increasing sharply from 0.1000 to 0.7914. Running all the above referred simulation process I obtained the results below.

Table 3.8: Simulation results for an ABS CDO with 100 BB+ Mezzanine tranches as Collateral

Tranche	Attachment Point (a)	Detachment Point (b)	PD	EL	Asset Correlation	MSE
Equity	0.0%	7.8%	35.58%	24.03%	0.9564	0.000074
Junior	7.8%	9.3%	17.78%	16.87%	0.9904	0.000001
Mezzanine	9.3%	14.3%	16.13%	14.66%	0.9909	0.000005
Senior	14.3%	21.3%	13.15%	11.79%	0.9909	0.000002
Super-Senior	21.3%	100.0%	10.44%	4.13%	0.8793	0.000069
Pool	0.0%	100.0%	35.58%	6.94%	0.7907	0.000000

As mentioned, this is a fully theoretical exercise to show how increasing systematically risky assets included in the pool contributed to an even increasing systematic risk. The result obtained allows the conclusion that the sensitivity of all tranches, except the super-senior, to the market factor is very close to 1, therefore being considered as catastrophe bonds, already referred previously as the cheapest-to-supply securities. This exercise lacks adherence to reality as the practice was to mix ABS tranches with other traditional bonds and to reset the attachment and detachment points in order to fulfil the market demand for AAA securities. That's obviously impossible given the super-senior tranche PD obtained via simulation in this exercise. Nevertheless we can conclude from this section that the inclusion of high systematically risky assets increases drastically the sensitivity of tranches to market factor.

ABS CDO Tranches Valuation

To a better approximation to the instruments in the source of the crisis let's take a look to ABS CDOs (also called Structured Finance CDO). Our intention will be to examine the arbitrage opportunities when selling ABS CDO tranches with subprime exposure – containing RMBS mezzanine tranches. The argument for a further increasing of the mispricing is the inclusion of already systematically risky collateral in the pool. Indeed, this was many times the case with the mortgages being of the same geography.

Lets assume that instead of using a homogeneous collateral pool of BB+ assets, all with sensitivity of 0.10 to the market factor, it is used a pool composed by the following assets:

	PD	LGD	#Securities	ρ
BB+ Mezzanine Tranche	6.92%	100%	70	79.14%
BB+ Bonds	7.00%	60%	30	10.00%

The parameters of the BB+ mezzanine tranches are based on the previous simulation, including the sensitivity parameter ρ. The new simulation intends to reflect the high systematic risk in the pool of assets. For the sake of comparability the attachment and detachment point are redefined in order to guarantee a rating based on PD for the mezzanine tranche to be same as of the inner CDO (BB+).

The results of the simulation are in the following table.

Table 3.9: Simulation results for an ABS CDO with 70 BB+ Mezzanine tranches and 30 BB+ bonds as Collateral

Tranche	Attachment Point (a)	Detachment Point (b)	PD	EL	Asset Correlation	MSE
Equity	0.0%	11.5%	80.47%	26.66%	0.8346	0.000870
Junior	11.5%	31.0%	13.66%	9.24%	0.9752	0.000009
Mezzanine	31.0%	66.0%	6.18%	3.34%	0.9500	0.000021
Senior	66.0%	80.0%	1.13%	0.44%	0.7602	0.001797
Super-Senior	80.0%	100.0%	0.00%	0.00%	0.4512	0.002091
Pool	0.0%	100.0%	80.47%	6.10%	0.6147	0.001591

The asset correlation result shows that a further increase of sensitivity of the parameter is observed. Therefore, assuming again investors' "blindness" to systematic risk, there will be further arbitrage gains as for the same rated tranches the "fair" value will be even lower.

Table 3.10: Comparison between prices obtained assuming 10% sensitivity to common market factor, ABS "Bond Representation" and ABS CDO "Bond Representation"

Comparable Rating Tranches	Bond Representation ABS CDO	Bond Representation ABS	Correlation=10%
Junior	69.79	57.27	54.94
Mezzanine	77.31	70.39	68.10
Senior	81.31	79.74	79.19
Super-Senior	81.87	81.86	81.87

Note that I didn't structure the CDO in order to optimize the arbitrage gains. In other words, since there are asset correlation values lower than one, there is a further margin to the optimization of gains for the arrangers.

Other Sources of Increasing Systematic Risk

The arbitrage gains can also be optimized by increasing the number of tranches or by introduction of thin tranches, also called trancheletes. From traditional portfolio theory it is known that pooling a large number of assets reduces the idiosyncratic loss variance but it is questionable whether this benefits the investor. Donhauser, Hamerle and Plank (2010) concludes that diversification in collateral pools of CDO tranches implies increasing concentration risk. The introduction of thin tranches was very popular prior to the crisis. It allowed, for example, the split between a senior and super-senior tranche, turning this last even safer but increasing the risk sensitivity factor.

