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Essays on Wavelets in Economics

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Resumo

O objectivo deste trabalho é realçar a utilidade da análise de onduletas em Economia. A análise de onduletas é uma ferramenta muito promissora, pois representa um refinamento da análise de Fourier. Em particular, permite ter em consideração quer o domínio do tempo quer o domínio da frequência de forma unificada, ou seja, é possível avaliar simultaneamente como é que as variáveis estão relacionadas em diferentes frequências e como é que essa relação tem evoluído ao longo do tempo. Apesar do potencial interesse, a análise de onduletas constitui ainda uma ferramenta relativamente pouco utilizada no estudo de fenómenos económicos. O trabalho aqui apresentado pretende contribuir para essa vertente da literatura.

Em particular, a análise de onduletas é usada para avaliar a relação entre o crescimento monetário e a inflação na área do euro, dado que o Banco Central Europeu atribui um papel fundamental para a moeda no contexto da sua estratégia de política monetária. Adicionalmente, é proposta uma medida de co-movimento baseada em onduletas sendo utilizada para estudar o co-movimento, em termos de crescimento, entre as maiores economias da área do euro. Com base nesta medida, também é proposta uma medida de coesão que é usada por sua vez para aferir a coesão entre os países da área do euro e nos Estados Unidos, quer a nível regional quer estadual. No contexto da literatura de economia financeira, a relação entre retornos de acções no mercado internacional é avaliada e é proposta uma abordagem baseada em onduletas para a medição de risco de mercado. Finalmente, a utilidade das onduletas para efeitos de previsão é investigada e a sua interacção com os modelos de factores é explorada.

Palavras-Chave: Onduletas; Tempo-frequência; Co-movimento; Coesão; Modelos de factores; Previsão.

Abstract

The aim of this work is to highlight the usefulness of wavelet analysis in Economics. Wavelet analysis is a very promising tool as it represents a refinement of Fourier analysis. In particular, it allows one to take into account both the time and frequency domains within a unified framework, that is, one can assess simultaneously how variables are related at different frequencies and how such relationship has evolved over time. Despite the potential value of wavelet analysis, it is still a relatively unexplored tool in the study of economic phenomena. The work herein presented intends to contribute to such strand of literature.

In particular, wavelet analysis is used to assess the link between money growth and inflation in the euro area, as the European Central Bank attributes a key role to money within the monetary policy strategy. Additionally, a wavelet-based measure of comovement is proposed and used to study the growth comovement among the major euro area countries. Based on this measure, a measure of cohesion is developed and used to investigate the cohesion among euro area countries and the cohesion within US at both the regional and state levels. Within the financial economics literature, the relationship among international stock market returns is assessed and a wavelet-based approach for measuring market risk is proposed. Finally, one also investigates the usefulness of wavelets for forecasting purposes while bridging such approach with factor-augmented models.

Keywords: Wavelets; Time-frequency; Comovement; Cohesion; Factor models; Forecasting.

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Contents

1 Introduction

Time domain analysis is, far from doubt, the most widespread approach in the literature to study time series. Through such approach, the evolution of individual variables is modelled and multivariate relationships are assessed over time. Another strand of literature, focus on the frequency domain. Frequency domain analysis is a complementary tool to time domain analysis. In particular, with spectral analysis, one can investigate the importance of different frequency components for the behaviour of a variable and the relationship between variables at the frequency level. Recent work resorting to Fourier analysis includes, for example, A'Hearn and Woitek (2001), Pakko (2004), Rua and Nunes (2005) and Breitung and Candelon (2006).

Wavelets analysis reconciles both approaches, in the sense that both time and frequency domains are taken into account. Hence, wavelets are a very promising tool as they represent a refinement in terms of analysis. As mentioned by Ramsey (2002), "Wavelets are treated as a 'lens' that enables the researcher to explore relationships that previously were unobservable" while "... the ability to apply a new 'lens' to inspect the relationships in economics and finance provides great promise for the development of the discipline". As Lau and Weng (1995) put it, the wavelet transform has the ability to make a time series sing as it decomposes a signal into localized frequencies with the corresponding measure of intensity and duration, analogous to the bass/treble clef, the crescendo/decrescendo and the tempo in a piece of music.

Despite its potential usefulness, wavelets have been more popular in fields other than economics. For example, in geophysics, for the analysis of oceanic and atmospheric flow phenomena, seismic signals and climatic data; in medicine, for heart rate monitoring, breathing rate variability and blood flow and pressure; in engineering, for the assessment of machine process behaviour;

just to name a few (see, for example, Adisson (2002) for a comprehensive overview).

Although there are still relatively few papers in economics resorting to wavelet analysis, such analysis has already shown to provide fruitful insights about several economic phenomena. For instance, the pioneer work of Ramsey and Lampart (1998a,b) draws on wavelets to study the relationship between several macroeconomic variables, namely money supply and output in the first case and consumption and income in the second. Recent work using wavelets includes, Kim and In (2003, 2005) who investigate the relationship between financial variables and industrial production and between stock returns and inflation, respectively, Gençay *et al.* (2003, 2005) and Fernandez (2005, 2006) study the Capital Asset Pricing Model at different scales, Connor and Rossiter (2005) focus on commodity prices, Wong *et al.* (2003), Conejo *et al.* (2005) and Fernandez (2007) use wavelets for forecasting purposes, In and Kim (2006) examine the relationship between the stock and futures markets, Gallegati and Gallegati (2007) provide a wavelet variance analysis of output in G-7 countries, Crivellini *et al.* (2004), Crowley and Mayes (2008), Gallegati *et al.* (2008) and Yogo (2008) resort to wavelets for business cycle analysis, Aguiar-Conraria *et al.* (2008) assess the time-frequency effects of monetary policy in the US and Aguiar-Conraria and Soares (2010) study the relationship between oil prices and industrial production, among others (see also, for example, Crowley (2007) for a recent survey).

Despite the growing literature in the last few years, there is clearly scope to widen further the application of wavelet analysis while highlighting the usefulness of such tools in economics. In this respect, Ramsey (2002) and Crowley (2007) stress that wavelet analysis has a huge potential as an exploratory tool. For instance, wavelet analysis allows one to unveil relationships between economic variables in the time-frequency space, that is, it allows one to assess simultaneously how variables are related at different frequencies and how such relationship has evolved over time. Hence, one is enabled to capture both time and frequency varying features within an unified framework. Following this line of research, several essays have been done within this dissertation.

Another field of research where wavelets can also be particularly useful is for forecasting purposes. In particular, the wavelet multiresolution approach for forecasting purposes consists in several steps. First, the series to be forecast is decomposed into its constituent time-scale components. Then, for each time-scale a model is fitted and used for forecasting. Finally, an overall forecast is obtained after recombining the components. This multiresolution approach can outperform the traditional single resolution approach for forecasting as it is possible to tailor specific forecasting models to each time-scale component and thereby enhance the forecasting performance of the series as a whole. Although the potential usefulness of wavelets in forecasting has been recognized, there are very few applications of wavelets for forecasting in economics. This dissertation also intends to contribute in that respect.

The remainder of the text is organised as follows. Firstly, an overview of the main building blocks of wavelet analysis is provided. In particular, in section 1.1, wavelet analysis is motivated by comparing it with the well-known Fourier analysis. Afterwards, in section 1.2, the continuous wavelet transform is discussed as it constitutes the basis for most of the work done while a short presentation of the discrete wavelet transform and multiresolution analysis is done in section 1.3. Note that the following discussion does not intend to be an exhaustive description of wavelet analysis. Instead, the aim is to provide an intuitive and brief overview of the main tools used in this dissertation. The interested reader can find, for example, in Percival and Walden (2000) an extensive introduction to wavelets whereas Gençay *et al.* (2002) discuss the use of wavelets for specific purposes in economics and finance and Crowley

(2007) provide a guide for economists with several illustrations and examples (see also Schleicher (2002)). In section 1.4, the dissertation plan is sketched whereas the corresponding essays are presented in chapters 2 up to 7.

1.1 From Fourier to wavelet analysis

The well-known Fourier transform is the conventional method for studying the frequency content of a signal. It involves the projection of a series onto an orthonormal set of trigonometric components (see, for example, Priestley (1981)). In particular, the Fourier transform uses a basis of sines and cosines of different frequencies to determine how much of each frequency the signal contains. The Fourier transform of the time series $x(t)$ is given by

$$F_x(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-i\omega t}dt \tag{1}$$

where ω is the angular frequency and $e^{-i\omega t} = \cos(\omega t) - i\sin(\omega t)$ according to Euler's formula. In addition, one can write $x(t)$ as

$$x(t) = \int_{-\infty}^{+\infty} F_x(\omega)e^{i\omega t}d\omega \tag{2}$$

Despite its usefulness, the Fourier transform provides no information about how the frequency content of the signal changes with time, that is, it tells us how much of each frequency exists in the signal but it does not tell us when in time these frequency components exist. To overcome such limitation it has been suggested the so-called short-time Fourier transform (also known as Gabor or windowed Fourier transform). As the name suggests, the basic idea is to use the Fourier transform for short periods of time. It consists in applying a short-time window to the signal and performing the Fourier transform within this window as it slides across all the data.

However, the time-frequency analysis is limited by the Heisenberg uncertainty principle. In quantum physics, the uncertainty principle, formulated by Heisenberg, states that the velocity and the position of a moving particle cannot be simultaneously known to arbitrary precision. In the current context, it implies that one cannot exactly know what frequency exists at what time instance. The best one can do is to investigate what spectral components exist at any given interval of time. Since the resolution in time and frequency can not be arbitrarily small, because their product is lower bounded, it results in a trade-off between time and frequency resolution. This means that for narrow windows one gets good time-resolution but poor frequency resolution whereas for wide windows one gets good frequency resolution and poor time-resolution.

The problem with the windowed Fourier transform is that it uses constant length windows. These fixed length windows give the uniform partition of the time-frequency plane. When a wide range of frequencies is involved, the fixed time window tends to contain a large number of high frequency cycles and a few low frequency cycles which results in an overrepresentation of high frequency components and an underrepresentation of the low frequency components. Hence, as the signal is examined under a fixed time-frequency window with constant intervals in the time and frequency domains, the windowed Fourier transform does not allow an adequate resolution for all frequencies.

In contrast, the wavelet transform uses local base functions that can be stretched and translated with a flexible resolution in both frequency and time. In the case of the wavelet transform, the time resolution is intrinsically adjusted to the frequency with the window width narrowing when focusing on high frequencies while widening when assessing low frequencies. Allowing for windows of different size, it enables to improve the frequency resolution of the low frequencies and the time resolution of the high frequencies. In other words, higher frequencies are better resolved in time while lower frequencies are better resolved in frequency. This means that, a certain high frequency component can be located better in time than a low frequency component. On the contrary, a low frequency component can be located better in frequency compared to a high frequency component. However, one should bear in mind that the Heisenberg uncertainty principle is also valid here. So, one cannot expect an improvement of both time and frequency resolution for a given point in the time-frequency plane but one can vary the ratio between the time and frequency uncertainty. As it enables a more flexible approach in time series analysis, wavelet analysis is seen as a refinement of Fourier analysis.

The above discussion can be illustrated through Figure (1). For a time series in the time domain each point contains information about all frequencies. In contrast, in the case of the Fourier transform, every point in the frequency domain contains information from all points in the time domain. For the short-time Fourier transform, the time-frequency plane is divided using a constant length window whereas for the wavelet transform the window width is adjusted to the frequency.

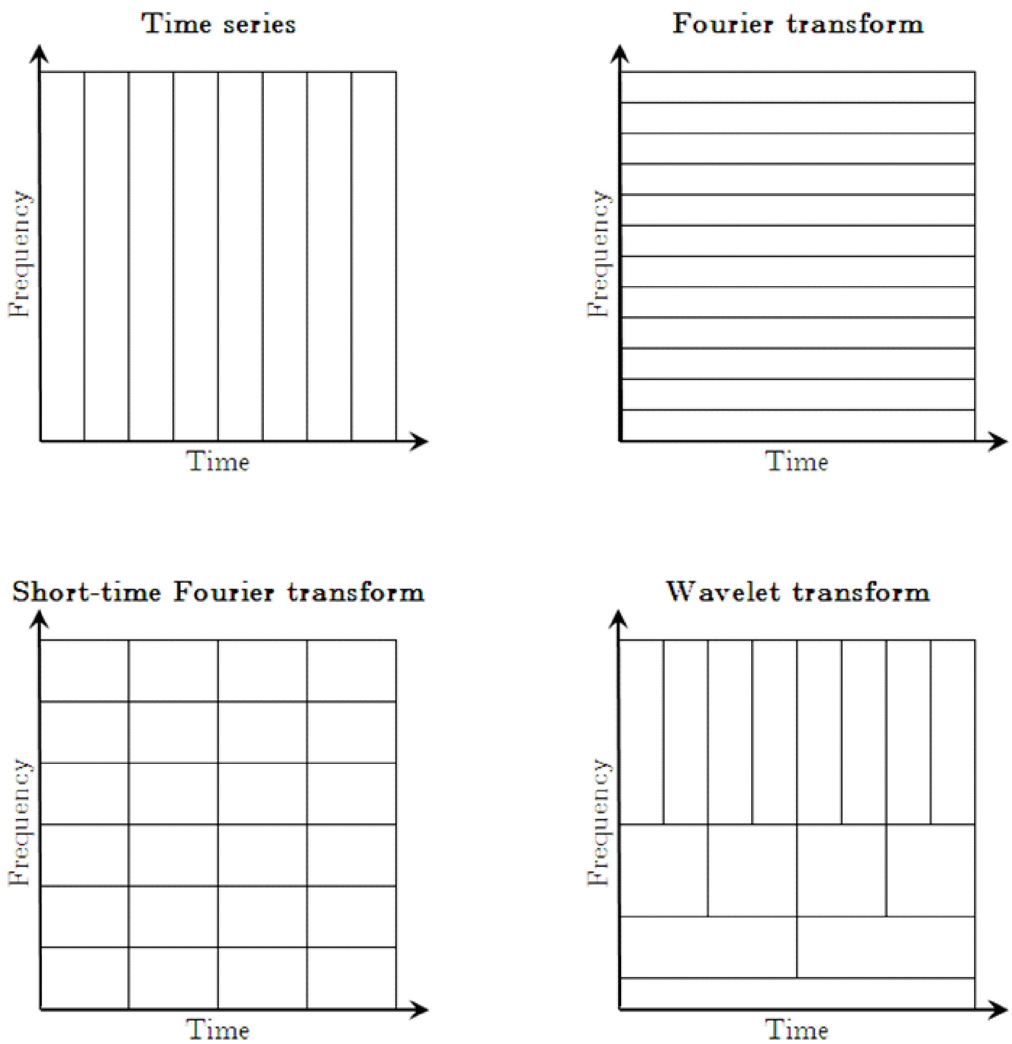


Figure 1: Comparison of time-frequency properties for a time series, its Fourier transform, short-time Fourier transform and wavelet transform.

1.2 The continuous wavelet transform

The continuous wavelet transform of a time series $x(t)$ can be written as

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t)\psi^*_{\tau,s}(t)dt \tag{3}$$

where * denotes the complex conjugate. Hence, the wavelet transform decomposes a time series $x(t)$ in terms of some basis functions (wavelets), $\psi_{\tau,s}(t)$, analogous to the use of sines and cosines in Fourier analysis. The term wavelet means a small wave. The smallness refers to the condition that this (window) function is of finite length (compactly supported). The wave refers to the condition that this function is oscillatory. These basis functions are obtained by translation and dilation of the so-called mother wavelet $\psi(t)$ and are defined as

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right) \tag{4}$$

where τ determines the time position (translation parameter), s is the scale (dilation parameter) and $\frac{1}{\sqrt{s}}$ is for energy normalization across the different scales[1]. The term translation is related to the location of the window, as the window is shifted through the signal (see Figure (2)). The scale refers to the width of the wavelet. By changing the scale parameter, one gets compressed and stretched versions of the mother wavelet. If $s < 1$ then the wavelet one will get is compressed; the wavelet corresponding to $s = 1$ is the mother wavelet; if $s > 1$ then one gets a stretched version of the mother wavelet (see Figure (3)). In terms of frequency, low scales by a compressed wavelet function capture rapidly changing details, that is high frequencies, whereas higher scales by a stretched wavelet function capture slowly changing features, that is, low frequencies.

[1]In particular, the term $\frac{1}{\sqrt{s}}$ makes the variance of the scaled mother wavelet equal to the variance of the original mother wavelet.

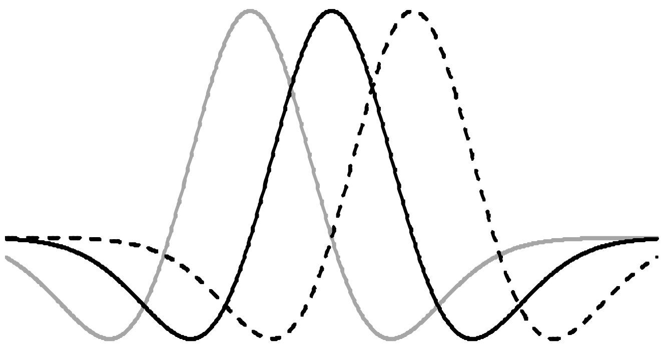


Figure 2: Wavelet translation: $\tau = 0$ (black solid line), $\tau < 0$ (gray solid line), $\tau > 0$ (black dashed line).

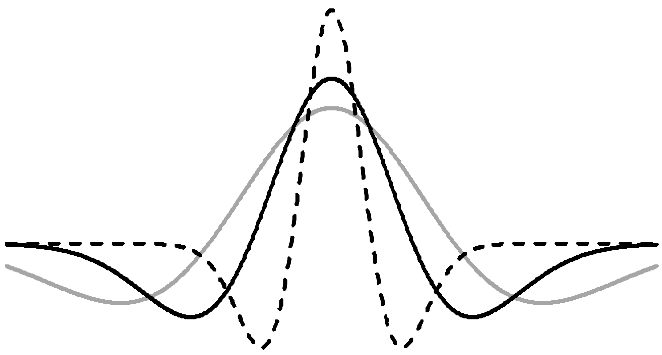


Figure 3: Wavelet dilation: $s = 1$ (black solid line), $s > 1$ (gray solid line), $s < 1$ (black dashed line).

To be a mother wavelet, $\psi(t)$, must fulfil certain criteria:
i) the integral of $\psi(.)$ is zero,

$$\int_{-\infty}^{+\infty} \psi(t)dt = 0 \tag{5}$$

that is, the average value of the wavelet in the time domain must be zero;
ii) its square integrates to unity,

$$\int_{-\infty}^{+\infty} \psi^2(t)dt = 1 \tag{6}$$

which means that $\psi(t)$ is limited to an interval of time;
iii) and it should also satisfy the so-called admissibility condition,

$$0 < C_\psi = \int_0^{+\infty} \frac{\left|\widehat{\psi}(\omega)\right|^2}{\omega} d\omega < +\infty \tag{7}$$

where $\widehat{\psi}(\omega)$ is the Fourier transform of $\psi(t)$, that is, $\widehat{\psi}(\omega) = \int_{-\infty}^{+\infty} \psi(t)e^{-i\omega\tau}dt$, so as to allow for the reconstruction of the signal without loss of information.

As with its Fourier counterpart, there is an inverse wavelet transform, defined as

$$x(t) = \frac{1}{C_\psi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \psi_{\tau,s}(t) W_x(\tau, s) \frac{d\tau ds}{s^2} \tag{8}$$

which allows to recover the original series, $x(t)$, from its continuous wavelet transform.

Likewise in Fourier analysis, several interesting quantities can be defined in the wavelet domain. For instance, one can define the wavelet power spectrum as $|W_x(\tau, s)|^2$. It measures the relative contribution at each time and at each scale to the time series' variance. In fact, the wavelet power spectrum can be integrated across τ and s to recover the total variance of the series as follows

$$\sigma_x^2 = \frac{1}{C_\psi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |W_x(\tau, s)|^2 \frac{d\tau ds}{s^2} \tag{9}$$

Another quantity of interest is the cross-wavelet spectrum which captures the covariance between two series in the time-frequency space. Given two time series $x(t)$ and $y(t)$, with wavelet transforms $W_x(\tau, s)$ and $W_y(\tau, s)$ one can define the cross-wavelet spectrum as $W_{xy}(\tau, s) = W_x(\tau, s)W_y^*(\tau, s)$. As the mother wavelet is in general complex, the cross-wavelet spectrum is also complex valued and it can be decomposed into real and imaginary parts.

In a similar fashion as in Fourier analysis, one can define the wavelet squared coherency as the absolute value squared of the smoothed cross-wavelet spectrum, normalized by the smoothed wavelet power spectra

$$R^2(\tau, s) = \frac{\left|S\left(s^{-1}W_{xy}(\tau, s)\right)\right|^2}{S\left(s^{-1}\left|W_x(\tau, s)\right|^2\right)S\left(s^{-1}\left|W_y(\tau, s)\right|^2\right)} \tag{10}$$

where $S(.)$ denotes smoothing in both time and scale (see, for example, Torrence and Webster (1999)) and the factor s^{-1} is used to convert to an energy density. As well as in Fourier analysis, smoothing is also required, otherwise squared coherency would be always equal to one (see, for example, Priestley (1981, p. 708)). The reason for this smoothing is that coherency should be calculated on expected values but in most cases this is impossible, as there is only one realisation of the time series and not a sample from the population.

The idea behind the wavelet squared coherency is similar to the one of squared coherency in Fourier analysis. The wavelet squared coherency measures the strength of the relationship between the two series over time and across frequencies (while the squared coherency in Fourier analysis only allows one to assess the latter). The $R^2(\tau, s)$ is between 0 and 1 with a high (low) value indicating a strong (weak) relationship. Hence, through the plot of the wavelet squared coherency one can distinguish the regions in the time-frequency space where the link is stronger and identify both time and frequency varying features.

Additionally, one can also compute the wavelet phase, which captures the lead-lag relationship between the variables in the time-frequency space. The wavelet phase is given by

$$\phi(\tau, s) = \tan^{-1}\left(\frac{\Im\left(S\left(s^{-1}W_{xy}(\tau, s)\right)\right)}{\Re\left(S\left(s^{-1}W_{xy}(\tau, s)\right)\right)}\right) \tag{11}$$

where $\Re$ and $\Im$ are the real and imaginary parts, respectively. The resemblance with the analogue measure in Fourier analysis is clear. Likewise in Fourier analysis, the phase provides information about the lead-lag relationship between the two series. However, besides providing information about the lead-lag across frequencies as standard Fourier analysis, the wavelet phase also allows one to assess how such lead-lag relationship has changed over time.

As the wavelet transform at a given point in time uses information of neighbouring data points, the values of the wavelet transform are generally less accurate as the wavelet approaches the edges of the time-series (this region is known as the cone of influence (see Torrence and Compo (1998)). The region affected increases with the temporal support (or width) of the wavelet and to minimize such effects several methods have been developed (zero padding, value padding, decay padding, periodization, polynomial fitting, among others (see, for example, Addison (2002)). Hence, the results should be read carefully close to the beginning or the end of the time series.

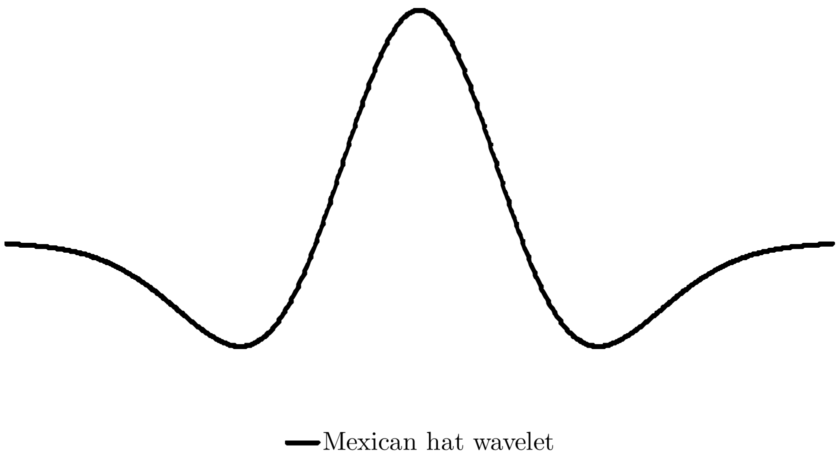


Figure 4: Mexican hat wavelet.

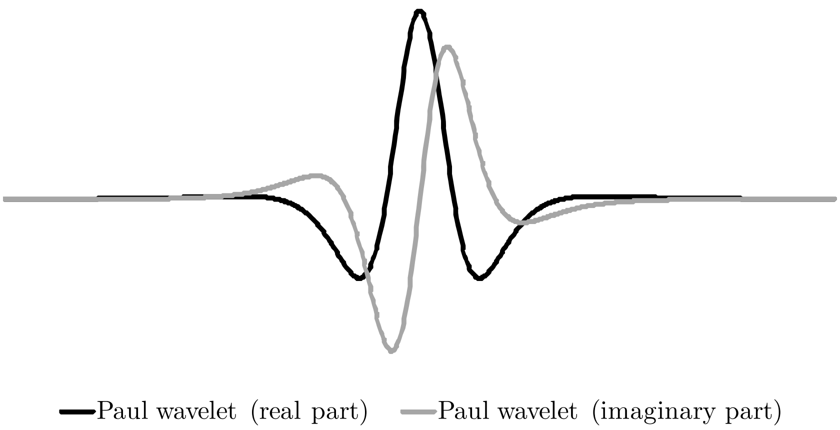


Figure 5: Paul wavelet.

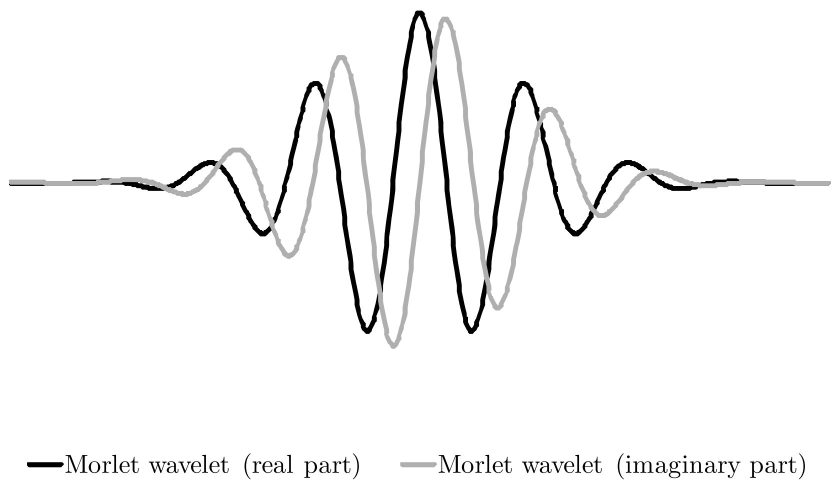


Figure 6: Morlet wavelet.

As mentioned earlier, all wavelets used in wavelet analysis are obtained by translation and dilation of the mother wavelet. There are a number of functions that can be used for this purpose. The most commonly used mother wavelet for the continuous wavelet transform is the Morlet wavelet (other mother wavelets include, for example, Paul and Mexican hat wavelets) (see Figures 4 to 6). One of the advantages of the Morlet wavelet is its complex nature which allows for both time-dependent amplitude and phase for different frequencies. The Morlet wavelet is defined as

$$\psi(t) = \pi^{-\frac{1}{4}} \left(e^{i\omega_0 t} - e^{-\frac{\omega_0^2}{2}} \right) e^{\frac{-t^2}{2}} \tag{12}$$

Since the term $e^{-\frac{\omega_0^2}{2}}$ (known as the correction term, as it corrects for the non-zero mean of the complex sine of the first term) becomes negligible for an appropriate ω_0, the Morlet wavelet can be simply defined as

$$\psi(t) = \pi^{-\frac{1}{4}} e^{i\omega_0 t} e^{\frac{-t^2}{2}} \tag{13}$$

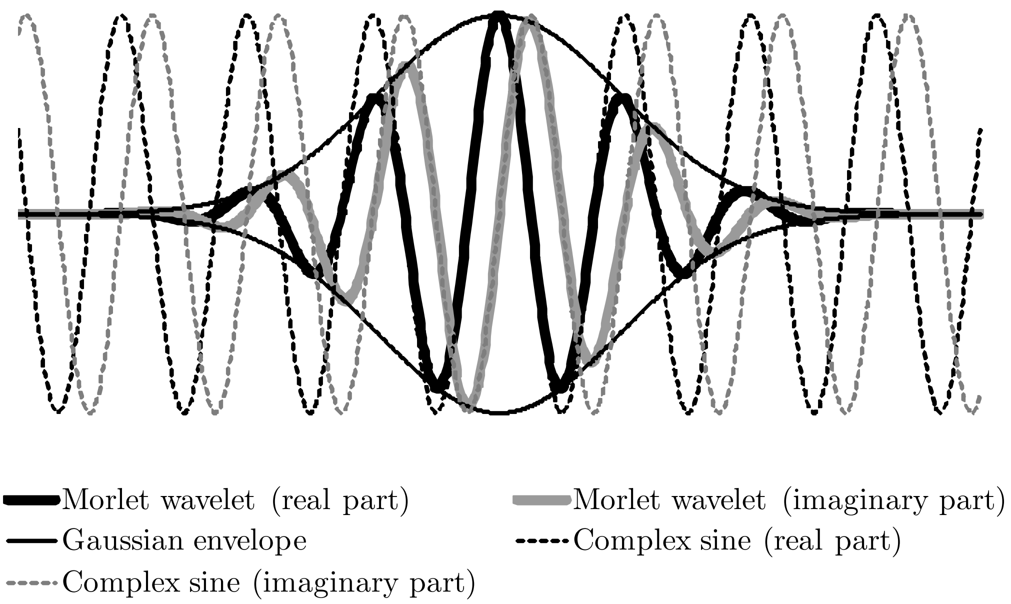


Figure 7: Real and imaginary parts of the Morlet wavelet for $\omega_0 = 6$.

One can see that the Morlet wavelet consists of a complex sine wave within a Gaussian (bell-shaped) envelope (see Figure (7)). The normalization factor, $\pi^{-\frac{1}{4}}$, ensures that the wavelet function has unit energy. The parameter ω_0 is the wavenumber and controls the number of oscillations within the Gaussian envelope. By increasing (decreasing) the wavenumber one achieves better (poorer) frequency localization but poorer (better) time localization. In practice, setting ω_0 to 6 provides a good balance between time and frequency localization. Since the wavelength for the Morlet wavelet is given by $\frac{4\pi s}{\omega_0 + \sqrt{2+\omega_0^2}}$ (see, Torrence and Compo (1998)), then for $\omega_0 = 6$, the wavelet scale s is almost equal to the Fourier period which eases the interpretation of wavelet analysis.

1.3 The discrete wavelet transform and multiresolution analysis

Up to now the discussion has been focused on the continuous wavelet transform. In the continuous wavelet transform, the wavelets used are not orthogonal and the data obtained by this transform are highly correlated. In fact, in a non-orthogonal wavelet analysis an arbitrary number of scales can be used to provide a complete picture. In contrast, the discrete wavelet transform decomposes the signal into a mutually orthogonal set of wavelets.[2] Hence, the discrete wavelet transform uses a limited number of translated and dilated wavelets. The idea is to choose τ and s so that the information contained in the signal can be summarized in a minimum of wavelet coefficients. Although the discrete wavelet transform can be derived without explicitly relating it to the continuous wavelet transform, one can think of the former as a "discretization" of the latter. The "discretization" of the wavelet representation in (4) can be written as

$$\psi_{j,k}(t) = \frac{1}{\sqrt{s_0^j}}\psi\left(\frac{t - k\tau_0 s_0^j}{s_0^j}\right) \tag{14}$$

where j and k are integers that control the wavelet dilation and translation, respectively. The scale discretization is given by $s = s_0^j$ and the translation discretization corresponds to $\tau = k\tau_0 s_0^j$ where $s_0 > 1$ and $\tau_0 > 0$. Note that the size of the translation step, $\Delta\tau = \tau_0 s_0^j$, is proportional to the wavelet

[2] There are other variants of the wavelet transform such as, for example, the maximal overlap discrete wavelet transform and the discrete wavelet packet transform (see, for example, Percival and Walden (2000)).

scale s_0^j. Typically, one sets $s_0 = 2$ and $\tau_0 = 1$ which renders a dyadic grid, that is, the sampling of both the frequency and time axis is dyadic. This implies that the discrete wavelet transform is calculated only at dyadic scales, that is, at scales 2^j, and within a given dyadic scale one picks times t that are separated by multiples of 2^j. In this case the maximum number of scales that can be considered is limited by the total number of observations T (that is, J is the maximum integer such that $J \leq \frac{\log(T)}{\log(2)}$). Naturally, the number of translations within a given dyadic scale is also upper bounded by the duration of the signal.

Substituting $s_0 = 2$ and $\tau_0 = 1$ into equation (14) one gets the following discrete wavelet

$$\psi_{j,k}(t) = \frac{1}{\sqrt{2^j}}\psi\left(\frac{t - 2^j k}{2^j}\right) \tag{15}$$

Discrete wavelets are usually chosen to be orthonormal through an appropriate choice of the mother wavelet, that is,

$$\int_{-\infty}^{+\infty} \psi_{j,k}(t)\psi^*_{m,n}(t) = \begin{cases} 1 \text{ if } j = m \text{ and } k = n \\ 0 \qquad\qquad \text{otherwise} \end{cases} \tag{16}$$

This means that the discrete wavelets are orthogonal to their own dilations and translations and normalized to have unit energy.

