Western Australia School of Mines

Stratified Deposit Production Schedule Optimisation Considering In-Pit Dumping and Haul Road Selection

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Declaration of authorship

I, Ranajit DAS, declare that this thesis titled, “Stratified Deposit Production Schedule Optimisation Considering In-Pit Dumping and Haul Road Selection” and the work presented in it are my own. I confirm that:

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Signed: ________________________

Date: 14th February 2024

Publications list

The publications arising from this current research are as follows:

Published Journal Papers

- Das, R., Topal, E., & Mardaneh, E. (2023). A Review of Open Pit Mine Waste Dump Management Planning. Resources Policy, 63(101438). doi:https://doi.org/10.1016/j.resourpol.2019.101438 (Chapter 2)
- Das, R., Topal, E., & Mardaneh, E. (2022). Improved Optimised Scheduling in Stratified Deposits in Open Pit Mines – Using In-Pit Dumping. International Journal of Mining, Reclamation and Environment, 1-18. doi:10.1080/17480930.2022.2036559 (Chapters 3 and 4)
- Das, R., Topal, E., & Mardaneh, E. (2024) Concurrent Optimisation of Open Pit Ore and Waste Movement with Optimal Haul Road Selection, Resources Policy, 91(104834) doi : https://doi.org/10.1016/j.resourpol.2024.104834 (Chapter 5)

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Abstract

Traditionally, the pit scheduling problem and dump scheduling are studied in isolation with a few exceptions in the literature. The aim of this thesis is to fill the research gap in pit production scheduling of stratified deposits considering concurrent in-pit waste dumping as well as haulage road optimisation. The research has two parts, the first part focuses on determining the waste dumping options including internal and external dumping options. The second part proposes concurrent optimisation of dumping as well as shortest haul road section from a network of block centroids and existing permanent haulage routes.

The purpose of the research in the first part was to develop a mathematical model which considers lag distance with the dynamically changing dump and the working face. It has been mathematically modeled on how to maintain a lag distance with the mining face and consider waste rock placements in optimal dumps (in-pit or external) so as to maximise Net Present Value(NPV). The output from the solutions obtained from the model have been verified in a 3D mine planning software for several case studies. One such case study has been published in Das, Topal and Mardaneh (2022) where a few significant saving were recorded between a schedule considering only external dump and one considering in-pit dumping.The distance of the external dump is more than the internal dump and hence the haulage cost reduces the margins, and thereby the NPV of the project reduces from \$121.9 Million to \$64.3 Million, a reduction of \$57.6 Million. It is also seen that the total haulage cost over the five years reduces by 27% with

optimal internal dumping versus a case with only external dumping. The surface area footprint of the dump was reduced to 47% in the first 5 years with the optimal and early use of the internal dump. Furthermore, the reduction of overall haulage distance will reduce the CO2 emission of the mining equipment as well.

The second part of the research focuses on extending the mathematical model, developed in the first phase, in order to include the selection of optimal haul roads from a database of shortest paths between pit blocks and all possible destinations including all dump blocks, stockpiles and process plants. The Dijkstra algorithm has been used to determine the shortest path between a source and destination. The methodology considers connections to neighbouring blocks from each block as edges. To be considered as a valid edge the slope of the connector has to be within a maximum limit. The same applies for adjacent dump blocks to create valid edges. An array of paths have been considered as options in the optimisation model for carrying the ore and waste from a block within a pit to destinations like dump locations, stockpile or process plant. The path with the least cost is chosen, while considering the other constraints used in the first part of the research. This model was implemented in OPL, CPLEX, where it was possible to solve cases with a limited number of pit and dump blocks, thereafter the model appeared to be NP Hard.

In order to solve a problem with a larger number of blocks within the pit as well as different possible dump locations, a meta-heuristic approach has been developed using Simulated Annealing and Topological Sort algorithms. For this meta-heuristic solution methodology the sequence of mining and dumping is determined using a topological sort algorithm. Once the source and destination of a block is determined the cost of hauling is determined using the selected path from the array of paths developed using the Dijkstra algorithm.

The proposed frameworks have been applied to several case studies. Comparisons have been made between the solutions achieved from the exact and meta-heuristic methods. Also comparisons have been made with a manual, non-optimized schedule using a mine planning software. Significant improvements have been achieved in terms of a 44% increase of back-filling achieved over the life of the mines, and decrease in overall haulage distances by 39% leading to significant cost savings. All these lead to a reduction in the environmental footprint of the mines along with decreased carbon footprint as well.

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Chapter 1

Introduction

1.1 Overview

Stratified deposits. such as phosphate, limestone, coal, and some types of iron ore, are often mined with a longer horizontal strike and cover a large and shallow area. They are also associated with large volumes of waste compared to ore, as the easier deposits with low stripping ratios have already been exploited. The approach to mining stratified deposits is often quite different to metallic or non-stratified deposits. Stratified deposits are mostly mined in strips and blocks. Strips mostly run along the strike direction of the deposit and follow down to all benches while maintaining the batter slopes. Each strip on each bench is divided into blocks or the smallest mining units. The direction of mining is perpendicular to strips, which are mined sequentially one after other. This method of mining enables early back-filling of the mined out voids which is known as in-pit dumping. Stratified deposit mines mostly have in-pit dumps. A minimum working room or safe lag distance is always maintained between the bottom of the mining face and the bottom of the dump. The earlier these mines can be back-filled, the less will be their environmental footprint and land will be back to the community for other intended use. Placing the waste back into voids created by mining helps to use less land area for external dumps and returning land to a usable state much earlier. In-pit dumping also minimises the exposure of dust from external dumps for people, agriculture, vegetation and animals.

With price of commodities improving over time the deposits with higher stripping ratio are gradually becoming economically mineable. However, this comes with a problem of handling large volumes of waste. Waste placement to in-pit or external dumps becomes an important part of mine production scheduling. There can be several waste dumping destinations available in a project including external and back-filled or in-pit dumps. The selection of dump destination at a point in time over the life of a mine depends on the availability of the waste dumps (in-pit or external) based on the void created at that point in time and the allowed lag distance from the mining face to provision for a safe working room. The hauling distance to the destination also plays a role in selecting the dump destination. The haulage cost can largely vary between in-pit and external dumps. Most of the time the in-pit dumps are closer and are often a cheaper option. Hence, it is crucial to incorporate the waste placement decisions into the pit schedule optimisation problem. Considering an optimal haulage option from several possible roads concurrently with selecting the optimal waste placement location is of utmost importance.

Typical mine production scheduling deals with deciding which mining blocks to extract and when, such that the discounted cash flow or net present value (NPV) of the project is maximised while adhering to the physical and production constraints (Johnson, 1968), (Osanloo, Gholamnejad and Karimi, 2008), (Topal and Ramazan, 2012*a*), (Mai, Topal, Erten and Sommerville, 2019), (Fathollahzadeh, Asad, Mardaneh and Cigla, 2021).

Conventional mine planning starts by finding the ultimate pit limit, which is the combination of blocks that maximises the total undiscounted cash flow of the project and respects slope constraints. Then the optimal extraction sequences of material within each time period is determined. The open pit mine-sequencing problem or production scheduling problem is defined as specifying the sequence in which material should be extracted from pits and then transferred to appropriate destinations in specific time periods (Xu, Xu and Li, 2018). Generally, material with no economic value is dumped into external waste dump areas while profitable material is processed at mills or stocked at stockpiles for future usage (Mai, Topal and Ertent, 2018). There has been significant research already done in this area (Li, Topal and Ramazan, 2014).

Among the core life of mine (LOM) planning considerations, waste management is a particularly important concern. Waste dumps and stockpiles represent significant volumes of material that substantially impact the local environment, while the available space for waste storage is often limited. Waste rock haulage and management is one of the costliest activities in any mining process (Das et al., 2022). Unless the waste is handled optimally it could create unwanted high backlog of rehandling expenses at later stages of the mine.

With stricter environmental regulations, open pit mines can no longer be left as pit voids and external dumps (Rimélé, Dimitrakopoulos and Gamache, 2018). The voids will need to be back-filled and vegetated to make them use-able land while the external dumps will need to be flattened to usable gradients and then vegetated. It should be the best attempt of any mining operation to back-fill the mining voids at the earliest, and preferably do it simultaneously while mining, to avoid any backlog. If simultaneous back-filling of the pits is not done, the backlog can cost a lot at the end of the mine life as a mine closure cost. For simultaneous back-filling, we need to know which dump locations the waste should be optimally directed to, depending on the availability of space in the dumps (external or in-pit) at a point in time. The cost of haulage will largely differ at different points in time over the life of the mine (Li et al., 2014). Thus, ignoring the proper placement of waste into external and internal dumps in a schedule could not only increase the mining footprint but also lead to a significant extra cost and a sub-optimal LOM schedule.

The haulage distance for in-pit dumps can be much less compared to external dumps. Hence, the overall cost of mining is reduced with increased in-pit dumping. An optimal solution with an objective to maximise discounted cash flow will always try to maximise in-pit dumping as a lower cost option. Furthermore, the benefit of prioritised in-pit dumping is in getting the mine rehabilitated progressively on the low wall side as the high wall side keeps progressing in mining. It is therefore, preferred to start internal dumping as early as possible during life of the mining operation so that the footprint of mining can be minimised in the earliest time frame, while reducing rehabilitation cost over the life of the mine.

Figure 1.1 shows a schematic diagram of a stratified deposit mine, which is progressively getting back-filled into the mined-out area and simultaneously getting rehabilitated with plantation. As can be seen from Figure 1.1, the effective footprint area is shown as the active mining and dumping area, which in the case of a stratified deposit shifts progressively, with the progressive return of back-filled and rehabilitated land to its original land-form suitable for vegetation. This way the disturbed area or mining footprint at any point in time is limited to only a small extent.

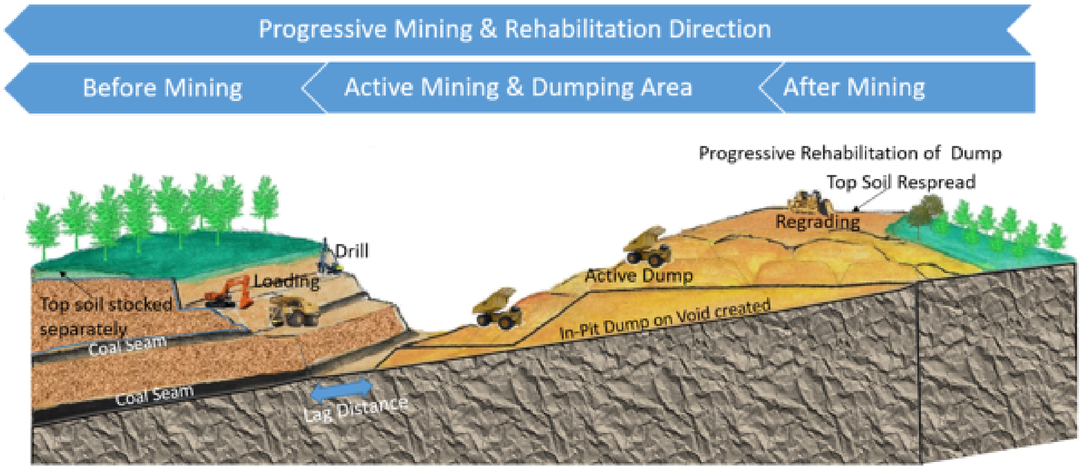


Figure 1.1: An artist's impression of progressive rehabilitation of a stratified deposit mine and dump (based on image from Mt Owen Mining Schematic, Glencore)

The rehabilitation practice in mines such as shown in Figure 1.2 includes handing over the mined-out areas back in original land form fit for grazing. The process involves back-filling the pits, dozing the dump to flatten slopes to mostly within 20 degrees, re-spread topsoil, seeding, and then monitoring plant growth. While external dumps also need to be rehabilitated, it is a requirement to back-fill and rehabilitate the mined voids. If more material is sent to external dumps, then it will need to be re-handled and brought back to the pits to use for back-filling. It also makes more sense as the haulage distances to in-pit dumps are mostly shorter than those to external dumps.

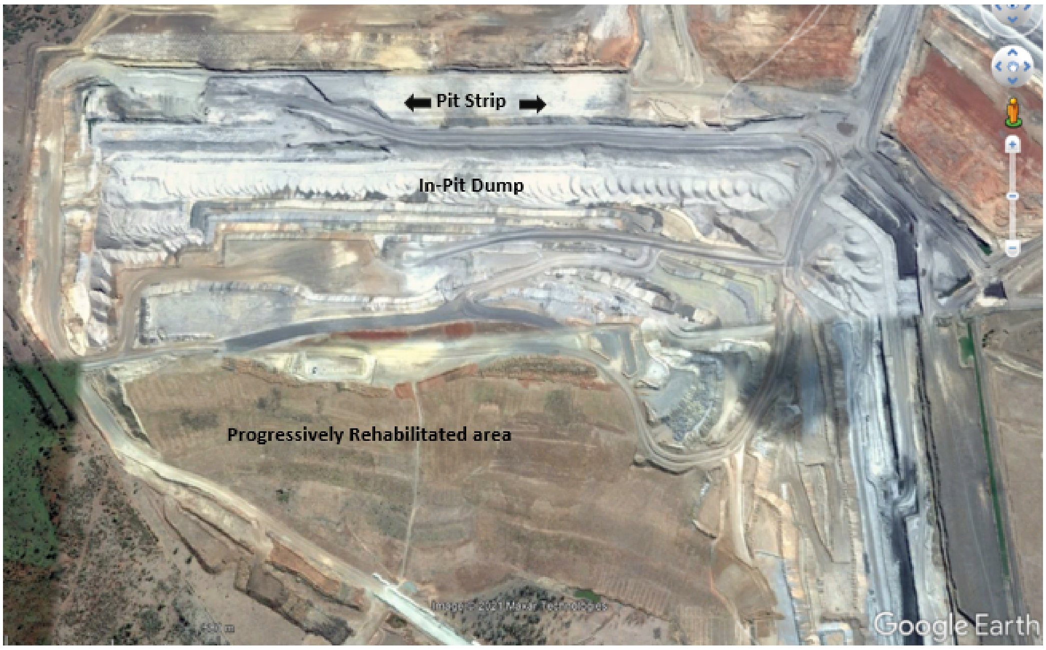


Figure 1.2: Example of progressive rehabilitation of part of Rollestone mine in Queensland (Source : Google Earth)

In this thesis, a mixed integer programming model has been developed to optimise dumping the waste dynamically into the mined out void area so that the cost of waste haulage can be minimised as well as the area can be returned for rehabilitation sooner.

With high volumes of waste encountered in most stratified deposits today a significant portion of the cost of mining is in removing and hauling the waste. Unless we know which dump the waste will be directed to, depending on the availability of space in the dump (external or in-pit), the cost of haulage will largely differ at different points in time (Li et al., 2013). The in-pit dump is a factor of the lag to be considered with the working face, while, the haulage cost depends on whether the optimal haul road option has been chosen (Caccetta and Hill, 2003). Thus, ignoring the proper placement of waste into external and in-pit dumps in a schedule could lead to a flawed result.

The haulage distances for in-pit dumps are often much less compared to transportation to external dumps, thereby the cost of mining is reduced with increased in-pit dumping. Hence, it is preferred to have internal dumping as early as possible in the mine life to be able to do early rehabilitation, thereby, reducing the footprint of environmental impact.

Haulage distances are one of the key factors that guide the selection of a destination. Often in mine scheduling problems haulage distances are approximated and often Euclidean distances are considered for the haulage from pit block to the pit exit or from the dump entry to the dump block. This can lead to flawed estimations of haulage distances. The haulage optimisation problem has so far been viewed as a separate subject dealt in isolation from a mine or waste dump scheduling problem (Thompson and Visser, 1997), (Yarmuch Guzmán, Brazil, Rubinstein and Thomas, 2020) . This thesis presents a pioneering study in the area of considering haulage in the model so that the sequence of mining and waste allocation in dumps are based on optimal haulage considerations.

1.2 Differences between stratified deposits and non-stratified deposits

Although the production scheduling problem has a similar structure with a stratified deposit as compared to massive or vein deposit, a few key differentiators can be listed as follows:

(i) **The geological model** : The geological modelling process in a stratified deposit is often different. Instead of a three dimensional (3D) block model of several blocks of fixed sizes a two dimensional (2D) grid model is developed. This model has a fixed grid size in X and Y directions. There are two parts in the model, which are a structural model and a quality model. The z in a structural model represents the roof and floor (highwall and footwall) of the ore body. The ore body is divided into layers or seams based on the borehole seam interpretation and correlation. In the quality model the z represents the value of the quality attribute for a layer or seam. Thus, the block model instead of being a single file consists of multiple files containing x,y and z values, where z represents the seam roof elevation, or seam floor elevation, or seam thickness, or seam quality parameters in different files.

(ii) **Pit geometry and layout**: For moderately dipping stratified deposits, the base of the layer of ore/coal forms the base of the pit. Unlike, in a non-stratified deposit such as vein type deposit or a massive deposit we may have to mine the footwall to create a stable pit wall. Stratified deposits are large in lateral extent and normally shallow in vertical extent, hence mines like coal are known to extend several kilometres, as shown in Figure 1.3 below. Since the extents are large, almost all stratified mines try to do back-filling by in-pit dumping as a priority in order to decrease haulage cost and expedite reclamation. It is mostly not the case in vein type or massive deposits.

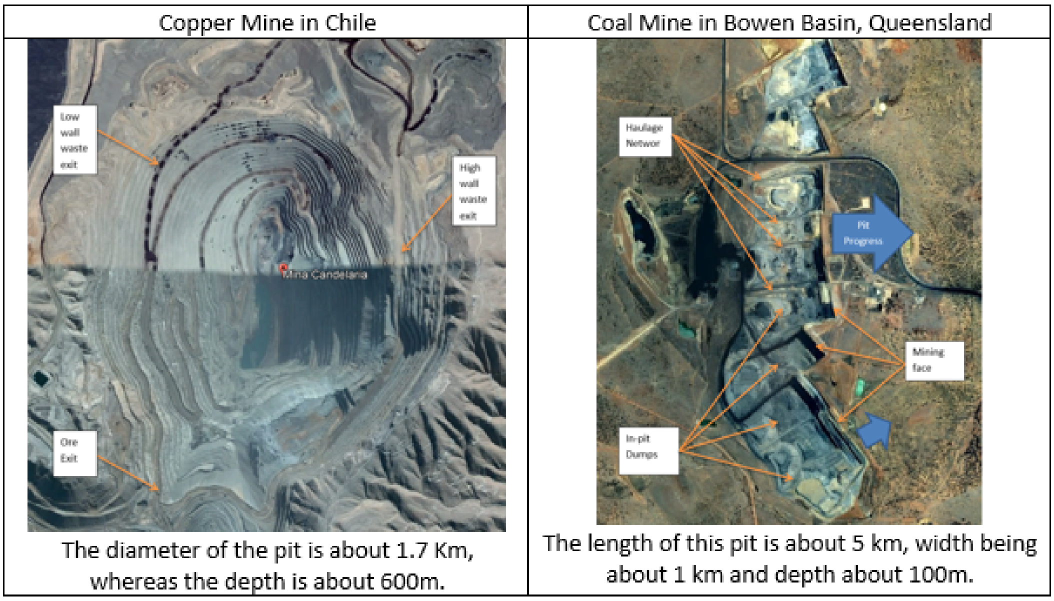


Figure 1.3: Difference in pit geometry of stratified and non-stratified open pit mines using Google Earth images

(iii) **Layered blocks**: A block represents a cell in a 3D representation of the ore body known as a block model. A dump is also often represented as a block-model in order to identify locations within the space allocated for the dump. There are differences in how blocks are defined in geological modelling and mine planning packages for stratified deposits. For metalliferous deposits a block in a block model can be either waste or ore depending on its cut-off grade. Anything above the cut-off grade will be treated as ore and the destination would be a process plant. However, in the case of a stratified deposit such as coal each block could consists of multiple coal and waste layers. Further, depending on the thickness of the coal layers and the quality, the layers could be combined with adjacent waste or coal layers while mining and such a model is often referred to as a run-off-mine(ROM) model with working sections.

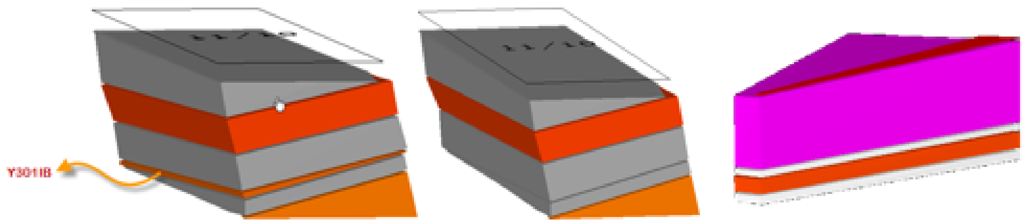


Figure 1.4: Typical coal reserves individual block - containing both coal and waste layers

Figure 1.4 presents the typical blocks in a coal mine. The resource model could contain thin layers such as the Y301IB layer of inter-burden. While converting the in-situ model to a ROM model it is considered as coal and combined with the adjacent coal. [Minex Reserves Database tutorial, 2018]. In order to mine the coal in a block the waste above it has to be removed.

(iv) **Irregular shape of the blocks**: Another significant difference of a stratified deposit model is the shape of the blocks which are not perfect cubes or cuboids as in non-stratified. As seen in Figure 1.4 blocks can have a definite slope angle which follow the ultimate wall or the strip wall angle. The blocks in plan view need not necessarily be a perfect rectangular shape, as commonly known. The block model referred here is only within the pit, where the pit can take any shape laterally. Hence, in order to fit in blocks at the edges or turns in a strip the blocks can be of any shape or size as opposed to regular shaped block models in metalliferous deposits.

(v) **Pre-designed pit** : The design of these blocks are normally done after the pit optimisation process, hence all blocks in the design are within the optimised pit shell obtained through ultimate pit limit(UPL) optimisation algorithms such as the Lerchs Grossman(LG) or other optimisation algorithms. The slope of the pit has already been considered during the design process. It may be observed that the sides of each block also has a slope as shown in Figure 1.4. This slope follows the slope specified for the strip or final wall.

(vi) **Pre-designed dump**: The space designated for dumps is divided into individual cells known as blocks. Similar to the pit, the dump blocks for a stratified deposit mine planning are also pre-designed with slopes. Dump blocks are designed and need to be considered to know their spatial location. The lag distances are checked from each pit block to corresponding dump blocks in the vicinity. Dump slopes are more important as they are flatter and normally in the range of 35-38 degrees. Hence in the same area the number of strips that can be fitted in the bottom bench may not be same as number of strips in top bench.

(vii) There are some differences in the mining strategy for typical open pit hard rock mining as compared to stratified deposits. Figure 1.5a and 1.5b show typical cross sections of a stratified deposit mine and a hard rock mine. While a stratified deposit mine is shallow in depth but is large in lateral extent, a hard rock mine is mostly deeper but smaller in lateral extent.

These design shapes are based on the geology of the ore body being mined.

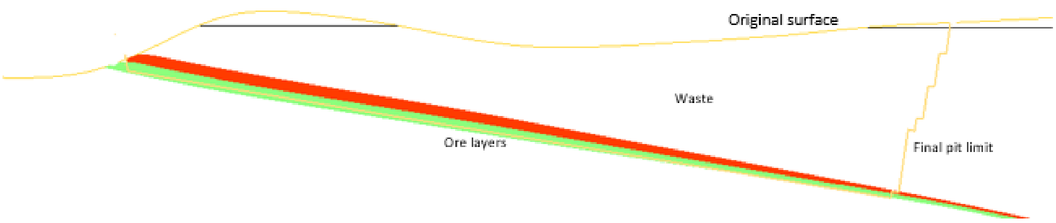


Figure 1.5a: Cross-section of a typical stratified deposit mine -showing two layers of ore and the final pit limit

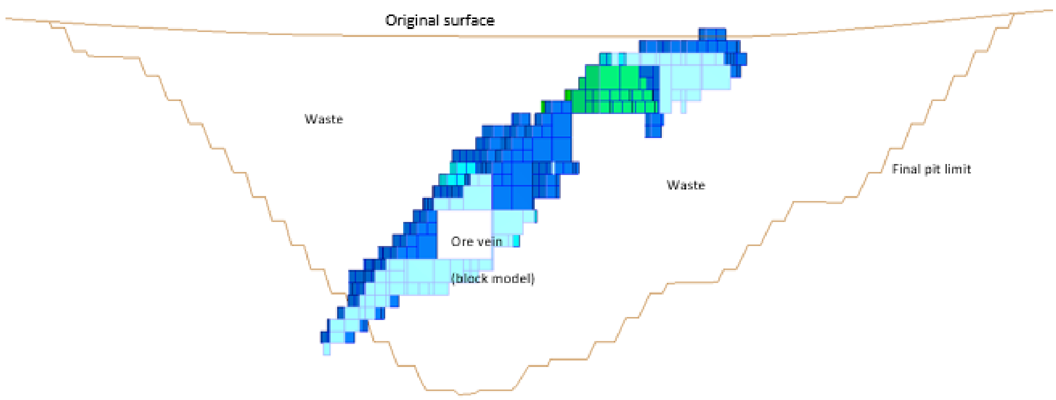
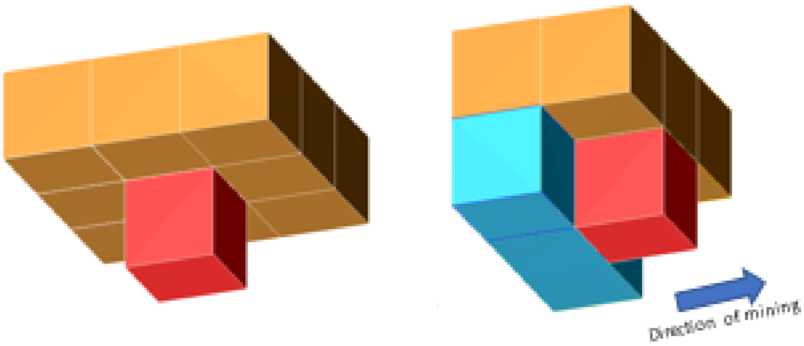


Figure 1.5b: Cross-section of a typical non-stratified deposit mine -showing the ore body and the final pit limit

(viii) In typical open pit mining applications, to mine a given block, at least nine blocks above it need to be mined in order to maintain a pit slope angle as shown in Figure 1.6 (Hustrulid and Kuchta, 2006). This representation of 9 blocks is just an illustrative example for explaining the concept behind the precedence of mining. However, in stratified deposits, since the precedence of mining is different and mostly directional, the same nine blocks are located differently depending on the direction of mining as shown in Figure 1.6b.



