

## ADAPTATION OF HYPERBOLOID STRUCTURE FOR HIGH-RISE BUILDINGS WITH EXOSKELETON

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### Abstract

In this paper implementing a hyperboloid structure as the exoskeleton of a high-rise building is studied. A DiaGrid exoskeleton with straight lines from the ground to top is used to set up the structure of the building and determine its hyperboloid architectural form within a parametric environment. The resulting structure is later subjected to lateral loads and linear-static structural analysis. The results demonstrate that the lateral drift of the structure with hyperboloid shape will become as low as half of a cylindrical shaped building with similar architectural and functional properties and the same structural system under similar loading conditions. The results of the preliminary analysis of the structure in the parametric environment are further verified with a more sophisticated structural model under seismic loads with static equivalent seismic loads and response spectrum modal analysis.

### Keywords

Parametric Modelling, DiaGrid Structures, Tall Building Design, Hyperboloid Structures

## 1 Introduction

The relationship between the architectural form of buildings and their structural behavior has always been of great interest both among architects and structural engineers. This matter becomes increasingly important in the design of high-rise buildings. This relationship becomes maximal when dealing with buildings with exoskeleton structures, where all or the majority of the load-bearing capacity of the structure is located within or peripheral to the building's envelope. In this paper which is part of an ongoing research project on the effect of the external shape of buildings with external structures and exoskeletons over their structural behavior, adopting a hyperboloid structure as the exoskeleton of high-rise buildings is investigated and comparison has been made with similar buildings with cylindrical shapes.

The hyperboloid exoskeleton of the structure is generated following a process through which corresponding nodes on the top and bottom circular sections of the building floor plans are connected with straight line subsequent to a 90 degrees shift. Thus, making an orthogonal diagonal grid (DiaGrid) for the exoskeleton structure of the building. In this fashion not only

all the structural members of the exoskeleton become straight lines, but also all the connecting structural elements at every joint will become coplanar.

Having considered that investigating the structural behavior of exoskeleton buildings with the curvilinear external surface may only be possible in a 3-dimensional environment; parametric design strategy has been incorporated in the design and structural analysis of the buildings. The resulting structure is then subjected to lateral loads, and following a linear static analysis, the structural response is generated for different values of design parameters such as total constructed area and height.

The comparison is later made with the response of similar building with cylindrical shapes and equal gross floor area. The results demonstrate that although the buildings with hyperboloid shape experience greater values of overall drift than their matching cylindrical ones, the values of the member forces remain significantly lower.

In order to verify the structural results obtained within the parametric environment, one of the models generated within the parametric environment is further investigated in a non-parametric structural analysis software. It is observed that although the response of static linear analysis shows the acceptable similarity between the two environments, the results of the response spectrum analysis show significant differences with static equivalent analysis.

## 2 Exoskeleton Buildings

Typically the structure of a building is designed to remain hidden within the architectural components of a building, however, in exoskeleton building, it is completely the other way around. In exoskeleton buildings, the structure overpasses the building envelope and becomes an essential part of the building architecture determining not only the buildings schematic shape but also the ornamental detailing of the building's exterior.

In an exoskeleton building, the structure is situated over the building exterior or within its envelope, and besides its expressiveness as an architectural feature of the building, it provides the advantage of the higher flexibility of internal spaces due to a reduction in spatial constraints within the interior spaces of a building. Among the most common structural system of exoskeleton building are DiaGrid structural systems, figure 1 shows some highlight examples of buildings with DiaGrid structures.



Figure 1. a) Hearst tower, New York. b) CCTV Headquarters, Beijing. c) Shukhov radio tower, Moscow.

## 2.1 Dia-Grid system

Diagonal Grid structural system – or DiaGrid system in short – is a type of exoskeleton structural system in which the structure of the building is composed of tension/compression members constituting a network of triangular, rhombus or parallelogram bracing all over the exterior of the building, hence the name DiaGrid. The behavior of DiaGrid modules is similar to the typical bracing frames used for lateral stiffness of steel buildings with the exception that the modules are not necessarily coplanar and the components and connections of the DiaGrid system are usually exposed. Another difference is that the diagonal members in DiaGrid systems are no more constrained within the structural frames, but rather through maneuvering around the building and covering the entire surface of the building exterior they will become a self-supporting structure, used to support both lateral and gravitational loads.

