A HYBRID METHOD FOR SOUND-HARD OBSTACLE RECONSTRUCTION

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Abstract

We are interested in solving the inverse problem of acoustic wave scattering to reconstruct the position and the shape of sound-hard obstacles from a given incident field and the corresponding far field pattern of the scattered field. The method we suggest is an extension of the hybrid method for the reconstruction of sound-soft cracks as presented in [11] to the case of sound-hard obstacles. The designation of the method is justified by the fact that it can be interpreted as a hybrid between a regularized Newton method applied to a nonlinear operator equation with the operator that maps the unknown boundary onto the solution of the direct scattering problem and a decomposition method in the spirit of the potential method as described in [5,6,7]. Since the method does not require a forward solver for each Newton step its computational costs are reduced. By some numerical examples we illustrate the feasibility of the method.

Key words: Inverse Scattering Problems
PACS: 81U40

1 Introduction

Nondestructive obstacle detecting through low frequency wave propagation motivates a number of challenging mathematical and numerical problems with several applications such as radar and sonar or medical imaging. Among these problems, we are interested in numerical methods for reconstructing unaccessible impenetrable scattering obstacles within a homogeneous background from the knowledge of

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the incident field and the scattered field at large distances (far field pattern). We confine ourselves to the case of time harmonic acoustic waves and the scattering from sound-hard obstacles.

Given an open bounded obstacle $D \subset \mathbb{R}^2$ with an unbounded complement and an incident field u^i, the direct scattering problem consists of finding the total field $u = u^i + u^s$ as the sum of the known incident field u^i and the scattered field u^s such that both the Helmholtz equation

$$\Delta u + k^2 u = 0 \quad \text{in } \mathbb{R}^2 \backslash D \tag{1}$$

with wave number $k > 0$ and the boundary Neumann condition

$$\frac{\partial u}{\partial \nu} = 0 \quad \text{on } \Gamma := \partial D \tag{2}$$

are satisfied, where ν stands for the exterior normal vector to Γ. To ensure well-posedness, at infinity one needs to impose the Sommerfeld radiation condition

$$\lim_{r\to\infty} \sqrt{r} \left(\frac{\partial u^s}{\partial r} - iku^s \right) = 0, \quad r = |x| \tag{3}$$

with the limit satisfied uniformly in all directions. Then it is known (e.g. [2, Ch.2]) that the solution u^s has an asymptotic behavior of the form

$$u^s(x) = \frac{e^{ik|x|}}{\sqrt{|x|}} \left(u_\infty(\hat{x}) + O\left(\frac{1}{|x|}\right)\right), \quad |x| \to \infty,$$

where $\hat{x} = x/|x|$. The function u_∞ defined on the unit circle Ω is denoted as the far field pattern of u^s. By Rellich's lemma (e.g. [2, Ch.2]) the scattered field u^s is completely determined by its far field pattern.

From the variety of methods that have been developed for solving inverse obstacle scattering problems, we would like to focus on regularized Newton iterations and decomposition methods. For a fixed incident field u^i, the solution of the direct scattering problem defines the operator

$$F : \gamma \mapsto u_\infty$$

that maps the closed curve γ onto the far field corresponding to scattering by the obstacle with boundary γ. In this sense, given the far field pattern u_∞, the inverse problem is equivalent to finding the solution of the nonlinear and ill-posed (e.g. [2, Ch.5]) operator equation

$$F(\Gamma) = u_\infty \tag{4}$$

for the unknown boundary Γ. Regularized Newton iterations applied to (4) have been studied and used for over two decades (see [4,13,14]). Their idea is to linearize (4), based on the Fréchet differentiability of the operator F (see [3,12]), and iterate this procedure. Due to the ill-posedness of F regularization is required in each iteration step. The main drawback of this method is that it requires the solution of the direct scattering problem at each iteration step and a reasonable initial guess to start the iterations.

On the other hand, decomposition methods take care of the ill-posedness and the nonlinearity of the inverse scattering problem separately. In a first step the function u is reconstructed from the given far field pattern u_∞, for example by representing the scattered field u^s as a single-layer, a double-layer or a combined potential with density on an approximate boundary γ. The requirement that the far field of the potential coincides with the given far field u_∞ leads to an ill-posed linear integral equation that can be approximately solved via Tikhonov regularization. Then in a second step one tries to find the boundary Γ as the location where the boundary condition (2) is satisfied in a least squares sense. Though this method does not need the solution of the forward problem, the reconstructions obtained are not as accurate as those obtained by Newton iterations. There is also a gap between the theoretical background and the numerical implementation of the method.

