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ScienceDirect

Procedia Structural Integrity 44 (2023) 1538–1545

Structural Integrity Procedia

www.elsevier.com/locate/procedia

XIX ANIDIS Conference, Seismic Engineering in Italy

Surrogate-based bayesian model updating of a historical masonry tower

Federico Ponsi[a], Elisa Bassoli[b], Ghita Eslami Varzaneh[b], Loris Vincenzi[b,*]

[a]*University of Bologna, Department of Civil, Chemical, Environmental, and Materials Engineering, Viale Risorgimento 2, Bologna 40126, Italy*
[b]*University of Modena and Reggio Emilia, Department of Engineering Enzo Ferrari, Via P. Vivarelli 10, Modena 41125, Italy*

Abstract

This paper presents the surrogate-based Bayesian model updating of a historical masonry bell tower. The finite element model of the structure is updated on the basis of the modal properties experimentally identified thanks to a vibration test. In a general context, model updating results are highly affected by several uncertainties, regarding both the experimental measures and the model. Stochastic approaches to model updating, as the one based on Bayes' theorem, enable to quantify the uncertainties associated to the updated parameters and, consequently, to increase the reliability of the identification. The major drawback of Bayesian model updating is the high computational effort requested to compute the posterior distribution of parameters. For this reason, the paper proposes to integrate the classical procedure with a surrogate model. A Gaussian surrogate is employed for the approximation of the posterior distribution of parameters and the performances of the proposed method are compared to those of an Bayesian numerical method proposed in literature.

Keywords: Bayesian model updating, masonry tower, damage function, surrogate model.

1. Introduction

The evaluation of the current structural health of historical masonry structures is a very important issue given their ageing and their poor seismic resistance (Bassoli et al. 2018, Rainieri et al., 2022, Sivori et al. 2022). Indeed, these structures have been generally conceived to bear static gravitational loads and their seismic behavior depends on

* Corresponding author. Tel.: +39-059-2056213; fax: +39-059-2056126.
E-mail address: loris.vincenzi@unimore.it

10.1016/j.prostr.2023.01.197

several factors, namely material properties, structural geometries, floor stiffness and connections between orthogonal walls and structural and non-structural elements (Barbieri et al. 2013). In this context, the use of reliable finite element (FE) models is of great importance for several purposes, such as seismic vulnerability assessment (Barbieri et al., 2013), evaluation of post-earthquake conditions (Bassoli et al. 2018), damage assessment (Ramos et al. 2010), evaluation of the bell-ringing effects (Vincenzi et al. 2019). However, the modelling of the structural behavior is characterized by a high level of uncertainty due to boundary conditions, complex geometries, material properties as well as the presence of damage and stiffness degradation. Vibration-based model updating is surely a widespread solution that allows to increase model accuracy by adjusting a set of uncertain structural parameters with the aim to minimize the difference between numerical and experimental modal properties (Vincenzi and Savoia 2015). Several deterministic approaches to model updating have provided satisfactory results in the structural assessment of masonry constructions (Boscato et al. 2015, Clementi et al. 2017, Ponsi et al. 2021). These approaches are focused on the determination of the optimal values of the updating parameters on the basis of the available experimental measures.

Another class of model updating approaches, named stochastic class of model updating, addresses the problem from a stochastic point of view and allows to quantify the uncertainty affecting the updating parameters. These uncertainties are not only related to the modeling but also to the measurements. For the specific case of experimental modal properties, uncertainties are mainly due to measurement noise, errors introduced by the modal extraction algorithms or changes in the environmental conditions that affect modal properties. The most diffused method of stochastic model updating is based on the Bayes' theorem, where parameter uncertainties are evaluated by combining information based on prior distribution and experimental data (Beck and Katafygiotis 1998). In the last twenty years, methods for structural identification with Bayesian inference have been widely investigated (see, for instance Yuen 2010). These works showed the high computational cost required by the method. Some studies presented in literature, such as Yan et al. (2020), García-Macías et al. (2021) and Ni et al. (2021), have demonstrated that the efficiency of Bayesian inference can be improved through the adoption of surrogate models.

This paper investigates the structural behavior of a masonry bell tower through a procedure based on experimental testing, dynamic identification and Bayesian model updating. Particular attention is paid to the uncertainty bounds for the updated stiffness of the tower. A surrogate model is also adopted as approximated solution in the Bayesian approach with the aim to reduce the computational cost. Results of the surrogate-based method are compared to those of the exact procedure and of a very diffused sampling algorithm for Bayesian updating, namely the Transitional Markov Chain Monte Carlo algorithm (Ching and Chen 2007).