It is noteworthy to mention that origins of DiaGrid structures can be found in the works of Vladimir Shukhov [1] the Russian Architect/Engineer whose name is also among the list of pioneers of hyperboloid structures.

## 3 Hyperboloid structures

Application of hyperboloid forms for building and construction structures and their elements dates back to the late 19th century with Antoni Gaudi's experiments with different geometrical forms for architectural spaces and architectural elements [2]. Even then the functional properties of hyperbolic forms, such as added stability, were evident along with their aesthetical characteristics. But it was through the works of the Russian polymath Vladimir Shukhov and his employment of this non-Euclidean geometry into the skeletal forms of structures that the full potential of hyperboloid structures in terms of stability and lightness was revealed [3]

Also, the ability that such double-curved surfaces can be generated from an orthogonal net of straight linear elements brings further attention to these types of structural forms since it will eliminate the need to curve the structural members.

Hyperboloid structures have traditionally been used as communication towers and industrial structures but during the past decade adaptation of hyperboloid forms for high-rise buildings has become of interest among architects and in some the hyperboloid shape of the building has taken a significant role in its structural behaviour, the most significant example being the almost 600 meters high Canton tower.

### 3.1 Negative Gaussian curvature

One important aspect of hyperboloid shapes is that all over the surfaces of a hyperboloid the Gaussian curvature has a negative value, meaning that at every point on the surface if you draw two perpendicular routes, while one is twirling in the inward direction the other twirls in the outward direction and vice versa.

Usually the negative Gaussian curvature is considered as an advantage against local buckling in the shell structures compared to surface shells with zero or positive Gaussian curvature under in-plane pressure [4]. This is mainly because while the in-plane pressure in one direction tends to terminate local stability by inward buckling in the other direction there exists a tendency for outward buckling and the reducing effect that these two have on each other will

result is further resistance in the shell member to avoid local buckling. This is the main reason for the hyperboloid shape of very large cooling towers where the absence of internal structural elements within the external shell of the tower increases the chance of failure due to local buckling. And since in the exoskeleton buildings the structural responsibilities are focused on the external surface of the building and through analogy with the aforementioned behavior it is assumed that the stability of the structure is increased by benefiting from a hyperboloid structure which will be later demonstrated in lower values of lateral drift.

## 4 Form generation process

The form generation process of the hyperboloid tower is similar to the technique used by the well-known architect Antoni Gaudi to create hyperbolic shape 3D ornaments, which is by connecting two circular rings with a set of strings and subsequently rotating one of the circular rings around its axis (Figure 2a).

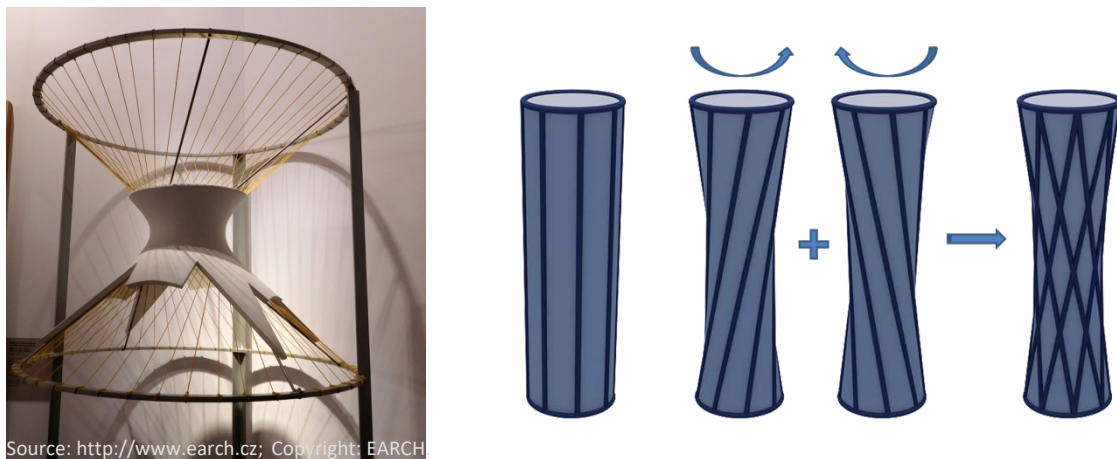


Figure 2. a) Gaudi Hyperboloid ornamentals. b) Form generation for hyperboloid tower.

