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Analysis and comparison in a real case

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CIO – Working Paper 11/2009

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October 20, 2009

Abstract

This work focus on hospital surgical suite optimization, mainly in the efficient use of the operating rooms when planning elective surgeries. We studied a real case in a hospital in Lisbon.

An integer linear programming model was developed to weekly schedule elective surgeries for the hospital surgical suite. The model was tested with real data collected from the hospital records. Non-optimal solutions obtained were improved with a simple and efficient improving heuristic. All solutions have actually improved through this process.

These results were finally analyzed and compared with the ones the hospital really performed. The analysis shows that the solutions obtained by our approach improve the use of the surgical suite while respecting the conditions imposed by the hospital. The analysis also shows that the plans obtained by the proposed approach can be implemented.

Keywords: Health Care, Integer Programming, Scheduling.

1 Introduction

Health sector has been progressively affected by increasingly restrictive budgets that necessarily lead to an urgent need of resources rationalization practice in hospitals.

Surgical suite is amply considered the hospital central engine as it has direct impact in many other hospital departments, such as surgical wards and recovery units. In fact, there is the need to rationalize resources so that, on the one hand, the number of surgeries performed increases, and on the other hand, their costs should reach the minimum possible level within current costs.

Increasing the number of surgeries performed, drastically reduces waiting lists for surgery. Costs for maintaining a patient on the waiting list for surgery are high on prevention and maintenance levels or even on usuary quality of live. In addition, surgery waiting list reduction is one of the priorities of the Portuguese National Health Service (SNS) according to the General Direction of Health (2004). To reduce waiting lists for surgery is largely beneficial both at human and scientific levels, and also in economic level.

In the literature, operating rooms planning has been considered as a three stage process. Magerlein and Martin (1978), Przasnyski (1986), Blake and Carter (1997) and, recently, Cardoen et al. (2009a) present literature reviews on operating room planning.

In a first stage, named *case mix planning*, operating room time is distributed among individual or groups of surgeons. In a strategic level of decision, this stage defines the hospital supply for surgery and is usually conducted on an annual basis, together with the annual hospital budget definition. There are some linear and integer linear programming approaches to solve the planning problem at this phase (Hughes and Soliman (1985), Robbins and Tuntiwongpiboon (1989), Kuo et al. (2003) and Testi et al. (2007)) and also a goal programming approach from Blake and Carter (2002).

The second phase involves the development of a master surgery schedule, a cyclic timetable that defines the number and type of operating rooms available, the hours that those rooms will be open, and determines the surgeons or surgical groups with priority in each operating room time periods. This phase is referred to as *master surgery planning* and relate to a tactical level of hospital management. There is a greater range of approaches for this stage of the operating room planning. Literature vary from integer or mixed integer linear programming models solved by general solvers (Blake et al. (2002), Vissers et al. (2005), Santibáñez et al. (2007) and Testi et al. (2007)) or heuristic methods (Blake and Donald, 2002), to a quadratic integer programming model solved by heuristic procedures, goal programming and simulated-annealing in a stochastic approach (Beliën and Demeulemeester (2007) and Beliën et al. (2009)). Another stochastic approach is due to van Oostrum et al. (2008).

In the third phase, each surgical case is scheduled to a specific operating room and day (sometimes referred to as *advance scheduling*) and to a specific period in the day or just ordering the surgeries scheduled for the same day (*allocation scheduling*). This phase is at an operational level and is referred to as *elective case scheduling*.

Studies focusing only on *advance scheduling* are due to Marcon et al. (2003), Hans et al. (2008) and Fei et al. (2009). Marcon et al. (2003) provide an integer linear programming model based on the multiple knapsack problem to assign elective surgeries to operating rooms minimizing the risk of no realization. Hans et al. (2008) address the problem of assigning elective surgeries to operating rooms in such a way that not only the utilization of surgical suite is optimized, but also the total overtime is minimized, preventing surgery cancelations. The authors developed a robust surgery loading by assigning a slack in addition to the planned surgeries on each operating room to cope with the stochastic durations of the planned surgeries. Fei et al. (2009) propose a column-generation-based heuristic in a deterministic approach to assign surgical cases to operating rooms and days. The integer programming model proposed by these authors complies with the availability of operating rooms and surgeons, and its objective seeks to minimize the cost of total unexploited opening hours and overtime.

For *allocation scheduling*, Hsu et al. (2003) present a tabu search approach for sequencing elective surgeries on a particular date such that the number of postanesthesia care unit nurses is minimized. Cardoen et al. (2009b,c) propose a multicriteria mixed integer linear model for sequencing elective surgeries. The authors use a weighted sum of the six objectives considered and solve the resulting problem by a branch-and-price approach. Lamiri and Xie (2006), Denton et al. (2007) and Lamiri et al. (2009) developed stochastic approaches for this part of the problem.

Guinet and Chaabane (2003) consider *elective case scheduling* in a unique problem. These authors propose an assignment model with resource and time-window constraints to assign patients to operating rooms, days and periods of time, in order to minimize patient intervention costs which are given by overtime cost and patient waiting time (defined by hospitalization cost). In this model, patients have a previous hospitalization date and thus planning surgery aims to minimize the patient waiting time in the hospital in face of this hospitalization date. So, the model allows overtime in order to assign all the surgeries considered. The model proposed was solved by a primal-dual heuristic.

Ozkarahan (1995) and Jebali et al. (2006) tackle both *advance scheduling* and *allocation scheduling* in two separate problems. Ozkarahan (1995) presents an integer linear programming model similar to the job shop makespan problem to assign elective surgeries to operating rooms in order to minimize makespan, ie the length of time required to complete all operations. A heuristic procedure is used to sequence the surgeries assigned in the first step. Jebali et al. (2006) propose a mixed integer linear programming model for each of the two steps.

We should note that, most of the approaches referred for the third stage do not address the earlier stages of the operating room planning. In addition, only Denton et al. (2007), Hans et al. (2008) and Cardoen et al. (2009b,c) present results applied in real cases.

