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## RESEARCH ARTICLE

# Variation Propagation Modeling for Multi-station Machining Processes with Fixtures based on Locating Surfaces

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Modeling the dimensional variation propagation in multi-station machining processes (MMPs) has been studied intensively in the past decade to understand and reduce the variation of product quality characteristics. Among others, the Stream-of-Variation (SoV) model has been successfully applied in a variety of applications, such as fault diagnosis, process planning and process-oriented tolerancing. However, current SoV model is limited to the MMPs where only fixtures with punctual locators are applied. Other types of fixtures, such as those based on locating surfaces, have not been investigated yet. In this paper, the derivation of SoV model is extended to model the effect of fixture- and datum-induced variations when fixtures with locating surfaces are applied. Due to the hyperstatic nature of these fixtures, different workholding configurations can be adopted. This will increase the dimension of the SoV model exponentially and thus, may make the model-based part quality prediction extremely complex. This paper presents how to reduce the complexity of the SoV model when fixtures based on locating surfaces are applied and how to evaluate the worst-case approach of the resulting part quality.

**Keywords:** Part quality; SoV; Fixtures; Variation Propagation Model; Locating Surfaces

## Nomenclature

### Acronyms

*SoV*: Stream-of-Variation.

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$MMP$: Multi-station Machining Process.
$CS$: Coordinate System.
$DCS$ ($D$): Design Coordinate System.
$FCS$ ($F$): Fixture Coordinate System.
$RCS$ ($R$): Reference Coordinate System.
$DMV$: Differential Motion Vector.
$DTV$: Differential Transformation Matrix.
$HTM$: Homogeneous Transformation Matrix.
$KPC$: Key Product Characteristic.

**Mathematical symbols for SoV model**

$\mathbf{A}_{k-1}$: Matrix that represents how the variations are transmitted by datum features generated before station $k$. Matrix defined by matrices $\mathbf{A}^1_{k-1}$, $\mathbf{A}^2_{k-1}$, $\mathbf{A}^4_{k-1}$ and $\mathbf{A}^5_{k-1}$.
$\mathbf{B}_k$: Matrix that represents how fixture and machining deviations affect part quality at station $k$. Matrix defined by matrices $\mathbf{B}^f_k$ and $\mathbf{B}^m_k$.
$\mathbf{C}_k$: Matrix that represents how the deviations of part surfaces are related to the deviations of KPCs inspected after station $k$.
$\mathbf{u}_k$: Fixture and machining deviations at station $k$.
$\mathbf{v}_k$: Measurement noise of the inspection process after station $k$.
$\mathbf{w}_k$: Un-modeled system noise and linearization errors at station $k$.
$\mathbf{x}_k$: Dimensional deviations of part surfaces at station $k$.
$\mathbf{y}_k$: Deviations of the KPCs inspected after station $k$.

**General mathematical symbols**

$\mathbf{d}^R_F$ : Small translational deviations of $F$ with respect to (w.r.t.) $R$, $\left[d^R_{Fx}, d^R_{Fy}, d^R_{Fz}\right]^T$.
$\mathbf{H}^D_F$ : HTM of $F$ w.r.t. $D$.
$\mathbf{p}^F_i$ : Position vector of point $i$ w.r.t. $F$, $\left[p^F_{ix}, p^F_{iy}, p^F_{iz}\right]^T$.
$\mathbf{x}^R_F$ : DMV of $F$ w.r.t. $R$, $\left[(\mathbf{d}^R_F)^T, (\boldsymbol{\theta}^R_F)^T\right]^T$.
$\boldsymbol{\theta}^R_F$ : Small orientational deviations of $F$ w.r.t. $R$, $\left[\theta^R_{Fx}, \theta^R_{Fy}, \theta^R_{Fz}\right]^T$.
$\delta\mathbf{H}^D_F$ : HTM for small position and orientation deviations from nominal values of $F$ w.r.t. $D$.
$\boldsymbol{\Delta}^D_F$ : DTM of $F$ w.r.t. $D$.

## 1. Introduction

Multi-station machining processes (MMPs) are widely applied in order to ensure modularity, flexibility, and reconfigurability of modern manufacturing systems. As abstractively illustrated in Figure 1, a MMP possesses the following common characteristics: (i) a series of machining operations are performed at multiple stations to sequentially generate designated features of products; (ii) some features generated from an upstream station are used as the datum features at downstream stations; and (iii) due to the inter-station operational dependency, variations of quality characteristics will be propagated from upstream stations to downstream stations.

The manufacturing variation propagation poses great challenges to the product and manufacturing process design for MMPs. The traditional product-oriented approach also called *over-the-wall design*, prevents the integration of design and manufacturing activities to improve product development (Ceglarek *et al.* 2004). In order to overcome this limitation and implement a process-oriented integrated product/process design and reduce the ramp-up time, it is desirable to establish a mathematical model of the 3-dimensional (3D) variation propagation along stations and evaluate the impacts of variation sources on product quality integrity.

There are mainly two types of 3D variation propagation modeling approaches for MMPs: Model of Manufacture Part (MoMP, (Villeneuve *et al.* 2001)) and Stream-of-Variation model (SoV, (Shi 2007, Liu 2010)). While both modeling approaches are fundamentally similar, SoV model is preferable to the MoMP for process-oriented activities, such as fault diagnosis and process planning, whereas the MoMP is preferable to the SoV model for product-oriented activities, such as product tolerance analysis and synthesis (Abellán-Nebot *et al.* 2012a).

The SoV model was developed for multi-station assembly processes (Jin and Shi 1999) and was later adapted for MMPs (Zhou *et al.* 2003). The well-known state space model from control theory (Ogata 2001) is adopted to mathematically represent the relationship between the variation sources (e.g., fixture mislocation) in MMPs and the quality deviations of the machined surfaces (e.g., the depth of a distance between two surfaces) generated at each station. More importantly, the SoV approach explicitly models the propagation of the quality deviations from upstream stations to downstream stations through datum schemes.

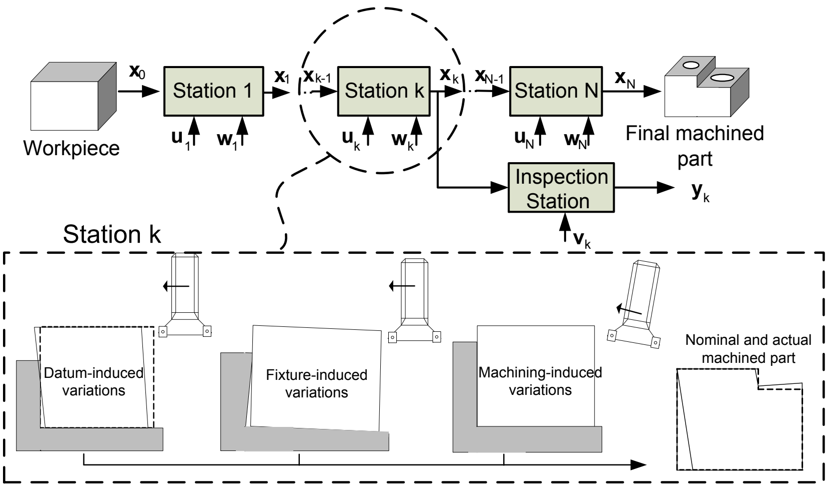



Figure 1. Manufacturing variation propagation in a MMP

SoV model is a mathematical representation of the relationship between dimensional deviations of part surfaces and their causes, and the propagation of such random deviations along multiple stations. The dimensional deviations of part surfaces from nominal values are represented by a state vector $\mathbf{x}_k$ where $k = 1, \cdots, N$ and $N$ is the number of stations in the MMP. As shown in Figure 1, the dimensional random deviations of part surfaces machined in a MMP are mainly caused by three types of variation sources:

fixture-induced variations, machining-induced variations and datum-induced variations. The impact of these three variation sources on part surface deviations in an N-station MMP can be defined from first principles with a generic state space model as (Abellán-Nebot *et al.* 2012b)

$$\mathbf{x}_k = \mathbf{A}_{k-1} \cdot \mathbf{x}_{k-1} + \mathbf{B}_k^f \cdot \mathbf{u}_k^f + \mathbf{B}_k^m \cdot \mathbf{u}_k^m + \mathbf{w}_k, \quad k = 1, 2, \cdots, N, \tag{1}$$

where $\mathbf{B}_k^f \cdot \mathbf{u}_k^f$ represents the fixture-induced variations within station $k$, denoting $\mathbf{u}_k^f$ as the fixture errors; $\mathbf{B}_k^m \cdot \mathbf{u}_k^m$ represents the machining-induced variations within station $k$, denoting $\mathbf{u}_k^m$ as the cutting-tool path deviation; $\mathbf{A}_{k-1} \cdot \mathbf{x}_{k-1}$ represents the variations transmitted by datum features generated at upstream stations; $\mathbf{w}_k$ is the un-modeled system noise and linearization errors. The general framework of building the state space model for a given $N$-station MMP is provided by Zhou *et al.* (2003), which presented the procedure of deriving the matrices $\mathbf{A}_{k-1}$, $\mathbf{B}_k^f$ and $\mathbf{B}_k^m$ at each station, according to given product and process information (i.e. part geometry and fixture layouts). The matrices $\mathbf{A}_{k-1}$, $\mathbf{B}_k^f$ and $\mathbf{B}_k^m$ are defined as:

$$\mathbf{A}_{k-1} = [\mathbf{A}_{k-1}^1 + \mathbf{A}_{k-1}^5 \cdot \mathbf{A}_{k-1}^4 \cdot \mathbf{A}_{k-1}^2 \cdot \mathbf{A}_{k-1}^1], \tag{2}$$

$$\mathbf{B}_k^f = [\mathbf{A}_{k-1}^5 \cdot \mathbf{A}_{k-1}^4 \cdot \mathbf{A}_{k-1}^3], \tag{3}$$

$$\mathbf{B}_k^m = [\mathbf{A}_{k-1}^5], \tag{4}$$

where $\mathbf{A}_{k-1}^1$ is the relocating matrix; $\mathbf{A}_{k-1}^2$ is the datum-induced variation matrix; $\mathbf{A}_{k-1}^3$ is the fixture-induced variation matrix; $\mathbf{A}_{k-1}^4$ is an auxiliary matrix; and $\mathbf{A}_{k-1}^5$ is the selector matrix (see Zhou *et al.* 2003). Figure 2 shows the methodology overview for the SoV model derivation with the matrices defined at each step. Recently, Abellán-Nebot *et al.* (2012b) expanded the matrix $\mathbf{B}_k^m$ to model machining-induced variations such as cutting-tool wear, spindle thermal expansion, deflections and kinematic and geometric machine-tool errors. However, one important limitation of the current SoV model is that it can only model the MMPs using fixtures equipped with punctual locators (Zhou *et al.* 2003, Loose *et al.* 2007). Reflected in the state space model, this limitation is related to the matrices $\mathbf{A}_{k-1}^2$ and $\mathbf{A}_{k-1}^3$, which are not generic to model other types of fixtures, such as those based on locating surfaces. It is desirable to provide the capability of modeling MMPs with general fixture devices, where surface-to-surface floating contacts exist between workpiece and fixture (Kamali Nejad *et al.* 2012, Abellán-Nebot *et al.* 2012a). In order to overcome this limitation, this paper proposes a methodology to derive the matrices $\mathbf{A}_{k-1}^2$ and $\mathbf{A}_{k-1}^3$ when fixtures based on locating surfaces are applied. Unlike fixtures based on 3-2-1 fixturing scheme and punctual locators, fixtures based on locating surfaces produce different workholding configurations corresponding to different variation scenarios, and thus, different $\mathbf{A}_{k-1}^2$ and $\mathbf{A}_{k-1}^3$ matrices should be defined. Furthermore, when a large number of stations are considered, the complexity of the resulting SoV model grows exponentially, making its derivation and the part quality prediction challenging. To address this challenge, a methodology to simplify the model and analyze the part quality according to the worst-case approach is required.

The rest of the paper is organized as follows. Section 2 and 3 present the procedures of deriving the matrices of the fixture- and the datum-induced variations, respectively, for the MMPs with surface locating surfaces. As the resulting model may have significantly

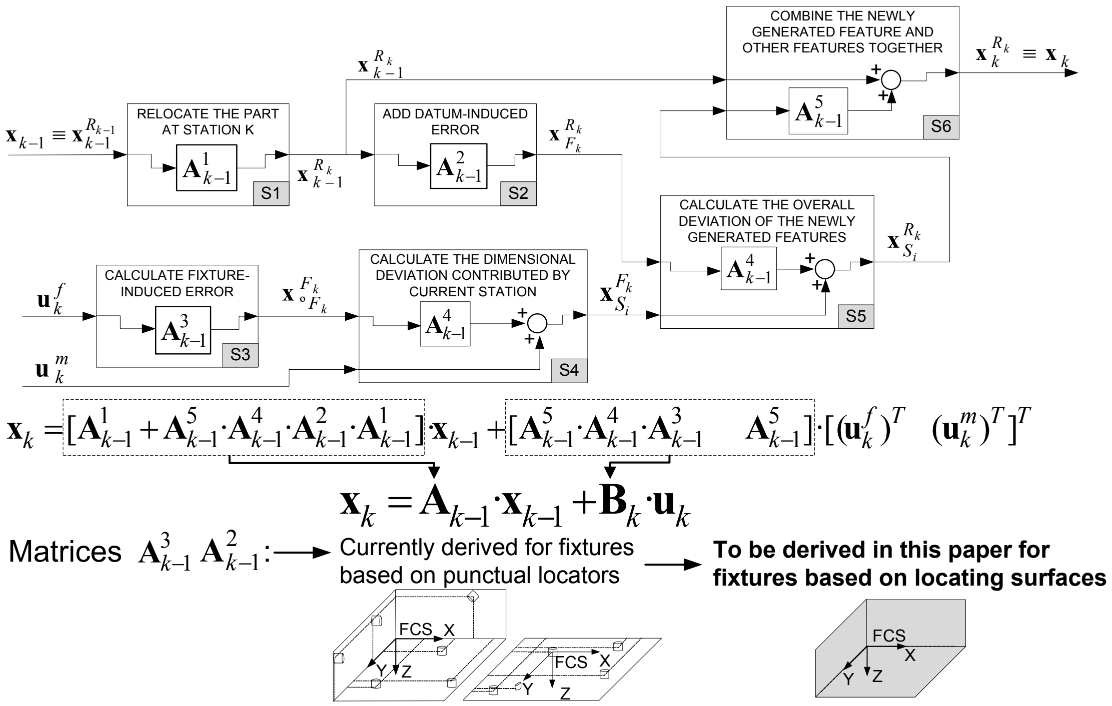


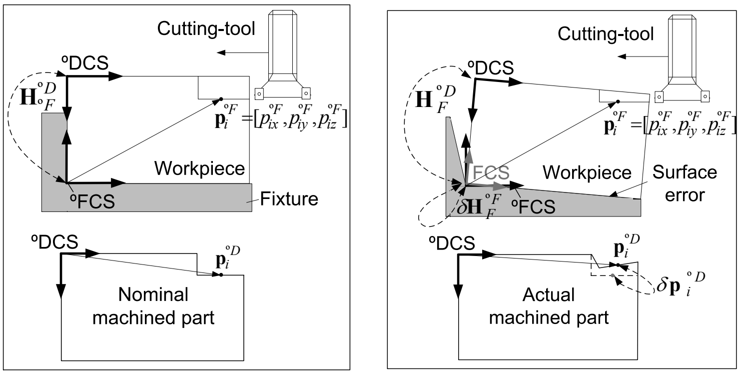

Figure 2. SoV model derivation and contribution of the paper

high dimension, Section 4 presents how to simplify the model and evaluate the part quality when the worst-case approach is analyzed. Section 5 shows a case study to validate the proposed methodology. Finally, Section 6 concludes the paper.

**General assumptions:** Datum surfaces and locating surfaces are assumed to be plane and their form errors are assumed negligible. Geometric errors are assumed to be small in comparison to nominal values and thus, the small-angle approximation can be applied. Deformations due to clamping forces are assumed negligible.

## 2. Fixture-induced variations with locating surfaces

Given a fixture, a workpiece should be located at a unique location, which is defined by its position and orientation and is considered to be deterministic if the workpiece cannot make an infinitesimal motion while maintaining contact with all the locating surfaces (Wang 2002). Due to fabrication and assembly imperfection, there are, however, random deviations of the locating surfaces from their nominal locations. Such imperfection can also be represented as the random deviations of the true fixture coordinate system (FCS) with respect to (w.r.t.) its nominal location (°FCS), as illustrated in Figure 3. Assuming that the position and orientation deviations are very small, the FCS deviation is expressed by a DMV, denoted as $\mathbf{x}_F^{\circ F}$. This vector will be composed of a position deviation vector, defined by $\mathbf{d}_F^{\circ F} = [d_{Fx}^{\circ F}, d_{Fy}^{\circ F}, d_{Fz}^{\circ F}]^T$, and an orientation deviation vector, defined by $\boldsymbol{\theta}_F^{\circ F} = [\theta_{Fx}^{\circ F}, \theta_{Fy}^{\circ F}, \theta_{Fz}^{\circ F}]^T$. Thus, the location deviation is defined as $\mathbf{x}_F^{\circ F} = \left[\left(\mathbf{d}_F^{\circ F}\right)^T, \left(\boldsymbol{\theta}_F^{\circ F}\right)^T\right]^T$. Since the cutting-tool trajectory is also referenced from °FCS, the deviation of FCS will lead to the deviation of the cutting-tool path w.r.t. the location of the workpiece, and thus, random deviations of machined surfaces w.r.t. their nominal locations, as shown in Figure 3.




(a) Nominal conditions (b) Conditions with fixture errors

Figure 3. Illustration of fixture-induced errors and coordinate systems.