4. Conclusions

The economic momentum, previous to the crisis, created a strong demand for AAA securities that was not available on the single bond market. The arrangers had, therefore, a strong incentive to create new investment products to fulfil these requirements. In order to exploit the market conditions and the focus of investors on ratings as the unique way to assess the riskiness of the instruments, arrangers were encouraged to create products with high levels of systematic risk. A natural candidate figuring as collateral was the subprime RMBSs because they were already systematically risky given its default rates and concentration in some specific geographical areas. These encourage further the generation of more subprime RMBSs with a decreasing quality of the underlyings. The abovementioned blindness of investors regarding the state of the economy in which the default could occur or, in other words, their blindness to the systematic risk was exploited by arrangers to generate arbitrage gains. These gains could be potentiated by the arrangers by keeping the first piece of loss, the equity tranche, as growing systematic risk tends to underestimate the value of these tranches (and overvalue the non-equity tranches). With the 2008 meltdown, the investors of highly rated SF securities stopped buying as there was a large downgrade of these product's ratings. They also understood that risk drivers of SF securities were systematically biased against them.

The biased risk drivers included high risk of model choice and parameter sensitivity, systematically biased asset pool risk parameters in rating models and high systemic risk factor sensitivity of structured debt. The first one can be fixed by adopting a Bayesian perspective, explicitly acknowledging the parameters uncertainty. With respect to the biased pool parameters, one can only conclude that there is a need for stronger estimation methodologies.
The main focus of this work was to explore the high systemic risk factor sensitivity of structured debt. So I applied the "bond representation" methodology to compare these debt products with traditional single-name products with the same rating. I performed three simulations: (i) A CDO backed by single bonds; (ii) A CDO backed by Mezzanine tranches and (iii) A CDO backed by single bonds and Mezzanine tranches. The finding that all tranches had a higher level of sensitivity to the market factor is the key point. The potential mispricing led to huge opportunities in generation of arbitrage gains and fed the subprime standards laxing that turned to be the cause of these products downgrades and losses.

The motivation of keeping the first piece of loss to show confidence collateral quality is strongly doubtful as also are the blindness of both rating agencies and supervisors.

The long-term incentives underlying securitizations are still valid but the second layer securitizations must be very carefully approached in the future. So I believe that a more transparent and standardized securitizations market will boost again the issuance in CDO market, but nothing compared to the level previously seen. In particular, the avoidance of high systematically risky exposures, second layer securitizations, thin tranches and very large collateral portfolios could help increasing investors' confidence, as well as maintain "controllable" the model risk of these instruments.

If CRAs uses TTC ratings, taking into account the systematic risk, then there is a potential to attract AAA investors again into the Structured Finance market. Most of the demand for these securities is driven by institutional investors, as pension funds, seeking for truly low risk products. The CRAs must guarantee that the stresses implied on ratings are rigorous enough to "cover" the worst phases of the economy. Moreover, it must also be taken into account empirical evidence that shows that extreme events tend to be more frequent than the values resulting from standard traditional distributions (typically the Gaussian).