Considering the definition of the operating room planning in a three stage process, this work falls within *elective case scheduling*.

This paper follows in section 2 with a description of the planning elective surgeries problem (PESP) and some hospital specifications. Section 3 proposes an integer linear programming model followed by the solution approach in section 4. Section 5 presents the four year historical data collected at the hospital under study and the results of the computational experiments undertaken with hospital data (section 6). Finally, the analysis of the solutions obtained is given in section 7 and section 8 reports the conclusions.

2 Problem description

2.1 Elective surgery

A surgery can be urgent or elective. Urgent surgeries are not in the aim of this paper. Hence, elective surgeries are not limited to a specific time and can be planned. An elective surgery can be either conventional or ambulatory. According to Administrative Regulation nb 45/2008 of 15 January (Ministry of Health, 2008), an ambulatory surgery is an elective surgery with hospital admission and discharge in a period less than 24 hours. Ambulatory surgery is also often called outpatient surgery and conventional surgery called inpatient surgery.

An elective surgery has a priority level associated which define its due date. Four priority levels are possible (Ministry of Health, 2008): Deferred Urgency (surgery must be completed in 72 hours); High Priority (surgery must be completed in 15 days); Priority (surgery must be completed in 2 months);

and Normal (surgery must be completed within 1 year).

2.2 Process and refund

A patient comes to the surgical suite from a hospital unit, even if it is an outpatient, and moves to a unit of recovery. In both units, for each patient, a bed, nurses and auxiliary staff in conformity are needed, with the main objective of maintaining a balanced flow of work.

The induction, the surgical procedure and the waking up all occur in the surgical suite. In the case study, these three procedures are made in the same space, the operating room. Hence, in this work, induction and waking up are included in surgery and treated as a single act.

Induction and waking up needs an anesthetic and a nurse with anesthetic functions (anaesthetist nurse), whereas surgical procedure requires one or more surgeons, two nurses (instrumentalist nurse and circulating nurse), auxiliary staff and equipment.

In the above context, maximizing the use of the surgical suite emerges as the key objective.

Surgery refund depends on the institution nature. If it is a public hospital, costs are borne by public finance under SNS, according to Administrative Regulation nb 132/2009 of 30 January (Ministry of Health, 2009), on the basis of patient classification on diagnosis-related groups. The type of surgical procedure contributes along with other variables, to classify a patient in a specific diagnosis-related groups. If it is a private hospital, costs are supported by the patient.

2.3 The case study

This work focuses on a general, central and university hospital in Lisbon, and integrated within the Portuguese National Health Service. It has no maternity or external urgencies and performs about 5 000 surgeries per year.

The hospital has five surgical specialties. Its surgical suite has six operating rooms, one reserved to ambulatory surgeries. All rooms in the surgical suite are equipped with the same basic equipment. Specialized equipment is mobile, however it must be moved few times due to its sensitivity and fragility. Practice in this hospital assigns rooms to surgical specialties. This leads to a situation in which the specialty can not be changed in the room during the day. Although the exchange of surgical specialty in a room during the day is technically possible, this would require a downtime of the operating room in about one hour.

Between two surgeries performed in the same room, cleaning and disinfecting protocols must take place, which are performed by some auxiliary staff and takes about 30 minutes.

Each operating room has a fixed and permanent nurse team assigned throughout the surgical suite regular time.

Each Patient is assigned to a surgeon at the time of waiting list inscription and, therefore, when planning, patient and surgeons are already assigned.

Currently, surgical suite regular work schedule is between 8 am and 8 pm, from Monday to Friday.

Surgery planning is performed on a weekly base and is finalized on Friday for the following week. Planning is independent per specialty. The planning maps for each specialty ward are sent to the surgical suite where they are conferred and the combined implementation is verified. Changes to the planning can occur during the week and must necessarily be proposed until 12 am of the previous day.

Note that, in the model developed, the problem was treated as a whole and all surgical specialties were considered together for the planning of surgeries in the surgical suite. In addition, the model does not consider the possibility of changing surgical specialty in any room during one day, according to a request from the hospital director. Changes to the planning lead to another problem of rescheduling that is not in the aim of this paper.

It is assumed that the resources do not limit the activity of the surgical suite, namely beds, nurses, auxiliary staff and equipments.

3 An ILP Model

On Friday, a set of surgeries C is considered for scheduling in the next planning week. The following subsets of C are defined: by specialty $j \in E$ (being E the set of surgical specialties) - C_j^e; and by priority level $i \in \{1,2,3,4\}$ - C_i^p, with $i = 1$ being Deferred Urgency, $i = 2$ High Priority, $i = 3$ Priority and $i = 4$ Normal priority level. The set $C^n = C \setminus C_1^p$ consists of surgeries considered for scheduling that are not classified as Deferred Urgency priority level.

To perform these surgeries there is a set of surgeons H.

Surgeries must be scheduled to a room in the set of rooms S, and a day in the set of days available for scheduling, D.

Time has a discrete representation as a set of time periods available for scheduling, T.

Surgery $c \in C$ has surgeon $h_c \in H$ assigned, belongs to surgical specialty $e_c \in E$ and is expected to last d_c minutes. The latter parameter leads to the number of time periods needed for the execution of surgery c - t_c -, and consequently to the possible time periods to start surgery c in order to be completed within the surgical suite regular time, given by $T_c \subseteq T$. Overtime is not allowed in planning and is unable by restricting the variables domain to the respective parameter T_c.

Daily and weekly operating time limits for each surgeon $h \in H$ are represented, respectively, by $T_{hd}^{MAX_D}$ and $T_h^{MAX_S}$.

The parameter i_{ctd} reflects the impossibility of surgery $c \in C$ start at period $t \in T_c$ and day $d \in D$ due to surgeon or patient unavailability.

Since surgeries are non-preemptive jobs, starting time variables were considered to problem formulation.