(a) (b)

Figure 1.6: (a) Blocks to be mined in order to mine the red block (b) Blocks to be mined in order to mine the red block for stratified deposits with pre-designed slope and direction of mining

Stratified deposits are mostly mined in strips. Strips are normally numbered making the direction of mining being the direction of increase in strip numbers. The strip numbers normally increase along the dip direction. Also once mined strip by strip, space can be ensured quicker for back filling, once a strip in an area has been mined out completely to create enough void or lag to be able to dump. Mining normally proceeds from the shallower area to the deeper area and the back-filling of the pit follows a similar sequence.

Furthermore, a block in a stratified deposit, being different from a normal hard rock block model, could contain both ore and waste layers within the same block (Das et al., 2022).

1.3 Motivation and scope of research

Previous research has focused on the sequence of mining without considering the haulage and dumping requirements of waste rock. (Williams, Topal, Zhang and Scott, 2008), focused on minimising the haulage cost for each open pit block of waste rock to be placed in the waste dump, with some allowance for the selective placement of benign and reactive waste rock, based on an open pit block model that delineates ore, and benign and reactive waste rock. (Li et al., 2013) took the work further in order to include trucking and minimise haulage costs. These works however did not simultaneously optimise ore and waste pit schedules based on the dumping requirements. They were based on existing optimal ore schedules.

A more recent study by (Fu et al., 2019) considered the simultaneous optimisation of material mining and dumping schedule and formulated the problem as a mixed integer programming(MIP) problem. This is a unique first attempt in this regard to bring together the two areas of mining and dumping which were so far being considered separately.

The literature on the in-pit dumping topic is limited, and some related work is found in (Zuckerberg, Stone, Pasyar and Mader, 2007), who present an extended version of BHP's mine planning software Blasor, named Blasor- In-Pit Dumping (BlasorIPD).

The mathematical model proposed in the first phase of the thesis has some similarity with Blasor IPD software, a proprietary software of BHP, which also deals with in-pit dumping. However, a notable difference is that Blasor IPD fills back the existing mining blocks whereas this model uses a separate block model for dumps which is available for internal as well as external dumping purposes simultaneously and also modelling of dynamic lag distance (mining and filling) for internal dumping purposes. The model described in this thesis is based on pre-designed solid shapes of blocks of pit and dump. The pit blocks and dump blocks have different pre-designed slopes to honour geotechnical considerations.

Solution methodologies have been attempted in the thesis for several cases of stratified deposits using exact and meta-heuristic methods. The meta-heuristic method was looked into as the model proved to be NP hard when applied to datasets with gradually increasing number of pit and dump blocks. The exact method included CPLEX whereas the meta-heuristic included the simulated annealing(SA) and topological sort algorithms.

1.4 Problem statement

While the application of Lerchs Grossman or Pseudo Flow algorithms have been widely used in the industry to determine the UPL, the next stage involved is determining the optimal schedule involving the practical constraints. These planning steps form part of the strategic mine planning process which later evolves into life of mine plans, medium term and short term plans.

For solving the pit production-scheduling problem, it is a common practice to create a three dimensional block model of the ore body to describe the ore reserves. Based on the sample data collected from boreholes or otherwise, the geo-chemical and economic parameters of each block are estimated. The precedence or sequence of mining is influenced by the physical and operational constraints keeping in view the geo-technical stability of the slopes. This discrete approach creates huge combinatorial problems whose mathematical formulations are large-scale instances of integer programming (IP) optimization problems (Alvarez-Ellacuria, Orfila, Olabarrieta, Medina, Vizoso and Tintoré, 2010), (Caccetta and Hill, 2003). The mine production-sequencing problem may be solved for different levels of accuracy. For simplification reasons, some blocks are aggregated into bigger units to obtain extraction sequences of these units with less computational efforts (Askari-Nasab, Awuah-Offei and Eivazy, 2010; Mai et al., 2019,1).

In an open pit scheduling problem the mined out block is sent to one of the different available destinations such as stockpile, process plant or waste dumps (internal or external). Figure 1.7 shows typical material flow options available from the pit to stockpile. The content of a block is identified as ore and waste depending on either using a cut off grade or using lithology or seam boundaries.

For stratified deposits mostly the lithology boundary or seam interpretation separates the ore and waste.

A major part of the cost of mining comes from the haulage costs. Hence in order to optimise a schedule it is required to consider the haulage distance, which in turn includes considering the dump destination (the ore destinations of stockpile and process plant being considered as fixed).

A project is normally evaluated based on the cash flow it will generate, which includes the sale price of the ore, the cost of mining, processing, rehabilitation etc. The cash flow occurs over the life of the mine, and hence cash flows occurring later in the life are given less weight or are discounted. The sum of the discounted cash flows is the NPV. The cash flow in a mine varies largely depending on the sequence of mining thereby changing the NPV. The sequence of mining can have impact on the following :

- ore and waste volumes in different periods or stripping ratio – which impacts the cost of mining
- quality of ore mined
- internal dumping area or void created, impacting haulage distance
- sequence of dumping – impacting haulage distance

Finding the optimal blocks to mine, and the optimal destination for dumping in a period of time is a challenge with several possible options. However, each option of mining or dumping sequence will result in a different NPV. Most schedule optimisation problems hence attempt to maximise the NPV by altering the mining and dumping sequence.

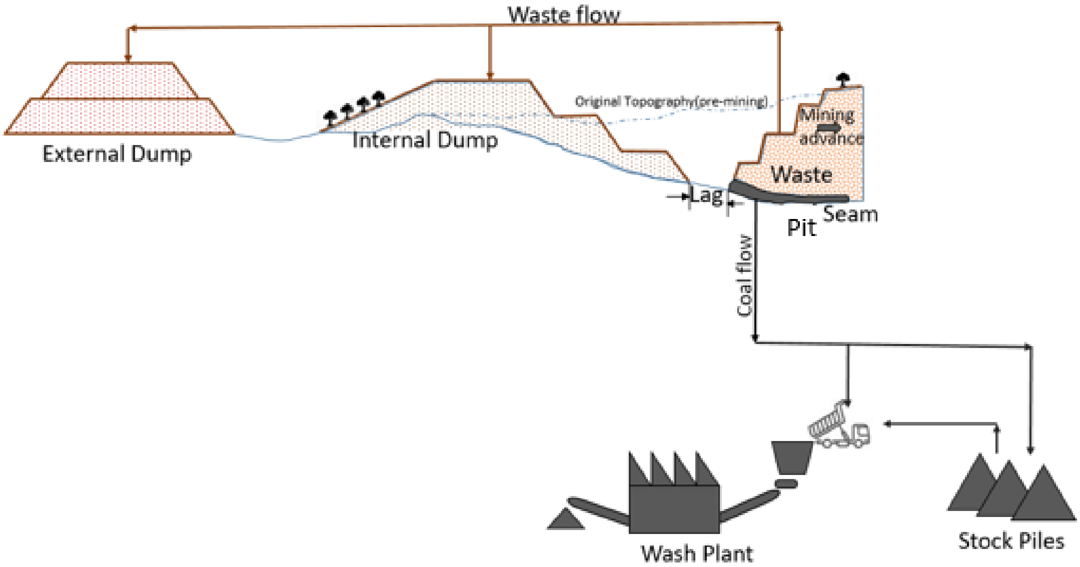


Figure 1.7: Schematic diagram to present material flow considered in the model

The current problem being addressed in this thesis is about finding the best block schedule sequence and the best dumping block sequence by periods, as well as the best haulage option to connect from the source to the destination. As a unique problem practical internal dumping including maintaining room for working of the mine, or lag distance, has also been considered in the research along with optimal haul road selection.

1.5 Thesis objectives

The objectives of the research are to develop mathematical models and solution methodologies to:

i) determine the optimal mining and waste dumping schedule including in-pit dumping, where the in-pit dumps are required to maintain adequate lag distance or working room with the working face of the pit,

ii) determine the optimal haulage route option from a pit block to its possible destination like dumps (external or in-pit for waste in the pit block) and stock-piles or plants (for ore in the pit block), and

iii) solve real case studies using data from various mining projects.

Stratified deposits such as coal are known to have high stripping ratios, thereby the waste handling part consists several times (often 5 to 10 times) the cost of handling ore, and a large part of the waste handling cost is the hauling cost itself (Hartman and Mutmansky, 2002). Also, placement of the waste could be in the external dump or the in-pit dump – depending on whether enough room in the in-pit is available at a certain point of time with the advancing working face. The destination of the waste could therefore change dynamically over the life of the mine, and the destination bears a direct impact on the haulage cost. If this cost is ignored it is likely that an optimal schedule obtained is not a true reflection of reality. Since hauling cost is a large part of the mining cost, any reduction in the haul road will lead to savings and reduction in carbon footprint.

A new mixed inteer linear programming (MILP) Model and methodology for optimizing the mining and dumping schedule simultaneously has been developed. The methodology and model can be used to create a long-term schedule for open pit stratified mining for LOM plan, which provides the optimal NPV and takes into consideration new factors discussed in subsequent chapters.

The investment decisions are largely based on NPV calculated on a project. The NPV of a stratified deposit mining project largely depends on the schedule – which includes mining, waste dumping, hauling among other major expenses. This will therefore bring in a much informed investment decision capability.

The reserach was conducted in two phases which are explained below and shown in Figure 1.8:

- In the first phase the following has been included in the MILP model:

  - Waste handling and placement to in-pit and external dumps,
  - Adequate lag distance being maintained for in-pit dumps,
  - Haulage plan using Eucledian distances from pit blocks and dump blocks to exit/entry points, to include approximate haulage costs,

- In the second phase, roads with minimum distance were generated using the Dijkstra algorithm :
  - A haulage system was created from a mesh of network points consisting of block centroids,
    - Considering maximum haulage gradient
    - Connect to main/existing haul road points
    - Create a database of road options based on constraints above for each source and destination
  - The MILP model was extended to select the best haulage option from the available options so as to maximise the NPV.
  - Exact solver CPLEX-OPL was used to solve the model. The exact method was limited by the size of data. With an increase in the number of pit and dump blocks the exact method could no longer be used. Hence, a SA method was used where multiple sequences of mining and dumping were generated using weighted topological sort method.

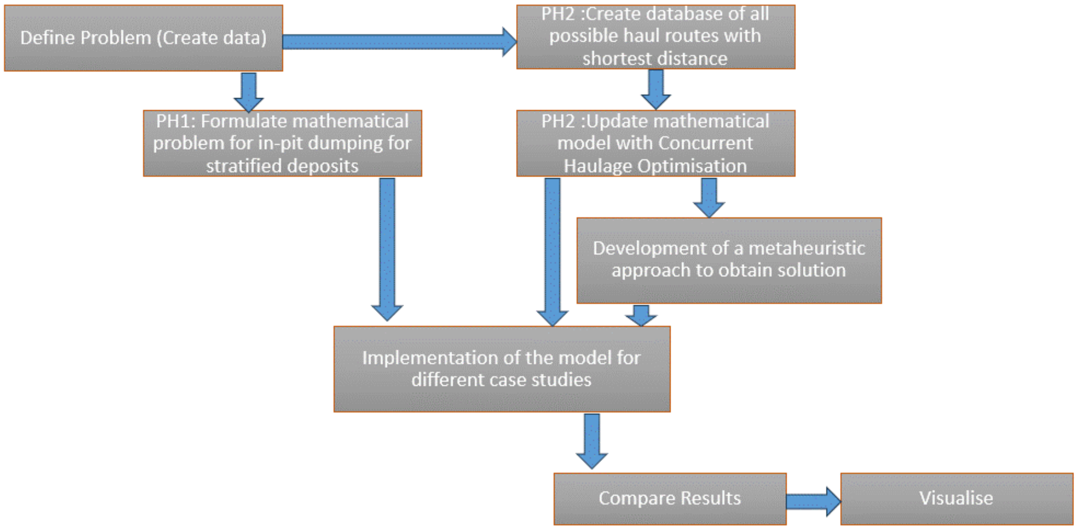


Figure 1.8: Schematic diagram to present phases of the research showing PH1 (Phase 1) and PH2 (Phase2)

As a part of the study a model has been developed which can concurrently optimize ore and waste handling along with haulage for stratified deposits. Since the waste handling cost is about 5-10 times of the ore handling cost in a stratified deposit any amount of savings in that will be of significance. This study will help engineers to evaluate and do long-term and LOM planning with much higher accuracy, saving projects from planning and investing based on inaccurate facts.

1.6 Organisation of the thesis

The remainder of this thesis, which consists of six chapters, is structured as follows:

Chapter 2 presents a literature review and background study of the topics covered in this thesis. This chapter is divided into two parts; the first part discusses the schedule optimisation models and solutions in general and have been captured and reorganised from the following papers published.

- Das, R., Topal, E., & Mardaneh, E. (2022). Improved Optimised Scheduling in Stratified Seposits in Open Pit Mines – Using In-Pit Dumping. International Journal of Mining, Reclamation and Environment, 1-18. doi: 10.1080/17480930.2022.2036559 (Chapter 3 and 4)

- Das, R., Topal, E., & Mardaneh, E. (2019). Optimised Pit Scheduling Including In-Pit Dumps for Stratified Deposit. Springer Series in Geomechanics and Geoengineering. Springer, Cham. https://doi.org/10.1007/978-3-030-33954-8_4.

- Das, R., Topal, E., & Mardaneh, E. (2023). Sustainable Open Pit Mine Planning With In-Pit Dumping and Haul Road Selection. 26th World Mining Congress, Brisbane Australia , 26-29 June 2023, (Chapter 4)

The second part focuses in particular on papers published on schedule optimisation that include waste handling and dumping and with their solution methodologies used. This chapter is mainly captured and reorganized from the following paper:

- Das, R., Topal, E., & Mardaneh, E. (2023). A Review of Open Pit Mine Waste Dump Management Planning. Resources Policy, 63(101438). doi:https://doi.org/10.1016/j.resourpol.2019.101438 (Chapter 2)

Chapter 3 addresses the first phase of the research where schedule optimisation considering internal dumping and lag distance were the key focus. A mathematical model developed for the problem has been discussed here. Case studies with exact solution methodologies using CPLEX have been discussed.

This chapter is captured and reorganized from the following papers.

- Das, R., Topal, E., & Mardaneh, E. (2019). Optimised Pit Scheduling Including In-Pit Dumps for Stratified Deposit. Springer Series in Geomechanics and Geoengineering. Springer, Cham. https://doi.org/10.1007/978-3-030-33954-8_4.

- Das, R., Topal, E., & Mardaneh, E. (2022). Improved optimised scheduling in stratified deposits in open pit mines – using in-pit dumping. International Journal of Mining, Reclamation and Environment, 1-18. doi:10.1080/17480930.2022.2036559 (Chapter 3 and 4)

Chapter 4 is an extension to the past research described in Chapter 3 and includes haulage. Here the model discussed in Chapter 3 has been improved upon to include selection of a haulage option from a database of options connecting all sources to all destinations. The options have been created using the Dijkstra algorithm, considering the pit and dump blocks, and existing permanent haul road points as nodes. Valid edges were created between possible connection between adjacent nodes within maximum slope gradient limits. The code for the model has been attached in Appendix A. Exact solution methodologies have been used to limited size of data using CPLEX. This chapter is captured and reorganized from the following paper:

- Das, R., Topal, E., & Mardaneh, E. (2023). Sustainable Open Pit Mine Planning With in-pit dumping and Haul Road Selection. 26th World Mining Congress, Brisbane Australia , 26-29 June 2023, (Chapter 4)

Chapter 5 provides a meta-heuristic solution methodology to the problem and its comparison with exact solutions. The solution methodology discussed in Chapter 4 is limited by the number of blocks, hence the meta-heuristic approach has been attempted using a weighted topological search algorithm and SA. Randomly generated sequences of mining and dumping using weighted topological search algorithm were generated. The pit sequence considers a weighted topological sorting methodology, weighted by the quality of the ore blocks. The best sequence has been arrived at using SA. The chapter also discusses several case studies solved using the two approaches and their comparison. This chapter is captured and reorganized from the following paper:

- Das, R., Topal, E., & Mardaneh, E. (2024) Concurrent Optimisation of Open Pit Ore and Waste Movement with Optimal Haul Road Selection, Resources Policy, 91(104834) doi : https://doi.org/10.1016/j.resourpol.2024.104834 (Chapter 5)

Chapter 6 summarises the main findings of the thesis and possible future research possibilities are discussed.

Chapter 2

Literature review

Production sequencing of a mineral deposit, among several constraints, to obtain the best value is a challenging task. Although, there have been numerous researches in this area since the late sixties, it is still evolving with newer concepts and methodologies. The key objective in almost all the past research has been the maximisation of NPV of a project or to minimise cost or minimise the production deviation from set targets of quality or quantity, with the main focus being on the scheduling of the ore. Considering waste dumps in the production schedule optimisation is a relatively newer concept but an integral part of a mining schedule, which had been omitted in earlier research. This chapter reviews the studies on the deterministic and stochastic based models for open-pit production schedule optimisation when waste dump consideration is incorporated into the scheduling. There has been a gradual evolution in the concept of considering waste dumping in a mine production schedule from considering ore and waste schedules separately to simultaneous optimisation with the consideration of the waste rock acidity as well as in-pit and ex-pit dumping strategies.

2.1 Introduction

Open pit mines constitute a pit or several pits that despatches uneconomic material as waste to dumps and uses the valuable raw materials for further processing to produce marketable products. This structure is a closed system of material flow from different sources to particular destinations over a certain time period, which is formulated as a mine production schedule (Fathollahzadeh, Asad, Mardaneh and Cigla, 2021).

The mine planning process, a well-recognised component in the mining value chain is the key for understanding the value of the project and then implementing it – spread over from a strategic to a tactical level. A mining production schedule specifies the extraction sequence of blocks, over a time frame, along with the destination of the material. Destinations can include processing plants, leach pads or stockpiling of the ore and sequence of placement of the waste rock in different waste dump locations. Production schedules need to comply with certain constraints, significant among them are the capacity constraints such as equipment or process capacity and spatial constraints to maintain overall pit slope. There are further constraints which are often added to address grade blending or stockpiling as well as available reserve. Creation of a 3D block model is the first step in any mine planning workflow, which represents the geology and quality of a deposit. This is a three-dimensional array of blocks, the parameters or grades of which have been estimated from the known borehole data using available resource estimation techniques. Such models can be either block models which are commonly used in metallic deposits or grid models which are commonly used in stratified deposits as shown in Figure 2.1.

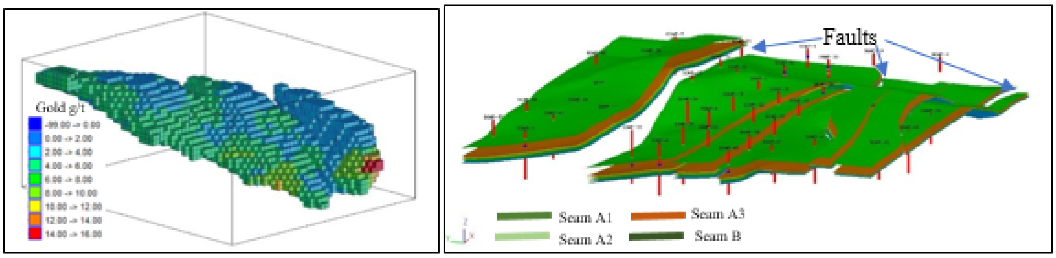


Figure 2.1: Example of block models for metallic(left) showing only ore blocks and stratified deposit(right) showing seams.

Thereafter, pit limits are derived using optimisation algorithms including LG (Lerchs, 1965) or Pseudo Flow techniques (Chandran and Hochbaum, 2009), based on a series of economic and geotechnical inputs. These are often adjusted to generate nested shells or pushbacks, as part of scenario or range analysis. Thereafter, trial schedules are created to sequence ore and waste block extraction, employing mining and processing constraints. Such strategic optimisation facilitates the creation of LOM economic plans, using peak NPV returns.

2.2 Classical approaches to open pit schedule optimisation

Commencing in the 1960's, numerous researchers have undertaken mine optimisation studies, with the incremental development of mathematical and computational treatments based on: linear programming (LP), pure IP; MILP; and dynamic programming (DP), with variably exact or heuristic approaches. As a noted forerunner, (Johnson, 1968) recognised the need for NPV maximisation as constrained by mining and processing capacity, ore grades and pit slopes, amongst other key input parameters. Since this premier study, mathematical modelling together with hardware and software development have progressively evolved wherein iterative mine plans and schedules are produced daily at mine sites, using industry standard software packages. Such advancements are well documented within applied literature, with notable examples including: (Johnson, 1968); (Gershon, 1983); (Dagdelen, 1986); (Dowd and Onur, 1993); (Tolwtnski and Underwood, 1996); (Ramazan and Dimitrakopoulos, 2013); (Caccetta and Hill, 2003); (Topal and Ramazan, 2012*b*); (Groeneveld, Topal and Leenders, 2019); (Mai et al., 2018).

Significantly, detailed waste management treatments are less well understood and often include a cursory assignment of waste blocks onto dumps, with multiple researchers observing that dedicated studies and particularly those facilitating in-pit dumping / back-filling options were rare (Osanloo et al., 2008) and (Fathollahzadeh, Mardaneh, Cigla and Asad, 2021). As waste haulage may comprise as much as 50-60% of total trucking expenditure depending on the stripping ratio (Alarie and Gamache, 2002), this study offers insights into its management, particularly with respect to cost and environmental constraints.

2.3 Approaches including waste rock handling

Early investigation into waste management and scheduling commenced in the 2000's with specialist software development, with examples including the Blasor-In-Pit Dumping (BlasorIPD) package (Zuckerberg et al., 2007). Subsequently acquired by BHP, this unconventional package creates optimal extraction sequences around which ultimate pit shells are designed, with blocks and panels aggregated for both ore and waste scheduling. Significantly, waste can be scheduled for dumping ex-pit or in-pit, using optimised haulage networks. However, not much detail about the package is available in the public domain, and nothing is available on the mathematical model behind the same.

Initial researchers (Williams et al., 2008) proposed MILP models to optimise waste haulage against cost and environmental impact, based on:

- the integration of open pit waste extraction and dump design
- a selective placement of benign and reactive waste to reduce any environmental impact,
- a maximum ramp gradient of 10% (Fig. 2.2)

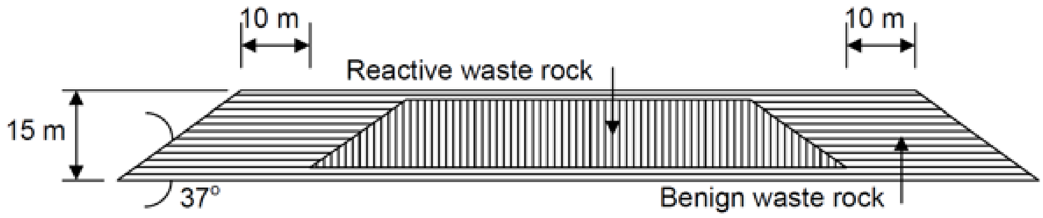


Figure 2.2: Covering reactive waste in a dump using benign waste material

Its objective function minimised waste haulage costs ex-pit by:

1. Stressing non-repetitive single waste block selection.
2. Assigning all reserve waste, including benign and reactive varieties, to dumps.
3. Ensuring lifts respect bottom-up, construction norms, with reactive waste encapsulated by benign waste.

As this model did not permit simultaneous ore and waste movement, multi- source and period optimisation was not feasible. However, with enhancements this model was able to manage multiple pits and waste dumps, variable dump sequencing strategies, as well as equipment optimisation for multiple time periods (Li et al., 2013) (Li, Topal and Ramazan, 2016). Thereafter, (Li et al., 2016) went onto create three MIP models for waste management which included:

1. LOP: Location optimisation algorithms to minimise haulage costs;
2. TBA: Truck balancing algorithms to maximise the truck fleet utilisation, and;

3. VCM: Combination algorithms which merged LOP and TBA functionality, offering variable options for location and balancing scenarios.

With reactive-benign waste encapsulation functionality, single and multi-level dump options were added, together with pit-exit and dump-entry selectivity. These three models were tested using conceptual dump sites, over a five-year schedule with large fluctuations in material movement observed using both manual and LOP models, against a higher cost TBA result (Figure 2.3 - (Li et al., 2014)(Li et al., 2016).