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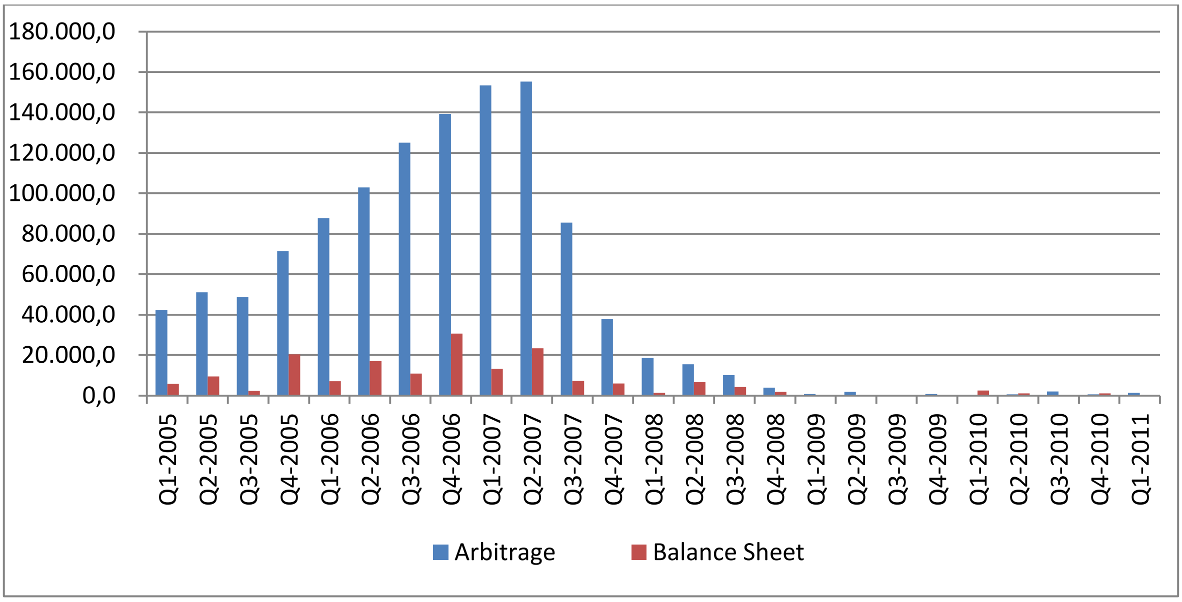
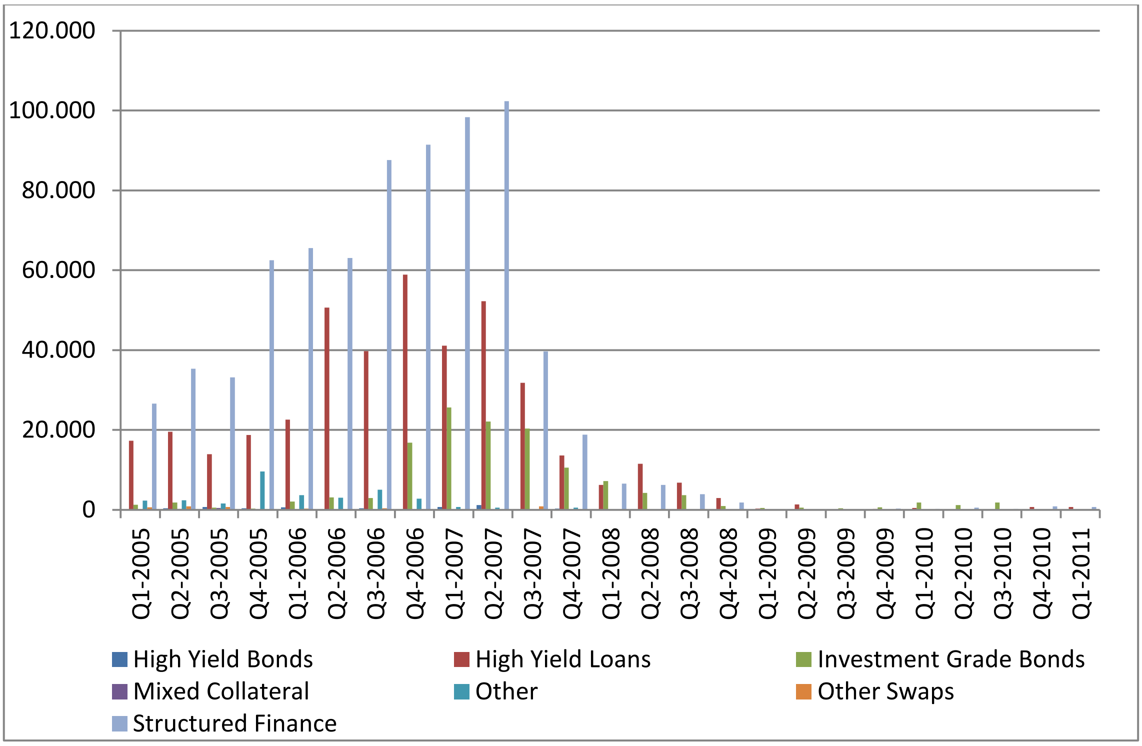
List of Figures

Figure 1: CDO Issuance by motivation (millions$)