As the planning for the following week is finalized on Friday, surgeries with Deferred Urgency priority level must be completed on Monday in order to respect the deadline of 72 hours, and thus have the following distinguished set of variables:

$$x_{cst1} = \begin{cases} 1, & \text{if Deferred Urgency priority surgery } c \text{ starts at period } t \text{ on day 1 in room } s \\ 0, & \text{otherwise} \qquad\qquad (c \in C_1^p,\ s \in S,\ t \in T_c) \end{cases}$$

For the remaining surgeries,

$$x_{cstd} = \begin{cases} 1, & \text{if non-Deferred Urgency priority surgery } c \text{ starts at period } t \text{ on day } d \text{ in room } s \\ 0, & \text{otherwise} \qquad\qquad (c \in C^n,\ s \in S,\ t \in T_c,\ d \in D) \end{cases}$$

Additional variables were also considered to daily register the surgical specialty assigned to each room:

$$y_{jsd} = \begin{cases} 1, & \text{if a surgery of specialty } j \text{ starts in room } s \text{ on day } d \\ 0, & \text{otherwise} \qquad\qquad\qquad\qquad (j \in E,\ s \in S,\ d \in D) \end{cases}$$

Variables y_{jsd} can be avoided in the formulation of the problem. However, the introduction of these variables in the model, although slightly increasing the total number of variables, significantly reduces the number of constraints that prevent the use of any operating room for more than one surgical specialty on the same day. In addition, preliminary experiments showed better results using these additional variables.

The integer linear programming model to plan elective surgeries becomes:

$$\max \quad \sum_{c \in C_1^p} \sum_{s \in S} \sum_{t \in T_c} t_c\ x_{cst1} + \sum_{c \in C^n} \sum_{s \in S} \sum_{t \in T_c} \sum_{d \in D} t_c\ x_{cstd} \tag{1}$$

subject to:

$$\sum_{s \in S} \sum_{t \in T_c} x_{cst1} = 1\ ,\ \forall\ c \in C_1^p \tag{2}$$

$$\sum_{s \in S} \sum_{t \in T_c} \sum_{d \in D} x_{cstd} = 1\ ,\ \forall\ c \in C_2^p \tag{3}$$

$$\sum_{s \in S} \sum_{t \in T_c} \sum_{d \in D} x_{cstd} \leq 1\ ,\ \forall\ c \in C \setminus (C_1^p \cup C_2^p) \tag{4}$$

$$\sum_{c \in C} \sum_{\substack{t'=t-t_c-1 \\ t' \in T_c}}^{t} x_{cst'1} \leq 1\ ,\ \forall\ s \in S,\ t \in T \tag{5}$$

$$\sum_{c \in C^n} \sum_{\substack{t'=t-t_c-1 \\ t' \in T_c}}^{t} x_{cst'd} \leq 1\ ,\ \forall\ s \in S,\ t \in T,\ d \in D \setminus \{1\} \tag{6}$$

$$\sum_{s \in S} x_{cst1} \leq i_{ct1}\ ,\ \forall\ c \in C_1^p,\ t \in T_c \tag{7}$$

$$\sum_{s \in S} x_{cstd} \leq i_{ctd}\ ,\ \forall\ c \in C^n,\ t \in T_c,\ d \in D \tag{8}$$

$$\sum_{j \in E} y_{jsd} \leq 1\ ,\ \forall\ s \in S,\ d \in D \tag{9}$$

$$\sum_{c \in C_j^e} \sum_{t \in T_c} x_{cst1} \leq y_{js1}\ |T|\ ,\ \forall\ j \in E,\ s \in S \tag{10}$$

$$\sum_{c \in (C_j^e \cap C^n)} \sum_{t \in T_c} x_{cstd} \leq y_{jsd}\ |T|\ ,\ \forall\ j \in E,\ s \in S,\ d \in D \setminus \{1\} \tag{11}$$

$$\sum_{\substack{c \in C: \\ h_c = h}} \sum_{\substack{t'=t-t_c+1 \\ t' \in T_c}}^{t} \sum_{s \in S} x_{cst'1} \leq 1\ ,\ \forall\ h \in H,\ t \in T \tag{12}$$

$$\sum_{\substack{c \in C^n: \\ h_c = h}} \sum_{\substack{t'=t-t_c+1 \\ t' \in T_c}}^{t} \sum_{s \in S} x_{cst'd} \leq 1\ ,\ \forall\ h \in H,\ d \in D \setminus \{1\},\ t \in T \tag{13}$$

$$\sum_{\substack{c \in C: \\ h_c = h}} \sum_{s \in S} \sum_{t \in T_c} t_c x_{cst1} \leq T_{h1}^{MAX_D} \ , \ \forall \ h \in H \tag{14}$$

$$\sum_{\substack{c \in C^n: \\ h_c = h}} \sum_{s \in S} \sum_{t \in T_c} t_c x_{cstd} \leq T_{hd}^{MAX_D} \ , \ \forall \ d \in D \setminus \{1\}, \ h \in H \tag{15}$$

$$\sum_{\substack{c \in C_1^p: \\ h_c = h}} \sum_{s \in S} \sum_{t \in T_c} t_c x_{cst1} + \sum_{\substack{c \in C^n: \\ h_c = h}} \sum_{d \in D} \sum_{s \in S} \sum_{t \in T_c} t_c x_{cstd} \leq T_h^{MAX_S} \ , \ \forall \ h \in H \tag{16}$$

$$x_{cst1} \in \{0,1\} \ , \ \forall \ c \in C_1^p, \ s \in S, \ t \in T_c \tag{17}$$

$$x_{cstd} \in \{0,1\} \ , \ \forall \ c \in C^n, \ s \in S, \ t \in T_c, \ d \in D \tag{18}$$

$$y_{jsd} \ \in \{0,1\} \ , \ \forall \ j \in E, \ s \in S, \ d \in D \tag{19}$$

In the above model, objective function (1) maximizes surgical suite occupation.