In the case of the hyperboloid tower, the top and lower circles respectively represent the roof plan and the ground floor plan of the building. Having divided each of them into 32 segments, results in 32 inclined but yet straight columns which shape the form of the building. The same process is again repeated but this time the direction of rotation is in the opposite direction. Superimposing the two hyperboloid forms generated through the aforementioned process (Figure 2b) will result in a hyperboloid shaped tower with 64 inclined yet straight columns that pass through one another at every connection point and create an orthogonal net with semi-regular rhomboid modules in the form of a DiaGrid structure that make up both the architectural form of the building as well as its structure.

As for the functional parameters of the building under study in this research; the top and bottom circles – constituting the roof and ground floor – each has a diameter of 47 meters and are with a distance of 154.8 meters from each other providing room for 36 stories with a height of 4.3 meters. This in total results in a total constructed area of 43244 m<sup>2</sup>.

For evaluating the behaviour of the hyperboloid structure generated by the above approach, a similar building with identical functional properties such as total constructed area, total height and number of storeys, and with the same structural system (exoskeleton DiaGrid) but with cylindrical form is also modelled and the results of the structural analysis of the two structures has been subject to comparison later in this paper. In order to obtain the same

functional properties for the two structures, the diameter of the cylindrical structures is considered to be equal to 38.6 meters resulting in an almost equal 43297 m<sup>2</sup> total constructed area. The area of every floor and the total sum along with other data related to the horizontal loads applied to each structure is presented in table 1.

## 5 Structural analysis

For the purpose of variations in the form and geometry of the buildings, the entire process of generating the building form and its structure has been performed within the parametric environment of *Grasshopper 3D* [5] plug-in for *Rhinoceros* [6]. This not only facilitates the creation of the 3D complex model, but also serves the broader objectives of this ongoing research, which is to investigate the effect of shape parameters such as; ratio between width and height; ratio between ground floor and roof diameters; horizontal and vertical segmentation of the DiaGrid system and etc. on the structural response of the building to lateral loads.

A linear static analysis is then performed on the generated structural model using *Karamba* add-on for *Grasshopper*, which is a Finite Element three-dimensional software with the ability to model beam and shell elements and working in *Grasshopper* parametric environment [7]. The linear analysis performed with *Karamba* is later verified in one case with an identical numerical model constructed with the software *SAP2000*; once subjected to a similar static linear analysis and once with a semi-dynamic response spectrum analysis. This will be further discussed in section 7 of this paper.

### 5.1 Structural mapping

As described earlier in the previous section the definition of the structure of the hyperboloid tower is carried out simultaneously along with the form generation process of the building architecture and there is no need for mapping a structural system over the designed building, nevertheless, there is still a need for an internal structure. In this research it is assumed that the exoskeleton DiaGrid explained in the previous section is solely responsible for resisting the entire lateral loads applied on the building. However, for the sake of a realistic design approach and avoiding extremely large interior spaces without vertical support within the building (with distances varying from 33 up to 47 meters), a secondary internal structure is added just to withstand vertical and gravitational loads and evading extremely large internal spans. This secondary structure is composed of 16 vertical columns in a circular arrangement with a diameter of 17 meters and situated in the center of the building with the non-rigid connection between the vertical columns of the secondary structure and the floor beams (Figure 3).

For the case of the cylindrical building, the same segmentation has been applied to external DiaGrid structure. The 32 horizontal segments at every story are situated with half a phase shift with respect to adjacent stories and a zigzag connection from bottom to top at every segment create the entire exoskeleton of the building. The same internal structure for vertical loads is also positioned in the middle of the building and identical structural elements are used in both structures (Figure 3).