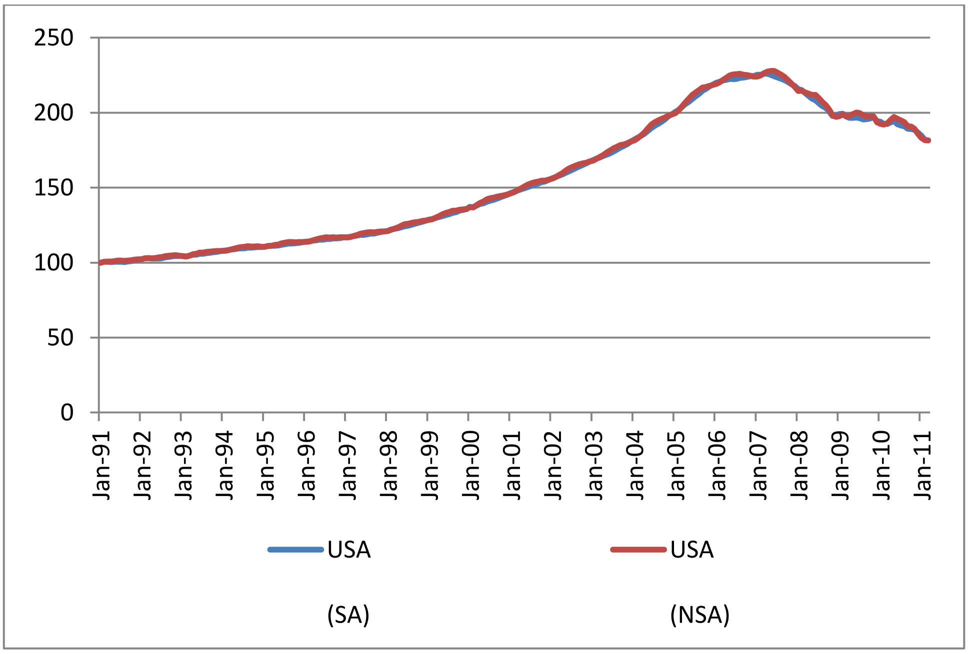
Source: SIFMA

Figure 2: CDO Issuance by collateral type (millions$)



Source: SIFMA

Figure 3: Monthly House Price Indexes for Census Divisions and U.S.



Purchase-Only Index (Only Index available with Monthly Frequency)