Constraint set (2) forces Deferred Urgency level priority surgeries to be scheduled on Monday to meet the deadline of 72 hours for their completion. Constraint set (3) imposes High Priority surgeries to be scheduled during the planning week. Constraints (4) state that the remaining surgeries, classified as Priority or Normal, may be scheduled or not during the planning week.

Constraints (5) and (6) guarantee that surgeries do not overlap in the same room. These constraints also impose two empty periods for cleaning the room at the end of each surgery.

Constraints (7) and (8) give the possibility to consider surgeons or patients unavailability periods.

Constraint set (9) prevents the assignment of more than one surgery specialty to each room and day. Therefore, it is not allowed to exchange surgery specialty in the room during the day.

Constraints (10) and (11) are the linking constraints for variables x and y.

Constraints (12) and (13) assure that surgeons do not overlap between rooms in the same time period and day.

Constraint sets (14), (15) and (16) impose daily and weekly operating time limit to each surgeon.

Finally, constraints (17), (18) and (19) express variables domain.

4 Solving approach

Model (1) - (19) is highly complex and attains a large dimension in real instances. In fact it has $o(|C| \times |S| \times |T| \times |D|)$ variables and $o(|C| \times |T| \times |D|)$ constraints.

Hence, the elective surgeries planning problem was decomposed into two hierarchical phases. Representing a greater number of surgeries and the planning of five operating rooms, conventional surgeries are planned in the first phase. Considering the plan obtained in the first phase, ambulatory surgeries are planned in the second phase.

In each phase, an integer linear programming (ILP) solver was used with limited time. In case of stopping without optimality, the best feasible integer solution obtained was improved using a simple improving heuristic.

The improving heuristic developed can be summarized in the following four steps:

1. Re-schedule surgeries as early as possible in the day, keeping the same ordering.

2. Try to schedule unscheduled surgeries in the time available at the end of each day, respecting each room surgery specialty and assuring that each surgery is completed within surgical suite regular time.

3. Try to exchange two or three consecutive scheduled surgeries by one unscheduled surgery with a non superior duration.

4. Try to exchange the last surgery scheduled at the end of the day by one unscheduled surgery fulfilling the remainder of the regular time in the day.

Every heuristic step must be performed taking into account feasibility, defined by constraints (2) - (19).

The first step rearranges surgeries scheduled to enable scheduling more surgeries in step 2, allowing a better utilization of the surgical suite and thus an improvement in the value of the objective function.

Step 3 reports directly to the objective function, since the exchange of two or three consecutive surgeries by another one with a non superior duration avoids the empty periods for cleaning the operating room between any pair of surgeries and, therefore, the value of the objective function increases.

The last step aims to complete the daily use of each operating room in the surgical suite, replacing the last scheduled surgery by another occupying the room until the end of regular time.

Note that while the first step only serves to support the second, not contributing directly to change the solution value, each of the following three steps directly allows the increase of the objective function value.

5 Data available

The hospital under study provided a historical record containing information on all surgeries performed in the hospital surgical suite from 1 January 2004 to 28 December 2007. In this period, 21 050 surgeries were performed.

Tables 1 and 2 describe, respectively, duration of conventional and ambulatory surgeries performed, aggregated by surgical specialty.

Table 1: Descriptive of conventional surgeries duration in the historical record (in minutes)

Surgical specialty	Mean	Median	St. deviation	Minimum	Maximum	Number
Otorhinolaryngology	92	77	61	0	410	2 371
Digestive and General Surgery	75	61	53	0	562	6 529
Thorax Surgery	110	96	62	5	476	2 148
Urology	72	54	58	2	473	4 165
Angiology and Vascular Surgery	72	54	54	1	413	1 652

These tables show that the median value is lower than the mean value for all surgical specialties. The difference is less significant in the case of ambulatory surgeries.

In addition to this historical record, another one was provided referring to the waiting list for surgery in seven different moments in which decisions about the week planning occurred (Friday) as

Table 2: Descriptive of ambulatory surgeries duration in the historical record (in minutes)

Surgical specialty	Mean	Median	St. deviation	Minimum	Maximum	Number
Otorhinolaryngology	28	27	13	2	107	1 484
Digestive and General Surgery	41	40	22	1	155	1 359
Urology	31	27	18	0	147	412
Angiology and Vascular Surgery	40	38	18	5	132	488

well as the respective hospital week planning and hospital record of each of those weeks. The weeks provided are the ones starting on 12 and 26 February 2007, 5, 12, 19 and 26 March 2007 and 2 April 2007.

Table 3: Descriptive of waiting lists for conventional and ambulatory surgeries in two weeks

	9 February		23 February	
	Conventional	Ambulatory	Conventional	Ambulatory
Number of surgeries	2043	264	1984	274
Surgical specialty (%)				
Otorhinolaryngology	9,9	15,5	11,7	19,0
Digestive and General Surgery	52,9	36,8	53,1	37,6
Thorax Surgery	5,0	-	4,6	-
Urology	18,2	3,4	16,9	2,9
Angiology and Vascular Surgery	14,0	44,3	13,7	40,5
Priority level (%)				
Deferred Urgency	0,98	0,7	1,2	0
High Priority	0,05	0	0	0
Priority	3,67	2,3	2,8	2,9
Normal	95,3	97,0	96	97,1

Table 3 presents a brief description of the waiting list for surgery in the respective moments of decision (Friday) for the first two weeks provided. These two weeks were selected because they were considered representative of all weeks provided by the hospital, namely on the level of surgeries classified as Deferred Urgency.

This table shows that Digestive and General Surgery is the surgical specialty most widely represented on the waiting list for conventional surgeries, corresponding to more than 50% of surgeries and more than ten times Thorax Surgery specialty, the least represented on the waiting list for conventional surgery.

On waiting lists for ambulatory surgery, Angiology and Vascular Surgery specialty is slightly more represented than Digestive and General Surgery, being Urology specialty the least represented on the waiting lists. Note that Thorax Surgery specialty does not perform ambulatory surgeries.

