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WP 18/2009/DE/SOCIUS

WORKING PAPERS

ISSN Nº 0874-4548

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Abstract

We study a situation in which, owing to the exhaustion of non-renewable energy sources, conventional motor vehicles will turn out of use. We consider two scenarios: recycling or dismantling these motor vehicles. $M|G|\infty$ queue system is used to study the process. Through it, we conclude that if the rate of dismantling and recycling of motor vehicles is greater than the rate at which they become idle, the system will tend to get balanced. The model allows also performing a brief study about the recycling or dismantling economic interest.

Introduction

Many resources on Earth got overexploited. Oil, gas and coal exploitation posed many questions about the way humans use resources. The exploitation of non-renewable sources of energy and their use brought serious problems. We are now observing strong climatic changes, a fast extinction of species and many other critical problems that may have an end. Governments and institutions may change things. If not, very strong changes in our way of living and in the balances of the planet will happen soon.

The generalized access to all goods by the majority of populations around the world has created new needs to people and has created a global consumption economy. In general, a significant part of the countries' populations got access to the generality of industrial goods and it seems that people would not be prepared in the future to abdicate from it.

However, the non-renewable energy, as we know it today, may not be enough for many decades more, from now on. And even if it would be possible to keep the production of this kind of energy, the effective changes on Earth balances would establish that life would not be compatible with the existing way of life for people. Consequently, people need to adjust behaviors and governments need to prepare their economies for the new era of globalization.

New situations may occur very suddenly. The oil production has already got its peak and new oil productions will occur in the future with decreasing rents until its complete depletion. The other non-renewable sources of energy will have the same end.

Therefore we may have to reorder the priorities and to reorganize structures in societies. We have now to produce the new kind of energies (clean energies) at a major scale. The big problem that remains is to know if the transition period from non-renewable sources of energy to the renewable sources is enough to overcome the big problems related with the destruction of Earth. All the wastes that people have made for so many decades must be overcome, as well. Anyway many kinds of new problems will occur. However what is important now is to know how quickly changes may happen while we develop the new sources of energy in order to create a new economy and a reorganized society.

Too many motor vehicles will become idle if conventional energy misses or even when conventional energy becomes replaced by a renewable one. Motor vehicles dismantling or recycling will become very usual because there will not be a way to get them functional with conventional oil, since the moment it gets depleted.

Just before oil becomes really scarce, it will get very high prices. At that moment, many motor vehicles' owners must find out an alternative solution for energy. In our study we'll see how to model a situation in which motor vehicles become idle and then recycled or dismantled, depending on their specific situation and depending on the will of motor vehicles' owners. Our aim is to show that any kind of equipments that work on the basis of oil may have an alternative use when this conventional energy collapses; or simply they may become dismantled. Then other uses for their materials and components will be made.

We apply our model to motor vehicles.

A motor vehicles' dismantling and recycling queue model

We consider the M|G|∞ queue system (Kelly, 1979) where customers arrive according to a Poisson process at rate λ (Ferreira, 1996). They receive a service whose length is a positive random variable with distribution function G(.) and mean α. Each customer as soon as it arrives at the system, immediately finds an available server. Each customer service is independent from the others customers' services and from the arrival process. The traffic intensity is given by $\rho = \lambda\alpha$.

With this model we intend to analyze a situation in which motor vehicles arrive at the system getting idle and leave the system as soon as they are recycled or dismantled. Both situations are modeled with the same purpose in the model. Our interest in studying this situation is precisely to see how the system may recover to a balanced situation in which motor vehicles get operational or get dismantled (in this situation materials would become employed as components in other applications).

Let $N(t)$ be the number of busy servers (or, what is the same, the number of customers being served) in the instant t, in a M|G|∞ system. If we consider $p_{0n}(t) = P[N(t) = n|N(0) = 0]$, $n = 0,1,2,...$, we may have (Carrillo, 1991):

$$p_{0n}(t) = \frac{\left(\lambda\int_0^t [1 - G(v)]dv\right)^n}{n!} e^{-\lambda\int_0^t [1-G(v)]dv}, \quad n = 0,1,2,... \quad (1).$$

So if the initial instant is a moment at which the system is empty, the transient distribution is Poisson with mean $\lambda\int_0^t [1 - G(v)]dv$.

The stationary distribution is the limit one:

$$\lim_{t\to\infty} p_{0n}(t) = \frac{\rho^n}{n!} e^{-\rho}, \quad n = 1,2,... \quad (2).$$

This queue system, as any other, has a sequence of busy periods and empty periods. A busy period begins when a customer arrives at the system, finding it empty.