NSA=Not Seasonally Adjusted; SA=Seasonally Adjusted

Figure 4: Long Interest Rate GS10 - 10-Year Treasury Constant Maturity Rate

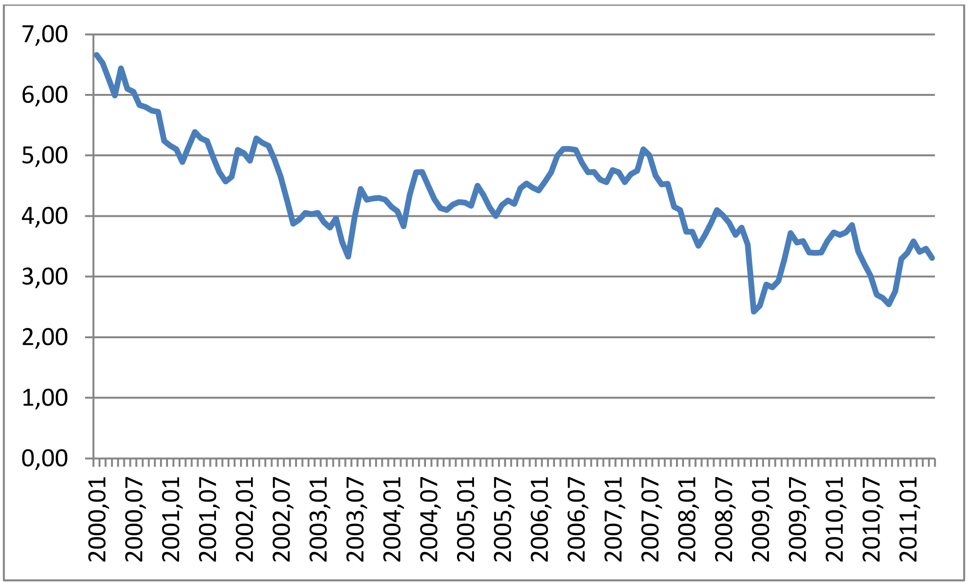


Figure 5: Expected Loss profiles of a single BB+ Bond and a Mezzanine tranche

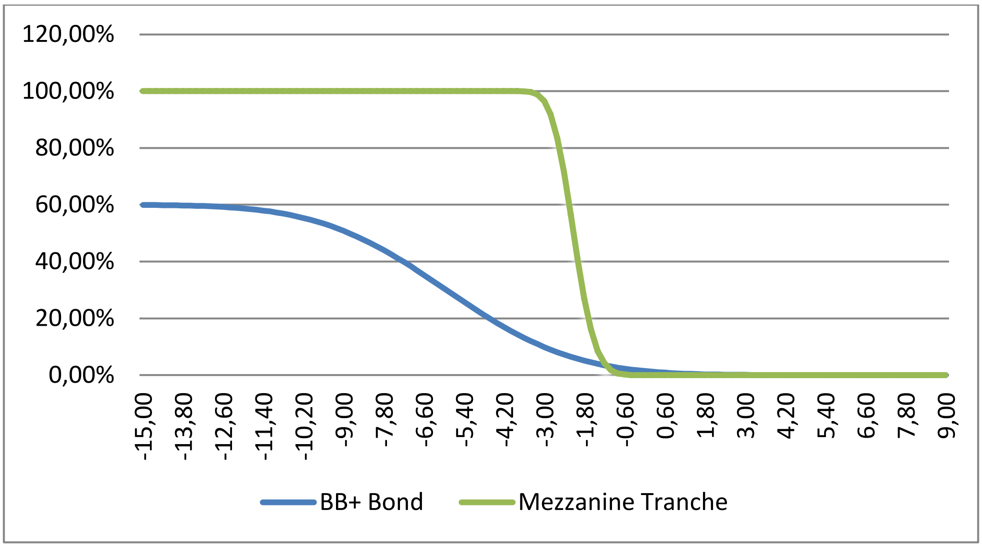
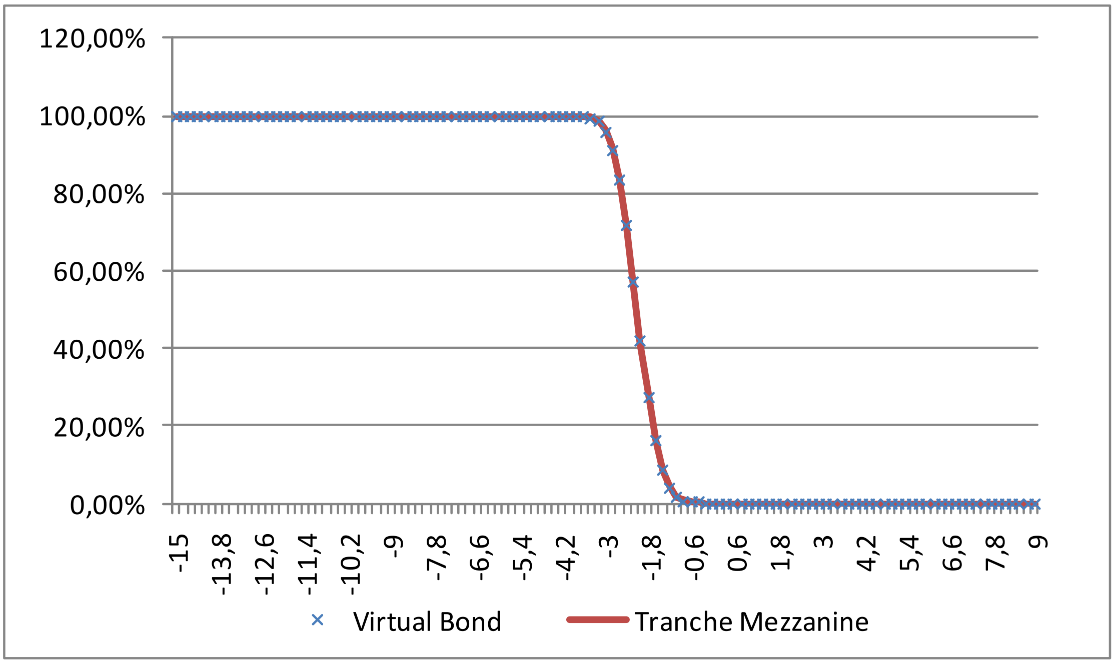


Figure 6: Goodness of fit of Virtual Bond and Mezzanine tranche



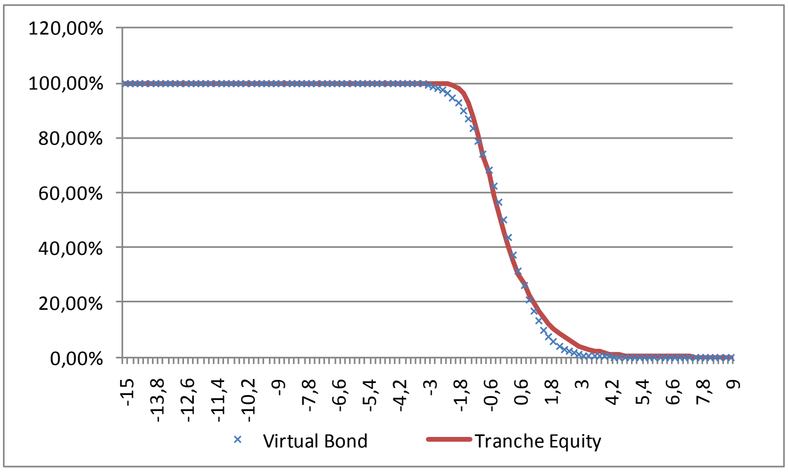
Annex I: S&P Default Rates

%	Default Rate	
	From	To
AAA	0,000	0,061
AA+	0,061	0,098
AA	0,098	0,219
AA-	0,219	0,276
A+	0,276	0,371
A	0,371	0,459
A-	0,459	0,686
BBB+	0,686	1,391
BBB	1,391	2,323
BBB-	2,323	5,179
BB+	5,179	7,020
BB	7,020	10,424
BB-	10,424	14,595
B+	14,595	18,571
B	18,571	24,463
B-	24,463	34,333
CCC+	34,333	55,809
CCC	55,809	70,042
CCC-	70,042	85,513