2. Bayesian model updating

Bayesian model updating provides a stochastic framework for parameter updating by considering the model parameters $\mathbf{x}$ and the prediction error as random variables. In this way, different sources of uncertainties can be included in the method. The general principle involves the updating through a set of measured data $\mathbf{d}$ of the prior probability distribution of the model parameters $p(\mathbf{x}|M)$ into the posterior distribution $p(\mathbf{x}|\mathbf{d},M)$:

$$p(\mathbf{x} \mid \mathbf{d}, M) = c^{-1} p(\mathbf{d} \mid \mathbf{x}, M)\, p(\mathbf{x} \mid M) \tag{1}$$

where c is the Bayesian evidence, a constant ensuring that the posterior distribution of parameters integrates to one, and is computed as the integral of the product $p(\mathbf{d}|\mathbf{x},M)p(\mathbf{x}|M)$ over the parameter domain. $p(\mathbf{d}|\mathbf{x},M)$ is the likelihood function representing the plausibility that model M parameterized by $\mathbf{x}$ provides the measured data $\mathbf{d}$. It reflects the contribution of data in the determination of the updated posterior distribution of parameters.

The formulation of the likelihood function depends on the definition of the prediction error, that represents the discrepancy between the N_m experimentally measured and predicted features, in this case frequencies f and mode shapes $\boldsymbol{\varphi}$. The usual practice is to assume a zero-mean Gaussian distribution for frequency and mode shape prediction error (Simoen et al. 2015). For a generic mode m, it is possible to write:

$$e_{f_m} = f_{\exp,m} - f_{\mathrm{num},m} : \mathcal{N}\left(0, \sigma_{f_m}\right); \qquad e_{\boldsymbol{\varphi}_m} = \frac{\boldsymbol{\varphi}_{\exp,m}}{\left\|\boldsymbol{\varphi}_{\exp,m}\right\|_2} - l_m \frac{\boldsymbol{\varphi}_{\mathrm{num},m}}{\left\|\boldsymbol{\varphi}_{\mathrm{num},m}\right\|_2} : \mathcal{N}\left(\mathbf{0}, \boldsymbol{\Sigma}_{\boldsymbol{\varphi}_m}\right) \tag{2}$$

with the scaling factor l_m defined as:

$$l_m = \frac{\boldsymbol{\varphi}_{\exp,m}^T \boldsymbol{\varphi}_{\mathrm{num},m}}{\left\|\boldsymbol{\varphi}_{\exp,m}\right\|_2 \left\|\boldsymbol{\varphi}_{\mathrm{num},m}\right\|_2} \tag{3}$$

Under the assumption of statistical independence of the identified modal properties, the likelihood function can be written as the product among the N_{m} Gaussian distributions with mean $f_{\exp,m}$ and standard deviation σ_{fm} and the N_{m} multivariate Gaussian distributions with mean vector $\boldsymbol{\varphi}_{\exp,m}$ and covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\varphi}m}$ (m =1, …, N_{m}). The covariance matrix is usually assumed to be diagonal meaning that no correlation is considered between different mode shape components. The variance of the frequency prediction error is expressed as $\sigma_{fm}^2=\varepsilon_f^2 f_{\exp,m}^2$, while the covariance matrix of the mode shape prediction error is expressed as $\boldsymbol{\Sigma}_{\varphi m}= \varepsilon_\varphi^2 \, \|\boldsymbol{\varphi}_{\exp,m}\|_2^2 \, \mathbf{I}$, where $\mathbf{I}$ is the identity matrix.

According to the previous assumptions, the likelihood function can be defined as:

$$p\left(\mathbf{d} \mid \mathbf{x}, M\right) = q_1 \exp\left[-\frac{1}{2} J\left(\mathbf{x}\right)\right] \tag{4}$$

where q_1, that is a function of the coefficients of variation ε_f and ε_φ, is a normalizing factor. $J(\mathbf{x})$ is a discrepancy function defined as:

$$J\left(\mathbf{x}\right) = \frac{1}{\varepsilon_f^2} \sum_{m=1}^{N_m} \left(\frac{f_{\mathrm{num},m}\left(\mathbf{x}\right) - f_{\exp,m}}{f_{\exp,m}} \right)^2 + \frac{1}{\varepsilon_\varphi^2} \sum_{m=1}^{N_m} \frac{1}{\left\|\boldsymbol{\varphi}_{\exp,m}\right\|_2^2} \left\| \frac{\boldsymbol{\varphi}_{\exp,m}}{\left\|\boldsymbol{\varphi}_{\exp,m}\right\|_2} - l_m \frac{\boldsymbol{\varphi}_{\mathrm{num},m}\left(\mathbf{x}\right)}{\left\|\boldsymbol{\varphi}_{\mathrm{num},m}\left(\mathbf{x}\right)\right\|_2} \right\|_2^2 \tag{5}$$