Drawing on its spectral properties, the wavelet function can be seen as a band-pass filter. Since for every stretch of the wavelet in the time domain with a factor of 2 the corresponding spectrum bandwidth is halved, one would need an infinite number of wavelets to cover the entire spectrum. To overcome such problem, one has to resort to the so-called scaling function (or father wavelet). The scaling function acts like a low-pass filter and, therefore, it is associated with the smoothing of the signal. In this way, the smooth and low-frequency part of the series is captured by the father wavelet $\phi(t)$, while the detail and high-frequency components are described by the mother wavelet $\psi(t)$.

Hence, the discrete wavelet transform makes it possible to decompose a time series into its constituent multiresolution components. The orthogonal wavelet series approximation to a time series $y(t)$ is defined by

$$y(t) = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + \cdots + \sum_k d_{1,k}\psi_{1,k}(t) \tag{17}$$

where J is the number of multiresolution levels (or scales) and k ranges from one to the number of coefficients in the corresponding component. The coefficients $s_{J,k}$, $d_{J,k}$, $d_{J-1,k}$, ..., $d_{1,k}$ are the wavelet transform coefficients, which are given by

$$s_{J,k} = \int y(t)\phi_{J,k}(t)dt \tag{18}$$

$$d_{j,k} = \int y(t)\psi_{j,k}(t)dt, \qquad j = 1, 2, ..., J. \tag{19}$$

These coefficients give a measure of the contribution of the corresponding wavelet function to the signal.

The functions $\phi_{J,k}(t)$ and $\psi_{j,k}(t)$ are generated from ϕ and ψ through scaling and translation as follows

$$\phi_{J,k}(t) = 2^{-J/2}\phi\left(\frac{t - 2^J k}{2^J}\right) \tag{20}$$

and

$$\psi_{j,k}(t) = 2^{-j/2}\psi\left(\frac{t - 2^j k}{2^j}\right), \qquad j = 1, 2, ..., J. \tag{21}$$

In a more synthetic way, equation (17) can be rewritten as

$$y(t) = S_J(t) + D_J(t) + D_{J-1}(t) + ... + D_1(t) \tag{22}$$

where $S_J(t) = \sum_k s_{J,k}\phi_{J,k}(t)$ and $D_j(t) = \sum_k d_{j,k}\psi_{j,k}(t)$ for $j = 1, 2, ..., J$ are the smooth and detail components, respectively. The expression (22) represents the decomposition of $y(t)$ into orthogonal components, $S_J(t)$, $D_J(t)$, $D_{J-1}(t)$,..., $D_1(t)$, at different resolutions and constitutes the so-called wavelet multiresolution decomposition. For a level J multiresolution analysis, the wavelet decomposition of the variable y consists of J wavelet details ($D_J(t)$, $D_{J-1}(t)$,..., $D_1(t)$) and a single wavelet smooth ($S_J(t)$). The wavelet smooth captures the low-frequency dynamics while the wavelet details represent the higher-frequency characteristics of y.

Concerning the wavelet families used in the discrete wavelet transform, there are several alternatives in the literature, namely, Haar, Daubechies, Symmlets, Coiflets, among others (see Figure (8)). As stressed by Crowley (2007), the Haar wavelet is not appropriate for the study of economic variables, due to the discontinuous nature of its waveform, but beyond this, there is no clear-cut choice. Naturally, a sensitivity analysis may help to assess the robustness of the results.

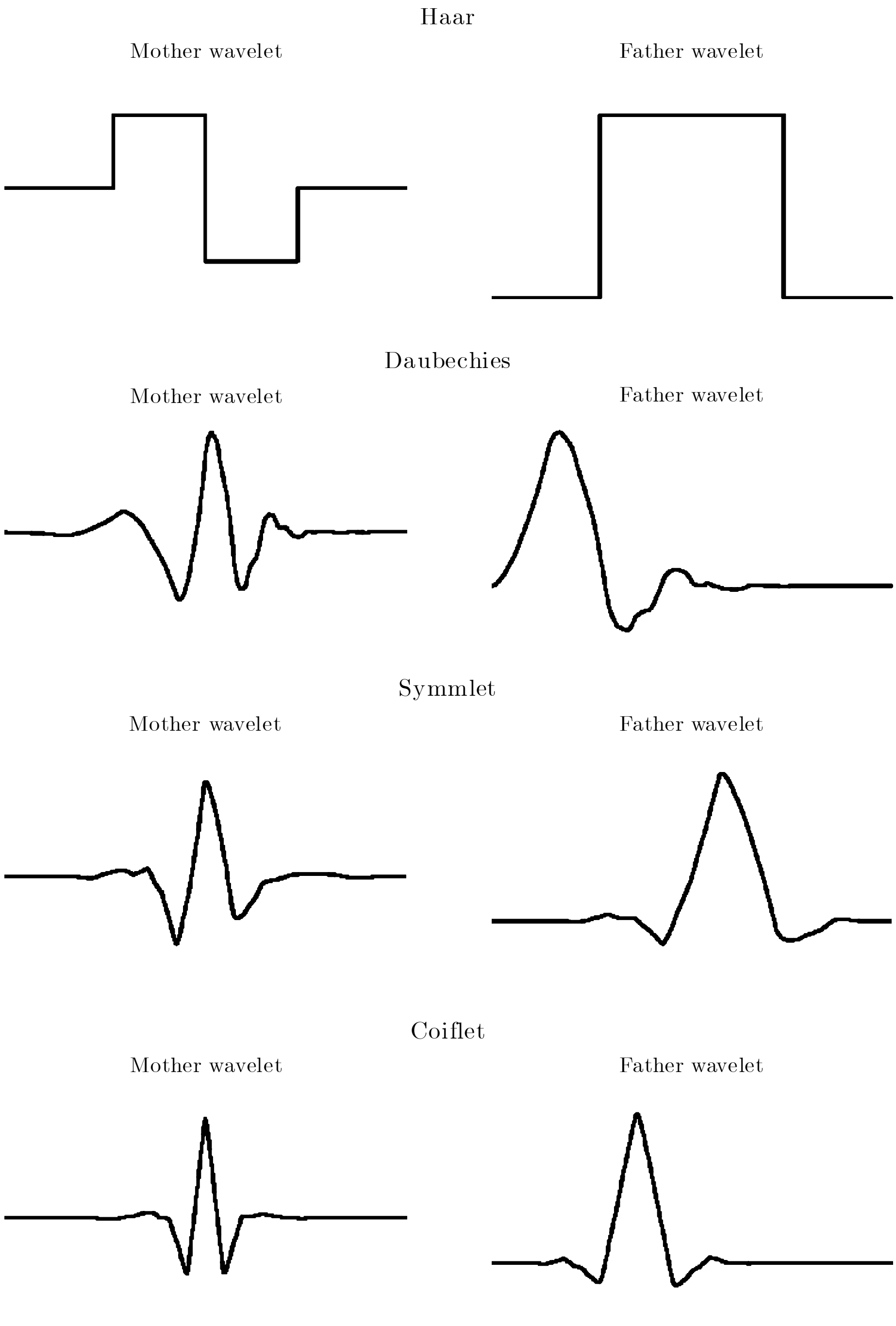


Figure 8: Several wavelet families.

1.4 Dissertation overview

As mentioned earlier, this dissertation intends to contribute to the literature by providing several essays where wavelet anaysis is used to address important economics problems. In the first essay, "Money growth and inflation in the euro area: a time-frequency view" (see chapter 2), a new look into the relationship between money growth and inflation is provided. In the euro area, there is a particular focus on the link between money growth and inflation, as the European Central Bank gives a prominent role to the monetary aggregate M3 within the monetary policy strategy. The assessment of such link over the last forty years is done resorting to wavelet analysis. Both frequency and time-varying interesting features are found concerning the link between money growth and inflation in the euro area. On one hand, the relationship between inflation and money growth is stronger at low frequencies than at the typical business cycle frequency range. On the other hand, such link is more robust at low frequencies over the whole sample period whereas there is only supporting evidence at the business cycle frequencies up to the beginning of the 80's. Concerning the leading properties of money growth, there seems to be a recent deterioration of money growth as leading indicator of inflation in the euro area. All in all, these results highlight the importance of a regular assessment of the privileged role attributed to the monetary aggregate M3 by the ECB within the two-pillar monetary policy framework for tracking medium to long-term developments in inflation.

Another topic of noteworthy interest is the measurement of comovement among economic variables. In the essay "Measuring comovement in the time-frequency space" (published in the *Journal of Macroeconomics*, 32 (2010) 685–691), a measure of comovement in the time-frequency space is proposed (see chapter 3). This wavelet-based measure allows one to assess simultaneously the comovement at the frequency level and over time, which constitutes a refinement to previous approaches. To illustrate the use of such measure, the comovement of growth cycles among the major euro area countries over the last three decades is assessed. It is found that the strength of comovement of growth cycles among the major euro area countries depends on the frequency and has changed over time.

In order to take on board more than two series when assessing comovement, in the essay "Cohesion within the euro area and United States: a wavelet-based view" (with Artur Silva Lopes) (see chapter 4), the measure proposed in the previous essay is extended to the more general case in order to obtain a measure of cohesion in the wavelet domain. Focusing on output growth, the cohesion among euro area countries and the cohesion within US at both the regional and state levels over the last decades is investigated. It is found that cohesion within euro area has been higher at the long-term and business cycle frequencies and it has increased since the mid-90s across all frequencies. On the other hand, cohesion within the US is higher at the typical business cycle frequency range but seems to have decreased since the beginning of the 90's. Note that this finding holds at both the regional and state levels. Moreover, it is found that US cohesion is higher at the regional level than at the state level across frequencies and over time. In addition, besides taking into account the spatial perspective when assessing cohesion, an analysis at the sectoral level is also conducted. Resorting to disaggregated data by eleven sectors for the euro area countries and US regions and states, a noteworthy heterogeneity in the results at the sectoral level is found.

It has already been acknowledged that wavelet analysis can contribute in a non-negligible way to the financial economics literature. In this respect, several work has been carried out. In the essay "International comovement of stock market returns: a wavelet analysis" (with Luis Catela Nunes, published in the *Journal of Empirical Finance*, 16 (2009) 632–639) the relationship between international stock market returns is assessed (see chapter 5). Naturally, this is a key issue in finance as it has important practical implications in asset allocation and risk management. In particular, the focus is on the major developed economies, namely Germany, Japan, United Kingdom and United States over the last four decades. Moreover, by considering the decomposition of the aggregate index in ten sectors, several insights at the sectoral level are also provided. Another contribution to financial economics is the essay "A wavelet-based assessment of market risk" (with Luis Catela Nunes) where a wavelet-based approach is proposed for measuring market risk (see chapter 6). In this way, market risk is allowed to vary both through time and at the frequency level within a unified framework. The wavelet counterparts of well-known measures of risk are obtained and to illustrate such analysis the emerging markets case over the last twenty years is considered.

Finally, the essay "A wavelet approach for factor-augmented forecasting" (forthcoming in *Journal of Forecasting*) intends to contribute to the use of wavelets in forecasting (see chapter 7). In particular, the aim of this essay is to bridge the wavelet approach and factor-augmented models. Factor-augmented models have become quite popular in the recent literature as they allow to handle large data sets in a straightforward and parsimonious way for forecasting purposes. The illustrate such approach, one focus on the short-term forecasting of GDP growth for the major euro area countries, namely Germany, France, Italy and Spain. Among the several alternatives considered, it is found that the best performing procedure is to combine the wavelet approach and factor-augmented models. In particular, for the one-quarter ahead horizon, the forecasting gains are quite noteworthy and the findings are supported by forecast accuracy and encompassing tests.

2 Money growth and inflation in the euro area: a time-frequency view

2.1 Introduction

As it is well known, the role of money for central banks has changed over the last decades. After the breakdown of the Bretton-Woods system of fixed exchange rates which provided a nominal anchor for monetary policy, several industrialized countries, including the United States, Canada, the United Kingdom, Germany and Switzerland, adopted the monetary targeting in the 1970s (see, for example, Bernanke and Mishkin (1992) and Mishkin and Posen (1997)). However, the monetary targeting pursued by most central banks was not successful in controlling inflation which led them to abandon such strategy starting in the early 1980s. Germany and Switzerland are noteworthy exceptions, with the Bundesbank and the Swiss National bank engaging in monetary targeting for over twenty years (see, for example, Bernanke and Mihov (1997) and Issing (1997) for the former and Rich (1997, 2003) for the latter). The abandon of explicit monetary targeting and the growing instability of the relationship between monetary aggregates and goal variables such as inflation (see, for example, Estrella and Mishkin (1997)) led to a less influential role of money in the policy process. In the early 1990s, inflation targeting was first adopted by industrialized countries such as New Zealand, Canada and the United Kingdom (see, for example, Leiderman and Svensson (1995) and Bernanke *et al.* (1999)) and since the late 1990s, it has been adopted in a number of emerging market and developing countries (see, for example, Batini *et al.* (2006)).

In contrast with the US Federal Reserve and many inflation targeting central banks, which give no special role to monetary aggregates, the European Central Bank (ECB) has kept an important role for money in its monetary policy strategy. As it is well known, the primary objective of the European Central Bank is to maintain price stability which is defined as a year-on-year increase in the Harmonised Index of Consumer Prices for the euro area of below, but close to, 2 per cent over the medium term (ECB (2003a)). The assessment of risks to price stability is based on the so-called two-pillar framework (see, for example ECB (2003b, 2004) and Gerlach (2003, 2004)). The first pillar relies on economic analysis, where the focus is on real activity and financial conditions in the economy, to identify short to medium-term risks. The second pillar refers to monetary analysis to assess medium to long-term developments in inflation. Hence, for tracking inflation shorter-run fluctuations ECB puts emphasis on the economic analysis while for medium to long-term trends the focus is on money growth. In this respect, the monetary aggregate M3 has a prominent role in the ECB's monetary policy strategy.

One of the reasons for the key role of M3 growth in the ECB's monetary policy framework is its leading properties regarding inflation in the euro area. Supporting evidence of such relationship can be found, for example, in Trecroci and Vega (2002) and Nicolleti-Altimari (2001).[3] However, several authors have argued that such evidence does not hold anymore when more up-to-date data is used (see, for example, Hofmann (2006), OECD (2007), Kahn and Benolkin (2007) and Alves *et al.* (2007)). In other words, M3 may be loosing its information content about future price developments in the euro area.

Time domain analysis is, far from doubt, the most widespread approach in the literature to study time series. Through such approach, the evolution of individual variables and multivariate relationships are assessed over time. Another strand of literature focus on the frequency domain analysis which is

[3] In contrast, in other countries, like the United States, there is no much evidence in favour of the usefulness of money for anticipating future inflation (see, for example, Friedman and Kuttner (1992), Estrella and Mishkin (1997) and Stock and Watson (1999)).

a complementary tool to time domain analysis. In fact, with Fourier analysis, one can obtain additional insights through the study of the individual and multivariate behaviour at the frequency level (recent work includes, for example, A'Hearn and Woitek (2001), Pakko (2004), Rua and Nunes (2005) and Breitung and Candelon (2006)).

In particular, the pioneer work of Lucas (1980) highlighted the importance of the frequency level when assessing the link between money growth and inflation. The idea that the determinants of inflation vary across frequencies with money growth and inflation more closely tied in the long-run, *i.e.*, low frequencies, set path to a growing literature assessing such relationships across frequency bands. This includes Summers (1983), Geweke (1986), Thoma (1994), Jaeger (2003), Haug and Dewald (2004), Benati (2005, 2009), Bruggeman *et al.* (2005), Assenmacher-Wesche and Gerlach (2007a, 2007b, 2008a, 2008b) among others.[4]

While on one hand there is evidence that the link between money growth and inflation can vary across frequencies on the other hand, it has been found that such relationship may also change over time (see, for example, Rolnick and Weber (1997), Christiano and Fitzgerald (2003), Benati (2005, 2009) and Sargent and Surico (2008)).

In this essay, one resorts to wavelet analysis which allows both time and frequency domains to be taken into account. Hence, one can assess simultaneously how variables are related at different frequencies and how such relationship has evolved over time through an unified framework. One should note that most of the above mentioned papers (on money growth and inflation) that take into account the frequency perspective end up conditioning the analysis on a somehow arbitrary cut-off of the frequency bands[5], whereas

[4] Assenmacher-Wesche and Gerlach (2008a) provide a nice summary of the mentioned contributions.

[5] For example, Jaeger (2003) considers low frequencies as those associated with fluctua-

those who provide evidence of time-varying relationship resort to the analysis of sub-samples with split dates more or less *ad-hoc*. Wavelet analysis avoids such problems as it provides a continuous assessment of the relationship between money growth and inflation in the time-frequency space.

Even though wavelets have been more popular in fields such as signal and image processing, meteorology, physics, among others, such analysis can also provide fruitful insights about several economic phenomena (see, for example, the seminal work of Ramsey and Zhang (1996, 1997) and Ramsey and Lampart (1998a, 1998b)). Recent work drawing on wavelets includes, Kim and In (2005) who assess the link between stock returns and inflation, Gençay *et al.* (2005) and Fernandez (2005) focus on the Capital Asset Pricing Model, Conejo *et al.* (2005) use wavelets for forecasting, Rua and Nunes (2009) assess the international comovement of stock market returns, Yogo (2008) and Rua (2010) resort to wavelets for business cycle analysis, among others (see, for example, Crowley (2007) for a survey).

In fact, wavelet analysis constitutes a very promising tool as it represents a refinement in terms of analysis and, as Lau and Weng (1995) put it, can be thought as being able to decompose a signal into localized frequencies with the corresponding measure of intensity and duration, analogous to the bass/treble clef, the crescendo/decrescendo and the tempo in a piece of music. So, the question is what do we hear regarding money growth and inflation in the euro area.

In particular, the aim of this essay is to assess, through a wavelet analysis, the information content of money growth for tracking developments in euro area inflation. In particular, the strength of the relationship between money

tions longer than 8 years, Benati (2005, 2009) consider as low frequencies those associated with fluctuations longer than 30 years, Assenmacher-Wesche and Gerlach (2008a, 2008b) define the long run as fluctuations with a periodicity of more than 4 years (although the latter authors perform a sensitivity analysis), etc.

growth and inflation is assessed simultaneously across frequencies and over time resorting to wavelet analysis in order to unveil possible frequency and time-varying features. This essay also tries to shed some more light on the leading properties of the monetary aggregate M3 which have been put in stake recently as mentioned earlier.

It is found both frequency and time-varying features concerning the link between money growth and inflation in the euro area. On one hand, the relationship between inflation and money growth is stronger at low frequencies than at the typical business cycle frequency range. On the other hand, such link is more robust at low frequencies over the whole sample period whereas there is only supporting evidence at the business cycle frequencies up to the beginning of the 80's. Concerning the leading properties of money growth, there seems to be a recent deterioration of money growth as leading indicator of inflation in the euro area. These results stress the importance of a regular assessment of the privileged role attributed to the monetary aggregate M3 by the ECB within the two-pillar monetary policy framework for tracking medium to long-term developments in inflation.

The remainder of the chapter is organised as follows. In section 2.2, an overview of wavelet analysis is provided. In section 2.3, the empirical results for the euro area are presented. Finally, section 2.4 concludes.

2.2 Wavelet analysis

The well-known Fourier transform involves the projection of a series onto an orthonormal set of trigonometric components (see, for example, Priestley (1981)). Despite its usefulness, under the Fourier transform, all the time information about the signal is lost which makes it unsuitable to study time-varying phenomena. To overcome such limitation it has been suggested the so-called short-time or windowed Fourier transform. The basic idea is to break the time-series into smaller sub-samples and apply the Fourier transform to each sub-sample. However, a major drawback of the windowed Fourier transform is that the window width is constant which does not allow an adequate resolution for all frequencies. In contrast, the wavelet transform uses local base functions that can be stretched and translated with a flexible resolution in both frequency and time domains. Hence, wavelets can be a particular useful tool when the signal shows a different behaviour in different time periods or when the signal is localized in time as well as frequency. As it enables a more flexible approach in time series analysis, wavelet analysis is seen as a refinement of Fourier analysis.

The wavelet transform decomposes a time series in terms of some elementary functions, the daughter wavelets or simply wavelets $\psi_{\tau,s}(t)$. Wavelets are 'small waves' that grow and decay in a limited time period, that is, they have finite energy and compact support. These wavelets result from a mother wavelet $\psi(t)$, that can be expressed as function of the time position τ (translation parameter) and the scale s (dilation parameter), which is related with the frequency. While the Fourier transform decomposes the time series into infinite length sines and cosines, discarding all time-localization information, the basis functions of the wavelet transform are shifted and scaled versions of the time-localized mother wavelet. More explicitly, wavelets are defined as

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right) \tag{23}$$

where $\frac{1}{\sqrt{s}}$ is a normalization factor to ensure that wavelet transforms are comparable across scales and time series. To be a mother wavelet, $\psi(t)$, must fulfil several conditions (see, for example, Percival and Walden (2000)): it must have zero mean, $\int_{-\infty}^{+\infty} \psi(t)dt = 0$; its square integrates to unity, $\int_{-\infty}^{+\infty} \psi^2(t)dt = 1$, which means that $\psi(t)$ is limited to an interval of time; and it should also satisfy the so-called admissibility condition, $0 < C_\psi =$

$\int_0^{+\infty} \frac{|\widehat{\psi}(\omega)|^2}{\omega} d\omega < +\infty$ where $\widehat{\psi}(\omega)$ is the Fourier transform of $\psi(t)$, that is, $\widehat{\psi}(\omega) = \int_{-\infty}^{+\infty} \psi(t)e^{-i\omega\tau}dt$. The latter condition allows the reconstruction of a time series $x(t)$ from its continuous wavelet transform, $W_x(\tau, s)$. Thus, it is possible to recover $x(t)$ from its wavelet transform through the following formula

$$x(t) = \frac{1}{C_\psi} \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} \frac{1}{\sqrt{s}} \psi \left(\frac{t-\tau}{s} \right) W_x(\tau, s) d\tau \right] \frac{ds}{s^2} \tag{24}$$

The continuous wavelet transform of a time series $x(t)$ with respect to $\psi(t)$ is given by the following convolution

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t)\psi^*_{\tau,s}(t)dt = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} x(t)\psi^* \left(\frac{t-\tau}{s} \right) dt \tag{25}$$

where * denotes the complex conjugate. For a discrete time series, $x(t)$, $t = 1, ..., N$ we have

$$W_x(\tau, s) = \frac{1}{\sqrt{s}} \sum_{t=1}^{N} x(t)\psi^* \left(\frac{t-\tau}{s} \right) \tag{26}$$

Although it is possible to compute the wavelet transform in the time domain using equation (26), a more convenient way to implement it is to carry out the wavelet transform in Fourier space (see, for example, Torrence and Compo (1998)).

The most commonly used mother wavelet is the Morlet wavelet and can be simply defined as

$$\psi(t) = \pi^{-\frac{1}{4}} e^{i\omega_0 t} e^{\frac{-t^2}{2}} \tag{27}$$

with the corresponding Fourier transform given by

$$\widehat{\psi}(\omega) = \pi^{\frac{1}{4}} \sqrt{2} e^{-\frac{1}{2}(\omega - \omega_0)^2} \tag{28}$$

The Morlet wavelet is a complex sine wave within a Gaussian envelope whereas ω_0 is the wavenumber. In practice, ω_0 is set to 6 as it provides a good balance between time and frequency localization (see, for example, Grinsted *et al.* (2004))[6]. One of the advantages of the Morlet wavelet is its complex nature which allows for both time-dependent amplitude and phase for different frequencies (see, for example, Adisson (2002) for further details).

Likewise in Fourier analysis, several interesting measures can be computed in the wavelet domain. For instance, one can define the wavelet power spectrum as $|W_x(\tau, s)|^2$. It gives a measure of the time series' variance at each time and at each scale.

Given two time series $x(t)$ and $y(t)$, with wavelet transforms $W_x(\tau, s)$ and $W_y(\tau, s)$ one can define the cross-wavelet spectrum as $W_{xy}(\tau, s) =$ $= W_x(\tau, s)W_y^*(\tau, s)$. In a similar fashion as in Fourier analysis, one can define the wavelet squared coherency as the absolute value squared of the smoothed cross-wavelet spectrum, normalized by the smoothed wavelet power spectra

$$R^2(\tau, s) = \frac{\left|S\left(s^{-1}W_{xy}(\tau, s)\right)\right|^2}{S\left(s^{-1}\left|W_x(\tau, s)\right|^2\right) S\left(s^{-1}\left|W_y(\tau, s)\right|^2\right)} \tag{29}$$

where $S(.)$ denotes smoothing in both time and scale (see, for example, Torrence and Webster (1999)). As well as in Fourier analysis, smoothing is also required, otherwise squared coherency would be always equal to one (see, for example, Priestley (1981)).

The idea behind the wavelet squared coherency is similar to the one of squared coherency in Fourier analysis (see, for example, Croux *et al.* (2001)).

[6]According to the Heisenberg uncertainty principle there is always a trade-off between resolution in time and frequency when measuring a signal. That is, to get a better picture of the frequency composition one needs to consider a long period of the signal whereas if one wants to pinpoint a small sample period then it becomes difficult do determine the frequency makeup of the signal in that period. In this respect, it has been argued in the related literature that the Morlet wavelet with $\omega_0 = 6$ is a good choice.

The wavelet squared coherency measures the strength of the relationship between the two series over time and across frequencies (while the squared coherency in Fourier analysis only allows one to assess the latter). The $R^2(\tau, s)$ is between 0 and 1 with a high (low) value indicating a strong (weak) relationship. Hence, through the plot of the wavelet squared coherency one can distinguish the regions in the time-frequency space where the link is stronger and identify both time and frequency varying features.

Additionally, one can also compute the wavelet phase, which captures the lead-lag relationship between the variables in the time-frequency space. The wavelet phase is given by

$$\phi(\tau, s) = \tan^{-1}\left(\frac{\Im\left(S\left(s^{-1}W_{xy}(\tau, s)\right)\right)}{\Re\left(S\left(s^{-1}W_{xy}(\tau, s)\right)\right)}\right) \tag{30}$$

where $\Re$ and $\Im$ are the real and imaginary parts, respectively. The resemblance with the analogue measure in Fourier analysis is clear. Likewise in Fourier analysis, the phase provides information about the lead-lag relationship between the two series. However, besides providing information about the lead-lag across frequencies as standard Fourier analysis, the wavelet phase also allows one to assess how such lead-lag relationship has changed over time.

2.3 Empirical results for the euro area

In this section, one proceeds into the computation of the above mentioned measures for money growth and inflation in the euro area. Data regarding the monetary aggregate M3 is provided by the ECB and a long time series for the euro area harmonized consumer price index was possible to obtain from the OECD Main Economic Indicators database. Data is available monthly and the sample ranges from January 1970 up to December 2007, covering almost forty years (see figure 9).

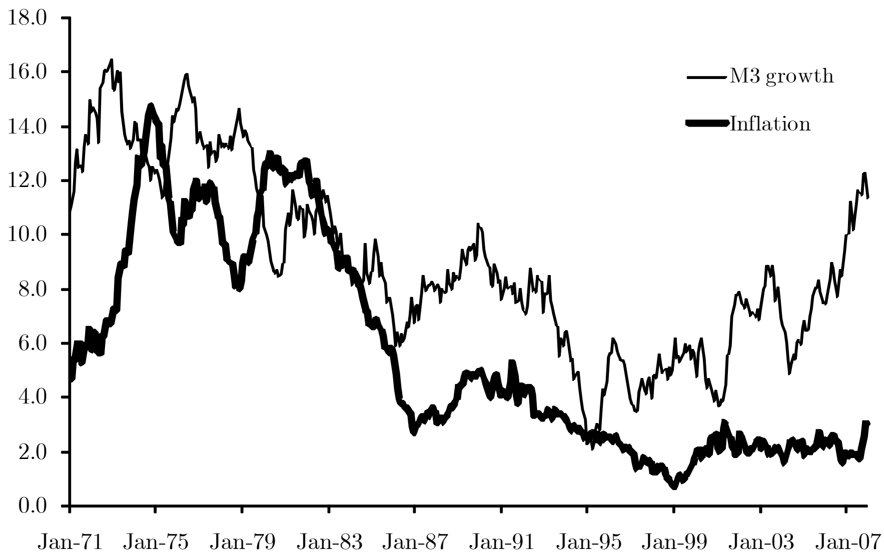


Figure 9: M3 growth and inflation in the euro area

In figures 10 and 11, the wavelet spectra for the corresponding standardized series is presented. The wavelet spectrum is presented through a contour plot as there are three dimensions involved. The horizontal axis refers to time while the vertical axis refers to frequency. To ease interpretation, the frequency is converted to time units (years). The grey scale is for the wavelet spectrum where increasing darkness corresponds to an increasing value and mimics the height in a surface plot. The bold line delimits the statistical significant area at the usual significance level of five per cent[7]. All computations have been done using Matlab.

One can see that both money growth and inflation have higher power at the typical business cycle frequency range, that is, between two and eight

[7] The critical values are based on the results of Torrence and Compo (1998) who have shown that the wavelet spectrum follows a Chi-square distribution.

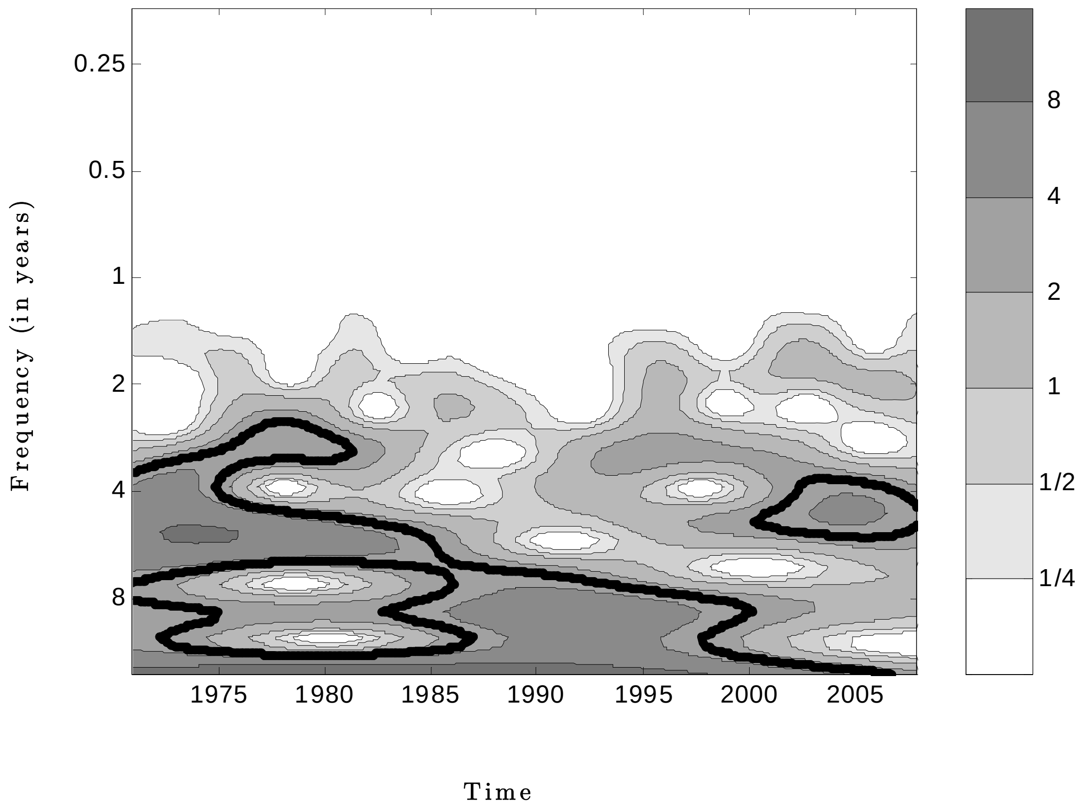


Figure 10: Wavelet spectrum of M3 growth

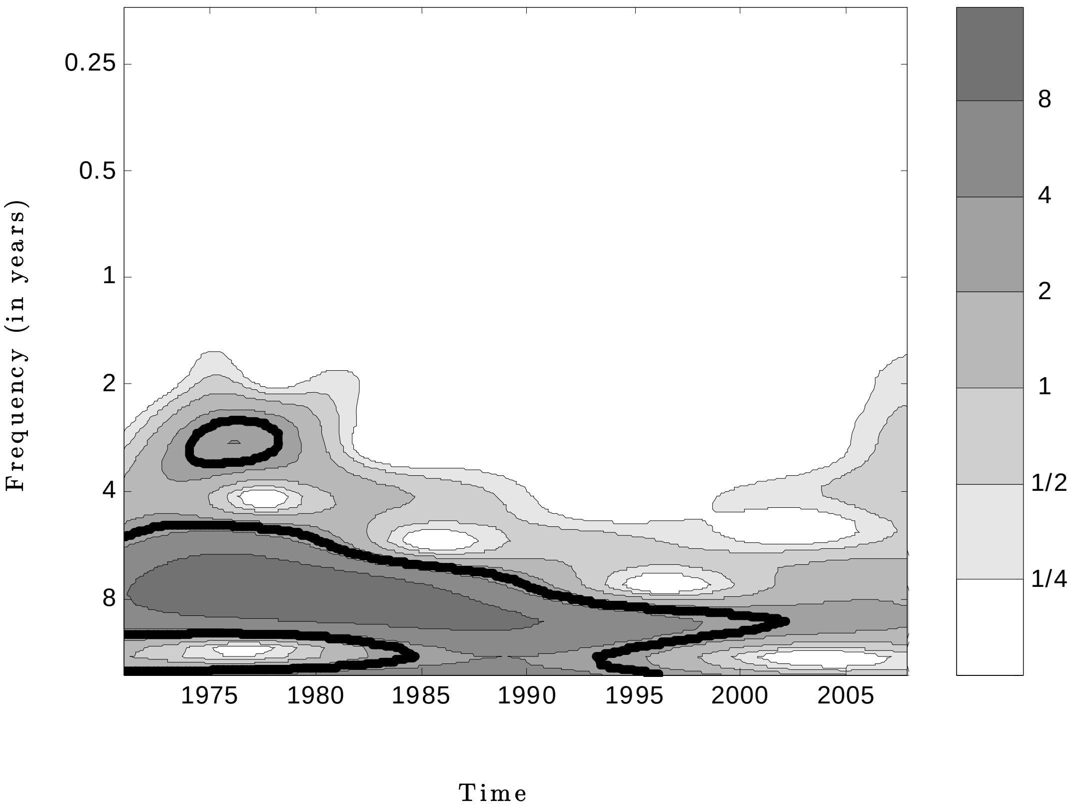


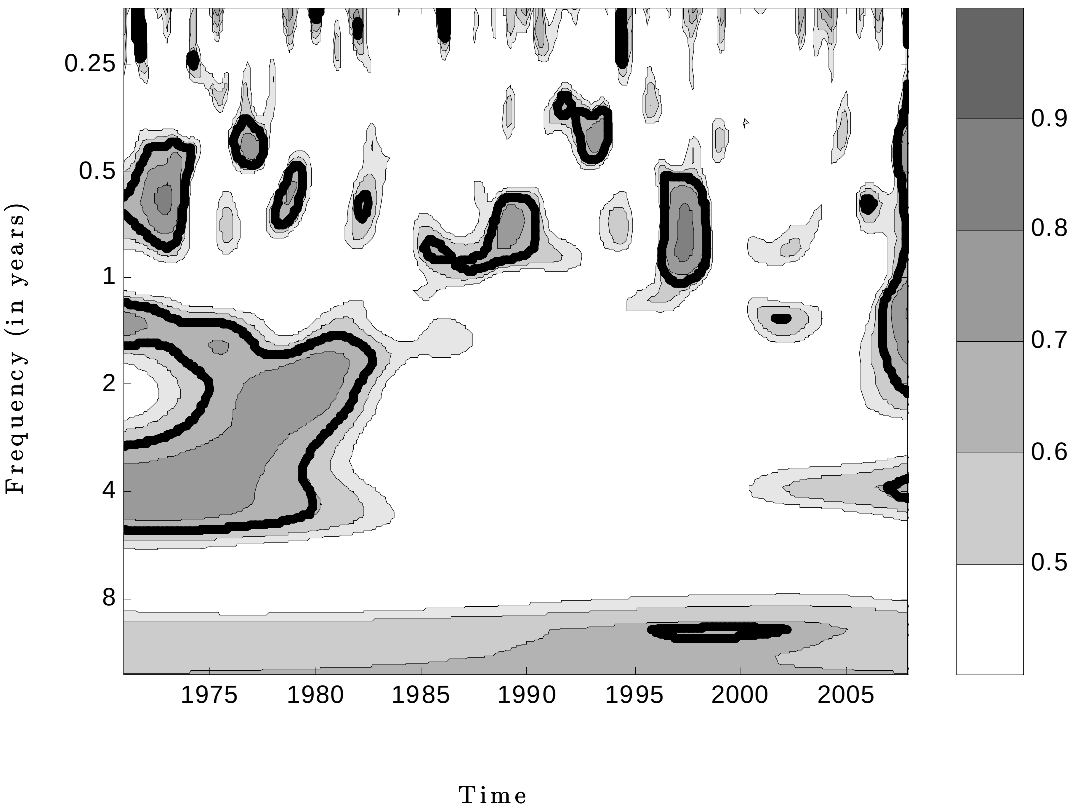
Figure 11: Wavelet spectrum of inflation

years, as well as at frequencies higher than eight years. For instance, Haug and Dewald (2004), using annual data from 1880 up to 2001 for several countries, also found that lower frequencies are more important in explaining the variance of money growth and inflation. Interestingly, there seems to be an increasing importance of low frequencies in detriment of the remaining ones in both series over time. However, regarding inflation, the power is not statistically significant at any frequency at the latter part of the sample. This reflects the flattening of inflation in the more recent period (see figure 9).