In [9] one of us suggested combining ideas of both of these two classes of reconstruction methods into a hybrid method for sound-soft obstacles. The same idea was applied to an inverse boundary value problem for the Laplace equation in [1] and to inverse scattering from sound-soft cracks in [11]. This new method does not need a forward solver and the accuracy of the reconstructions is as satisfactory as for the Newton iterations, provided the initial guess is close enough to the exact boundary. In the present paper we describe an extension to scattering from sound-hard obstacles.

2 The Direct Problem

To introduce notations, we briefly discuss the solution of the direct scattering problem via the double-layer potential approach. For details we refer, for example, to [2, Ch.3]. Given the domain D with boundary Γ of class C^2 and the incident field u^i, we want to find the uniquely determined scattered field u^s such that (1)–(3) are satisfied. By

$$\Phi(x,y) = \frac{i}{4}\, H_1^{(0)}(k|x-y|)$$

we denote the fundamental solution to the two-dimensional Helmholtz equation in terms of the Hankel function $H_1^{(0)}$ of the first kind and order zero. After introducing the double-layer potential with density φ on a closed C^2 curve γ by

$$(K_\gamma \varphi)(x) := \int_\gamma \frac{\partial \Phi(x,y)}{\partial \nu(y)}\, \varphi(y)\, ds(y), \quad x \in \mathbb{R}^2, \tag{5}$$

we represent u^s in the form

$$u^s = K_\Gamma \varphi \quad \text{in } \mathbb{R}^2 \backslash D. \tag{6}$$

Since u^s given by (6) satisfies the Helmholtz equation and the Sommerfeld radiation condition φ has to be determined such that the boundary condition (2) is satisfied.

By the jump relations, considering the double-layer potential (6) as defined in $\mathbb{R}^2 \backslash \Gamma$, its trace on Γ is given by

$$u^s_\pm = K_\Gamma \varphi \pm \frac{\varphi}{2} \quad \text{on } \Gamma,$$

where $\pm$ stands for the limit when approaching Γ from outside and inside D, respectively. The normal trace of u^s has no jump and is given by

$$\frac{\partial u^s}{\partial \nu} = T_\Gamma \varphi \quad \text{on } \Gamma$$

with the hyper-singular operator

$$(T_\gamma \varphi)(x) := \frac{\partial}{\partial \nu(x)} \int_\gamma \frac{\partial \Phi(x,y)}{\partial \nu(y)} \, \varphi(y) \, ds(y), \quad x \in \gamma. \tag{7}$$

In terms of this operator, to satisfy the boundary condition (2), the density φ has to be obtained as solution of

$$T_\Gamma \varphi = -\frac{\partial u^i}{\partial \nu} \quad \text{on } \Gamma. \tag{8}$$

Provided that k^2 is not an interior Neumann eigenvalue, equation (8) is uniquely solvable. An approximate solution can be obtained, for example, by a collocation method as described in [8].

We note that via the asymptotics of the Hankel function the far field pattern of the double-layer potential (5) is given by

$$u_\infty = K_{\infty,\gamma} \varphi \quad \text{on } \Omega$$

with the far field operator

$$(K_{\infty,\gamma} \varphi)(\hat{x}) = \frac{e^{i\pi/4}}{\sqrt{8\pi k}} \int_\gamma \frac{\partial e^{-ik\hat{x}\cdot y}}{\partial \nu(y)} \, \varphi(y) \, ds(y), \quad \hat{x} \in \Omega. \tag{9}$$

3 Differentiability of the Normal Trace

For the further analysis, a parameterization of the boundary curves is required. We assume that

$$\gamma = \{z(s) : s \in [0, 2\pi]\},$$

with a 2π periodic C^2 function $z : \mathbb{R} \to \mathbb{R}^2$ and counter-clockwise orientation such that $z|_{[0,2\pi)}$ is injective. Our hybrid method is based on the linearization of the operator G that for a given C^2-smooth field u, defined in a neighborhood of γ, maps the parameterization z of the contour γ onto the normal derivative of u on γ, that is,

$$G : z \mapsto \frac{z'^{\perp}}{|z'|} \cdot (\operatorname{grad} u \circ z)\,, \tag{10}$$

where $z^{\perp} = (z_2, -z_1)$. Note that because of $\nu \circ z = z'^{\perp}/|z'|$ indeed G parameterizes the normal derivative of u on γ. If u is the total field, then the inverse scattering problem can be posed as finding z such that $G(z) = 0$.