	1	2	3	4	5
Manual method	2,931.8	5,828.9	4,156.3	3,691.8	3,353.7
LOP model	2,019.2	5,249.7	4,523.7	3,696.2	2,946.4
TBA model	3,000.0	4,000.0	4,212.7	4,000.0	4,000.0
Available trucking capacity	3,000.0	4,000.0	4,000.0	4,000.0	4,000.0

Figure 2.3: Comparison of volume X meters for manual method, LOP model, and TBA model (Li et al., 2013)

Predictably, waste optimisation models require predefined ore schedules as input to support parallel waste movement, as they are inevitably impacted by changes in dumping sequence and haulage costs.

More recently, researchers have focussed on pit and dump schedule optimisation algorithms that maximised NPV by including cumulative ex- pit material economics ($/tonne) by destination including ore, potentially acid forming (PAF) waste rock, non-acid forming (NAF) ore to stockpile or dump, and stockpile rehandle (Fu et al., 2019). These contemporary models required multiple constraints comprising: reserve tonnages, mining fleet, plant capacity, ore bins / grades for blending, maximum available dump volumes, mining and dumping precedence and stockpile capacity (Figure 2.4).

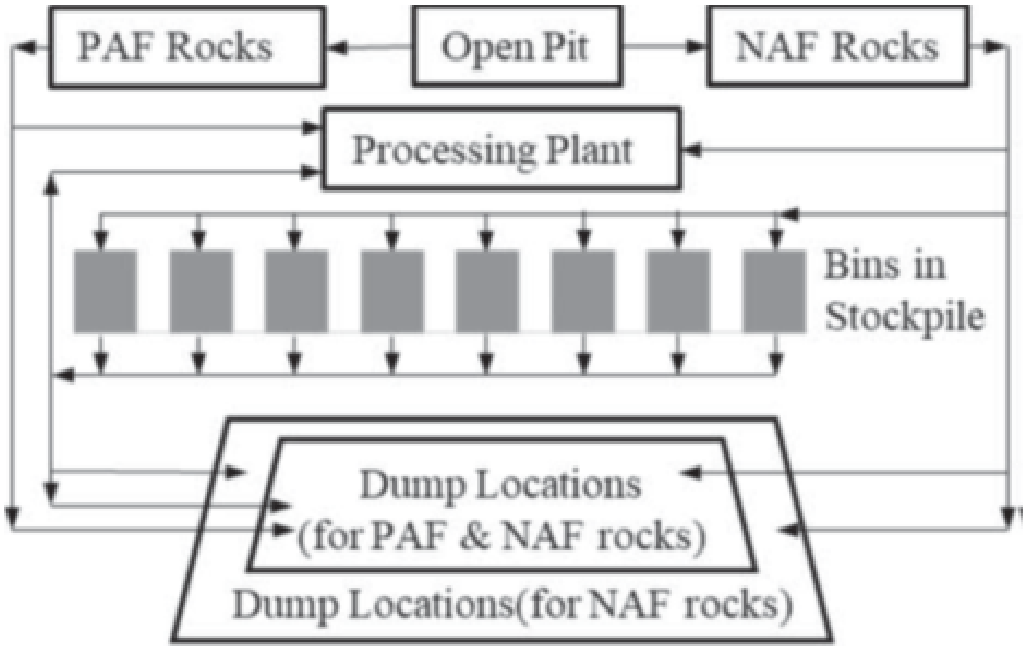


Figure 2.4: Structure of the model showing PAF, NAF and grade bins

Unsurprisingly, such MIP models classify as NP-hard with elevated constraint and variable numbers increasing computational complexity for exact methods. In this instance, an optimal CPLEX solution was achieved using a 4.92% gap over 29 hours for a four-year schedule, comprising 1688 mining blocks and 6265 binary variables for a hypothetical gold deposit (Figure 2.5). A MIP trial produced a combined ore and waste dump schedule (WDS) for a 5-year cash flow for comparison against traditional and two-step MIP (TSMIP). This MIP model generated a project NPV of \$43M (1.02% increase), with \$21M higher revenue (0.5% increase) than that predicted by traditional or TSMIP schedules.

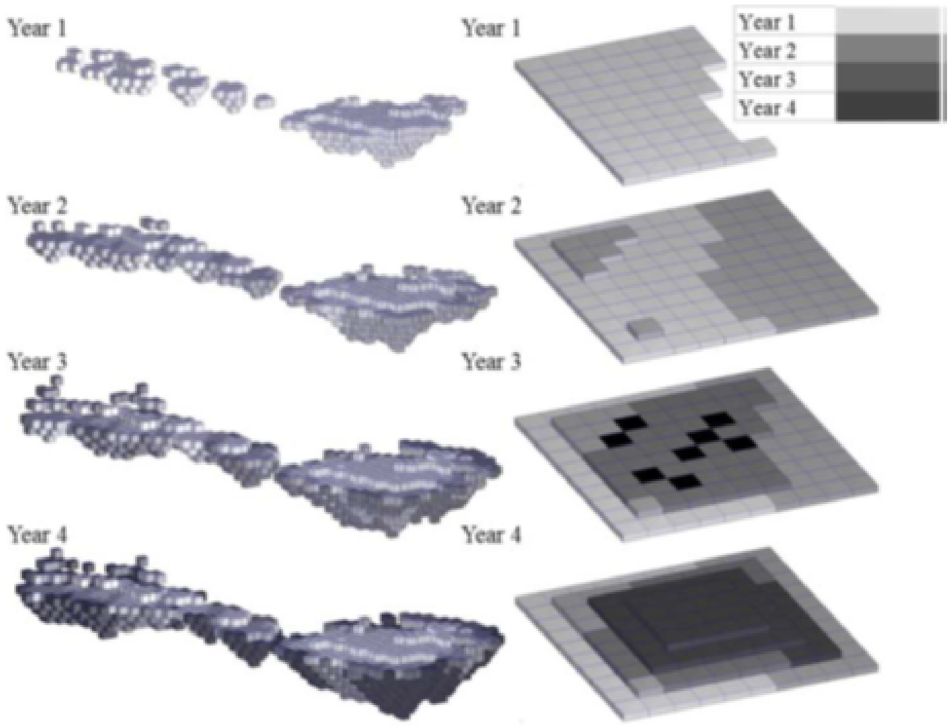


Figure 2.5: Open-pit mining schedule and dump over 4 years. (Fu et al., 2019)

Badiozamani and Askari-Nasab (2013) modelled waste movement in an oil sands operation. Stratified in nature, these deposits generate large volumes of pond tailings which warrant reclamation prior to agricultural redevelopment. These particular researchers recognised the need for mining and environmental milestone integration, where holistic schedules combine ore movement with the material requirements of waste landform rehabilitation. As input, such all-purpose models expect spatial block precedence, capacity limits for mining equipment, processing and tailings, together with reclamation material requirements and mining and fill directions per period. In this trial, real world data was optimised for NPV using MILP techniques which scheduled both oil sand and reclamation activities. Project revenue was estimated using ore and waste movement, ore processing and handling, as well as reclamation costs. These latter most expenditures were based on tailings storage facility (TSF) construction, overburden and interburden material (OI) movement, as well as tailing coarse sand (TCS) volumes.

Source data included 45,648, reserves blocks with dimrnsions of $50m \times 50m \times 15m$ from two operational stages, separated by a river. As ore grades varied, directional mining was used to access early high-grade material, as well as better manage TSF and reclamation activities. Mining was initiated at a corner of the first stage and thereafter directed to enable early in-pit tailings storage. This MILP was solved in four directions (N-S, S-N, E-W and W-E), which demonstrated the impact that this constraint had on NPV.

To reduce the impact of acid rock drainage (ARD), (Vaziri, Sayadi, Parbhakar-Fox, Mousavi and Monjezi, 2022) created a MILP model which minimised deviation from an optimal blending requirement of NAF and acid-neutralizing capacity (ANC) materials with PAF rocks, within short-term mine plans. To reduce potential environmental liabilities and closure costs, two constraints were used to control vertical and horizontal dump construction, together with dump cell dependency constraints as recommended by Li et al. (2013). Their results suggested that most mine operators should be capable of reducing their respective environmental footprints using an appropriately designed LOM schedule.

Regardless of the use of simultaneous waste and ore scheduling across the extractive sector, researchers continue to identify opportunities to advance in-pit waste dumping. This is especially relevant for stratified deposits, in light of their contrasting horizontal and vertical extents.

2.4 Model that considers in-pit dumping

Das et al. (2022) created a stratified deposit model which included an in-pit dumping strategy, with a production schedule which respected key geotechnical structures. Unlike hard rock operations, a transitive precedence constraint was created to ensure that overlying blocks were mined first (Figure 1.6). To facilitate in-pit dumping, these researchers tested different mining directions by using a cut-strip precedence, with each strip comprising multiple blocks. This model also employed a horizontal precedence to generate an optimal solution, which mined the pit from a specific side and direction, thereby facilitating early in-pit dumping and rehabilitation.

An MILP model created by Das et al. (2022) expanded on work by Fu et al. (2019), with the examination of in-pit dumping in stratified deposits, necessitating the use of dynamic lag distances, individual dump cells and geometric sequencing (i.e. Figure 1.6). This lag constraint ensured a safe distance was maintained between mining and in-pit dumping fronts for both operators and machinery working on the lowest mineralised seams (Figure 1.1).

This same constraint similarly generated a set of pit blocks B' which occurred within a particular radius, or lag distance. For in-pit dumping, this lag constraint was used to determine whether a block B' had been mined previously, thereby permitting waste dumping onto block d. This in-pit dumping or lag constraint was modelled using an objective function and some constraints, part of the objective function for NPV and some variables used have been shown below, while a more detailed version of the equations have been elaborated in other chapters:

$$\begin{aligned} npv = &\sum_{b=1}^{B}\sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{t=1}^{T} K_{\text{bimt}}\mathrm{X}_{\text{bimt}} + \mid \sum_{b=1}^{B}\sum_{i=1}^{I}\sum_{e=1}^{E}\sum_{d=1}^{D}\sum_{t=1}^{T} K_{\text{biedt}}\,\mathrm{X}_{\text{biedt}} \\ &+ \sum_{b=1}^{B}\sum_{i=1}^{I}\sum_{s=1}^{S}\sum_{t=1}^{T} K_{\text{bist}}\mathrm{X}_{\text{bist}} + \sum_{s=1}^{S}\sum_{m=1}^{M}\sum_{t=1}^{T} K_{\text{smt}}\mathrm{X}_{\text{smt}} \end{aligned} \tag{2.1}$$

Where,

B = the set of all pit blocks

B_d = Set of pit blocks which will have to be removed to allow dump block d to maintain the specified lag distance from all blocks in B_d, $B_d \in B$

l' = the index of the blocks that need to be extracted for dumping block d while maintaining the specified lag distance.

Location of l' is within the radial distance equivalent of the specified lag distance measured from the dump block d

$n_{l'}$ = density of block l' in g/cc

$X_{l'iedt'}$ = the volume of waste rock, from block l' transported through pit exit i and entering the dump through entry e, dumped in block d during period t', being a continuous variable with units in bank cubic meters

$X_{l'ist'}$ = a continuous variable in bank cubic metres, posting the volume of ore hauled ex-pit through exit i and taken to stockpile s during period t'

$X_{l'\ \mathrm{imt}}$ = the volume of ore hauled out of pit through exit i and taken to process plant m during period t', being a continuous variable with units in bank cubic meters

Y_{dt} is a binary variable where 1 presents the availability of a waste-dump location d over period t.

K_{bimt} Discounted value of mining and hauling unit ore(coal) tonne from block b through pit exit i to plant m in period t

K_{biedt} Discounted value of mining and hauling unit waste volume from block b through pit exit i to dump block d, entering through dump entry e in period t

K_{bist} Discounted value of mining and hauling unit ore(coal) tonne from block b through pit exit i to stockpile s in period t

K_{smt} Discounted value of hauling unit ore(coal) tonne from stockpile s to plant m in period t

This algorithm was used to schedule a coal mine, stressing in-pit / internal dumping with a concomitant reduction in external dumping (Figure 2.6). Results demonstrated that as haulage distances decreased, their associated costs fell markedly (-27%), together with the dimensions of external dumps (-47%.)

Lower trucking hours led to substantial improvements in project NPV, revised to \$121.9M from \$64.3M, without including the benefits of lower CO_2 emissions.

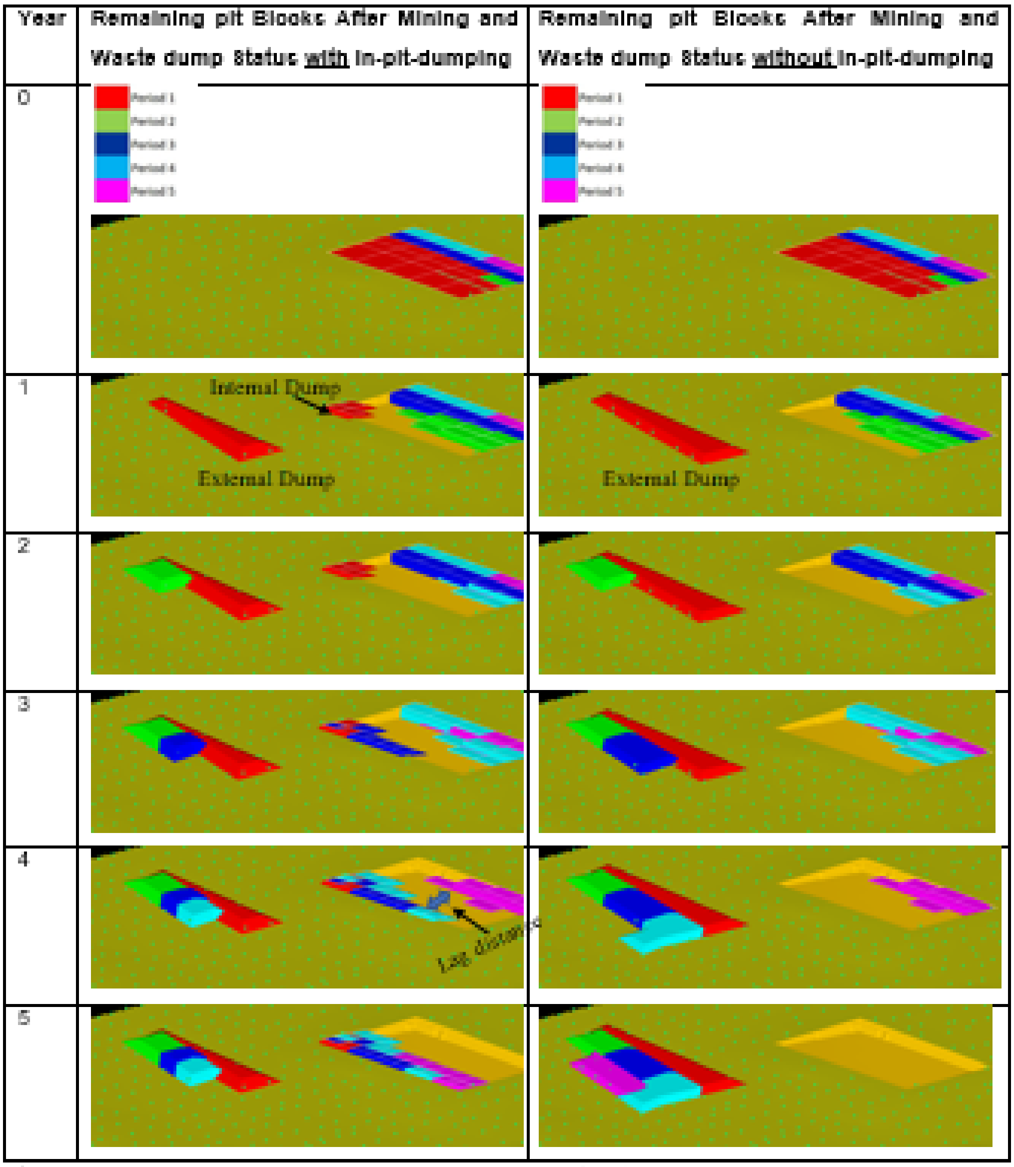


Figure 2.6: Surface area comparison for out of pit dump with and without in-pit dumping

As this model employed Euclidean distances for both in- and ex-pit road designs, actual haul road locations might introduce minor differences, however for this computationally large LP, as estimated, these significant cost savings remain realistic.

2.5 Stochastic models including waste management

Rimélé et al. (2018) generated stochastic models for managing waste rock in a two-stage stochastic integer LP, with fixed recourse for an iron ore deposit. This model simultaneously optimised extraction and destination sequences, including in-pit storage volumes. The objective function was divided into two parts with an initial optimisation of average discounted cash flows (DCF) for multiple scenarios, followed by a second which penalised deviations from minimum production targets.

For an in-pit dumping strategy, strip constraints were created, including two limiting zones for waste storage capacity in the void, both of which were made available for dumping, by period. Back-filled dumps were only allowed to increase in size and once filled, no waste rehandle was allowed. As mine and dump locations move dynamically over time, a limiting northern strip was allowed to progress northwards and a limiting southern strip, southwards both with waste. These researchers demonstrated that accessing in-pit storage saved considerable rehandling costs during rehabilitation, while reducing the impact on the local environment at comparable costs to external waste dump development.

Levinson and Dimitrakopoulos (2020) produced a stochastic model to maximise project NPV from a gold complex, while minimising the risk of meeting LOM production targets and environmental constraints, including deviations on waste dump and stockpile capacities. This model considered incremental mining cost against pit depth. Penalties were included to ensure proximal blocks (within 60 m) were mined together by period, as well as by bench (within 120 m). Although this model did not manage waste placement in dumps, it was shown to effectively minimise waste generation by treating it as a mining product, while considering material uncertainty and variability.

Whereas comparatively few waste optimisation studies currently exist, Table 2.1 summarises and comments on the attributes of the prominent examples discussed heretofore. These models are MILP-based and employ exact and/or heuristic techniques, with most developed for strategic scheduling purposes. In this respect, an opportunity exists for the development of parallel short-term plans, with waste management functionality. As these latter schedules often involve detailed excavator and truck movement, together with shift management and even grade blending, their source models often become overly large and complex.

Table 2.1: Models on waste dump schedule optimisation

Model Type	Name of Researcher	Year	Advantages of the Model	Disadvantages of the model
Deterministic, MILP Models				
Software product Blasor (MILP)	Zuckerberg et al. (2007)	2007	• Uses aggregate of blocks and bins • Considers in-pit dumping and lag distance between pit and dump • Considers a road network • Considers water table constraint	• No details of the model is available in the public domain
	Stone, Froyland, Menabde, Law, Pasyar and Monkhouse (2018)	2018	• Considers capacity of the downstream supply chain infrastructure; and • Considers market tonnage, blended ore quality, and grade constraints.	
MILP	Williams et al. (2008)	2008	• First publicly available model with the integration of open pit mining with dumping • Waste haulage costs for a 10% ramp is five times costlier than cost of hauling on flat road, causing dumps to expand laterally rather than going up to subsequent lifts; and • Selective dumping of benign and reactive rock, for covering the reactive waste with benign rock, to have minimum environmental impact.	• No concurrent ore and waste schedule • No multi time scheduling just optimise the allocation of waste block • Did not consider in-pit filling option.
MILP	Li et al. (2013)	2013, 2014	• Three original models were presented. The first one is a model for location optimisation (LOP), the second model is for truck balance (TBA), and the third is a combination of both models. • Multi-time period schedule for the waste rock management • Single-level and multi-level waste dump design formulations	• Did not simultaneously optimise ore and waste rock movement • No in-pit dumping option.
MILP	Badiozamani and Askari-Nasab (2013)	2013	• Looks at the material required for reclamation • Considers volume of tailings produced • Directional mining has been considered • Tailings constraints and material required for reclamation are considered in the model	• Blocks have been aggregated together and called cuts • Does not consider waste or tailings placement in the pit voids
MILP	Li et al. (2016)	2016	• Multiple pits and dumping locations have been considered • Three different optimisation models introduced, OP, TBA, VCM and compared with respect to overall haulage distance and, truck productivity perspective • The number of truck requirements optimised for the haulage requirement. • Euclidean distances considered from each pit block to all destination dump locations	• Did not simultaneously optimise ore and waste rock movement • No in-pit dumping option.

MILP	Fu et al. (2019)	2019	• Simultaneously optimised the pit and waste dump schedule. • A series of grade bins have been considered, which allows the model to allocate material with varying grades (including NAF rocks) to the relevant bins for blending purposes. • The model allows dynamic cut-off grade optimisation.	• Did not consider in-pit dumping option • Creates computationally large-scale model as it concurrently schedules the ore and waste movements.
MILP	Das et al. (2022)	2019/2022	• Considers stratified deposits, and horizontal precedence • Considers both external (out of pit) and in-pit dumping options concurrently. • Lag distance considered between working face and in-pit waste dump. • Stockpiling and blending requirements considered • A mining block has been assumed to contain both ore and waste	• Haulage distances are Euclidian from pit/dump blocks to exit/entry • Creates computationally large-scale models as it concurrently schedules ore mining and waste dumps
MILP	Sayadi, Karimzadeh, Naghavi and Monfared (2022)	2022	• Blending of reactive and benign waste material for minimizing ARD formation • Waste dump cell spatial precedence is considered in both horizontal and vertical directions. • Simultaneously balancing the production of acid-neutralizing and acid-producing waste to maximise the efficiency and prevent ARD formation.	• Haulage has not been considered in the model. • In-pit dumping has not been considered
Stochastic, MILP	Rimélé et al. (2018)	2018	• First stochastic nature of problem formulation. • The objective function is in two parts, the first part aims to maximise the average discounted cash flow for a set of scenarios, while the second part penalizes any deviations in targets for production	• The storage area is limited by two strips, one top and one bottom, during a time period. Individual blocks within the strips are not considered • Dumping in multiple benches in different strips at the block level is not considered • Pit blocks are also used as dump blocks (both having vertical walls from the block model)- designs not considered with appropriate wall slope for pit wall and dump wall • Minimum lag distance between working face and dump not treated explicitly

Stochastic,MILP	Levinson and Dimitrakopoulos (2020)	2020	• Simultaneously optimised pit and waste generation • Considered uncertainty in the input factors • The impact of changes in the cut-off grades to the volume of waste production have been considered • Attempted to create practically feasible mining solutions - with constraints restricting vertical and horizontal drop • Multi-neighbourhood simulated annealing with adaptive neighbourhood search has been used for solving the formulation.	• Haulage distances have not been considered in the model • in-pit filling has not been considered.

2.6 Chapter summary

The cost of waste removal and handling in open pit mines accounts for some of the largest operational expenses, as prescribed by stripping ratios. With the inevitable depletion of world class ore bodies, such ratios can only increase and with them, the importance of waste management optimisation and environmental footprint minimisation. This study reaffirms the need for carefully staged, in-pit dumping and back-filling. It also documents how open pit schedule optimisation has evolved from relatively simple ore blending models to the use of highly sophisticated MIP/MILP techniques, particularly over the past two decades. In this regard, it highlights the step change which has clearly occurred, with the progressive optimisation and scheduling of all material types and in particular, waste.

Whereas the authors recognise that such holistic operational scheduling continues to lag at most mine sites, research interest has clearly grown with both deterministic and other stochastic models increasingly available in applied literature. With a gradual replacement of separate ore and waste scheduling by inclusive, multi-purpose models, improved economic and environmental outcomes are assured. While continuing to develop solutions for large real-world applications, the authors note that future research opportunities should address as to how commercial software packages might best incorporate such functionality for both strategic, long-term and tactical short-term, mine plans.

Chapter 3

Mine waste dump schedule optimisation including in-pit dumping option

Reducing the footprint of mining by back-filling and rehabilitating open pit mines progressively reduces the amount of land exposed at any point in time. Optimal dump destination (in-pit or external) depends on their availability, which for in-pit is determined by the mined-out void created and available lag distance between the mining face to the dumping face. A mathematical model has been developed to maximise value of the mine while not only considering external dumping option but internal dumping as well while maintaining the lag distance with the working face. The model has been applied to a stratified deposit mine with promising results.

3.1 Introduction

Typical mine production scheduling deals with deciding which mining blocks to extract and when, such that the discounted cash flow or NPV of the project is maximised while adhering to the physical and production constraints. (Johnson, 1968); (Osanloo et al., 2008); (Topal and Ramazan, 2012*b*); (Mai et al., 2019); (Fathollahzadeh, Mardaneh, Cigla and Asad, 2021).

Conventional mine planning starts by finding ultimate pit limits, which is the combination of blocks that maximises the total undiscounted cash flow of the project and respects slope constraints. Then the optimal extraction sequences of material within each period is determined. The open pit mine-sequencing problem or production scheduling problem is defined as specifying the sequence in which material should be extracted from pits and then transferred to appropriate destinations in specific time periods (Xu, Gu, Wang, Gao, Liu, Wang and Wang, 2018). Generally, material with no economic value is dumped into external waste dump areas while profitable material is processed at mills or stocked at stockpiles for future usage (Mai et al., 2018). There has been significant research already done in this area (Li et al., 2014).

For solving the pit production-scheduling problem, it is a common practice to create a three dimensional block model of the ore body to describe the ore reserves. Based on the sample data collected from boreholes or otherwise, the geo-chemical and economic parameters of each block are estimated. The precedence or sequence of mining is influenced by the physical and operational constraints keeping in view the geotechnical stability of the slopes. This discrete approach creates huge combinatorial problems whose mathematical formulations are large-scale instances of IP optimization problems (Alvarez-Ellacuria et al., 2010), (Caccetta and Hill, 2003). The mine production-sequencing problem may be solved for different levels of accuracy. For simplification reasons, some blocks are aggregated into bigger units to obtain extraction sequences of these units with less computational efforts (Askari-Nasab et al., 2010); (Mai et al., 2019); (Mai et al., 2018).