Almost all of both types of surgeries on the waiting list are Normal priority surgeries. The second week has a higher percentage of surgeries classified as Deferred Urgency on the waiting list for conventional surgeries.

6 Computational experiments

A computational experiment was developed to test the solution approach described in section 4 with real data from the hospital. Tests focused on the two weeks described above.

Surgeons and patient unavailability, represented by constraints (7) and (8), were not included in conventional surgeries planning since it had not been possible to obtain data in accordance. These sets of constraints were included in ambulatory surgery planning to ensure feasibility of the whole week planning, linking conventional and ambulatory surgeries planning. In addition, total time spent by each surgeon in conventional surgeries schedule was reduced in total daily and weekly operating time limit to be used in constraints (14), (15) and (16) for ambulatory surgeries planning.

Despite surgical suite regular working hours begin at 8 am, hospital planning do not consider surgeries starting before 8h30 am. Hence this is the time defined to start scheduling when testing the model.

Conventional surgeries week planning includes five rooms while ambulatory surgeries week planning includes only one, totalizing six rooms in the surgical suite.

Expected duration for each surgery was based on mean and median values obtained in the historical data for the same surgical procedure. If no surgery in the historical data was performed with the same surgical procedure, expected surgery duration gets the mean and median values for its surgery specialty.

Time periods of 15 minutes were used thus creating 46 daily time periods in regular working time. Overtime is not allowed in planning.

Daily and weekly operating time limit to each surgeon was based in a percentage of their daily and weekly working hours. For all surgeons the operating time limit of 75% of a 8h working day and 60% of a 42h working week was considered.

Instances tested denomination includes a reference to the number of surgeries considered for planning ($|C|$) and, in subscript, a reference to the type of expected duration used (respectively, 1 and 2 for mean and median values). Symbols $\mathcal{A}$ and $\mathcal{C}$ denote, respectively, ambulatory and conventional planning.

Since the waiting list for conventional surgery is too long, a subset of surgeries was considered as input for the model proposed for PESP, constituting set C. The selection was made based on the surgeries included in hospital planning, in order to enable a comparative basis, followed by an order of priority until fulfill the required number of surgeries ($|C|$).

The ILP models were solved using CPLEX 11.0 with CONCERT 2.5 (ILOG, 2003, 2004). The improving heuristic was coded in C++ language. Tests were performed in a Core2 Duo, 2.53 GHz computer with 4GB of RAM. Time limit to run ILP model with CPLEX was set to 30 000 seconds (8 hours).

The results obtained are shown in tables 4 and 5. Column 1 refers to the instance tested and columns 2 and 3 to its dimension given by the number of variables and constraints, respectively. *LP Time* is the time to solve the linear relaxation. Columns 5 and 6 refer to the gap obtained within Cplex time limit and to the time needed to reach this gap. *H Time* is the time used by the improving heuristic and *Final Gap* the gap associated to the heuristic solution.

In fact, the model reaches high dimensions in real instances. The instances that used the median value for the expected surgery duration have more variables than the instances that used the mean value. This observation is justified by the fact that the median value is, in general, lower than the mean value, thus increasing the number of time periods in T_c. Ambulatory surgery instances have

Table 4: Results obtained for conventional surgeries planning

Instance	Variables	Constraints	LP Time (sec.)	IP Gap (%)	Time to Gap (sec.)	H Time (sec.)	Final Gap (%)
Planning week: 12-16 February 2007							
$Pw1\mathcal{C}\_250_1$	242 200	11 934	1 571,76	9,97	13 895,50	0	6,55
$Pw1\mathcal{C}\_250_2$	243 525	11 934	435,71	5,27	12 865,30	0	5,17
$Pw1\mathcal{C}\_300_1$	294 225	11 984	616,66	10,02	6 526,52	0	3,78
$Pw1\mathcal{C}\_300_2$	295 800	11 984	595,17	7,65	9 131,55	0	5,76
$Pw1\mathcal{C}\_500_1$	503 625	12 184	444,52	3,74	18 743,20	0	3,46
$Pw1\mathcal{C}\_500_2$	506 725	12 184	355,71	12,98	9 677,82	0	5,83
$Pw1\mathcal{C}\_1000_1$	1 035 500	13 864	746,54	o.m.	-	-	-
$Pw1\mathcal{C}\_1000_2$	1 041 525	13 864	672,01	o.m.	-	-	-
Planning week: 26-2 March 2007							
$Pw2\mathcal{C}\_250_1$	240 370	11 698	459,21	8,03	5 592,39	0	6,13
$Pw2\mathcal{C}\_250_2$	241 720	11 698	1 704,54	-	30 000	-	-
$Pw2\mathcal{C}\_300_1$	293 570	11 984	619,68	-	30 000	-	-
$Pw2\mathcal{C}\_300_2$	295 195	11 984	613,91	7,01	8 017,33	0	5,19
$Pw2\mathcal{C}\_500_1$	500 845	12 420	363,84	o.m.	-	-	-
$Pw2\mathcal{C}\_500_2$	503 620	12 420	678,96	12,19	11 933,2	0	6,36
$Pw2\mathcal{C}\_1000_1$	1 030 895	13 864	825,46	o.m.	-	-	-
$Pw2\mathcal{C}\_1000_2$	1 037 095	13 864	821,22	o.m.	-	-	-

o.m. - out of memory

Table 5: Results obtained for ambulatory surgeries planning

Instance	Variables	Constraints	LP Time (sec.)	IP Gap (%)	Time to Gap (sec.)
Planning week: 12-16 February 2007					
$Pw1\mathcal{A}\_264_1$	57 768	69 595	1,92	0	7,72
$Pw1\mathcal{A}\_264_2$	57 803	69 630	1,75	0	19,78
Planning week: 26-2 March 2007					
$Pw2\mathcal{A}\_274_1$	60 455	72 056	2,20	0	56,38
$Pw2\mathcal{A}\_274_2$	60 475	72 076	2,22	0,76	202,40

less variables than conventional surgery instances since they have fewer surgeries and include only one operating room. However, the number of constraints in ambulatory surgery instances increases significantly due to the inclusion of constraints (7) and (8) in ambulatory surgeries planning, not considered in conventional surgeries planning.