Regarding the relationship between money growth and inflation, the wavelet squared coherency is shown in figure 12. The wavelet squared coherency is also presented through a contour plot where increasing darkness corresponds to an increasing value ($0 \leq R^2(\tau, s) \leq 1$). Hence, through the inspection of the graph one can identify both frequency bands (in the vertical axis) and time intervals (in the horizontal axis) where the series move together. For example, a dark area at the bottom (top) of the graph means strong link at low (high) frequencies whereas a dark area at the left-hand (right-hand) side denotes strong relationship at the beginning (end) of the sample period. Moreover, through such wavelet analysis one can also assess if the strength of the link has increased or decreased over time and across frequencies capturing possible varying features in the relationship between the two series in the time-frequency space. The black bold line in the graph delimits the statistical significant area at the usual significance level of five per cent[8].

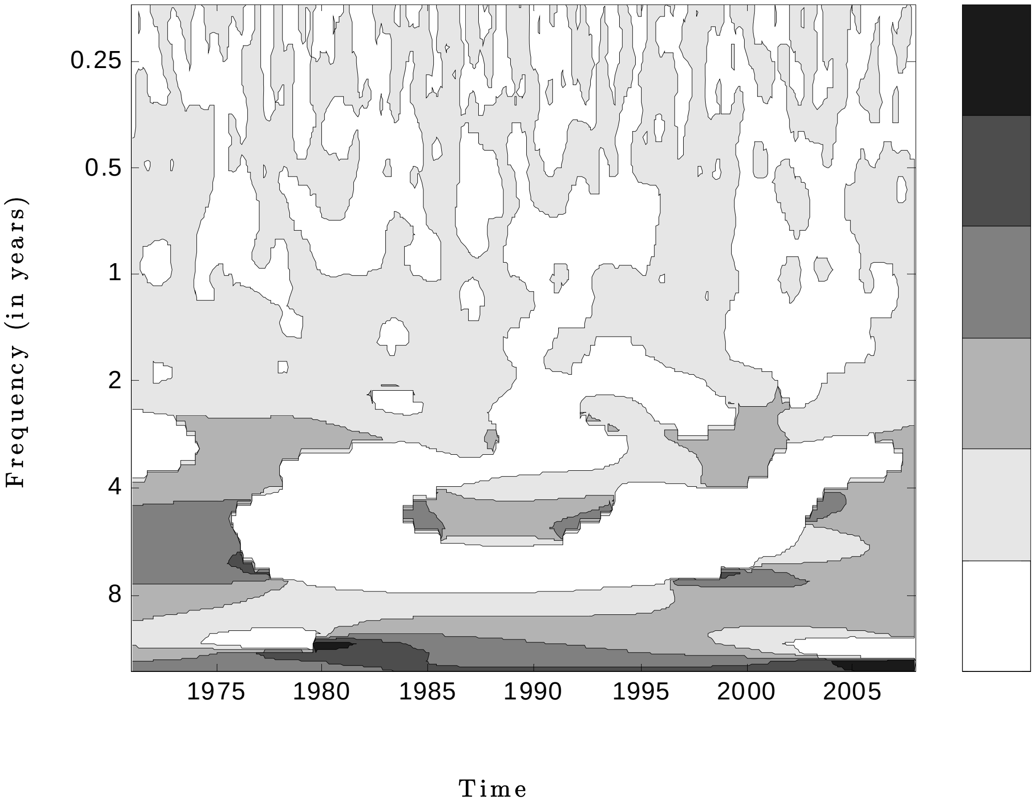
Disregarding higher frequencies, which as mentioned earlier do not receive much attention from the ECB when looking at the money growth and inflation relationship, one can see that money growth and inflation presented

[8]In this case, as the distribution is not known, the five per cent significance level was determined from a Monte Carlo simulation of wavelet squared coherency between 10 000 sets of two white noise time series with the same length as the series under analysis (see, for example, Torrence and Webster (1999)).



Note: White area corresponds to a squared coherency lower than 0.5 with increasing darkness corresponding to increasing squared coherency.

Figure 12: Wavelet squared coherency



Note: White area corresponds to a negative lead, that is, money growth lags inflation. Increasing darkness corresponds to increasing lead. Black area corresponds to a lead higher than four years.

Figure 13: Wavelet phase

a high and significant link at business cycle frequencies only up to the beginning of the 80's. Hence, it is not surprising that Alves *et al.* (2007) using data only from 1980 onwards did not find any evidence of a link between the two variables at business cycle frequencies.

Concerning long-term movements, there seems to be a more robust link between money growth and inflation in the euro area over the whole period considered (see also Benati (2005, 2009)). The results are also in line with Jaeger (2003) who found that, using a frequency domain analysis and pre-EMU data since 1961, the relationship between inflation and money growth is stronger at low frequencies than at frequencies associated with business cycle fluctuations. However, one should note that such link at low frequencies is stronger during the 90's up to the beginning of 2000's while weakening in the more recent period. This may explain why Alves *et al.* (2007) found evidence in favour of such link at low frequencies using data up to the beginning of 2000's which disappears when more recent data is included.

Hence, it seems that money growth has lost information content for tracking inflation medium term movements while for longer term developments the relationship is not as strong as it was when the euro area was launched. Therefore, one should be cautious about the role played by money growth as an indicator of inflation developments in the euro area.

Concerning the lead-lag relationship, the wavelet phase between the two series in the time-frequency space is presented in figure 13. To ease the reading of the figure, the phase is presented in time units (years) and only the lead time of money growth is discriminated, as this is the focus of the ECB. One can see that, at the business cycle frequency range and at the period where the link is stronger, money growth is leading inflation by a lead time up to around two years. For longer-term developments, money growth also presents leading properties. For instance, Haug and Dewald (2004) and Bruggeman *et al.* (2005) found that money growth leads inflation by 1 to

3 years and around 1.5 years, respectively. However, one should note that since the end of the 90's such characteristic seems to be weakening. This is also in line with the finding of the recent deterioration of money growth as leading indicator of inflation in the euro area.

So, not only the strength of the relationship between money growth and inflation in the euro area seems to have decreased but also the lead time. These results highlight the importance of a regular assessment of the role of money growth in tracking inflation developments in the euro area since such relationship varies across frequencies and over time. Furthermore, this should also be taken into account in the modelling of the link between money growth and inflation.

2.4 Conclusions

In this essay, the link between money growth and inflation is assessed through wavelet analysis. Wavelet analysis constitutes a very promising tool as it represents a refinement in terms of analysis in the sense that both time and frequency domains are taken into account. The motivation for using wavelet analysis to assess such link comes naturally as there is by now evidence that such relationship varies across frequencies and over time. In particular, the focus is on the euro area as the ECB attributes a privileged role to money growth within the two-pillar monetary policy framework. Hence, a time-frequency view of the relationship between money growth and inflation in the euro area is provided over the last forty years.

It is found that the link between monetary growth and inflation in the euro area presents both frequency and time-varying features. On one hand, the relationship between inflation and money growth has been stronger at the long-term developments than at frequencies associated with business cycle fluctuations. On the other hand, such link is more robust at low frequencies over the whole sample period whereas there is only supporting evidence at the business cycle frequency range up to the beginning of the 80's. In terms of the lead-lag relationship, in time-frequency areas where the link is stronger money growth has shown leading properties. However, there seems to be a deterioration of money growth as leading indicator of inflation in the more recent period. These results highlight the importance of a regular monitoring of the usefulness of money growth for tracking medium to long-term developments in euro area inflation since such relationship is frequency and time-varying.

3 Measuring comovement in the time-frequency space[9]

3.1 Introduction

The measurement of comovement among economic variables is key in several areas of economics and finance. From the innumerous fields where such assessment is crucial, one can mention business cycle analysis or asset allocation and risk management, just to name a few.

Traditionally, comovement is assessed in the time domain. The most popular measure of comovement is the well-known correlation coefficient. The contemporaneous correlation coefficient provides in a single number the degree of comovement between the series over the sample period. However, being a synthetic measure it can be rather limited unfolding the relationship between economic variables. For instance, it has been long acknowledged that the degree of comovement may vary over time. To take this feature into account, it has been current practice in the literature to compute a rolling window correlation coefficient or to consider non-overlapping sample periods to evaluate the evolving properties of comovement.

Another strand of literature focus on the frequency domain analysis which is a complementary tool to time domain analysis. In fact, with Fourier analysis, one can obtain additional insights through the study of the relationship between variables at the frequency level (see, for example, A'Hearn and Woitek (2001), Pakko (2004), Breitung and Candelon (2006) and Lemmens, Croux and DeKimpe (2008)). In this respect, Croux, Forni and Reichlin (2001) have proposed a spectral-based measure, the dynamic correlation, which allows one to measure the comovement between two series at each individual frequency. This measure, which ranges between -1 and 1, is concep-

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tually similar to the contemporaneous correlation between two series in the time domain. However, unlike the correlation coefficient in the time domain, one now obtains a comovement measure that can vary across frequencies. Several applications of the dynamic correlation can be found in recent literature (see, Crone (2005), Rua and Nunes (2005), Camacho, Perez-Quiros and Saiz (2006), Eickmeier and Breitung (2006), Lemmens, Croux and DeKimpe (2007), among others). However, as the dynamic correlation is defined in the frequency domain it disregards the time dependence of comovement. That is, it provides only a snapshot of the comovement at the frequency level not allowing to capture time-varying features.

Wavelet analysis merges both approaches, in the sense that both time and frequency domains are taken into account. Through wavelet analysis one can assess simultaneously how variables are related at different frequencies and how such relationship has evolved over time, allowing to capture non-stationary features. This is a distinct and noteworthy aspect as both time- and frequency-varying behaviour cannot be captured using previous approaches. Hence, wavelet analysis constitutes a very promising tool as it represents a refinement in terms of analysis which can provide rich insights about several economic phenomena (see, for example, the pioneer work of Ramsey and Zhang (1996, 1997) and Ramsey and Lampart (1998a,b)). Recent work drawing on wavelets includes, Kim and In (2005) who investigate the relationship between stock returns and inflation, Gençay *et al.* (2005) and Fernandez (2005) study the Capital Asset Pricing Model, Gallegati *et al.* (2008) and Yogo (2008) resort to wavelets for business cycle analysis, Rua and Nunes (2009) focus on international stock market returns, among others (see, for example, Crowley (2007) for a survey).

In this essay, it is proposed a measure of comovement in the time-frequency space by resorting to wavelet analysis. The wavelet-based measure of comovement herein suggested allows one to assess the extent to which two variables move together over time and across frequencies within an unified framework. To illustrate the use of such measure, the comovement of growth cycles among the major euro area countries over the last three decades is assessed. Through such empirical application, the usefulness of the proposed wavelet-based measure of comovement is highlighted as it allows to unveil both time- and frequency varying features. In fact, it is found that the strength of comovement of growth cycles among the major euro area countries depends on the frequency and has changed over time.

The chapter is organised as follows. In section 3.2, the wavelet-based measure of comovement is presented while in section 3.3, the empirical application is discussed. Finally, section 3.4 concludes.

3.2 A wavelet-based measure of comovement

The well-known Fourier transform involves the projection of a series onto an orthonormal set of trigononometric components (see, for example, Priestley (1981)). In particular, it uses sine and cosine base functions that have infinite energy (do not fade away) and finite power (do not change over time). Hence, the Fourier transform does not allow for any time dependence of the signal and therefore cannot provide any information about the time evolution of its spectral characteristics. To circumvent such limitation it has been suggested the so-called short-time or windowed Fourier transform. It consists in applying a short-time window to the signal and performing the Fourier transform within this window as it slides across all the data. A caveat of the windowed Fourier transform is that the window width and thus the time resolution is constant for all frequencies. When a wide range of frequencies is involved, the fixed time window tends to contain a large number of high frequency cycles and a few low frequency cycles which results in an overrepresentation of high frequency components and an underrepresentation of the low frequency components. Hence, as the signal is examined under a fixed time-frequency window with constant intervals in the time and frequency domains, the windowed Fourier transform does not allow an adequate resolution for all frequencies. In contrast, the wavelet transform uses local base functions that can be stretched and translated with a flexible resolution in both frequency and time. In the case of the wavelet transform, the time resolution is intrinsically adjusted to the frequency with the window width narrowing when focusing on high frequencies while widenning when assessing low frequencies. As it enables a more flexible approach in time series analysis, wavelet analysis is seen as a refinement of Fourier analysis.

Mathematically, the wavelet transform decomposes a time series in terms of some elementary functions, $\psi_{\tau,s}(t)$, which are derived from a time-localized mother wavelet $\psi(t)$ by translation and dilation (see, for example, Percival and Walden (2000)). Wavelets have finite energy and compact support, that is, they grow and decay in a limited time period and are defined as

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right) \tag{31}$$

where τ is the time position (translation parameter), s is the scale (dilation parameter), which is related with the frequency, and $\frac{1}{\sqrt{s}}$ is a normalization factor to ensure that wavelet transforms are comparable across scales and time series. To be a mother wavelet, $\psi(t)$, must fulfil certain criteria: it must have zero mean, $\int_{-\infty}^{+\infty}\psi(t)dt = 0$; its square integrates to unity, $\int_{-\infty}^{+\infty}\psi^2(t)dt = 1$, which means that $\psi(t)$ is limited to an interval of time; and it should also satisfy the so-called admissibility condition, $0 < C_\psi = \int_0^{+\infty}\frac{\left|\widehat{\psi}(\omega)\right|^2}{\omega}d\omega < +\infty$ where $\widehat{\psi}(\omega)$ is the Fourier transform of $\psi(t)$, that is, $\widehat{\psi}(\omega) = \int_{-\infty}^{+\infty}\psi(t)e^{-i\omega\tau}dt$.

The continuous wavelet transform of a time series $x(t)$ with respect to $\psi(t)$ is given by the following convolution

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t)\psi^*_{\tau,s}(t)dt = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} x(t)\psi^* \left(\frac{t-\tau}{s} \right) dt \qquad (32)$$

where * denotes the complex conjugate.

As with its Fourier counterpart, there is an inverse wavelet transform, defined as

$$x(t) = \frac{1}{C_\psi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \psi_{\tau,s}(t) W_x(\tau, s) \frac{d\tau ds}{s^2} \qquad (33)$$

This allows to recover the original series, $x(t)$, from its wavelet transform by integrating over all scales and time positions.

Likewise in Fourier analysis, several interesting quantities can be defined in the wavelet domain. For instance, one can define the wavelet power spectrum as $|W_x(\tau, s)|^2$. It measures the relative contribution at each time and at each scale to the time series' variance. In fact, the wavelet power spectrum can be integrated across τ and s to recover the total variance of the series as follows

$$\sigma_x^2 = \frac{1}{C_\psi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} |W_x(\tau, s)|^2 \frac{d\tau ds}{s^2} \qquad (34)$$

Another quantity of interest is the cross-wavelet spectrum which captures the covariance between two series in the time-frequency space. Given two time series $x(t)$ and $y(t)$, with wavelet transforms $W_x(\tau, s)$ and $W_y(\tau, s)$ one can define the cross-wavelet spectrum as $W_{xy}(\tau, s) = W_x(\tau, s)W_y^*(\tau, s)$. As the mother wavelet is in general complex, the cross-wavelet spectrum is also complex valued and it can be decomposed into real and imaginary parts.

In a similar fashion to Croux, Forni and Reichlin (2001), one can obtain the following measure

$$\rho_{xy}(\tau, s) = \frac{\Re\left(W_{xy}(\tau, s)\right)}{\sqrt{|W_x(\tau, s)|^2 |W_y(\tau, s)|^2}} \qquad (35)$$

where $\Re$ denotes the real part of the cross-wavelet spectrum which measures the contemporaneous covariance. The wavelet-based measure $\rho_{xy}(\tau, s)$ allows one to quantify the comovement in the time-frequency space and assess over which periods of time and frequencies is the comovement higher. Basically, it plays a role as a contemporaneous correlation coefficient around each moment in time and for each frequency. Likewise the standard correlation coefficient and the dynamic correlation proposed by Croux, Forni and Reichlin (2001), $\rho_{xy}(\tau, s)$ ranges between -1 and 1. While the dynamic correlation measure suggested by Croux, Forni and Reichlin (2001) is the Fourier counterpart of the standard correlation coefficient allowing to assess in which frequencies is the contemporaneous comovement higher, the measure herein proposed can be seen as a generalisation of such measure in the sense that $\rho_{xy}(\tau, s)$ provides information about the comovement not only at the frequency level but also over time.[10] This feature is of striking importance as it has been long acknowledged that the strength of the comovement may vary over time. In particular, by inspecting the contour plot of the above measure, one can identify the regions in the time-frequency space where the two time series co-move and assess both time and frequency varying features of the comovement. Hence, the suggested wavelet-based measure allows for a richer description on the comovement between the variables of interest.

3.3 An empirical application

For illustration purposes, we assess the comovement of growth cycles among the major euro area countries, namely Germany, France, Italy and Spain. As usual, the growth cycle is defined as the quarter-on-quarter real GDP growth

[10]Note that, the herein proposed measure is closely related with the wavelet squared coherency, likewise dynamic correlation of Croux, Forni and Reichlin (2001) is related with squared coherency. However, as also mentioned by Croux, Forni and Reichlin (2001), the phase differences between the variables are entirely disregarded in the latter case.

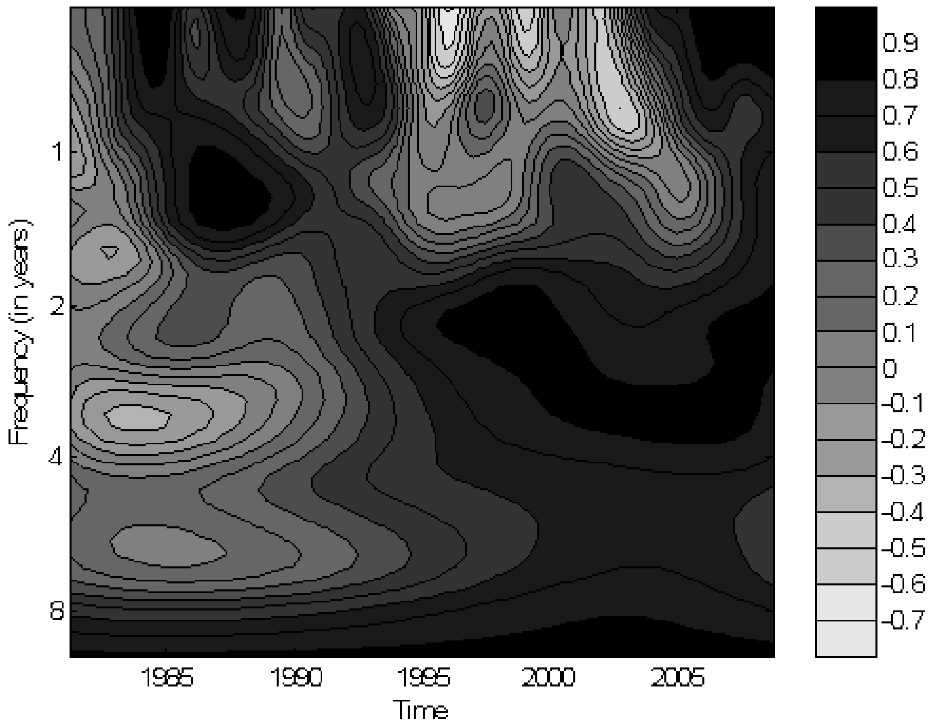

Figure 14: Comovement between the growth cycles of Germany and France

rate. The data are from Thompson Financial Datastream and the sample period for GDP runs from the first quarter of 1981 up to the fourth quarter of 2008.[11] Concerning the mother wavelet, the most frequent choice is the Morlet wavelet (see, for example, Adisson (2002) for further details) which will be the one used here.[12] All computations are done using Matlab.

[11] In the case of Germany, the growth rate before 1991 refers to West Germany.

[12] The Morlet wavelet can be defined as $\psi(t) = \pi^{-\frac{1}{4}} e^{i\omega_0 t} e^{\frac{-t^2}{2}}$, *i.e.* the Morlet wavelet is a complex sine wave within a Gaussian envelope whereas ω_0 is the wavenumber. In practice, ω_0 is set to 6 as it provides a good balance between time and frequency localization. In this case, the wavelet scale is almost equal to the Fourier period.

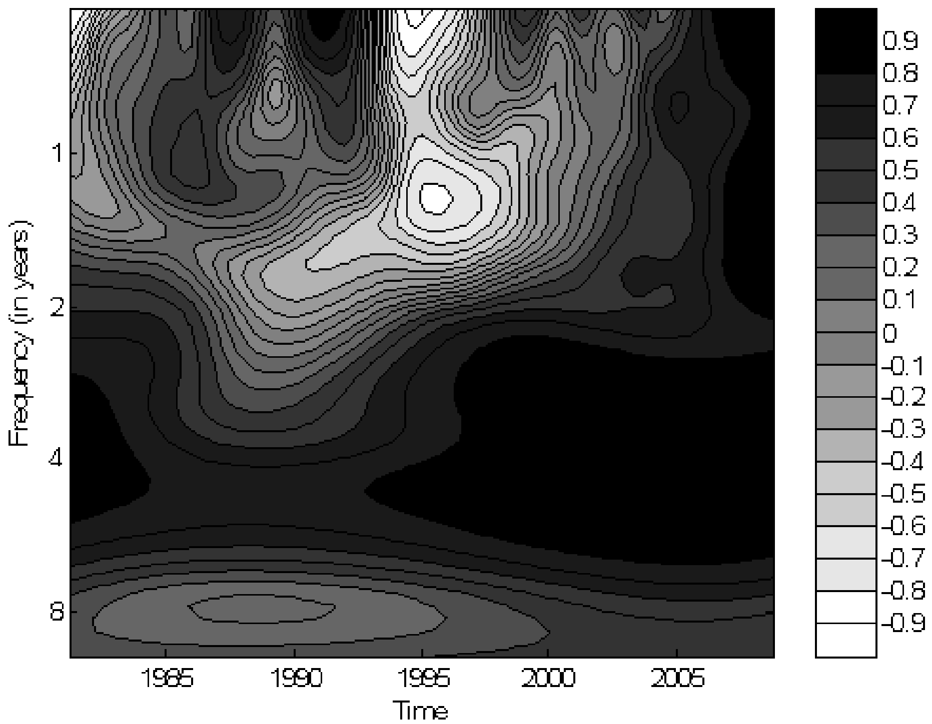

Figure 15: Comovement between the growth cycles of Germany and Italy

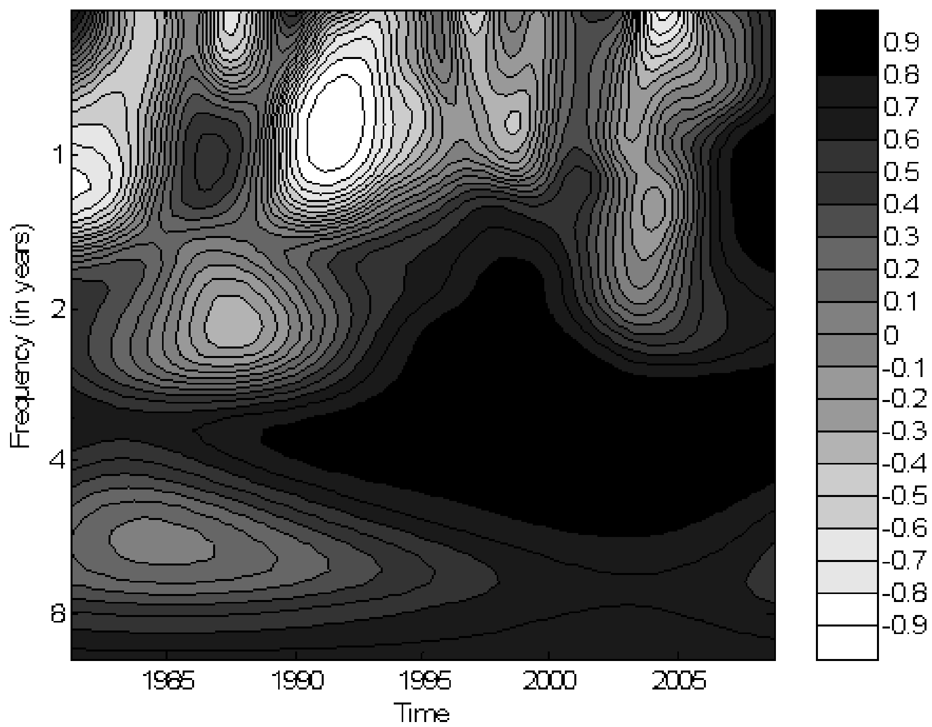

Figure 16: Comovement between the growth cycles of Germany and Spain

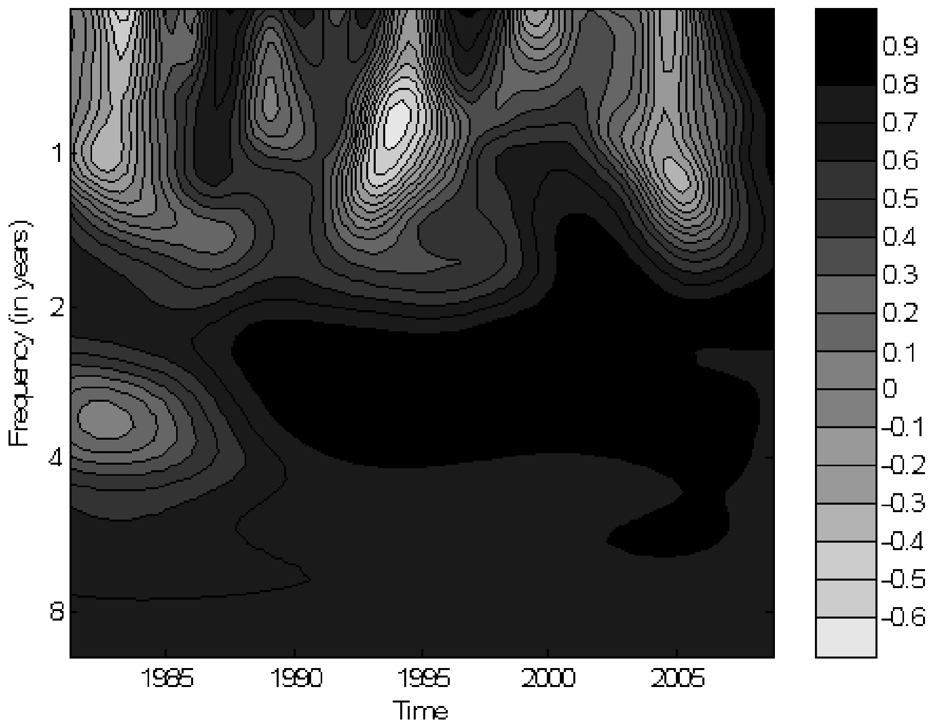

Figure 17: Comovement between the growth cycles of France and Italy

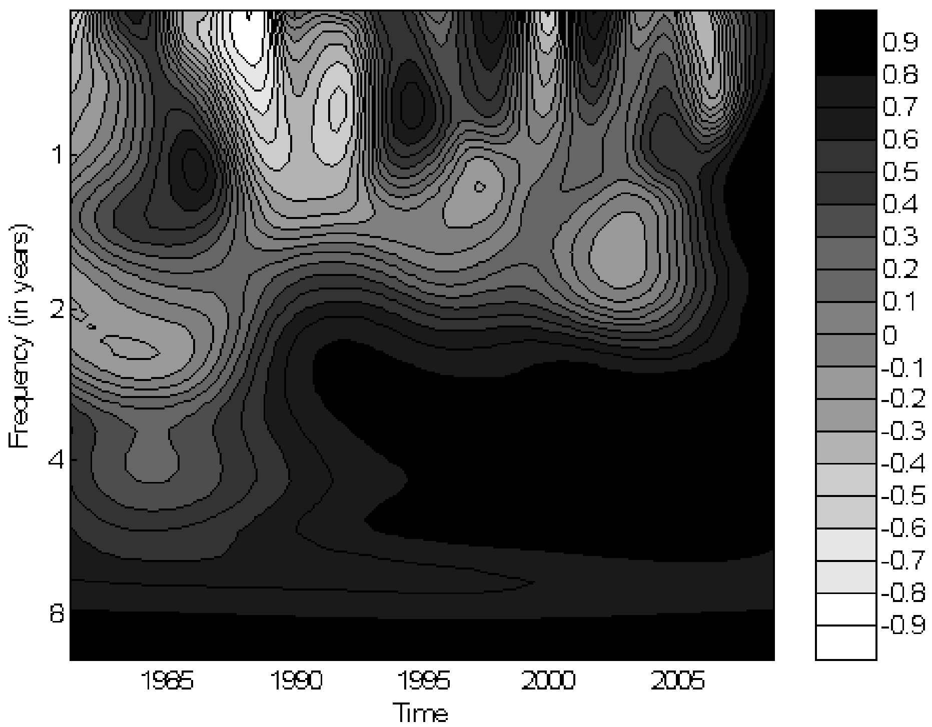

Figure 18: Comovement between the growth cycles of Italy and Spain

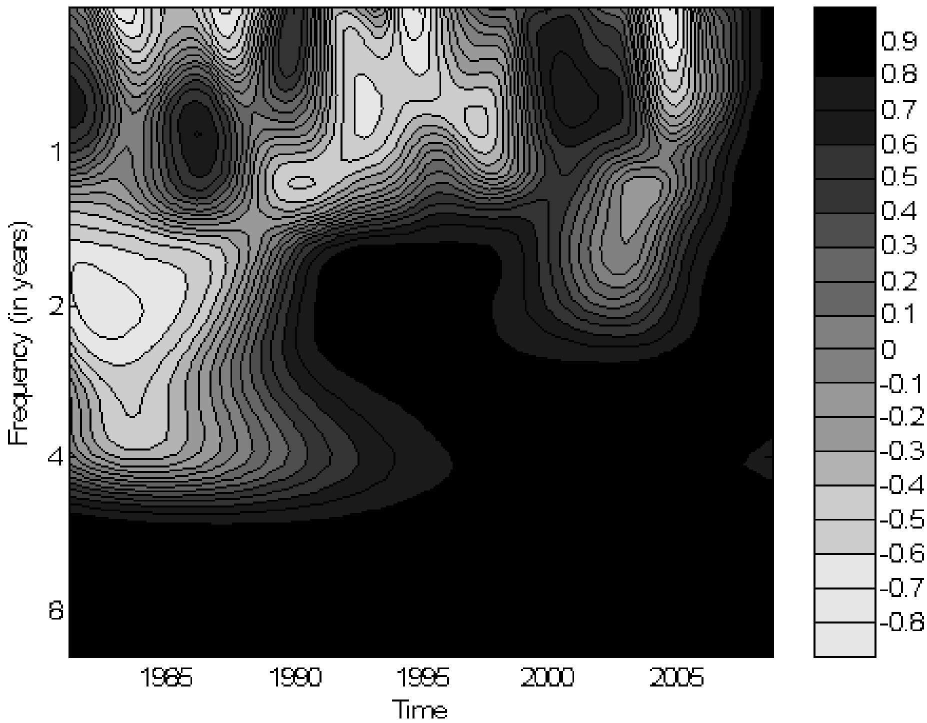

Figure 19: Comovement between the growth cycles of France and Spain

The results for all possible country pairs are presented in Figures 14 up to 19. The wavelet-based measure of comovement is presented through a contour plot as there are three dimensions involved. The horizontal axis refers to time while the vertical axis refers to frequency. To ease interpretation, the frequency is converted to time units (years). The gray scale is for the wavelet-based measure whereas increasing darkness corresponds to an increasing value and mimics the height in a surface plot. Hence, by inspecting the contour plot one can identify both frequency bands (in the vertical axis) and time intervals (in the horizontal axis) where the series move together and assess if the strength of the comovement changes across frequencies and over time.

