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An Arbitrage Model for the Stock Price Adjustment in the Dividend Period

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# An Arbitrage Model for the Stock Price Adjustment in the Dividend Period

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## Abstract

Following a dividend distribution, investors expect the stock price to decrease on the ex-dividend day. With no market imperfections, the price decrease should exactly match the amount of the dividend, thus eliminating all opportunities for profitable arbitrage. Allowing for different taxes on dividends and on capital gains results in a stock price adjustment ratio different from one, but there is still a unique equilibrium. With a simple model, considering four types of investors, we show that the consideration of transaction costs results in multiple possible equilibria (equilibrium zone), defined by the arbitrage boundaries of each type of investors. We also show that trading activity by the different types of investors is reflected in abnormal trading volume.

Key words: Dividend; Arbitrage; Market equilibrium; Transactions costs; Taxes

JEL classification: G12; G14

#### Introduction

The model developed in this paper analyzes the equilibrium conditions for the stock price adjustment after a dividend distribution, in the context of imperfect markets, considering the existence of taxes and transaction costs, and assuming that arbitrage activity leads prices to equilibrium. This arbitrage model is inspired by the pioneering work of Elton and Gruber (1970) as well as other authors including Kalay (1982), Eades Hess and Kim (1984), Lakonishok and Vermaelen (1986) and Michaely (1991).

An investor that sells his stock before the payment of the dividend looses the right to receive that dividend. If he sells his stock after the payment of the dividend, he receives the dividend but, under arbitrage, he expects the stock price to decrease to a level that would make him indifferent to sell before or after the dividend distribution. With no market imperfections, the price decrease would exactly match the amount of the dividend. If we allow for taxes and transaction costs in a market with rational arbitrage, the price decrease after the dividend event should reflect the relative taxation of dividends and capital gains, as well as the costs inherent to stock transactions.

Several papers study the effect of dynamic trading strategies around the ex-dividend day. These strategies imply that investors trade around the ex-dividend day in order to avoid or to capture the dividend, depending on their preferences for dividends or capital gains. Kalay (1982) argued that, without risk or transaction costs, investors with the same taxes on dividends and capital gains would buy the stock cum-dividend and sell ex-dividend, forcing the stock price down by the amount of the dividend. This is consistent with the findings of Elton and Gruber (1970) because the marginal investors in the stock would be investors facing the same taxes on dividends and capital gains. Of course, the income tax of these arbitrageurs cannot be inferred, as proposed by Elton and Gruber (1970). Kalay (1982)

recognizes that transaction costs should be taken into consideration with the implication that the price adjustment ratio would no longer be constrained to being equal to one. The author argues that only within the boundaries defined by transaction costs would it be possible to infer the marginal investor's income tax as beyond those limits the price change would be affected by arbitrage from investors seeking to take advantage of the opportunity to obtain excessive returns. Only within the boundaries defined by (3), there are no arbitrage opportunities. Miller and Scholes (1982) presented a similar argument in which the detected relationship between dividend yield and price change subsequent to a dividend distribution can be explained by short term trading as an alternative to the clientele effect explanation.

Eades et al. (1984) studied the behavior of prices around the ex-dividend day and showed the existence of abnormal returns on days other than the ex-day. This is contrary to the tax-induced clientele hypothesis as the tax effect explains only the price change on the exday and not the behavior of prices around the ex-day. The results of Kalay (1982) are consistent with the findings of Karpoff and Walking (1988) who detect a significant relationship between ex-day returns and transaction costs. Boyd and Jagannathan (1994) develop a model with different types of investors facing different transaction costs before showing how the relationship between the ex-day price change and dividend-yield is nonlinear. Lakonishok and Vermaelen (1986) confirm the presence of short-term traders in the market around the ex-dividend day, detectable because of high or abnormal volumes. These are more pronounced in high-yield stocks and Michaely and Vila (1995) set out an inverse relation between transaction costs and abnormal volume. The evidence of high or abnormal volumes around the ex-day is contrary to the static clientele models. Naranjo et al. (1998) reexamined and extended the work of Eades et al. (1984) and find that the high-yield stock exday returns are highly influenced by corporate dividend capture. Other insightful studies include Kadapakkam (2000), Bartholdy and Brown (2004) and Borges (2004).