2.1. Bayesian selection of the optimal coefficients of variation

When dealing with the construction of the likelihood function, incorrect assumptions regarding the characteristic of the prediction error, namely the value of the coefficients of variation ε_f and ε_φ, may unfairly influence the Bayesian updating results. In this regard, the Bayesian inference framework enables to make use of the available data and to include the error parameters in the updating process in order to identify the characteristics of the prediction error (Simoen et al. 2015). Bayesian model class selection (BMCS) is an additional level of model updating where the focus is addressed to the selection of the most plausible model class from a set of alternatives according to the measured data **d**. In our case, a model class is defined by a specific value of the coefficients of variation ε_f and ε_φ.

Considering a discrete set of model classes $\mathbf{M}=\{M_{\mathrm{k}} : \mathrm{k}=1,2,...,N_{\mathrm{MC}}\}$ the Bayes' theorem expressed at model class level updates the prior probability $P(M_{\mathrm{k}}|\mathbf{M})$ into the posterior $P(M_{\mathrm{k}}|\mathbf{d},\mathbf{M})$ through the information contained in **d**:

$$P\left(M_k \mid \mathbf{d}, \mathbf{M}\right) = \frac{p\left(\mathbf{d} \mid M_k\right) P\left(M_k \mid \mathbf{M}\right)}{p\left(\mathbf{d} \mid \mathbf{M}\right)} \tag{6}$$

If all the model classes are considered equally plausible a priori, the posterior probability depends exclusively on the factor $p(\mathbf{d} \mid M_{\mathrm{k}})$, which is the Bayesian evidence for the model class M_{k}, previously introduced as c in section 2 for a generic model class M. The denominator of Eq. (6) is a constant ensuring that the sum of the posterior probabilities related to all model classes gives 1.

2.2. Computation of the posterior distribution

In accordance with Eq. (1), for the determination of the posterior distribution of parameters $p(\mathbf{x}|\mathbf{d},M)$ the evaluation of the Bayesian evidence is needed. In practice, the direct computation of the Bayesian evidence involves numerical integration of the product between prior distribution and likelihood function over the parameter domain discretized through a mesh. In the following, we indicate this procedure as the "exact" procedure, despite the unavoidable approximations related to the numerical integration. The numerical integration becomes unfeasible when the number of updating parameters is high since the number of evaluations of the likelihood function grows exponentially with the number of parameters. For this reason, several approximated methods have been developed for the determination of the updated distribution of parameters and the evidence. The algorithm developed by Ching and Chen (2007), named Transitional Markov Chain Monte Carlo (TMCMC)-algorithm, is one of the most diffused in the context of Bayesian model updating of structural models. The TMCMC uses a series of intermediate distributions p_j that converge from the prior distribution to the posterior one. At each step, the Metropolis-Hastings algorithm (Hastings 1970) generates a fixed number of samples according to the distribution p_j. Plausibility weights are introduced to assess if the samples generated in the previous step can be employed also in the current one. Finally, the TMCMC also allows to estimate the Bayesian evidence as the product among the expected values of the plausibility weights computed at each step. All the details of the algorithm can be found in Ching and Chen (2007).

2.3. Proposed surrogate-based solution

The main problem of the numerical sampling methods for Bayesian model updating is the high number of samples required to characterize the posterior distribution. In this work, the authors propose the use of a Gaussian surrogate, that approximates the posterior distribution and also allows the computation of the Bayesian evidence.

