PROCEEDINGS OF SCIENCE

Structure functions and perturbative hysteresis

V. Bertone,[a] **G. Bozzi**[b,c] **and F. Hautmann**[d,e]

[a] *IRFU, CEA, Université Paris-Saclay, F-91191 Gif-sur-Yvette*
[b] *Dipartimento di Fisica, Università di Cagliari, Cittadella Universitaria, I-09042 Monserrato*
[c] *INFN, Sezione di Cagliari, Cittadella Universitaria, I-09042 Monserrato*
[d] *University of Oxford, Theoretical Physics Department, Oxford OX1 3PU*
[e] *Universiteit Antwerpen, Elementaire Deeltjes Fysica, B 2020 Antwerpen*
E-mail: valerio.bertone@cern.ch, giuseppe.bozzi@unica.it, hautmann@thphys.ox.ac.uk

We discuss hysteresis effects in the perturbative solution of renormalization group equations for the strong coupling and parton distribution functions, and study their impact on precision determinations of proton's deep-inelastic structure functions F_2 and F_L.

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Hadron structure constitutes one of the main sources of systematic uncertainties in theoretical predictions for high-energy collider physics. Besides the available experimental information from deep inelastic scattering (DIS) measurements and from Large Hadron Collider (LHC) and Tevatron data, a substantial improvement in our knowledge of hadrons' parton distribution functions (PDFs) is expected from future hadronic colliders FCC-hh [1], FCC-he and LHeC [2], EIC [3]. A major effort is ongoing to extend the current perturbative accuracy of theoretical calculations for the partonic structure to four-loop splitting functions and DIS coefficient functions [4–7], and develop corresponding next-to-next-to-next-to-leading-order (N^3LO) phenomenology [8].

With this dramatic increase in precision, theoretical systematic uncertainties arising from the perturbative solution of renormalization group equations (RGEs) become an important factor in determining the overall accuracy of theory predictions for collider processes. In Ref. [9] we have investigated this systematics in the case of the RGEs controlling the evolution of the Quantum Chromodynamics (QCD) running coupling α_s and PDFs. We have analyzed "hysteresis effects" in perturbative RGEs, associated with the difference between solutions which are formally equivalent at the nominal logarithmic accuracy but differ by subleading terms, and thus source RGE theory uncertainties. We have shown that these effects can be recast in terms of emergent "resummation scales", by employing techniques frequently used in the soft-gluon resummation literature [10–12].

The relevance of such effects in double-logarithmic problems has been noted in [13, 14] for analytic resummation calculations and in [15] for parton branching calculations. In [9, 16] we have stressed their role in single-logarithmic problems, in particular noting their importance in kinematic regions which are essential for present (and future) PDF determinations.

In this article we report results of implementing the resummation scales of [9, 17] in the evolution code APFEL++ [18], computing the DIS structure functions F_2 and F_L, and comparing numerically the size of theoretical systematic uncertainties encoded in the resummation scales with those encoded in the factorization and renormalization scales. In this calculation we take the value of α_s at the Z-boson mass ($\alpha_s(M_Z) = 0.118$ [19]) as an input to the RGE for QCD running coupling and the MSHT20 [20] PDF set at $Q_0 = 2$ GeV as an input to the RGE for PDFs.

In Figs. 1 and 2 we show the x-dependence of, respectively, the F_2 structure function and the longitudinal structure function F_L, for different values of Q, at next-to-next-to-leading order (N^2LO) using three-loop splitting functions [21, 22] and order-α_s^2 coefficient functions [23] (the order-α_s^3 F_L coefficient [24] is not yet implemented in the figures). We show theoretical uncertainties obtained from the "standard" variations of the renormalization scale μ_R (purple band) and factorization scale μ_F (green band) about the hard-scattering scale Q, and from variations of the resummation scales [9, 17] (blue band). The effects due to the resummation scales associated with perturbative hysteresis in the RGEs for α_s and PDFs are in principle distinct; in Figs. 1 and 2 we combine these effects according to the approach [17]. The blue band shows the combined effect, expressed through variations of the resummation scale parameter ξ.

