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José Sebastião e Silva (1914–1972)

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Introduction

José Sebastião e Silva was the foremost Portuguese mathematician of the 20th century. He was a remarkable man and scientist, with multifarious interests, whose influence in Portuguese society was tremendous and is still felt nowadays.

Born on 12 December 1914 in Mértola (a small town in Portugal's deep South), he graduated in mathematics in 1937 from the University of Lisbon. After a period of five years when he was essentially unemployed, barely subsisting with private lessons and occasional teaching duties in private schools, he was hired as a teaching assistant at the Faculty of Sciences of the University of Lisbon in 1942 and, in the following year, he went to Rome with a scholarship from the *Instituto para a Alta Cultura*. He stayed at the *Istituto di Alta Matematica* until December 1946.

Sebastião e Silva in 1951

Before leaving for Rome, he was an active member of the *Movimento Matemático* (Mathematical Movement), an informal gathering of young Portuguese mathematicians that was very important in the promotion of a deep renewal in mathematical teaching and research in Portugal [2]. The group was active from about 1937 until the late 1940s and, during this time, made an important public intervention. At the end of the 1940s, the repressive backlash by the right-wing, dictatorial regime, in the wake of the rigged 1945 general elections, led to the firing from university teaching positions, the imprisonment and the exile of a large number of its members.

In about one decade, the *Movimento* accomplished several milestones in Portugal's scientific landscape, such as the creation in 1937 of the scientific journal *Portugaliæ Mathematica* (nowadays published by the EMS Publish-

A group of mathematicians of the *Movimento Matemático* at the 1942 visit of Maurice Fréchet (5th from the right) to Lisbon, at the invitation of the Centro de Estudos Matemáticos de Lisboa. Sebastião e Silva is the 4th from the right, next to Frechét.

ing House), the publication, from 1939 onwards, of the *Gazeta de Matemática*, a magazine devoted to mathematical culture at large and, in particular, to what is presently called "raising public awareness of mathematics" (and is still published nowadays by the Portuguese Mathematical Society) and the founding of the Portuguese Mathematical Society (Sebastião e Silva was its member number 10) in 1940.

At the start of the 1940s, several research centres were created as part of the initiatives of the *Instituto para a Alta Cultura* (a governmental body) and the research activities of the *Movimento Matemático* were integrated into these officially supported bodies. Sebastião e Silva was a member of one of these research centres, the *Centro de Estudos Matemáticos de Lisboa*, and it was with a studentship from this *Centro* that he went to Rome in February 1943.

After returning to Portugal, he obtained his PhD at the University of Lisbon in 1949, where he became *1º assistente* (an extinct position nowadays). From 1951 until 1960 he was a full professor at the *Instituto Superior de Agronomia* (Agronomy School of the Technical University of Lisbon) and from this year until his untimely death in 1972 he was a full professor at the Faculty of Sciences of the University of Lisbon.

Sebastião e Silva received the Artur Malheiros Prize of Lisbon's Academy of Sciences in 1956. He became a corresponding member of the academy in 1959 and a permanent member in 1966.

The influence of Sebastião e Silva in the renewal of the mathematics curriculum at the secondary and university levels was extraordinary. He played a leading part in the establishing of a number of research institutions in Lisbon where, with his example of daily research and his role in research supervision and coordination, he inspired a younger generation of mathematicians throughout the 1950s and 1960s, and was focal in the developments of the preconditions that led to the unprecedented flourishing of mathematical research in Portugal from the 1980s onwards.

The 100th anniversary of Sebastião e Silva's birth is being commemorated with an evocative exhibition at the University of Lisbon. Most of his writings (in particular his research papers mentioned below), which were published at the beginning of the 1980s as a three volume collected works, are now digitally available at
www.sebastiaoesilva100anos.org.

Sebastião e Silva's research

Sebastião e Silva's main research area was functional analysis but his first papers, in 1940, 1941 and 1946, involved a numerical method for the zeros of algebraic equations. Although these were the first research efforts of a young mathematician, their quality and importance is attested by the fact that they were still central in the area of numerical factorisation of polynomials 30 years later [3].