Previous research has focused on the sequence of mining without considering the haulage and dumping requirements of waste rock. Williams et al. (2008) focused on minimising the haulage cost for each open pit block of waste rock to be placed in the waste dump, with some allowance for the selective placement of benign and reactive waste rock, based on an open pit block model that delineates ore, and benign and reactive waste rock. Li et al. (2013) took the work further in order to include trucking and minimise haulage costs. These works however did not simultaneously optimise ore and waste pit schedules based on the dumping requirements. They were based on existing optimal ore schedules.
A recent study by Fu et al. (2019) considered the simultaneous optimisation of material mining and dumping schedule and formulated the problem as a MIP problem. This is a unique first attempt in this regard to bring together the two areas of mining and dumping which were so far being considered separately.

Zuckerberg et al. (2007), presented some work on in-pit dumping as an extended version of BHP's mine planning software Blasor, named Blasor- In-Pit Dumping (BlasorIPD). This works by first finding an ultimate pit limit boundary from the blocks provided. This is followed by aggregating blocks and panels in order to find phases of the mine. These phases are then used to schedule the mine. Blasor IPD choses some portion of the waste to go into external dumps or to in-pit dumps. Blasor IPD choses a path along a road network upon which to send the waste. The path along the network terminates at an external waste dump, or back into the space once occupied by some block which Blasor IPD classifies as empty and available for dumping. A location (say A) can only be classified as available for dumping if all blocks within a user defined radius of A have already been mined out, and if additionally all locations (say B) within the ore body that lie below A, for which the slope angle of the line connecting A and B is greater than the maximum angle of repose for waste have already been refilled.

Blasor IPD has been developed such that a space cannot be made available for dumping if that sits on top of material classified as ore that has not yet been cleared. Blasor IPD is the only software known to do schedule optimisation including in-pit waste dumping. However, not much detail of the mathematical model behind the software is available in the public domain.

The mathematical model proposed in this chapter has a similar concept as the Blasor IPD software. However, a notable difference is that Blasor IPD fills back the existing mining blocks whereas this model uses a separate block model for dumps which is available for internal as well as external dumping purposes simultaneously and also modelling of dynamic lag distance (mining and filling) for internal dumping purposes. The model described in this chapter is based on predesigned solid shapes of blocks of pit and dump. The pit blocks and dump blocks have different pre-designed slopes. As seen in the artists impression in Figure 1.5a, the slopes for pit face and the in-pit dump face are in opposite directions from the working area where we have the lag distance maintained.

In stratified deposits, instead of phases as used in Blasor IPD, mining and dumping are mostly done in strips. As seen in Figure 1.1, stratified deposits can be mined and progressively back-filled in order to minimise external land usage and create rehabilitated surfaces. Hence, back filling is an essential part of stratified deposit mining because such pits have a greater lateral extent rather than vertical extent. Hence, the nature of stratified deposits is distinctly different and it may not be possible to apply optimisation models developed for hard rock deposits onto stratified deposits.

Rimélé et al. (2018) have used strips and mining blocks for mining and back-fill dumping. Their paper assumes that the same blocks in a strip can be back-filled after they have been mined. Normally while scheduling a pit and dump in a deposit the mine planner has a separate design and block model for the pit and the dump, as they follow different bench geometry and slope. The paper presents an interesting concept on which blocks can be made available for dumping once they have been mined. Unlike in (Zuckerberg et al., 2007) where a distance has been considered between the operating face and the dump, here it is based on the order that the top strip should be further north of the bottom strip. The paper demonstrates acceptable results for an iron ore project.

Levinson (2019) have presented simultaneous stochastic optimisation considering waste management and cut-off grade optimisation. This is applicable for mines that contain potentially acid forming waste, and need to be suitably covered by non-acid forming waste. The model aims to define the extraction sequence, destination policy, and processing stream decisions while simultaneously managing the targets and capacities at waste, processing, and stockpile facilities. The mathematical model considers and restricts the allowable flow of materials to either the waste dumps, stockpiles, or processing facilities based on the material characteristics, while managing the related risk. However, their model does not deal with the placement of the waste into a particular location.

Although swell factor is not explicitly mentioned by many researchers, it is an underlying consideration. Fu et al. (2019) in their paper mention 1.25 as the swell factor considered. In this model swell factor has been used as a variable and depending on the material in the case study it has been changed.

3.1.1 Model description

The model has been built typically for open pit stratified deposits which have some differences in mining strategy with open pit hard rock. The model considers the requirement of early back-filling of stratified deposit mines which are mined over a large lateral extent.

3.2 The mathematical model

The model extends the previous work in the area (Fu et al., 2019) with the objective function of maximising NPV, thereby prioritising the low-cost haulage to the internal dump and expedite rehabilitation. The equations considered in this model are in line with previous studies, with the following additional considerations and extensions:

i. There are modifications in order to consider the stratified nature of the deposit. Consideration of a block to be mined is as shown in Figure 1.5b,

ii. Blocks used in this model could contain both ore and waste layers,

iii. Every pit and dump blocks here have been considered to have a location with a coordinate for their centroid, with separate designs for the pit and dump,

iv. The centroids have been used to find a set of pit blocks that fall within a certain radius, and need to be mined out, for each dump block, so as to maintain a lag distance with the pit blocks,

v. As a stratified deposit pit is deeper on the dip side, the number of benches increase as we mine the strips one by one, hence consideration has been made for different number of benches in different areas for both pit and dump.

Only equations from that model that are of significance to stratified deposits have been provided below, as most other equations are common to previous studies in the field.

t = Period, t $\in$ T Set of time periods in the scheduling horizon

b = Pit block, $b \in B$ Set of all mining blocks

$B_b = B_b \in B$; Set of mining blocks to be mined in order to mine block b

$i =$ Pit exit, $i \in |$ Set of pit exits

$e =$ Dump entry, e $\in E$ Set of dump entries

d = Dump block, d $\in$ D Set of dump blocks

$D_d =$ $D_d \in D$; Set of dump blocks that need to be filled to dump on block d

$m =$ Process plants, $m \in M$ Set of processing plants

$s =$ Stockpiles, $s \in S$ Set of stockpiles

$B_d = B_d \in B$; Set of mining blocks that need to be mined until the dump block d can be dumped

Parameters

p_t Selling price of ore(coal) per tonne in period t

c_{t} Per tonne ore(coal) mining cost in period t

w_1 Per cubic meter waste mining cost in period t

h_{t} Ore(coal) and waste haulage cost in period t for every meter hauled

f_t Ore(coal) processing cost per tonne feed in period t

C_{st} Tonnes of ore(coal) in stockpile s in period t

$\bar{c}_{st}$ Maximum capacity of stockpile s in period t in tonnes

$\underline{c_{st}}$ Minimum capacity of stockpile s in period t in tonnes

$\bar{M}_{\mathrm{t}}$ Maximum mining capacity in period t in bcm

$\underline{M}_t$ Minimum mining capacity at period t in bcm

$\bar{P}_{\mathrm{mt}}$ Maximum process capacity of plant m in period t in tonnes

$\underline{P}_{\mathrm{mt}}$ Minimum process capacity of plant m in period t in tonnes

$\underline{G}_{\mathrm{mt}}$ Minimum grade of ore(coal) fed to process plant m in period t

r_{b} Wash plant recovery(yield %) of block b

g_b Grade of mining block b

v_{b} Waste tonnes of block b o_b Ore(coal) tonnes of block b

t_{b} Total tonnes of block b

$\mathrm{n_b}$ Density of block b in tonnes/cu.m

g_s Average grade of ore(coal) in stockpiles

u_{d} Volume of dump block d

j_s Re-handling cost of ore(coal) from stockpiles s

u_{d} Volume of dump block d in cu.m

Z_{bi} Euclidean distance between block b's centroid and pit exit i

Z_{im} Euclidean distance between pit exit i and process plant m

Z_{ie} Euclidean distance between pit exit i and dump entry e

Z_{ed} Euclidean distance between dump entry e and centroid of dump block d

Z_{is} Euclidean distance between pit exit i and stockpile s

Z_{sm} Euclidean distance between stockpile s and process plant m

a Discount rate in SW Swell factor - ratio of loose cubic meter to bank cubic meter

dc Total dump capacity including external and in-pit dumps in cu.m

K_{bimt} Discounted value of mining and hauling unit ore(coal) tonne from block b through pit exit i to plant m in period t

K_{biedt} Discounted value of mining and hauling unit waste volume from block b through pit exit i to dump block d, entering through dump entry e in period t

K_{bist} Discounted value of mining and hauling unit ore(coal) tonne from block b through pit exit i to stockpile s in period t

K_{smt} Discounted value of hauling unit ore(coal) tonne from stockpile s to plant m in period t

$$K_{bimt} = \mathrm{p}_t \cdot \mathrm{g}_b \cdot \mathrm{r}_b - \mathrm{c}_t - \mathrm{f}_t - \mathrm{h_t} \cdot (\mathrm{Z}_{bi} + \mathrm{Z}_{im}) \cdot \left(\frac{1}{(1+a)^t}\right) \tag{3.1a}$$
$$\forall \mathrm{b} \in \mathrm{B}, \mathrm{i} \in \mathrm{I}, \mathrm{m} \in \mathrm{M}, \mathrm{t} \in \mathrm{T}$$

$$K_{biedt} = \mathrm{w}_t \cdot \mathrm{n_b} - \mathrm{h}_t \cdot (\mathrm{Z}_{bi} + \mathrm{Z}_{ie} + \mathrm{Z}_{ed}) \cdot \left(\frac{1}{(1+a)^t}\right) \tag{3.1b}$$
$$\forall \mathrm{b} \in \mathrm{B}, \mathrm{i} \in \mathrm{I}, \mathrm{e} \in \mathrm{E}, \mathrm{d} \in \mathrm{D}, \mathrm{t} \in \mathrm{T}$$

$$K_{bist} = -\mathrm{c}_t - \mathrm{h}_t \cdot (\mathrm{Z}_{bi} + \mathrm{Z}_{is}) \cdot \left(\frac{1}{(1+a)^t}\right) \quad \forall \mathrm{b} \in \mathrm{B}, \mathrm{i} \in \mathrm{I}, \mathrm{s} \in \mathrm{S}, \mathrm{t} \in \mathrm{T} \tag{3.1c}$$

$$K_{smt} = \mathrm{p}_t \cdot \mathrm{g}_s \cdot \mathrm{j}_b - \mathrm{f}_t - \mathrm{h}_t \cdot (\mathrm{Z}_{sm}) \cdot \left(\frac{1}{(1+a)^t}\right) \quad \forall \mathrm{m} \in \mathrm{M}, \mathrm{s} \in \mathrm{S}, \mathrm{t} \in \mathrm{T} \tag{3.1d}$$

Equations 3.1a to 3.1d are used to calculate the discounted value for material movement from pit to dump, stockpile or plant. This considers the revenue, the costs and discount factor. The revenue is a function of the grade and recovery at the process plant. The haulage cost is a factor of the distance to the destination. The values have been discounted by a known discount factor ‘a’ in order to get the cumulative discounted cash flow

Variables

X_{bimt} Continuous variable, which represents the amount of ore(coal) in tonnes mined from block b, hauled through pit exit i, and to processing plant m in period t;$\mathrm{X_{bimt}} \geq 0$

X_{biedt} Continuous variable, which represents the amount of waste in cubic meters mined from block b, hauled through pit exit i, and entry e to dump location d in period t; $X_{\mathrm{biedt}} \geq 0$

X_{bist} Continuous variable, which represents the amount of ore (coal) in tonnes mined from block b, hauled through pit exit i, to stockpile s, in period t;$\mathrm{X_{bist}} \geq 0$

X_{smt} Continuous variable, which represents the amount of ore(coal) in tonnes hauled from stockpile s, to processing plant m in period t;$\mathrm{X}_{\mathrm{smt}} \geq 0$

Binary variables

Y_{bt} Binary variable, which represents whether all overlying blocks of block b are mined out by end of period t

Y_{dt} Binary variable, which represents whether the waste-dump location d is available to be filled in period t

ζ_{bt} Binary variable, which represents if all waste above the ore layer in block b is removed

The performance metric of interest is the NPV of the mining schedule, which is defined previously in Equation 2.1

The NPV should be maximised subject to the constraints described below: Some of the unique constraints that differentiate this model from others are listed below:

- A lag constraint is used to ensure that there is a distance between the mining face and the dump. This is one of the key differentiators of this model from models developed in the past and making it capable of scheduling for back-filling or in-pit dumping. A set of pit blocks are determined, for each dump block, which fall within the radius of a defined lag distance. The lag constraint checks whether these pit blocks have been already mined. The dump block can be filled only if these selected set of pit blocks have been entirely mined in any previous period.

$$\sum_{l'\in Bd}^{Bd}\sum_{i=1}^{I}\sum_{e=1}^{E}\sum_{d=1}^{D}\sum_{t'=1}^{t}\mathrm{X}_{\mathrm{liedt}} + \sum_{l'\in Bd}^{Bd}\sum_{i=1}^{I}\sum_{s=1}^{S}\sum_{t=1}^{t}\mathrm{X}_{\mathrm{rist}\ /\mathrm{nl}'} + \sum_{t/\in Bd}^{Bd}\sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{t'=1}^{t}\mathrm{X}_{\mathrm{rimt}\ /\mathrm{nr}} \geq \mathrm{Y}_{\mathrm{dt}}\sum_{l\in Bd}^{Bd}\mathrm{tr} \qquad (3.2)$$

$\forall t \in T,\ \mathrm{d} \in D$

Where l' are the blocks that need to be mined in order to dump in a dump block d and maintain a certain lag distance. These blocks fall within the pre-defined radius from the dump block d.

- The volume of waste that can be dumped into a block is limited by available empty volume in that dump block d, considering the swell factor of sw.

$$\sum_{b=1}^{B}\sum_{i=1}^{l}\sum_{e=1}^{E}\sum_{t''=1}^{t} sw.X_{\mathrm{biedt}} \leq \mathrm{ud}\ \forall d \in D\ |, t \in T \qquad (3.3)$$

- A dump block d can only be filled once the block below has been filled, where d' are the dump blocks that need to be filled to fill dump block d

$$\sum_{b=1}^{B}\sum_{i=1}^{I}\sum_{e=1}^{E}\sum_{d'\in D_d}\sum_{t'=1}^{t} sw \cdot X_{\mathrm{bied}}\mathrm{t}' - Y_{\mathrm{dt}}\sum_{d'\epsilon Dd} u\ \mathrm{d}' \geq 0 \qquad (3.4)$$

- It needs to be ensured that total volume of block is mined, as it contains layers of ore and waste.

$$\sum_{i=1}^{I}\sum_{m=1}^{M}\sum_{t'=1}^{t} X_{\mathrm{bimt}} + \sum_{i=1}^{I}\sum_{s=1}^{s}\sum_{t'=1}^{t} X_{\mathrm{bist'}} + \sum_{i=1}^{l}\sum_{e=1}^{E}\sum_{d=1}^{D}\sum_{t'=1}^{t} X_{\mathrm{biedt}} - Y_{bt.}\mathrm{tb} \leq 0$$

$\forall b \in B, t \in T$

(3.5)

- To ensure that all waste above the ore layer in a block are removed

$$\sum_{i=1}^{I}\sum_{e=1}^{E}\sum_{d=1}^{D}\sum_{t'=1}^{t} X_{\mathrm{biedt}}{}' \geq v_{\mathrm{b}} - M\left(1-\zeta_{\mathrm{bt}}\right) \quad \forall b \in B, t \in T \tag{3.6}$$

M is a very large number

$$\zeta_{\mathrm{bt}}{}^{*}O_{\mathrm{b}} \geq \sum_{i=1}^{I}\sum_{m=1}^{M} X_{\mathrm{bimt}} + \sum_{i=1}^{I}\sum_{s=1}^{S} X_{bist} \quad \forall b \in B, t \in T \tag{3.7}$$

Other constraints which are common in most models are as below:

- Maximum and minimum equipment capacity available for mining the pit to be honored.
- Maximum and minimum process plant (wash plant) capacity to be honored.
- Minimum grade limits of ore(coal) mined to be honored
- Mining slope requirement to be honored, as shown in Figure 1.5b are to be mined before mining each block
- Stockpiles have a limited capacity and should not be exceeded

3.3 Conceptual implementation of model

The model was first implemented for a synthetic block model with 18 blocks to demonstrate the proof of concept. It was then applied to a stratified mine data for which pit and dump designed with strips and blocks in a 3D mine planning software were available. The volumes, tonnes, quality, and block coordinates were reported out of the software and tabulated in an Excel workbook. A mixed-integer programming based model has been developed using CPLEX OPL to optimise the production schedule for each period and the corresponding dump locations to be used for each block.

The model was first tested on an 18 block (Figure 3.1) test data being mined in three periods. The 18 blocks were assumed in two benches, with each bench having three strips and three blocks in each strip, each block being a 100m X 100m X 100m cube. An in-pit dump with 18 blocks was also considered of equal size and design and at the same footprint. Figure 3.2 presents the production schedule for three periods. Figure 3.3 represents the beginning of the third period where in-pit dumping has started while maintaining a lag with the mining face. The pit and dumps were designed in a 3D Mine planning software and the details including pit and dump block centroid coordinates were exported to Excel, which formed an input data to the optimisation model in CPLEX OPL. The dump has been pre-designed to contain individual blocks each with a designed shape. Both pit and dump blocks have been considered to have a location with a coordinate for each block centroid. A lag distance of 100m was considered between the pit and dump.

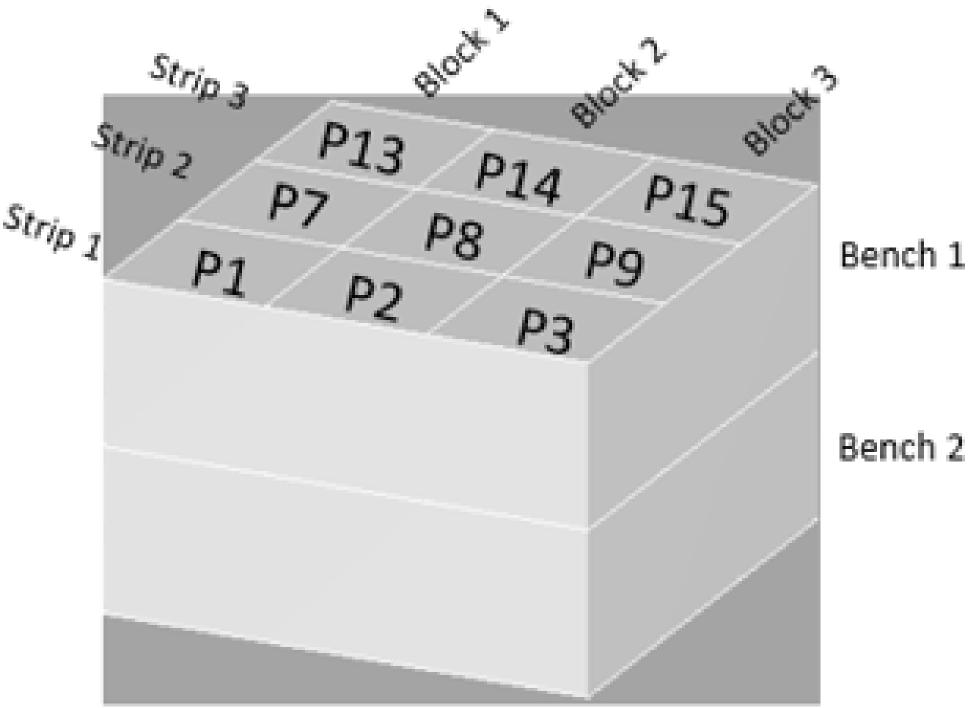


Figure 3.1: Block test model (100m X 100m X 100m)

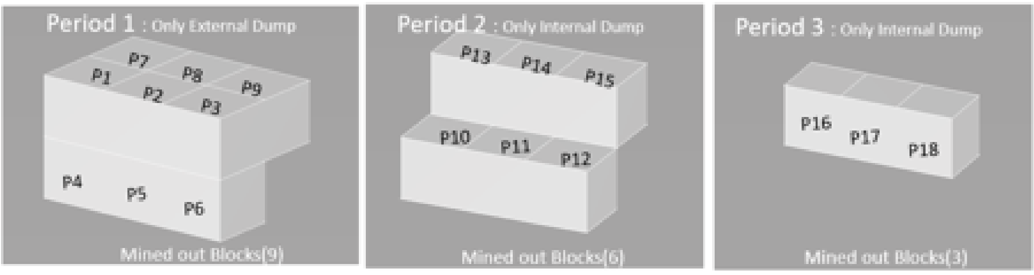


Figure 3.2: Sequence of blocks being mined in each of the three periods

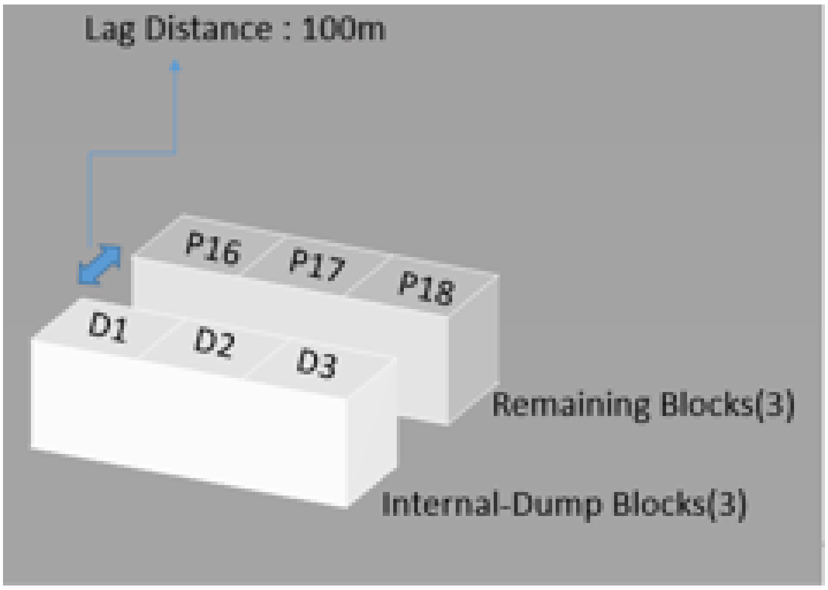


Figure 3.3: Status of the dump at the end of Period 2 with maintaining lag distance

3.4 Case study implementation

The model was applied to a real stratified deposit mining dataset. The dataset is a part of a widely used tutorial dataset of GEOVIA Minex software and belongs to an old mine in Hunter Valley, New South Wales, Australia. A schematic diagram of the explanation of the material flow in the data is shown in Figure 1.7. It has a single pit with 9 benches. A total of 118 blocks of average size 120meters X 250 meters – covering an area of 1240m X 740m was chosen for the study as shown in Figure 3.4. This area covers a 5-year mine life. The number of blocks may seem less here for a coal mine, however, it is typical in stratified deposit mines to have larger block dimensions in X and Y, while Z can often be small compared to hard rock deposit mines. Hence the number of blocks compared to a hard rock mine of the same size is much smaller. The coal is transported to any of the three stockpiles – based on the quality or can be directly fed to the wash plant. Coal is also fed to the wash plant from the stockpile by rehandling. The data used have been described in Table 3.1.

Table 3.1: Input parameters for the optimisation model for stratified deposit

Input parameter	Variable name	Maximum	Minimum	Unit
Wash Plant capacity	$\bar{P}_{\mathrm{mt}}$ and $\underline{P}_{\mathrm{mt}}$	$6,000,000$	$2,000,000$	Tonnes/Yr
Mining capacity	$\bar{M}_{\mathrm{t}}$ and $\underline{M}_t$	$16,589,047$	$16,589,047$	Cubic m / Yr
Specific energy	g_b	-	15	MJ/Kg
Coal sale price	p_t	-	\$150.00	\$ per t
Coal mining cost	c_{t}	-	\$4.00	\$ per t
Waste mining cost	w_1	-	\$3.00	\$ per BCM
Waste haulage	h_{t}	-	\$0.0002	\$ per BCM meters
Washing cost	f_t	-	\$6.00	\$ per t ROM feed
Rehandling cost	j_s	-	\$0.75	\$ per t ROM feed
Swell factor	SW	-	1.25	Volume to BCM ratio
Discount Rate	a	-	10	%
Recovery/Yield	r_{b}	-	90	%

Note: the \$ refers to teh Australian dollar.

The in-pit dump has the same footprint as the pit at the bottom-most bench floor. This dump has two benches and 104 dump blocks were considered, as shown in Figure 3.4, for the current problem. One external dump area has been considered as a single block with an equivalent capacity. The pit blocks have multiple layers of coal and waste in them as evident from the colours of the layers in Figure 3.4:. The lowest bench is mostly coal and has two coal seams. The schedule has been solved for five periods.