The main contribution of our model is to extend theoretically on this discussion, by defining the equilibrium conditions for different types of investors, considering taxes, transaction costs and rational arbitrage. The general equilibrium will be defined by a range of potential equilibrium points (the equilibrium zone), that will depend on the relative weights of the different types of investors. We show that there is a relation between investor typologies and the pattern of trading volume in the dividend period, which may help identifying the types of investor that are present in the market.

## Hypothesis

The hypotheses of the model are the following: (i) We assume that prices are stationary around the dividend event. This implies that daily returns are expected to be zero and that new information does not arrive to the market in that period. (ii) Investors maximize expected return, when deciding to buy or sell stocks. Any expected profit opportunity will be explored by investors. (iii) Investors are rational and form unbiased expectations about future prices. Investors do not commit systematic errors in their predictions about future price behavior. (iv) Investors are risk neutral. Decisions to buy and sell are only determined by expected returns. Between two alternatives, investors will always choose that which has higher expected return, independently of risk. Investors are indifferent between two alternative strategies if they have the same expected return, whatever the risk of each alternative. (v) Information is public and free. In the dividend period, all investors are fully informed about the dividend amount, the distribution date, transaction costs, and taxation on dividends and capital gains.

Under these hypotheses, and considering the absence of taxes and transaction costs, there is market equilibrium when the expected price adjustment on the ex-day equals the dividend amount, resulting in an expected return of zero.

$$\frac{\left(P_b - P_a^e\right)}{D} = 1 \tag{1}$$

$$R^{e} = \frac{P_{a}^{e} - P_{b} + D}{P_{b}} = 0$$
<sup>(2)</sup>

where  $R^e$  is the expected return,  $P_b$  is the stock price before the dividend,  $P_a^e$  is the expected stock price after the dividend and D is the dividend. Allowing for market imperfections, namely taxes and transaction costs, we add three other assumptions to our model: (vi) Dividends are taxed, and the tax rate is assumed to be equal for all investors. (vii) Capital gains are also taxed, and the tax rate is assumed to be equal for all investors. (viii) Every buy or sell transaction has a positive transaction cost tax, independent of the trading volume, and equal for all investors. With these additional assumptions, the equilibrium conditions (1) and (2) are no longer valid, as will be shown in the next sections.

#### Effects from the Behavior of Different Types of Investors

The price adjustment after the dividend event will depend on the types of investors that are present in the market during the dividend period. We consider four types of investors. The first two have already made the decision to buy or to sell the stock, independently of the dividend event. Investors type S want to sell, while investors type B have decided to buy stocks. These investors now have only to decide the timing of the transaction, before or after the dividend. The other two types of investors are arbitrageurs that will decide to make a round-trip transaction, if they expect to obtain a positive return. Investors type BS buy stocks before the dividend and sell them after, while investors type SB hold stocks in their portfolio

and consider selling before the distribution date and buy them again after the event. We assume that the actions of these four types of investors will force the stock price to adjust to a level where profit opportunities become inexistent, for all of them. We will now determine the equilibrium conditions for each type of investor.

## **Investors Type S**

Investors type S will sell before the dividend distribution if the expected result is positive:

$$\left[-P_{c}(1+t_{tc})+P_{b}(1-t_{tc})-t_{cg}(P_{b}-P_{c})\right]-\left[-P_{c}(1+t_{tc})+P_{a}^{e}(1-t_{tc})-t_{cg}(P_{a}^{e}-P_{c})+D(1-t_{d})\right]>0$$
(3)

where the new variables have the following meaning:  $P_c$  is the stock acquisition price,  $t_d$  is the dividend tax,  $t_{cg}$  is the capital gains tax and  $t_{tc}$  is the transaction cost tax. The first part of (3) is the net result of selling before the dividend, and the second part is the net result of selling after the dividend. From (3) we obtain:

$$\frac{\left(P_b - P_a^e\right)}{D} > \frac{1 - t_d}{1 - t_{cg} - t_{tc}} \tag{4}$$