Resummation scale ξ uncertainty bands are observed to be generally of comparable size to the renormalization scale μ_R and factorization scale μ_F uncertainty bands. For instance, in the top left panel of Fig. 1 for F_2 at $Q = 10$ GeV, the ξ contribution dominates in the low-x region while the μ_F contribution dominates at the highest x. The other panels of Fig. 1 illustrate that, as Q increases, the ξ uncertainties become larger relative to the μ_F and μ_R uncertainties, so that they eventually become important also in the high-x region.

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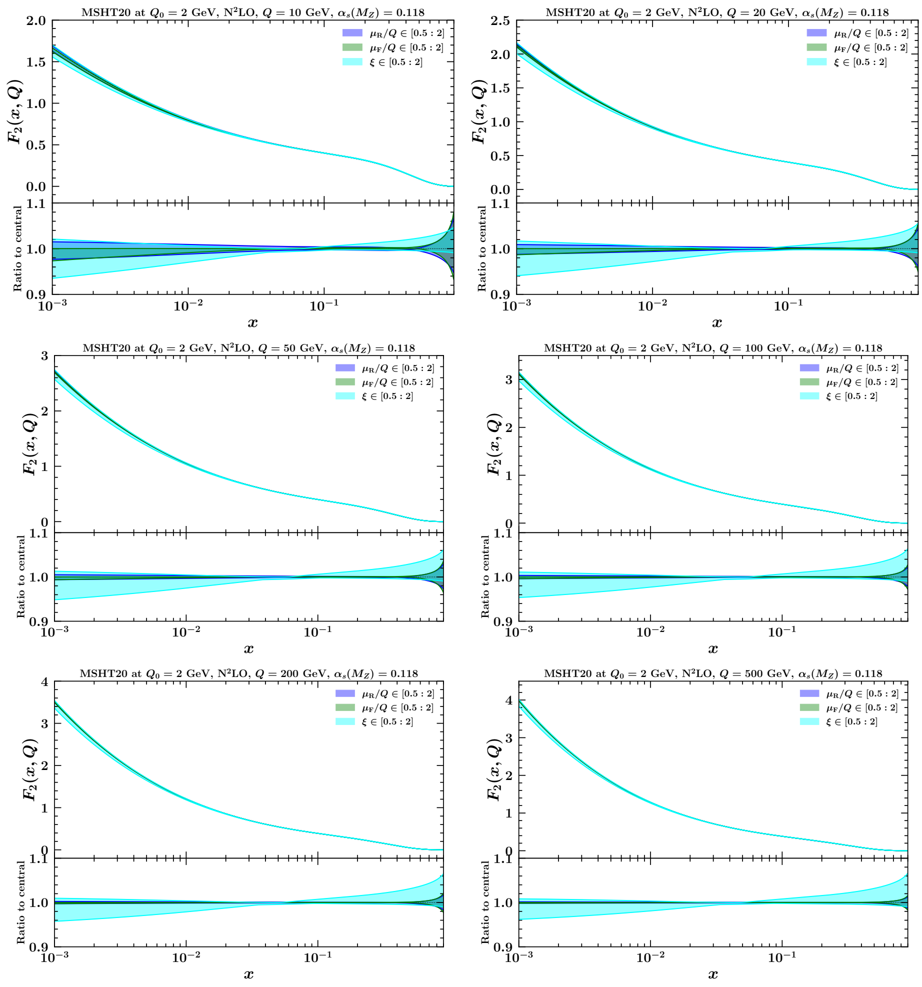

Figure 1: The F_2 structure function versus x for different values of Q, at N^{2}LO in perturbation theory, with the uncertainty bands associated with variations of renormalization and factorization scales, μ_R and μ_F, and resummation scale, $\mu_{\text{Res}} = \xi\mu$. We use MSHT20 PDFs at $Q_0 = 2$ GeV and QCD coupling at $\mu_0 = M_Z$ as RGE inputs.

In the case of the longitudinal structure function F_L in Fig. 2, the μ_F and μ_R uncertainties are larger than for F_2, and dominate the ξ contribution for low Q; however, the relative importance of the ξ contribution increases as Q increases, similarly to the case of F_2, first becoming comparable to the μ_F and μ_R contributions at low x, and then also at high x.