In conventional surgeries planning, the instances that considered 1000 surgeries were unsolved. In the second planning week, three more instances were not solved, two of which failed to obtain an integer solution at the end of the 30 000 seconds and the other stopped by out of memory. This is due to the fact that the percentage of surgeries classified as Deferred Urgency is higher in the second week tested. In all instances tested the improving heuristic used zero seconds and effectively improved the *IP Gap* obtained in all but one of the instances tested.

Since the ambulatory surgeries planning is of smaller dimension, the model obtained an optimal solution in less than two minutes in all instances tested, and therefore the use of the heuristic was not necessary.

7 Comparative analysis

To analyze week plans resulting from the approach proposed in this paper, the final solution from $Pw1\mathcal{C}\_300_1 + Pw1\mathcal{A}\_264_1$ instance was explored. It refers to the planning week from 12 to 16 February 2007. This planning week was arbitrarily chosen. The remaining showed similar behavior.

Figure 1 shows the proposed schedule as well as the respective hospital planning for the corresponding week. For each day, figure 1 shows the hospital plan (above) and the proposed plan (below).

It is possible to see that the proposed plan verifies all conditions imposed by the hospital, namely those of respecting surgical suite regular working hours, bounded by the black line at the end of each day, and periods for room cleaning after each surgery (30 minutes corresponding to two periods in white after each surgery). However, figure 1 also shows that hospital is planning without considering time to clean the room (see e.g. Monday rooms E and F) and using surgical suite overtime (see e.g. Monday room F).

Time to prepare the operating room for exchange of surgical specialty, about an hour, is also not being considered in the hospital plan (see e.g. Tuesday and Thursday room E). As requested by the hospital director, in the proposed plan this exchange is not allowed.

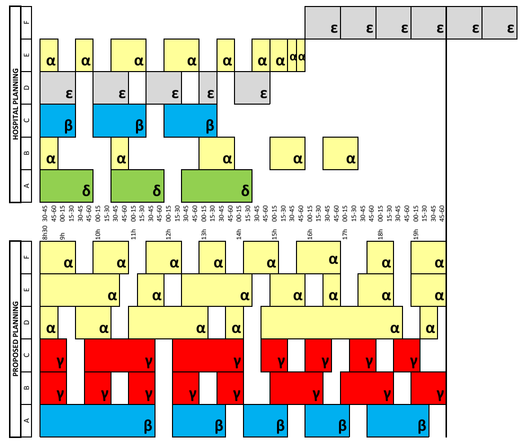
Figure 1 also shows many empty periods in the hospital plan indicating an inefficient use of the surgical suite.

In order to analyze the approach suggested in this paper, it is also interesting to have an insight on how the proposed plan can behave in "reality". With all data collected, the proposed plan viability can be analyzed by taking into account the difference between the real surgery duration and the expected duration used for planning and its impact on the plan implementation. So, the real duration of the scheduled surgeries was used for a proposed plan simulation. Surgeries start on the time scheduled unless previous surgeries took more time than expected. Starting surgeries in overtime was not allowed in this simulation, so when this would occur, surgeries were canceled and therefore would not be held. The exception is the surgeries classified as Urgency Deferred priority level. Another possibility requiring delay of a surgery is when the respective surgeon was performing another delayed surgery in another operating room at the time expected to begin.

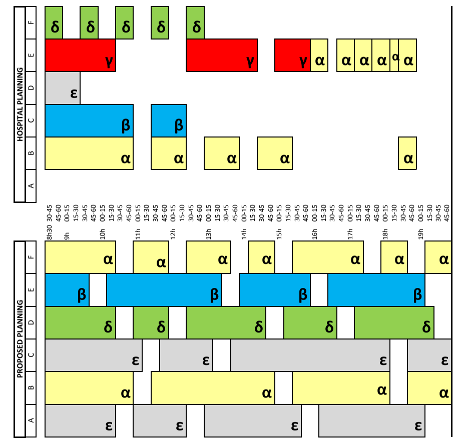
The resulting simulation can be seen in figure 2 together with the hospital record, both for the