The complete procedure used in this work to define the surrogate can be summarized as follows:

1) Minimization of the function defined as the negative logarithm of the product $p(\mathbf{d}|\mathbf{x},M)p(\mathbf{x}|M)$ and identification of the Maximum a Posteriori (MAP) solution. Indeed, this solution is the point that maximizes the product $p(\mathbf{d}|\mathbf{x},M)p(\mathbf{x}|M)$. The minimization is performed with a surrogate-assisted evolutionary algorithm (Vincenzi and Gambarelli 2017).
2) Creation of a database containing all the points $\mathbf{x}_i$ evaluated by the optimization algorithm in the previous step and the corresponding values of the product $p(\mathbf{d}|\mathbf{x}_i,M)p(\mathbf{x}_i|M)$.
3) Normalization of the values $p(\mathbf{d}|\mathbf{x}_i,M)p(\mathbf{x}_i|M)$ of the database based on the maximum value obtained at step 1). Collection of these values in the vector $\bar{\mathbf{s}}$.
4) Definition of a Gaussian distribution as the surrogate for the posterior distribution of the updating parameters. The mean vector of the distribution is the MAP solution computed at step 1). The covariance matrix is calibrated by minimizing the error function:

$$f\left(\mathbf{\Sigma}_\mathbf{x}\right)=\left\|\bar{\mathbf{g}}\left(\mathbf{\Sigma}_\mathbf{x}\right)-\bar{\mathbf{s}}\right\|_1 \tag{7}$$

where the symbol $\|\cdot\|_1$ denotes the generalized 1-norm and $\bar{\mathbf{g}}\left(\mathbf{\Sigma}_\mathbf{x}\right)$ is the vector collecting the normalized values of the Gaussian distribution with covariance matrix $\mathbf{\Sigma}_\mathbf{x}$. These values are computed for the points $\mathbf{x}_i$ contained in the database created at step 2). The calibration of the covariance matrix is performed with a penalty approach since the matrix needs to be positive defined.

5) Computation of the evidence as the ratio between the maximum value of the product $p(\mathbf{d}|\mathbf{x}_i,M)p(\mathbf{x}_i|M)$ and the corresponding value of the Gaussian distribution.

Based on this procedure, the Bayesian model updating method allows to determine the updating parameters and their uncertainties with a limited number of modal analyses. Moreover, the integration operation is not necessary, so the computational cost is significantly lower than that of the exact procedure. The use of a surrogate-assisted evolutionary algorithm allows the characterization of the region close to the MAP solution with enough points. In this way, the calibration of the Gaussian covariance matrix is mainly based on the points with the highest values of probably density.

3. The Ficarolo bell tower

The structure, showed in Fig. 1, is a masonry bell tower located in the city of Ficarolo (Veneto, Italy). Its construction started in 1777 and it presents an impressive vertical inclination with a mean tilt angle of about 3°. The tower is about 68 m high and has a variable cross section whose dimension is variable, from about 8.50 m at the base up to 5.30 m at the cusp level. Two intermediate masonry cross-vaulted floors are located at the level of 45.0 m and 53.0 m, the first of those support the belfry. Due to the Emilia earthquake occurred in 2012, the structure has suffered serious damage, thus retrofitting interventions where planned and performed in 2014. The dynamic behavior of the Ficarolo bell tower has been characterized thanks to two ambient vibration tests performed before and after the strengthening interventions. Since the model calibration is performed with reference to the actual condition of the tower (namely after the strengthening), the identification of modal parameters presented in the following refers to that condition. The first five identified modes are listed in the first two columns of Table 1. More detail about the tower geometry, the instrumentation and the modal extraction can be found in Ponsi et al. (2022).

3.1. FE model

A FE model of a three-dimensional cantilever beam discretized in 32 elements with flexural and shear deformability has been created for the bell tower. The distribution of the stiffness along the height of the tower is highly uncertain due to the deformability of the soil-foundation system, the presence of a rock basement and the presence of a masonry vault at the height of 45 m. The calibration of the stiffness distribution is thus necessary in order to match the experimental modal properties as well as possible. The variation of stiffness along the tower height is taken into account by implementing the so-called damage function approach as proposed by Teughels et al. (2002).

The external side of the square cross section, that affects both flexural and shear stiffness, is described by a piece-wise linear function, while the internal side of the cross section is supposed to have a constant value equal to 3.9 m. A reference value for the external side of the cross-section B_0 of 7 m has been considered and for each element e of the FE model the updated side B^e is computed through the parameter a^e, that represents the relative variation with respect to the reference value:

$$B^e = B^0\left(1 - a^e\right) \tag{8}$$

Fig. 1. The Ficarolo bell tower.

Table 1. Comparison between experimental and numerical modal properties (MAP solution).