From the analysis of the results several interesting findings emerge. In general, there is a high degree of comovement at lower frequencies, *i .e.* long-term fluctuations, among the major euro area countries. Only in the case of Germany and Italy, the comovement is weak at low frequencies. Regarding the typical business cycle frequency range, *i .e.* fluctuations between two and eight years, one can see that, for example, Germany and France only show signs of relatively high comovement since the mid-90's while Germany and Italy present a high degree of comovement almost over the entire sample period although it has also become stronger since the mid-90s. In the remaining country pairs, there seems to be a high degree of comovement since the 90's at the business cycle frequency range. In contrast, the comovement is in general weak at frequencies associated with fluctuations that last less than two years. Note, however, that the strength of comovement seems to have increased at the latter part of the sample at higher frequencies. This feature is more clear in the case of Germany and France and in the case of Germany and Italy.

The above findings suggest that the synchronization of growth cycles among the major euro area countries has always been high at low frequencies, *i.e.* long-term developments. In turn, at the typical business cycle frequency range, the synchronization was, in general, relatively low at the beginning of the sample period but increased since the mid-90s attainning a high degree of synchronization thereafter. This may reflect the deepening of European economic integration reinforced with the establishment of the monetary union in 1999. In contrast, the comovement of growth cycles at high frequencies is, with a few minor exceptions, rather low throughout the whole sample period. This may result from the fact that very short-term fluctuations are essentially idiosyncratic. Furthermore, the synchronization of growth cycles seems to be higher among the major euro area countries than between those countires and countries like, for example, the United States[13].

All these results highlight the usefulness of the proposed wavelet-based measure of comovement. In fact, the degree of comovement can change across frequencies and over time and being able to capture such evolving features is crucial for a richer comovement assessment.

3.4 Conclusions

The assessment of the comovement among economic variables is of key importance in several strands of the literature. One can distinguish two main approaches for measuring comovement, the more traditional approach in the time domain and the one based on spectral analysis. While the first approach discards all the information concerning the relationship at the frequency level, the second one does not take into account the possible time dependence of such relationship. To overcome such caveats one can resort to wavelet analysis. Wavelet analysis allows one to take into account both the time and frequency domains within an unified framework.

In this essay, it is proposed a wavelet-based measure of comovement. The

[13]The results are available from the author upon request.

wavelet-based measure allows one to quantify the degree of comovement in the time-frequency space. That is, it allows one to assess simultaneously over which time periods and at which frequencies is comovement higher. Besides allowing one to identify the regions in the time-frequency space where the two time series comove, one can also assess if the degree of comovement has been changing across frequencies and over time. Hence, the suggested wavelet-based measure allows for a richer description on the comovement between the variables of interest and can be seen as a refinement of previous approaches.

An empirical application is provided to illustrate the use of such measure. In particular, the comovement of growth cycles among the major euro area countries is assessed. The results highlight the usefulness of the wavelet-based measure of comovement, as it is found that the degree of comovement depends on the frequency and has changed over time.

4 Cohesion within the euro area and U. S.: a wavelet-based view[14]

4.1 Introduction

The analysis of business cycle comovement has long been a topic of interest in economics. Several measures have been used in the literature to assess the synchronization of business cycles but the Pearson correlation coefficient remained the most popular because it summarizes the degree of comovement through time in a single value. However, due to its synthetic nature it can be rather limited describing the relationship between the variables. Alternatively, one can resort to spectral analysis to obtain further insights about the relationship at the frequency level (see, for example, A'Hearn and Woitek (2001), Pakko (2004) and Breitung and Candelon (2006)). Croux *et al.* (2001) have suggested a spectral-based measure, the dynamic correlation, which is conceptually similar to the contemporaneous correlation but allows to measure comovement at the frequency level (empirical work drawing on this measure includes, for example, Tripier (2002), Rua and Nunes (2005) and Camacho *et al.* (2006)). However, while the Pearson correlation coefficient disregards completely the relationship at the frequency level, with the dynamic correlation proposed by Croux *et al.* (2001) all the time dependence of comovement is lost.

To overcome such caveats, Rua (2010) has proposed a measure of comovement by resorting to wavelet analysis. Wavelet analysis constitutes a very promising tool as it represents a refinement in terms of analysis in the sense that both time and frequency domains are taken into account. Although wavelets have been more popular in fields such as signal and image processing, meteorology, physics, among others, such analysis can also provide fruitful in-

[14] Joint work with Artur Silva Lopes.

sights about several economic phenomena (see, for example, the pioneer work of Ramsey and Zhang (1996, 1997) and Ramsey and Lampart (1998a,b)). Recent applications of wavelets in economics can be found, for instance, in Gallegati and Gallegati (2007), Gallegati *et al.* (2008), Yogo (2008), Crowley and Mayes (2008), Rua and Nunes (2009) and Aguiar-Conraria and Soares (2010) (see Crowley (2007) for a survey). In particular, the wavelet-based measure of comovement suggested by Rua (2010) allows one to assess simultaneously how variables are related at different frequencies and how such relationship has evolved over time. In this way it is possible to capture both time and frequency varying features within an unified framework.

In order to take on board more than two series when assessing comovement, Croux *et al.* (2001) have extended the dynamic correlation to the multivariate case and named this generalised measure as cohesion. Cohesion is based on the dynamic correlations between all possible pairs of series within a group of variables and has been used by Croux *et al.* (2001) to assess the comovement of output fluctuations between European countries and across U.S. states and regions. As stressed by de Haan *et al.* (2008), this measure provides a useful summary statistic on the degree of comovement across countries or regions while avoiding the problem of choosing a base country or region. Cohesion has also been applied by Carlino and DeFina (2004) to study the comovement in employment across U.S. states and sectors, by Crone (2005) to evaluate the business cycle cohesion within U.S. regions, by Eickmeier and Breitung (2006) for assessing output growth and inflation cohesion between European countries, among others.

The assessment of comovement among different countries or regions now involves a huge literature. In fact, the degree of synchronization of macroeconomic fluctuations across countries or regions plays a key role on the discussion about the attractiveness of economic integration. In this respect, the debate about the European monetary union has dominated the literature over the past decade.[15] In particular, building on the work of Mundell (1961) concerning Optimum Currency Areas, it has been argued that the cost of joining a monetary union will be low if countries have highly synchronized business cycles. However, it has also been pointed out that economic integration itself can affect the synchronization of macroeconomic fluctuations.[16] Hence, besides the frequency level perspective of comovement, capturing its time-varying dimension is also a worthwhile purpose.

Following Croux *et al.* (2001), we extend the bivariate measure proposed by Rua (2010) to the more general case in order to obtain a measure of cohesion in the wavelet domain. The resulting measure allows one to assess how cohesion has evolved over time and across frequencies simultaneously. Therefore, it can provide a detailed and rich picture, with additional insights in the time dimension. Focusing on output growth, we investigate the cohesion among euro area countries and the cohesion within the U.S. at both the regional and state levels over the last decades.

We find that cohesion within euro area has been higher at the long-run and business cycle frequencies and it has increased since the mid-90s across all frequencies. For the U.S., we find that cohesion is higher at the typical business cycle frequency range but seems to have decreased since the beginning of the 90's, and we note that this finding holds at both the regional and state levels. Moreover, we also find that U.S. cohesion is higher at the regional level than at the state level, both across frequencies and over

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[15] See, for example, de Haan *et al.* (2008) for a literature survey regarding business cycle synchronization in the euro area.

[16] For instance, Frankel and Rose (1998) claim that the removal of trade barriers induces a higher symmetry of output fluctuations while Rose (2000) provides evidence that a common currency results in more trade. In contrast, Krugman (1993) argues that a higher level of trade can lead to a higher economic specialization and less synchronized business cycles. See, for example, Kalemli-Ozcan *et al.* (2001) for an overview of the effects of economic integration on output fluctuations symmetry.

time. In addition, besides taking into account the spatial perspective when assessing cohesion we also conduct an analysis at the sectoral level. Resorting to disaggregated data by eleven sectors for the euro area countries and U.S. regions and states, we find a noteworthy heterogeneity in the results at the sectoral level. This analysis allows us to unveil the sectors underlying the previously mentioned overall results.

The chapter is organised as follows. In section 4.2, the main building blocks are discussed and the wavelet-based measure of cohesion is presented. In section 4.3, a data description is provided while in section 4.4, the empirical application is carried out for both the euro area and the U.S.. Finally, section 4.5 concludes.

4.2 Measuring cohesion in the wavelet domain

While the well-known Fourier transform decomposes the time series into infinite length sines and cosines (see, for example, Priestley (1981)), discarding all time-localization information, the wavelet transform uses local base functions that can be stretched and translated with a flexible resolution in both frequency and time. In particular, the wavelet transform decomposes a time series in terms of some elementary functions, the daughter wavelets or simply wavelets $\psi_{\tau,s}(t)$. These wavelets result from a mother wavelet $\psi(t)$, that can be expressed as function of the time position τ (translation parameter) and the scale s (dilation parameter), which is related with the frequency, that is,

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right). \tag{36}$$

To be a mother wavelet $\psi(t)$ must fulfil several conditions (see, for example, Percival and Walden (2000) for further discussion). The continuous wavelet transform of a time series $x(t)$ with respect to $\psi(t)$ is given by the following convolution

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t)\psi^*_{\tau,s}(t)dt = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} x(t)\psi^* \left(\frac{t-\tau}{s}\right) dt, \tag{37}$$

where * denotes the complex conjugate.

The most commonly used mother wavelet is the Morlet wavelet, which can be simply defined as

$$\psi(t) = \pi^{-\frac{1}{4}} e^{i\omega_0 t} e^{\frac{-t^2}{2}}. \tag{38}$$

One can observe that the Morlet wavelet is a complex sine wave within a Gaussian envelope whereas ω_0 is the wavenumber. In practice, ω_0 is set to 6 as it provides a good balance between time and frequency localization (see, for example, Adisson (2002) for further details on the Morlet wavelet).

Given two time series $x_i(t)$ and $x_j(t)$, with wavelet transforms $W_{x_i}(\tau, s)$ and $W_{x_j}(\tau, s)$ one can define the cross-wavelet spectrum as $W_{x_i x_j}(\tau, s) = W_{x_i}(\tau, s) W^*_{x_j}(\tau, s)$. As the mother wavelet is in general complex, the cross-wavelet spectrum is also complex valued and it can be decomposed into real and imaginary parts. The measure proposed by Rua (2010) is given by

$$\rho_{x_i x_j}(\tau, s) = \frac{\Re\left(W_{x_i x_j}(\tau, s)\right)}{\sqrt{\left|W_{x_i}(\tau, s)\right|^2 \left|W_{x_j}(\tau, s)\right|^2}} \tag{39}$$

where $\Re$ denotes the real part of the cross-wavelet spectrum. This wavelet-based measure $\rho_{x_i x_j}(\tau, s)$ allows one to evaluate the degree of comovement in the time-frequency space and assess over which periods of time and over which frequencies is the comovement higher. Basically, it plays a role as a contemporaneous correlation coefficient around each moment in time and for each frequency. Since it provides information about the comovement not only at the frequency level but also over time, it can be seen as a generalisation of the dynamic correlation measure suggested by Croux *et al.* (2001).

In a similar fashion to Croux *et al.* (2001), who extended the dynamic correlation measure to the multivariate case providing a measure of cohesion in the frequency domain, we extend the bivariate measure proposed by Rua (2010) to the more general case in order to obtain a measure of cohesion in the wavelet domain. In particular, cohesion is defined as the weighted average of the wavelet-based measure $\rho_{x_i x_j}(\tau, s)$ between all possible pairs of series:

$$coh(\tau, s) = \frac{\sum_{i \neq j} \varpi_{ij} \rho_{x_i x_j}(\tau, s)}{\sum_{i \neq j} \varpi_{ij}}, \tag{40}$$

where ϖ_{ij} is the weight attached to the pair of series (i, j). Given that the $\rho_{x_i x_j}(\tau, s)$ ranges between -1 and 1, the wavelet-based cohesion also varies between -1 and 1. This measure allows one to quantify the extent of cohesion among several series at different frequencies and investigate if such global relationship has changed over time. Hence, it enables a richer analysis than the one which is possible with the cohesion measure suggested in Croux *et al.* (2001), which focus only at the frequency level. This is of particular importance as there is by now evidence that the comovement can vary across frequencies as well as over time. Moreover, the suggested wavelet-based cohesion allows to capture both features within an unified framework.

4.3 Data

Regarding the United States, we considered data at the regional and state levels provided by the Bureau of Economic Analysis (BEA) of the U.S. Department of Commerce.[17] Annual real GDP by region and state is available

[17] The 8 BEA regions are: New England, Mideast, Great Lakes, Plains, Southeast, Southwest, Rocky Mountain, Far West. The 51 BEA states are: Alabama, Alaska, Arizona, Arkansas, California, Colorado, Connecticut, Delaware, District of Columbia,

from 1977 up to 2008. However, as nominal data is available from 1963, in order to cover a longer time span we resorted to the U.S. GDP deflator prior to 1977. The disaggregation of regional and state output by sectors is also provided by BEA for the same sample periods, and again we used the U.S. corresponding deflator, so as to obtain volume series since 1963 up to 2008. In order to ease the comparison between the U.S. and the euro area, we considered eleven sectors, namely: (1) Agriculture, hunting, forestry, and fishing; (2) Mining; (3) Manufacturing; (4) Utilities; (5) Construction; (6) Wholesale and retail trade; (7) Transport and storage; (8) Information; (9) Finance, insurance, real estate and rental and leasing; (10) Services including professional and business services, educational services, health care, social assistance, arts, entertainment, recreation, accommodation and food services and other services; (11) Government.

Concerning the euro area, we considered all the member countries as of 1 January 2001, namely, Austria, Belgium, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Netherlands, Portugal and Spain. Data regarding annual real GDP has been collected from the European Commission AMECO database.[18] Concerning sectoral data for the euro area countries, we used the EU KLEMS database provided by Groningen Growth and Development Centre (GGDC which is financially supported by the European Commission).[19] As this data ranges from 1970 up to 2007, we update it with the year 2008 resorting to the AMECO database.

Florida, Georgia, Hawaii, Idaho, Illinois, Indiana, Iowa, Kansas, Kentucky, Louisiana, Maine, Maryland, Massachusetts, Michigan, Minnesota, Mississippi, Missouri, Montana, Nebraska, Nevada, New Hampshire, New Jersey, New Mexico, New York, North Carolina, North Dakota, Ohio, Oklahoma, Oregon, Pennsylvania, Rhode Island, South Carolina, South Dakota, Tennessee, Texas, Utah, Vermont, Virginia, Washington, West Virginia, Wisconsin, Wyoming. See http://www.bea.gov/regional .

[18] See http://ec.europa.eu/economy_finance/db_indicators/ameco .

[19] See http://www.euklems.net .

The weights used in (40) correspond to the share of output of the pair (i, j) in the year 2000.[20] As usual, all series are taken in logs and first differenced.

4.4 Cohesion within euro area and U.S.

4.4.1 Spatial cohesion

In this section, we proceed into the computation of the above suggested measure to assess the cohesion among euro area countries and the cohesion within the U.S. at both the regional and state levels. As three dimensions are involved, the wavelet-based cohesion is presented through a contour plot. The horizontal axis refers to time while the vertical axis refers to frequency. To ease interpretation, the frequency is converted to time units (years). The gray scale is for the wavelet-based cohesion where increasing darkness corresponds to an increasing value and mimics the height in a surface plot. Hence, through the inspection of the graph one can identify both frequency bands (in the vertical axis) and time intervals (in the horizontal axis) where the cohesion is higher and whether it has changed over time.[21]

In Figure 20, we present the wavelet-based measure of cohesion among euro area countries in terms of GDP growth. Several findings seem to be particularly relevant. First, one can see that cohesion is typically larger at low frequencies, *i .e.* long-run dynamics, than in the remaining frequencies. Moreover, at low frequencies, cohesion has increased in the late 70's and early 80's and has kept high thereafter. Concerning the standard business cycle frequency range - that is, fluctuations between two and eight years -, although

[20] As both the euro area and US data are released with 2000 as the base year, this year was selected to compute the weights. Nevertheless, the results are not sensitive to the chosen year.

[21] All computations are done using Matlab and the codes are available from the authors upon request.

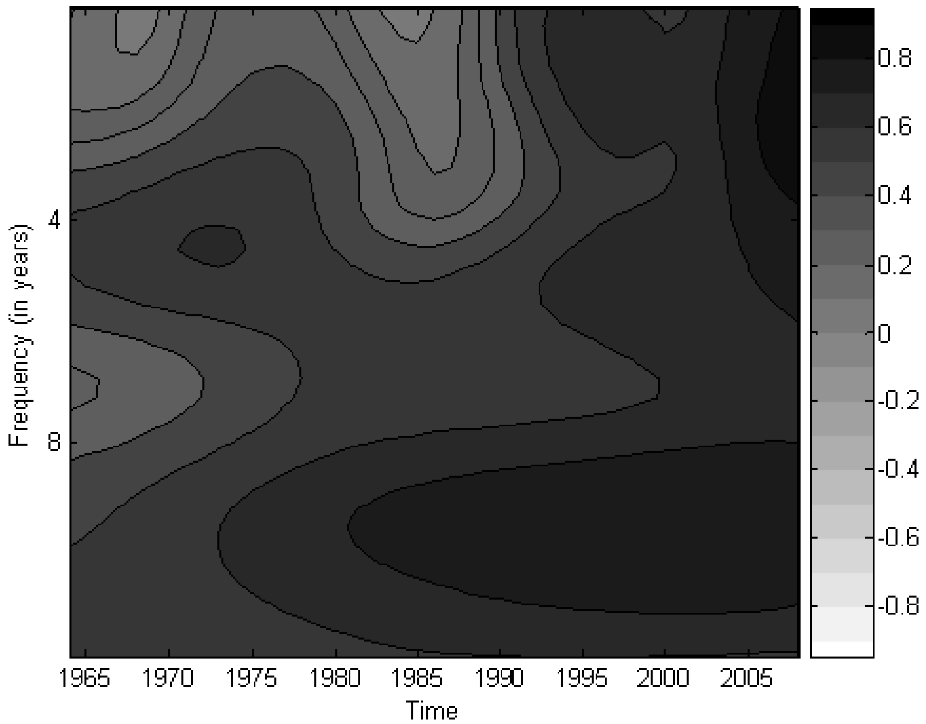


Figure 20: Cohesion within euro area

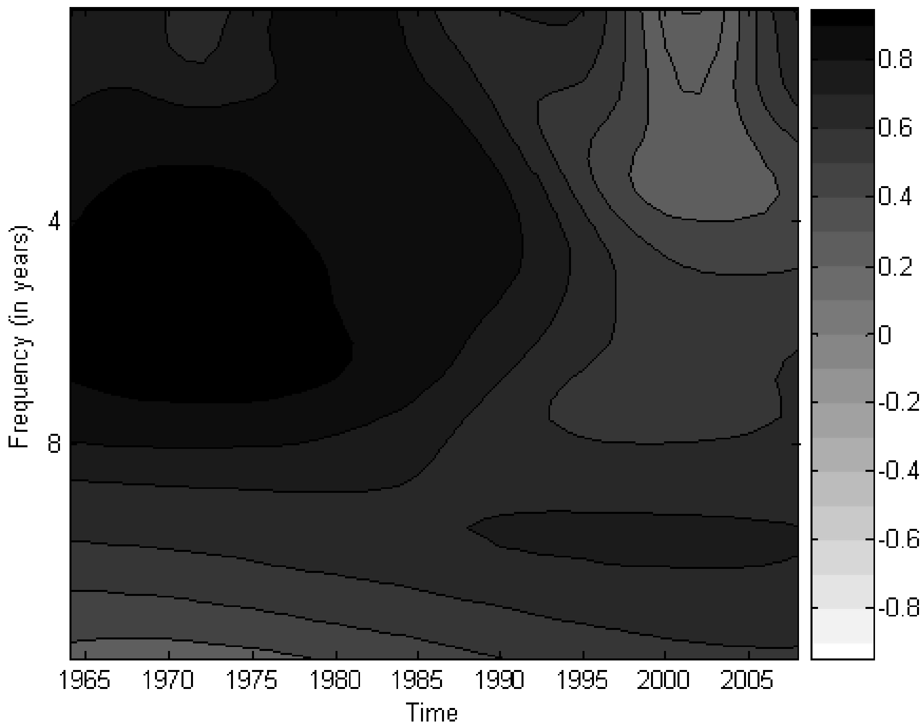


Figure 21: Cohesion within US (at the regional level)

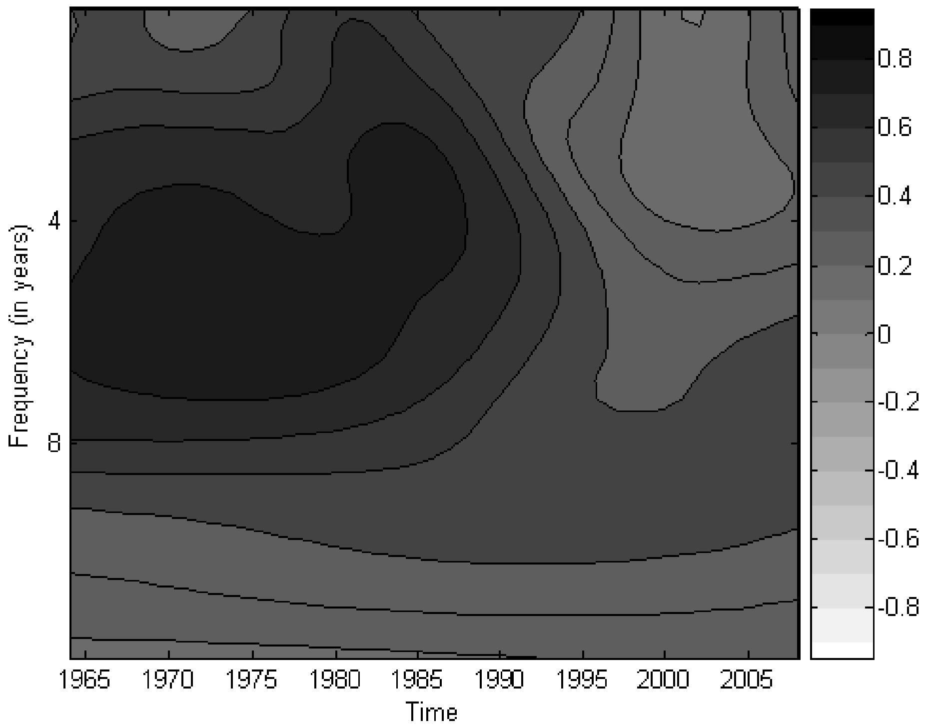


Figure 22: Cohesion within US (at the state level)

cohesion has been lower than at low frequencies, it has also increased since the mid-90s. At high frequencies, that is, for very short-run fluctuations, cohesion has always been weak with the exception of the latter part of the sample. Hence, cohesion within euro area has been larger at long-term and business cycle dynamics and it has increased for several frequencies since the mid-90s.[22]

Regarding the U.S., we compute the cohesion at both the regional and state levels (see Figures 21 and 22, respectively). Likewise as in the euro area case, cohesion is positive within the U.S. at both the regional and state levels (see also Carlino and DeFina (2004)). Comparing Figures 21 and 22, one can observe that the U.S. cohesion at the regional level is higher than at the state level, whatever the frequency and/or the time period. This is in line with the results of Croux *et al.* (2001) who argue that this is due to the fact that by aggregating the states, the idiosyncratic sources of variations are diminished. In contrast with the euro area, the frequency range where cohesion is higher in the U.S. is at the business cycle frequency range. A noteworthy and distinct finding is that, while in the euro area cohesion seems to have increased, reflecting most probably the deepening of the process of European economic integration, in the U.S. there seems to be evidence of a decrease in cohesion since the beginning of the 90's. This holds both at the regional and state levels.

To shed more light on this latter issue as well as on all the previous results, in the next section we conduct a cohesion analysis at the sectoral level.

4.4.2 Cohesion at the sectoral level

Up to now the focus has been on assessing the spatial cohesion within euro area and the U.S.. To complement and provide further insights about the

[22] All these findings are broadly in line with the results of Rua (2010), who considers only the major euro area countries and assesses all possible country pairs individually.

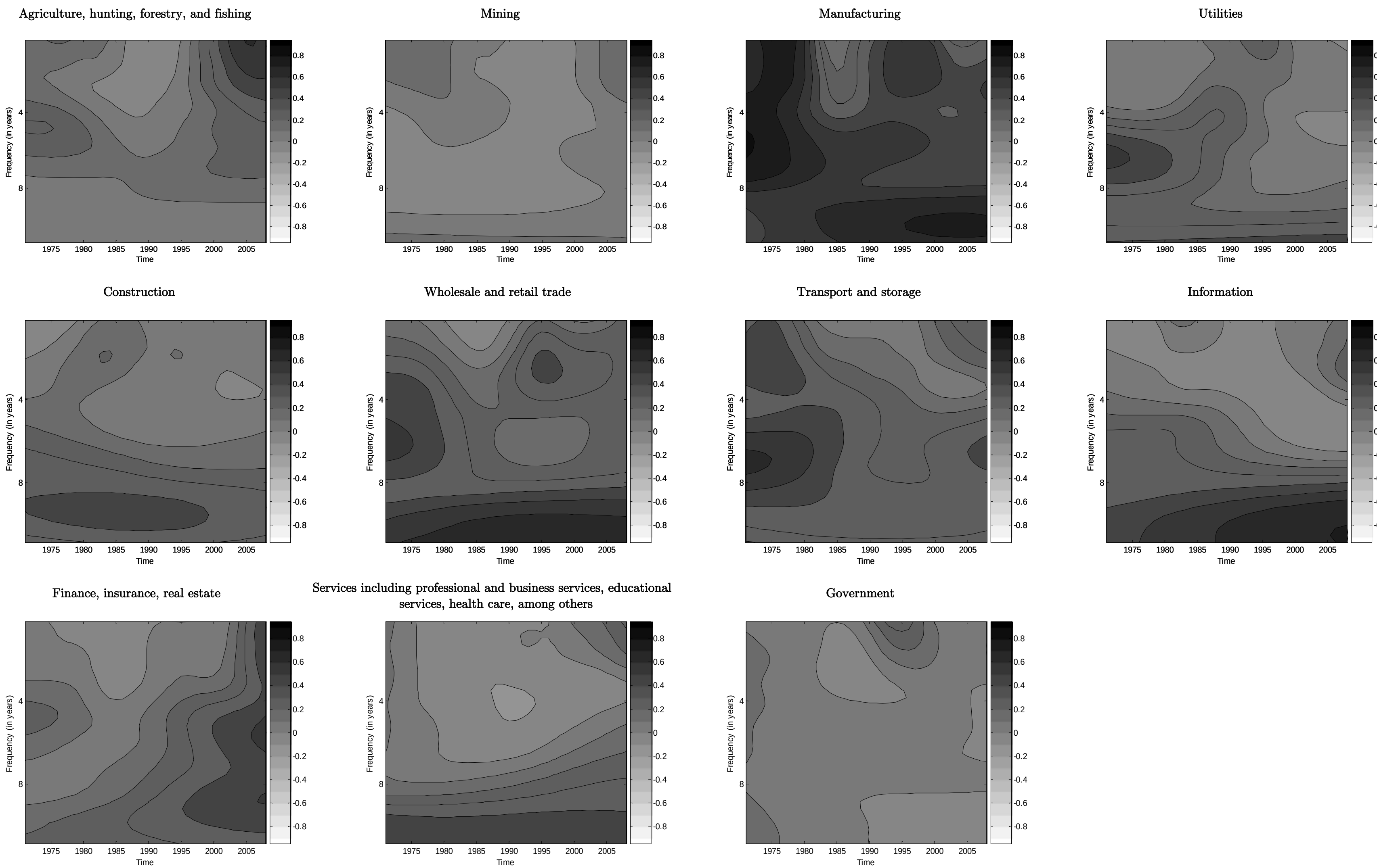


Figure 23 - Cohesion across euro area countries by sector

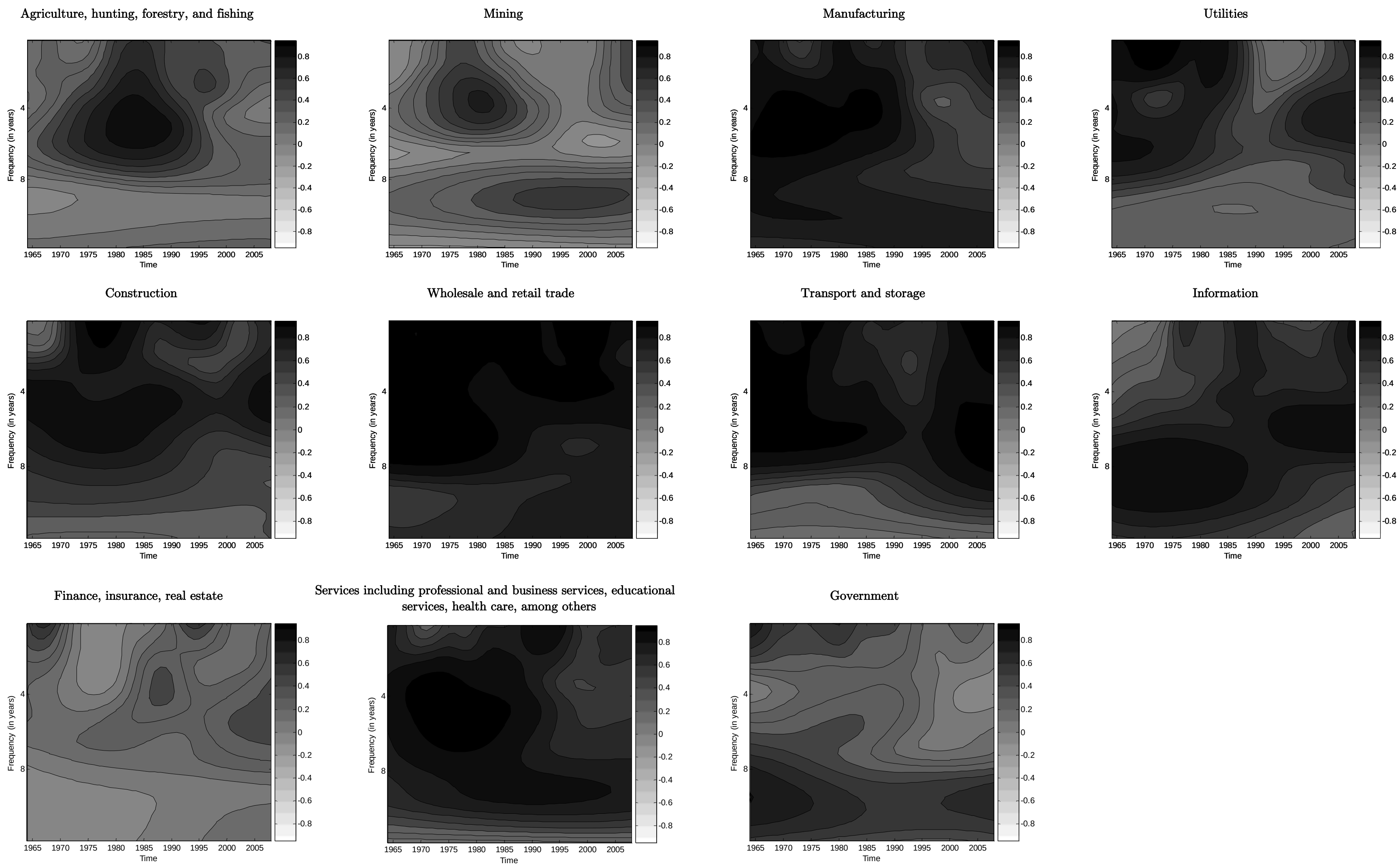


Figure 24 - Cohesion across US regions by sector

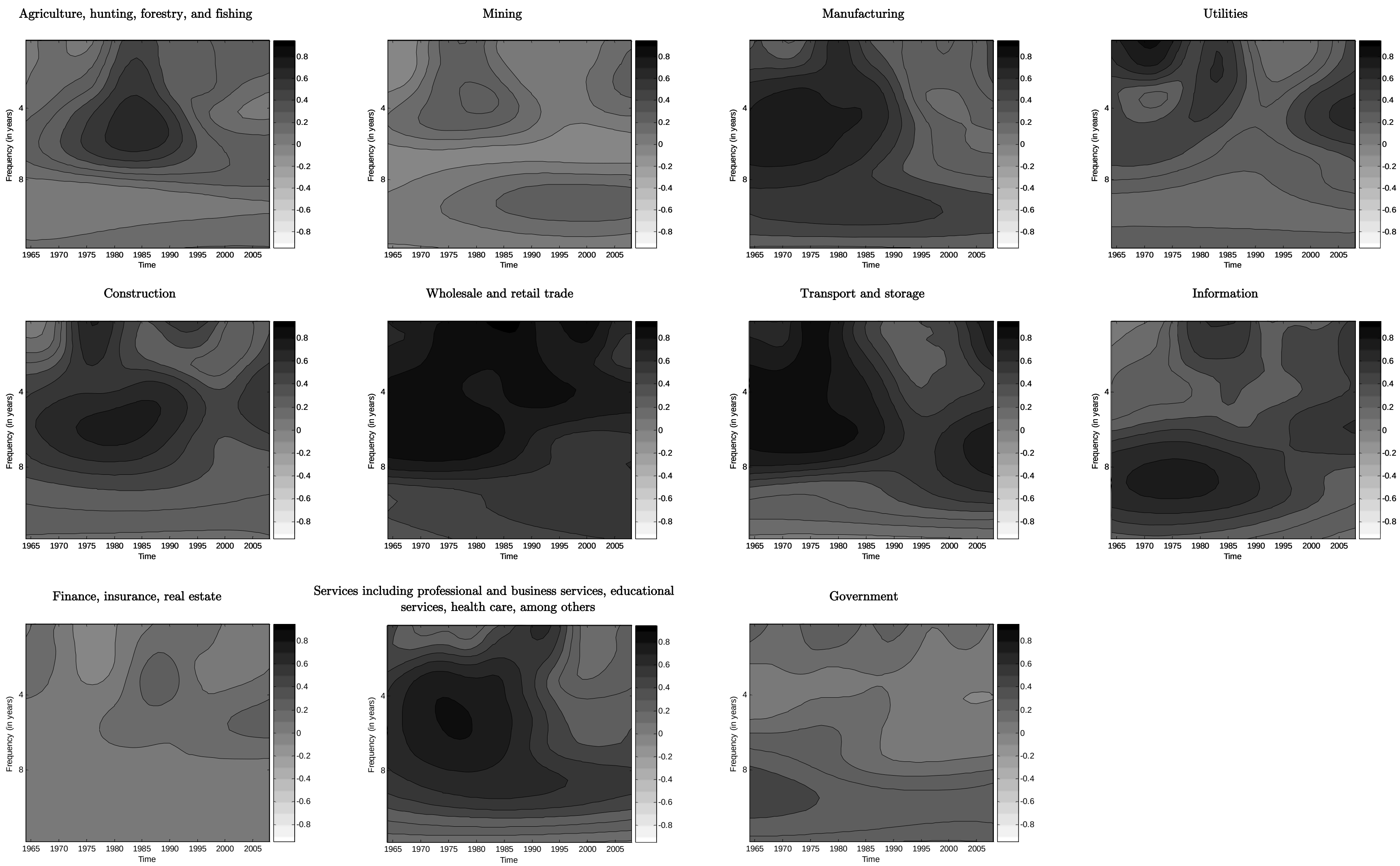


Figure 25 - Cohesion across US states by sector

previous results, we also investigate cohesion at the sectoral level. This analysis allows us to identify if the above findings are broadly based or if they are being driven by any particular sector.