In these early years, Sebastião e Silva also devoted attention to point set topology, then a very active field for the members of the *Movimento Matemático*.

After moving to Italy, Sebastião e Silva started two lines of work: one in mathematical logic and another in functional analysis.

In functional analysis, Sebastião e Silva worked with the Italian mathematician Luigi Fantappiè in the theory of analytic functionals. Sebastião e Silva improved Fantappiè's definition of an analytic functional and, with his new definition, was able to introduce a linear structure on the set of analytic functionals. Next, he introduced the notion of a convergent sequence of analytic functionals. In order to define a topology in the linear space of analytical functionals, he was led to the study of topological vector spaces and, in particular, locally convex spaces. At the same time, he was interested in Laurent Schwartz's theory of distributions.

In the theory of locally convex spaces, Sebastião e Silva studied special cases of inductive and projective limits, defining the spaces LN* and M*, which are known today as Silva spaces.

In the theory of distributions, Sebastião e Silva introduced an axiomatic construction of finite order distributions. This construction was suggested by a previous model of finite order distributions: they are entities of the form D^nF, where F is a continuous function and n is a multi-index. The finite order space of distributions is the quotient space of the Cartesian product of the sets of continuous functions and of non-negative integers by a suitable equivalence relation.

Sebastião e Silva was very proud of his axiomatic construction of distributions. We would like to quote one of his remarks:

"Dans le cas de la théorie des distributions, comme dans d'autres cas, les modèles se sont présentés avant l'axiomatique. Et c'est justement la pluralité de concepts concrets, ontologiques, de distribution (comme fonctionnelles, comme séries formelles, comme classes de suites de fonctions, comme couples de fonctions analytiques, etc.), qui suggère d'en extraire la forme abstraite, par axiomatisation. Une définition en plus? Oui et non: il s'agit alors de faire une synthèse des définitions "concrètes"; ce qui en résulte sera plutôt la

vraie définition." [4] (In the case of distribution theory, as in other cases, the models came before the axiomatic construction. And it is precisely the plurality of concrete, ontological, concepts of distribution (as functionals, as formal series, as classes of sequences of functions, as pairs of analytic functions, etc.), that suggests extracting the abstract form by axiomatization. One more definition? Yes and no: it is rather a synthesis of "concrete" definitions; the result will be the true definition.)

Still in the theory of distributions, Sebastião e Silva introduced the concepts of limit of a distribution at a point, of order of growth of a distribution and of integral of a distribution. The notion of integral allowed him to write the convolution of distributions by the usual formula for functions; a similar situation occurs with the Fourier transform of distributions.

In some problems of differential equations, there is the need for complex translations; they do not exist in the theory of distributions. Also in quantum mechanics there is the need for multipole series; these series are not convergent (unless they reduce to a finite sum) in the theory of distributions. These problems led Sebastião e Silva to the study of ultradistributions. In a paper of 1958 [5], he introduced, using the Stieltjes transform, two new spaces: the space of tempered ultradistributions and the space of exponential growth ultradistributions. The first one contains the space of tempered distributions and the second one the space of distributions of exponential growth. In these spaces, besides the operators of derivation and of product by polynomials, Sebastião e Silva defined the complex translation operator and proved the convergence of some multipole series.

Sebastião e Silva also introduced the space of ultradistributions of compact support. It is a subspace of the space of tempered ultradistributions. In this new space, he conjectured a necessary and sufficient condition for a multipole series to be convergent, the proof of which was completed later by one of his pupils Silva Oliveira.

Sebastião e Silva generalised the Fourier transform to the space of ultradistributions of exponential growth. In that space, this new Fourier transform is a linear and topological endomorphism. This is a beautiful generalisation of Schwartz's result for tempered distributions.

The last research paper written by Sebastião e Silva, when he was hospitalised with a terminal illness, was published posthumously. It deals with an application of tempered distributions with values in a Hilbert space to the Boltzmann equation [6].