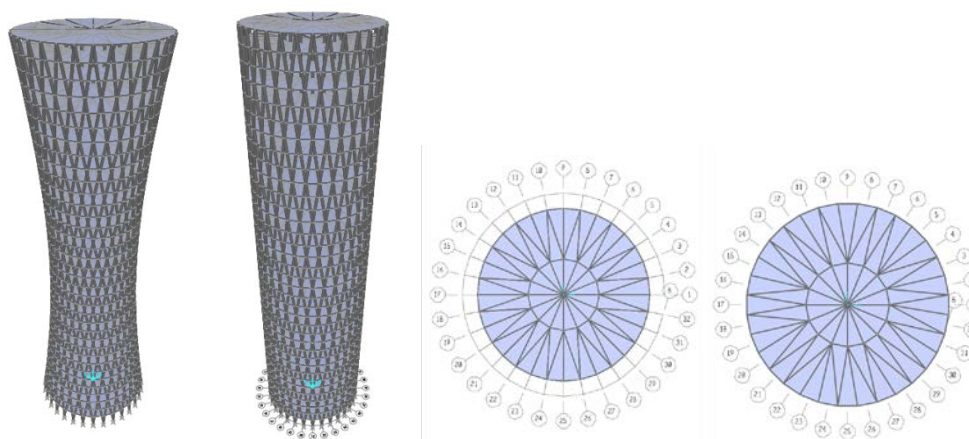


Figure 3. Structure of the Hyperboloid and Cylindrical structures.

Besides transferring the vertical loads, this will offer an internal core for positioning vertical access such as elevators and staircases as well as building services. Figure 4 demonstrates the arrangement of the secondary structure as well as its role in the architectural plans. It needs to be further emphasized that due to the non-rigid connection between the internal columns and floor slabs they play an insignificant role in the building's lateral response.

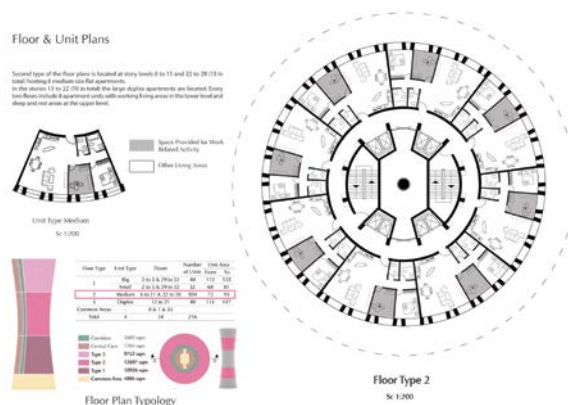


Figure 4. Architectural floor plan of the Hyperboloid tower.

For both the DiaGrid structure and the internal columns a steel tube section with an outer diameter of 80cm and thickness of 2cm has been assigned. And the radial floor beams of the building are a generalized section with a cross-section area of 250cm<sup>2</sup> and moment of inertia equal to 7400cm<sup>4</sup>. The total weight of the steel structure of the two buildings with described profile sections is presented in table 2 along with the results of the structural analysis performed.

## 5.2 Application of lateral loads

For the linear static analysis of the structure, a total horizontal load of 60'000 KN is applied on the structure representing the likely seismic load. This value is derived from the assumption of a total mass of 60'000 tons participating in seismic action (1.3 tons pre-meter square) and a base shear coefficient of 0.1. The 60'000 KN base shear which is the total lateral load acting on the structure is distributed in accordance with the assumptions of "static equivalent seismic load" and following the equation below:

$$f_i = \frac{W_i h_i}{\sum W_i h_i} V \quad (1)$$

Where  $f_i$  is the applied load on the  $i^{\text{th}}$  floor,  $W_i$  is the weight of the  $i^{\text{th}}$  floor and  $h_i$  is the height of the  $i^{\text{th}}$  floor from ground level and  $V$  is the total horizontal equivalent seismic load acting on the structure. Table (1) shows the value of the loads applied to each structure at the center of every floor level.