The results at the sectoral level for the euro area are reported in Figure 23. Each plot presents the cohesion among the euro area countries for a given sector. In general, sectors like Agriculture, Mining, Government and to a lesser extent, Utilities, Construction, Transport and storage present relatively weak cohesion across frequencies and over time. Contrasting with these, sectors such as Manufacturing, Wholesale and retail trade, Information denote a relatively high cohesion at low frequencies, one which has increased throughout time. These sectors seem to be responsible for the time-varying behaviour discussed earlier of cohesion within the euro area at low frequencies. It is also interesting to note that the sector where cohesion has been increasing over the last decade and across all frequencies is the sector related to Finance, insurance and real estate. This clearly reflects the financial integration that has been taking place in the euro area.

We now turn to the U.S. results (see Figures 24 and 25). As pointed out in the previous section, cohesion at the regional level is higher than at the state level and this finding seems to hold for all the sectors across all the frequencies and over time. Since the results at the regional and state levels are qualitatively similar, we focus henceforth on the results obtained at the state level. The sectors that present lower cohesion are Agriculture, Mining, Finance, insurance and real estate and Government. In sectors like Construction, Transport and storage and to a lesser extent, Utilities, cohesion seems to be higher at business cycle frequencies, whereas in the Information sector cohesion is more marked at low frequencies. On the other hand, in Manufacturing, Wholesale and retail trade, Services including professional and business services and others, where cohesion has been high at the business cycle frequency range, have recorded a decrease in the last years Hence, these appear to be the main sectors behind the recent deterioration of cohesion within the U.S..

4.5 Conclusions

The assessment of output synchronization across countries or regions has been of key importance in several strands of the literature, as for example, in the discussion of economic integration. Though comovement is traditionally measured in the time domain resorting to the well-known Pearson correlation coefficient, there has been an increasing focus on frequency domain analysis. However, while the former approach ignores the relationship at the frequency level, the latter disregards the fact that comovement can change over time. To overcome such shortcomings one can resort to wavelet analysis. In particular, in this essay we propose a wavelet-based measure of cohesion which allows one to assess how synchronization has evolved over time and across frequencies simultaneously, within a set of countries or regions.

To illustrate its empirical application we focus on the output growth synchronization within the euro area and the U.S.. We study cohesion among euro area countries and within the U.S. both at the regional and state levels over the last decades. The results obtained highlight the usefulness of a wavelet-based measure of cohesion so as to uncover both frequency and time-varying features. We find that cohesion within the euro area has been higher at the long-run frequencies and it has increased since the mid-90s across all frequencies. In contrast, cohesion within the U.S. is higher at the typical business cycle frequency range but seems to have decreased since the beginning of the 90's. These findings for the U.S. hold both at the regional and state levels. Furthermore, we find that the U.S. cohesion at the regional level is higher than at the state level across all frequencies and over the whole sample period. Additionally, we find a noteworthy heterogeneity in the results at the sectoral level. The sectors that seem to lie behind the overall results are identified.

5 International comovement of stock market returns: a wavelet analysis[23]

5.1 Introduction

The analysis of the comovement of stock market returns is a key issue in finance as it has important practical implications in asset allocation and risk management. Since the seminal work of Grubel (1968) on the benefits of international portfolio diversification (see also, Levy and Sarnat (1970) and Agmon (1972)) this topic has received a lot of attention in international finance. In fact, a growing body of literature has emerged more recently on studying the comovement of international stock prices (see, for example, King *et al.* (1994), Lin *et al.* (1994), Longin and Solnik (1995, 2001), Karolyi and Stulz (1996), Forbes and Rigobon (2002), Brooks and Del Negro (2005, 2006)). In particular, most of those studies have found that the comovement of stock returns is not constant over time. For instance, Brooks and Del Negro (2004) and Kizys and Pierdzioch (2008) found evidence of increasing international comovement of stock returns since the mid-90's among the major developed countries. It has been current practice to evaluate the comovement of stock returns through the correlation coefficient while the evolving properties have been investigated either through a rolling window correlation coefficient (see, for example, Brooks and Del Negro (2004)) or by considering non-overlapping sample periods (see, for example, King and Wadhwani (1990) and Lin *et al.* (1994)).

However, the comovement analysis should also take into account the distinction between the short and long-term investor (see, for example, Candelon *et al.* (2008)). From a portfolio diversification view, the first kind of investor

[23] Joint work with Luis Catela Nunes which has been published in the Journal of Empirical Finance, 16 (2009) 632–639.

is naturally more interested in the comovement of stock returns at higher frequencies, that is, short-term fluctuations, whereas the latter focus on the relationship at lower frequencies, that is, long-term fluctuations. Hence, one has to resort to the frequency domain analysis to obtain insights about the comovement at the frequency level (see, for example, A'Hearn and Woitek (2001) and Pakko (2004)). One should note that, despite its recognized interest, analysis in the frequency domain is much less found in the financial empirical literature (see, for example, Smith (2001)).

In this essay, we re-examine the stock returns comovement among the major developed economies through a novel approach, wavelet analysis. Wavelet analysis constitutes a very promising tool as it represents a refinement in terms of analysis in the sense that both time and frequency domains are taken into account. Although wavelets have been more popular in fields such as signal and image processing, meteorology, physics, among others, such analysis can also provide fruitful insights about several economic phenomena (see, for example, Ramsey and Zhang (1996, 1997)). The pioneer work of Ramsey and Lampart (1998a, 1998b) draws on wavelets to study the relationship between several macroeconomic variables (see, for example, Crowley (2007) for a survey). In particular, wavelet analysis provides a unified framework to measure comovement in the time-frequency space.

The study of the comovement of stock market returns is crucial for risk assessment of portfolios. A higher comovement among the assets of a given portfolio implies lower gains, in terms of risk management, stemming from portfolio diversification. Hence, the evaluation of the comovement is of striking importance to the investor so that he can best assess the risk of a portfolio. On one hand, it has been acknowledged that the comovement of stock returns varies over time. Hence, one has to be able to capture this time-varying feature as it implies an evolving risk exposure. On the other hand, the distinction between short and long-term investors should not be ignored as the first is more interested on short-run movements whereas the latter on long-run fluctuations. That is, if the degree of the comovement of stock returns varies across frequencies the risk for each type of investor will also be different. In contrast with time or frequency domain approaches which allow one to focus only on one of these issues, wavelet analysis encompasses both. In particular, through wavelets one can assess simultaneously the strength of the comovement at different frequencies and how such strength has evolved over time. In this way it is possible to identify regions in the time-frequency space where the comovement is higher and the benefits of portfolio diversification in terms of risk management are lower.

In addition, we also extend such analysis to the sectoral level. That is, besides considering the aggregate stock returns, we also distinguish ten sectors for each country. For the international diversification of equity portfolios, the assessment of the comovement at the sectoral level also plays a role (see, for example, Roll (1992), Heston and Rouwenhorst (1994) and Griffin and Karolyi (1998)). For instance, it is important to assess if the evidence of greater interdependence of international stock markets is confined or not to a small set of sectors (see, for example, Berben and Jansen (2005)). Again, wavelet analysis can provide interesting insights on how such international comovement has evolved over time across frequencies for the different sectors.

Hence, this essay provides a fresh new look into the characterisation of the comovement among international stock returns. We focus on the major developed economies, namely Germany, Japan, United Kingdom and United States over the last four decades. Moreover, by considering the decomposition of the aggregate index in ten sectors, we also provide insights at the sectoral level.

This chapter is organised as follows. In section 5.2, the comovement measure in the wavelet domain is presented. In section 5.3, a data overview is provided and in section 5.4 the empirical results for the major developed economies are discussed. Finally, section 5.5 concludes.

5.2 Wavelet analysis

The wavelet transform decomposes a time series in terms of some elementary functions, the daughter wavelets or simply wavelets $\psi_{\tau,s}(t)$. Wavelets are 'small waves' that grow and decay in a limited time period. These wavelets result from a mother wavelet $\psi(t)$, that can be expressed as function of the time position τ (translation parameter) and the scale s (dilation parameter), which is related with the frequency. While the Fourier transform decomposes the time series into infinite length sines and cosines, discarding all time-localization information, the basis functions of the wavelet transform are shifted and scaled versions of the time-localized mother wavelet. More explicitly, wavelets are defined as

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right) \tag{41}$$

where $\frac{1}{\sqrt{s}}$ is a normalization factor to ensure that wavelet transforms are comparable across scales and time series. To be a mother wavelet, $\psi(t)$, must fulfil several conditions (see, for example, Gençay *et al.* (2002), Percival and Walden (2000) and Bruce and Gao (1996)): it must have zero mean, $\int_{-\infty}^{+\infty}\psi(t)dt = 0$; its square integrates to unity, $\int_{-\infty}^{+\infty}\psi^2(t)dt = 1$, which means that $\psi(t)$ is limited to an interval of time; and it should also satisfy the so-called admissibility condition, $0 < C_\psi = \int_0^{+\infty}\frac{\left|\widehat{\psi}(\omega)\right|^2}{\omega}d\omega < +\infty$ where $\widehat{\psi}(\omega)$ is the Fourier transform of $\psi(t)$, that is, $\widehat{\psi}(\omega) = \int_{-\infty}^{+\infty}\psi(t)e^{-i\omega\tau}dt$. The latter condition allows the reconstruction of a time series $x(t)$ from its continuous wavelet transform, $W_x(\tau,s)$. Thus, it is possible to recover $x(t)$ from its wavelet transform through the following formula

$$x(t) = \frac{1}{C_\psi}\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty}\frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right)W_x(\tau,s)d\tau\right]\frac{ds}{s^2} \tag{42}$$

The continuous wavelet transform of a time series $x(t)$ with respect to $\psi(t)$ is given by the following convolution

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t)\psi^*_{\tau,s}(t)dt = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} x(t)\psi^* \left(\frac{t-\tau}{s} \right) dt \qquad (43)$$

where * denotes the complex conjugate. For a discrete time series, $x(t)$, $t = 1, ..., N$ we have

$$W_x(\tau, s) = \frac{1}{\sqrt{s}} \sum_{t=1}^{N} x(t)\psi^* \left(\frac{t-\tau}{s} \right) \qquad (44)$$

Although it is possible to compute the wavelet transform in the time domain using equation (44), a more convenient way to implement it is to carry out the wavelet transform in Fourier space (see, for example, Torrence and Compo (1998)).

The most commonly used mother wavelet is the Morlet wavelet and is defined as

$$\psi(t) = \pi^{-\frac{1}{4}} \left(e^{i\omega_0 t} - e^{-\frac{\omega_0^2}{2}} \right) e^{\frac{-t^2}{2}} \qquad (45)$$

Since the term $e^{-\frac{\omega_0^2}{2}}$ becomes negligible for an appropriate ω_0, the Morlet wavelet is simply defined as

$$\psi(t) = \pi^{-\frac{1}{4}} e^{i\omega_0 t} e^{\frac{-t^2}{2}} \qquad (46)$$

with the corresponding Fourier transform given by

$$\widehat{\psi}(\omega) = \pi^{\frac{1}{4}} \sqrt{2} e^{-\frac{1}{2}(\omega-\omega_0)^2} \qquad (47)$$

One can see that the Morlet wavelet is a complex sine wave within a Gaussian envelope whereas ω_0 is the wavenumber (see, for example, Adisson (2002) for further details). In practice, ω_0 is set to 6 as it provides a good balance between time and frequency localization (see, for example, Grinsted *et al.* (2004)).

Given two time series $x(t)$ and $y(t)$, with wavelet transforms $W_x(\tau, s)$ and $W_y(\tau, s)$ one can define the cross-wavelet spectrum as $W_{xy}(\tau, s) = = W_x(\tau, s)W_y^*(\tau, s)$. In a similar fashion as in Fourier analysis, one can define the wavelet squared coherency as the absolute value squared of the smoothed cross-wavelet spectrum, normalized by the smoothed wavelet power spectra

$$R^2(\tau, s) = \frac{\left|S\left(s^{-1}W_{xy}(\tau, s)\right)\right|^2}{S\left(s^{-1}\left|W_x(\tau, s)\right|^2\right) S\left(s^{-1}\left|W_y(\tau, s)\right|^2\right)} \tag{48}$$

where $S(.)$ denotes smoothing in both time and scale (see, for example, Torrence and Webster (1999)). Likewise in Fourier analysis, smoothing is also necessary, otherwise squared coherency would be always equal to one (see, for example, Priestley (1981)).

The intuition behind the wavelet squared coherency is similar to the one of squared coherency in Fourier analysis. As it can be seen from (48), the wavelet squared coherency is essentially the ratio of the squared cross-wavelet spectrum to the product of two wavelet spectra, analogously to the squared coefficient of correlation. In other words, the wavelet squared coherency plays a role as a correlation coefficient around each moment in time and for each frequency. Therefore, one can use wavelet squared coherency to measure the extent to which two time series move together over time and across frequencies (while the squared coherency in Fourier analysis only allows one to assess the latter). Likewise the squared coefficient of correlation, $R^2(\tau, s)$ is between 0 and 1 with a high (low) value indicating a strong (weak) comovement. Hence, through the graph of the wavelet squared coherency one can detect the regions in the time-frequency space where the two time series co-vary and capture both time and frequency varying features. In this way, it is possible to provide a richer picture on the comovement between two series[24].

5.3 Data

Stock prices data for the major developed economies, namely, Germany, Japan, United Kingdom and United States are from Thompson Financial Datastream. For each country, we collected Datastream constructed data for the broad-based market price index as well as for the ten economic sectors that make up the index, namely: Oil and gas; Basic materials; Industrials; Consumer goods; Healthcare; Consumer services; Telecommunications; Utilities; Financials and Technology. Apart from a few exceptions, the sample period runs from January 1973 up to December 2007 comprising 420 monthly observations, end of period figures. The exceptions are: Oil and Gas for Germany was not included in the analysis as the time series span was too short; the Technology index for Germany only starts at December 1988; the Telecommunications index for the UK is available from December 1981 onwards and the Utilities index for the UK only begins at January 1987. We focus on monthly stock returns, defined as the log first difference of monthly stock price indices and we use returns denominated in the home currency of each respective country.[25] In Table 1 some descriptive statistics are presented.

[24] Additionally, one can also compute the wavelet phase, which captures the lead-lag relationship between the variables in the time-frequency space. However, the results are not reported here as no noteworthy lead-lag relationship was found in the empirical application.

[25] We also performed the analysis using stock prices converted to a common currency and the results do not change qualitatively. The bilateral exchange rates are also from Thompson Financial Datastream.

5.4 Empirical results

In this section, wavelet squared coherency is presented for all possible country pairs in order to assess cross-country comovement (namely, US and Germany; UK and Germany; US and UK; Japan and Germany; Japan and US; Japan and UK). The wavelet squared coherency is presented through a contour plot as we have three dimensions involved. The horizontal axis refers to time while the vertical axis refers to frequency. To ease interpretation, the frequency is converted to time units (years). The gray scale is for the wavelet squared coherency where increasing darkness corresponds to an increasing value and mimics the height in a surface plot. Hence, through the inspection of the graph one can identify both frequency bands (in the vertical axis) and time intervals (in the horizontal axis) where the series move together[26]. For example, a dark area at the bottom (top) of the graph means strong comovement at low (high) frequencies whereas a dark area at the left-hand (right-hand) side denotes strong comovement at the beginning (end) of the sample period. Moreover, through such wavelet analysis one can also assess if the comovement has increased or decreased over time and across frequencies capturing possible varying features in the relationship between stock returns in the time-frequency space. The black bold line in the graph delimits the statistical significant area at the usual significance level of five per cent, *i.e.*, the wavelet squared coherency is statistically significant within such delimited time-frequency area. In particular, the five per cent significance level was determined from a Monte Carlo simulation of 10 000 sets of two white noise time series with the same length as the series under analysis. All computations have been done using Matlab.

From the analysis of the results obtained for the broad-based market stock

[26] As the continuous wavelet transform at a given point in time uses information of neighbouring data points, results should be read carefully close to the beginning or the end of the time series.

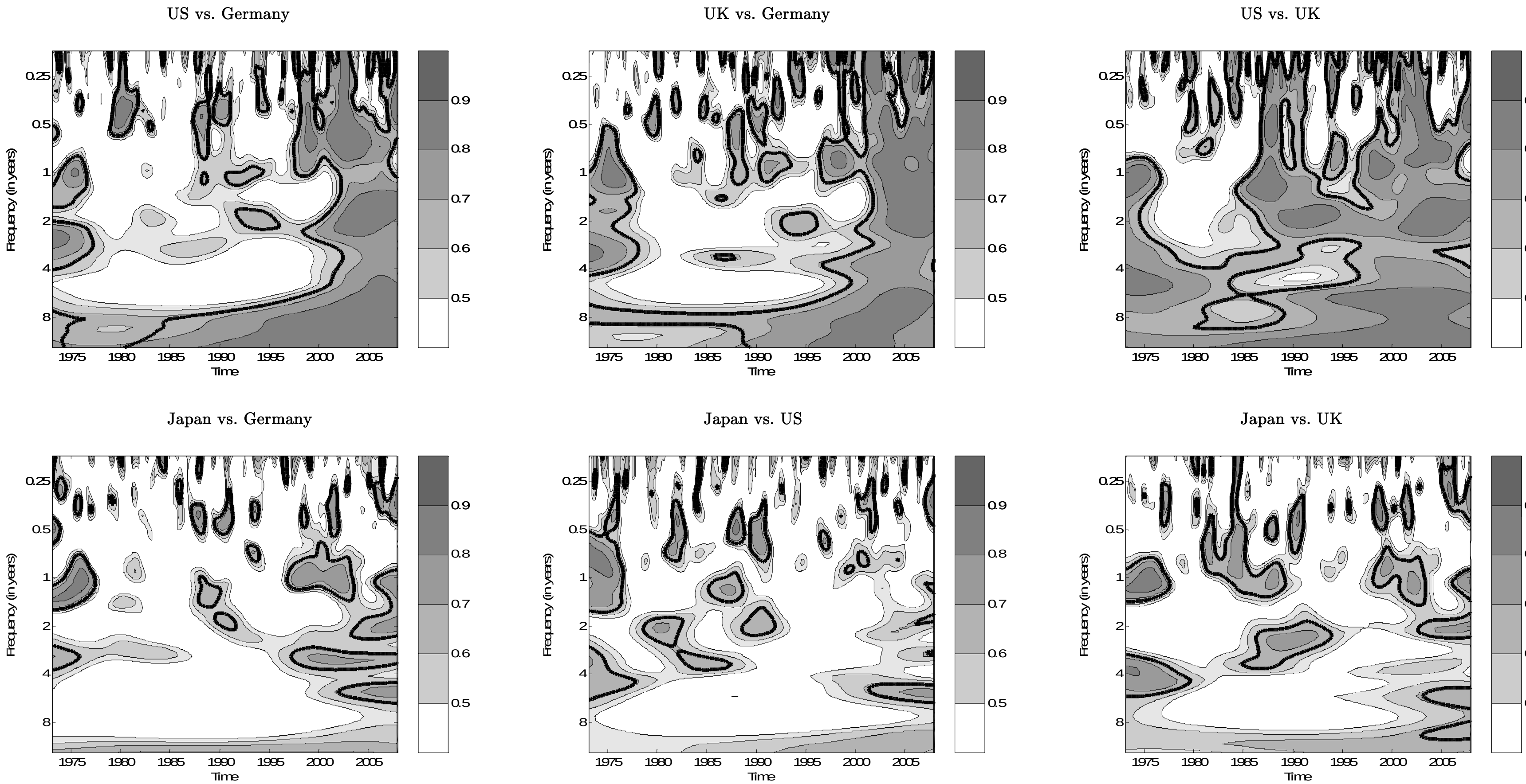


Figure 26 - Wavelet squared coherency for the overall stock market

returns several interesting findings arise (see Figure 26). First, Germany presents a relatively high degree of comovement at lower frequencies with US and UK over the whole sample period. Interestingly, since the end of the 90's, this high degree of comovement has been extended to all frequencies. Hence, there seems to be a change in the pattern of the relationship at the end of the 90's, whereas prior to that date the strong comovement is confined only to long-run fluctuations while afterwards it is visible for all sort of fluctuations. This finding provides an additional insight on the fact, commonly found in the recent literature, that there has been an overall increasing comovement.

Second, the US and UK stock markets seem to present a high degree of comovement over the last four decades (see also, for example, Kizys and Pierdzioch (2008)). One can see that this evidence is true for all frequencies but the highest ones. In particular, for fluctuations with a duration less than a year the comovement is weaker. However, even at those frequencies one can observe episodes where it is also high, namely around the 1987 US stock market crash and at the turn of the century with the technology bubble.

Third, one can conclude that Japan presents, in general, a low comovement with all the other countries considered. This low comovement of the Japanese stock market with the other major stock markets has also been found elsewhere (see, for example, Berben and Jansen (2005) and Longin and Solnik (1995)). From the current analysis it becomes clear that such evidence seems to be robust to the sample period and across all frequencies.

We now turn to the sectoral analysis.[27] Through an overview of the results some findings immediately emerge. First, the results obtained support the idea of a weak correlation between the Japanese stock market and the other major developed stock markets both across sectors and across time

[27] For each of the ten sectors considered, we computed the squared wavelet coherency for the same country pairs as in Figure 26. To save space such figures are not presented here but are available from the authors upon request.

and frequency (see also Berben and Jansen (2005)). However, there are two noteworthy exceptions, namely, in the consumer goods and technology sectors. In both sectors there is evidence of a strong comovement at lower frequencies between Japan and the other countries and in the technology sector, in particular, the comovement between Japan and US and between Japan and UK seems to have increased at other frequencies since the mid-90's.

Focusing now only on Germany, US and UK, one can see that in several sectors there is a strong comovement at long term fluctuations and interestingly, in most sectors, the US and UK stock markets present a temporary strong comovement at higher frequencies around the time of the 1987 US market crash.

Let us now run through the sectoral results in more detail. Regarding the oil and gas sector, there is a strong comovement at several frequencies between US and UK over the whole sample period. In the basic materials sector, we find a significant comovement at lower frequencies between US and Germany over the entire sample period as well as between UK and Germany and US and UK since the 90's. Note that between US and UK there is also a strong comovement at fluctuations longer than half a year and shorter than three years since the mid-80's. Concerning industrials, we find evidence of an increasing comovement between US and Germany and between US and UK at lower to medium frequencies range since the 90's (and to a less extent, between UK and Germany since the turn of the century). In the consumer goods sector, there is a significant comovement at lower frequencies for all country pairs. In the healthcare sector, the strongest comovement seems to be between US and UK comprising several frequencies. Regarding consumer services, Germany presents an increasing comovement with both US and UK at the turn of the century for almost all frequencies while US and UK show a strong comovement at the typical business cycle frequency range. In the telecommunications sector, we find a strong comovement at fluctuations longer than four years between UK and Germany and US and UK over almost the whole sample period and between US and Germany since late 80's. Concerning utilities, we find a significant comovement between UK and the other countries at lower frequencies. In the financials sector, Germany presents an increasing comovement with both US and UK since the mid-90's at several frequencies while there is a significant comovement between US and UK over most sample. Finally, in the technology sector, there is evidence of a strong comovement at long-term fluctuations since the 90's for all country pairs whereas US and UK present an increasing comovement at all frequencies since the mid-90's.

In summary, in terms of the aggregate index, among all the country pairs considered, the US and UK stock markets seem to present the highest comovement across time and frequencies while the Japanese market shows a low degree of comovement with any other major stock market in the time-frequency space. Regarding Germany, we find a high degree of comovement at lower frequencies with US and UK over the whole sample period and since the end of the 90's this is also observed for all the other frequencies. At the sectoral level, the weak comovement of Japan with the other countries is also, in general, present while Germany, US and UK show a significant comovement in several sectors at lower frequencies.

Let us illustrate the importance of wavelet analysis for risk management (see also Gençay *et al.* (2005) and Fernandez (2005, 2006)). To highlight the implications of the above findings, we perform a Value at Risk (VaR) analysis. The VaR is a widely known tool for risk assessment and it can be interpreted as the maximum loss of a portfolio not exceeded with a given probability over a period of time. The VaR at the $(1-\alpha)$ percent confidence level of a portfolio of k assets can be written as

$$VaR(\alpha) = V_0\Phi^{-1}(1-\alpha)\sigma_p \quad (49)$$

where V_0 is the value of the initial investment, $\Phi(.)$ is the cumulative distribution function of the standard Normal and σ_p is the square root of the portfolio variance. For a portfolio of k assets, the portfolio variance is given by

$$\sigma_p^2 = \sum_i \omega_i^2\sigma_i^2 + \sum_i \sum_{j\neq i} \omega_i\omega_j Cov(r_i, r_j) \qquad i, j = 1, ..., k \quad (50)$$

where ω_i is the weight of asset i in the portfolio, r_i is the return of asset i and σ_i^2 is the corresponding variance. From (50) one can see that the portfolio variance can be decomposed into two terms whereas the first one is strictly related with the variance of stock returns while the second term reflects the comovement. To make clear the importance of the comovement for risk assessment, we compute the VaR of a portfolio assuming that there is no comovement between the assets and the VaR of the same portfolio but without this restriction. In practice, we compute the portfolio variance discarding the second term of (50) in the former case while considering both terms in the latter case. To ease the comparison of the two VaR, we compute the ratio between them which resumes to the ratio of portfolio variances. In this way, one can assess the percentage increase/decrease in the VaR due to comovement. If the ratio is equal to one then it means that the comovement does not change the VaR while if the ratio is higher (lower) than one then it means that the comovement implies a higher (lower) VaR.

Let us consider an equally-weighted portfolio, whose value is measured in US dollars, comprised by the four country broad-based market indices. Resorting to the wavelet counterparts of variance and covariance in (50), we computed the above mentioned ratio in the time-frequency space (see Figure 27). Firstly, one can see that the ratio is almost always higher than one.

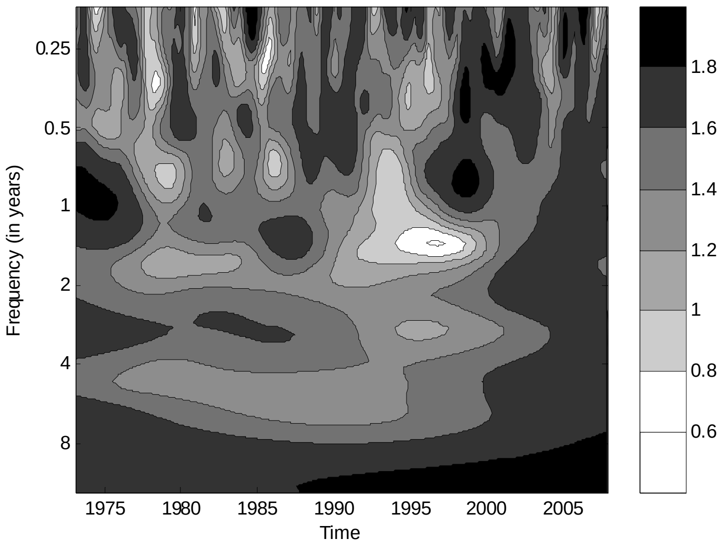


Figure 27: Ratio between the VaR of a multi-country portfolio with and without comovement

That is, whatever the frequency or the moment in time, the comovement among country stock returns implies a higher VaR. Naturally, from a portfolio perspective, a positive comovement among stock returns increases risk. Moreover, as expected, given the results above discussed, the comovement has a different impact in the VaR across frequencies and time. One can see that the ratio is higher at low frequencies over the whole sample period (the VaR is higher around 80 per cent) and it has increased over time attainnig higher values at all frequencies in the latter part of the sample period. This evidence reinforces the above findings regarding the fact that the benefits of international portfolio diversification vary across frequencies and over time.

5.5 Conclusions

The assessment of the international comovement of stock returns is crucial so as to shed light, for example, on the potential benefits of international portfolio diversification. This essay provides a new look into the comovement measurement of stock returns by resorting to wavelet analysis. Wavelet analysis allows one to assess the time- and frequency-varying comovement within an unified framework. This analysis is of particular interest in the context of the study of stock returns comovement as it is by now a stylized fact that the degree of comovement has changed over time and because one should be able to take into consideration the distinction between the short- and long-term investor, that is, the frequency domain. In fact, with wavelet analysis one can take into account the time and frequency domains simultaneously.