Renewal of secondary school instruction in Portugal

Sebastião e Silva's ideas and criticisms about the mathematics being taught in Portuguese schools first appeared in print in the 1940s, in a series of articles published in the *Gazeta de Matemática*. In the 1950s, he wrote two high school textbooks, on algebra and plane analytic geometry, which were later selected by the Ministry of Educa-

tion for adoption in every school in the country. In this decade, also under his inspiration, Portugal joined the international forums where the renewal of mathematics teaching was being debated. Sebastião e Silva thus represented Portugal in the ICMI (International Commission on Mathematical Instruction) and the CIEAEM (International Commission for the Study and Improvement of Mathematics Teaching).

From 1963 onwards, Sebastião e Silva oversaw a national commission for the modernisation of mathematics teaching in Portuguese secondary schools, which started by dealing with what are nowadays known as the 10th to 12th grades (that is, the three years prior to university entrance). This commission operated under an agreement between the National Ministry of Education and the European Organization for Economic Cooperation (OECE, nowadays OECD). The purpose was to construct a project of reform, part of an international proposal that was considered necessary and urgent to keep up with the evolution of science and technology. The teaching of mathematics was supposed to mirror that evolution, which implied restructuring all the curricula. But, more than that, in the words of Sebastião e Silva [7]:

> *The modernization of the teaching of Mathematics must be done not only regarding curricula but also with respect to teaching methods. The teacher must abandon, inasmuch as possible, the traditional expository method, in which the students' role is almost one hundred percent passive, and try, in contrast, to follow an active method, establishing a dialogue with the students and stimulating their imagination, in such a way as to lead them, whenever possible, to rediscovery.*[1]

The experience of Modern Mathematics, the name by which this project became known, started in the school year 1963/64 in three class groups, taught in three schools (in Lisbon, Porto and Coimbra) by senior teachers who supervised teacher training as part of the commission's work. In the following years, the project was progressively broadened to other schools; new teachers were trained and texts and support materials were written both for students and for teachers of the experimental classes. These texts, *Compêndio de Matemática*[2] and *Guia para a Utilização do Compêndio de Matemática*[3] [7], were written by Sebastião e Silva and are still reference works for the studying and teaching of mathematics at pre-university level.

Lasting influence in Portuguese mathematics

Sebastião e Silva was the foremost Portuguese mathematician in the 20th century and also, undoubtedly, the most influential one. As a professor he was a strong leader of mathematics reform, having a pivotal role in the modernisation of mathematics teaching at all levels. As a researcher, he was at the forefront of several fields in analysis and had a brilliant international career, in particular as the author of an alternative route to developing the theory of distributions. His standing as a researcher may be assessed from the number of items he authored: 49 items listed in ZentralBlatt and 45 in MathSciNet, plac-

ing him as the most productive Portuguese mathematician up to his time.

Most importantly, he was the first Portuguese mathematician of international stature who developed a school of thought, in the sense of leaving a set of disciples capable of carrying on his work and developing his ideas [1]. This is not immediately apparent from the number of PhD students (2) and descendants (10) found in Mathematics Genealogy [8]. There is, however, a reason for this underestimation. Until the late 1960s, a PhD was not mandatory for university professorships in Portugal and so several of his disciples did not earn the degree – but they, in turn, supervised many PhDs. In reality, the number of Sebastião e Silva's direct mathematical descendants is much larger than 10, since, by the next generation, the PhD was already necessary for an academic career.

The impact of the School of Functional Analysis is still felt strongly today, having spread to several Portuguese research universities beyond his original Universidade de Lisboa and Universidade Técnica de Lisboa (which, incidentally, fused in 2013 to form the new ULisboa). In fact, it may be argued, without great exaggeration, that the origins of functional analysis in Portugal may be traced back to a single point: Sebastião e Silva.

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[2] Compendium of Mathematics.
[3] Guide for the Use of the Compendium of Mathematics.