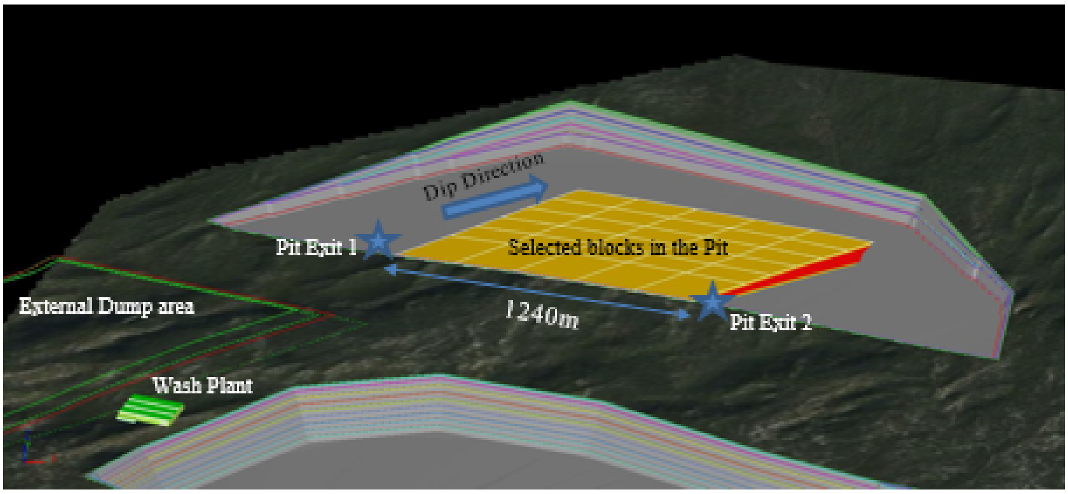


Figure 3.4: Complete pit and topography surface (5-year planned pit blocks coloured by layers of coal and waste).

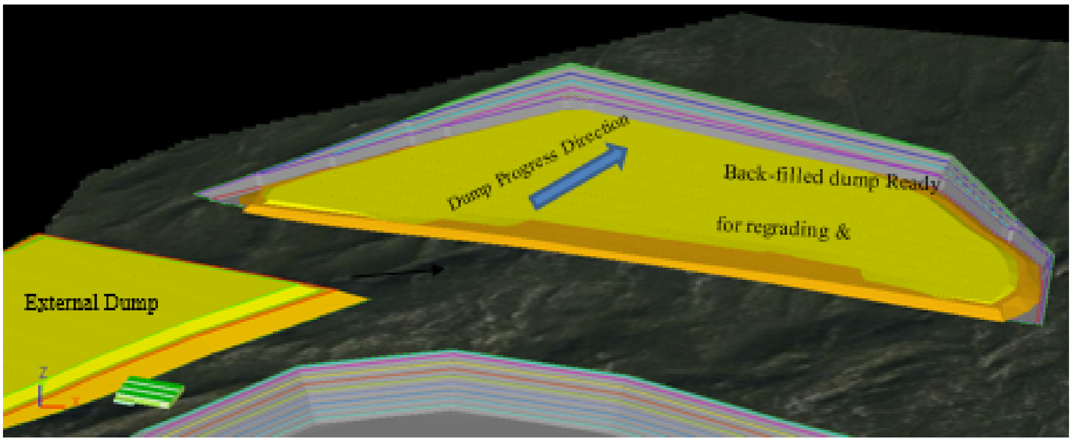


Figure 3.5: In-pit dump blocks for the life of mine

The dumps have been pre-designed with blocks at 37-degree slope angles as can be seen in Figure 3.5. Due to the slope in the design, although at the bottom bench of the dump the first strip #1 exists all along, there is not enough room for strip #1 on the upper bench, hence it has lesser strips and blocks (lesser area). A swell factor of 1.25 has been considered for waste placed into dump blocks. Table 3.2 presents the pit data which contains a snapshot of the block details. Blocks could contain both coal and waste whereas some blocks have only waste. There are other tables like- one having coordinates of each block, and one for the dump blocks and their coordinates.

Table 3.2: An example format of the pit block details [The complete list has 118 blocks]

Block_Id	Bench	Strip	Block	Coal Quality	Coal Tons	Recovery	Waste Vol	Total Volume
P4	1	1	4	-	-	-	155	155
P5	1	1	5	-	-	-	156	156
P6	1	1	6	-	-	-	154	154
P7	1	1	7	-	-	-	151	151
P8	1	1	8	-	-	-	149	149
P326	8	1	4	-	-	-	1,128,414	1,128,414
P327	8	1	5	-	-	-	1,246,694	1,246,694
P328	8	1	6	-	-	-	1,377,228	1,377,228
P329	8	1	7	-	-	-	1,423,040	1,423,040
P330	8	1	8	-	-	-	1,287,632	1,287,632
P439	9	1	4	28	612,148	76.92	-	612,148
P440	9	1	5	28	669,341	76.92	-	669,341
P441	9	1	6	28	681,694	76.92	-	681,694
P442	9	1	7	28	664,436	76.92	-	664,436
P443	9	1	8	28	628,271	76.92	-	628,271
P17	1	2	4	-	-	-	4,668	4,668
P18	1	2	5	-	-	-	66,374	66,374
P19	1	2	6	-	-	-	92,947	92,947
P20	1	2	7	-	-	-	57,713	57,713

The size and shape of the blocks are not uniform – which is a common feature in stratified deposit strip – block design. The blocks have a pre-designed angle to maintain the high-wall angle. The height of the benches are also not uniform and is largely a function of the seam geometry. Figure 3.6 presents the annual material flows for the mine for the next five periods, coloured by period. It can be seen that the mining sequence proceeds gradually strip-by-strip making room for input dumping as early as possible.

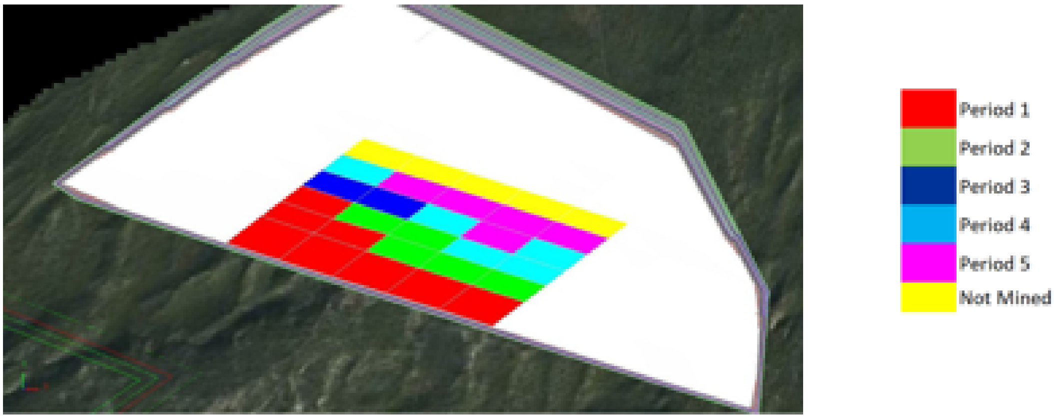


Figure 3.6: View of pit coloured by periods for bottom bench (Bench 9)

The lag distance is calculated at the base of the pit between the last pit bench and the first dump bench. Pit Bench 9 in this case shown in Figure 3.6 is the bottom-most pit Bench while dump Bench 1 shown in Figure 3.7 is the bottom-most in the in-pit dump. Lag has been considered between pit Bench 9 and dump Bench 1.

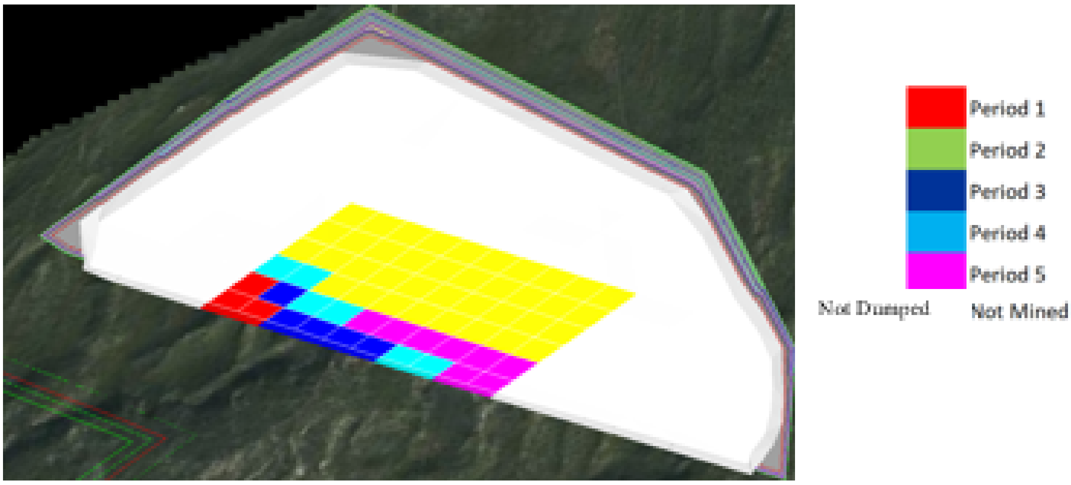


Figure 3.7: Internal dump coloured by periods for the bottom bench of dump (Bench 1)

3.5 Analysis of results

The material flow of the schedule can be seen in Table 3.3, which includes waste movement to both external and internal dumps as well as for coal movement to the plant and stockpile.

Table 3.3: Results obtained from the model for waste placement from pit to dump blocks for each period (All figures are in '000)

Year	Waste Volume ('000 cum)	Waste Dump('000 cum)		Coal To Plant from Mine('000 t)	Coal to Stockpile from Mine('000 t)	Stockpile to Plant('000 t)	Total to Plant('000 t)
		External	In-pit				
1	12,179	10,505	1,674	3,227	1,183	-	3,227
2	3,898	3,898	-	2,000	102	-	2,000
3	5,175	2,418	2,757	825	-	1,175	2,000
4	5,358	2,164	3,193	1,890	-	110	2,000
5	4,210	-	4,210	2,000	-	-	2,000
Total	30,820	18,985	11,834	9,942	1,285	1,285	11,227

As can be seen from Table 3.3 and Figure 3.8 waste has been directed to both external and in-pit dumps. This has been decided based on the shortest distance (lowest cost) and based on blocks available for in-pit dumping, in order to maintain the lag distance with the pit blocks being mined.

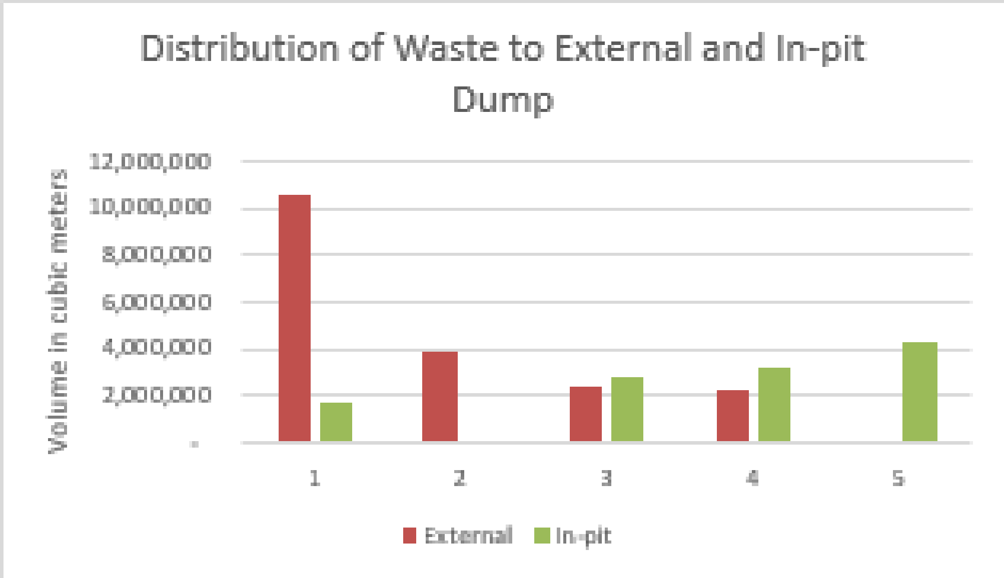


Figure 3.8: Distribution of waste to external and in-pit dump

In the early periods when mostly the first and second strips are being mined there is not enough room for starting the in-pit dump. Hence, initially, the waste is mostly sent to the external dump block. After the 4th year, there is enough room in the in-pit dump hence all waste is sent there. As soon as a strip or two in the in-pit dump has been back-filled, it could be ready for regrading and rehabilitation. In this way, the strips could continue to be rehabilitated optimally, while keeping in mind maximising project value or decreasing costs.

Coal has been sent directly either to the wash plant or to the stockpile. Coal sent to the stockpile has been drawn down in subsequent years. Although long-term stockpiling of coal may not be practical, the problem has to be managed with short-term planning. Figure 3.9 represents the area to be mined showing the blocks and also the area available for external dumping. Figure 3.10 presents the annual progress of mining and dumping coloured by periods.

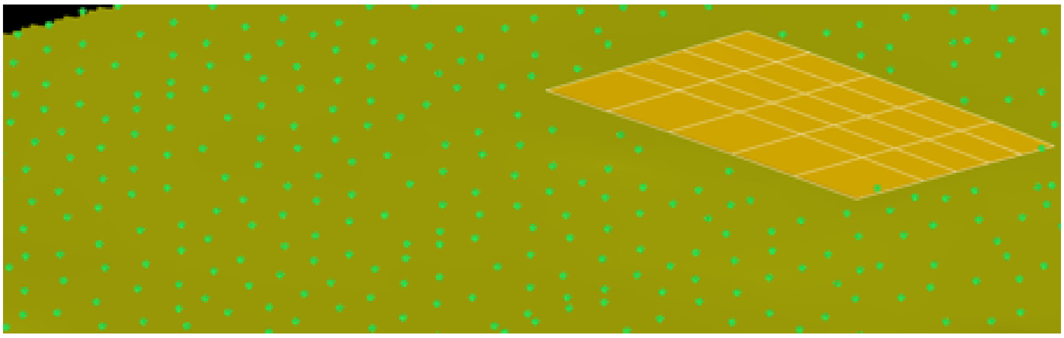


Figure 3.9: Area designated for mining in 5 years and vegetated land surface available for dumping

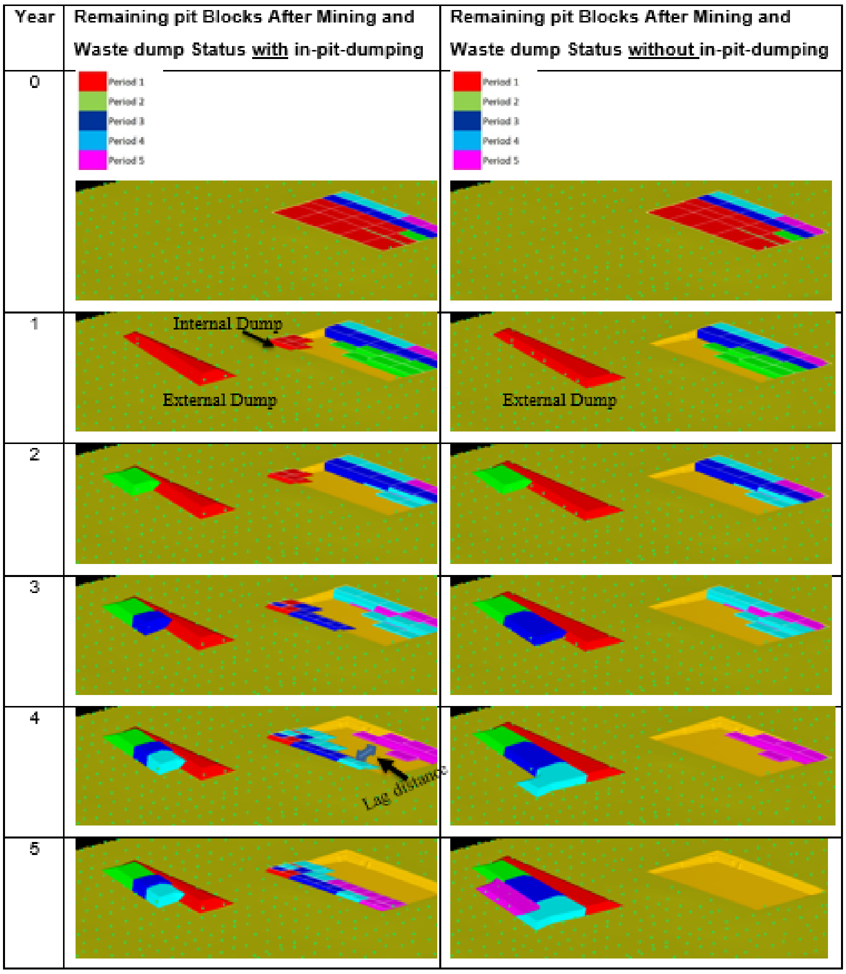


Figure 3.10: Comparison of external dump area with and without in-pit dumping

The external dump needs tree clearing and has a surface area required as a footprint. It is best to be able to minimise this area while keeping the project economics robust. In Figure 3.10 – Year 5 a side-by-side comparison is available for the footprint required if the internal dump as optimised by the model is used versus if the internal dump is not used at all in these five years. In the latter case without an in-pit dump, the footprint of the external dump is 775.5 thousand square meters, which is reduced to 405.6 thousand square meters with the use of an internal dump.

For the calculation of NPV, each period has been assumed to be a year. The NPV of the current schedule at a 10% discount is $121.9 million as seen in Table 3.4, under the given assumptions, as shown in Table 3.1.

Table 3.4: Cash flow and NPV calculated from the schedule including in-pit dumping

Year	Waste Mining Cast $'000	Waste Haulage Cost		Coal Mining & Washing Cost			Wash Cost $'000	Re-handling Cost $'000	Revenue $'000	Margin $'000
		Ext $'000	In-pit $'000	Mining $'000	Haulage to plant $'000	Haulage to Stock $'000				
1	36,538	164,431	976	17,638	278,839	1,907	19,361	-	435,628	−82, 156
2	11,694	16,779	-	8,409	172,823	48	12,000	-	270,000	48,295
3	15,525	9,461	11,602	3,300	71,284	-	12,000	881	270,000	145,947
4	16,073	3,668	18,972	7,561	163,339	-	12,000	82	270,000	48,305
5	12,629	-	41,908	8,000	172,823	0	12,000	-	270,000	22,639

NPV in $'000 : 121,928

Table 3.5: Cash flow and NPV calculated from the schedule – excluding in-pit dumping

Year	Waste Mining Cast $'000	Waste Haulage Cost		Coal Mining & Washing Cost			Wash Cost $'000	Rehandling Cost $'000	Revenue $'000	Margin $'000
		Ext $'000	In-pit $'000	Mining $'000	Haulage to plant $'000	Haulage to Stock $'000				
1	36,538	197,734	-	17,638	278,839	1,907	19,361	-	435,628	−114, 483
2	11,694	16,779	-	8,409	172,823	48	12,000	-	270,000	48,295
3	15,525	42,026	-	3,300	71,284	-	12,000	881	270,000	124,984
4	16,073	40,911	-	7,561	163,339	-	12,000	82	270,000	30,035
5	12,629	41,908	-	8,000	172,823	0	12,000	-	270,000	22,639

NPV in $'000 : 64,312

An estimation has been made for a scenario considering only an external dump, where all the waste from the pit is assumed to be dumped out of the pit in the external dump only. This is represented in the third column of Figure 3.10, whereas the second column represents a scenario of using both external and in-pit dumping optimally. The distance of the external dump is more than the internal dump and hence the haulage cost reduces the margins, and thereby the NPV of the project reduces from \$121.9 Million to \$64.3 Million, a reduction of \$57.6 Million (Table 3.5). It is also seen that the total haulage cost over the five years reduces by 27% with optimal internal dumping versus a case with only external dumping. The surface area footprint of the dump was reduced to 47% in the first 5 years with the optimal and early use of the internal dump. Furthermore, the reduction of overall haulage distance will reduce the CO_2 emission of the mining equipment as well.

3.6 Chapter summary

This chapter presents the integration of pit and dump scheduling which includes an in-pit dumping strategy for stratified deposits. The ability to decide the optimal destination of waste blocks into in-pit and external dumps along with the optimal mining sequence can add significant value to mining operations. Not only does it allow maximising the value of the project but it also allows for quicker back-filling of the in-pit dumps and making them available for rehabilitation, thereby decreasing the footprint of external dumps. The chapter also presents how to consider in a mathematical model the lag spacing for mining and in-pit filling with the dynamically changing mining face. This model could be applied to different mines around the world reducing the impact of mining on the environment while not impacting the cash flow. It is possible to enhance the model to give it the shape of user-friendly software in the future. As a future research work, the haulage distances using a network of haul roads would be considered to optimise the mining haul roads. Furthermore, the optimisation model can be established more robust and different solution methodologies can be developed to be able to obtain quick solutions for large scale models.

Chapter 4

Concurrent optimisation of open pit ore and waste movement with optimal haul road selection

Open pit mine production scheduling should include an optimal sequence of material movement to different possible destinations. Depending on the geological structure of the deposit, waste material can be allocated to ex-pit dumping as well as in-pit dumping locations. In-pit dumping is becoming increasingly important in open pit mines, with decreasing availability of land for large external dumps, the need for acceleration of the rehabilitation process, reduction of the footprint of mining activities and cost-saving purposes. The truck cycle time is a factor of the choice of a road from several possible haul road options from a network of roads that could be available between a pair of sources and destinations. A mathematical model has been developed to perform concurrent optimisation of ore and waste rock movement to different available destinations which include in-pit back-filling options. Furthermore, the model considers the selection of optimal haul road paths concurrently directing material to the most suitable destination along a path. The originality of the proposed methodology comes from a unique approach where the shortest path haul road selection and dumping in-pit while maintaining a constant lag distance between the advancing working face and in-pit dump are all considered simultaneously. The model has been implemented on a dataset from a coal mine resulting in a significant demonstrated improvement of 27% in NPV and a 30% increase usage of in-pit dumping that can be achieved, compared to a schedule run in a mine planning software.

4.1 Introduction

Open-pit mines, in particular the stratified deposit mines like limestone, coal, phosphate, and some iron ore deposits are typically spread over large areas with relatively shallow depths and substantial volumes of material being moved from the ground to different destinations such as stockpiles, process plant, and waste dumps. Among the LOM planning considerations, waste management is of particular importance. Waste is hauled to either external dumps - outside the pit or to internal dumps (in-pit) within the post-mining void. Back-filling these voids decreases the land requirement for large external dumps and also enables early return of the mined-out land for rehabilitation. Waste dumps and stockpiles consist of large volumes of material which could also be potentially acid forming at times that could potentially impact the local environment. Furthermore, it is common that the space available for external dumping is restricted. Unless the waste is handled optimally and on time, it has the potential to create a backlog of accumulated re-handling expenses for the future.

Material movements in open-pit mines are normally made using large dump trucks. Truck haulage cost is the highest cost component in a truck-shovel mining operation, in some cases accounting for around 50% of operating costs (Thompson and Visser, 1997). This cost is a direct factor of the haulage distance travelled to haul the materials. There are different possible paths that a mining block can travel along from a source to a destination node. Such paths consist of temporary ramps that connect source node or mining blocks in the pit to main haul roads which in turn connects to the temporary ramps in the dumps, or stockpiles and process plants. Selecting the right path from the source to the destination is of great importance in mine planning.

Mining schedule optimisation models in the past did not consider in-pit dumping with lag distance from the working face and optimised haulage selection. Also, the common practice of estimating haulage costs in a mine optimisation problem involves straight-line Euclidean distances from a source pit block to a pit exit point, then a connecting line with gradient adjustment to the waste dump entry point, and finally another connecting line from the waste dump entry point to the destination dump location. (Das, Topal and Mardenah, 2020), (Das et al., 2022), (Fu et al., 2019) utilizes the straight-line Euclidean distance method described for estimating haulage distances. While this method could be acceptable at a scoping level of mine project study, it will create a bias on cost estimation as the project moves towards feasibility study as well as operational level. This is because the Euclidean distance generally underestimates the actual road travelling distance. Hence it is important to determine a more accurate distance from a source block within a pit to different destinations such as stockpile, plant or dumps by following actual possible paths consisting of permanent haul roads and linking ramps passing through exposed pit blocks and dump blocks.

4.2 Past studies

Many researchers have worked on the mine production scheduling or sequencing problem since the 1960s. Pure integer programming; dynamic programming; Linear programming; mixed integer linear programming ; with exact or heuristic approaches are some of the mathematical formulation approaches that have been considered for the problem over time.

(Johnson, 1968) was one of the first papers in this area that described the mine production scheduling as one that deals with the extraction sequence of mining blocks such that the accumulated discounted cash flow of the project is maximised without compromising on the production or spatial constraints. There has been ongoing significant research in this area resulting in several mathematical models and software made available for optimal planning and scheduling of open pit mining (Johnson, 1968); (Gershon, 1983); (Dagdelen, 1986); (Dowd and Onur, 1993); (Tolwtnski and Underwood, 1996); (Ramazan and Dimitrakopoulos, 2013); (Caccetta and Hill, 2003); (Topal and Ramazan, 2012*b*); (Groeneveld et al., 2019); (Mai et al., 2018). The review of such research has been covered earlier by Osanloo et al. (2008) and Fathollahzadeh, Asad, Mardaneh and Cigla (2021). However, these models only schedule the pit blocks to the destination waste dump but ignore the detailed placement of the waste into individual dump block locations. Scheduling the dump by location could impact one of the most important cost centres in a mining project constituting 50-60% of operating cost, depending on the stripping ratio.