The ξ contribution staying comparatively significant in the large-Q, small-x regions in Figs. 1 and 2 corresponds to higher-order perturbative corrections to PDF anomalous dimensions dominating the small-x region [25, 26] for sufficiently large Q. In this sense, taking into account theoretical uncertainties by exploiting perturbative hysteresis and the associated resummation scale gives an estimate of the size of effects to be expected from phenomenological analyses [27–32] of small-x resummations.

On the other hand, as emphasized in [9] the method presented in this article is general. It can

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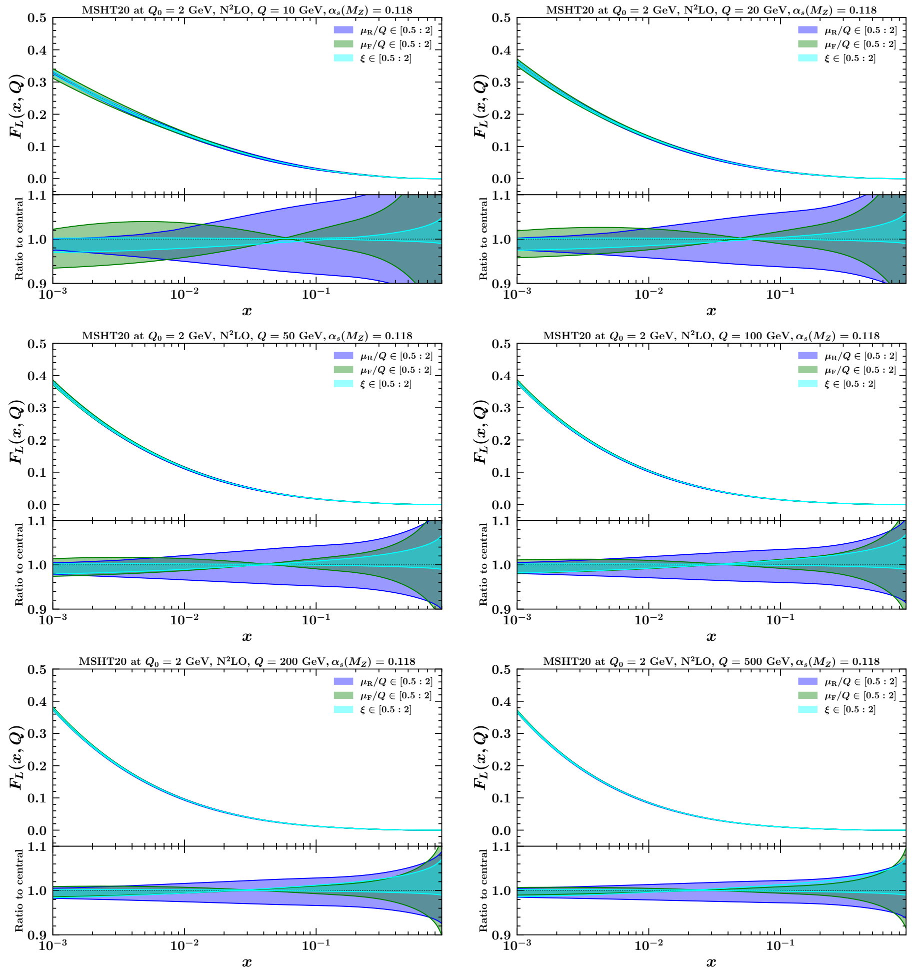

Figure 2: The longitudinal structure function F_L versus x for different values of Q, at N^{2}LO in perturbation theory, with the uncertainty bands associated with variations of renormalization and factorization scales, μ_R and μ_F, and resummation scale, $\mu_{\text{Res}} = \xi\mu$. We use MSHT20 PDFs at $Q_0 = 2$ GeV and QCD coupling at $\mu_0 = M_Z$ as RGE inputs.

be used to estimate theoretical uncertainties in PDF determinations for any value of x, and also applied to precision studies of transverse momentum distributions [33, 34] in Drell-Yan production.

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