Table 1: constructed area and applied load at every story level

		Hyperboloid		Cylindrical				Hyperboloid		Cylindrical	
floors No.	height above ground	Floor area	Seismic load	Floor area	Seismic load	floors No.	height above ground	Floor area	Seismic load	Floor area	Seismic load
	m	m <sup>2</sup>	KN	m <sup>2</sup>			m	m <sup>2</sup>	KN	m <sup>2</sup>	
0	0	1724	0	1170	0	19	81,7	869	1272,21	1170	1711,71
1	4,3	1637	126,2	1170	90,09	20	86	874	1348,02	1170	1801,80
2	8,6	1550	238,96	1170	180,18	21	90,3	890	1439,93	1170	1891,89
3	12,9	1462	338,18	1170	270,27	22	94,6	908	1538,99	1170	1981,98
4	17,2	1389	428,12	1170	360,36	23	98,9	932	1651,75	1170	2072,07
5	21,5	1317	507,73	1170	450,45	24	103,2	961	1778,50	1170	2162,16
6	25,8	1245	575,9	1170	540,54	25	107,5	996	1918,48	1170	2252,25
7	30,1	1189	641,73	1170	630,63	26	111,8	1036	2076,25	1170	2342,34
8	34,4	1131	697,39	1170	720,72	27	116,1	1082	2250,92	1170	2432,43
9	38,7	1082	750,31	1170	810,81	28	120,4	1131	2440,85	1170	2522,52
10	43	1036	798,56	1170	900,9	29	124,7	1189	2658,58	1170	2612,61
11	47,3	996	844,13	1170	990,99	30	129	1245	2879,49	1170	2702,70
12	51,6	961	889,25	1170	1081,08	31	133,3	1317	3147,93	1170	2792,79
13	55,9	932	933,6	1170	1171,17	32	137,6	1389	3424,94	1170	2882,88
14	60,2	908	979,36	1170	1261,26	33	141,9	1462	3719,97	1170	2972,97
15	64,5	890	1028,52	1170	1351,35	34	146,2	1550	4062,39	1170	3063,06
16	68,8	874	1078,41	1170	1441,44	35	150,5	1637	4416,91	1170	3153,15
17	73,1	869	1138,29	1170	1531,53	36	154,8	1724	4783,41	1170	3243,24
18	77,4	862	1195,85	1170	1621,62						
<b>Total Sum</b>								<b>43245</b>	<b>60000</b>	<b>43298</b>	<b>60000</b>

## 6 Analysis results and comparison

Having applied the horizontal loads to the hyperboloid and cylindrical shaped buildings according to table (1) and performing the linear static structural analysis, it can be observed that the horizontal displacement of the structure under lateral loads at the top floor in the hyperboloid structure becomes equal to 0.776 meters, which is about 60% of the 1.291m displacement at the top floor of the cylindrical structure subject to the same amount of horizontal action. This concludes that the lateral stiffness of the hyperboloid structure is 1.66 times the stiffness of the cylindrical shaped building with DiaGrid exoskeleton structure.

Although the considerable difference is witnessed in the stiffness of the two structures, there has not been a considerable difference in the value of the resulting forces in the structural members of the two structures. While the maximum utilization ratio (resulting force in the structural member divided by its load bearing capacity) of the diagonal members of the cylindrical structure at the lowest storey level reaches a maximum of 148%, the same value for diagonal members of the hyperboloid structures is up to a maximum of 139%, which is only 6% smaller than that of cylindrical tower.

The resulting data of the maximum displacement, maximum utilization ratio, maximum tensile and compressive forces, and maximum story drift ratio (relative displacement between adjacent stories divided by the story height) for the two structures are provided in table 2.

The displacement of the floor center of each structure due to lateral loads is also presented in figure 5.

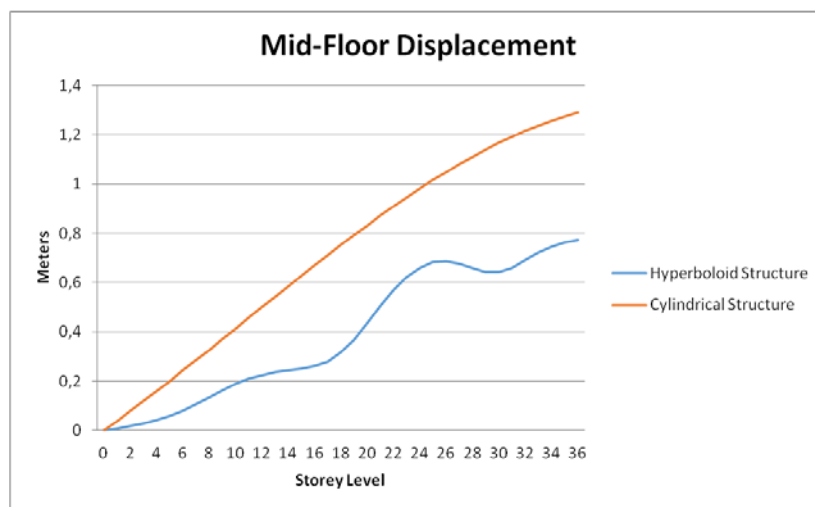


Figure 5. Mid-floor displacements for Hyperboloid and Cylindrical Structures subject to lateral loads.