Let's see the distribution of the number of customers being served in the instant t in the M|G|∞ system, when the initial instant is the moment at which a busy period begins, that gets relevant for our purposes.

Be $p_{1'n} = P[N(t) = n|N(0) = 1']$, $n = 0,1,2,...$, and $N(0) = 1'$ the initial instant at which a customer arrives at the system and the number of customers being served turns from 0 to 1. This means that a busy period has just begun (Ferreira, 1988).

So at the instant $t \geq 0$, we may have a situation that represents (Ferreira, 1998):

1. the customer that arrived at the system at the initial instant has left the system with a probability $G(t)$, or he remains in the system, with probability $1-G(t)$;
2. the other servers, which were empty at the beginning (initial instant), may be now empty or busy with 1, 2, ... customers, with probabilities given by $p_{0n}(t), \quad n = 0,1,2,....$

The two subsystems, the one of the initial customer and the one of servers initially empty, are independent. Consequently:

$$p_{1'0}(t) = p_{00}(t)G(t)$$

$$p_{1'n}(t) = p_{0n}(t)G(t) + p_{0n-1}(t)(1-G(t)), \; n = 1,2,... \qquad (3).$$

We also have for this situation:

$$\lim_{t\to\infty} p_{1'n}(t) = \frac{\rho^n}{n!}e^{-\rho}, \quad n = 0,1,2,... \qquad (4).$$

For the M|M|∞ system (exponential service times), the equations 3 are applicable even when $N(0)=1$ (when the initial instant is a moment at which there is a customer in the system; that does not enforce that this is the moment at which we are turning from 0 to 1 customer to be served). This results from the lack of memory of the exponential distribution.

If $g(t)$ is the probability density function related to $G(t)$, and if we call $h(t)$ the hazard rate function, we'll have (Ross, 1983):

$$h(t) = \frac{g(t)}{1-G(t)} \qquad (5).$$

The function $h(t)$ is the rate at which services end.

So,

Proposition 1:

If $G(t) < 1, \quad t > 0$, continuous and differentiable and if

$$h(t) \geq \lambda G(t), \; t > 0 \qquad (6)$$

$p_{1'0}(t)$ is a non-decreasing function.

Dem:

It's enough to observe that $\frac{d}{dt}p_{1'0}(t) = p_{00}(t)(1-G(t))\left(\frac{g(t)}{1-G(t)} - \lambda G(t)\right)$.

Besides, we may note that

$$h(t) \geq \lambda, \quad t > 0 \qquad (7)$$

is a sufficient condition for the result in 6.
So, if the rate at which the services end is greater or equal than the rate of arrivals, we conclude that $p_{1'0}(t)$ does not decrease.

For the system M|M|∞, the equation 7 is equivalent to

$$\rho \leq 1 \qquad (8).$$

Considering $\mu(1',t)$ and $\mu(0,t)$ the mean values of the distributions given by 3 and 1, respectively, we'll have

$$\mu(1',t)=\sum_{n=1}^{\infty} np_{1'n}(t)=\sum_{n=1}^{\infty} nG(t)p_{00}(t)+\sum_{n=1}^{\infty} np_{0n-1}(t)(1-G(t))=$$

$$=G(t)\mu(0,t)+(1-G(t))\sum_{j=0}^{\infty}(j+1)p_{0j}(t)=\mu(0,t)+(1-G(t)).$$

So,

$$\mu(1',t)=1-G(t)+\lambda\int_0^t[1-G(v)]dv \qquad (9).$$

Proposition 2:

If $G(t)<1, \quad t>0$, continuous and differentiable and if

$$h(t)\leq\lambda, \quad t>0 \qquad (10).$$

$\mu(1',t)$ is a non-decreasing function.

Dem:

It's enough to observe that, considering equation 9, $\frac{d}{dt}\mu(1',t)=(1-G(t))(\lambda-h(t))$.

Besides, if the rate at which services end is lesser or equal than the rate at which customers arrive $\mu(1',t)$ is a non-decreasing function. We can note additionally that, for the M|M|∞ system, the equation 10 is equivalent to

$$\rho \geq 1 \qquad (11).$$

Results and comments

According to our study interests, the customers are the motor vehicles that become idle. The arrival rate is the rate at which the motor vehicles become idle. The service time for each one is the time that goes from the instant they get idle until the instant they become recycled or dismantled. The service time hazard rate function is the rate at which the motor vehicles become recycled or dismantled.

An idle period for our M|G|∞ system should be a one at which there were no motor vehicles idle. In a busy period there are always continuously idle motor vehicles.