In order to model the effects of fixture errors on the quality of a machined surface, i.e., its deviation from nominal location, we consider an ideal machining operation and a point $i$ on the surface generated by the cutting-tool, as shown in Figure 3 (a). This point is defined w.r.t. °FCS, and is denoted as $\mathbf{p}_i^{\circ F} = [p_{ix}^{\circ F}, p_{iy}^{\circ F}, p_{iz}^{\circ F}]$. With the vector $\tilde{\mathbf{p}}_i^{\circ F} = [\mathbf{p}_i^{\circ F}, 1]^T$, the point $i$ w.r.t. the part design coordinate system (DCS, the reference system for all the surfaces on a part), denoted as °DCS, can be expressed as:

$$\tilde{\mathbf{p}}_i^{\circ D} = \mathbf{H}_{\circ F}^{\circ D} \cdot \tilde{\mathbf{p}}_i^{\circ F}, \tag{5}$$

where $\mathbf{H}_{\circ F}^{\circ D}$ is the homogeneous transformation matrix (HTM) of °FCS w.r.t. °DCS. However, due to fixture-induced errors (neglecting any other errors), the actual FCS may deviate from °FCS and thus, the point on the machined surface will deviate w.r.t. °DCS, as shown in Figure 3 (b). The actual position of a point $i$ in °DCS, given the position of point $i$ in °FCS is:

$$\tilde{\mathbf{p}}_i^{\circ D} = \mathbf{H}_{F}^{\circ D} \cdot \delta\mathbf{H}_{\circ F}^{F} \cdot \tilde{\mathbf{p}}_i^{\circ F}, \tag{6}$$

where $\delta\mathbf{H}_{\circ F}^{F}$ is the HTM for small position and orientation deviations of °FCS w.r.t. FCS (see Appendix A). Thus, the deviation of the point $i$ on the machined surface from nominal values is:

$$\begin{aligned} \delta\tilde{\mathbf{p}}_i^{\circ D} &= \mathbf{H}_{F}^{\circ D} \cdot \delta\mathbf{H}_{\circ F}^{F} \cdot \tilde{\mathbf{p}}_i^{\circ F} - \mathbf{H}_{F}^{\circ D} \cdot \mathbf{I}_{4\times 4} \cdot \tilde{\mathbf{p}}_i^{\circ F}, \\ &= \mathbf{H}_{F}^{\circ D} \cdot \left(\delta\mathbf{H}_{\circ F}^{F} - \mathbf{I}_{4\times 4}\right) \cdot \tilde{\mathbf{p}}_i^{\circ F}. \end{aligned} \tag{7}$$

According to Appendix A, Eq. (7) can be rewritten as

$$\delta\tilde{\mathbf{p}}_i^{\circ D} = -\mathbf{H}_{F}^{\circ D} \cdot \boldsymbol{\Delta}_{F}^{\circ F} \cdot \tilde{\mathbf{p}}_i^{\circ F}, \tag{8}$$

where $\boldsymbol{\Delta}_{F}^{\circ F}$ is the differential transformation matrix (DTM) and can be derived from the DMV $\mathbf{x}_F^{\circ F} = \left[\left(\mathbf{d}_F^{\circ F}\right)^T, \left(\boldsymbol{\theta}_F^{\circ F}\right)^T\right]^T$. Therefore, the fixture-induced deviation can be defined as a function of $\mathbf{x}_F^{\circ F}$.

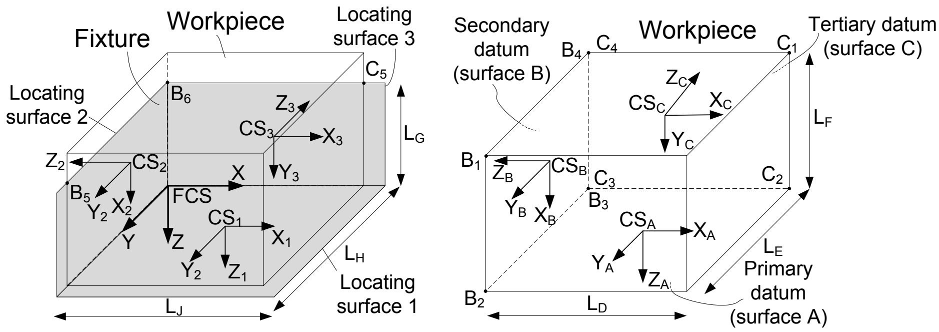



Figure 4. A generic 3-2-1 locating scheme based on locating surfaces.

In order to derive $\mathbf{x}_F^{\circ F}$ when fixtures based on locating surfaces are applied, the 3-2-1 locating scheme and the workpiece shown in Figure 4 is considered. The locating surfaces 1, 2 and 3 are in contact with the datum $A$ (primary datum), $B$ (secondary datum) and $C$ (tertiary datum) on the workpiece, respectively, and are used to constrain three degrees of freedom (d.o.f.), two d.o.f. and one d.o.f., respectively. For this locating scheme, the following restrictions apply:

(1) R1. There is a full contact between locating surface 1 and the primary datum surface $A$, and thus, any point on datum surface $A$, denoted as $A_l$, will also lie on locating surface 1. Considering the CS of the surfaces with $Z$-axis pointing normal to the surface, the condition $\left[\tilde{\mathbf{p}}_{A_l}^1\right]_{(z)} = 0$ holds for any $A_l$, where $[\cdot]_{(z)}$ refers to the $Z$ component of the vector.

(2) R2. There are at least two contact points between locating surface 2 and the workpiece datum surface $B$. Thus, the constraints $\left[\tilde{\mathbf{p}}_{B_i}^2\right]_{(z)} = 0$ and $[\tilde{\mathbf{p}}_{B_j}^2]_{(z)} = 0$ can be defined, where $B_i$ and $B_j$ are two contact points on the workpiece surface $B$. For any other points on surface $B$ that are potential contact points, denoted as $B_l$ for $l \neq i, j$, the condition $\left[\tilde{\mathbf{p}}_{B_l}^2\right]_{(z)} \leq 0$ holds.

(3) R3. There is at least one contact point between locating surface 3 and the workpiece tertiary datum surface $C$. For the contact point $C_i$ on the workpiece surface $C$, the condition $\left[\tilde{\mathbf{p}}_{C_i}^3\right]_{(z)} = 0$ holds. Also, the condition $\left[\tilde{\mathbf{p}}_{C_l}^3\right]_{(z)} \leq 0$ holds, where $C_l$, $l \neq i$, is any potential contact point between surfaces $C$ and 3.

These restrictions ensure a deterministic contact between fixture surfaces and datum surfaces. Note that the clamping order should keep the same sequence as the workpiece mounting sequence, clamping firstly the primary datum with the fixture surface, and then clamping the secondary and the tertiary datum. One can easily expect that any random deviation of locating surfaces may generate a deviation of the FCS, defined by the DMV $\mathbf{x}_F^{\circ F}$. Analyzing step by step the workpiece mounting sequence (see Figure 5), $\mathbf{x}_F^{\circ F}$ can be obtained as follows.

Step 1: The workpiece datum surface $A$ is placed on the locating surface 1 (see Figure 5). In this step, the deviation of the locating surface 1 along the three constrained

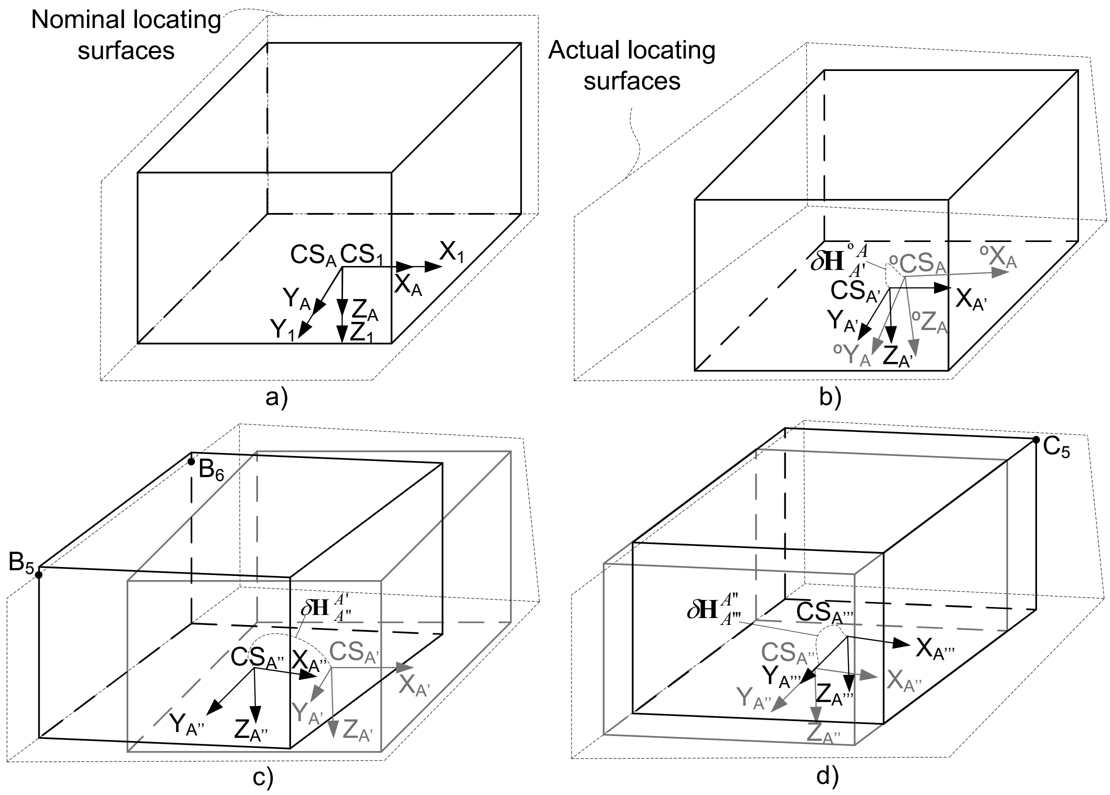



Figure 5. a) Nominal workpiece-fixture assembly. Sequence of small movements for locating the workpiece on the actual fixture over the: b) primary datum, c) secondary datum, and d) tertiary datum.