## 7 Verification of structural analysis

In order to validate the structural analysis performed in *Karamba* in the parametric environment, both the hyperboloid and cylindrical structures were again modeled in *SAP2000* structural analysis software. The same section properties have been assigned to structural members and analysis with static equivalent seismic load with a base shear coefficient of 0.1 and modal response spectrum analysis have been performed. The input data for the response spectrum analysis are as follows: design ground acceleration 0.4g; spectrum type 1 of Eurocode 8, 2004; ground type B; and target mass participation of 99%. The resulting base shear for each analysis is presented in table 2.



Table 2: Analysis results and comparison data for Hyperboloid and Cylindrical buildings

		Weight of Structure	Constructed area	Maximum displacement	Total horizontal load	Maximum resultant force in structural members
		kg	m <sup>2</sup>	m	KN	KN
Hyperboloid Structure	Parametric Analysis In Karamba	12,64 e6	43244	0,776	60000	9823
	Static Equivalent Seismic			0,732	59015	
	Response Spectrum			0,495	49792	
Cylindrical Structure	Parametric Analysis In Karamba	12,07 e6	43297	1,291	60000	16164
	Static Equivalent			1,218	61407	15522
	Response Spectrum			0,884	49100	11770

Also, the displacement of a center point at every floor level for analyses performed in Karamba, Static equivalent force in *SAP2000* and modal response spectrum analysis for both structures is presented in figure 6.

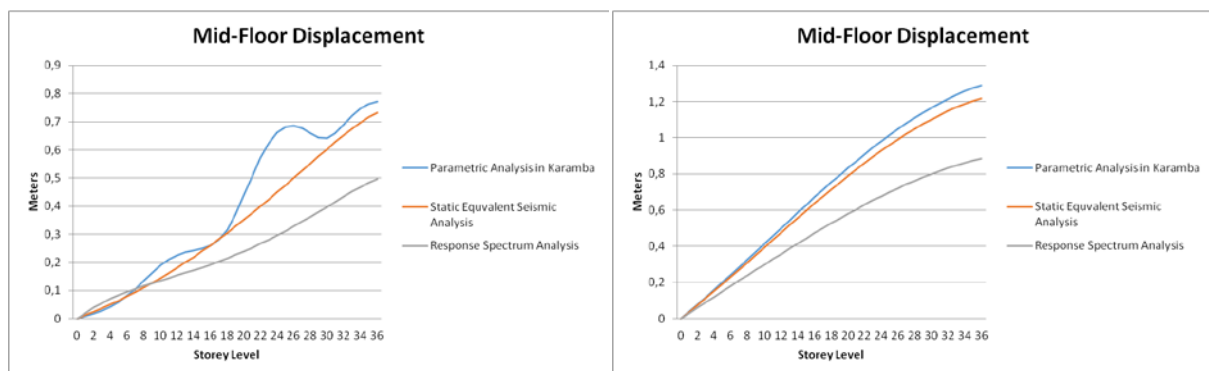


Figure 6. Mid-floor displacements for a) Hyperboloid Structure and b) Cylindrical Structure in different analysis environments.

## 8 Conclusions

In this study, based on the findings and experimentations of Architect/Engineers of the early modern era, Shukhov and Gaudi, with hyperboloid forms, a form generation process for tall buildings within the parametric environment is presented that simultaneously determine the shape of the building and its supporting structure. Further on, static lateral loads representing seismic actions have been applied to the structure, and the behavior of the structure against

lateral loads investigated. The comparison has also been made a cylindrical shaped tall building with similar functional and architectural properties and structural system. It was observed that while significant difference does not occur in the resulting forces of the structural members of the two structures (no more than 6%), a considerable increase in the lateral stiffness of the structure can be observed in the hyperboloid structure in the order of 60%. Finally, the analyses performed in *sap2000* software for evaluating the structural analysis executed in the parametric environment demonstrated that the results of the analysis performed in parametric environment are in very close proximity with the results of the analysis with static equivalent seismic forces with base shear ration equal to 0.1, and in close range of semi-dynamic response spectrum modal analysis.

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