If condition (4) holds, investors type *S* will prefer to sell before the dividend, forcing the price  $P_b$  to decrease. But if the inequality (4) is the opposite, investors type *S* will prefer to sell after the dividend, forcing  $P_a^e$  down. Arbitrage by this type of investors will force prices to adjust until the following equilibrium is reached:

$$\frac{\left(P_{b} - P_{a}^{e}\right)}{D} = \frac{1 - t_{d}}{1 - t_{cg} - t_{tc}}$$
(5)

#### Investors Type **B**

Investors type *B* will buy stocks before the dividend, if they expect (6) to be positive:

$$\left[D(1-t_{d})-P_{b}(1+t_{tc})-t_{cg}(P_{s}^{e}-P_{b})+P_{s}^{e}(1-t_{tc})\right]-\left[-P_{a}^{e}(1+t_{tc})-t_{cg}(P_{s}^{e}-P_{a}^{e})+P_{s}^{e}(1-t_{tc})\right]>0$$
(6)

where,  $P_s^e$  is the price that these investors will sell the stock in the future<sup>1</sup>. From (6) we obtain the following condition:

$$\frac{\left(P_b - P_a^e\right)}{D} < \frac{1 - t_d}{1 - t_{cg} + t_{tc}} \tag{7}$$

Investors type *B* will buy stocks before the dividend if condition (7) holds, forcing  $P_b$  to increase. If inequality (7) does not hold, investors type *B* will buy stocks after the dividend and, in this case,  $P_a^e$  will increase. Again, the actions of these investors will lead to an equilibrium condition:

$$\frac{\left(P_{b} - P_{a}^{e}\right)}{D} = \frac{1 - t_{d}}{1 - t_{cg} + t_{tc}}$$
(8)

## Investors Type BS

Investors type *BS* are short term traders following a strategy of buying before the dividend and selling after, if they believe they can make a profit:

$$-P_b(1+t_{tc}) + D(1-t_d) + P_a^e(1-t_{tc}) - t_{cg}(P_a^e - P_b) > 0$$
(9)

From (9), we obtain a necessary condition for these short term traders to enter the market:

$$\frac{P_b - P_a^e}{D} < \frac{1 - t_d}{1 - t_{cg}} - \frac{t_{tc}}{1 - t_{cg}} \frac{P_b + P_a^e}{D}$$
(10)

<sup>&</sup>lt;sup>1</sup> Or we could think of  $P_s^e$  as representing all future inflows from the stock, including dividends and capital gains. We do not need to worry about the precise calculation of  $P_v^e$  because this variable cancels out in (7), and it is independent from the investors decision to buy, or not.

If there is this an arbitrage opportunity, these investors will buy before the dividend, forcing an increase in  $P_b$ , and sell after the dividend, pushing down  $P^e_a$ . The combination of this opposite movements in prices will remove profit opportunities, until:

$$\frac{P_b - P_a^e}{D} \ge \frac{1 - t_d}{1 - t_{cg}} - \frac{t_{tc}}{1 - t_{cg}} \frac{P_b + P_a^e}{D}$$
(11)

## Investors Type SB

Finally, investors type *SB* will sell before the dividend and buy after, if the expected result of this behavior is higher than to the alternative of holding the stock in their portfolio:

$$\left[P_{a}(1-t_{tc})-t_{cg}\left(P_{b}-P_{c}\right)\right]+\left[-P_{a}^{e}(1+t_{tc})-t_{cg}\left(P_{s}^{e}-P_{a}^{e}\right)\right]+P_{s}^{e}(1-t_{tc})-\left[-t_{cg}\left(P_{s}^{e}-P_{c}\right)+D(1-t_{d})+P_{s}^{e}(1-t_{tc})\right]>0$$
(12)

Investors type SB obtain a profit if (13) holds:

$$\frac{P_b - P_a^e}{D} > \frac{1 - t_d}{1 - t_{cg}} + \frac{t_{tc}}{1 - t_{cg}} \frac{P_b + P_a^e}{D}$$
(13)