d.o.f. will generate a deviation of the workpiece surface $A$ in its three corresponding constrained d.o.f. As small deviation magnitudes are assumed, CS of surface $A$ deviates to that of $A'$. This small movement can be evaluated as:

$$\delta\mathbf{H}^{\circ A}_{A'} = \mathbf{H}^{\circ A}_{\circ 1} \cdot \delta\mathbf{H}^{\circ 1}_{1} \cdot \mathbf{H}^{1}_{A'}. \tag{9}$$

According to restriction (R1), $\mathbf{H}^{1}_{A'} = \mathbf{H}^{\circ 1}_{\circ A}$, and thus Eq. (9) becomes:

$$\delta\mathbf{H}^{\circ A}_{A'} = \mathbf{H}^{\circ A}_{\circ 1} \cdot \delta\mathbf{H}^{\circ 1}_{1} \cdot \mathbf{H}^{\circ 1}_{\circ A}, \tag{10}$$

where $\mathbf{H}^{\circ A}_{\circ 1}$ and $\mathbf{H}^{\circ 1}_{\circ A}$ are defined according to the nominal part dimensions and nominal fixture layout. Note that these small deviations are only carried out on the three constrained d.o.f. and for the others, null deviations apply.

Step 2: The workpiece is moved over the primary datum to touch the locating surface 2 with its secondary datum surface $B$ (see Figure 5). The two contact points, defined in restriction (R2) ensure:

$$\left[\tilde{\mathbf{p}}^{2}_{B_i}\right]_{(z)} = \left[\mathbf{H}^{2}_{B} \cdot \tilde{\mathbf{p}}^{B}_{B_i}\right]_{(z)} = 0, \tag{11}$$

$$\left[\tilde{\mathbf{p}}^{2}_{B_j}\right]_{(z)} = \left[\mathbf{H}^{2}_{B} \cdot \tilde{\mathbf{p}}^{B}_{B_j}\right]_{(z)} = 0, \tag{12}$$

$$\left[\tilde{\mathbf{p}}^{2}_{B_l}\right]_{(z)} = \left[\mathbf{H}^{2}_{B} \cdot \tilde{\mathbf{p}}^{B}_{B_l}\right]_{(z)} \leq 0, \tag{13}$$

As the workpiece is moved to make contact, the HTM $\mathbf{H}^{2}_{B}$ is defined as:

$$\begin{aligned}\mathbf{H}^{2}_{B} &= \mathbf{H}^{2}_{1} \cdot \mathbf{H}^{1}_{A} \cdot \mathbf{H}^{A}_{B}, \\ &= (\delta\mathbf{H}^{2}_{\circ 2} \cdot \mathbf{H}^{\circ 2}_{\circ 1} \cdot \delta\mathbf{H}^{\circ 1}_{1}) \cdot \mathbf{H}^{1}_{A} \cdot (\delta\mathbf{H}^{A}_{\circ A} \cdot \mathbf{H}^{\circ A}_{\circ B} \cdot \delta\mathbf{H}^{\circ B}_{B}).\end{aligned} \tag{14}$$

Since only fixture variations are considered, there is no deviation of $A$ ($B$) from $\circ A$ ($\circ B$) and thus, matrices $\delta\mathbf{H}^{A}_{\circ A}$ and $\delta\mathbf{H}^{\circ B}_{B}$ are $4 \times 4$ identity matrices. Furthermore, after step 1 the restriction (R1) satisfies the condition $\mathbf{H}^{1}_{A} = \mathbf{H}^{1}_{A'} = \mathbf{H}^{\circ 1}_{\circ A}$. Considering that over the primary datum the workpiece is moved to make contact with the secondary datum (and thus there is an additional HTM $\delta\mathbf{H}^{A'}_{A''}$ that defines the small translational and rotational movement of the workpiece conducted over the primary datum to make the workpiece contact at points $B_i$ and $B_j$), Eq. (14) can be rewritten as:

$$\mathbf{H}^{2}_{B} = (\delta\mathbf{H}^{2}_{\circ 2} \cdot \mathbf{H}^{\circ 2}_{\circ 1} \cdot \delta\mathbf{H}^{\circ 1}_{1}) \cdot (\mathbf{H}^{\circ 1}_{\circ A} \cdot \delta\mathbf{H}^{A'}_{A''}) \cdot \mathbf{H}^{\circ A}_{\circ B}. \tag{15}$$

Note that matrix $\delta\mathbf{H}^{A'}_{A''}$ defines the small translational and orientation deviations that are constrained by the secondary datum surface, the other deviations defined in this HTM are zero. By solving Eqs. (11), (12) and (15) constrained by Eq. (13), matrix $\delta\mathbf{H}^{A'}_{A''}$ can be evaluated.

Step 3: The workpiece is moved over the primary datum while maintaining the contact between surfaces $B$ and 2 until the workpiece is in touch with the locating surface 3 with its tertiary datum surface $C$. The contact point, $C_i$, ensures:

$$\left[\tilde{\mathbf{p}}^{3}_{C_i}\right]_{(z)} = \left[\mathbf{H}^{3}_{C} \cdot \tilde{\mathbf{p}}^{C}_{C_i}\right]_{(z)} = 0, \tag{16}$$

$$\left[\tilde{\mathbf{p}}^{3}_{C_l}\right]_{(z)} = \left[\mathbf{H}^{3}_{C} \cdot \tilde{\mathbf{p}}^{C}_{C_l}\right]_{(z)} \leq 0, \tag{17}$$

where $C_i$ and $C_l$ are the points defined in restriction (R3). As the workpiece is moved over the primary and secondary datum to make contact, the HTM $\mathbf{H}^{3}_{C}$ is defined as:

$$\begin{aligned}\mathbf{H}^{3}_{C} &= \mathbf{H}^{3}_{1} \cdot \mathbf{H}^{1}_{A} \cdot \mathbf{H}^{A}_{C}, \\ &= (\delta\mathbf{H}^{3}_{\circ 3} \cdot \mathbf{H}^{\circ 3}_{\circ 1} \cdot \delta\mathbf{H}^{\circ 1}_{1}) \cdot \mathbf{H}^{1}_{A} \cdot (\delta\mathbf{H}^{A}_{\circ A} \cdot \mathbf{H}^{\circ A}_{\circ C} \cdot \delta\mathbf{H}^{\circ C}_{C}).\end{aligned} \tag{18}$$

In this case, $\mathbf{H}^{1}_{A}$ is equal to $\mathbf{H}^{\circ 1}_{\circ A} \cdot \delta\mathbf{H}^{A'}_{A''} \cdot \delta\mathbf{H}^{A''}_{A'''}$, where $\delta\mathbf{H}^{A''}_{A'''}$ is the HTM that defines the small translational movement of the workpiece performed over the primary and secondary datum to make the workpiece contact at point $C_i$. Considering only fixture errors, Eq.

(18) can be rewritten as:

$$\mathbf{H}_C^3 = (\delta\mathbf{H}_{\circ 3}^3 \cdot \mathbf{H}_{\circ 1}^{\circ 3} \cdot \delta\mathbf{H}_1^{\circ 1}) \cdot (\mathbf{H}_{\circ A}^{\circ 1} \cdot \delta\mathbf{H}_{A''}^{A'} \cdot \delta\mathbf{H}_{A'''}^{A''}) \cdot \mathbf{H}_{\circ C}^{\circ A}. \tag{19}$$

By solving Eqs. (16) and (19) constrained by Eq. (17), matrix $\delta\mathbf{H}_{A'''}^{A''}$ can be evaluated.