In this essay, we consider the stock returns for the major developed countries, namely Germany, Japan, United Kingdom and United States over the last four decades and besides the aggregate index we also consider its decomposition in ten main sectors, so as to provide insights at the sectoral level. A noteworthy finding of this essay is that the strength of the comovement of international stock returns depends on the frequency. In general, we find that comovements between markets is stronger at the lower frequencies suggesting that the benefits from international diversification may be relatively less important in the long-term than in the short-term. Therefore, the nature of the investor, in terms of short or long-term profile, should be taken into account when addressing the international portfolio diversification problem. We also found that the strength of the comovement in the time-frequency space varies across countries as well as across sectors. For instance, even though the Japanese stock market is generally weakly correlated with the other developed countries stock markets considered (as in Berben and Jansen, 2005), there are some sectors (technology and consumer goods) displaying strong comovements at particular frequencies and time periods. Finally, it was also found that the degree of comovement has changed over time, in line with the findings of Brooks and Del Negro (2004), among others. However, such changes are found to be confined, in several cases, to particular frequency ranges. Moreover, the detected changes are of different natures regarding their persistence in time. For example, the degree of comovement of the German market with the US and UK markets is characterized by some permanent changes over time: a gradual but steady increase of the comovement at the lower frequencies, and also a sudden increase after the end of the nineties for the other frequencies. On the other hand, the episodes of stronger comovement at higher frequencies between the US and UK markets around the 1987 crash and at the end of the century technological bubble are clearly of a distinct transitory nature. The first phenomena may be explained by the increased integration of financial markets whereas the latter may be associated with contagion. All these results highlight the importance of taking into consideration the time and frequency-varying properties of stock returns comovement in designing international portfolios as it may influence the benefits of international portfolio diversification in a non-negligible way.

6 A wavelet-based assessment of market risk[28]

6.1 Introduction

The assessment of market risk has long posed a challenge to many types of economic agents and researchers (see, for instance, Granger (2002) for an overview). Market risk arises from the random unantecipated changes in the prices of financial assets and its measurement is crucial for investors. Besides portfolio managers, the assessment of market risk is also relevant for the overall management of risk within banks as well as for bank supervisors. Although bank failures are traditionally related with an excess of non-performing loans (the so-called credit risk), the failure of the Barings Bank in 1995 has shown how market risk can lead to bankruptcy. Furthermore, market risk has received increasing attention in recent years as banks' financial trading activities have grown.

Although the measurement of market risk has a long tradition in finance, still there is no universally agreed upon definition of risk. The modern theory of portfolio analysis dates back to the pioneering work of Harry Markowitz in the 1950s. The starting point of portfolio theory rests on the assumption that investors choose between portfolios on the basis of their expected return, on the one hand, and the variance of their return, on the other. The investor should choose a portfolio that maximises expected return for any given variance, or alternatively, minimises variance for any given expected return. The portfolio choice is determined by the investor preferred trade-off between expected return and risk. Hence, in his seminal paper, Markowitz (1952) implicitly provided a mathematical definition of risk, that is, the variance of returns. In this way, risk is thought in terms of how spread-out the distribution of returns is.

[28] Joint work with Luis Catela Nunes.

Later on, the Capital Asset Pricing Model (CAPM) emerged with the work of Sharpe (1964) and Lintner (1965a, 1965b). According to the CAPM the relevant risk measure in holding a given asset is the systematic risk, since all other risks can be diversified away through portfolio diversification. The systematic risk, measured by the beta coefficient, is a widely used measure of risk. In statistical terms, it is assumed that the variability in each stock's return is a linear function of the return on some larger market with the beta reflecting the responsiveness of an asset to movements in the market portfolio. For instance, in a context of international portfolio diversification, the country risk is defined as the sensitivity of the country return to a world stock return. Traditionally, it is assumed that beta is constant through time. However, empirical research has found evidence that betas are time varying (see, for example, the pioneer work of Blume (1971, 1975)). Such finding led to a surge of literature on this issue (see, for example, Fabozzi and Francis (1977, 1978), Sunder (1980), Alexander and Benson (1982), Collins *et al.* (1987), Harvey (1989, 1991), Ferson and Harvey (1991, 1993), Ghysels (1998) among others). One natural implication of such result is that risk measurement has to be able to account for this time-varying feature.

Besides the time-variation, risk management should also take into account the distinction between the short and long-term investor (see, for example, Candelon *et al.* (2008)). In fact, the first kind of investor is naturally more interested in risk assessment at higher frequencies, that is, short-term fluctuations, whereas the latter focus on risk at lower frequencies, that is, long-term fluctuations. Analysis at the frequency level provides a valuable source of information, considering that different financial decisions occur at different frequencies. Hence, one has to resort to the frequency domain analysis to obtain insights about the risk at the frequency level.

In this essay, we re-examine risk measurement through a novel approach, wavelet analysis. Wavelet analysis constitutes a very promising tool as it represents a refinement in terms of analysis in the sense that both time and frequency domains are taken into account. In particular, one can resort to wavelet analysis to provide a unified framework to measure risk in the time-frequency space. As both time and frequency domains are encompassed, one is able to capture the time-varying feature of risk while disentangling its behaviour at the frequency level. In this way, one can simultaneously measure the evolving risk exposure and distinguish the risk faced by short and long-term investors. Although wavelets have been more popular in fields such as signal and image processing, meteorology, physics, among others, such analysis can also provide fruitful insights about several economic phenomena (see, for example, the pioneer work of Ramsey and Zhang (1996, 1997) and Ramsey and Lampart (1998a, 1998b)). Recent work using wavelets includes, for example, Kim and In (2003, 2005) who investigate the relationship between financial variables and industrial production and between stock returns and inflation, Gençay *et al.* (2003, 2005) and Fernandez (2005, 2006) who study the CAPM at different frequency scales, Connor and Rossiter (2005) focus on commodity prices, In and Kim (2006) examine the relationship between the stock and futures markets, Gallegati and Gallegati (2007) provide a wavelet variance analysis of output in G-7 countries, Gallegati *et al.* (2008) and Yogo (2008) resort to wavelets for business cycle analysis, Rua (2010b) focus on forecasting GDP growth in the major euro area countries, among others (see also Crowley (2007) for a survey). However, up to now, almost all of the work drawing on wavelets has been based on the discrete wavelet transform. Although the discrete wavelet transform can be useful in dividing the original series into components of different frequency so that each component may be studied separately, the use of the continuous wavelet transform can enhance the analysis by allowing capturing both time and frequency-varying features within an unified framework (see, for example, Rua and Nunes (2009) and Rua (2010a)). Hence, in order to assess market risk we will draw on the continuous wavelet transform.

We provide an illustration of such analysis by considering the emerging markets case. The new equity markets that have emerged around the world have received a considerable attention in the last two decades leading to an extensive recent literature on this topic (see, for example, Harvey (1995), Bekaert and Harvey (1995, 1997, 2000, 2002, 2003), Garcia and Ghysels (1998), Estrada (2000), De Jong and De Roon (2005), Chambet and Gibson (2008), among others). The fact that the volatility of stock prices changes over time has been known for a long time (see, for example, Fama (1965)), and such feature has also been documented for the emerging markets. The time variation of risk comes even more naturally in these countries due to the changing economic environment resulting from capital market liberalizations or the increasing integration with world markets and the evolution of political risks. In fact, several papers have acknowledged time varying volatility and betas for the emerging markets (see, for example, Bekaert and Harvey (1997, 2000, 2002, 2003), Santis and Imrohoroglu (1997) and Estrada (2000)). Moreover, the process of market integration is a gradual one as emphasized by Bekaert and Harvey (2002). Therefore, methods that allow for gradual transitions at changing speeds, such as wavelets, are preferable to segmenting the analysis to different subperiods. Hence, the emerging markets case constitutes an interesting example for the application of risk measurement with the continuous wavelet transform.

This chapter is organised as follows. In section 6.2, the main building blocks of wavelet analysis are presented. In section 6.3, we provide the wavelet counterpart of well-known risk measures. In section 6.4, an application to the emerging markets case is provided. Finally, section 6.5 concludes.

6.2 Wavelet analysis

The wavelet transform decomposes a time series in terms of some elementary functions, the daughter wavelets or simply wavelets $\psi_{\tau,s}(t)$. Wavelets are 'small waves' that grow and decay in a limited time period. These wavelets result from a mother wavelet $\psi(t)$, that can be expressed as function of the time position τ (translation parameter) and the scale s (dilation parameter), which is related with the frequency. While the Fourier transform decomposes the time series into infinite length sines and cosines (see, for example, Priestley (1981)), discarding all time-localization information, the basis functions of the wavelet transform are shifted and scaled versions of the time-localized mother wavelet. In fact, wavelet analysis can be seen as a refinement of Fourier analysis. More explicitly, wavelets are defined as

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right) \tag{51}$$

where $\frac{1}{\sqrt{s}}$ is a normalization factor to ensure that wavelet transforms are comparable across scales and time series. To be a mother wavelet, $\psi(t)$, must fulfil several conditions (see, for example, Percival and Walden (2000)): it must have zero mean, $\int_{-\infty}^{+\infty}\psi(t)dt = 0$; its square integrates to unity, $\int_{-\infty}^{+\infty}\psi^2(t)dt = 1$, which means that $\psi(t)$ is limited to an interval of time; and it should also satisfy the so-called admissibility condition, $0 < C_\psi = \int_0^{+\infty}\frac{\left|\widehat{\psi}(\omega)\right|^2}{\omega}d\omega < +\infty$ where $\widehat{\psi}(\omega)$ is the Fourier transform of $\psi(t)$, that is, $\widehat{\psi}(\omega) = \int_{-\infty}^{+\infty}\psi(t)e^{-i\omega\tau}dt$. The latter condition allows the reconstruction of a time series $x(t)$ from its continuous wavelet transform, $W_x(\tau, s)$. Thus, it is possible to recover $x(t)$ from its wavelet transform through the following formula

$$x(t) = \frac{1}{C_\psi}\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty}\frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right)W_x(\tau, s)d\tau\right]\frac{ds}{s^2} \tag{52}$$

The continuous wavelet transform of a time series $x(t)$ with respect to

$\psi(t)$ is given by the following convolution

$$W_x(\tau, s) = \int_{-\infty}^{+\infty} x(t)\psi^*_{\tau,s}(t)dt = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} x(t)\psi^*\left(\frac{t-\tau}{s}\right)dt \tag{53}$$

where * denotes the complex conjugate. For a discrete time series, $x(t)$, $t = 1, ..., N$ we have

$$W_x(\tau, s) = \frac{1}{\sqrt{s}} \sum_{t=1}^{N} x(t)\psi^*\left(\frac{t-\tau}{s}\right) \tag{54}$$

Although it is possible to compute the wavelet transform in the time domain using equation (54), a more convenient way to implement it is to carry out the wavelet transform in Fourier space (see, for example, Torrence and Compo (1998)).

The most commonly used mother wavelet is the Morlet wavelet and is defined as

$$\psi(t) = \pi^{-\frac{1}{4}}\left(e^{i\omega_0 t} - e^{-\frac{\omega_0^2}{2}}\right)e^{\frac{-t^2}{2}} \tag{55}$$

Since the term $e^{-\frac{\omega_0^2}{2}}$ becomes negligible for an appropriate ω_0, the Morlet wavelet is simply defined as

$$\psi(t) = \pi^{-\frac{1}{4}}e^{i\omega_0 t}e^{\frac{-t^2}{2}} \tag{56}$$

with the corresponding Fourier transform given by

$$\widehat{\psi}(\omega) = \pi^{\frac{1}{4}}\sqrt{2}e^{-\frac{1}{2}(\omega-\omega_0)^2} \tag{57}$$

One can see that the Morlet wavelet is a complex sine wave within a Gaussian envelope whereas ω_0 is the wavenumber. In practice, ω_0 is set to 6 as it provides a good balance between time and frequency localization. One of the advantages of the Morlet wavelet is its complex nature which allows for both time-dependent amplitude and phase for different frequencies (see, for example, Adisson (2002) for further details on the Morlet wavelet).

As in Fourier analysis, several interesting measures can be defined in the wavelet domain. For instance, one can define the wavelet power spectrum as $|W_x(\tau, s)|^2$. It measures the time series' variance at each time and at each scale. Another measure of interest is the cross-wavelet spectrum which captures the covariance between two series in the time-frequency space. Given two time series $x(t)$ and $y(t)$, with wavelet transforms $W_x(\tau, s)$ and $W_y(\tau, s)$ one can define the cross-wavelet spectrum as $W_{xy}(\tau, s) = W_x(\tau, s)W_y^*(\tau, s)$. As the mother wavelet is complex, the cross-wavelet spectrum is also complex valued and it can be decomposed into real and imaginary parts.

6.3 Measuring risk with wavelets

The variance is the most famous moment-based measure of risk in finance after the seminal work of Markowitz. The variance is a particularly appropriate measure of risk in segmented markets or if one is interested in a single asset. While the variance measures total risk, the beta captures the systematic risk. In contrast with the variance which treats an asset in isolation, the beta reflects the idea that any asset can be viewed as a part of a portfolio. In light of this, the asset's risk can be thought in terms of the contribution to the variability of the portfolio. According to the CAPM developed by Sharpe (1964), Lintner (1965a, 1965b) and Mossin (1966), as it is well known,

$$E[R_i] = R_f + \beta_i [E[R_m] - R_f] \tag{58}$$

where R_i is the return on asset i, R_f is the risk-free asset return and R_m is the market return. Equation (58) states that the expected return on asset i is equal to the risk-free rate (compensanting investors for delaying consumption) plus a risk premium (compensating them for taking the risk associated with the investment). The risk premium can be broken into two parts. The term in brackets is the risk premium for the market portfolio, which can be thought as the risk premium for an average, or representative, asset. To obtain the risk premium for asset i, one has to multiply the risk premium for the average asset by the other term, the risk measure for asset i, that is, the beta. The beta is defined as

$$\beta_i = \frac{Cov(R_i, R_m)}{\sigma^2_{R_m}} \tag{59}$$

where $Cov(R_i, R_m)$ is the covariance between the return on asset i and the return on the market portfolio and $\sigma^2_{R_m}$ is the variance of the portfolio return. For instance, in the context of the world CAPM, a country's beta is defined as the covariance of the country's returns with the world market portfolio divided by the variance of the world market return.

The *rationale* of using beta as a measure of risk is also motivated by the index model developed by Sharpe (1963). Each asset is assumed to respond to the pull of a single factor, which is usually taken to be the market portfolio. The return on asset i can be written as

$$R_{it} = \alpha + \beta_i R_{mt} + \varepsilon_t \tag{60}$$

This model implicitly assumes that two types of events determine the period-to-period variability in the asset's return. On the one hand, events that influence the return on the market portfolio, and through the pull of the market, they induce changes in the return on individual assets. On the other hand, events that have impact on asset i but no effect on the other assets. Hence, the total risk of asset i can be decomposed as

$$\sigma^2_{R_i} = \beta^2_i \sigma^2_{R_m} + \sigma^2_{\varepsilon} \tag{61}$$

that is, the variance of the return on asset i can be written as the sum of two terms. The first one is called the systematic risk and accounts for that part of the variance which cannot be diversified away while the second term is called the unsystematic risk and represents the part of the variance that disappears with diversification.

We now discuss the wavelet counterpart of the above risk measures[29]. The natural wavelet counterpart of the variance, that is total risk, is the wavelet spectrum. As mentioned earlier, the wavelet spectrum for the return on asset i can be obtained as $|W_{R_i}(\tau,s)|^2$ and it measures variance in the time-frequency space. Regarding beta, the wavelet counterpart of (59) is given by

$$\beta_i(\tau,s) = \frac{\Re\left(W_{R_i,R_m}(\tau,s)\right)}{|W_{R_m}(\tau,s)|^2} \tag{62}$$

where $\Re$ denotes the real part of the cross-wavelet spectrum which measures

[29]Another popular measure of risk is what is known as the Value-at-Risk (VaR) (see, for example, Jorion (1997)). The VaR is the minimal potential loss that a portfolio can suffer in the 100α per cent worst cases over a fixed time horizon. Suppose X is a random variable denoting the loss of a given portfolio. The VaR at the $1-\alpha$ confidence level can be written as

$$VaR_\alpha(X) = \sup\left\{x \mid P\left[X \geqslant x\right] > \alpha\right\}$$

where $\sup\{x \mid A\}$ is the upper limit of x given event A. Since VaR has several limitations (see, for example, Tasche (2002)), it has been proposed an alternative measure, the expected shortfall (also called conditional VaR). The expected shortfall is defined as the conditional expectation of loss when the loss exceeds the VaR level.

$$ES_\alpha(X) = E\left[X \mid X \geqslant VaR_\alpha(X)\right]$$

Under the Normal distribution, both the VaR and the expected shortfall are scalar multiples of the standard deviation (see, for example, Yamai and Yoshiba (2005)). Hence, both measures provide essentially the same information as the standard deviation.

the contemporaneous covariance. Additionally, one can also assess the importance of systematic risk for explaining total risk of asset i. This can be done by computing the ratio between systematic risk and total risk, $\frac{\beta_i^2 \sigma_{R_m}^2}{\sigma_{R_i}^2}$, which corresponds to the well-known measure of fit , the R^2, for model (60). The wavelet R^2 can be computed as

$$R^2(\tau, s) = \frac{\beta_i(\tau, s)^2 \left|W_{R_m}(\tau, s)\right|^2}{\left|W_{R_i}(\tau, s)\right|^2} \tag{63}$$

In this way, it is possible to quantify the fit of model (60) in the time-frequency space and assess over which periods of time and frequencies is the fit higher. Naturally, $R^2(\tau, s)$ is between 0 and 1, where a value close to 0 can be interpreted as the systematic risk having a small contribution to total risk while a value close 1 denotes a high importance of systematic risk in determining total variability.

6.4 The emerging markets case

To illustrate the above suggested measures, we assess the risk faced by an investor in emerging markets over the last twenty years. We use the Morgan Stanley Capital International (MSCI) all country world index and the MSCI emerging markets index taken from Thompson Financial Datastream. The MSCI emerging markets index is a free float-adjusted market capitalization index and consists of the following 23 emerging market country indices: Argentina, Brazil, Chile, China, Colombia, Czech Republic, Egypt, Hungary, India, Indonesia, Israel, Korea, Malaysia, Mexico, Morocco, Peru, Philippines, Poland, Russia, South Africa, Taiwan, Thailand, and Turkey. The MSCI all country index is also a free float-adjusted market capitalization weighted index and consists of 46 country indices comprising the above 23 emerging market countries and 23 developed country indices. The de-

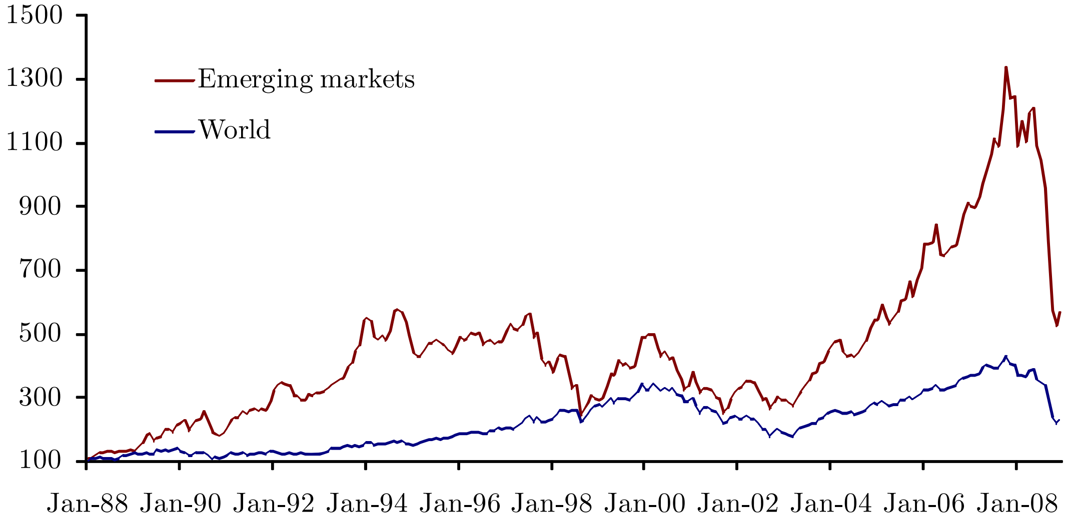


Figure 28: Monthly stock price indices

veloped market country indices included are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, the United Kingdom and the United States. The data sample ranges from January 1988 up to December 2008, whereas monthly returns are computed as the percentage change of the stock price indices considering end of month figures. The returns on both the MSCI stock indices are expressed in US dollars, that is, we consider the case of an American investor investing in emerging markets. In figures 28 and 29, we plot the monthly stock indices and the monthly returns, respectively. In Table 1, we report some descriptive statistics. The results are in line with well-known facts about emerging markets, namely that average returns are higher as well as volatility.

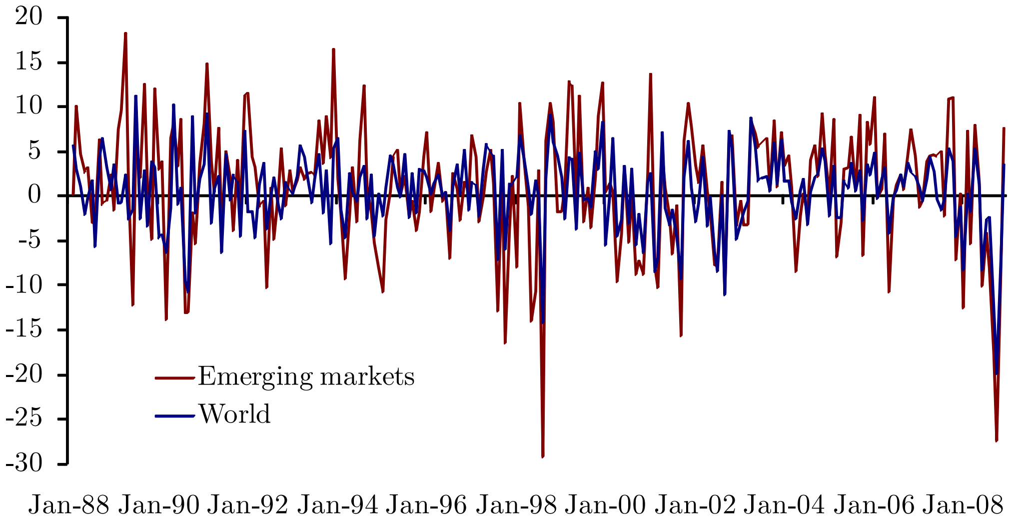


Figure 29: Monthly stock returns (in percentage)

	Emerging markets	World
Mean	0.9	0.4
Standard deviation	6.9	4.3
Minimum	-29.3	-19.9
Maximum	18.1	11.1
Beta	1.2	-
R^2	0.53	-

Table 1: Descriptive statistics of monthly stock returns (in percentage)

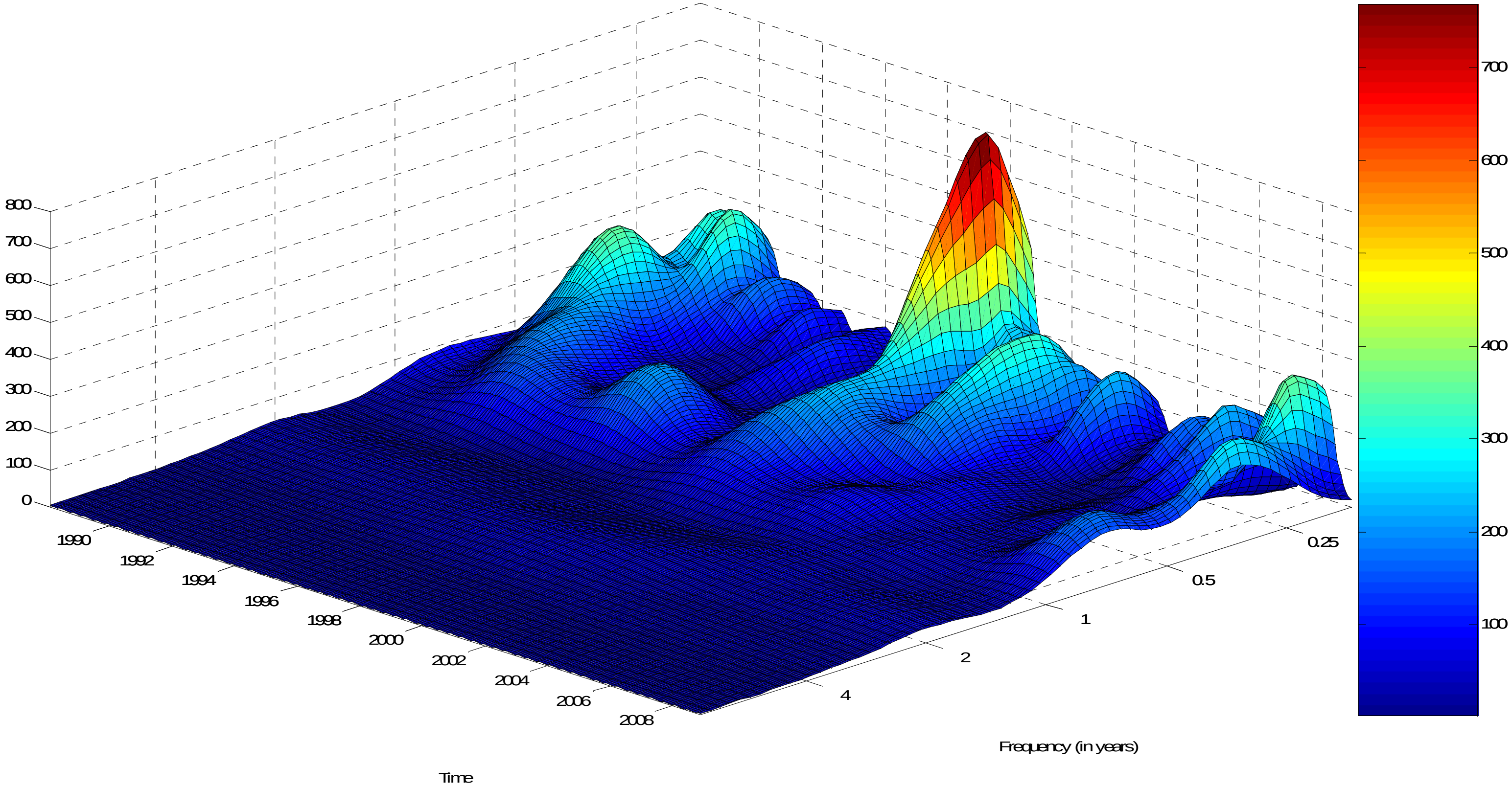


Figure 30 - Wavelet spectrum for the emerging markets returns

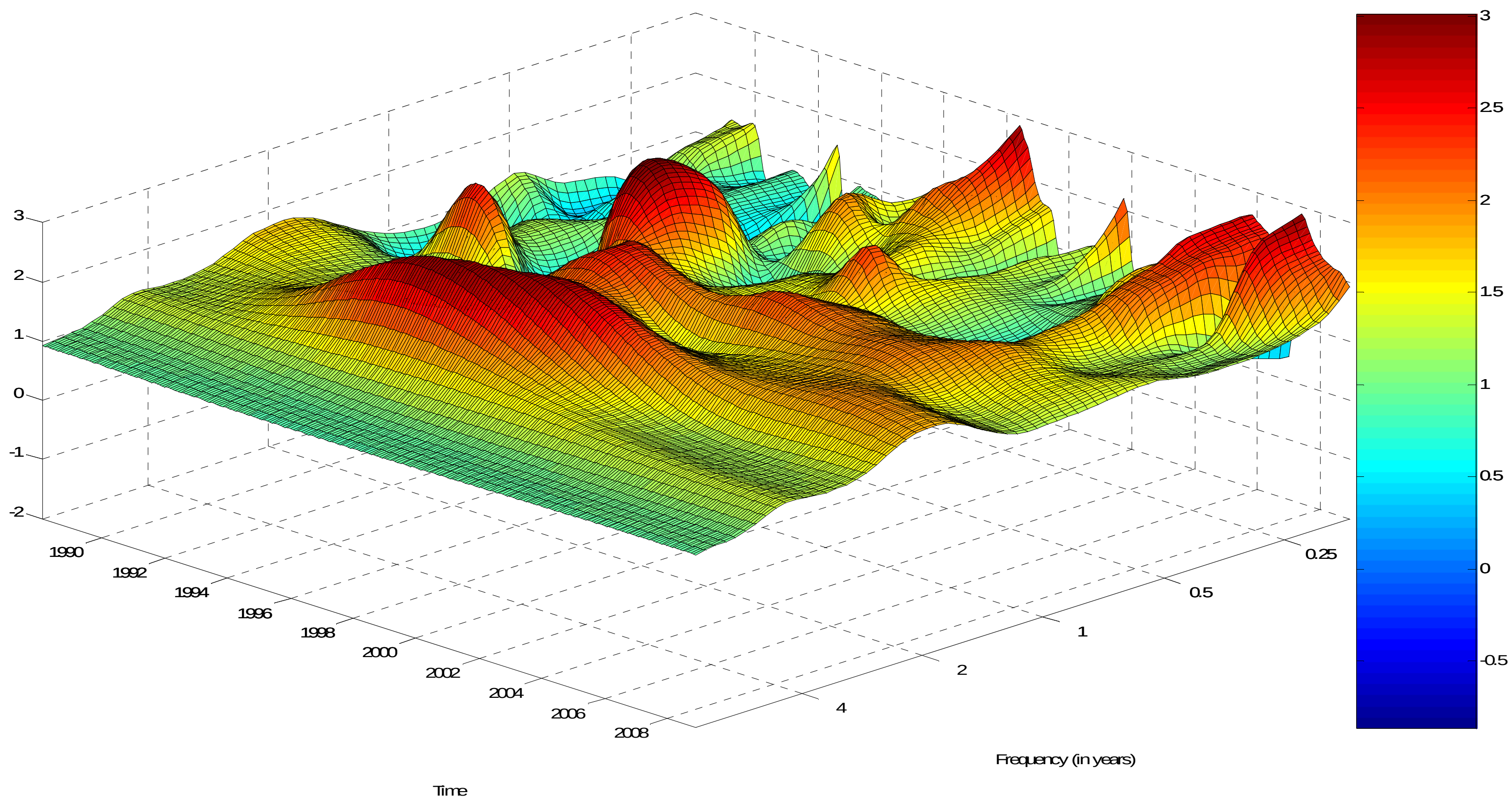


Figure 31 - Wavelet beta for emerging markets

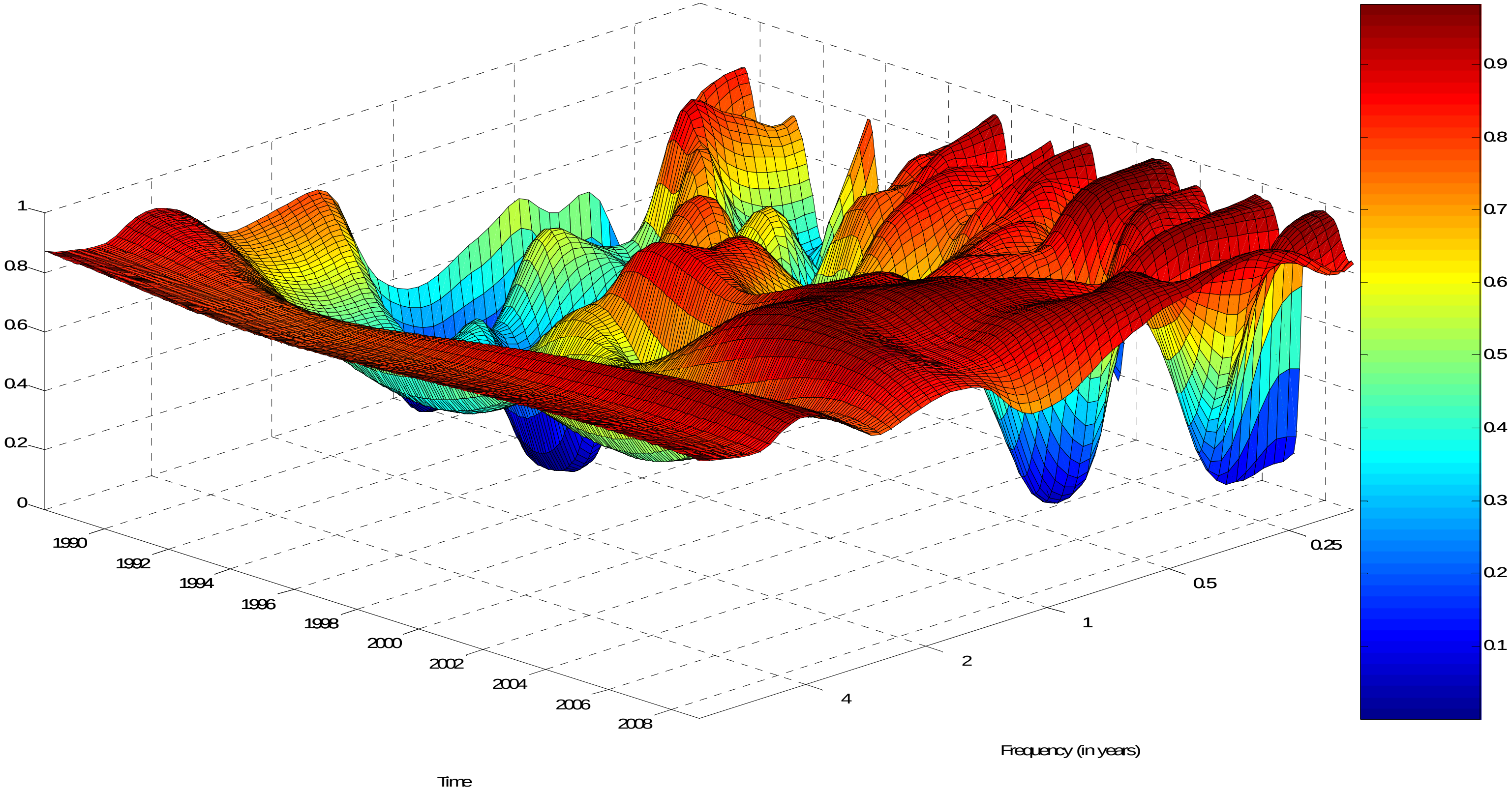


Figure 32 - Wavelet R^2 for emerging markets

First, we focus on total risk as measured by the variance. In figure 30, we present the wavelet spectrum for the return on the emerging markets index[30]. As three dimensions are involved, we get a 3-D surface plot. The x-axis refers to time while the y-axis refers to frequency. To ease interpretation, the frequency is converted to time units (years). The z-axis, that is, the height of the surface corresponds to the value of the wavelet spectrum whereas the colour scale helps to visualize the magnitude. From the analysis of the wavelet spectrum several findings emerge. First, one can see that the volatility of monthly stock returns is concentrated at high frequencies, that is, the short-term fluctuations dictate the variance of the series. In fact, frequencies associated with movements longer than one year are almost negligible in terms of contribution to total variance. Besides the varying feature across frequencies, one can see that variance has also changed over time. In particular, one can detect clearly periods of higher volatility, namely around 1990, 1998, 2001 and 2008. The volatility at the end of the 1980s and beginning of the 1990s is related with the crisis involving several Asian countries, namely Taiwan, the Philippines, Korea, Thailand and Indonesia. The highest volatility period is around 1998 when several crises occurred, starting with the East Asian crisis during 1997 and 1998, when Thailand, Malaysia, the Philippines, Indonesia and Korea underwent severe financial and currency crises, the Russian default in August 1998 and the Brazilian crisis at the beginning of 1999. The volatility in 2001 reflects the Argentinean and Turkish crises while the more recent episode is related with the subprime mortgage crisis and the subsequent liquidity squeeze in the US which started in mid-2007 and sent shock waves around the world. One should mention that despite the variance has changed over time, it is not visible any trend in terms of volatility in the emerging markets (see also Bekaert and Harvey (1997, 2003)). Indeed, as discussed by Bekaert and Harvey (2003), it is not

[30] All computations are done using Matlab.

clear from finance theory that volatility should increase or decrease when markets are liberalized. In fact, if on the one hand equity market liberalization may lead to higher volatility due to an increase of the importance of short-term fluctuations, on the other hand, one may expect lower volatility coming from long-term swings. However, our results suggest that no trend is present for both high and low frequencies.