If this arbitrage opportunity exists, the actions of investors type *SB* force  $P_b$  to decrease and  $P^e_a$  to increase, reducing profits which will disappear when:

$$\frac{P_b - P_a^e}{D} \le \frac{1 - t_d}{1 - t_{cg}} + \frac{t_{tc}}{1 - t_{cg}} \frac{P_b + P_a^e}{D}$$
(14)

#### Arbitrage and Equilibrium

## **Boundaries to Arbitrage Opportunities**

Now we can look at the effects of the joint behavior of the four types of investors. If we assume that transaction costs are null ( $t_{tc}=0$ ) the condition for the nonexistence of profit

opportunities for each type of investor, given by (5), (8), (11) e (14), simplifies to the equilibrium condition of Elton and Gruber (1970):

$$\frac{\left(P_b - P_a^e\right)}{D} = \frac{1 - t_d}{1 - t_{cg}} \tag{15}$$

Positive transaction costs have, however, a significant impact on the equilibrium conditions, with different boundaries for the arbitrage opportunities. Figure 1 identifies profit opportunities for the different types of investors and the arrows show the impact of their actions on the expected price adjustment.



Figure 1 Boundaries of Arbitrage Zones

All boundaries are defined in relation to the central value given by (15). Between boundaries B and S, investors types S and B, force the price adjustment in opposite directions. The location of the equilibrium depends on the relative forces of the two types of investors, but it certainly will fall in the interval define by these two boundaries:

$$\left[\frac{1-t_{d}}{1-t_{cg}+t_{tc}};\frac{1-t_{d}}{1-t_{cg}-t_{tc}}\right]$$
(16)

For investors types *SB* and *BS*, there are no profit opportunities inside boundaries *SB* and *BS*, that is, in:

$$\left[\frac{1-t_{d}}{1-t_{cg}} - \frac{t_{tc}}{1-t_{cg}}\frac{P_{b} + P_{a}^{e}}{D}; \frac{1-t_{d}}{1-t_{cg}} + \frac{t_{tc}}{1-t_{cg}}\frac{P_{b} + P_{a}^{e}}{D}\right]$$
(17)

Figure 1 assumes that (16) is narrower than (17). There is no general proof of this result. However, if we restrict ourselves to combinations of  $t_d$ ,  $t_{cg}$ ,  $t_{tc}$  and D/P which are close to values observed in the real world, interval (16) will be much smaller than interval (17). In Figure 2, using  $t_d=25\%$ ,  $t_{cg}=15\%$ , D/P=1%, interval (17) is more than 200 times wider than interval (16), for any transaction cost,  $t_{tc}$ , between 0% and 5%.

Figure 2 Sensitivity of Arbitrage Boundaries to Transaction Costs



This result is not surprising, because transaction costs have a very different impact on investors type B and S, on one side, and investors BS and SB, on the other. While investors B and S only face costs from one transaction, investors SB and BS will pay the costs of two transactions. Furthermore, investors B and S compare two alternatives which imply, both of them, the costs from one transaction. Transaction costs is thus a fixed cost and almost

irrelevant for their decision. The difference in transaction costs, between their two alternatives, to transact before or after the dividend, is given by  $t_{tc}(P_b - P_a^e)$ .

#### Equilibrium

If profit opportunities are totally explored the price adjustment will be inside boundaries S and B, where all points are a possible equilibrium solution. Here, the actions of investors S and B push the price in opposite directions, and the final equilibrium will depend on the relative strengths of these two types of investors. Note that none of the investors is individually in equilibrium, because profit opportunities have not been completely exhausted. In any case, both types of investors choose to delay buying/selling until after the dividend distribution. This means that, *ceteris paribus*, we should observe a positive abnormal transaction volume after the distribution event and a negative abnormal volume before. If the price adjustment falls outside boundaries S and B, this means that the actions of investors Sand B are not sufficient to explore all profit opportunities, by reasons unrelated to taxes and transaction costs. This may happen if these types of investors exist in small numbers in the market, and so they would not have a strong enough influence over prices.