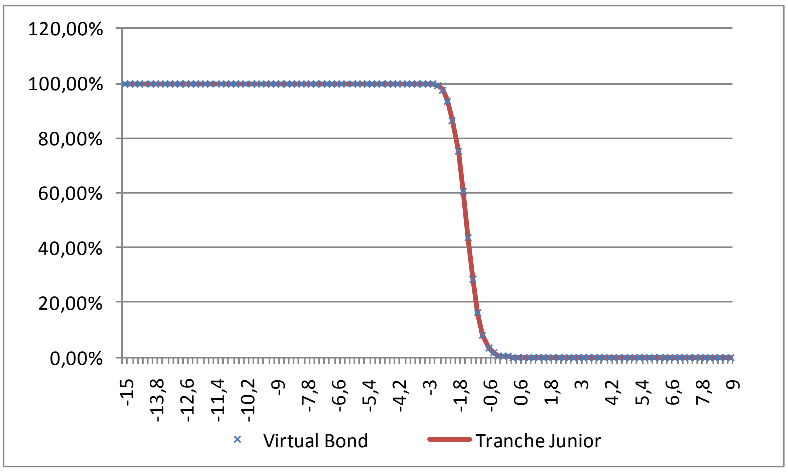
Source: Standard & Poor's. *CDO Evaluator Version 3.0: Technical Document*. December 2005