The development in waste dump handling along with ore scheduling started in the early 2000s (Zuckerberg et al., 2007). Following this, there have been several other papers by (Williams et al., 2008) that propose the first publicly available mathematical formulation of a model based on MILP. The developed model optimises the haulage of waste from an open pit mine to a dumping location while minimising the cost as well as causing the lowest impact on the environment. The model integrated mining and dumping along with handling benign and reactive waste in dump layers. (Williams et al., 2008)'s model was further improved by (Li et al., 2013), (Li et al., 2016), which considered multiple waste dump options, different dump strategies as well as optimisation of equipment utilisation for multiple time periods. (Fu et al., 2019) presented a model that simultaneously optimised the pit and dump schedule. The model aims at NPV maximisation of an open pit operation by including cumulative product material values ($/tonne) hauled from the pit to each possible destination including ore to the processing plant, PAF rock to waste dumps, NAF ore to the stockpile or NAF waste to dumps, material re-handled from stockpiles to the processing plant. The model also considers several constraints including reserve constraints, mining equipment capacity constraints, processing plant capacity constraints, required ore grade from blending, maximum available volume of waste dump constraint, precedence of mining and dumping, and stockpile capacity constraints. In addition, a series of grade bins have been considered for blending purposes, which allocates material with grades in particular ranges to relevant grade bins. (Das et al., 2022) worked further on the model and included in-pit dumping mainly for stratified deposits. In-pit dumping involves maintaining a lag distance between the advancing pit and the dump to maintain a working room for equipment. (Badiozamani and Askari-Nasab, 2013) considered the capacity of tailings and the requirement of material for reclamation of tailing ponds in oil sand mines. Oil sands are stratified in nature, and their processing generates waste tailings in large volumes which are sent to tailings ponds.

4.3 Model description

(Das et al., 2022) proposed a model for open-pit mine schedule optimization for stratified deposits, considering in-pit dump scheduling. A constant distance, called the lag distance, is maintained between the advancing pit face and the in-pit dump face leaving safe room for working. The paper presented a mathematical model and applied it to a real data set from a mine. The paper however considered Euclidean distances between a pit block and the exits, as well as between a dump block and the dump entries. The concept of central haul roads and multiple paths feasible options were not considered from pit blocks to dump blocks or stockpiles and plants. The model presented by (Das et al., 2022) has been modified in this thesis to include the selection of optimal haul road design and layout from all the possible path (road) options. The proposed methodology in this thesis utilises the Dijkstra algorithm and Mixed Integer Linear Programming formulation to create all the possible paths and selection of the optimal layout which then can be used in the production scheduling model.

Dutch computer scientist Edsger W. Dijkstra presented the algorithm popularly known as the Dijkstra algorithm to find the shortest path from a single source to a single destination (Dijkstra, 1959). This algorithm is commonly applied to find the shortest path between two locations considering the roads to the destination. Dijkstra's algorithm is a dynamic programming approach for general shortest path problems, where all the edge lengths are non-negative. Given a graph G = (V, E) with a source node s and a sink node t and with edge costs c(e), we want to find the shortest path from s to t. Since all the edge lengths are non-negative, we can immediately find the length of the shortest path to the closest neighbor to s: we go there directly. It must take longer if we go via another node.

Past research on determining mining haul roads has been independent of the mining schedule optimization problem. In this research, we combine the optimal haul road selection with the production schedule optimization into a single MILP based mathematical model. The model enables us to create an optimal production schedule with optimal selection of a path between the mining blocks and the possible destinations like stockpiles, process plants, and in-pit and external dumps. This is an important step that was missing in earlier work, as material was being routed either without considering haulage paths or even if considered they were Euclidean distances between the pit exit or dump entry to the pit or dump block.

The model developed generates possible haulage connections from each block to adjacent blocks or the nearest defined haul road point. All connections from a block to the adjacent block or a haul road point are checked to be within a maximum allowable gradient. If the connection is within the defined gradient, then this connection is considered an edge. The distance of this edge is recorded, and the connection nodes (pit block, road point, or dump block) are recorded in the list of edges. For each combination of pit block, dump block, stockpile, and plant, the Dijkstra algorithm is run using the list of edges. For each combination of source and destination, the shortest path is determined. The path found is further broken into pit blocks along the path, road points along the path, and dump blocks along the path or stockpile/plant as shown in Table 4.1. The process followed is described in Figure 4.1.

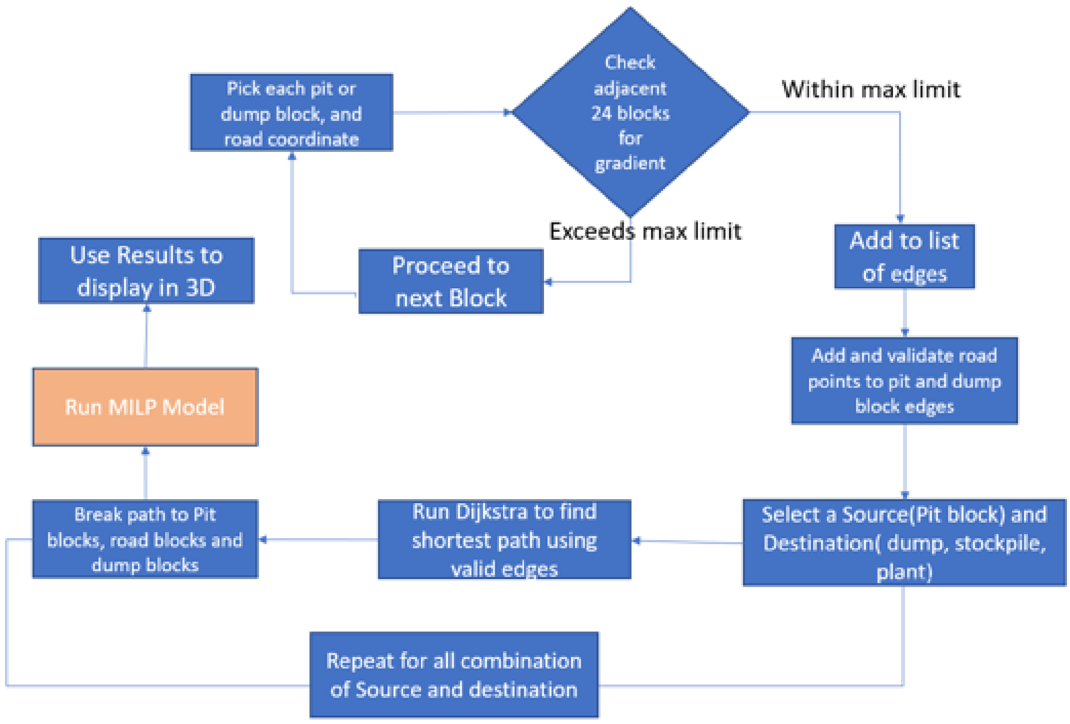


Figure 4.1: Data preparation for the MILP model

In the process described in Figure 4.1 we are testing for all surrounding blocks to a block. A block could be surrounded in 3D by 26 blocks. Out of this there is one block right above the block being considered and one right below. These blocks are excluded from the test as they would obviously result in a steep gradient. Hence only 24 blocks are tested.

Table 4.1 provides a snapshot of the data used for paths connecting the different sources and destinations. The table contains at least one path from each source pit block to each destination dump block, stockpiles, or plants. Each path has a unique ID with the prefix PT. These paths sometimes pass through other pit blocks and connect to a point in the nearest haul road (Road points are denoted with the prefix R). For connecting to a dump destination, they may also travel along some intermediate dump blocks and onto the destination dump block. The distances determined using Dijkstra algorithm are in the last column and are in meters.

Table 4.1: A snapshot of paths data connecting source and destination

Path Id	Source	Dest.	Pit Blocks in path	Road Points in Path	Dump Blocks in Path	Stock/Plant	Distance(m)
PT1	P1	D1	P1	R57,R58,R59,R63,R20	D6,D1	-	404.5913
PT2	P1	D1	P1	R57,R58,R59,R63,R20,R21	D5,D1	-	453.1978
PT3	P10	D1	P10, P12, P14	R59,R63,R20	D6,D1	-	572.9086
PT4	P10	D1	P10,P9	R58,R59,R63,R20	D6,D1	-	587.3566
PT5	P10	D1	P10,12	R58,R59,R63,R20	D6,D1	-	584.7852
PT6	P10	D1	P10,P12,P14	R59,R63,R20	D6,D1	-	572.9086

The model uses a data set which includes mining blocks in the pit, dump blocks in the dumps (internal and external), processing plants, stockpiles, set of all possible paths between sources and destinations for a given period. Several parameters are considered such as financial ones including the selling price of the ore, mining cost of ore and waste, processing cost of ore and waste, haulage cost of ore and waste, re-handling cost from the stockpile to plant, and discount rate, capacity limits i.e., limits of stockpiles, maximum and minimum mining, and processing capacities, block properties such as wash plant recovery of each block, grade of each block, waste volume, ore tonnes and total volume of each block, the density of each pit block, swell factor, volume capacity available in each dump block and haulage distance of each path.

Continuous linear variables have been considered for the volume of waste hauled from each block in a period and to a destination dump block, similarly, ore tonnes hauled from each pit block to the stockpile or processing plant in a period, and ore fed from stock pile to process plant in a period. Another continuous variable is used to represent the volume of ore or waste being hauled in a particular period through a particular path. Binary integer variables represent whether a mining block is being mined in a period and whether a dump block is being dumped in a period. Another binary variable is used to check if all layers above a certain layer of ore/waste in a block have been mined.

The objective of the MILP model is to maximise NPV of the project and report the blocks being mined by period, along with their destination and the path used to reach the destination. CPLEX OPL has been used for solving the MILP.

The NPV of the project is a factor of the discounted cost of ore being processed and the waste being dumped in each period and the discounted revenue generated from the processed ore in each period. The common constraints of capacity, grade limitations, and stockpile limitations in most Mixed Integer Linear Programing models have been discussed in several papers (Das et al., 2020) (Fu et al., 2019), hence they are not being discussed here. (Das et al., 2020) proposed a lag constraint for maintaining a lag distance between the advancing face of the pit and the dump, so as not to dump on unmined areas. Some additional constraints have been developed in the model pertaining to selecting the optimal haulage route from an exhaustive set of possible roads generated by the process described in Figure 4.1.

For selecting the optimal haulage route, the tonnages from a block are distributed to the corresponding possible destinations connected by a path. The total tonnage travelling from a block using different paths cannot exceed the tonnage of the block itself. Material mined in a period should be moved by any of the roads available in the period. The total material flowing through all paths should be the same as the total quantity of all material that has been moved out of the pit. Blocks forming part of a path need to provide a surface. Such a surface could be either a mined-out pit surface at any point in time or the original mine surface in the pit side. Similarly, on the dump side, it must be either a surface created by filling dump blocks or the mined-out base surface of the pit. Therefore, it is necessary to see that each set of pit blocks within a selected path, or each set of dump blocks within a selected path are on a surface.

4.3.1 Model implementation

The model was demonstrated on data where the pit was pre-designed with economic mining limits determined using the LG algorithm (Lerchs, 1965). The dataset is from a mined-out pit in Hunter Valley, New South Wales, Australia. It is a part of the entire project and only one pit has been considered here with nine benches, one external dump, and one internal dump. The data has 118 pit blocks of average size 120meters X 250 meters which cover an area of 1240m X 740m as shown in Figure 4.2. The block sizes are relatively large in X and Y directions compared to Z, which is typical for stratified deposits.//

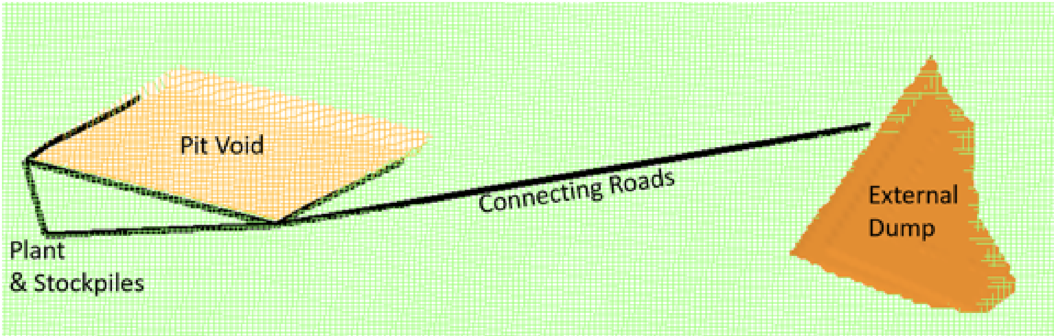


Figure 4.2: Pit void, roads, and external dump by a hillside

Three stockpiles have been considered for the coal, based on the coal quality. Coal can be either directly fed to the wash plant or stocked for re-handling and feeding the plant in the future. The parameters used for this dataset are described in Table 4.2.

Table 4.2: Input parameters for the optimisation model for coal deposit

Input parameter	Variable name	Maximum	Minimum	Unit
Wash Plant capacity	$\bar{P}_{\mathrm{mt}}$ and $\underline{P}_{\mathrm{mt}}$	2,750,000	1,500,000	Tonnes/Yr
Mining capacity	$\bar{M}_{\mathrm{t}}$ and $\underline{M}_t$	14,250,000	10,000,000	Cubic m / Yr
Specific energy	g_b	-	25	MJ/Kg
Coal sale price	p_t	-	\$99.90	\$ per t
Coal mining cost	c_{t}	-	\$4.00	\$ per t
Waste mining cost	w_1	-	\$3.00	\$ per BCM
Waste haulage	h_{t}	-	\$0.001	\$ per BCM meters
Washing cost	f_t	-	\$6.00	\$ per t ROM feed
Rehandling cost	j_s	-	\$0.50	\$ per t ROM feed
Swell factor	SW	-	1.25	Volume to BCM ratio
Discount Rate	a	-	10	%
Recovery/Yield	r_{b}	-	90	%

Note: the \$ refers to teh Australian dollar.

As can be seen from Figure 4.2 or Figure 4.5, the waste from the pit can either be hauled to the external dump or can be dumped in the pit void through the connecting roads. A lag distance of 160m is considered between the in-pit dump face and the pit face as both progress over time. The dumping destinations are selected by the model to maximise the NPV. The NPV has a reverse relationship with the haulage distance or hauling cost, hence the model tries to choose the path with the least cost. The objective here is to determine the best sequence to extract the pit blocks and dump the material into either destination dump blocks, stockpile, or feed to plant, and select the best haulage option while respecting the constraints of capacity and grade to maximise the Net Present Value of the operations.

The footprint of the pit at the lowest bench matches the footprint of the dump at the lowest bench by maintaining the same orientation of the strips for the pit and the internal dump at the pit floor. There are 104 blocks in the internal dump in two benches, each block having an individual capacity and coordinate. One external dump area has been considered, in this case, as a single block with an equivalent capacity has been considered, as this dump is not affected by lag distance. Some of the pit blocks could contain multiple layers of coal and waste in them, while others could be only coal or only waste. No cut-off coal quality has been used as coal and waste in a coal mine are pre-determined at the geological modelling stage. The lowest bench is mostly coal and has two consecutive coal seams. The maximum life of the project considered is five years.

Both pits and dumps have been pre-designed and divided into benches, strips, and blocks, considering the pit face angle for block walls – commonly referred to as block solids. The dump blocks have an angle of repose of 37-degree slope considered. A swell factor of 1.25 has been considered on the bulk waste for determining dump volume requirements.

There are several data tables considered below:

- for the pit blocks with details of bench, strips, blocks, quantity, and quality, coordinates,
- dump blocks with bench, strip, block, volume, and coordinates,
- and a table of paths connecting all sources with all destinations along with distance.

Since the pit has been divided into benches, strips, and blocks, the size, and shape of the pit blocks are not uniform – this is a common practice in stratified deposits with strip – block design. The pit blocks have been designed to maintain the wall slope angle. The bench height for the benches close to the seam is often less and is largely a function of the seam geometry (dip angle). Figure 4.3 presents the pit blocks mined over five years, coloured by year. It can be seen that the mining sequence proceeds gradually strip-by-strip making room for in-pit dumping as early as possible. Figure 4.4 shows the progressive mining and back-filling of the in-pit dump over the five periods.

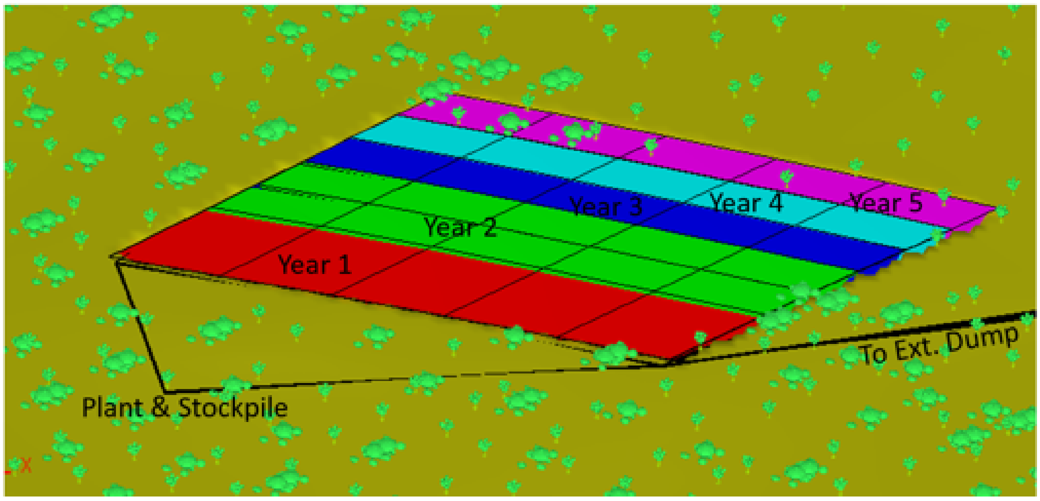


Figure 4.3: View of pit coloured by periods at the top bench

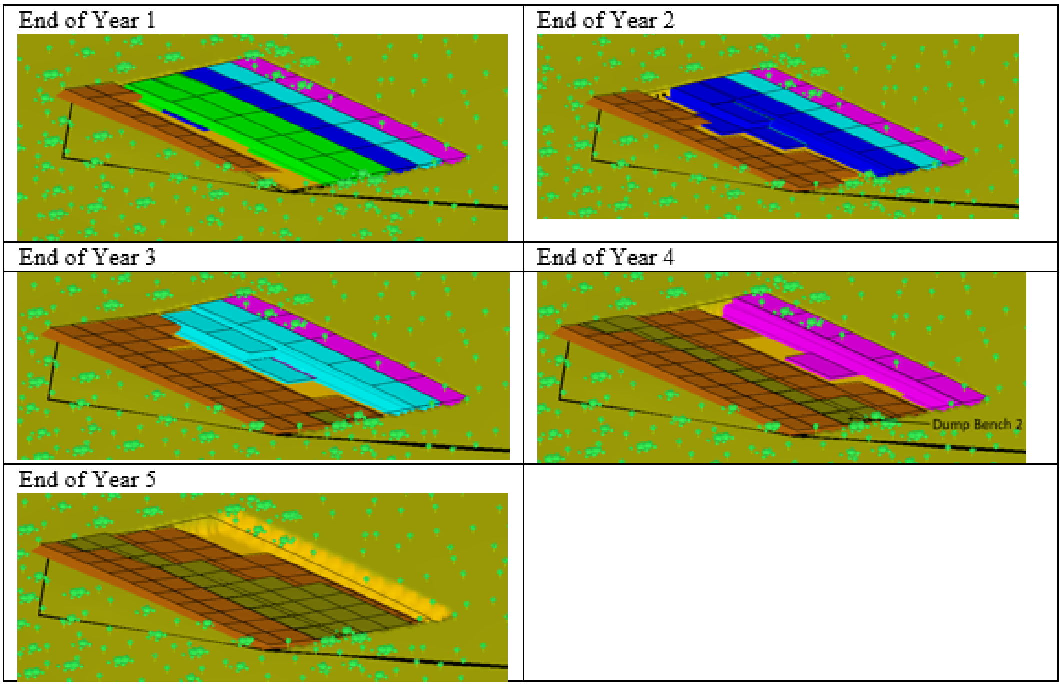


Figure 4.4: Progressive pit and internal dump at the end of each year

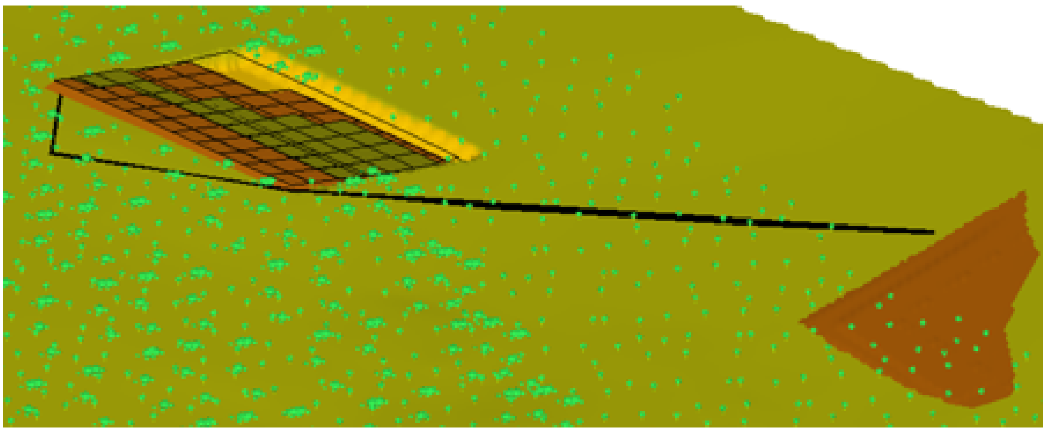


Figure 4.5: Final shape at the end of five years with external and internal dump

Table 4.3 and Table 4.4 present the comparison of coal and waste volume movement based on the proposed model as well as a commercial software available to the industry to use. As can be seen from the tables, the proposed model utilises the in-pit dumping option significantly as compared to software-based schedule within the five years – 32.3 million cubic meters vs 20.5 million cubic meters out of a total waste of 38.7 million cubic meters, which is a 30% increase in in-pit dumping using the model. This will provide a significant reduction of the mining footprint of external dumps as well as much quicker rehabilitation for post-mining voids without incurring extra re-handling expenses.

Table 4.3: Volumes of coal and waste moved in each period using the model

Year	Waste Volume(cu.m)	Destination Dump		Coal to Plant from Mine (Tons)
		External (cu.m)	In-pit (cu.m)	
1	6,463,755	1,497,850	4,965,905	2,750,000
2	8,283,831	2,463,846	5,819,985	2,672,830
3	6,774,701	416,720	6,357,981	2,499,990
4	9,297,463	2,071,950	7,225,513	2,500,001
5	7,913,964	-	7,913,964	2,499,990
Total	38,733,713	6,450,366	32,283,347	12,922,811

Table 4.4: Volumes of Coal and Waste moved in each period using a scheduling software

Year	Waste Volume(cu.m)	Destination Dump		Coal to Plant from Mine (Tons)
		External(cu.m)	In-pit(cu.m)	
1	6,463,773	6,980,872	-	2,750,120
2	10,309,500	11,134,264	-	2,750,225
3	6,648,765	340,703	6,839,965	2,750,010
4	7,397,682	1,277,956	6,711,546	2,750,001
5	7,913,964	1,593,921	6,953,161	1,922,484
Total	38,733,684	21,327,716	20,504,672	12,922,840

Furthermore, the proposed model provides significant NPV improvement for the project at $614 million as compared to $485 million for the software-based schedule. This is mainly due to the software schedule doing a late start of internal dumping only in the 3rd year. In addition, there is a cost involved in rehabilitating

the external dump which has not been considered in the NPV estimation.
The same data and assumptions were used in a mine scheduling software for scheduling the project and estimating the haulage distances.

4.4 Chapter summary

This chapter presents an integration of pit and dump scheduling with a focus on stratified deposits and includes an in-pit dumping strategy while considering the selection of haulage options from available shortest paths. The sequence of mining plays an important role in maximising the value of a mining operation. Haulage costs are a significant part of truck-shovel operations, hence the ability to decide the optimal destination of waste into the right dumps can add significant value to mining operations. Effective utilisation of in-pit dumps allows for quicker back-filling of the mining voids making them available quicker for rehabilitation, thereby decreasing the footprint of mining and external dumps as early as possible in the life of the project. A mathematical model has also been described in the chapter which includes maintaining a lag distance between the dynamically advancing mining face and in-pit fill dumping face, while simultaneously finding the optimal haulage road and maximising the project NPV. This model is applicable to different mining projects around the world with the potential to reduce the impact of mining on the environment without compromising on the cash flow. The model has been successfully applied to a coal mine dataset to get early and increased in-pit dumping to be able to do progressive rehabilitation earlier in the life of the project, as well as have an improved NPV by 27% compared to a schedule done in a mine planning software using similar parameters. It is possible to enhance the model to give it the shape of user-friendly software in the future. The optimisation model can be further enhanced and established for more robust and swifter solutions using different solution methodologies for large-scale models.

Chapter 5

A new methodology for concurrent optimisation of production schedule with haul road design and new meta-heuristic solution method

Optimal open pit ore and waste scheduling with a network of haulage options create a large combinatorial problem. In this chapter, a mathematical model has been proposed to simultaneously optimise pit and waste dump schedules with the shortest haulage selection from possible haul-road networks. The model also determines the optimal quantity of material to be sent through the shortest haulage path for all combinations of sources and destinations. This is a new approach of simultaneous optimization as earlier studies had optimized pit, dumps and haulage separately missing their combined impact on optimality. The new model has been implemented on several case studies detailed in the chapter. A comparison has been performed for a case study which revealed 39% savings in haulage distances and 44% savings on back-filling to in-pit dumps over a schedule using a mine planning software. Furthermore, different case studies have been solved using an exact as well as a meta-heuristic method developed for the purpose and the results of both methods match within close limits and whereas the meta-heuristics showed a considerable improvement in solution time as data size increased.