Investors types *BS* and *SB* do not intervene if the price adjustment is within boundaries *BS* and *SB*. The profit opportunities for these investors exist only below boundary *BS* and above boundary *SB*, respectively. The activity of these investors will lead to abnormal transaction volume both before and after the dividend. An important aspect is the extreme sensitivity of arbitrage opportunities for investors *SB* and *BS*, to the dividend yield, when transaction costs are positive. If we take  $t_d=t_{cg}$ , D/P=1% e  $t_{tc}=0.5\%$ , the transaction costs of a round-trip exactly match the amount of the dividend. With this scenario, profit opportunities

exist for investors BS if  $P_b - P_a^e < 0$ , and for investors SB if  $(P_b - P_a^e)/D > 2$ . Figure 3 shows how boundaries SB and BS get wider as dividend yield lowers and transaction costs increase.



Figure 3 Sensitivity of Arbitrage Boundaries *SB* and *BS* to *t<sub>tc</sub>* and *D/P* 

## **Expected Pre-tax Returns in Equilibrium**

In equilibrium, after-tax returns are zero. With taxes and transaction costs, investors will demand positive pre-tax returns. We obtain these equilibrium pre-tax returns by equaling (5), (8), (11) and (14) to zero, for all types of investors:

Type S: 
$$R_{S}^{e} = \frac{P_{a}^{e} - P_{b} + D}{P_{b}} = \frac{\left(t_{d} - t_{cg} - t_{tc}\right)D}{\left(1 - t_{cg} - t_{tc}\right)P_{b}}$$
 (18)

Type B: 
$$R_{B}^{e} = \frac{P_{a}^{e} - P_{b} + D}{P_{b}} = \frac{(t_{d} - t_{cg} + t_{tc})}{(1 - t_{cg} + t_{tc})} \frac{D}{P_{b}}$$
 (19)

,

Type BS: 
$$R_{BS}^{e} = \frac{P_{a}^{e} - P_{b} + D}{P_{b}} = \frac{(t_{d} - t_{cg})}{(1 - t_{cg})} \frac{D}{P_{b}} + \frac{t_{tc}(1 + P_{a}^{e}/P_{b})}{1 - t_{cg}}$$
 (20)

Type SB: 
$$R_{SB}^{e} = \frac{P_{a}^{e} - P_{b} + D}{P_{b}} = \frac{(t_{d} - t_{cg})}{(1 - t_{cg})} \frac{D}{P_{b}} - \frac{t_{tc}(1 + P_{a}^{e}/P_{b})}{1 - t_{cg}}$$
 (21)

These conditions are the minimum pre-tax returns demanded by investors, and occur when all arbitrage opportunities are exhausted. For investors types *SB* and *BS*, these conditions represent the minimum return demanded to enter the market. If the following condition does not hold:

$$R_{BS}^{e} = \frac{P_{a}^{e} - P_{b} + D}{P_{b}} > \frac{\left(t_{d} - t_{cg}\right)}{\left(1 - t_{cg}\right)} \frac{D}{P_{b}} + \frac{t_{tc}\left(1 + P_{a}^{e}/P_{b}\right)}{1 - t_{cg}}$$
(22)

investor type *BS* will prefer not to enter the market. On the other hand, investor type *SB* will stay out of the market unless:

$$R_{SB}^{e} = \frac{P_{a}^{e} - P_{b} + D}{P_{b}} < \frac{\left(t_{d} - t_{cg}\right)}{\left(1 - t_{cg}\right)} \frac{D}{P_{b}} - \frac{t_{tc}\left(1 + P_{a}^{e}/P_{b}\right)}{1 - t_{cg}}$$
(23)