Finally, the deviation of the FCS w.r.t. nominal values can be obtained by deriving the position and orientation of CS $A'''$ w.r.t. $A$ by the following equation:

$$\delta\mathbf{H}_F^{\circ F} = \mathbf{H}_{\circ A}^{\circ F} \cdot \delta\mathbf{H}_{A'''}^{\circ A} \cdot \mathbf{H}_F^{A'''}, \tag{20}$$

which can also be presented as:

$$\delta\mathbf{H}_F^{\circ F} = \mathbf{H}_{\circ A}^{\circ F} \cdot \delta\mathbf{H}_{A'}^{\circ A} \cdot \delta\mathbf{H}_{A''}^{A'} \cdot \delta\mathbf{H}_{A'''}^{A''} \cdot \mathbf{H}_F^{A'''}. \tag{21}$$

Due to solid rigid movement, FCS is deviated in the same way as CS of the datum surface $A$, i.e., $\mathbf{H}_F^{A'''} = \mathbf{H}_{\circ F}^{\circ A}$, and assuming that the deviations are small (second and higher order small values can be neglected), Eq. (21) becomes:

$$\delta\mathbf{H}_F^{\circ F} = \mathbf{H}_{\circ A}^{\circ F} \cdot \left(\mathbf{I}_{4\times 4} + \mathbf{\Delta}_{A'}^{\circ A} + \mathbf{\Delta}_{A''}^{A'} + \mathbf{\Delta}_{A'''}^{A''}\right) \cdot \mathbf{H}_{\circ F}^{\circ A}. \tag{22}$$

The terms $\mathbf{\Delta}_{A'}^{\circ A}$, $\mathbf{\Delta}_{A''}^{A'}$ and $\mathbf{\Delta}_{A'''}^{A''}$ are DTMs obtained from $\boldsymbol{\delta}\mathbf{H}_{A'}^{\circ A}$, $\boldsymbol{\delta}\mathbf{H}_{A''}^{A'}$ and $\boldsymbol{\delta}\mathbf{H}_{A'''}^{A''}$ (see Appendix A) which are calculated from Eqs. (10), (15) and (19), respectively. The term $\delta\mathbf{H}_F^{\circ F}$ from Eq. (22) can be rewritten in vector form as:

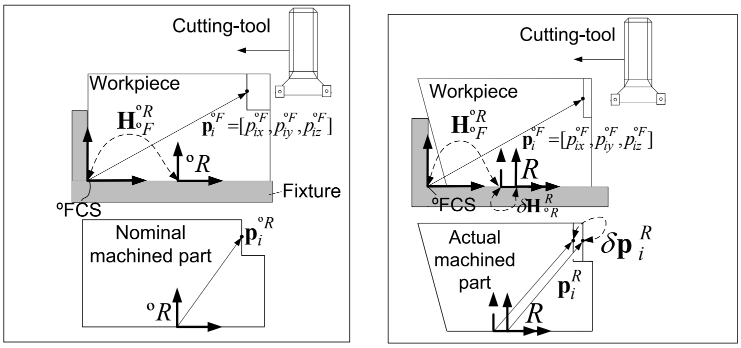
$$\mathbf{x}_F^{\circ F} = [\mathbf{\Upsilon}_1 \quad \mathbf{\Upsilon}_2 \quad \mathbf{\Upsilon}_3] \cdot [(\mathbf{x}_1^{\circ 1})^T \quad (\mathbf{x}_2^{\circ 2})^T \quad (\mathbf{x}_3^{\circ 3})^T]^T, \tag{23}$$

where $\mathbf{x}_1^{\circ 1}$, $\mathbf{x}_2^{\circ 2}$ and $\mathbf{x}_3^{\circ 3}$ represent the DMVs of the CS of the locating surfaces 1, 2 and 3, respectively; $\mathbf{\Upsilon}_1$, $\mathbf{\Upsilon}_2$ and $\mathbf{\Upsilon}_3$ are the resulting matrices from reordering and rewriting using DMVs. Note that the matrix related to fixture-induced variations used in Eq. (1) to derive the SoV model, denoted as $\mathbf{A}_{k-1}^3$, is equal to $-[\mathbf{\Upsilon}_1 \quad \mathbf{\Upsilon}_2 \quad \mathbf{\Upsilon}_3]$. Also note that the rank of $\mathbf{A}_{k-1}^3$ should be 6 to ensure that the 6 d.o.f. of the workpiece are constrained.

## 3. Datum-induced variations with locating surfaces

As the fixture-induced variations will exert onto the part dimension through datum surfaces and will propagate along multiple machining stations with datum schemes, datum-induced variations corresponding to fixture based on locating surfaces should be explicitly modeled. Datum surfaces used for locating the workpiece may always present some degree of geometric imperfection due to manufacturing variability in previous stations. Due to this imperfection, the part reference coordinate system (RCS) of the workpiece in the fixture setup will deviate from its nominal location, denoted as °RCS, and thus a dimensional variation of the machined part will present.

In order to model the impacts of deviations of datum surfaces, it is easy to consider an ideal machining operation and a point $i$ on the surface generated by a cutting-tool, as




(a) Nominal conditions (b) Conditions with datum errors

Figure 6. Example of datum feature errors.

shown in Figure 6 (a). Without loss of generality, the CS of the primary datum surface is defined as the RCS, denoted as $^\circ R$, from which the machined surfaces are referred. A point $i$ w.r.t. $^\circ R$ is defined as:

$$\tilde{\mathbf{p}}_i^{\circ R} = \mathbf{H}_{\circ F}^{\circ R} \cdot \tilde{\mathbf{p}}_i^{\circ F}. \tag{24}$$

However, $^\circ R$ may deviate from its nominal location due to datum-induced variations (neglecting any other types of variations). This deviation can be modeled by a HTM $\delta\mathbf{H}_{\circ R}^{R}$ and thus a point on the machined surface may deviate, as shown in Figure 6 (b). The actual position of a point $i$ w.r.t. $R$, knowing the position of point $i$ w.r.t. the °FCS, can be defined as:

$$\tilde{\mathbf{p}}_i^{R} = \delta\mathbf{H}_{\circ R}^{R} \cdot \mathbf{H}_{\circ F}^{\circ R} \cdot \tilde{\mathbf{p}}_i^{\circ F}. \tag{25}$$

If there are no datum-induced variations, $\delta\mathbf{H}_{\circ R}^{R}$ will reduce a $4 \times 4$ identity matrix. Thus, the deviation of a point $i$ on the machined surface due to datum-induced variations can be obtained as:

$$\delta\tilde{\mathbf{p}}_i^{R} = (\delta\mathbf{H}_{\circ R}^{R} \cdot \mathbf{H}_{\circ F}^{\circ R} - \mathbf{H}_{\circ F}^{\circ R}) \cdot \tilde{\mathbf{p}}_i^{\circ F}. \tag{26}$$

According to Eq. (26), the deviation of a point on the machined surface is a function of the deviation of $R$, which is modeled by the term $\delta\mathbf{H}_{\circ R}^{R}$.

In order to derive $\delta\mathbf{H}_{\circ R}^{R}$ for fixtures based on locating surfaces, we consider the fixture layout illustrated in Figure 4, the primary, secondary and tertiary datums are $A$ (which is the same as $R$), $B$, and $C$, respectively. According to the workpiece and clamping mounting sequence explained in the previous subsection, the first step is to place the datum surface $A$ on locating surface 1. As only datum-induced variations are assumed and the restriction (R1) applies, $\mathbf{H}_1^A = \mathbf{H}_{\circ 1}^{\circ A}$, and $\delta\mathbf{H}_{A'}^{A}$ is an identity matrix, $\mathbf{I}_{4\times 4}$. For the second step, the workpiece is moved to make a contact between datum surface $B$ and locating surface 2, which is defined by the HTM $\delta\mathbf{H}_{A''}^{A'}$. There are at least two contact points $i$ and $j$ that ensure Eqs. (11-13). Thus, the HTM $\mathbf{H}_B^2$ can be defined as:

$$\mathbf{H}_B^2 = \mathbf{H}_{\circ 1}^{\circ 2} \cdot (\mathbf{H}_{\circ A}^{\circ 1} \cdot \delta\mathbf{H}_{A''}^{A'}) \cdot (\mathbf{H}_{\circ B}^{\circ A} \cdot \delta\mathbf{H}_{B}^{A}). \tag{27}$$

From Eqs. (11-13) and (27), matrix $\delta\mathbf{H}_{A''}^{A'}$ can be evaluated.

Similarly, the third step moves the workpiece to make a contact between datum surface $C$ and locating surfaces 3, and then, Eqs. (16) and (17) hold. As the workpiece is moved over the primary and secondary datum to make contact, the HTM $\mathbf{H}_C^3$ is defined as:

$$\mathbf{H}_C^3 = \mathbf{H}_{\circ 1}^{\circ 3} \cdot (\mathbf{H}_{\circ A}^{\circ 1} \cdot \delta\mathbf{H}_{A''}^{A'} \cdot \delta\mathbf{H}_{A'''}^{A''}) \cdot (\mathbf{H}_{\circ C}^{\circ A} \cdot \delta\mathbf{H}_C^A), \tag{28}$$

where $\delta\mathbf{H}_{A'''}^{A''}$ is the HTM that defines the small translational movement of the workpiece over the primary and secondary datum surfaces to make the workpiece contact at point $C_i$. From Eqs. (16), (17) and (28), matrix $\delta\mathbf{H}_{A'''}^{A''}$ can be evaluated.

By neglecting second-order small values (and higher), the deviation of $A$ w.r.t. 1 is defined as:

$$\begin{aligned}\mathbf{H}_A^1 &= \mathbf{H}_{\circ A}^{\circ 1} \cdot \mathbf{H}_A^1 \\ &= \mathbf{H}_{\circ A}^{\circ 1} \cdot \left(\mathbf{I}_{4\times 4} + \boldsymbol{\Delta}_{A''}^{A'} + \boldsymbol{\Delta}_{A'''}^{A''}\right),\end{aligned} \tag{29}$$

where $\boldsymbol{\Delta}_{A''}^{A'}$ and $\boldsymbol{\Delta}_{A'''}^{A''}$ are the DTMs obtained from $\boldsymbol{\delta}\mathbf{H}_{A'}^{\circ A}$, $\boldsymbol{\delta}\mathbf{H}_{A''}^{A'}$ and $\boldsymbol{\delta}\mathbf{H}_{A'''}^{A''}$ which are calculated from Eqs. (27) and (28), respectively. From Eq. (29), it can be observed that

$$\delta\mathbf{H}_A^{\circ A} = \delta\mathbf{H}_A^1 = \mathbf{I}_{4\times 4} + \boldsymbol{\Delta}_{A''}^{A'} + \boldsymbol{\Delta}_{A'''}^{A''}, \tag{30}$$

since $\delta\mathbf{H}_{\circ 1}^1$ is $\mathbf{I}_{4\times 4}$.