Mode shape	Exp. Freq. [Hz]	Num. Freq. [Hz]	MAC [%]
1st bending Y	0.55	0.56	96
1st bending X	0.57	0.56	97
2nd bending X	2.16	2.14	96
2nd bending Y	2.18	2.14	96
1st torsional	3.10	3.08	95

The correction parameter a^e is computed by the linear combination of the damage functions N_i:

$$a^e = \sum_{i=1}^{N_F} p_i N_i \left(\mathbf{y}^e\right) \quad (9)$$

where N_F is the number of functions N_i used in the discretization, p_i are their multiplication factors and $\mathbf{y}^e$ is the vector containing the centroid coordinate of the element e. In summary, the FE model is divided into substructures and the variation of the generic parameter a^e inside a substructure is described through the damage functions.

For this case study, three substructures, each one characterized by linear or piece-wise linear damage functions, are defined. The first substructure includes the FEs n° 1-3 and takes into account the deformability of the soil-foundation system and the presence of a rock basement. The second substructure includes the FEs n° 4-18 and is characterized by the decrement of the cross-section dimension with height. The last substructure includes the FEs n° 19-22 and considers the increment of stiffness due to the presence of a masonry vault at the height of about 45 m. The upper part of the bell tower does not significantly affect the modal behavior of the tower. It is accurately modeled in term of masses and stiffeners, but no updating parameters are considered for this part of the model.

3.2. Bayesian model updating and model class selection

Bayesian model updating and model class selection (see section 2) have been carried out in order to determine the optimal coefficients of variation ε_f and ε_φ for the likelihood function and the posterior distribution of parameter vector $\mathbf{x}$. The updating parameters are the four multiplication factors p_i that appear in Eq. (9). The considered prior distribution is a non-informative uniform distribution defined in a four-dimensional hyper-cubic domain where each updating parameter p_i belongs to the interval [-0.5, 0.4]. This domain is discretized into a regular grid employing a step size of 0.025 for each parameter.

Fig. 2a shows the contour plot of the posterior probability for the coefficients ε_f and ε_φ. The probabilities have been computed for values of ε_f and ε_φ in the range [1%, 10%] with step-size 0.5%. The optimal pair of coefficients, that corresponds to the pair with the maximum posterior probability, is ε_f =2.5% and ε_φ =3%. Moving away from the maximum, the slope of the distribution is steeper in the ε_φ direction, highlighting the more sensitivity of the posterior probability towards the mode shape coefficient.

Fixed the optimal coefficients of variation, the posterior distribution of updating parameters is calculated. Considering the MAP solution, a very good agreement between experimental and numerical modal properties is obtained, as shown in Table 2. As concerns the parameter uncertainty, the posterior marginal distributions are reported in Fig. 2b. The corresponding updated stiffness distribution is illustrated in Fig. 3a with the uncertainty bounds [μ_{EI} - σ_{EI}; μ_{EI} + σ_{EI}]. We can note the high uncertainty that characterizes the updated values at the base of the tower with differences of about $2.5 \cdot 10^{11}$ Nm^2.

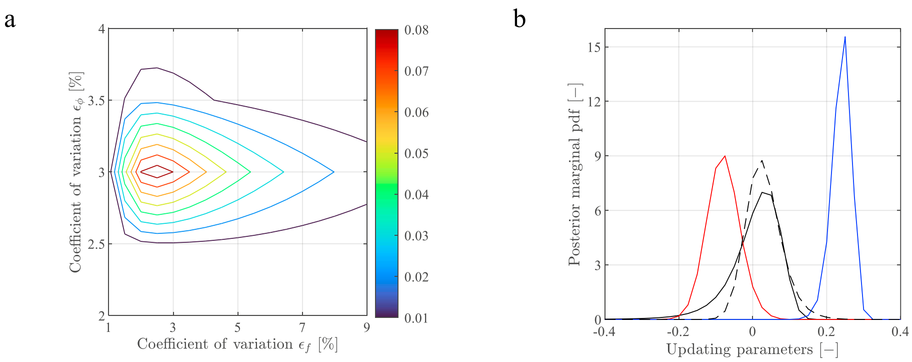

Fig. 2. (a) contour plot of the posterior probability for different values of the coefficients of variation ε_f and ε_φ; (b) marginal posterior distributions of the updating parameters p_1 (black), p_2 (red), p_3 (blue) and p_4 (dashed black)

Table 2. Comparison between the results of the exact procedure, the TMCMC and the proposed solution.