Generally, expected pre-tax equilibrium returns will be positive if  $t_d > t_{cg}$ , and negative if  $t_d < t_{cg}$ , although we have to also consider transaction costs. Table I shows the impact on pretax equilibrium returns for all types of investors, from changes in  $t_d$ ,  $t_{cg}$ ,  $t_{tc}$  and D/P. For all investors, expected pre-tax returns increase with the tax rate on dividends. Conversely, expected pre-tax returns will be lower as the tax on capital gains increases, for all investors except for type *BS*, where the sign is ambiguous. The impact of transaction costs is negative on the expected pre-tax return of investors *S* and *SB*, and positive on the expected pre-tax return of investors *B* and *BS*. Finally, the relationship between expected pre-tax returns and dividend yields will be positive for investors *SB* and *BS*, if  $t_d > t_{cg}$ , for investors *S*, if  $t_d > t_{cg} + t_{tc}$ , and for investors *B*, if  $t_d > t_{cg} - t_{tc}$ . Normally,  $t_{tc}$  is much smaller than  $t_d$ and  $t_{cg}$ , so we will have a positive relationship between the expected pre-tax return and the dividend yield, if  $t_d > t_{cg}$ .

Table IImpact of Changes in  $t_d$ ,  $t_{cg}$ ,  $t_{tc}$  and D/P on Pre-tax Equilibrium Returns

	Type S	Туре <i>В</i>	Type SB	Type BS
$\frac{\partial R^{e}}{\partial t_{d}} =$	$\frac{D}{(1-t_{cg}-t_{tc}).P_b}$	$\frac{D}{(1-t_{cg}+t_w).P_b}$	$\frac{D}{(1-t_{cg}).P_b}$	$\frac{D}{(1-t_{cg}).P_b}$
	(>0)	(20)	(>0)	(>0)
$\frac{\partial R^{e}}{\partial t_{m}} =$	$-\frac{(1-t_d)}{(1-t_{cg}-t_{tc})^2}\cdot\frac{D}{P_b}$	$-\frac{(1-t_d)}{(1-t_{cg}+t_{tc})^2}\cdot\frac{D}{P_b}$	$-\frac{(1-t_{d})}{(1-t_{cg})^{2}}\cdot\frac{D}{P_{b}}-\frac{t_{cr}(1+P_{a}^{e}/P_{b})}{(1-t_{cg})^{2}}$	$-\frac{(1-t_d)}{(1-t_{cg})^2} \cdot \frac{D}{P_b} + \frac{t_{ic}(1+P_a^e/P_b)}{(1-t_{cg})^2}$
cg	(<0)	(<0)	(<0)	(ambiguous)
$\frac{\partial R^{e}}{\partial t} =$	$-\frac{(1-t_d)}{(1-t_d-t_d)^2}\cdot\frac{D}{P_b}$	$\frac{\left(1-t_{d}\right)}{\left(1-t_{ce}+t_{te}\right)^{2}}\cdot\frac{D}{P_{b}}$	$-\frac{1+P_a^{\epsilon}/P_b}{1-t_a}$	$\frac{1+P_a^e/P_b}{1-t}$
OI <sub>IC</sub>	(<0)	(>0)	(<0)	(>0)
$\frac{\partial R^{\epsilon}}{\partial (D/P)} =$	$\frac{\left(t_d - t_{cg} - t_{tc}\right)}{\left(1 - t_{cg} - t_{tc}\right)}$	$\frac{\left(t_d - t_{cg} + t_{tc}\right)}{\left(1 - t_{cg} + t_{tc}\right)}$	$\frac{\left(t_d - t_{cg}\right)}{\left(1 - t_{cg}\right)}$	$\frac{\left(t_d - t_{cg}\right)}{\left(1 - t_{cg}\right)}$
	$(>0, \text{ if } t_d > t_{cg} + t_{tc})$	$(>0, \text{ se } t_d > t_{cg} - t_{tc})$	$(>0, se t_d > t_{cg})$	$(>0, \text{ se } t_d > t_{cg})$
	$(<0, \text{ se } t_d < t_{cg} + t_{tc})$	$(<0, \text{ se } t_d < t_{cg} - t_{tc})$	$(<0, se t_d < t_{cg})$	$(<0, se t_d < t_{cg})$