It should be noted that in the current SoV model derivation, the deviation due to datum-induced variations is added into the model through the deviation of $FCS$ w.r.t. the CS of datum surface $A$, represented by the DMV $\mathbf{x}_F^A$. According to Eqs. (31) and (32)

$$\mathbf{H}_A^F = \mathbf{H}_1^F \cdot \mathbf{H}_A^1, \tag{31}$$

$$\mathbf{H}_{\circ A}^{\circ F} \cdot \delta\mathbf{H}_A^F = \mathbf{H}_{\circ 1}^{\circ F} \cdot \mathbf{H}_{\circ A}^{\circ 1} \cdot \delta\mathbf{H}_A^{\circ A}, \tag{32}$$

it can be observed that $\delta\mathbf{H}_A^F = \delta\mathbf{H}_A^{\circ A}$, and rewriting $\delta\mathbf{H}_A^{\circ A}$ from Eq. (30) in vector form, $\delta\mathbf{H}_A^F$ can be expressed by its DMV as:

$$\mathbf{x}_F^A = \mathbf{T}_1 \cdot \mathbf{x}_B^A + \mathbf{T}_2 \cdot \mathbf{x}_C^A, \tag{33}$$

where $\mathbf{x}_B^A$ and $\mathbf{x}_C^A$ are the DMVs of the CSs of the locating datum surfaces $B$ and $C$ w.r.t. locating datum A, respectively, and $\mathbf{T}_1$ and $\mathbf{T}_2$ are the resulting matrices from ordering and rewriting Eq. (30) using DMVs. By regrouping the terms, Eq. (33) becomes

$$\mathbf{x}_F^A = \mathbf{A}_{k-1}^2 \cdot [\ldots \quad \mathbf{x}_B^A \quad \ldots \quad \mathbf{x}_C^A \quad \ldots]^T, \tag{34}$$

where $\mathbf{A}_{k-1}^2 = [\mathbf{0} \quad \ldots \quad \mathbf{T}_1 \quad \ldots \quad \mathbf{0} \quad \ldots \quad \mathbf{T}_2 \quad \ldots \quad \mathbf{0}]$ is the matrix related to datum-induced variations used in Eq. (1) to derive the SoV model.

Table 1. Resulting matrices and inequalities for workholding configuration $A$-$(B_5B_6)$-$C_5$

$$\boldsymbol{\Upsilon}_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_G & 0 & 0 \\ 0 & 0 & -1 & L_E/2 & -L_D/2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \boldsymbol{\Upsilon}_2 = \begin{pmatrix} 0 & 0 & 1 & -L_E/2 & -L_G/2 & 0 \\ 0 & 0 & 0 & L_D & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \end{pmatrix}, \quad \boldsymbol{\Upsilon}_3 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -L_G/2 & -L_D/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

$$\mathbf{T}_1 = \begin{pmatrix} 0 & 0 & -1 & 0 & L_F/2 & 0 \\ 0 & 0 & 0 & -L_D/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}, \quad \mathbf{T}_2 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & -(L_F/2 - L_G) & L_D/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

When does this configuration hold?

$L_D \cdot \theta_{C_y}^{A} - L_D \cdot \theta_{3_y}^{\circ 3} + L_D \cdot (-\theta_{B_x}^{A} + \theta_{2_x}^{\circ 2}) \leq 0$; $\theta_{1_x}^{\circ 1} \cdot L_G + \theta_{C_x}^{A} \cdot L_G - \theta_{3_x}^{\circ 3} \cdot L_G \leq 0$

$L_D \cdot \theta_{C_y}^{A} + \theta_{1_x}^{\circ 1} \cdot L_G - L_D \cdot \theta_{3_y}^{\circ 3} + L_D \cdot (-\theta_{B_x}^{A} + \theta_{2_x}^{\circ 2}) + \theta_{C_x}^{A} \cdot L_G - \theta_{3_x}^{\circ 3} \cdot L_G \leq 0$; $-L_G \cdot (\theta_{B_y}^{A} + \theta_{1_y}^{\circ 1} - \theta_{2_y}^{\circ 2}) \leq 0$;

## 4. Part quality prediction

Following the state space model formulation from control theory (Ogata 2001), a virtual inspection after the $k$th machining station can be conducted and represented as a measurement equation as:

$$\mathbf{y}_k = \mathbf{C}_k \cdot \mathbf{x}_k + \mathbf{v}_k, \tag{35}$$

where $\mathbf{y}_k$ is a vector containing the deviations of the $M$ inspected key product characteristics (KPCs) after station $k$; $\mathbf{y}_k$ is represented as a linear combination of the deviations of the workpiece surfaces at the $k$th station, i.e., $\mathbf{C}_k \cdot \mathbf{x}_k$; and $\mathbf{v}_k$ is the measurement noise of the inspection process. Considering Eqs. (1) and (35), the SoV model can be expressed in an input-output form as:

$$\mathbf{Y} = \boldsymbol{\Gamma} \cdot \mathbf{U} + \boldsymbol{\varepsilon}, \tag{36}$$

where $\mathbf{Y} = \mathbf{y}_N = [y_1, \ldots, y_M]^T$ and $\mathbf{U} = [\mathbf{u}_1^T, \mathbf{u}_2^T, \ldots, \mathbf{u}_N^T]^T$, where $\mathbf{u}_k = [(\mathbf{u}_k^f)^T \quad (\mathbf{u}_k^m)^T]^T$. Matrix $\boldsymbol{\Gamma}$ is built based on matrices $\mathbf{A}_{k-1}$, $\mathbf{B}_k^f$, $\mathbf{B}_k^m$ and $\mathbf{C}_k$. The procedure of deriving $\boldsymbol{\Gamma}$ and $\boldsymbol{\varepsilon}$ is explained in detail in Shi (2007).

When applying isostatic fixtures based on 3-2-1 punctual locators, Eq. (36) is unique. However, if fixtures are based on surfaces, different workholding configurations may exist, corresponding to the different variations of fixture and datum surfaces from current and previous machining stations. Each workholding configuration is defined by different matrices $\mathbf{A}_{k-1}^2$ and $\mathbf{A}_{k-1}^3$ according to the contact points between fixture and workpiece surfaces. Thus, at each workholding configuration different $\boldsymbol{\Gamma}$ matrices are defined, and Eq. (36) becomes

$$\begin{pmatrix} \mathbf{Y}_1 \\ \vdots \\ \mathbf{Y}_P \end{pmatrix} = \begin{pmatrix} \boldsymbol{\Gamma}_1^f \\ \vdots \\ \boldsymbol{\Gamma}_P^f \end{pmatrix} \cdot \mathbf{U}^f + \boldsymbol{\Omega} \cdot \boldsymbol{\Gamma}^m \cdot \mathbf{U}^m + \boldsymbol{\Omega} \cdot \boldsymbol{\varepsilon}, \tag{37}$$

where $P$ is the number of potential workholding configurations in the MMP, and $\mathbf{Y}_{\bullet}$ refers to deviations of the KPCs when the MMP presents the configuration $\bullet$. That is, at each station, a specific workholding configuration presents. Note that each configuration will present only if the sources of variation produce this specific workpiece-fixture assembly. In other words, a specific workpiece-fixture assembly holds if a set of contact points exist, and thus, this configuration is subjected to a set of inequalities that can be denoted as $\mathbf{\Phi} \cdot \mathbf{U} \leq 0$. From Eq. (37), $\mathbf{U}^f$ and $\mathbf{U}^m$ are the staking vector of the deviations of locating fixture surfaces and machining errors, respectively; $\mathbf{\Gamma}^f_\bullet$ and $\mathbf{\Gamma}^m$ are the corresponding block matrices from $\mathbf{\Gamma}$; and $\mathbf{\Omega}$ is a $M \cdot P \times 1$ vector of ones.

It can be noted that the complexity of the SoV model increases notably when fixtures based on surfaces are applied. Compare to the common SoV model for locator-based fixtures, the size of the resulting SoV model in its input-output form as shown in Eq. (37) increases exponentially with the number of stations. Furthermore, this SoV model is subjected to a large number of inequalities that indicates which of the $P$ configurations apply for a given sources of variations, making its analysis extremely difficult.