Annex II: "Bond Representation" results for Tranches and Collateral Pool

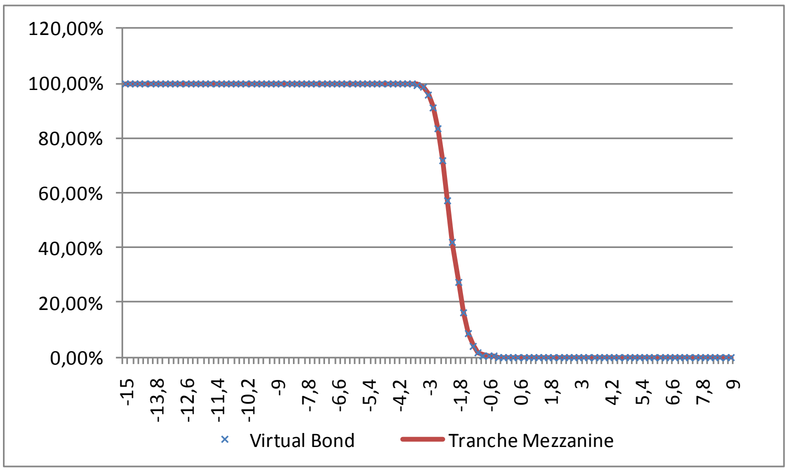
Equity Tranche



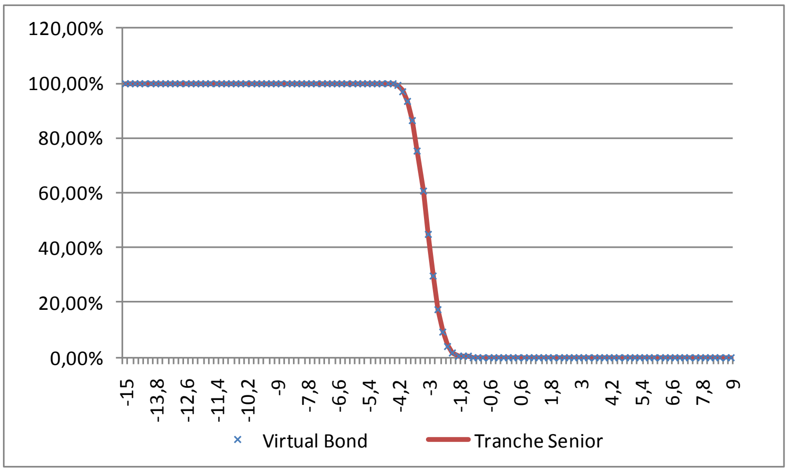
Junior Tranche



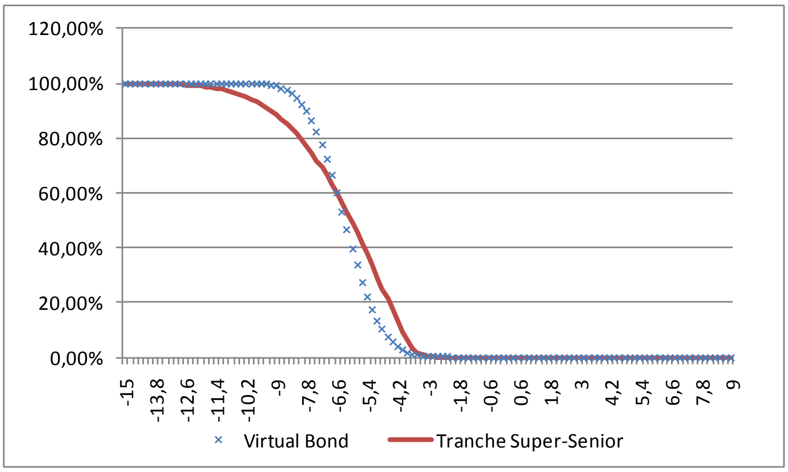
Mezzanine Tranche



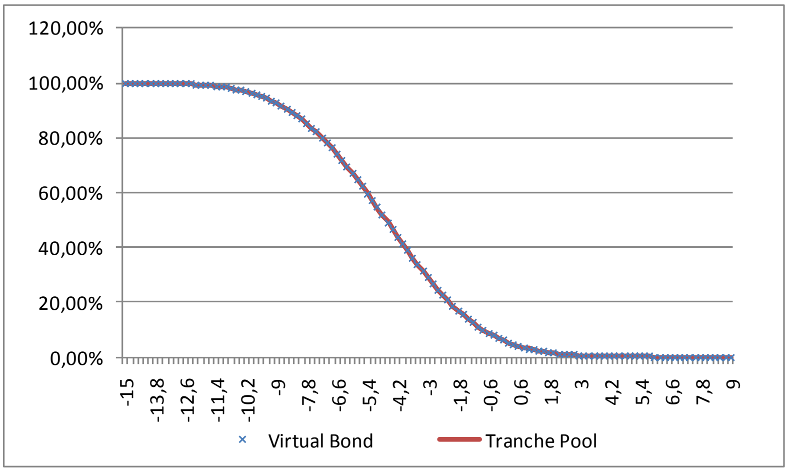
Senior Tranche



Super-Senior Tranche



Collateral Pool



Annex III: Simulation Codes

Inconditional PD and EL Simulator Code

Source: this function is based on Loeffler and Posch codes. Some changes to the basic version were implemented.

```
Sub simCDO()

'PD e EL incondicional para as tranches

Dim M As Long, N As Long, K As Integer, i As Long, j As Long, a As Integer, y As Integer, auxRO As Double

M = Range("c3")  'Number of simulations

N = Application.Count(Range("B10:B65536"))   'Number of loans

K = Application.Count(Range("G3:G65536"))   'Number of tranches

Dim d(), LGD() As Double, EAD() As Double, w() As Double, w2() As Double

Dim tranchePD() As Double, trancheEL() As Double, attach() As Double

Dim poolPD As Double, poolEL As Double

Dim factor As Double, loss_j As Double, sumEAD As Double

ReDim d(1 To N), LGD(1 To N), EAD(1 To N), w(1 To N), w2(1 To N)

ReDim tranchePD(1 To K), trancheEL(1 To K), attach(1 To K + 1)

'Read in attachment points and sum of loan exposures

For a = 1 To K

   attach(a) = Range("G" & a + 2)

Next a

attach(K + 1) = 1

sumEAD = Application.WorksheetFunction.Sum(Range("D3:D65536"))

'Write loan characteristics into arrays

For i = 1 To N

   d(i) = Application.WorksheetFunction.NormSInv(Range("B" & i + 9))

   LGD(i) = Range("C" & i + 9)

   EAD(i) = Range("D" & i + 9) / sumEAD

   auxRO = Range("E" & i + 9)

   w(i) = (auxRO) ^ 0.5

   w2(i) = ((1 - auxRO) ^ 0.5)
```

```
Next i
'Conduct M Monte Carlo trials
For j = 1 To M
   factor = NRND()
   'Compute portfolio loss for one trial
   loss_j = 0
   For y = 1 To 1
      For i = 1 To N
         If w(i) * factor + w2(i) * NRND() < d(i) Then
            loss_j = loss_j + LGD(i) * EAD(i)
         End If
      Next i
   Next y
 'Record losses for tranches
   a = 1
   Do While loss_j - attach(a) > 10 ^ -15
      tranchePD(a) = tranchePD(a) + 1 / M
      trancheEL(a) = trancheEL(a) + Application.WorksheetFunction.Min _
                 ((loss_j - attach(a)) / (attach(a + 1) - attach(a)), 1) / M
      a = a + 1
   Loop
   'Record losses for collateral pool
   If loss_j - 0 > 10 ^ -15 Then
      poolPD = poolPD + 1 / M
      poolEL = poolEL + Application.WorksheetFunction.Min _
                 (loss_j, 1) / M
   End If
Next j
Range("H3:H" & K + 2) = Application.WorksheetFunction.Transpose(tranchePD)
Range("I3:i" & K + 2) = Application.WorksheetFunction.Transpose(trancheEL)
```

```
Range("H" & K + 3) = poolPD
Range("I" & K + 3) = poolEL
Range("F" & K + 3) = "Pool"
End Sub
```