5.1 Introduction

Open-pit mines, in particular the stratified deposits, can be spread over several hectares where large volumes of material are moved from the ground to suitable destinations such as process streams, stockpiles, and waste dumps. Production scheduling in general involves creating a three-dimensional model of the ore body which describes the estimated physical and chemical properties of the ore. Boreholes or surface samples help in the estimation of the geo-chemical and economic parameters of each block in the model. Mine scheduling involves finding the location of the blocks of ore and waste to be removed at a certain time to maximize the Net NPV of the project, considering several constraints like the production capacity and process plant capacity or despatch capacity, ore grade blending, block precedence among others (Groeneveld, Topal and Leenders, 2012). This also involves moving the waste to particular dump locations and the ore to the different process locations along haul roads. This approach creates a large number of possibilities creating a large combinatorial problem. The mathematical formulations of such large combinatorial problems are large-scale instances of Integer Programming.

In open-pit mines, there could be various paths that a mining block can travel along from a source node, which is a block within a pit to a destination node, which can be a particular dump location, stockpiles or process plants. The path could consist of certain temporary ramps which connect with the mining blocks in the pit (pit-blocks), or blocks in the dump (dump-blocks). The haul roads connect these temporary ramps to destinations like dumps, stockpiles, or process plants.

The material movements on these haul roads are normally accomplished by using large mining trucks. Truck haulage is the highest cost component in a mining operation, in some cases accounting for up to half of the cost of mining operations (Thompson, 2000) (Fu, Topal and Erten, 2014). This cost is a direct factor of the haulage distance travelled to haul the materials. Higher road gradients and long haulage distances result in increased cycle time and decreased haulage equipment availability (Richardson and Nicholls, 2011). As haul road length significantly impacts mine operating cost, reduction of haulage distance in the initial years of the mining project would lead to a higher NPV. Therefore, along with optimizing a mining and dumping schedule, it is necessary to optimize the haulage distance throughout the life of a mine plan to maximise returns from a mining project. Open pit mine schedule optimisation problems have been dealt with in several research works in the past mainly focusing on the optimal scheduling of ore extraction (Johnson, 1968); (Gershon, 1983); (Dagdelen, 1986); (Dowd and Onur, 1993); (Tolwtnski and Underwood, 1996); (Ramazan and Dimitrakopoulos, 2013); (Caccetta and Hill, 2003); (Topal and Ramazan, 2012*a*); (Groeneveld et al., 2019); (Mai et al., 2018).

A review of the past research on production schedule optimisation approaches has been presented in Osanloo et al. (2008), further recent approaches including waste dump planning has been included in Das, Topal and Mardaneh (2024).

Williams et al. (2008), for the first time included haulage cost waste rock blocks in the pit for placing in waste dumps. This also included selective dumping of benign and reactive waste. Li et al. (2013), Li et al. (2014), Li et al.

(2016) extended this research further to maximize equipment utilization and include haulage costs for multiple periods and dump sequence profiles for multiple pits and waste dump options. These works however included dumping sequence optimisation based on an existing optimal ore schedule and did not include simultaneous ore and waste production scheduling. They also considered Euclidean distances for the haul roads which were predetermined. A more recent study by (Fu et al., 2019) considered simultaneous mine and dump scheduling and included formulating the problem as a Mixed Integer Problem model. This is the first attempt in simultaneous optimization of mine and dump scheduling which was so far being considered separately. However, it still considers Euclidean distances as haulage distances from pit or dump locations with fixed entry/exit points over a predefined haul road distances between the pit and dump.

Das et al. (2022) proposed a model for open-pit mine schedule Optimisation for stratified deposits, considering in-pit dump strategy along with external dumping options. A distance is maintained between the in-pit dump face and the advancing pit face, termed as the lag distance. The paper presented a mathematical model and applied it to data from a mining project. This paper considered 3D mining blocks in the pit as well as in the dump. The mining blocks have pre-designed batter slopes and shapes. Such blocks from the designed pit are referred to as "pit blocks" and those from the dumps are referred to as "dump blocks". The paper however considered Euclidean distances between a pit block and the exits, as well as between a dump block and the dump entries, along with predefined surface haulage distances. The concept of central haul roads and multiple feasible paths were not considered from pit blocks to dump blocks or stockpiles and plants. Production schedule optimisation models in the past did not consider concurrent optimised haulage road selection.

Das et al. (2024) have made the initial attempt on the research in this area to include the selection of optimal haul road design and layout from all the possible path (road) options. The original contribution of this paper includes the concurrent optimisation model for multiple time periods, multiple destinations for ore and waste including internal and external dump options with optimised haul road selections. The proposed methodology in this paper briefly describes the use of the Dijkstra algorithm to create all the possible paths which are then used in the schedule optimization MIP model.

Accurate haul roads give rise to a huge number of combinatorial road options between all source nodes (pit blocks), dump nodes (dump blocks), stockpiles, and process plants. Solving such large optimisation problems using exact methods becomes computationally expensive or impossible with larger problem sizes as demonstrated with multiple cases in Section 5.2. Hence, this thesis further proposes a new alternative meta-heuristic approach using weighted topological sort and simulated annealing methods to obtain a feasible solution for the large-scale models. SA has been used successfully in the past for various problems in mining, (Kumral and Dowd, 2005), (Goodfellow and Dimitrakopoulos, 2013), (Danish, Khan, Muhammad, Ahmad and Salman, 2021), while its introduction in the current context of pit scheduling, dump scheduling and haul road selection is new. SA involves searching the solution space with random feasible solutions, which in this case interprets to random sequences of mining and dumping. The random feasible solutions have been created using the Toposort algorithm, which is also a new application in this area. Comparative real case studies have been undertaken between exact and meta-heuristic approaches to obtain solutions for the concurrent optimisation model. The rest of the chapter is structured as follows. Section 5.2 explains the method used to create the shortest haulage path between source and destination nodes. Section 5.3 describes the proposed mathematical model, Section 5.4 illustrates the meta-heuristic approaches to solve the problem, 5.4.1 details the use of early start method to eliminate some variables and decrease the problem size. Section 5.4.2 discusses the weighted topological sort algorithm to find random sequences of mining and dumping and Section 5.4.3 presents the simulated annealing method. Section 5.5 implements the proposed methodology into real case studies and presents the results obtained using both exact and meta-heuristic approaches.

5.2 Finding the shortest haulage path for open pit mines

Dutch computer scientist Edsger Dijkstra presented the algorithm popularly known as the Dijkstra algorithm to find the shortest path from a single source to a single destination (Dijkstra, 1959). Dijkstra's algorithm is a dynamic programming approach for finding the shortest path, where the lengths of edges are non-negative. In the past, the Dijkstra's algorithm has been utilized by a few researchers in mining. Souza, Câmara, Torres, Nader and Galery (2019) applied the Dijkstra algorithm to determine optimized paths for mining blocks to different destinations. The pit blocks are assumed as nodes of the tree for the graph. A non-operational route was generated by the Dijkstra algorithm, which was solved using non-parametric equations. A block model for transportation was developed, assuming a single destination for a block deeper region of the pit had higher Euclidean distance and transport time, hence it is important to identify potentially high transport cost areas for the pit advance and correctly quantifying values to help in efficient mine planning.

Zhang, Tian, Yang, Yang and Yan (2015) worked on optimal path analysis on roadway network for a network of underground mining tunnels, considering the Dijkstra as the classic algorithm and developed their own search algorithm for two source optimal path analysis and single source two destination path analysis. A graph of the road network was built consisting of nodes, arcs, sites, weight, etc.

Optimal paths with minimum costs were determined between source and destination considering situations where certain nodes in a path must be included and certain nodes should be avoided.

Yarmuch Guzmán et al. (2020) attempted to minimize ramp construction and operation costs by designing an optimal open pit haulage ramp between two points of a mine, honouring constraints like a ramp width and a maximum ramp gradient, and curvature. Since ramps within a pit require removal of a lot of waste material (stripping), they suggested two approaches: one for high stripping (or in-pit) ramp design and the other for low stripping (or ex-pit) road design. For the stripping ramp design problem, they presented an integer programming model; in the low stripping ramp design problem, they presented a shortest-path approach. The proposed formulation was tested on data from a mine, resulting in significant improvement in cost reduction.

All the above work on haul roads has been independent of the mine production schedule optimisation problem. In this thesis, an attempt has been made to combine the optimal haul road selection with the production schedule optimisation which includes scheduling of ore for different processing destinations as well as internal and external dumping of the waste into a single MIP-based mathematical model. The model creates an optimal production schedule with optimal selection of haul road path between the source and destination of any material flow from the open pit mining operation. This is a significant step that was missing in earlier studies, as the material was being routed to its destination either without considering practical haulage distances, or even if considered they were Euclidean distances between an exit to a block in the pit or dump. The difference in concept is evident from the schematic diagram in Figure 5.1 below showing Euclidean distances vs. Connector distances to a permanent haul road from different pit locations.

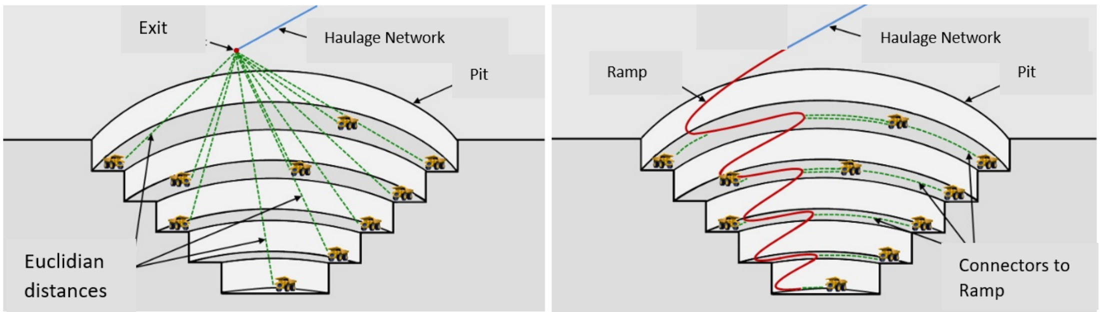


Figure 5.1: Conceptual representation of Euclidean distances (left) and detail haul road distances (right)

5.3 Mathematical model for pit-dump schedule and haulage optimisation

In order to implement the mathematical model, a database of haulage options for all blocks in the pit and to all destinations are created and made available to the MILP mathematical model to choose from. The process followed is described below in Algorithm 1.

Algorithm 1: Pseudo code to create a database of road options to use in MILP

Algorithm 1: Determine a database of roads for use in MILP	
1.	Input: Coordinates for pit blocks, dump blocks, permanent haul road points, plant and stockpiles, Max Gradient
2.	Output: Database of path options for all sources to all destinations
3.	For each pit block or dump block
4	Edge-Create connector to adjacent 24 blocks (exclude block above and below)
5	If the gradient of the connector > max gradient
6	Exclude edge
7	Else
8	Add edge to list of Edges
9	Next pit block or dump block
10	For Each Road Point
11	Create connectors to adjacent pit/dump blocks process plants and stockpiles
12	Exclude edge if it exceeds max gradient
13	Add to the list of edges
14	Next road point
15	For each pit block to each destination (dump block, stockpiles, plants)
16	Run the Dijkstra algorithm (use the list of edges created earlier as input)
17	Add the output path generated between the source and destination to the list of paths(database)
18	Next pit block

Since the paths contain connections through pit blocks and dump blocks, few alternative paths are also generated by considering scenarios like what if one of the pit blocks in the path were to be already mined out, or what if one of the dump blocks in the path is yet to be filled. Therefore between a source and a destination more than one possible path option is generated to give flexibility to the optimisation model. Each path has been subdivided into attributes which include: path id, source, destination, pit block numbers on the path, permanent haul road point numbers in the path, dump block numbers on the path (for destination dump), and a total distance of the path in metres.

Figure 5.2 shows some of the paths generated using the Algorithm 1, for a steeply dipping phosphate mine data located in Queensland, Australia. The mine has one pit and two dumps (one external and one internal). A main haul road passing between the pit and the dump connects the process plant and stock piles located in the north. The case has 267 pit blocks and 79 external dump blocks. There were 87 points in the main haul road. Algorithm 1 was used to generate 36,711 road options, out of which 619 paths were used in the solution. As an example, pit blocks P170 connected to dump block D61 (white) and pit block P10 connected to dump block D61 and stockpile S1(blue) mine have been shown in Figure 5.2 . It needs to be mentioned that in order to maintain the maximum gradient limit of 10% a small part of the road to the stockpile passes over the waste dump.

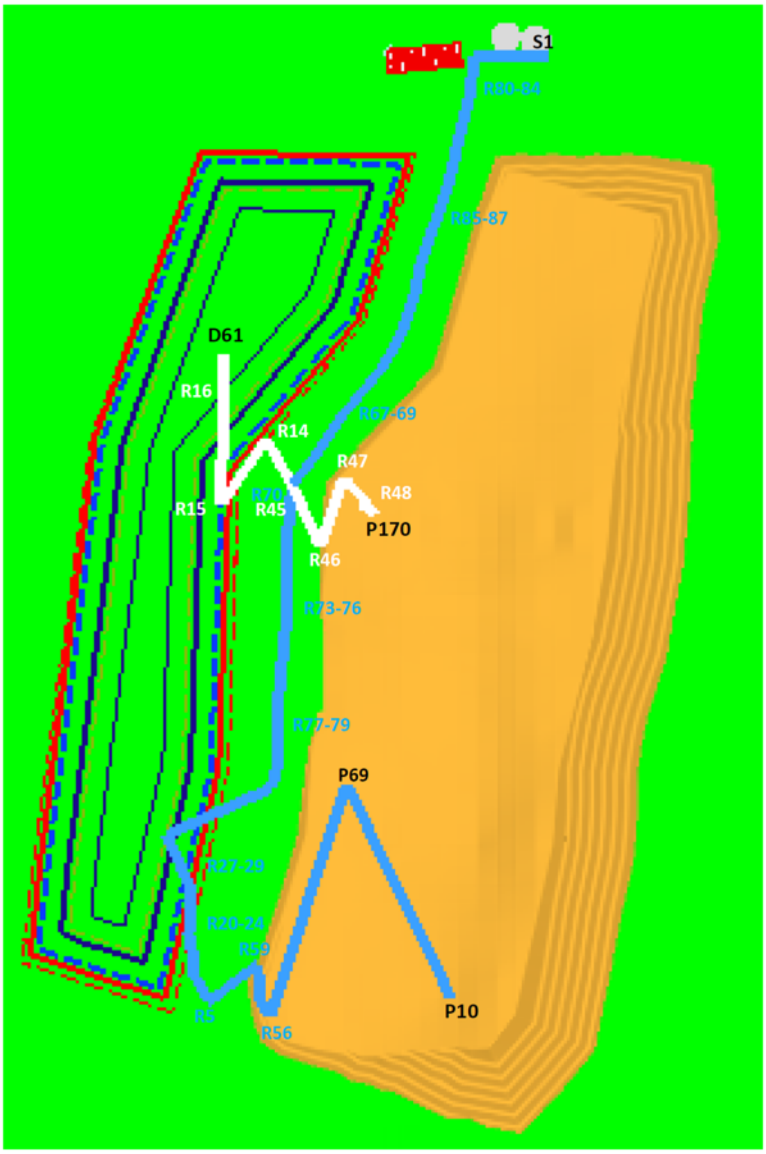


Figure 5.2: Two paths shown as an example from the 619 paths used in the schedule

Table 5.1: Sample part of a database of paths between source and destinations

Path Id	Source	Dest	Pit Blocks on Path	Road points on path	Dump blocks on path	Distance(m)
PT1	PI	D1	P1	R57,R58,R59,R63,R20	D6,D1	404.5913
PT2	P1	D1	P1	R57,R58,R59,R63,R20,R21	D5,D1	453.1978
PT3	P10	D1	P10, P12, P14	R59,R63,R20	D6,D1	572.9086
PT4	P10	D1	P10,P9	R58,R59,R63,R20	D6,D1	587.3566
PT5	P10	D1	P10,P12	R58,R59,R63,R20	D6,D1	584.7852
PT6	P10	D1	P10,P12,P14	R59,R63,R20	D6,D1	572.9086
...	...	...	...	...	...	
PT26041	P170	D61	P170	R48,R47,R46,R45,R14,R15,R16	D61	647.8979
...	...	...	...	...	...	
PT35680	P10	S1	P10,P61	R56-59,R20-24,R27,R29,R77-79,R73-76,R70-72,R67-69,R85-87,R80-84		2459.274

Table 5.1 shows a sample list of paths from various sources to destinations. The paths PT26041 and PT35680 in Table 5.1 have been shown in Figure 5.2 . The list of paths is divided into three types based on their destination – dumps, stockpiles or process plants, as follows :

- one set connecting from pit blocks b to dump blocks $d\,(P_{bd})$,
- next from pit blocks b to stockpiles s (P_{bs}),
- and a third from pit blocks b to process plants $m\,(P_{bm})$.

These paths connecting all sources to all destinations along shortest paths have been used in the MILP described below. The MILP selects the destinations for a source block to be mined in a certain period t, and using one of the paths.

The MILP model is described below:

Sets

T = Set of periods in the scheduling horizon

B = Set of all mining blocks, $b \in B$

B_b = Set of mining blocks to be mined to mine block b; $B_b \in B$

D = Set of dump blocks, $d \in D$

D_d = Set of dump blocks to be filled to dump on block d;

$D_d \in D$ M = Set of processing plants, m $\in$ M

$S =$ Set of stockpiles, $s \in S$

$E =$ Set of all path IDs, $e \in E$

$P_b = P_{bm} \cup P_{bs} \cup P_{bd}$ set of all possible paths from b

As seen in Table 5.1 each path in P_b has the following attributes:

- Pit blocks,
- road points,
- destination dump blocks,
- or destination: - Stockpiles or Plants
- Total Haulage distance of path $p = Z_p$

Parameters

p_t Selling price of ore (or coal) per tonne in period t

c_t Cost per tonne ore (or coal) mining cost in period t

w_t Per cubic meter waste mining cost in period t

h_t Ore (or coal) and waste haulage cost in time period t for every meter hauled

f_t Ore (or coal) processing cost per tonne feed in period t

C_{st} Minimum process capacity of plant m in period t

$\bar{cst}$ Maximum capacity of stockpiles in period t

$\underline{cst}$ Minimum capacity of stockpiles in period t

$\bar{M}_t$ Maximum mining capacity in period t

$\underline{M}_t$ Minimum mining capacity at period t

$\bar{P}_{mt}$ Maximum process capacity of plant m in period t

P_{mt} Minimum process capacity of plant m in period t

G_{mt} Minimum grade of ore (or coal) fed to process plant

m in period t

r_b Wash plant recovery (yield %) of block b

g_b Grade of mining block b

v_b Waste volume of block b

o_b ore (or coal) tonnes of block b

t_b Total volume of block b

n_b Density of block b in g/cc

g_s Average grade of ore (or coal) in stockpiles

j_s Re-handling cost of ore (or coal) from stockpiles

u_d Volume of dump block d

a Discount rate %

r_b Wash plant recovery (yield %) of block *b*

sw Swell factor - the ratio of loose cubic meter to bank cubic meter

dc Total dump capacity including external and in-pit dumps

Z_p Haulage distance of Path p

C_{pt} The haulage cost per tonne for each path p in time period t is calculated as:

$C_{pt} = h_t.Z_p$

b$*$ Pit blocks that are part of the unique list of source-destination combination

bb Pit blocks below any pit block that are in a path, (P_{bm}, P_{bs}, P_{bd})

dd Dump blocks below a dump block (P_{bd}) along a path

K_{bmt} Discounted value of mining and hauling ore (or coal) from block b to plant m in period t (per tonne)

K_{bdt} Discounted value of mining and hauling unit waste volume from block b to dump block d in period t

K_{bst} Discounted value of mining and hauling unit ore (or coal) tonne from block b to stockpile s in period t

K_{smt} Discounted value of hauling unit ore (or coal) tonne from stockpile s to plant m in period t (per tonne)

K_{bdt} Discounted value of mining and hauling unit waste volume from block b to dump block d in period t

K_{bst} Discounted value of mining and hauling unit ore (or coal) tonne from block b to stockpile s in period t

K_{smt} Discounted value of hauling unit ore (or coal) tonne from stockpile s to plant m in period *t*

$$K_{bmt} = (p_t \cdot g_b \cdot r_b - c_t - f_t) \cdot \left(\frac{1}{(1+\alpha)^t} \right) \forall b \in B, m \in M, t \in T \tag{5.1}$$

$$K_{bdt} = w_t \left(\frac{1}{(1+a)^t} \right) \quad \forall b \in B, d \in D, t \in T \tag{5.2}$$

$$K_{bst} = -c_t \cdot \left(\frac{1}{(1+a)^t} \right) \quad \forall b \in B, s \in S, t \in T \tag{5.3}$$

$$K_{smt} = (p_t \cdot g_s \cdot j_b - f_t) \cdot \left(\frac{1}{(1+a)^t} \right) \quad \forall m \in M, s \in S, t \in T \tag{5.4}$$

Continuous variables

X_{bmt} Continuous variable, which represents the amount of ore (coal) in tonnes mined from block b, to processing plant m in period t; $X_{bmt} >= 0$

X_{bdt} Continuous variable, which represents the amount of waste in cubic meters mined from block *b*, to dump location *d* in period *t*; $X_{bdt} >= 0$

X_{bst} Continuous variable, which represents the amount of ore (coal) in tonnes mined from block b, to stockpile s, in period t; $X_{bist} >= 0$

X_{smt} Continuous variable, which represents the amount of ore (coal) in tonnes hauled from stockpile *s* to processing plant m in period t; $X_{smt} >= 0$

λ_{pt} Continuous variable, which represents the amount of material (ore/coal and waste) in tonnes hauled through path p, in period t; $\lambda_{pt} >= 0$

The objective function is the cumulative discounted cash flow of the project achieved from operating according to the mining schedule, and is defined as follows:

$$MaximiseNPV = \sum_{b=1}^{B}\sum_{m=1}^{M}\sum_{t=1}^{T} K_{bmt}X_{bmt} + \sum_{b=1}^{B}\sum_{d=1}^{D}\sum_{t=1}^{T} K_{bdt}X_{bdt} + \sum_{b=1}^{B}\sum_{s=1}^{S}\sum_{t=1}^{T} K_{bst}X_{bst}$$
$$+ \sum_{s=1}^{S}\sum_{m=1}^{M}\sum_{t=1}^{T} K_{smt}X_{smt} - \sum_{t=1}^{T}\sum_{p\in P_b}^{P_b} \mathrm{C_{pt}}\,\lambda_{pt}\frac{1}{(1+a)^t} \tag{5.5}$$

The objective, represented by Equation 5.5, is to maximise the NPV subject to the constraints described in Equation 5.6 to 5.9. NPV maximisation is a factor of reduced cost. Among the costs, the haulage cost is a major component and given an option, the model selects the least cost path.

Other than the haulage cost, this model also respects constraints which are common to other models in the past and include those of mining and processing capacity, blend grade, and available dumping space. The other constraints are common to most MILP models that have been discussed in previous papers (Fu et al., 2019), (Das et al., 2020) (Das et al., 2022)., hence they are not repeated here.

Further, the following constraints have been added to the MILP model for selecting the optimal haulage route. The tonnages from a block are distributed to possible destinations connected by a feasible path from the list of paths already pre-determined. The total tonnage travelling from a block using different paths cannot exceed the tonnage of the block itself, as shown in constraints (Equation 5.6 to 5.8) below.

$$X_{b^*dt} >= \sum_{p \in P_{bd}}^{P_b} \lambda_{pt} \quad \forall b^* \in B, t \in T, d \in D \tag{5.6}$$

Where b^* is a part of the unique set of source-destination. Similarly in the following equations for destination plant(m) or stockpile (s)

$$X_{b^*mt} = \sum_{p \in P_{bm}}^{P_b} \lambda_{pt} \quad \forall b^* \in B, t \in T \tag{5.7}$$

$$X_{b^*st} = \sum_{p \in P_{bs}}^{P_b} \lambda_{pt} \quad \forall b^* \in B, t \in T \tag{5.8}$$

$$\sum_{e \in E}^{E} \sum_{t'=1}^{t} \lambda_{et'} = \sum_{b \in B} \sum_{d \in D} \sum_{t=1}^{t} X_{bdt'} + \sum_{b \in B} \sum_{m \in M} \sum_{t=1}^{t} X_{bmt'} + \sum_{b \in B} \sum_{s \in S} \sum^{t} X_{bst'} \quad \forall t \in T \tag{5.9}$$

All material mined in a period should be moved by any of the roads in a period. The total material flowing through all paths should be the same as the total of all material that has been moved out of the pit, as shown in Equation 5.9.

5.4 The meta-heuristic approach to solve the problem

The meta-heuristic approach used variable elimination, weighted topological sorting and simulated annealing as described below.