For illustrative purposes, consider the fixture and workpiece shown in Figure 4. After conducting the procedure explained in Subsection 2 and 3, matrices $\mathbf{A}^3_{k-1}$ and $\mathbf{A}^2_{k-1}$ (defined by sub-matrices $\mathbf{\Upsilon}_1$, $\mathbf{\Upsilon}_2$, $\mathbf{\Upsilon}_3$, $\mathbf{T}_1$ and $\mathbf{T}_2$) can be obtained according to each potential workholding configuration. Note that this fixture-workpiece assembly presents up to 8 potential configurations if the 3-2-1 workpiece and clamping sequence explained above is applied. Which one of the 8 potential configurations is presented depends on the existing sources of variation. According to these sources of variation, the secondary datum may block the workpiece movement through 2 different pair of contact points (points $B_2$ and $B_3$, and points $B_5$ and $B_6$) and the tertiary datum may block the workpiece movement through 4 different contact points (points $C_2$, $C_3$, $C_5$ and $C_6$), resulting in 8 different workpiece-fixture assemblies. Table 1 shows the resulting matrices for the workpiece-fixture assembly defined by $A$-$B_5B_6$-$C_5$, following the 3-2-1 locating scheme notation, and the corresponding inequalities that make this configuration hold. As shown in Table 1, this configuration will be subjected to 4 inequalities, which are obtained by Eqs. (13) and (17). Different matrices and inequalities will be defined for the other 7 potential configurations. Then, $P$ is equal to 8, and Eq. (37) is composed of $8 \cdot M$ linear equations. Furthermore, if one considers a two station process with fixtures based on surfaces, the 8 potential workholding configurations at station 1 will be combined with the other 8 potential configurations at station 2, resulting in 64 potential configurations. In this case, each configuration is subjected to 8 inequalities, 4 for each workholding configuration. As a conclusion, an $N$-station machining process based on the fixture shown in Figure 4 will be defined by Eq. (37), which is composed of $M \cdot P$ linear equations, subjected to $4N \cdot P$ inequalities, with $P$ equal to $8^N$.

### 4.1. *Reduction of number of workholding configurations*

As shown above, the fixtures based on workholding surfaces significantly increase the complexity of the SoV model and the dimension of the model will be unmanageable for MMPs with large number of stations. In order to reduce the dimension of the SoV model, the following two-step methodology is proposed.

(1) Eliminating non-impacting sources of variation. The SoV model defined in Eq. (37) can be simplified by eliminating the components of the DMV of the locating surfaces that have no impacts on the d.o.f. constrained by the

fixture. For instance, for a plane with a local CS defined with $Z$ axis pointing normal to the place, any translational deviation along $X$ and $Y$ axis or orientation deviation along $Z$ axis keeps the plane invariant. Thus, the DMVs $\mathbf{x}_1^{\circ 1}$, $\mathbf{x}_2^{\circ 2}$ and $\mathbf{x}_3^{\circ 3}$ at each station can be simplified from $6 \times 1$ to $3 \times 1$ vectors, and the number of sources of error related to the fixtures is decreased from $18N$ to $9N$.

(2) Combining workholding configurations. The SoV model defined in Eq. (37) can also be simplified by combining those workholding configurations that produce the same deviation on the KPCs into a single one with a set of inequalities that result from compounding their inequalities. As a result, the potential workholding configurations will decrease from $P$ to $\bar{P}$ configurations. Assuming that the machining operations generate planar surfaces, this combining procedure can be conducted by applying two rules:

- **Rule 1**: At each station, if the nominal primary datum is parallel to the machined surface, the effect of deviations of locating surfaces that locate the secondary and tertiary datums can be neglected. If one considers $Q$ potential workholding configurations at this station, any one out of this $Q$ workholding configurations can be used for analyzing the resulting part quality. The inequalities that apply in this case only refer to the upper ($\mathbf{u}_b$) and lower ($\mathbf{l}_b$) boundaries of the DMV that defines the deviations of the locating surface for the primary datum.
- **Rule 2**: At each station, if the nominal secondary datum is parallel to the machined surface, the effect of deviations of the locating surface that locates the tertiary datum can be neglected. If one considers $Q$ potential workholding configurations at this station, only the ones that show different contact points at the secondary datum are considered, discarding those with different contact points only at the tertiary datum. As the locating surface at the tertiary datum has no influence, the resulting inequalities include: (i) those from the configurations considered according to the contact points at the secondary datum, and (ii) those related to the boundaries $\mathbf{u}_b$ and $\mathbf{l}_b$ of the DMVs that define the deviations of the locating surfaces at primary and secondary datums, discarding those related to the tertiary datum.

## 4.2. *Worst-case analysis*

Given the SoV model expressed by Eq. (36), part quality prediction can be conducted according to two common approaches: the worst-case analysis and the statistical analysis (Abellán-Nebot *et al.* 2012a). The worst-case analysis can be conducted assuming that all coefficients increase the KPC variation. Thus, the worst-case deviation is defined as

$$\mathbf{Y}_{wc} = \pm \left( |\mathbf{\Gamma}| \cdot |\mathbf{U}| + |\boldsymbol{\varepsilon}| \right). \tag{38}$$

It is straightforward to solve Eq. (38) if the MMP is composed of isostatic 3-2-1 fixtures based on locators. However, for MMPs with fixtures based on locating surfaces, multiple workholding configurations can arise, with each one subjected to a set of inequalities. This, makes it extremely complex to solve Eq. (38).

As a result, the worst-case variations of the KPCs have to be estimated for each potential set of workholding configurations, subjected to its inequalities (i.e. $\mathbf{Y}_{1_{wc}}, \ldots, \mathbf{Y}_{\bar{P}_{wc}}$ should be obtained). Since the inequalities are linear and the worst-case analysis refers to obtain the maximum and minimum values of the KPCs, the worst-case analysis becomes a set of $2M \cdot \bar{P}$ simplex optimization problems. The optimization result will indicate the worst-case value of each KPC, and the values of the sources of variations at which the worst-case value is produced.

## 5. Case Study

The proposed methodology is applied to generate the SoV model for a 3-station machining system shown in Figure 7. This MMP is used to manufacture the part shown in Table 2. After station 3, the machined part is moved to an inspection station to measure $KPC_1$, $KPC_2$ and $KPC_3$, as marked in the figure of Table 2. In order to validate the proposed methodology and its worst-case analysis, the same MMP was analyzed using a Computer-Aided-Design (CAD) software named Pro/Engineer Wildfire 5.0.

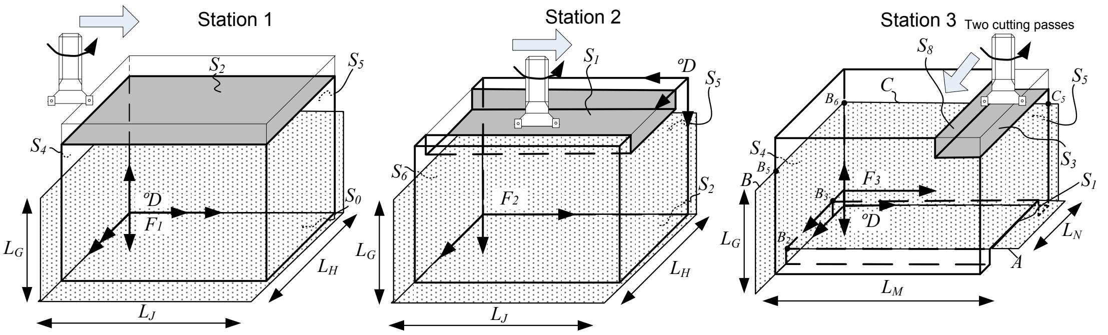
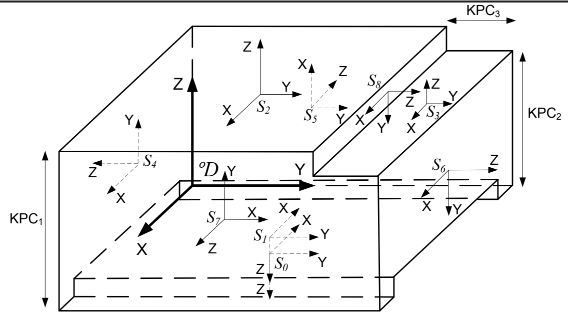



Figure 7. A three-station machining process. For all stations, $L_G = 35$, $L_H = 95$, $L_N = 90$ and $L_J = L_M = 95$ (in mm).