Conditional PD and EL Simulator Code

Source: this code was also based on (...)

```
Sub simulationCDO_M_Norm()

'PD's, EL's e IPD's condicionais ao factor M

Dim M As Long, N As Long, K As Integer, i As Long, j As Long, a As Integer, auxRO As Double

Dim L As Long, x As Long, myRange As Range, Lmax As Double

Dim poolEL As Double, poolPD As Double, poolLGD As Double, poolIPD As Double

M = Range("c3")  'Number of simulations

N = Application.Count(Range("B10:B65536"))   'Number of loans

K = Application.Count(Range("G3:G65536"))   'Number of tranches

L = Range("C4")

Lmax = 0.6

Dim d(), LGD() As Double, EAD() As Double, w() As Double, w2() As Double

Dim tranchePD() As Double, trancheEL() As Double, attach() As Double

Dim marketFact() As Double

Dim trancheLGD() As Double

Dim trancheIPD() As Double

Dim factor As Double, loss_j As Double, sumEAD As Double

ReDim d(1 To N), LGD(1 To N), EAD(1 To N), w(1 To N), w2(1 To N)

ReDim tranchePD(1 To K), trancheEL(1 To K), attach(1 To K + 1)

ReDim trancheLGD(1 To K)

ReDim trancheIPD(1 To K)

'Read in attachment points and sum of loan exposures

For a = 1 To K

   attach(a) = Range("G" & a + 2)

Next a
```

```
attach(K + 1) = 1

sumEAD = Application.WorksheetFunction.Sum(Range("D3:D65536"))

'Write loan characteristics into arrays

For i = 1 To N

   d(i) = Application.WorksheetFunction.NormSInv(Range("B" & i + 9))

   LGD(i) = Range("C" & i + 9)

   EAD(i) = Range("D" & i + 9) / sumEAD

   auxRO = Range("E" & i + 9)

   w(i) = (auxRO) ^ 0.5

   w2(i) = (1 - auxRO) ^ 0.5

Next i

'Jumps to the output sheet

Sheet4.Select

For x = 1 To L

factor = Cells(1, x + 1)

'Conduct M Monte Carlo trials

For j = 1 To M

   'factor = NRND()

   'Compute portfolio loss for one trial

   loss_j = 0

   For i = 1 To N

      If w(i) * factor + w2(i) * NRND() < d(i) Then

         loss_j = loss_j + LGD(i) * EAD(i)

      End If

   Next i

   'Record losses for tranches

   a = 1
```

```
    Do While loss_j - attach(a) > 10 ^ -15
        tranchePD(a) = tranchePD(a) + 1 / M
        trancheEL(a) = trancheEL(a) + Application.WorksheetFunction.Min _
                ((loss_j - attach(a)) / (attach(a + 1) - attach(a)), 1) / M
        trancheLGD(a) = (Application.WorksheetFunction.Min(Lmax, attach(a + 1)) - attach(a)) /
(attach(a + 1) - attach(a))
        trancheIPD(a) = trancheEL(a) / trancheLGD(a)
        a = a + 1
    Loop
    'Record losses for pool
    If loss_j - 0 > 10 ^ -15 Then
    poolPD = poolPD + 1 / M
    poolEL = poolEL + Application.WorksheetFunction.Min _
                (loss_j, 1) / M
    poolLGD = Lmax
    poolIPD = poolEL / poolLGD
    End If
Next j

'Range("H3:H" & K + 2) = Application.WorksheetFunction.Transpose(tranchePD)
Set myRange = Range(Cells(3, x + 1), Cells(3 + K - 1, x + 1))
myRange = Application.WorksheetFunction.Transpose(trancheEL)
Cells(3 + K, x + 1) = poolEL
Set myRange = Range(Cells(3 + K + 2, x + 1), Cells(3 + K + 2 + K - 1, x + 1))
myRange = Application.WorksheetFunction.Transpose(tranchePD)
Cells(3 + K + 2 + K, x + 1) = poolPD
Set myRange = Range(Cells(3 + 2 * (K + 2), x + 1), Cells(3 + 2 * (K + 2) + K - 1, x + 1))
myRange = Application.WorksheetFunction.Transpose(trancheIPD)
```

```
Cells(3 + 2 * (K + 2) + K, x + 1) = poolIPD

Set myRange = Range(Cells(3 + 3 * (K + 2), x + 1), Cells(3 + 3 * (K + 2) + K - 1, x + 1))

myRange = Application.WorksheetFunction.Transpose(trancheLGD)

Cells(3 + 3 * (K + 2) + K, x + 1) = poolLGD

ReDim trancheEL(1 To K)

ReDim tranchePD(1 To K)

ReDim trancheLGD(1 To K)

ReDim trancheIPD(1 To K)

poolEL = 0

poolPD = 0

poolLGD = 0

poolIPD = 0


Next x


End Sub
```