Method	p_1		p_2		p_3		p_4	
	Mean [-]	St. Dev. [-]	Mean [-]	St. Dev. [-]	Mean [-]	St. Dev. [-]	Mean [-]	St. Dev. [-]
Exact	0.006	0.071	-0.079	0.044	0.240	0.026	0.031	0.049
TMCMC	-0.014	0.087	-0.062	0.049	0.226	0.033	0.049	0.057
Gaussian surr.	0.034	0.043	-0.093	0.037	0.249	0.021	0.016	0.043

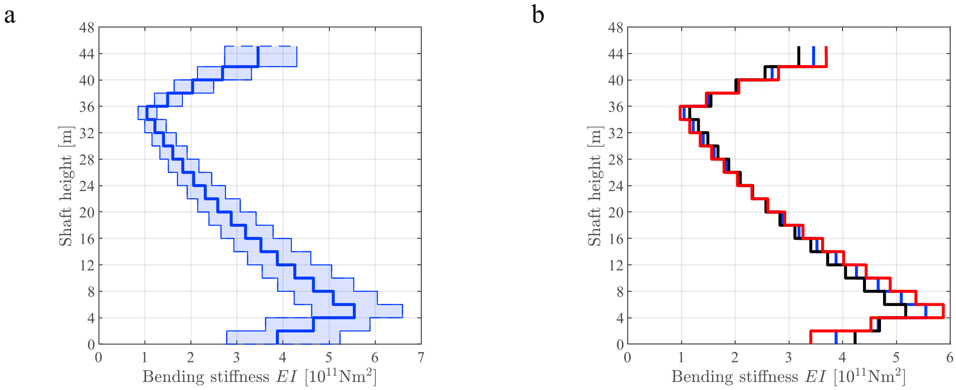

Fig. 3. (a) bending stiffness distribution with the uncertainty bounds [μ_{EI}-σ_{EI}; μ_{EI}+σ_{EI}]. (b) mean values of the updated stiffness distribution for the exact method (blue line), the TMCMC (black line) and the surrogate-based method (red line).

3.3. Approximated methods for Bayesian model updating

In this section, the approximated surrogate-based method proposed in section 2.3 is applied for the Bayesian updating of the tower FE model. Updating results are presented in Table 2 together with the results of the exact procedure and those of the TMCMC. Focusing on the comparison of the TMCMC and of the surrogate-based method results with the exact ones, the mean values of the updating parameters p_1, p_2, p_3 and p_4 are quite similar. The largest relative difference is found for the parameter p_1 in both TMCMC and surrogate-based method. As regards the standard deviation of all the parameters, it is lightly overestimated by the TMCMC and lightly underestimated by the surrogate-based method. There are significant differences in the number of modal analyses required to perform the updating: about 1.9 million for the exact procedure, 9000 for the TMCMC and 265 for the surrogate-based method.

The mean values of the updated stiffness distribution obtained with the three methods are represented in Fig. 3b. All the distributions are characterized by the same trend, proving the good approximations obtained by proposed method. The major differences in terms of stiffness values are noted for the elements located at the base of the model. This is in line with the observations of the results presented in Table 2.

4. Conclusions

In this paper, the structural identification of the FE model of the Ficarolo bell tower has been presented. The FE model is a simple cantilever beam where the stiffness variation along the longitudinal axis is parametrized with the so-called damage function approach. The parameter identification is based on the experimental modal properties extracted from the acceleration response of the structure acquired during an ambient vibration test and it is performed with a Bayesian approach. This approach allows to obtain the optimal values of the prediction error coefficients of variation from the experimental data and to quantify the uncertainty of the updated parameters.

Results show a large uncertainty for the updated stiffness value at the base of the tower. Indeed, the deformability of the soil-foundation system and the presence of a rock basement represent significant uncertainty sources for the model.

The posterior distribution of the updating parameters is computed in an approximate way through a surrogate-based method. The comparison with the exact results and with those obtained by means of the TMCMC algorithm reveals that the proposed method allows to compute a sufficiently accurate solution for this problem with a computational cost significantly reduced if compared to that of the exact procedure or of the TMCMC. The main drawback deals with the uncertainty of the updated parameters, that is lightly underestimated with respect to the exact values. In the authors opinion, it can depend on the discrepancy between the exact posterior distribution and the Gaussian approximator in the tail areas.

Acknowledgment

The research was partially supported by the ReLUIS-DPC 2022-2024 Project (Line WP6 – Monitoring and satellite data). The financial support of the Civil Protection Department and the Reluis Consortium is gratefully acknowledged.

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