From now on τ denotes the unit tangent vector to γ and is given by $\tau \circ z = z'/|z'|$. In a slight abuse of notation we will denote $\nu = \nu \circ z$ and $\tau = \tau \circ z$. We should also clarify that $s \in [0, 2\pi]$ is the parametrization variable and so for a point $x = z(s) \in \gamma$ one has

$$\frac{1}{|z'(s)|} \frac{\partial u(z(s))}{\partial s} = \frac{\partial u(x)}{\partial \tau}.$$

The following theorem is the basis for the implementation of our hybrid method. We note that as opposed to the Fréchet derivative of the operator F from equation (4) a $h \cdot \tau$ component occurs in the Fréchet derivative of G since, in general, the function u in definition (10) does not satify the boundary condition.

Theorem 1 *The operator $G : C^2[0, 2\pi] \to C[0, 2\pi]$ is Fréchet differentiable and its derivative is given by*

$$\begin{aligned} G'(z)h = {} & \frac{1}{|z'|}\Big(h'^{\perp} - \nu\,(\tau \cdot h')\Big) \cdot \operatorname{grad} u \circ z + \left(\frac{\partial^2 u}{\partial \nu^2} \circ z\right) h \cdot \nu \\ & + \left(\frac{z'' \cdot \nu}{|z'|^2} \frac{\partial u}{\partial s} \circ z + \frac{\partial^2 u}{\partial s \partial \nu} \circ z\right) \frac{h \cdot \tau}{|z'|}\,. \end{aligned} \tag{11}$$

Proof. Let h be sufficiently small to ensure that

$$\gamma_{z+h} = \{z(t) + h(t) : t \in [0, 2\pi]\}$$

describes a closed curve. We decompose

$$G(z+h)-G(z)=\Big(\frac{z'^{\perp}+h'^{\perp}}{|z'+h'|}-\frac{z'^{\perp}}{|z'|}\Big)\cdot\Big(\mathrm{grad} u\circ(z+h)\Big)$$
$$+\frac{z'^{\perp}}{|z'|}\cdot\Big(\mathrm{grad} u\circ(z+h)-\mathrm{grad} u\circ z\Big) \tag{12}$$

and treat both terms on the right hand side separately. Using Taylor's formula, we begin by noting that

$$\frac{z'^{\perp}+h'^{\perp}}{|z'+h'|}-\frac{z'^{\perp}}{|z'|}=\frac{h'^{\perp}}{|z'|}-\frac{z'^{\perp}(z'\cdot h')}{|z'|^3}+O(|h'|^2)$$
$$=\frac{1}{|z'|}\Big(h'^{\perp}-\nu\,(\tau\cdot h')\Big)+O(|h'|^2).$$

Since

$$\mathrm{grad}\,u\circ(z+h)-\mathrm{grad}\,u\circ z=O(|h|)$$

we consequently have

$$\Big(\frac{z'^{\perp}+h'^{\perp}}{|z'+h'|}-\frac{z'^{\perp}}{|z'|}\Big)\cdot\Big(\,\mathrm{grad}\,u\circ(z+h)\Big)$$
$$=\frac{1}{|z'|}\Big(h'^{\perp}-\nu\,(\tau\cdot h')\Big)\cdot\mathrm{grad}\,u\circ z+O(|h'|^2)+O(|h'h|). \tag{13}$$

For the second term on the right hand side of (12) we perform a change of variables in a neighborhood of γ by

$$x(s,\varepsilon)=z(s)+\varepsilon\nu(s),\quad s\in[0,2\pi],\,\varepsilon\in(-\varepsilon_0,\varepsilon_0)$$

and set

$$v(s,\varepsilon)=u(z(s)+\varepsilon\nu(s)).$$

In the new coordinate system we have that

$$\mathrm{grad}\,v(s,\varepsilon)=\frac{1}{|z'(s)+\varepsilon\nu'(s)|^2}\frac{\partial v}{\partial s}(s,\varepsilon)\,[z'(s)+\varepsilon\nu'(s)]+\frac{\partial v}{\partial\varepsilon}(s,\varepsilon)\nu(s).$$