Figure 1: Hospital plan and proposed plan

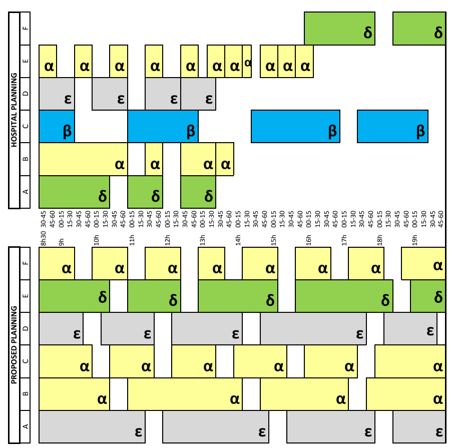
(a) Monday - 12 February 2007



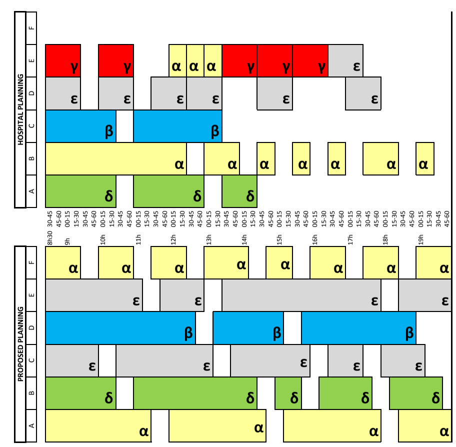
(b) Tuesday - 13 February 2007



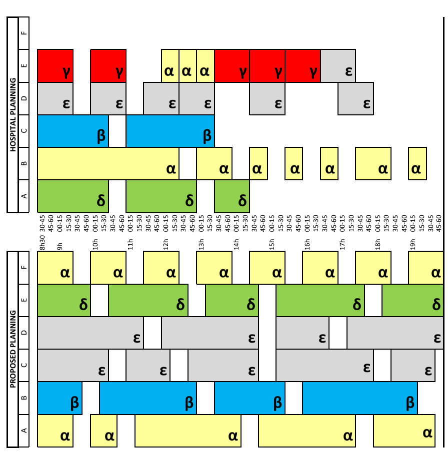
(c) Wednesday - 14 February 2007



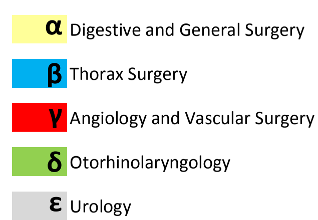
(d) Thursday - 15 February 2007



(e) Friday - 16 February 2007



(f) Legend



same planning week as figure 1. Similarly to the previous figure, each day figure shows hospital record (above) and the simulation of the proposed plan (below). Note that in the hospital record some surgeries not planned were added during the week and these are not considered in the proposed plan simulation.

Empty periods on the simulation diagram indicate operating room inactivity caused by surgeries that took less than expected, except for the two time periods after each surgery reserved for the room cleaning.

It is possible to see in figure 2 that some surgeries go beyond the regular time black line, indicating the use of time periods in overtime both for the hospital record and the simulation of the solution proposed. However, while in hospital record overtime can be used to start up surgeries (see e.g. Monday room C and Thursday room D), in the simulation of the solution proposed this is used only to complete the surgeries already started in regular time.

Based on the same instance in analysis, tables 6, 7 and 8 provide a balance of the week under review based on some indicators. The analysis was performed on the week planning and record/simulation for the information of the hospital (*Hospital plan* and *Hospital record*), the solution obtained from the model developed (without using improving heuristic - *IP plan* and *IP simulation*), and for the proposed solution (solution obtained from the model with the improving heuristic - *Proposed plan* and *Prop. plan simulation*).

Table 6 shows the number of time periods planned and used in regular and overtime, as well as the weight of each of the contributors to the difference between the plan and the record/simulation. These contributors are the differences between the real duration and the expected duration, canceled surgeries duration and added surgeries duration in the case of hospital information.

It is easy to verify that the plan resulting from the approach proposed in this paper allows a better use of the hospital surgical suite, planning more than twice the regular time periods compared with hospital plan.

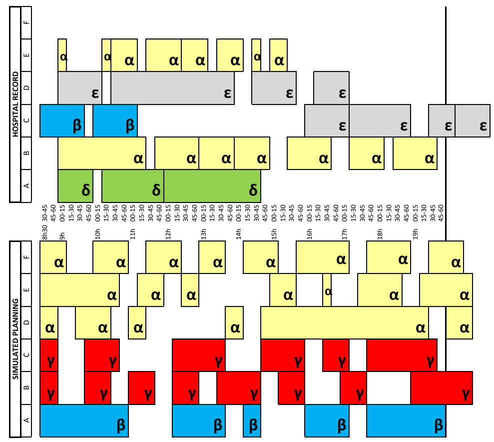
In the hospital information, table shows that the number of time periods used is higher than the number of time periods planned. This is justified not only by the possibility of adding surgeries during the week, but also by under prediction of surgeries duration used in hospital planning. Indeed, among the three cases compared, the hospital plan/record is the only one with positive balance (67 time periods) in the total difference between the real and expected duration of the surgeries performed, ie the expected duration is below the respective real duration in the week balance.

Either in the approach without using the improving heuristic, or in the proposed approach, the number of time periods in simulation was lower than in the plan due not only to canceled surgeries, but also to the negative balance of the total difference between the real and expected duration of the surgeries performed (-48 and -19, respectively). This suggests an over prediction of surgeries duration in this planning approach. Note that the instance analyzed used the mean value for the same type of surgical procedure as expected duration for each surgery, and the median is lower than the mean.

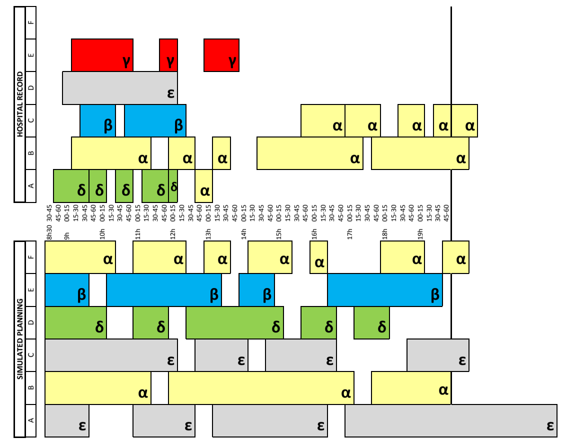
The use of the improving heuristic enables an increase on the number of time periods booked and used, being in accordance with the problem objective function, although in this case the number of surgeries performed is slightly lower than in the simulated solution obtained without using the improving heuristic.