The equation 6 shows that if the dismantling and recycling rate is greater or equal than the rate at which motor vehicles get idle, the probability that the system gets empty (that is, there are no idle motor vehicles) does not decrease with time. This means that the system has a tendency to become balanced as far as time goes on.

The equation 10 shows that if the dismantling and recycling rate is lesser or equal than the rate at which motor vehicles get idle, the mean number of motor vehicles in the system does not decrease with time. This means that the system has a tendency to become unbalanced as far as time goes on.

Consequently, we conclude that when the rate of dismantling and recycling of motor vehicles is greater than the rate at which they become idle, the system has a tendency to get balanced. In this situation, the motor vehicles that become unused with the conventional energy turn useful with another kind of energy or get included in other useful devices.

We must note that it is important the recycling or the dismantling of motor vehicles but, more than that, it is essentially relevant the cadence at which these actions are performed. Moreover, we give a reference for this cadence: λ, the rate at which motor vehicles get idle.

An economic analysis as a complement to the model

We have seen that rates λ and *h(t)* are determinant to monitor the way that the system of motor vehicles recycling and dismantling may be managed.

We consider now additionally p as the probability for the motor vehicles arrivals destined to recycling and (1-p) as the probability for the motor vehicles arrivals destined to dismantling. Let $h_i(t)$, $c_i(t)$ and $b_i(t)$, $i=1,2$ be the hazard rate function, the mean cost and the mean benefit, respectively for recycling when $i=1$ and dismantling when $i=2$.

With these new variables we can perform an economic analysis (beyond other considerations that may be posed) to evaluate about the interest of recycling and dismantling.

We will analyze this situation in a global approach and not in a selfish way, considering the individuals point of view.

So, we may consider the total cost per unit of time for motor vehicles recycling and dismantling as:

$$C(t) = pc_1(t)\lambda + (1-p)c_2(t)\lambda \qquad (12).$$

Furthermore, the benefit per unit of time resulting from recycling and dismantling is given by:

$$B(t) = b_1(t)h_1(t) + b_2(t)h_2(t) \qquad (13).$$

From an economic point of view, it must be B(t)>C(t).

To conclude about the advantage of recycling, we have the following:

$$b_1(t) > \max\left[\ \frac{p\lambda c_1(t) + (1-p)\lambda c_2(t) - b_2(t)h_2(t)}{h_1(t)}, 0\ \right] \qquad (14).$$

$G_1(t)$ and $G_2(t)$ are both exponential, equation 14 becomes:

$$b_1(t) > \max\left[\ (pc_1(t) + (1-p)c_2(t))\rho_1 - \frac{\alpha_1}{\alpha_2} b_2(t), 0\ \right] \qquad (15).$$

To conclude about the advantage of dismantling we have, in the same conditions:

$$b_2(t) > \max\left[\ \frac{p\lambda c_1(t) + (1-p)\lambda c_2(t) - b_1(t)h_1(t)}{h_2(t)}, 0\ \right] \qquad (16)$$

and

$$b_2(t) > \max\left[\ (pc_1(t) + (1-p)c_2(t))\rho_2 - \frac{\alpha_2}{\alpha_1} b_1(t), 0\ \right] \qquad (17).$$

So, there are minimum benefits above which, from an economic point of view both, recycling and dismantling, are interesting. The most interesting is the one for which this minimum benefit is the least. By other words: in a global perspective, it is more efficient the activity that corresponds to a lower level for the minimum interesting benefit.

Recycling seems to be as much interesting as far as it is more economically profitable and our inequalities 14 to 17 are tools that may be applied to evaluate this interest.

Strengths and limitations

Our model contributes for a better understanding of this kind of problems and it (or some modified versions of it) may be applied to study some other social and economic phenomena, such as unemployment, health or projects of investment, for example, with interesting results.

An extension of our model has also permitted to get conclusions about the economic advantages of recycling and dismantling.

The model application to the phenomenon studied in our paper shows that it is very useful and that its conclusions and results are quite simple to understand. Just through its theoretical analysis, we evidence some remarkable topics in analyzing the evolution of the studied system. Or another one, whichever it is, since it is according the assumptions of the model.

In practice, it is essential to estimate λ and *h(t)* to get conclusive particular results for the available data about the system. This will give us the tools to monitor the situation and to suggest solutions. A correct estimation of λ will depend on the arrivals process to be Poisson, in real.

Additionally, in general, it is correct to admit that with very large populations, such as the one we are dealing with, the estimation of *h(t)* is usually technically complicated. So, frequently, the best to do is to estimate directly *h(t)* instead of estimating first the service time distribution and then computing *h(t)*.

A particular situation at which the computation is easier is the exponential service time one, for which $h(t) = 1/\alpha$.

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