Now Taylor's formula and the relations $\nu\cdot z'=0$ and $\nu'\cdot\nu=0$ imply that

$$\nu(s)\cdot[\mathrm{grad}\,v(s+\sigma,\epsilon)-\mathrm{grad}\,v(s,0)]$$

$$=\left[\frac{z''(s)\cdot\nu(s)}{|z'(s)|^2}\frac{\partial v}{\partial s}(s,0)+\frac{\partial^2 v}{\partial s\partial\varepsilon}(s,0)\right]\sigma+\frac{\partial^2 v}{\partial\varepsilon^2}(s,0)\,\epsilon+O\left(\sigma^2+\epsilon^2\right)$$

In view of the the second term on the right hand side of (12) we want to choose the pair (σ, ϵ) such that

$$z(s) + h(s) = z(s+\sigma) + \epsilon\nu(s+\sigma).$$

For this, by Taylor's formula, we note that

$$h(s) - \epsilon\nu(s) + O(\sigma\epsilon) = z(s+\sigma) - z(s) = z'(s)\sigma + O(\sigma^2)$$

and therefore

$$h(s) = z'(s)\sigma + \epsilon\,\nu(s) + O(\sigma\epsilon) + O(\sigma^2).$$

Comparing the previous expression with the decomposition

$$h(s) = \left(\frac{h(s)\cdot\tau(s)}{|z'(s)|}\right) z'(s) + \Big(h(s)\cdot\nu(s)\Big)\nu(s),$$

we need to choose

$$\sigma = \frac{h(s)\cdot\tau(s)}{|z'(s)|} \quad \text{and} \quad \epsilon = h(s)\cdot\nu(s).$$

Therefore, finally, we can write the second term on the right hand side of (12) as

$$\begin{aligned} &\nu(s)\cdot\Big(\operatorname{grad} u\big(z(s)+h(s)\big) - \operatorname{grad} u\big(z(s)\big)\Big) \\ &= \left[\frac{z''(s)\cdot\nu(s)}{|z'(s)|^2}\frac{\partial u}{\partial s}\big(z(s)\big) + \frac{\partial^2 u}{\partial s\partial\nu}\big(z(s)\big)\right]\frac{h(s)\cdot\tau(s)}{|z'(s)|} \\ &\quad + \frac{\partial^2 u}{\partial\nu^2}\big(z(s)\big)h(s)\cdot\nu(s) + O\left(|h|^2\right) \end{aligned} \tag{14}$$

Inserting (13) and (14) into (12) and observing the definition of the Fréchet derivative

$$|G(z+h) - G(z) - G'(z)h| = O(\|h\|_{C^2}^2), \quad \|h\|_{C^2} \to 0,$$

now finishes the proof. □

In practice one wants to avoid computing the term $\partial^2 u/\partial^2\nu$ appearing in (11). Therefore, in the following corollary this term is eliminated by using the fact that u satisfies the Helmholtz equation.

Corollary 2 *Provided u satisfies the Helmholtz equation, the Fréchet derivative of*

$G : C^2[0,2\pi] \to C[0,2\pi]$ is given by

$$
\begin{aligned}
G'(z)h = {} & \frac{1}{|z'|}\Big(h'^{\perp} - \nu\,(\tau\cdot h')\Big)\cdot \operatorname{grad} u\circ z \\
& - \left(k^2\, u\circ z - \frac{z'.z''}{|z'|^4}\frac{\partial u}{\partial s}\circ z + \frac{1}{|z'|^2}\frac{\partial^2 u}{\partial s^2}\circ z - \frac{z''.\nu}{|z'|^2}\frac{\partial u}{\partial \nu}\circ z\right) h\cdot\nu \\
& + \left(\frac{z''\cdot\nu}{|z'(s)|^2}\frac{\partial u}{\partial s}\circ z + \frac{\partial^2 u}{\partial s\partial\nu}\circ z\right)\frac{h\cdot\tau}{|z'|}\ .
\end{aligned}
$$

Proof. Using the change of variables as in the previous proof for the Laplace operator we have that

$$
\begin{aligned}
\Delta v(s,\varepsilon) = {} & \frac{1}{|z'(s)+\varepsilon\nu'(s)|}\left\{\frac{\partial}{\partial s}\left(\frac{1}{|z'(s)+\varepsilon\nu'(s)|}\frac{\partial v}{\partial s}(s,\varepsilon)\right)\right. \\
& \left. + \frac{\partial}{\partial\varepsilon}\left(|z'(s)+\varepsilon\nu'(s)|\,\frac{\partial v}{\partial\varepsilon}(s,\varepsilon)\right)\right\}.
\end{aligned}
$$