The number of surgeries performed was always less than the number of surgeries planned. The biggest difference verified in the case of proposed approach makes the percentage of canceled surgeries

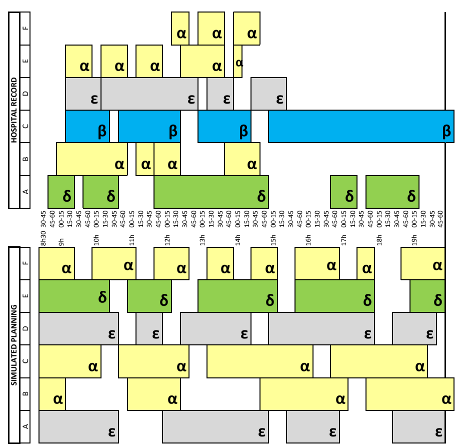
Figure 2: Hospital record and proposed plan simulation



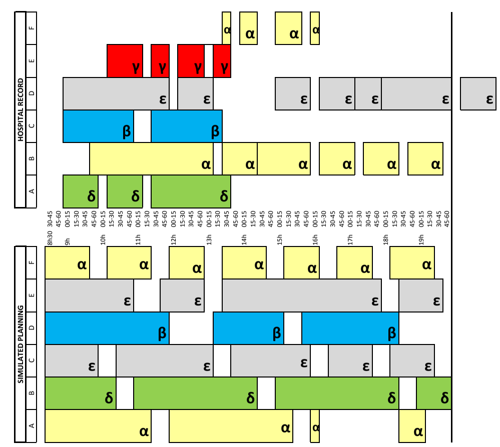
(a) Monday - 12 February 2007



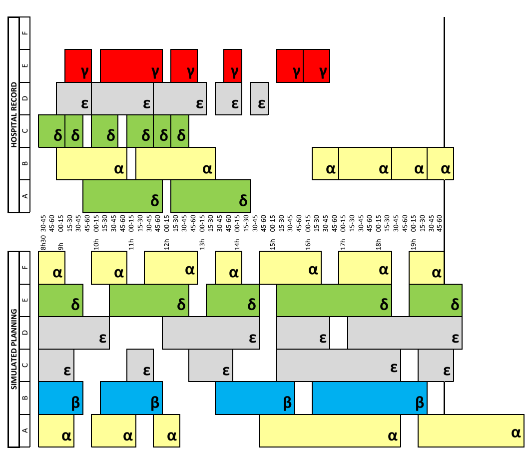
(b) Tuesday - 13 February 2007



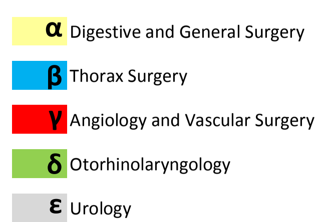
(c) Wednesday - 14 February 2007



(d) Thursday - 15 February 2007



(e) Friday - 16 February 2007



(f) Legend

Table 6: Week balance: production indicators

	Time periods booked/used			Cause			Surgeries
	Regular time	Overtime	Total	Different duration	Surgery canceled	Surgery added	
Hospital plan	518	10	528				131
Hospital record	590	16	606	28,9%	33,2%	37,9%	127
IP plan	1 039	0	1 039				162
IP simulation	920	52	972	71,6%	28,4%	-	158
Proposed plan	1 088	0	1 088				161
Prop. plan simulation	991	42	1 033	34,5%	65,5%	-	154

Table 7: Week balance: operating surgeon service indicators

	Surgeons with no surgeries scheduled	Operating time periods per surgeon		
		Minimum	Mean	Maximum
Hospital plan	34,4%	2	12,9	48
Hospital record	36,1%	1	15,2	54
IP plan	34,4%	3	25,98	97
IP simulation	34,4%	3	24,3	102
Proposed plan	34,4%	3	27,2	98
Prop. plan simulation	36,1%	3	26,5	97

Table 8: Week balance: general surgical suite performance indicators

	Regular time occupancy rate (%)	Waiting list reduction rate (%)
Hospital plan	37,54	5,68
Hospital record	42,75	5,47
IP plan	75,29	7,02
IP simulation	66,67	6,85
Proposed plan	78,84	6,98
Prop. plan simulation	71,81	6,68

higher than in the approach without using the improving heuristic.

Table 7 summarizes surgeon analysis in respect to surgical service distribution. The table shows the percentage of surgeons who do not have surgeries scheduled throughout the week and, for the remaining, the minimum and maximum number of time periods planned/performed and the respective mean value.

It is possible to verify that the percentage of surgeons with no surgeries planned during the week is almost the same for the three comparisons presented.

In the case of surgeons with surgical service planned during the week, table shows that the proposed approach significantly increases both the average and the maximum week operating time periods. Note that mean operating time per surgeon in hospital solution is about 42 minutes on average per day and in proposed approach this increases to 1h21m on average per day.

Table 8 provides generally used indicators for surgical suite performance, namely the occupancy rate in regular time and waiting list reduction rate. These are generic indicators that derive naturally from the indicators presented in table 6.

8 Conclusions

This paper considers jointly *advance scheduling* and *allocation scheduling*, and thus assigns elective surgeries to an operating room, a day and a specific period of time. Guinet and Chaabane (2003) is the only work found in the literature that also considers these two steps in the same problem. The major difference between the two approaches is that, in our case nothing is known about the possible date for each surgical case on the waiting list, while in the approach from Guinet and Chaabane (2003) each patient has a previous hospitalization date. Thus, in the latter approach there are costs associated with the time from the hospitalization date to the surgical intervention date. In the present approach, the hospitalization date is defined in accordance to intervention date and therefore the objective is to maximize operating rooms utilization. Hence, the methodologies and results are not comparable.

The elective surgery plan produced by this approach is feasible and follows the necessary requirements imposed by the hospital in study. Furthermore, this approach includes more conditions that in practice have not been considered up to now.

As shown in table 8, the approach proposed in this paper allows a more efficient use of the hospital surgical suite, increasing the surgery waiting list reduction rate.

The analysis performed also shows that the proposed plan simulation uses much periods in overtime. However, this can easily be overcome by using accurate estimates for surgery duration. Therefore, this is not resulting from the approach proposed.

Thus, the approach proposed in this work provides an effective and innovative response to the current requirements in this specific area of health care, proposing an alternative method for planning elective surgeries in a real case.

Acknowledgements

The authors would like to thank Dr. Manuel Delgado for his enthusiastic interest in this work, Dra. Margarida Baltazar for her patient provision of all necessary data and Enfª Fátima Menezes for the friendly and patient description of all the process.

This research is partially supported by Portuguese Foundation for Science and Technology under project POCTI/ISFL-1/152.

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