To assess the systematic risk of stock market returns in emerging countries, we present in figure 31 the wavelet beta for the emerging markets index considering as a proxy for the market portfolio the world index. For the sample as a whole, the beta is slightly higher than one (see table 1) in line with the findings of Estrada (2000). However, one can find noteworthy variation in the results across frequencies and over time. First, the beta coefficient seems to be more stable over time at low frequencies and more time-varying at high frequencies. Second, at low frequencies the beta is around one while at high frequencies one can identify regions in the time-frequency space where the beta is near three. The periods where the beta is highest include the Mexican crisis in 1994, the year 1998 when crisis hit several emerging markets, late 2005 through 2006 encompassing the Turkish crisis in the Spring 2006 and the most recent period since mid-2007 with the awakening of a global financial crisis.

The importance of the systematic risk in explaining total risk in emerging markets can be assessed through figure 32 where we plot the wavelet R^2. Once again, the time-frequency analysis can provide worthwhile insights. For instance, although the R^2 of equation (60) for the sample period as a whole is close to 0.5 which means that the systematic part is as important as the unsystematic one for total variance (see table 1), one can see that the value of R^2 changes a lot across frequencies and over time. A finding is that the importance of the systematic risk in emerging markets is relatively high and stable over time at low frequencies. The proportion of the total variance explained by the systematic component is around 80 per cent at frequencies associated with fluctuations that last longer than four years. In contrast, for higher frequencies it has been observed a time-varying influence of the systematic part. In particular, we have values around 30 per cent up to the mid-1990s followed by a relatively steady increase thereafter, though clearly interrupted during 2004 and at the beginning of 2007, attaining values nearby 80 per cent at the end of the sample. This finding supports the idea that the increase of the correlation between the emerging markets and world indices at the end of the 1990s highlighted by Bekaert and Harvey (2003) may be of a permanent nature.

6.5 Conclusions

Although most textbook models assume volatilities and covariances to be constant, it has been long acknowledged among both finance academics and practitioners that market risk varies over time. Besides taking into account such time-varying feature, the risk profile of an investor, in terms of investment horizon, makes it also crucial to assess risk at the frequency level. Naturally, a short-term investor is more interested in the risk associated with high frequencies whereas a long-term investor focus on lower frequencies. This essay provides a new look into market risk measurement by resorting to wavelet analysis as it allows one to evaluate the time and frequency-varying features within an unified framework. In particular, we derive the wavelet counterpart of well-known measures of market risk. We consider total risk, as measured by the variance of returns, the systematic risk, captured by the beta coefficient and we provide the tools to assess the importance of systematic risk on total risk in the time-frequency space.

To illustrate such analysis, we consider the emerging markets case, which has received a lot of attention in the literature, over the last twenty years.

As those countries have experienced a changing economic environment, it is particularly interesting to see how market risk has changed across frequencies and over time. We find that the variance of monthly returns is determined essentially by short-run fluctuations and that the volatility has changed over time. In particular, the periods of higher volatility are associated with several economic crises that hit the emerging markets. Regarding the systematic risk, we find that the beta coefficient is relatively stable at low frequencies presenting a value around one. In contrast, at higher frequencies, the beta coefficient varies a lot attaining values as high as three in some economic episodes. Additionally, we also assessed the importance of the systematic risk in explaining total risk in emerging markets. Again, we find noteworthy variation in the results across frequencies and over time. We conclude that the importance of systematic risk in emerging markets is relatively high and stable over time at low frequencies. At higher frequencies, the influence of the systematic part was relatively low before the mid-1990s but increased gradually thereafter attaining values also relatively high at the end of the sample. All these results highlight the importance of considering the time and frequency-varying features in risk assessment. Hence, wavelet analysis can be a valuable tool to obtain additional insights which may influence risk-taking decisions.

7 A wavelet approach for factor-augmented forecasting[31]

7.1 Introduction

In a context of growing data availability, there has been an increasing focus on factor models as such models allow exploiting large data sets in a simple and parsimonious way. The literature on factor models in economics goes back to Geweke (1977), Sargent and Sims (1977), Geweke and Singleton (1981) and Watson and Engle (1983). In the conventional factor model, the data generating process of each variable is the sum of two components, a component associated with factors common to all series and an idiosyncratic term. The underlying idea is that one can summarize the large information set into a small number of variables, the common factors, which retain the main features of the original data set. In practice, this means that, for forecasting purposes, a large number of predictors can be replaced by a reduced number of variables without a significant loss of information (see Stock and Watson, 1998). The use of those factors as regressors in forecasting equations provides what is known as factor-augmented forecasts. Several work has been done in this line of research, including Stock and Watson (1999, 2002a, 2002b) for the US, Marcellino *et al.* (2003) for the euro area, Artis *et al.* (2005) for the UK, Schumacher (2007) for Germany, Bruneau *et al.* (2007) for France, among others.

From a different perspective, a relatively unexplored tool for forecasting is wavelets. Wavelet multiresolution analysis allows one to decompose a time series into a low-frequency base scale and higher-frequency scales. Those frequency components can be analysed individually or compared across variables. For example, the pioneer work of Ramsey and Lampart (1998a,b)

[31] This chapter is forthcoming in Journal of Forecasting.

draws on wavelets to study the relationship between several macroeconomic variables, namely money supply and output in the first case and consumption and income in the second. Recent work includes Kim and In (2005), who investigate the relationship between stock returns and inflation, Gençay *et al.* (2005) and Fernandez (2005, 2006) study the Capital Asset Pricing Model at different frequency scales, Crivellini *et al.* (2004), Gallegati *et al.* (2008) and Yogo (2008) resort to wavelets for business cycle analysis, among others. Although it has been acknowledged the potential usefulness of wavelets in forecasting, there are very few applications of wavelets for forecasting in economics. In particular, Arino (1995) focus on car sales forecasts, Wong *et al.* (2003) provide an application to exchange rates, Conejo *et al.* (2005) forecast electricity prices and Fernandez (2007) focus on forecasting shipments of US manufactured items. In general, the results obtained in terms of forecasting performance seem to be promising.

The wavelet multiresolution approach for forecasting purposes consists in several steps. First, the series to be forecast is decomposed into its constituent time-scale components. In particular, through wavelets, a time series is decomposed into orthogonal components of different frequencies, which in turn are localized in time. Then, for each time-scale a model is fitted and used for forecasting. Finally, an overall forecast is obtained after recombining the components. This multiresolution approach can outperform the traditional single resolution approach for forecasting as it is possible to tailor specific forecasting models to each time-scale component and thereby enhance the forecasting performance.

Up to now, the literature concerning forecasting with wavelets has been restricted to univariate models for modelling each time-scale component (Conejo *et al.* (2005) is an exception, although they only consider one independent variable). Hence, there is scope for extending the current modelling framework. In particular, one can extend the information that is taken on board for forecasting purposes by considering factor-augmented models. The ability to handle large data sets in a straightforward and parsimonious way has contributed to the popularity of such models both in the literature and among practitioners. The aim of this essay is to bridge the wavelet approach and factor-augmented models.

We focus on the short-term forecasting of GDP growth for the major euro area countries, namely Germany, France, Italy and Spain. Resorting to large data sets for those countries over the last twenty years, we evaluate the out-of-sample performance of several alternatives for forecasting one- and two-quarters ahead GDP growth. Within the single resolution approach, we consider two models, an autoregressive model as the usual benchmark and the factor-augmented model. Regarding the wavelet approach, we consider the corresponding two variants. In the first, an autoregressive model is fitted to each time-scale component whereas in the second, a factor-augmented model is considered for each time-scale.

It is found that the factor-augmented model outperforms, in general, the benchmark for short-term forecasting, in line with the results found in the related literature. When one follows a wavelet approach and considers a univariate model for each time-scale, one also improves on the benchmark. But the best performing procedure is to combine the wavelet approach and factor-augmented models. We find that such approach outperforms all the above-mentioned alternatives for forecasting GDP growth in all countries and for all forecast horizons. In particular, for the one-quarter ahead horizon, the forecasting gains are quite noteworthy. Moreover, the findings are supported by forecast accuracy and encompassing tests.

The chapter is organised as follows. In section 7.2, the wavelet multiresolution decomposition is addressed. In section 7.3, the wavelet approach for forecasting with factor-augmented models is presented. In section 7.4, a brief description of the data for the major euro area countries is provided and the results of the out-of-sample forecasting exercise are discussed. Finally, section 7.5 concludes.

7.2 Wavelet multiresolution decomposition

The well-known Fourier transform involves the projection of a series onto an orthonormal set of trigonometric components. In particular, Fourier series have infinite energy (they do not fade away) and finite power (do not change over time). In contrast, wavelets have finite energy and compact support, that is, they grow and decay in a limited time period. Wavelets can be a particular useful tool when the signal shows a different behaviour in different time periods or when the signal is localized in time as well as frequency. As it enables a more flexible approach in time series analysis, wavelet analysis is seen as a refinement of Fourier analysis.

In particular, the discrete wavelet transform (DWT) makes it possible to decompose a time series into its constituent multiresolution components (see, for example, Percival and Walden (2000)).[32] High-frequency components reflect the short-term behaviour, whereas the low-frequency component captures the long-term dynamics of the variable. There are two types of wavelets, father wavelets ϕ and mother wavelets ψ, where

$$\int \phi(t)dt = 1 \tag{64}$$

and

$$\int \psi(t)dt = 0. \tag{65}$$

The smooth and low-frequency part of the series is captured by the father wavelet, while the detail and high-frequency components are described by

[32] Recent work drawing on the continuous wavelet transform (CWT) include, for example, Rua and Nunes (2009) and Rua (2010).

the mother wavelet.

The orthogonal wavelet series approximation to a series $y(t)$ is defined by

$$y(t) = \sum_k s_{J,k}\phi_{J,k}(t) + \sum_k d_{J,k}\psi_{J,k}(t) + \sum_k d_{J-1,k}\psi_{J-1,k}(t) + \cdots + \sum_k d_{1,k}\psi_{1,k}(t) \tag{66}$$

where J is the number of multiresolution levels (or scales) and k ranges from one to the number of coefficients in the corresponding component. When the number of observations, T, is divisible by 2^J there are $T/2^j$ $d_{j,k}$ coefficients at scale $j = 1, ..., J-1$, while at scale J there are $T/2^J$ $d_{J,k}$ coefficients and $T/2^J$ $s_{J,k}$ coefficients. In total, there are T wavelet coefficients, that is, $T = T/2^1 + T/2^2 + ... + T/2^{J-1} + T/2^J + T/2^J$. The coefficients $s_{J,k}$, $d_{J,k}$, $d_{J-1,k}$, ..., $d_{1,k}$ are the wavelet transform coefficients, which are given by

$$s_{J,k} = \int y(t)\phi_{J,k}(t)dt \tag{67}$$

$$d_{j,k} = \int y(t)\psi_{j,k}(t)dt, \qquad j = 1, 2, ..., J. \tag{68}$$

These coefficients give a measure of the contribution of the corresponding wavelet function to the signal.

The functions $\phi_{J,k}(t)$ and $\psi_{j,k}(t)$ are the approximating wavelet functions, generated from ϕ and ψ through scaling and translation as follows

$$\phi_{J,k}(t) = 2^{-J/2}\phi\left(\frac{t-2^J k}{2^J}\right) \tag{69}$$

and

$$\psi_{j,k}(t) = 2^{-j/2}\psi\left(\frac{t-2^j k}{2^j}\right), \qquad j = 1, 2, ..., J. \tag{70}$$

The DWT allows to obtain the coefficients of the wavelet series approximation in (66) for a discrete signal of finite extent. The DWT maps the vector $y = (y_1, y_2, ..., y_T)'$ to a vector of T wavelet coefficients that contains the smooth coefficients $s_{J,k}$ and the detail coefficients $d_{j,k}$. In other words, the DWT maps a time series from its original representation in the time domain to a representation in the time-scale domain.

Equation (66) can be rewritten as

$$y(t) = S_J(t) + D_J(t) + D_{J-1}(t) + ... + D_1(t) \tag{71}$$

where $S_J(t) = \sum_k s_{J,k}\phi_{J,k}(t)$ and $D_j(t) = \sum_k d_{j,k}\psi_{j,k}(t)$ for $j = 1, 2, ..., J$ are the smooth and detail components, respectively. The expression (71) represents the decomposition of $y(t)$ into orthogonal components, $S_J(t)$, $D_J(t)$, $D_{J-1}(t)$,..., $D_1(t)$, at different resolutions and constitutes the so-called wavelet multiresolution decomposition. Note that for a level J multiresolution analysis, the wavelet decomposition of the variable y consists of J wavelet details ($D_J(t)$, $D_{J-1}(t)$,..., $D_1(t)$) and a single wavelet smooth ($S_J(t)$). The wavelet smooth captures the low-frequency dynamics while the wavelet details represent the higher-frequency characteristics of y. The maximum number of scales that can be considered in the analysis is limited by the number of observations ($T \geq 2^J$).

7.3 Wavelet-based forecasting with factor-augmented models

In the conventional factor model representation, each variable is assumed to be the sum of two components, a common component, driven by a small number of latent common factors, and an idiosyncratic component. Let X_t be a N-dimensional stationary time series observed for $t = 1, \cdots, T$. Consider the static factor representation

$$X_t = \Lambda F_t + e_t \quad (t = 1, \cdots, T) \tag{72}$$

where F_t is a $(r \times 1)$ vector of non-observable factors, Λ is a $(N \times r)$ matrix of (unknown) loadings and e_t is a N-dimensional vector of the idiosyncratic components. When both $N \to \infty$ and $T \to \infty$, Stock and Watson (1998, 2002b), Bai and Ng (2002), Bai (2003) and Amengual and Watson (2007) have shown that, under slightly different sets of assumptions regarding the data generating processes of the factors and the idiosyncratic components[33], the first k principal components $\hat{F}^{(k)} = \left[\hat{F}_1 \cdots \hat{F}_k\right]$ span the factor space.

Suppose that one is interested in forecasting the value of a stationary (or previously stationarized) variable y for period $T+h$, y_{T+h}. The standard factor-augmented regression to forecast y_{T+h} is given by (see, for example, Stock and Watson (2002a))

$$y_{t+h} = \alpha_0 + \sum_{i=1}^{k} \alpha_i \hat{F}_{t,i} + \sum_{j=1}^{p} \gamma_j y_{t+1-j} + \varepsilon_{t+h} \quad (t = p, \cdots, T-h) \tag{73}$$

where the number of estimated factors k to be included in the forecasting equation can be determined by minimizing a modified version of the Bayesian information criteria (BIC) suggested by Stock and Watson (1998)[34], whereas the number of autoregressive terms p is usually chosen according to the standard BIC criterion. Through an extensive comparison of several methods for forecasting with many predictors, Stock and Watson (2005a) found that the factor-augmented model (73) performs best.

Instead of fitting a model to the variable y as a whole as done in the standard factor-augmented approach, what we propose here is to fit a model like (73) to each time-scale component of the wavelet multiresolution decomposition of y (see equation (22)). Then, a forecast for the variable y can be

[33] The typical assumptions allow for some heteroskedasticity and limited dependence of the idiosyncratic components in both the time and cross-section dimensions, as well as for moderate correlation between the latter and the factors.

[34] Alternatively, one can use, for example, the criteria proposed by Bai and Ng (2002).

obtained by aggregating the forecasts for the orthogonal components using the corresponding estimated models. As far as we know, this has never been done up to now.

Let us sketch in more detail the several steps involved. Firstly, a wavelet multiresolution decomposition is performed to the variable to be forecasted, y, as well as for all the N predictors, as described in section 2. As a result, one obtains $S_J^y(t)$, $D_J^y(t)$, $D_{J-1}^y(t)$, ..., $D_1^y(t)$ for variable y, $S_J^{x_1}(t)$, $D_J^{x_1}(t)$, $D_{J-1}^{x_1}(t)$, ..., $D_1^{x_1}(t)$ for the first predictor, $S_J^{x_2}(t)$, $D_J^{x_2}(t)$, $D_{J-1}^{x_2}(t)$, ..., $D_1^{x_2}(t)$ for the second predictor, and so on. Secondly, for each resolution level, the first principal components are computed from the corresponding components of the N predictors, after being, as usual, standardized. Then, a search for the the values of k and p that minimize the above mentioned metrics is performed, with the search done up to $k_{\max}$ and $p_{\max}$, which denote the maximum number of factors and autoregressive terms allowed in equation (73) respectively. Once a model like (73) has been estimated for each resolution level, it can be used to produce the h-step ahead forecast of the corresponding component of the variable y. Finally, the h-step ahead forecast for the variable y as a whole can be obtained by adding up those forecasts. Hence, this constitutes the wavelet approach for factor augmented forecasting, where a factor-augmented model is tailored to each time-scale component of y.

7.4 Forecasting GDP growth in the major euro area countries

In this section, the performance of the wavelet approach for factor-augmented forecasting is evaluated. In particular, we focus on the short-term forecasting of quarterly GDP growth in the major euro area countries namely, Germany, France, Italy and Spain.

7.4.1 Data

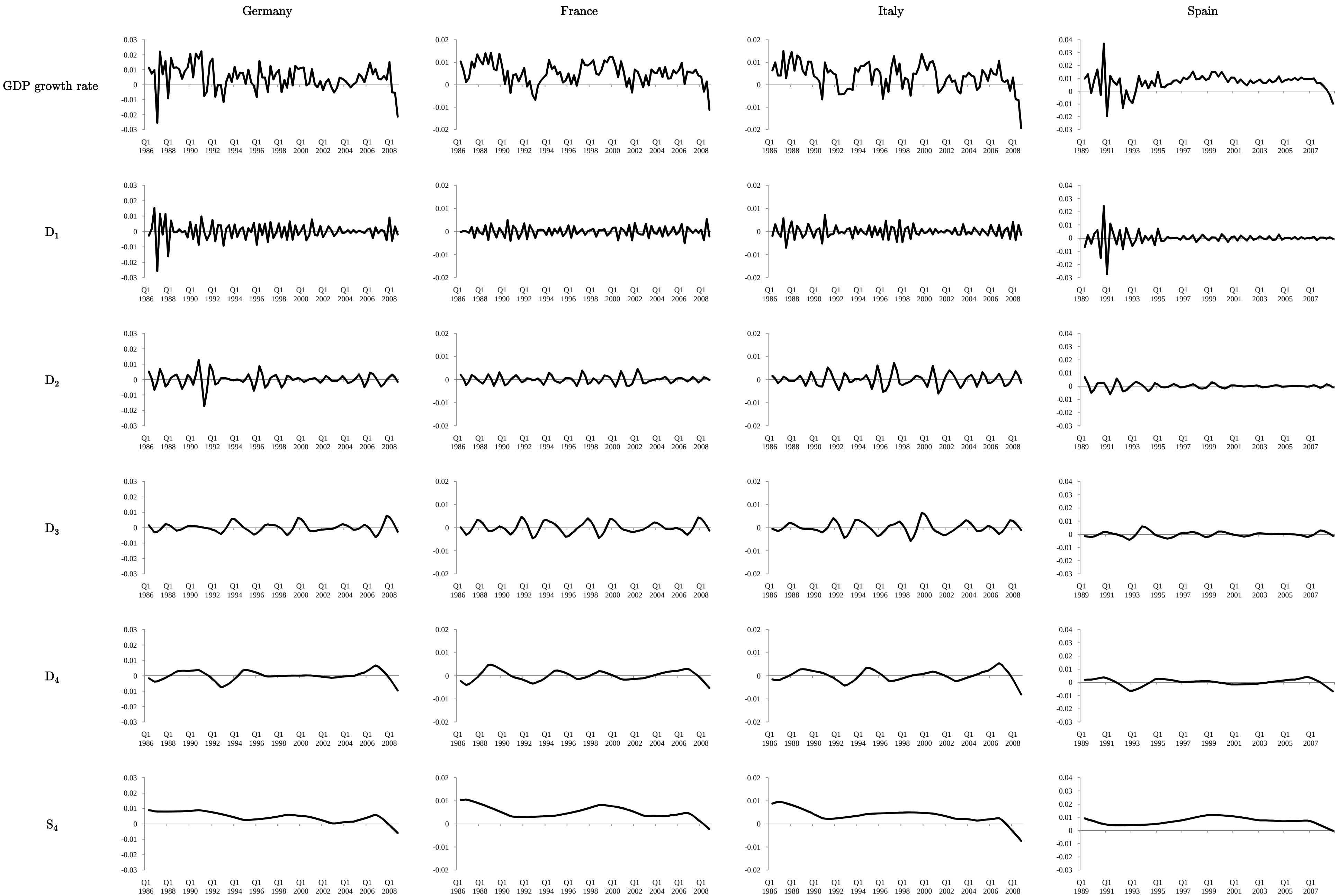
Resorting to the Thomson Financial Datastream database, which covers both international and national data sources, large panel sets of macroeconomic series were compiled for Germany, France, Italy and Spain. For each country, besides GDP series, it was collected a comprehensive panel data set including a wide range of variables, namely industrial production and sales, labour market variables, price series, monetary aggregates, business and consumer surveys, among others (corresponding to 76 series for Germany, 81 for France, 63 for Italy and 72 for Spain).[35] For all countries but Spain, the sample covers the period from the first quarter of 1986 up to the fourth quarter of 2008 while for Spain it starts on the first quarter of 1989. As usual, data are seasonally adjusted and transformed by taking logs and/or differences when necessary. Following Stock and Watson (2005b), outlier-adjusted series are used for the estimation of the factors[36].

7.4.2 Empirical results

In Figure 33, the wavelet multiresolution decomposition of quarterly real GDP growth is presented for all countries. Several comments are in order. Regarding the number of scales, taking into account the number of observations available for all countries and the out-of-sample period to be considered later on, we considered $J = 4$. Hence, the growth rate series is decomposed into four wavelet details (D_4, D_3, D_2, D_1) and a wavelet smooth (S_4). Note that the wavelet details and wavelet smooth form an additive decomposition (see equation (22)). That is, adding up the wavelet details and the wavelet smooth at each time t will result in the growth rate series at time t.

[35] See the Annex for the detailed list of series.

[36] The outlier adjustment corresponds to replacing observations of the transfomed series with absolute deviations larger than six times the interquartile range by the median value of the preceding five observations (see, for example Stock and Watson (2005b)).



Note: For each country, the GDP growth rate series is plotted in the top row while below it - from top to bottom - are the wavelet details D_1, D_2, D_3, D_4, and the wavelet smooth S_4.

Figure 1 - Wavelet multiresolution decomposition

The frequency interpretation of the multiresolution decomposition scale levels is the following. D_1 is associated with fluctuations between 2 and 4 quarters, D_2 is related with $4-8$ quarter dynamics, D_3 reflects $8-16$ quarter movements, D_4 captures $16 - 32$ quarter dynamics and S_4 reflects all the movements with periodicity above a $32-$ quarter period. As one can see from Figure 33, the wavelet smooth captures the low-frequency characteristics while the wavelet details reflect the higher-frequency dynamics.

Concerning the choice of the wavelet function for the multiresolution decomposition, the symmlet4 wavelet was used. The symmlet wavelet is commonly used in multiresolution analysis and a wavelet length of 4 has been argued to be an adequate choice for most macroeconomic data and when working with relatively short data sets as it is the case (see, for example, Crowley (2007)).[37]

To assess the performance of the wavelet-based forecasts with factor-augmented models an out-of-sample forecasting exercise is carried out. The out-of-sample period runs from the first quarter of 2004 up to the fourth quarter of 2008, corresponding to about one fourth of the sample period, which seems reasonable taking into account the dimension of the sample at hand. As usual, a recursive estimation process is implemented. This involves recursive factor estimation, parameter estimation, model selection, and so forth. Starting from the estimation period (up to the fourth quarter of 2003), in each round a new observation is added to the sample and the h-step ahead forecast is computed. In particular, we focus on short-term forecasting by considering one and two-quarter ahead forecasts, that is, $h = 1, 2$.[38]

[37] As a sensitivity analysis, we considered other wavelet families as, for example, daubechies and coiflets, as well as a wavelet length of 8. In general, the results do not change much.

[38] Higher forecast horizons were also investigated but, as found elsewhere for the euro area countries (see, for example, Runstler *et al.* (2009)), the forecasting gains of using factor-augmented models disappear when the forecast horizon increases. This is also found

For comparison, we consider other natural forecasting alternatives within our framework. Within the single resolution level approach, we consider, as usual, an autoregressive model as the benchmark and the factor-augmented model described earlier. That is, the variable y is forecasted as whole resorting to model (73) with $\alpha_i = 0$ for $i = 1, \cdots, k$ in the first case and without any restriction in the second case.[39] Within the wavelet multiresolution approach, besides using model (73) for forecasting each component, we compute wavelet-based forecasts by fitting an autoregressive model to each component. We set $k_{\max} = 6$ and $p_{\max} = 6$.

In Table 2, we present the mean squared error (MSE) for each of the forecasting models relative to the autoregressive benchmark. Several findings emerge. Focusing on $h = 1$, one can see that within the standard approach, factor-augmented models outperform the autoregressive benchmark in all countries but Italy. Note that there is some heterogeneity in the magnitude of the gains. The reduction of the relative MSE is 14 p.p. for Germany, only 3 p.p. for France and more than 65 p.p. for Spain. Within the wavelet approach, wavelet-based forecasts with autoregressive models outperform the benchmark in all countries but France. The reduction of the relative MSE is about 10 p.p. for Germany, more than 18 p.p. for Italy and 63 p.p. for Spain. However, the best forecasting results are obtained through the wavelet approach for factor-augmented forecasting. The wavelet-based forecasts with factor-augmented models outperform all the other methods for all countries. The reduction of the relative MSE is almost 22 p.p. for Germany, 17 p.p. for France, 33 p.p for Italy and more than 66 p.p. for Spain. Hence, there is

for the wavelet approach.

[39] Stock and Watson (2002a) considered the forecasting model (73) with and without the autoregressive terms (i.e. $\gamma_j = 0$ for $j = 1, \cdots, p$) and found that for forecasting real variables, the latter formulation performed, in general better. Hence, we assess the two variants for the standard and wavelet approaches while presenting the results only for the best one.

a noteworthy increase in the forecast accuracy when the wavelet approach is merged with factor-augmented models vis-à-vis all the other alternatives.

	Germany	France	Italy	Spain
h =1				
Standard approach				
AR model	1.000	1.000	1.000	1.000
Factor-augmented model	**0.860**	**0.970**	1.090	**0.347**
Wavelet approach				
AR model	**0.899**	1.295	**0.815**	**0.370**
Factor-augmented model	**0.783**	**0.831**	**0.673**	**0.335**
h = 2				
Standard approach				
AR model	1.000	1.000	1.000	1.000
Factor-augmented model	**0.995**	1.030	1.087	**0.963**
Wavelet approach				
AR model	1.277	**0.995**	**0.994**	**0.740**
Factor-augmented model	**0.934**	**0.934**	**0.974**	**0.665**

Note: Each entry of the table corresponds to the ratio between the MSE of each model and the MSE of the benchmark model (i.e. the standard approach AR model). The bold format corresponds to a value lower than one, that is, the model is better than the benchmark whereas the shaded area denotes the best performing model for each forecast horizon.

Table 2: Mean Squared Error (relative to the benchmark model)

When the forecast horizon increases to $h = 2$, one can see that the gains of the standard factor-augmented model almost disappear. Although wavelet-based forecasts with autoregressive models outperform the benchmark model in all countries but Germany, the reduction in the relative MSE is quite marginal (with Spain being an exception). Again, the wavelet approach for factor-augmented forecasting delivers the best forecasting results for all countries. However, the gains are quite smaller than the ones obtained for $h = 1$. The reduction of the relative MSE is almost 7 p.p. for Germany and France, only 3 p.p for Italy and more than 43 p.p. for Spain.

To assess the significance of the gains, we computed the well-known Granger-Newbold test (see, for example, Enders (2004)).[40] Suppose that e_{1t} and e_{2t} are sequences of forecast errors of models 1 and 2, respectively, of length H. Under the null hypothesis the models have equal forecast accuracy and the test statistic is given by

$$\frac{r_{xz}}{\sqrt{\left(1 - r_{xz}^2\right) / \left(H - 1\right)}} \sim t_{(H-1)} \tag{74}$$

where r_{xz} denotes the sample correlation coefficient between $x_t = e_{1t} + e_{2t}$ and $z_t = e_{1t} - e_{2t}$. If r_{xz} is positive and statistically different from zero, then model 1 has a larger MSE than model 2. If r_{xz} is negative and statistically different from zero, then model 2 has a larger MSE than model 1. In Table 3, we present the results for the Granger-Newbold test where model 1 is the benchmark model and model 2 corresponds to each of the other models considered. The results confirm the significance of the gains discussed earlier. When $h = 1$, the gains are significant for all countries using the wavelet approach for factor-augmented forecasting whereas when the forecast horizon increases, although there is an improvement, it is not enough to be considered statistically significant, except in the case of Spain.

[40] We also computed the Harvey *et al.* (1997) modified version of the Diebold and Mariano (1995) test but the results were inconclusive regarding the forecast accuracy of one model relative to another.

	Germany	France	Italy	Spain
h =1				
Standard approach				
AR model	-	-	-	-
Factor-augmented model	0.752	1.553 *	0.034	3.269 ***
Wavelet approach				
AR model	0.332	-1.879 **	0.395	4.246 ***
Factor-augmented model	1.561 *	1.400 *	2.383 **	3.968 ***
h = 2				
Standard approach				
AR model	-	-	-	-
Factor-augmented model	1.590 *	1.262	-0.321	0.010
Wavelet approach				
AR model	-1.184	-0.279	-0.345	2.759 ***
Factor-augmented model	0.549	-0.077	0.273	2.740 ***

Note: *, **, *** denote the rejection of the null hypothesis of equal forecast accuracy at a 10, 5 and 1 per cent significance level, respectively.

Table 3: Granger-Newbold test

In addition, we also perform a forecast encompassing analysis. In particular, we take the general specification approach proposed by Fair and Shiller (1989, 1990) and consider the following regression model

$$y_{t+h} = \alpha + \beta_1 \widehat{y}^1_{t+h} + \beta_2 \widehat{y}^2_{t+h} + u_{t+h} \tag{75}$$

where $\widehat{y}^1_{t+h}$ and $\widehat{y}^2_{t+h}$ are the h-step ahead forecasts of models 1 and 2, respectively.[41] If $\beta_1 \neq 0$ and $\beta_2 = 0$ model 1 forecast encompasses the

[41] Other approaches suggested in the literature consider particular cases of model (75). For example, Nelson (1972) and Granger and Newbold (1973) impose the restrictions

second while if $\beta_1 = 0$ and $\beta_2 \neq 0$ model 2 forecast encompasses the first. If both forecasts contain independent information then both β_1 and β_2 should be different from zero. In Table 4, we present the test statistic for β_i, $i = 1, 2$ under the null hypothesis $\beta_i = 0$. One can see that the results reinforce the above forecast accuracy evaluation. In fact, the forecast encompassing test results highlight the information content of the short-term forecasts provided by factor-augmented models, and in particular, through a wavelet approach. When $h = 1$, wavelet-based forecasts using factor-augmented models are statistically relevant for all countries while encompassing the benchmark in all countries but Spain. When $h = 2$, the usefulness of such forecasts is confirmed in the cases of Germany and Spain.

$\alpha = 0$ and $\beta_1 + \beta_2 = 1$, Chong and Hendry (1986) impose that $\alpha = 0$ and $\beta_1 = 1$ and Andrews *et al.* (1996) impose that $\beta_1 + \beta_2 = 1$.