Therefore

$$
\begin{aligned}
\Delta v(s,0) = {} & -\frac{z'(s).z''(s)}{|z'(s)|^4}\frac{\partial v}{\partial s}(s,0) + \frac{1}{|z'(s)|^2}\frac{\partial^2 v}{\partial s^2}(s,0) \\
& + \frac{z'(s).\nu'(s)}{|z'(s)|^2}\frac{\partial v}{\partial\varepsilon}(s,0) + \frac{\partial^2 v}{\partial\varepsilon^2}(s,0),
\end{aligned}
$$

and observing that u satisfies the Helmholtz equation we find that

$$
\begin{aligned}
\frac{\partial^2 v}{\partial\varepsilon^2}(s,0) = {} & -k^2\, v(s,0) + \frac{z'(s).z''(s)}{|z'(s)|^4}\frac{\partial v}{\partial s}(s,0) \\
& - \frac{1}{|z'(s)|^2}\frac{\partial^2 v}{\partial s^2}(s,0) - \frac{z'(s).\nu'(s)}{|z'(s)|^2}\frac{\partial v}{\partial\varepsilon}(s,0).
\end{aligned}
$$

Using this expression and the identity

$$\nu'(s)\cdot\tau(s) = -\frac{z''(s)\cdot\nu(s)}{|z'(s)|}$$

the statement of the corollary now follows from Theorem 1. □

4 The Hybrid Method

We are now in a position to present our hybrid method. For the sake of simplicity, we confine our presentation to the case of star shaped domains, that is, we assume that the boundary Γ can be parameterized in the form

$$z(t) = r(t)(\cos t, \sin t) : t \in [0, 2\pi]\} \tag{15}$$

with some 2π periodic positive C^2 function r. However, the method can be extended to more general domains, that, is to other parameterizations.

As already mentioned the hybrid method combines ideas of both Newton and decomposition methods. As in the latter, it consists of two steps. In the first step, one deals with the ill-posedness in the spirit of the potential method of Kirsch and Kress [5,6,7]. Given an approximation γ with parameterization z of the form (15), we start by solving the far field equation

$$K_{\infty,\gamma}\varphi = u_\infty \tag{16}$$

for φ. As $K_{\infty,\tilde{\Gamma}}$ is a compact operator its inversion is ill-posed and therefore stabilization is needed. For this, we suggest using the well established Tikhonov regularization. Settling the first step, with an approximate solution φ of (16) we then obtain an approximation of the total field by setting

$$u = u^i + K_\gamma\varphi \quad \text{in } \mathbb{R}^2\backslash\gamma. \tag{17}$$

We now recall the parameterization to the boundary condition operator G introduced in (10). In order to satisfy the boundary condition, we need to find an updated parameterization $z + h$ such that

$$G(z + h) = 0.$$

Therefore, in a second step, as in the classical Newton method, we solve the linearized equation

$$G(z) + G'(z)h = 0 \tag{18}$$

for h in a least squares sense. Our hybrid method then consists in repeating both steps iteratively until some stopping criteria is fulfilled.

We point out that this method does not need a forward solver at each iteration step which reduces the computational costs. As we will see in the next section, this does not deteriorate the reconstructions. Therefore the method combines the advantages of both Newton type and decomposition methods.

Remark 3 *Note that the approximation of the total field u given by* (17) *has a jump on γ. Therefore, at each collocation point considered for solving* (18) *in a least squares sense a choice has to be made whether to use the interior or exterior values of u to compute the Fréchet derivative of G given in Theorem 1. We consider the exterior field for the computations since we are interested in the exterior problem. In the decomposition method [6], this choice was more or less justified by the assumption that the initial guess γ lies inside the scatterer D. The reason* *was that according to this class of method's main idea, as referred in the introduction, one wants the solution Γ to lie in the domain of definition of the approximated total field. However, as well as for the hybrid method, this* a priori *knowledge on the scatterer has just a theoretical background and is not needed in practise. The important consequence of this naive choice is that it does not allow an immediate extension to the reconstruction of sound-hard cracks, as opposed to the approach for sound-soft ones as suggested in [11].*

5 Numerical Results

In this final section we describe some of the details of the numerical implementation of the method.

Our synthetic data were obtained through the process described in section 2. We used the far field pattern at 80 equidistant points on the unit circle Ω.

We solved (16) by Tikhonov regularization, considering 60 points over the boundary γ. For the regularization parameter, we used $0.9^n \times 10^{-6}$, where n is the number of iterations.