The analyzed MMP present 512 potential workholding configurations ($8^3$, 8 different workholding configurations at each station). Thus, the SoV model would be composed of $1,536$ $(3 \cdot 512)$ equations and $6,144$ $(4 \cdot 3 \cdot 512)$ inequalities, making its analysis highly complicated. However, the model can be simplified by applying rules 1 and 2 accordingly. At stations 1 and 2, rule 1 applies, and thus, only one workholding configuration (any of them, denoted as $*$) at these stations is required to be analyzed subjected to the inequalities of the boundaries of the primary locating surface. At station 3, rule 2 applies, and thus, only 2 of the 8 workholding configurations with different contact points at the secondary datum (contact points $B_2$, $B_3$, and $B_5$, $B_6$) are required to be analyzed, subjected to 8 inequalities plus those inequalities related to the boundaries of the DMV of the primary and secondary locating surface. Thus, the MMP can be simplified from 512 configurations to 2 configurations, where configuration 1 is denoted as $(*)/(*)/(A\text{-}B_2B_3\text{-}C_5)$ and configuration 2 is denoted as $(*)/(*)/(A\text{-}B_5B_6\text{-}C_5)$. In total, 6 equations $(3 \cdot 2)$ subjected to 8 inequalities (plus inequalities related to boundaries) define the simplified SoV model. Using this model, the worst-case values of the KPCs were obtained through a simplex optimization. Table 4 shows the results according to the methodology proposed

Table 2. Product design information. Nominal position and orientation of each surface.

| Part | Surface | $(\boldsymbol{\omega}^{\circ D}_{S_i})^T$ | $(\mathbf{t}^{\circ D}_{S_i})^T$ |
|---|---|---|---|
|  | $S_0$ | $[0, \pi, 0]$ | $[47.5, 47.5, 0]$ |
| | $S_1$ | $[0, \pi, 0]$ | $[47.5, 47.5, 2.5]$ |
| | $S_2$ | $[0, 0, 0]$ | $[47.5, 42.5, 45]$ |
| | $S_3$ | $[0, 0, 0]$ | $[47.5, 90, 40]$ |
| | $S_4$ | $[\pi/2, -\pi/2, -\pi/2]$ | $[47.5, 0, 22.5]$ |
| | $S_5$ | $[0, -\pi/2, 0]$ | $[0, 47.5, 22.5]$ |
| | $S_6$ | $[\pi/2, \pi/2, -\pi/2]$ | $[47.5, 95, 20]$ |
| | $S_7$ | $[0, \pi/2, \pi/2]$ | $[95, 47.5, 22.5]$ |
| | $S_8$ | $[\pi/2, \pi/2, -\pi/2]$ | $[47.5, 85, 42.5]$ |

$\mathbf{t}^{\circ D}_{S_i}$: Nominal position (mm); $\boldsymbol{\omega}^{\circ D}_{S_i}$: Nominal orientation (rad)

Table 3. Upper and lower boundaries of locating surfaces and machining deviations. Values apply for all machining stations (in mm and rad)

| Locating surface deviations |
|---|
| $[-0.04, -0.04, -0.04] \leq [d_x, d_y, d_z] \leq [0.04, 0.04, 0.04]$ |
| $[-0.004, -0.004, -0.004] \leq [\theta_x, \theta_y, \theta_z] \leq [0.004, 0.004, 0.004]$ |
| Machining deviations |
| $[-0.02, -0.02, -0.02] \leq [d_x, d_y, d_z] \leq [0.02, 0.02, 0.02]$ |
| $[-0.002, -0.002, -0.002] \leq [\theta_x, \theta_y, \theta_z] \leq [0.002, 0.002, 0.002]$ |

Table 4. Numerical resolution of the worst-case analysis. The pair numbers for each KPC refer to its extreme points at each configuration (in bold the worst-case value). Dimensions in -*mm*-.

| | | $KPC_1$ | $KPC_2$ | $KPC_3$ |
|---|---|---|---|---|
| Pro/E | Config. 1: $(*)/(*)/(A\text{-}B_2B_3\text{-}C_5)$ | **45.0796** | **40.8629** | **10.7771** |
| | | **44.9204** | 39.3127 | 9.8507 |
| | Config. 2: $(*)/(*)/(A\text{-}B_5B_6\text{-}C_5)$ | **45.0796** | 40.6873 | 10.1507 |
| | | **44.9204** | **39.1371** | **9.7938** |
| Proposed Methodology | | 45.0800 | 40.8700 | 10.7700 |
| | | 44.9200 | 39.1300 | 9.7900 |
| Average Error | | 0.5% | 0.82% | 1.37% |

and the CAD solution. The results shown that the methodology proposed in this paper is able to find the worst-case of the KPCs with an average error of 0.9%.

## 6. Conclusions and future work

Despite the well-known capability of the SoV model for modeling fixture-, datum- and machining-induced variations in MMPs, its application has been limited to processes where the fixture devices used only generate an unique isostatic configurations such as those referred to 3-2-1 fixture schemes based on punctual locators. This paper has shown in detail how to derive the effect of fixture- and datum- variations when applying fixtures based on locating surfaces and how to include them into the SoV model formulation. Due to the hyperstatic nature of this type of fixtures, the complexity of the SoV model grows exponentially, making the derivation of the model and the part quality prediction challenging when a large number of stations are considered. To address this challenge, a methodology to reduce the dimension of the model and analyze the part quality according to the worst-case approach has been also discussed and proved effective through a case study.

Potential future works may include improving the comprehensiveness of the model by including surface form errors and modeling other type of contacts between fixture and workpiece surfaces such as plane-cylinder contacts. It should be remarked that current technologies in industry for modeling process variation are numerical-based (based on a large number of simulations with CAD systems) rather than model-based. This is because of the complexity of the modeling derivation procedure. Meanwhile, practitioners are focused on numerically analyzing many isolated pieces of information rather than providing a comprehensive understanding of the manufacturing and production system behavior. However, it is expected that future production systems will be designed from a mathematical-model-based point of view, and some software providers have been recently investigating on the application of the SoV model for modeling, analysis and synthesis, and performance prediction of multi-station manufacturing processes (3DCS 2008). The mathematical derivation presented in this paper for fixture based on locating surfaces can be of interest in this future application.

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## Appendix A. Differential Transformation Matrix

A differential transformation matrix (DTM) in the 3D space is a $4 \times 4$ matrix that is used to represent the small position and orientation deviation of one CS w.r.t. another CS. For illustrative purposes, let us consider two CSs, 1 and 2, as shown in Figure A1. If CS 2 is deviated from nominal values by a small position and orientation deviation defined as $\mathbf{d}_2^{\circ 2} = [d_{2x}^{\circ 2}, d_{2y}^{\circ 2}, d_{2z}^{\circ 2}]^T$ and $\boldsymbol{\theta}_2^{\circ 2} = [\theta_{2x}^{\circ 2}, \theta_{2y}^{\circ 2}, \theta_{2z}^{\circ 2}]^T$, respectively, the HTM between the nominal CS $^\circ 1$ and the actual CS 2, named $\mathbf{H}_2^{\circ 1}$, is defined as:

$$\mathbf{H}_2^{\circ 1} = \mathbf{H}_{\circ 2}^{\circ 1} \cdot \delta \mathbf{H}_2^{\circ 2}, \tag{A1}$$

where $\mathbf{H}_{\circ 2}^{\circ 1}$ is the HTM between the nominal CSs $^\circ 1$ and $^\circ 2$, and $\delta \mathbf{H}_2^{\circ 2}$ is a HTM that defines a small deviation of the CS 2 from nominal values, and is defined as:

$$\delta \mathbf{H}_2^{\circ 2} = \begin{pmatrix} 1 & -\theta_{2z}^{\circ 2} & \theta_{2y}^{\circ 2} & d_{2x}^{\circ 2} \\ \theta_{2z}^{\circ 2} & 1 & -\theta_{2x}^{\circ 2} & d_{2y}^{\circ 2} \\ -\theta_{2y}^{\circ 2} & \theta_{2x}^{\circ 2} & 1 & d_{2z}^{\circ 2} \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{A2}$$

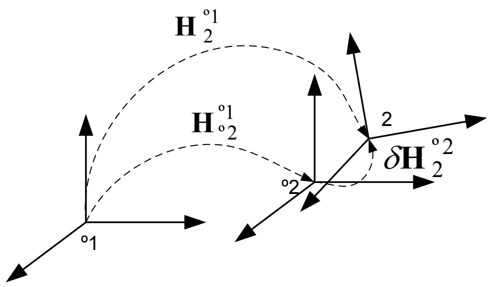


Figure A1. HTM from CS 2 to CS 1 if CS 2 is deviated from nominal values.

Eq. (A2) can be rewritten as:

$$\delta\mathbf{H}_2^{\circ 2} = \mathbf{I}_{4\times 4} + \mathbf{\Delta}_2^{\circ 2}, \tag{A3}$$

where $\mathbf{\Delta}_2^{\circ 2}$ is called the DTM which is defined as:

$$\mathbf{\Delta}_2^{\circ 2} = \begin{pmatrix} \hat{\boldsymbol{\theta}}_2^{\circ 2} & \mathbf{d}_2^{\circ 2} \\ \mathbf{0}_{1\times 3} & 0 \end{pmatrix}, \tag{A4}$$

where $\hat{\boldsymbol{\theta}}_2^{\circ 2}$ is the skew matrix of $\boldsymbol{\theta}_2^{\circ 2}$ and it is defined as:

$$\hat{\boldsymbol{\theta}}_2^{\circ 2} = \begin{pmatrix} 0 & -\theta_{2z}^{\circ 2} & \theta_{2y}^{\circ 2} \\ \theta_{2z}^{\circ 2} & 0 & -\theta_{2x}^{\circ 2} \\ -\theta_{2y}^{\circ 2} & \theta_{2x}^{\circ 2} & 0 \end{pmatrix}. \tag{A5}$$

It is important to remark that any DTM defines the small position and orientation deviation of one CS w.r.t. another CS, and these deviations can also be expressed in vector form as a differential motion vector (DMV). For instance, given the DTM of CS 2 w.r.t. 1, denoted as $\mathbf{\Delta}_2^1$, a DMV is straightforwardly defined as:

$$\mathbf{x}_2^1 = \begin{pmatrix} \mathbf{d}_2^1 \\ \boldsymbol{\theta}_2^1 \end{pmatrix}. \tag{A6}$$

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