	Germany		France		Italy		Spain	
	β_1	β_2	β_1	β_2	β_1	β_2	β_1	β_2
h =1								
Standard approach								
AR model	-	-	-	-	-	-	-	-
Factor-augmented model	0.026	2.31 **	-0.419	2.17 **	1.650	1.26	0.406	4.41 ***
Wavelet approach								
AR model	0.031	1.62	2.050 *	-1.36	1.330	-0.926	2.850 **	8.00 ***
Factor-augmented model	-0.884	2.6 **	-1.410	2.4 **	-1.330	2.73 **	3.310 ***	8.21 ***
h = 2								
Standard approach								
AR model	-	-	-	-	-	-	-	-
Factor-augmented model	-1.180	1.71	0.567	1.89 *	1.610	0.728	3.430 ***	1.03
Wavelet approach								
AR model	0.380	0.025	1.770 *	-0.799	2.68 **	-1.8 *	0.544	1.3
Factor-augmented model	-1.040	2.17 **	1.420	-0.481	1.460	-0.308	1.150	2.34 **

Note: *, **, *** denote the rejection of $\beta_i = 0$ at a 10, 5 and 1 per cent significance level, respectively.

Table 4: Forecast encompassing test

7.5 Conclusions

It has been acknowledged that multiresolution approaches can outperform the traditional single resolution approach for forecasting. In particular, through the wavelet multiresolution decomposition, a time series can be disentangled into different time-scale components and a model can be fitted to each component to improve the forecast accuracy of the series as a whole. Despite the potential usefulness of wavelets in forecasting, there are very few applications of the wavelet approach in economics. Moreover, the literature on forecasting with wavelets has mainly focused on univariate models.

Hence, extending the information set that is taken on board in the forecasting model seems to be a natural step in the development of the wavelet approach for forecasting. The aim of this essay is to bridge the wavelet approach and the recently developed literature on factor-augmented models. In a context of growing data availability, factor-augmented models have become quite popular in the literature and among practitioners as they can handle large panel data sets in a simple and parsimonious way. Furthermore, they have proved to be quite useful for forecasting purposes.

To assess the performance of the wavelet approach with factor-augmented models an out-of-sample forecasting exercise has been conducted. In particular, resorting to large data sets collected for the major euro area countries, we assessed the short-term forecasting of GDP growth. We found that merging the wavelet approach and factor-augmented models enhances, in a noteworthy magnitude, the performance of short-term forecasts. Moreover, this evidence is supported by forecast accuracy and encompassing tests and is cross-country based.

Annex

SERIES	Thomson Financial Datastream code
GERMANY	
BD PRODUCTION OF TOTAL INDUSTRY (EXCLUDING CONSTRUCTION) VOLA	BDOPRI35G
BD PRODUCTION IN TOTAL MANUFACTURING VOLA	BDOPRI38G
BD PRODUCTION OF TOTAL CONSTRUCTION VOLA	BDOPRI30G
BD PRODUCTION OF TOTAL MANUFACTURED INTERMEDIATE GOODS VOLA	BDOPRI61G
BD PRODUCTION OF TOTAL MANUFACTURED INVESTMENT GOODS VOLA	BDOPRI70G
BD ORDERS FOR TOTAL MANUFACTURED GOODS (VOLUME) VOLA	BDOODI45G
BD ORDERS FOR EXPORTED MANUFACTURED GOODS (VOLUME) VOLA	BDOODI54G
BD ORDERS FOR MANUFACTURED GOODS FROM DOM. MARKET (VOLUME) VOL	BDOODI53G
BD ORDERS FOR MANUFACTURED INTERMEDIATE GOODS (VOLUME) VOLA	BDOODI51G
BD ORDERS FOR MANUFACTURED INVESTMENT GOODS (VOLUME) VOLA	BDOODI52G
BD SALES OF MANUFACTURED INTERMEDIATE GOODS (VOLUME) VOLN	BDOSLI26H
BD SALES OF MANUFACTURED INVESTMENT GOODS (VOLUME) VOLN	BDOSLI27H
BD TOTAL WHOLESALE TRADE (VOLUME) VOLN	BDOSLI22H
BD TOTAL RETAIL TRADE (VOLUME) VOLA	BDOSLI15G
BD TOTAL CAR REGISTRATIONS VOLA	BDOSLI05O
BD PASSENGER CAR REGISTRATIONS SADJ	BDOSLI12E
BD PERMITS ISSUED FOR DWELLINGS VOLA	BDOODI15O
BD IMPORTS CIF CURA	BDOXT009B
BD EXPORTS FOB CURA	BDOXT003B
BD UNEMPLOYMENT: % CIVILIAN LABOUR(% DEPENDENT LABOUR TO DEC 196	BDUN%TOTQ
BD PERSONS IN EMPLOYMENT - MINING AND MANUFACTURINGVOLN	BDUUOA01P
BD UNFILLED VACANCIES VOLA	BDOOL015O
BD PPI - ALL ITEMS NADJ	BDOPP019F
BD PPI - MANUFACTURING INDUSTRY NADJ	BDOPP017F
BD PPI - FOOD, BEVERAGES & TOBACCO NADJ	BDOPP013F
BD PPI - INVESTMENT GOODS NADJ	BDOPP068F
BD PPI - INTERMEDIATE GOODS NADJ	BDOPP064F
BD WPI NADJ	BDOWP005F
BD CPI -HOUSING RENTAL SERVICES NADJ	BDOCP053F
BD CPI - ENERGY (EXCL. GASOLINE BEFORE 1991) NADJ	BDOCP041F
BD CPI - EXCLUDING FOOD & ENERGY NADJ	BDOCP042F
BD CPI - FOOD AND ALCOHOL-FREE DRINKS (EXCL. REST)NADJ	BDOCP019F
BD CPI NADJ	BDOCP009F
BD EXPORT PRICE INDEX SADJ	BDEXPPRCE
BD IMPORT PRICE INDEX SADJ	BDIMPPRCE
BD MONEY SUPPLY-GERMAN CONTRIBUTION TO EURO M1(PAN BD M0690)	BDM1....A
BD MONEY SUPPLY - M2 (CONTINUOUS SERIES) CURA	BDM2C...B
BD MONEY SUPPLY - M3 (CONTINUOUS SERIES) CURA	BDM3C...B
BD FIBOR - 3 MONTH (MTH.AVG.)	BDINTER3
BD YIELD 10-YEAR GOVT.BONDS(PROXY- 9-10+ YEAR FEDERAL SECUR NADJ	BDOIR080R
BD SHARE PRICES - CDAX NADJ	BDOSP001F
BD GERMAN MARKS TO US$ (MTH.AVG.)	BDXRUSD.
UK MARKET PRICE - UK BRENT CURN	UKI76AAZA
BD ECONOMIC SENTIMENT INDICATOR - GERMANY SADJ	BDEUSESIG
BD CONSTRUCTION CONFIDENCE INDICATOR - GERMANY SADJ	BDEUSBCIQ
BD CONSTRUCTION SURVEY: ACT.COMPARED TO LAST MONTH-GERMANY SADJ	BDEUSBACQ
BD CONSTRUCTION SURVEY: EMPLOYMENT EXPECTATIONS - GERMANY SADJ	BDEUSBEMQ
BD CONSTRUCTION SURVEY: ORDER BOOK POSITION - GERMANY SADJ	BDEUSBOBQ
BD CONSTRUCTION SURVEY: PRICE EXPECTATIONS - GERMANY SADJ	BDEUSBPRQ
BD CONSUMER CONFIDENCE INDICATOR - GERMANY SADJ	BDEUSCCIQ
BD CONSUMER SURVEY: ECONOMIC SITUATION LAST 12 MTH-GERMANY SADJ	BDEUSCECQ
BD CONSUMER SURVEY: ECONOMIC SITUATION NEXT 12 MTH-GERMANY SADJ	BDEUSCEYQ
BD CONSUMER SURVEY: FINANCIAL SITUATION LAST 12 MTH-GERMANY SADJ	BDEUSCFNQ
BD CONSUMER SURVEY: FINANCIAL SITUATION NEXT 12 MTH-GERMANY SADJ	BDEUSCFYQ
BD CONSUMER SURVEY: MAJOR PURCH.OVER NEXT 12 MONTHS-GERMANY SAD	BDEUSCPCQ
BD CONSUMER SURVEY: MAJOR PURCHASES AT PRESENT - GERMANY SADJ	BDEUSCMPQ
BD CONSUMER SURVEY: PRICES LAST 12 MONTHS - GERMANYSADJ	BDEUSCPRQ
BD CONSUMER SURVEY: PRICES NEXT 12 MONTHS - GERMANYSADJ	BDEUSCPYQ
BD CONSUMER SURVEY: SAVINGS AT PRESENT - GERMANY SADJ	BDEUSCSAQ
BD CONSUMER SURVEY: SAVINGS OVER NEXT 12 MONTHS - GERMANY SADJ	BDEUSCSYQ

BD CONSUMER SURVEY: STATEMENT ON FIN.SITUATION OF HOUSEHOLD SADJ	BDEUSCFHQ
BD CONSUMER SURVEY: UNEMPLOYMENT NEXT 12 MONTHS - GERMANY SADJ	BDEUSCUNQ
BD INDUSTRIAL CONFIDENCE INDICATOR - GERMANY SADJ	BDEUSICIQ
BD INDUSTRY SURVEY: EMP.EXPECTATIONS FOR MO.AHEAD -GERMANY SADJ	BDEUSIEMQ
BD INDUSTRY SURVEY: EXPORT ORDER BOOK POSITION - GERMANY SADJ	BDEUSIEBQ
BD INDUSTRY SURVEY: ORDER BOOK POSITION - GERMANY SADJ	BDEUSIOBQ
BD INDUSTRY SURVEY: PROD.EXPECTATION FOR MTH.AHEAD-GERMANY SADJ	BDEUSIPAQ
BD INDUSTRY SURVEY: PRODN. TRENDS IN RECENT MTH. - GERMANY SADJ	BDEUSIPRQ
BD INDUSTRY SURVEY: SELLING PRC.EXPECT.MTH. AHEAD -GERMANY SADJ	BDEUSISPQ
BD INDUSTRY SURVEY: STOCKS OF FINISHED GOODS - GERMANY SADJ	BDEUSIFPQ
BD RETAIL CONFIDENCE INDICATOR - GERMANY SADJ	BDEUSRCIQ
BD RETAIL SURVEY: CURRENT BUSINESS SITUATION - GERMANY SADJ	BDEUSRPBQ
BD RETAIL SURVEY: EMPLOYMENT - GERMANY SADJ	BDEUSREMQ
BD RETAIL SURVEY: FUTURE BUSINESS SITUATION - GERMANY SADJ	BDEUSREBQ
BD RETAIL SURVEY: ORDERS PLACED WITH SUPPLIERS - GERMANY SADJ	BDEUSROSQ
BD RETAIL SURVEY: STOCKS - GERMANY SADJ	BDEUSRSTQ
FRANCE	
FR PRODUCTION OF TOTAL INDUSTRY (EXCLUDING CONSTRUCTION) VOLA	FROPRI35G
FR PRODUCTION IN TOTAL MANUFACTURING VOLA	FROPRI38G
FR PRODUCTION OF TOTAL MANUFACTURED CONSUMER GOODS VOLA	FROPRI49G
FR PRODUCTION OF TOTAL MANUFACTURED INTERMEDIATE GOODS VOLA	FROPRI61G
FR PRODUCTION OF TOTAL MANUFACTURED INVESTMENT GOODS VOLA	FROPRI70G
FR PRODUCTION OF TOTAL ENERGY VOLA	FROPRI44G
FR PRODUCTION IN TOTAL AGRICULTURE VOLA	FROPRI47G
FR PRODUCTION OF TOTAL CONSTRUCTION VOLA	FROPRI30G
FR PRODUCTION OF TOTAL VEHICLES VOLA	FROPRI58G
FR PERMITS ISSUED FOR DWELLINGS VOLA	FROODI15O
FR WORK STARTED FOR DWELLINGS VOLA	FROWSI41O
FR TOTAL RETAIL TRADE (VOLUME) VOLA	FROSLI15G
FR HOUSEHOLD CONSUMPTION - MANUFACTURED GOODS CONA	FRHCONMGD
FR HOUSEHOLD CONSUMPTION - MANUFACTURED GOODS, RETAIL GOODS CONA	FRHCONMCD
FR HOUSEHOLD CONSUMPTION - AUTOMOBILES CONA	FRHCONAUD
FR HOUSEHOLD CONSUMPTION - DURABLE GOODS CONA	FRHCONDGD
FR HOUSEHOLD CONSUMPTION - TEXTILES & LEATHER CONA	FRHCONTLD
FR HOUSEHOLD CONSUMPTION - OTHER MANUFACTURED GOODSCONA	FRHCONOTD
FR HOUSEHOLD CONSUMPTION - FURNITURE CONA	FRHCONFND
FR HOUSEHOLD CONSUMPTION - HOUSEHOLD APPLIANCES CONA	FRHCONHAD
FR HOUSEHOLD CONSUMPTION - ELECTRICAL GOODS CONA	FRHCONELD
FR PASSENGER CAR REGISTRATIONS SADJ	FROSLI12E
FR TOTAL CAR REGISTRATIONS VOLA	FROSLI05O
FR IMPORTS FOB CURA	FROXT009B
FR EXPORTS FOB CURA	FROXT003B
FR UNEMPLOYMENT VOLA	FROUN010O
FR NEW UNEMPLOYMENT CLAIMS SADJ	FROUN007G
FR UNEMPLOYMENT RATE (% OF TOTAL LABOUR FORCE) SADJ	FROUN015Q
FR NEW JOB VACANCIES FULL & PART-TIME REGISTERED DURING MONTH	FRVACTOTO
FR PPI - AGRICULTURAL GOODS NADJ	FROPP004F
FR PPI - INTERMEDIATE GOODS EXCLUDING ENERGY NADJ	FROPP065F
FR PPI - CHEMICALS NADJ	FROPP054F
FR PPI - METAL PRODUCTS NADJ	FROPP037F
FR PPI - MANUFACTURED PRODUCTS NADJ	FROPP017F
FR CPI NADJ	FROCP009F
FR CPI - FOOD NADJ	FROCP019F
FR CPI - ENERGY NADJ	FROCP041F
FR CPI - EXCLUDING FOOD & ENERGY NADJ	FROCP042F
FR CPI - RENT NADJ	FROCP054F
FR CPI - SERVICES EXCLUDING RENT NADJ	FROCP064F
FR MONEY SUPPLY - M1 (NATIONAL CONTRIBUTION TO M1) CURN	FRM1....A
FR MONEY SUPPLY - M2 (NATIONAL CONTRIBUTION TO M2) CURN	FRM2....A
FR MONEY SUPPLY - M3 (NATIONAL CONTRIBUTION TO M3) CURN	FRM3....A
FR PIBOR / EURIBOR - 3-MONTH (MTH.AVG.)	FRINTER3
FR YIELD 10-YEAR GOVERNMENT BENCHMARK BONDS NADJ	FROIR080R

FR SHARE PRICES - SBF 250 NADJ	FROSP001F
FR FRENCH FRANC TO US $	FRXRUSD.
UK MARKET PRICE - UK BRENT CURN	UKI76AAZA
FR ECONOMIC SENTIMENT INDICATOR - FRANCE SADJ	FREUSESIG
FR CONSTRUCTION CONFIDENCE INDICATOR - FRANCE SADJ	FREUSBCIQ
FR CONSTRUCTION SURVEY: ACT.COMPARED TO LAST MONTH - FRANCE SADJ	FREUSBACQ
FR CONSTRUCTION SURVEY: EMPLOYMENT EXPECTATIONS - FRANCE SADJ	FREUSBEMQ
FR CONSTRUCTION SURVEY: ORDER BOOK POSITION - FRANCE SADJ	FREUSBOBQ
FR CONSTRUCTION SURVEY: PRICE EXPECTATIONS - FRANCESADJ	FREUSBPRQ
FR CONSUMER CONFIDENCE INDICATOR - FRANCE SADJ	FREUSCCIQ
FR CONSUMER SURVEY: ECONOMIC SITUATION LAST 12 MTH.- FRANCE SADJ	FREUSCECQ
FR CONSUMER SURVEY: ECONOMIC SITUATION NEXT 12 MTH.- FRANCE SADJ	FREUSCEYQ
FR CONSUMER SURVEY: FINANCIAL SITUATION LAST 12 MTH- FRANCE SADJ	FREUSCFNQ
FR CONSUMER SURVEY: FINANCIAL SITUATION NEXT 12 MTH- FRANCE SADJ	FREUSCFYQ
FR CONSUMER SURVEY: MAJOR PURCH.OVER NEXT 12 MONTHS- FRANCE SADJ	FREUSCPCQ
FR CONSUMER SURVEY: MAJOR PURCHASES AT PRESENT - FRANCE SADJ	FREUSCMPQ
FR CONSUMER SURVEY: PRICES LAST 12 MONTHS - FRANCE SADJ	FREUSCPRQ
FR CONSUMER SURVEY: PRICES NEXT 12 MONTHS - FRANCE SADJ	FREUSCPYQ
FR CONSUMER SURVEY: SAVINGS AT PRESENT - FRANCE SADJ	FREUSCSAQ
FR CONSUMER SURVEY: SAVINGS OVER NEXT 12 MONTHS - FRANCE SADJ	FREUSCSYQ
FR CONSUMER SURVEY: STATEMENT ON FIN.SITUATION OF HOUSEHOLD SADJ	FREUSCFHQ
FR CONSUMER SURVEY: UNEMPLOYMENT NEXT 12 MONTHS - FRANCE SADJ	FREUSCUNQ
FR INDUSTRIAL CONFIDENCE INDICATOR - FRANCE SADJ	FREUSICIQ
FR INDUSTRY SURVEY: EMP.EXPECTATIONS FOR MO. AHEAD - FRANCE SADJ	FREUSIEMQ
FR INDUSTRY SURVEY: EXPORT ORDER BOOK POSITION - FRANCE SADJ	FREUSIEBQ
FR INDUSTRY SURVEY: ORDER BOOK POSITION - FRANCE SADJ	FREUSIOBQ
FR INDUSTRY SURVEY: PROD.EXPECTATION FOR MTH.AHEAD - FRANCE SADJ	FREUSIPAQ
FR INDUSTRY SURVEY: PRODN. TRENDS IN RECENT MTH. - FRANCE SADJ	FREUSIPRQ
FR INDUSTRY SURVEY: SELLING PRC.EXPECT. MTH. AHEAD - FRANCE SADJ	FREUSISPQ
FR INDUSTRY SURVEY: STOCKS OF FINISHED GOODS - FRANCE SADJ	FREUSIFPQ
FR RETAIL CONFIDENCE INDICATOR - FRANCE SADJ	FREUSRCIQ
FR RETAIL SURVEY: CURRENT BUSINESS SITUATION - FRANCE SADJ	FREUSRPBQ
FR RETAIL SURVEY: EMPLOYMENT - FRANCE SADJ	FREUSREMQ
FR RETAIL SURVEY: FUTURE BUSINESS SITUATION - FRANCE SADJ	FREUSREBQ
FR RETAIL SURVEY: ORDERS PLACED WITH SUPPLIERS - FRANCE SADJ	FREUSROSQ
FR RETAIL SURVEY: STOCKS - FRANCE SADJ	FREUSRSTQ

ITALY

IT PRODUCTION OF TOTAL INDUSTRY (EXCLUDING CONSTRUCTION) VOLA	ITOPRI35G
IT PRODUCTION OF TOTAL MANUFACTURED CONSUMER GOODS VOLA	ITOPRI49G
IT PRODUCTION OF TOTAL MANUFACTURED INTERMEDIATE GOODS VOLA	ITOPRI61G
IT PRODUCTION OF TOTAL MANUFACTURED INVESTMENT GOODS VOLA	ITOPRI70G
IT SALES OF TOTAL MANUFACTURED GOODS (VALUE) NADJ	ITOSLI09F
IT SALES OF TOTAL MANUFACTURED CONSUMER GOODS (VALUE) NADJ	ITOSLI61F
IT SALES OF MANUFACTURED INTERMEDIATE GOODS (VALUE)NADJ	ITOSLI64F
IT SALES OF MANUFACTURED INVESTMENT GOODS (VALUE) NADJ	ITOSLI65F
IT ORDERS FOR TOTAL MANUFACTURED GOODS (VALUE) SADJ	ITOODI32E
IT TOTAL RETAIL TRADE (VOLUME) VOLA	ITOSLI15G
IT TOTAL CAR REGISTRATIONS VOLA	ITOSLI05O
IT PASSENGER CAR REGISTRATIONS SADJ	ITOSLI12E
IT IMPORTS CIF CURA	ITOXT009B
IT EXPORTS FOB CURA	ITOXT003B
IT STANDARDIZED UNEMPLOYMENT RATE SADJ	ITOUN014Q
IT PPI NADJ	ITOPP019F
IT CPI NADJ	ITOCP009F
IT CPI - FOOD NADJ	ITOCP019F
IT CPI - ENERGY NADJ	ITOCP041F
IT CPI - EXCLUDING FOOD & ENERGY NADJ	ITOCP042F
IT CPI - SERVICES LESS HOUSING NADJ	ITOCP064F
IT CPI - HOUSING NADJ	ITOCP057F
IT EXPORT UNIT VALUE INDEX NADJ	ITEXPPRCF
IT MONEY SUPPLY: M1 - ITALIAN CONTRIBUTION TO THE EURO AREA CURN	ITM1....A
IT MONEY SUPPLY: M2 - ITALIAN CONTRIBUTION TO THE EURO AREA CURN	ITM2....A

IT MONEY SUPPLY: M3 - ITALIAN CONTRIBUTION TO THE EURO AREA CURN	ITM3....A
IT TREASURY BOND NET YIELD -SECONDARY MKT. (EP)	ITGBOND.
IT SHARE PRICES - ISE MIB STORICO NADJ	ITOSP001F
IT ITALIAN LIRE TO US $ (MTH.AVG.)	ITXRUSD.
UK MARKET PRICE - UK BRENT CURN	UKI76AAZA
IT ECONOMIC SENTIMENT INDICATOR - ITALY SADJ	ITEUSESIG
IT CONSTRUCTION CONFIDENCE INDICATOR - ITALY SADJ	ITEUSBCIQ
IT CONSTRUCTION SURVEY: ACT.COMPARED TO LAST MONTH - ITALY SADJ	ITEUSBACQ
IT CONSTRUCTION SURVEY: EMPLOYMENT EXPECTATIONS - ITALY SADJ	ITEUSBEMQ
IT CONSTRUCTION SURVEY: ORDER BOOK POSITION - ITALYSADJ	ITEUSBOBQ
IT CONSTRUCTION SURVEY: PRICE EXPECTATIONS - ITALY SADJ	ITEUSBPRQ
IT CONSUMER CONFIDENCE INDICATOR - ITALY SADJ	ITEUSCCIQ
IT CONSUMER SURVEY: ECONOMIC SITUATION LAST 12 MTH.- ITALY SADJ	ITEUSCECQ
IT CONSUMER SURVEY: ECONOMIC SITUATION NEXT 12 MTH.- ITALY SADJ	ITEUSCEYQ
IT CONSUMER SURVEY: FINANCIAL SITUATION LAST 12 MTH.- ITALY SADJ	ITEUSCFNQ
IT CONSUMER SURVEY: FINANCIAL SITUATION NEXT 12 MTH.- ITALY SADJ	ITEUSCFYQ
IT CONSUMER SURVEY: MAJOR PURCH.OVER NEXT 12 MONTHS- ITALY SADJ	ITEUSCPCQ
IT CONSUMER SURVEY: MAJOR PURCHASES AT PRESENT - ITALY SADJ	ITEUSCMPQ
IT CONSUMER SURVEY: PRICES LAST 12 MONTHS - ITALY SADJ	ITEUSCPRQ
IT CONSUMER SURVEY: PRICES NEXT 12 MONTHS - ITALY SADJ	ITEUSCPYQ
IT CONSUMER SURVEY: SAVINGS AT PRESENT - ITALY SADJ	ITEUSCSAQ
IT CONSUMER SURVEY: SAVINGS OVER NEXT 12 MONTHS - ITALY SADJ	ITEUSCSYQ
IT CONSUMER SURVEY: STATEMENT ON FIN.SITUATION OF HOUSEHOLD SADJ	ITEUSCFHQ
IT CONSUMER SURVEY: UNEMPLOYMENT NEXT 12 MONTHS - ITALY SADJ	ITEUSCUNQ
IT INDUSTRIAL CONFIDENCE INDICATOR - ITALY SADJ	ITEUSICIQ
IT INDUSTRY SURVEY: EMP. EXPECTATIONS FOR MO. AHEAD- ITALY SADJ	ITEUSIEMQ
IT INDUSTRY SURVEY: EXPORT ORDER BOOK POSITION - ITALY SADJ	ITEUSIEBQ
IT INDUSTRY SURVEY: ORDER BOOK POSITION - ITALY SADJ	ITEUSIOBQ
IT INDUSTRY SURVEY: PROD.EXPECTATION FOR MTH. AHEAD- ITALY SADJ	ITEUSIPAQ
IT INDUSTRY SURVEY: PRODN. TRENDS IN RECENT MTH. - ITALY SADJ	ITEUSIPRQ
IT INDUSTRY SURVEY: SELLING PRC. EXPECT. MTH. AHEAD- ITALY SADJ	ITEUSISPQ
IT INDUSTRY SURVEY: STOCKS OF FINISHED GOODS - ITALY SADJ	ITEUSIFPQ
IT RETAIL CONFIDENCE INDICATOR - ITALY SADJ	ITEUSRCIQ
IT RETAIL SURVEY: CURRENT BUSINESS SITUATION - ITALY SADJ	ITEUSRPBQ
IT RETAIL SURVEY: EMPLOYMENT - ITALY SADJ	ITEUSREMQ
IT RETAIL SURVEY: FUTURE BUSINESS SITUATION - ITALYSADJ	ITEUSREBQ
IT RETAIL SURVEY: ORDERS PLACED WITH SUPPLIERS - ITALY SADJ	ITEUSROSQ
IT RETAIL SURVEY: STOCKS - ITALY SADJ	ITEUSRSTQ
SPAIN	
ES PRODUCTION OF TOTAL INDUSTRY (EXCLUDING CONSTRUCTION) VOLA	ESOPRI35G
ES PRODUCTION IN TOTAL MANUFACTURING VOLA	ESOPRI38G
ES PRODUCTION IN TOTAL MINING VOLN	ESOPRI36H
ES PRODUCTION OF TOTAL MANUFACTURED CONSUMER GOODS VOLN	ESOPRI49H
ES PRODUCTION OF TOTAL MANUFACTURED INTERMEDIATE GOODS VOLN	ESOPRI61H
ES PRODUCTION OF TOTAL MANUFACTURED INVESTMENT GOODS VOLN	ESOPRI70H
ES PRODUCTION OF CEMENT VOLA	ESOPRI01O
ES PRODUCTION OF ACCOMMODATION: NIGHTS IN HOTEL VOLA	ESOPRI21O
ES PASSENGER CAR REGISTRATIONS VOLA	ESOSLI12O
ES CONSUMPTION: PETROL - CARS (VOLA) VOLA	ESPCA313O
ES CONSUMPTION: DIESEL OIL (VOLA) VOLA	ESOIL562O
ES ELECTRICITY CONSUMPTION (VOLA) VOLA	ESECO312O
ES ELECTRICITY CONSUMPTION - INDUSTRIAL SECTOR (VOLA) VOLA	ESELE629G
ES CONSUMPTION: VISIBLE - CEMENT (VOLA) VOLA	ESCEM301O
ES IMPORTS CIF CURA	ESOXT009B
ES EXPORTS FOB CURA	ESOXT003B
ES STANDARDIZED UNEMPLOYMENT RATE SADJ	ESOUN014Q
ES PPI NADJ	ESOPP019F
ES PPI - AGRICULTURAL PRODUCTS NADJ	ESOPP004F
ES PPI - MANUFACTURING ALL ITEMS NADJ	ESOPP017F
ES PPI - INTERMEDIATE GOODS NADJ	ESOPP064F
ES PPI - CONSUMER GOODS NADJ	ESOPP062F
ES PPI - INVESTMENT GOODS NADJ	ESOPP068F

ES PPI - ENERGY NADJ	ESOPP022F
ES CPI NADJ	ESOCP009F
ES CPI - ENERGY NADJ	ESOCP041F
ES CPI - EXCLUDING FOOD & ENERGY NADJ	ESOCP042F
ES CPI - SERVICES EXCLUDING RENT NADJ	ESOCP064F
ES CPI - RENT NADJ	ESOCP057F
ES CONSTRUCTION COST INDEX NADJ	ESOOP005F
ES EXPORT UNIT VALUE INDEX NADJ	ESEXPPRCF
ES IMPORT UNIT VALUE INDEX NADJ	ESIMPPRCF
ES MONEY SUPPLY: M2 - SPANISH CONTRIBUTION TO EURO M2 CURN	ESM2....A
ES MONEY SUPPLY: M3 - SPANISH CONTRIBUTION TO EURO M3 CURN	ESM3....A
ES INTERBANK RATE - 3 MONTH (WEIGHTED AVERAGE, EP)	ESINTER3
ES YIELD 10-YEAR GOVERNMENT BONDS NADJ	ESOIR080R
ES SHARE PRICES - MSE GENERAL INDEX NADJ	ESOSP001F
ES SPANISH PESETAS TO US $ (MTH.AVG.)	ESXRUSD.
UK MARKET PRICE - UK BRENT CURN	UKI76AAZA
ES ECONOMIC SENTIMENT INDICATOR - SPAIN SADJ	ESEUSESIG
ES CONSTRUCTION CONFIDENCE INDICATOR - SPAIN SADJ	ESEUSBCIQ
ES CONSTRUCTION SURVEY: ACT.COMPARED TO LAST MONTH - SPAIN SADJ	ESEUSBACQ
ES CONSTRUCTION SURVEY: EMPLOYMENT EXPECTATIONS - SPAIN SADJ	ESEUSBEMQ
ES CONSTRUCTION SURVEY: ORDER BOOK POSITION - SPAINSADJ	ESEUSBOBQ
ES CONSTRUCTION SURVEY: PRICE EXPECTATIONS - SPAIN SADJ	ESEUSBPRQ
ES CONSUMER CONFIDENCE INDICATOR - SPAIN SADJ	ESEUSCCIQ
ES CONSUMER SURVEY: ECONOMIC SITUATION LAST 12 MTH.- SPAIN SADJ	ESEUSCECQ
ES CONSUMER SURVEY: ECONOMIC SITUATION NEXT 12 MTH.- SPAIN SADJ	ESEUSCEYQ
ES CONSUMER SURVEY: FINANCIAL SITUATION LAST 12 MTH.- SPAIN SADJ	ESEUSCFNQ
ES CONSUMER SURVEY: FINANCIAL SITUATION NEXT 12 MTH.- SPAIN SADJ	ESEUSCFYQ
ES CONSUMER SURVEY: MAJOR PURCH.OVER NEXT 12 MONTHS- SPAIN SADJ	ESEUSCPCQ
ES CONSUMER SURVEY: MAJOR PURCHASES AT PRESENT - SPAIN SADJ	ESEUSCMPQ
ES CONSUMER SURVEY: PRICES LAST 12 MONTHS - SPAIN SADJ	ESEUSCPRQ
ES CONSUMER SURVEY: PRICES NEXT 12 MONTHS - SPAIN SADJ	ESEUSCPYQ
ES CONSUMER SURVEY: SAVINGS AT PRESENT - SPAIN SADJ	ESEUSCSAQ
ES CONSUMER SURVEY: SAVINGS OVER NEXT 12 MONTHS - SPAIN SADJ	ESEUSCSYQ
ES CONSUMER SURVEY: STATEMENT ON FIN.SITUATION OF HOUSEHOLD SADJ	ESEUSCFHQ
ES CONSUMER SURVEY: UNEMPLOYMENT NEXT 12 MONTHS - SPAIN SADJ	ESEUSCUNQ
ES INDUSTRIAL CONFIDENCE INDICATOR - SPAIN SADJ	ESEUSICIQ
ES INDUSTRY SURVEY: EMP. EXPECTATIONS FOR MO. AHEAD- SPAIN SADJ	ESEUSIEMQ
ES INDUSTRY SURVEY: EXPORT ORDER BOOK POSITION - SPAIN SADJ	ESEUSIEBQ
ES INDUSTRY SURVEY: ORDER BOOK POSITION - SPAIN SADJ	ESEUSIOBQ
ES INDUSTRY SURVEY: PROD.EXPECTATION FOR MTH. AHEAD- SPAIN SADJ	ESEUSIPAQ
ES INDUSTRY SURVEY: PRODN. TRENDS IN RECENT MTH. - SPAIN SADJ	ESEUSIPRQ
ES INDUSTRY SURVEY: SELLING PRC. EXPECT. MTH. AHEAD- SPAIN SADJ	ESEUSISPQ
ES INDUSTRY SURVEY: STOCKS OF FINISHED GOODS - SPAIN SADJ	ESEUSIFPQ
ES RETAIL CONFIDENCE INDICATOR - SPAIN SADJ	ESEUSRCIQ
ES RETAIL SURVEY: CURRENT BUSINESS SITUATION - SPAIN SADJ	ESEUSRPBQ
ES RETAIL SURVEY: EMPLOYMENT - SPAIN SADJ	ESEUSREMQ
ES RETAIL SURVEY: FUTURE BUSINESS SITUATION - SPAINSADJ	ESEUSREBQ
ES RETAIL SURVEY: ORDERS PLACED WITH SUPPLIERS - SPAIN SADJ	ESEUSROSQ
ES RETAIL SURVEY: STOCKS - SPAIN SADJ	ESEUSRSTQ

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