In each step, the function u and its normal derivative $\partial u/\partial\nu$ have to be computed using their integral representation. We evaluate the integral operators K and T using the trigonometric quadrature rules described in [8]. For the tangential derivatives occurring in the expressions for the Fréchet derivative of G we use trigonometric differentiation, that is, we interpolate by a trigonometric polynomial and take its derivative as approximation.

As parameterization space for the radial function we considered even trigonometric polynomials

$$r(t) = \sum_{j=0}^{N} a_j \cos\, jt$$

of degree $N = 10$. For all the examples presented, we fixed the wave number $k = 1$ and the incident field

$$u^i(x) = e^{ikx_2},$$

that is, a plane wave with direction $d = (0, 1)$.

As stopping criteria we used the residual

$$\left\| T_\gamma\varphi + \frac{\partial u^i}{\partial\nu} \right\|_{L^2(\gamma)} + \|K_{\infty,\gamma}\varphi - u_\infty\|_{L^2(\Omega)}$$

associated with each iteration step in the following way. We computed the value of the residual for the current approximation. Then we solved (18) to get the candidate for a new approximation by a Marquardt-Levenberg step to improve on the stability of the method. As regularization parameter for the Marquardt-Levenberg step we started with 10^{-5} and if the residual for the new approximation was larger than for the current approximation, we would increase the Marquardt-Levenberg parameter by a factor of 10 and repeat the second step. Otherwise we would take the new approximation and proceed with a next iteration repeating both steps of the method. The method was stopped when the regularization parameter for the Marquardt-Levenberg step became equal to 1.

For a first example, we considered a domain that is contained in the approximation space and given by

$$\Gamma = \{(2 + 0.3\cos(3\,t))\{\cos(t), \sin(t)\} \,:\, t \in [0, 2\pi]\}\,.$$

As we can see in Figure 1, the reconstruction is perfect without noise. The red line is the correct boundary, the blue line is the approximation and the dashed line is the initial guess. Even with 6% artificial noise, the location of the obstacle is well recovered though the quality of the shape reconstruction is deteriorated. These results were achieved after 5 and 10 iterations, respectively.

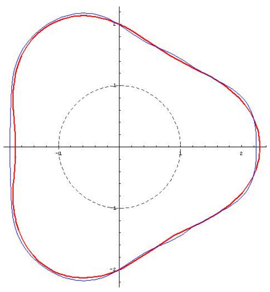

Figure 1. Reconst. without noise

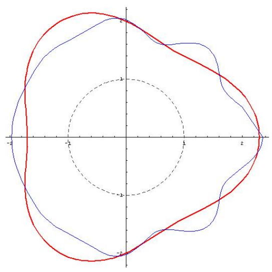

Figure 2. Reconst. with 6% noise

In Figures 3 and 4 we tested reconstructions for a domain that is not contained in the approximation space. We choose a peanut shaped obstacle given by

$$\Gamma = \left\{\sqrt{\cos^2 t + 0.25\,\sin^2 t}\,\{\cos(t), \sin(t)\} \,:\, t \in [0, 2\pi]\right\}.$$

The results are also very accurate and were obtained with 31 and 28 iterations for exact and noisy data, respectively. The convex part of the scatterer is well approximated after 10 iterations as in the previous case, but then the method takes about 20 iterations to also approximate the non-convex part of the boundary. This might be related to remark 3, in the sense that for the final approximation one should use the boundary values for u as obtained from the interior instead of the exterior.

In conclusion, the examples for the reconstructions exhibit the feasibility of the hybrid method. Roughly speaking on one hand the quality of the reconstructions can compete with those obtained by traditional Newton-type methods and on the other hand the computational costs are reduced since there is no forward solver

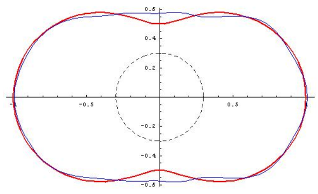

Figure 3. Reconst. without noise

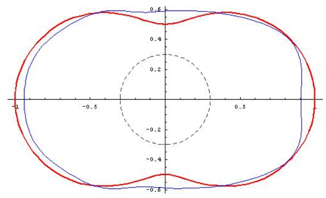

Figure 4. Reconst. with 3% noise

involved. However, more research has to be carried out on improving the performance by more sophisticated selections of the regularization parameters and the approximation spaces.

Acknowledgments

The second author would like to acknowledge the support of his work by *Fundação para a Ciência e a Tecnologia* through the scholarship SFRH/BD/14248/2003.

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