## **Effects of Differential Taxation**

The previous analysis is based on the assumption that taxes are equal for all investors. If we assume that investors may face different tax rates, there are some interesting points to show. Consider  $t_d > t_{cg}$  for all investors, except for investors type *BS*, who face equal tax rates,  $t_d = t_{cg}$ . With no transaction costs, arbitrage opportunities disappear for investors type *BS*, if the price adjustment equals the amount of the dividend,

$$\frac{\left(P_b - P_a^e\right)}{D} = \frac{1 - t_d}{1 - t_{cg}} = 1$$

For all other investors, we will have:

$$\frac{\left(P_{b} - P_{a}^{e}\right)}{D} = \frac{1 - t_{d}}{1 - t_{cg}} = e < 1.$$

Figure 4 shows the arbitrage boundaries under these assumptions. For transactions costs below A, there are divergent interests between investors BS and the other investors. While investors BS push the price toward 1, the other investors push the price towards e. The

final equilibrium will depend on the relative weight of all types of investors, but it will be located in the shadowed area at the left of A.





If transaction costs are between A and B, there will be divergent interests between investors BS, on one side, and investors S and B, on the other. Once again, equilibrium will depend on the relative weight of these investors and it will be located in the shadowed area between A and B. If transaction costs exceed B, we will have the same case as in the model with identical taxation for all investors, with the equilibrium located between boundaries B and S. This example illustrates conflicts of interest between different types of investors, resulting from different taxations. In the real world more complex equilibria certainly exist. This example is useful because it shows that it is possible to have a global equilibrium where none of the investors has explored all profit opportunities and so, is not individually in equilibrium. Another implication is that the income tax rate of the marginal investor can not be inferred from the price adjustment, as proposed by Elton and Gruber (1970).

#### **Trading Volume in the Dividend Period**

The trading volume will be affected by the actions of the different types of investors. Depending on the equilibrium zone, we will observe different behaviors by investors, which will affect transaction volume in different ways, as is graphically shown in Figure 5.

Let us consider the existence of profit opportunities for investors types BS and SB, that is, the price adjustment is expected to fall in Zones *Ia* or *Ib*. In this case, we observe a positive abnormal trading volume both before and after the dividend event, by these types of investors. Investors *BS* will be active in zone *Ia*, while investors *SB* will be active in zone *Ib*. If the price adjustment is expected to fall in zones *Ia* or *IIa*, investors *B* will anticipate transactions to before the dividend and this translates in abnormal positive volume before the dividend and negative after. Investors *S* will postpone their sales until after the dividend, thus having an opposite effect on abnormal trading volume. The combined effect on trading volume depends on the relative weights of the transactions made by these two types of investors. This rationale can be extended to equilibrium zones *IIb* and *Ib*, where investors *B* and *S* change positions, in terms of their impact on the abnormal trading volume, as investors *B* will sell after the dividend and investors *S* will buy before the dividend.

Again the combined effect on abnormal trading volume is ambiguous as it depends on the relative weights of both types of investors. If the price adjustment is expected to fall in zone *III*, both investors types *B* and *S* postpone their transactions until after the dividend, thus causing a negative abnormal volume before the dividend and a positive abnormal volume after. Thus, the observation of the trading volume during the dividend period may be an important indicator for the types of investors active in the market, affecting the level of price adjustment.

Figure 5 Abnormal Trading Volume Around the Dividend Event



#### Conclusions

With a simple model assuming very restrictive hypothesis, we show that the allowance for market imperfections such as taxes and transactions costs implies that there is not a unique equilibrium point for the level of stock price adjustment following a dividend distribution event, but rather there are many possible equilibria. The main reason for this is the fact that transaction costs limit the arbitrage opportunities and so there are boundaries below which (or above which) arbitrage becomes unprofitable and so there are no market forces pushing the price to a unique market equilibrium.

In the real world, where the spectrum of investors is more diverse than the four types allowed for in the model, where: tax rates are different between investors; transaction costs may be different for different agents; preferences regarding the trade-off between risk and return are also heterogeneous; and arbitrage may be limited by infrequent trading, we should expect to observe much more complex equilibria, and a wider range of possible values for the stock price adjustment.

Finally, and in the absence of direct data regarding the identification of the types of investors affecting the global equilibrium, we show that the observation of abnormal trading volume around the dividend event may give us some insights on the identification of which investors are present in the market.

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