5.4.1 Variable elimination using early starts

In the model, Y_{bt} is a binary variable, which represents whether all overlying blocks of block b have been mined out by the end of period t. Depending on the mining capacity, we can predetermine blocks that can never be uncovered within a period t for a certain mining capacity C. All such Y_{bt} can be assumed to have a preassigned value of 0 , thereby removing them from the problem. There have been several works done on early start to reduce the number of variables in mine scheduling problems notable among them are (Topal, 2004), (Topal, 2008), (Gaupp, 2008), (Amaya, Espinoza, Goycoolea, Moreno, Prevost and Rubio, 2009), (Chicoisne, Espinoza, Goycoolea, Moreno and Rubio, 2012), (Lambert and Newman, 2014). For a block b to be extracted in period t, all its predecessor blocks b' have to be removed, where t is the total tons and o the ore tons of the block b'. The earliest possible time this can be done is using the maximum mining capacity C.

Earliest time to mine
$(\mathrm{ES_b}) = \sum t/\mathrm{C}$

Hence block b may not be extracted before $\mathrm{ES_{b.}}$ If we also consider the process plant maximum capacity M, then the ore contained in the predecessor blocks b' can change the earliest start time to ES'_b. This revised early start time ES'_b is estimated without considering stockpiling.

Early start time to process
$(\mathrm{ES}'_\mathrm{b}) = \sum o/\mathrm{M}$

Enhanced Early start time = minimum $(\mathrm{ES_b}, \mathrm{ES}'_\mathrm{b})$

The above Enhanced early start method has been applied while solving the mathematical model.

5.4.2 Topological sorting

Topological sorting of a directed graph is linear ordering of the vertices of the graph so that for every directed edge uv from vertex u to vertex v, u precedes v. For example, the vertices of the graph may represent pit blocks to be mined, while the edges represent the sequence in which predecessor blocks are to be removed first; in this application, topological sorting is a valid sequence of mining. Topological sorting is about traversing a graph in which each node v is visited only after all its predecessors are visited. A topological sorting is not possible if there are directed cycles in the graph, that is, if it is a Directed Acyclic Graph (DAG). At least one topologically sorted option exists for any DAG which can be constructed using any known algorithm within a linear time.

One of these algorithms, described in Algorithm 2, first developed by Kahn (1962), works by choosing vertices in the same order as topological sorting. It starts with finding a list of possible "start nodes" that have no incoming edges and then inserts them into a set S; at least one such node must exist in a non-empty acyclic graph. Gerbner, Keszegh, Palmer and Pálvölgyi (2016) mention weighted topological sorting of directed acyclic graphs and have investigated whether a given weighted directed acyclic graph has a non-negative topological ordering.

Algorithm 2 Pseudocode for Topological Sort

Algorithm 2. Pseudocode for Topological Sorting Algorithm
E $\leftarrow$ Empty list for storing the sorted elements
S $\leftarrow$ Set of all possible start nodes
While S is not empty do
Remove a node n from S
add n to E
For each node m with an edge e from n to m do
Remove edge e from the graph
If m has no other incoming edges then
insert m into S
if edges are there in the graph then
return error (at least one cycle exists in the graph)
else
return E (topologically sorted)

The Weighted Topological sort algorithm uses the predetermined block precedence list for pits and dumps. It has been used to generate random sequences of mining and dumping for all possible start points and multiple runs of sorting along with simulated annealing.

5.4.3 Simulated annealing

Simulated annealing , is a concept which was first presented by Kirkpatrick (1984) for solving NP-hard scheduling problems, and it is a random-search algorithm, inspired by the process of annealing in solids.

Goodfellow and Dimitrakopoulos (2013) addressed the phase or push-back design problem under grade uncertainty for an open pit scheduling problem with an application of the SA algorithm. Kumral (2013) achieved simultaneous resolution of mining block sequencing and ore-waste discrimination (cut-off grade) problems under grade uncertainty using his proposed solution approach of a hybrid of goal programming and SA for open pit scheduling problems. Montiel and Dimitrakopoulos (2013) worked on an open pit mine and stockpiles with multiple sources of varying material types in terms of geo-metallurgical properties under grade uncertainty, by applying SA as a solution approach.

Del Castillo and Dimitrakopoulos (2019) applied SA to solve a multi-stage model for a mine consisting of multiple pits, several material types, and several processing destinations. The model also explicitly includes the flexibility of decision making for capital investments over the life of a mine, thus coming up with a strategic mine plan under dynamic scenarios of changes introduced through capital investments.

Earlier applications of SA algorithms (Kumral and Dowd, 2005) address the supply or demand uncertainty and the minimum grade average, respectively. Specifically, Kumral and Dowd (2005) demonstrated work on an iron ore mine with strict quality specification requirements and minimized the variations in grade blending requirements.

Danish et al. (2021) integrated stockpile policy in optimisation procedure by incorporating non-linear constraints. They also proposed to improve the search process using a greedy heuristic algorithm.

However, none of the above SA algorithms looked at the simultaneous scheduling of ore, and waste including in-pit dumping and haulage options. The Pseudo code of the SA algorithm can be seen in Algorithm 3

Algorithm 3 : Simulated Annealing algorithm

Algorithm 3: Pseudo code for Simulated Annealing(SA) algorithm

Set parameters for Simulated Annealing(Max Iteration, Grid, nodes, stopping temperature etc)

Create an initial solution for an initial sequence of mining and dumping, considering capacity constraints

Ct : Current iteration

Maxit: Maximum iterations

Tt: Current temperature

Tf: Final temperature

While $Tt >$ Tf ; Maxit $> Ct$

Update the current solution by using topological sort

F= Calculate change in objective function (a positive change means new solution is better than the current solution)

If new solution is better

Accept the new solution;

Endif

$p = \exp[-FTt]$

If $p > \text{rand}(0, 1)$

Accept the new solution;

Endif

Update the best solution and the best objective functions obtained so far;

Increment Iteration;

End while

5.5 Implementation of the proposed methodology

The model was implemented on several datasets. These are all stratified deposits where the size of pit blocks is greater in the lateral direction compared to the vertical direction. In these cases, the pit blocks vary from 50m to 250m laterally as shown in the "Avg. block dimension" column of Table 5.2. The number of blocks ranges from 27 to 549. These cases all have an internal dump option (in-pit filling), in some cases, the external dump has been considered as well (cases 3 and 5). For internal dumps, a lag distance has been maintained between the working and the dumping faces. There are one or more main haul roads in each case, and the number of points in these haul roads has been mentioned in the "Road points" column in Table 5.2. Using these road points, the pit and dump blocks, stockpile and plant nodes, several road options were generated. The number of road options generated is mentioned in the "No. of road options" in Table 5.2. All cases were scheduled for 5-year time periods.

Table 5.2: Details of case studies undertaken

Case No	Description	No. of Pit Blocks	Number of Dump Blocks			Avg. block dimension	Lag Distance	Road points	No. of road options	No of Periods
			Tot	In-pit	External					
1	Data set 1	27	54	27	27	100 m	75 m	15	1,567	5
2	Data set 2	49	68	32	36	250 m	275 m	52	3,529	5
3	Data set 3	117	105	104	1	100 m	160 m	13	12,754	5
4	Data set 4	117	137	50	87	100 m	160 m	15	16,380	5
5	Data set 5	147	94	93	1	50 m	60 m	14	14,259	5
6	Data set 6	341	410	264	146	60 m	110 m	101	141,174	5
7	Data set 7	549	458	338	120	110 m	150 m	51	253,638	5

Table 5.3: Number of variables and constraints for each case

Case no.	Binary variables	Pre-assigned Binary variables for ESS	Total variables	constraints	Non-Zero Coefficients
1	476	64	16,220	9,714	435,590
2	714	116	36,125	20,408	1, 215, 120
3	1,601	94	129,240	69,109	4, 043, 785
4	1,741	114	166,840	88,314	5, 072, 505
5	1,852	88	145,280	77,229	4, 551, 400
6	4,840	620	1, 417, 215	723,375	55, 261, 760
7	7,334	446	2, 544, 175	224,388	93, 463, 465

The proposed model has been developed using OPL programming language and solved using a meta-heuristic method and exact solver (CPLEX) on a computer with i7 2.30Hz 8 core, 32GB RAM. The results are in Table 5.4. The simulated annealing method provided results reasonably close but was able to run much faster and for cases that were determined as unsolvable using exact methods.

Table 5.4: Comparative results of simulated annealing vs. exact method using CPLEX

	Exact method- CPLEX			Meta-heuristic method - simulated annealing			
Case No	Gap	solution Time	Objective Value	No of iterations	Solution time	objective value	Difference with exact method
1	0.19%	0Min6.33Sec	8.696050E + 08	1000	1Min43.07Sec	8.5811278E + 08	1.32%
2	0.00%	0Min8.41Sec	1.760640E + 12	1000	3Min54.08Sec	1.6021827E + 12	9.00%−
3	0.01%	46Min24.31Sec	2.478300E + 10	1000	13 Min 28.58Sec	2.4449311E + 10	1.35%
4	0.01%	2Hrs2Min57.23Sec	2.478880E + 10	1000	47Min32.2sec	2.4458512E + 10	1.33%
5	0.01%	105Hrs42Min22 Sec	1.440620E + 09	8000	2Hrs26Mir1.84Sec	1.3518684E + 09	6.16%
6		-	No Solution	1000	15 min 53.597Sec	1.3407755E + 09	-
7		-	No Solution	250	12Hrs6min16.3Sec	1.9428155E + 09	-

As it can be observed from Table 5.4 that the solution time in both methods increases with the increase of the size of the data. The solution times for reaching a near 0% gap in CPLEX increased to over 100 hours as the data size increased. However, for a larger gap, solutions can be reached earlier. Similarly, in simulated annealing, the solution time increases with the number of iterations. About 1,000 iterations on average seem to provide solutions close enough to those obtained by the exact method.

5.5.1 Case study 5

One of the case studies (Case study 5 from Table 5.2) consisted of a coal deposit with a pit, an external dump area, an in-pit dump, and a network of haulage roads scheduled for five periods using both the exact method and simulated annealing algorithm. The pit design has been created for strip mining and the deposit has been divided into strips and blocks for each bench. The internal dump also follows a similar footprint of strips and blocks but has been designed with a batter angle of 37 degrees. The batter angle of the pit and the in-pit dump are angled away from each other like two arms of a V, the pit design is at a steeper angle. The blocks within the pit have quantity and quality information attached to them. All pit and dump blocks have their 3D coordinates of the centroids stored as northing, easting, and elevation. All points of the haulage roads also have coordinates. The same capacity and cost parameters and lag distances have been used in both solution methodologies.

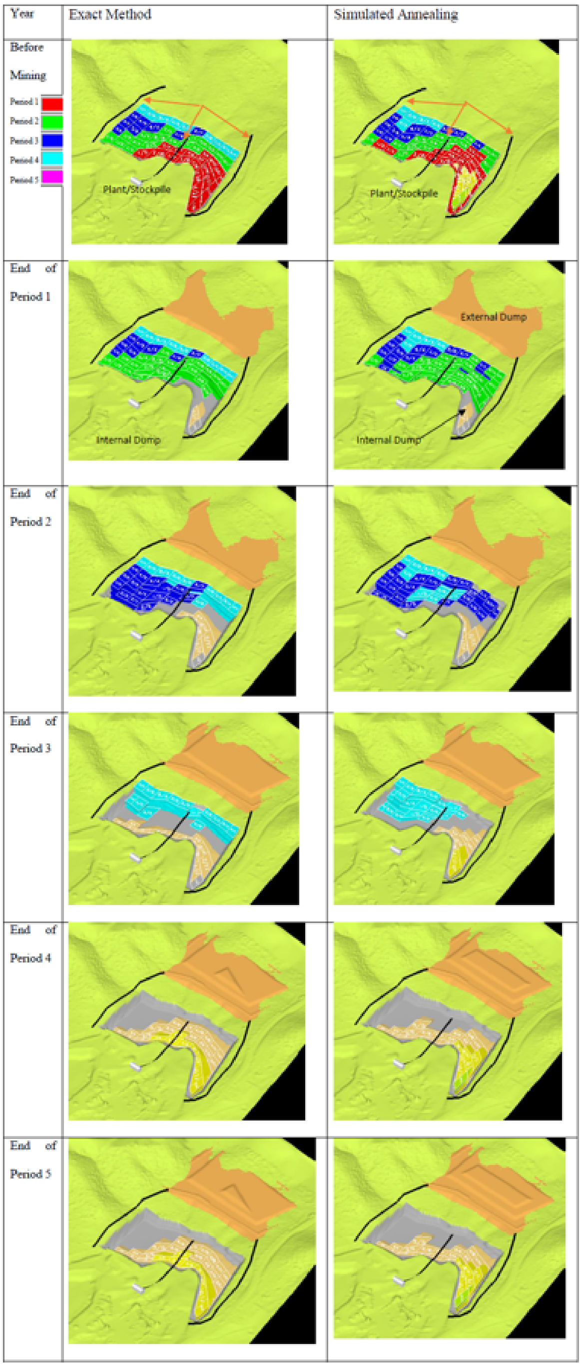


Figure 5.3: Case study 5, comparison of exact and meta-heuristic(simulated annealing) solutions by periods

Both methods produce the required coal production for 5 years, and both finished waste removal in 4 years. Although the blocks handled in individual periods are slightly different, but overall, their patterns match in both solutions as seen in Figure 5.3. Both solutions gradually move from external dumping to increased internal dumping as depicted in Figure 5.4

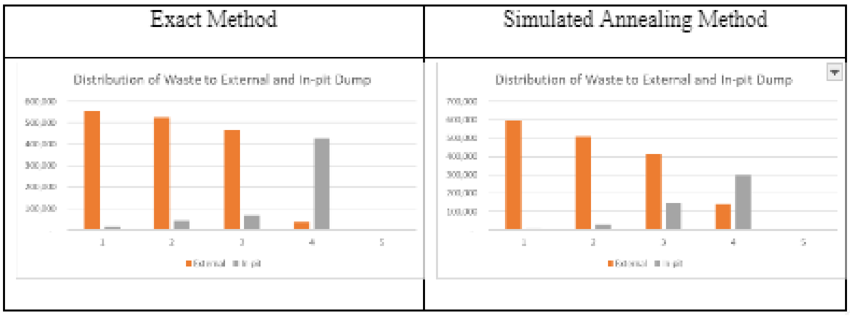


Figure 5.4: Comparison of the distribution of waste to external and in-pit dump for case study 5, exact solution and simulated annealing solution

The NPV attained by two methods is within a difference of 6.2%. The exact method however took a very long time (105+ hrs.) to find a solution with a gap of 0.01% while the Simulated Annealing method could be within 6.2% of the exact solution with 8000 iterations in 2hrs and 26min.

5.5.2 Case study 7

In order to demonstrate the benefit of the meta-heuristic method, another case has been described here in more detail from Table 5.5 (Case study 7 from Table 5.2) . This is a relatively larger and real dataset with in-pit and external dumps, and 253,638 haulage path options. This case is unsolvable using exact methods and hence has been solved using the meta-heuristic method. This data is for a coal mine with two adjoining seams. The ultimate pit limit for the area was identified using the LG algorithm for the given sale price, coal quality, cost of mining and processing, and overall pit slope. Detailed pit and in-pit dump were designed using strips and blocks within the ultimate pit limits identified by the LG algorithm. The external dump was also designed with strips, blocks and appropriate batter angles. There is a network of roads connecting the pit blocks with dump blocks, plants, and stockpiles. The schematic diagram of the pit, dump, and haul roads is shown in Figure 5.5.

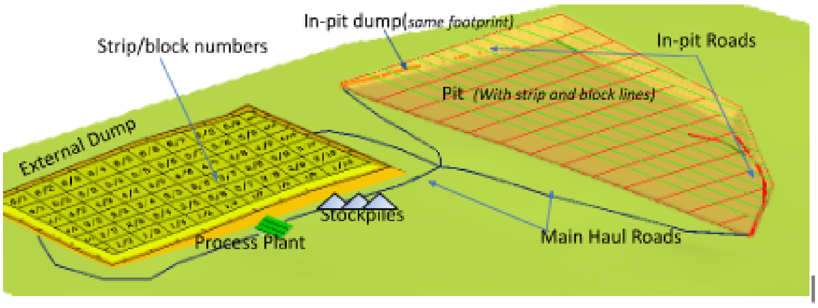


Figure 5.5: Haulage network from pit to dump, plant, and stockpiles

As this is a strip pit design, the shape of the blocks follows the structure of the deposit and is not necessarily cubic in shape (Das et al., 2020). The data contains 549 source nodes (pit blocks) and destination nodes consisting of 338 in-pit dump blocks, 120 external dump blocks, three stockpiles, and one process plant. There are 51 points on the center line of the permanent haul road including the in-pit roads, as seen in Figure 5.5.

The capacity and cost assumptions are in Table 5.5.

Table 5.5: Capacity and cost assumption parameter for case study 7

Input parameter	Variable name	Maximum	Minimum	Unit
Wash Plant capacity	$\bar{P}_{\mathrm{mt}}$ and $\underline{P}_{\mathrm{mt}}$	$7,000,000$	$5,000,000$	Tonnes/Yr
Mining capacity	$\bar{M}_{\mathrm{t}}$ and $\underline{M}_t$	$65,000,000$	$10,000,000$	Cubic m / Yr
Specific energy	g_b	-	15	MJ/Kg
Coal sale price	p_t	-	$72.00	$ per t
Coal mining cost	c_{t}	-	$4.00	$ per t
Waste mining cost	w_1	-	$4.00	$ per BCM
Waste haulage	h_{t}	-	$0.0001	$ per BCM meters
Washing cost	f_t	-	$6.00	$ per t ROM feed
Rehandling cost	j_s	-	$0.05	$ per t ROM feed
Swell factor	SW	-	1.25	Volume to BCM ratio
Discount Rate	a	-	10	%
Recovery/Yield	r_{b}	-	90	%

Note: the $ refers to teh Australian dollar.

The valid edges, connecting adjacent blocks, having a gradient within a maximum of 10% were considered. The length of each edge was recorded in meters, to be used as a weight in the Dijkstra algorithm. The road coordinate points, or nodes were also included in the process. Connections or edges were made from pit blocks to pit ramp points and dump blocks to dump ramp points. The stockpile and plant were also connected to the nearest points on the road. This resulted in 14,330 possible edges. Then the Dijkstra algorithm was utilized to determine all possible paths as 253,638 between each source and destinations. The model includes 7334 binary variables, 2.5 million total variables, and 93.4 million non-zero coefficients. It took 12 hours to solve using the meta heuristic method for 250 iterations as seen in Figure 5.6.

In this implementation of SA, the parameters for cooling schedule were decided based on trial and error and is specified for each problem instance considered. Finally, the run-time was controlled by the number of iterations in order to reach solutions as close as possible to the exact method.

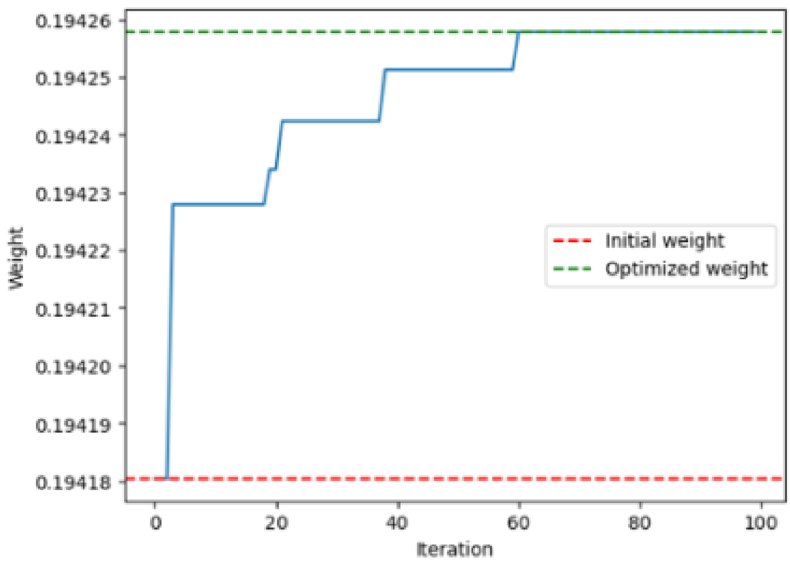


Figure 5.6: Simulated annealing graph for Case study 7

The results for the physical quantities per year reported from the optimized schedule are shown in Table 5.6. The entire waste is mined in the first 4 years while the coal supply in the fifth year is from the stockpile maintaining the required coal feed. Coal normally deteriorates with time and sometimes is subject to spontaneous combustion. In the current state of the model, deterioration due to long-term stocking has not been considered.

Table 5.6: Physical quantities from the solution obtained

Year	Waste volume	Dump vol. (cu.m)		Coal To plant	Coal to Stockpile	Coal from Stockpile	Total to Plant
	(cu.m)	External	In-pit	from Mine(t)	from Mine(t)	to Plant (t)	(t)
1	50, 391, 561	25, 615, 138	24, 776, 423	7, 000, 000	6, 451, 814	-	7, 000, 000
2	54, 983, 864	34, 723, 065	20, 260, 799	6, 799, 192	–	200,808	7, 000, 000
3	54, 198, 852	25, 178, 098	29, 020, 754	7, 000, 000	566,817	-	7, 000, 000
4	22, 383, 655	3, 598, 447	18, 785, 208	6, 921, 182	-	78,818	7, 000, 000
5	-	-	-	-	-	6, 739, 005	6, 739, 005
Total	181, 957, 933	89, 114, 748	92, 843, 185	27, 720, 374	7, 018, 631	7, 018, 631	34, 739, 005

The annual cost and revenue for the solution, leading to the NPV are in Table 5.7

Table 5.7: Financials for the solution obtained - in $'000

Year	Waste mining	Waste haulage cost		Coal mining & washing cost				Rehandling	Revenue ($)	Margin ($)
	cost	Ext	In-pit	Mining	Haulage to plant	Haulage to Stockpile	Coal wash cost	cost		
1	151,175	56	31	53,807	14	8	42,000	-	705,600	458,517
2	164952	85	32	27,197	16	-	42,000	100	705,600	471,219
3	162,597	85	47	30,267	18	1	42,000	-	705,600	470,586
4	67,151	11	28	27,685	19	-	42,000	39	705,600	568,666
5	-	-	-	-	-	-	40,434	3,370	679,292	635,488

The results obtained from the model have been taken to a planning software for 3D visualization. The schedule phases at the end of each period are shown in 3D in Figure 5.7.

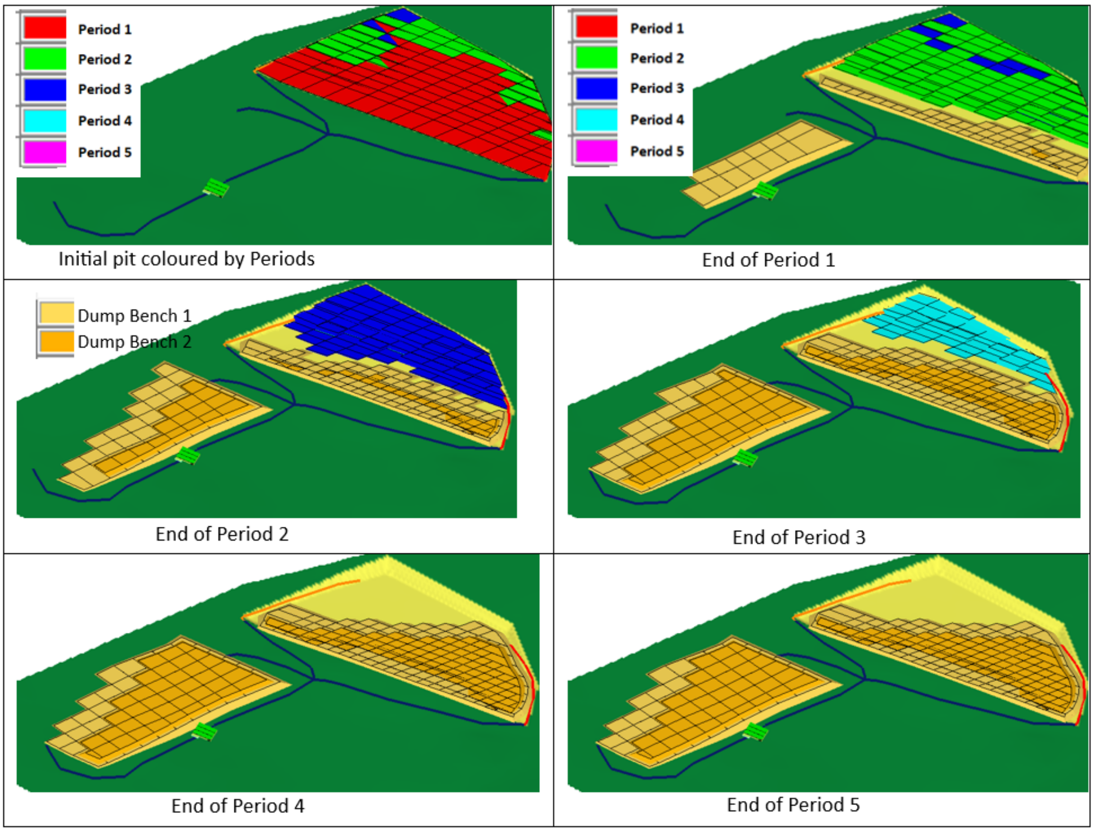


Figure 5.7: End of Period 3D phases of the pit and dump for case study 7 - pit coloured by periods

Out of the provided options of 253,638 paths, the model selected 881 paths to carry all the coal and waste to respective destinations. Figure 5.8 shows the distribution of waste to in-pit and external dumps. With early start of in-pit dumping rehabilitation for returning the land for pre-mining uses is expedited along with an optimal NPV of the project.

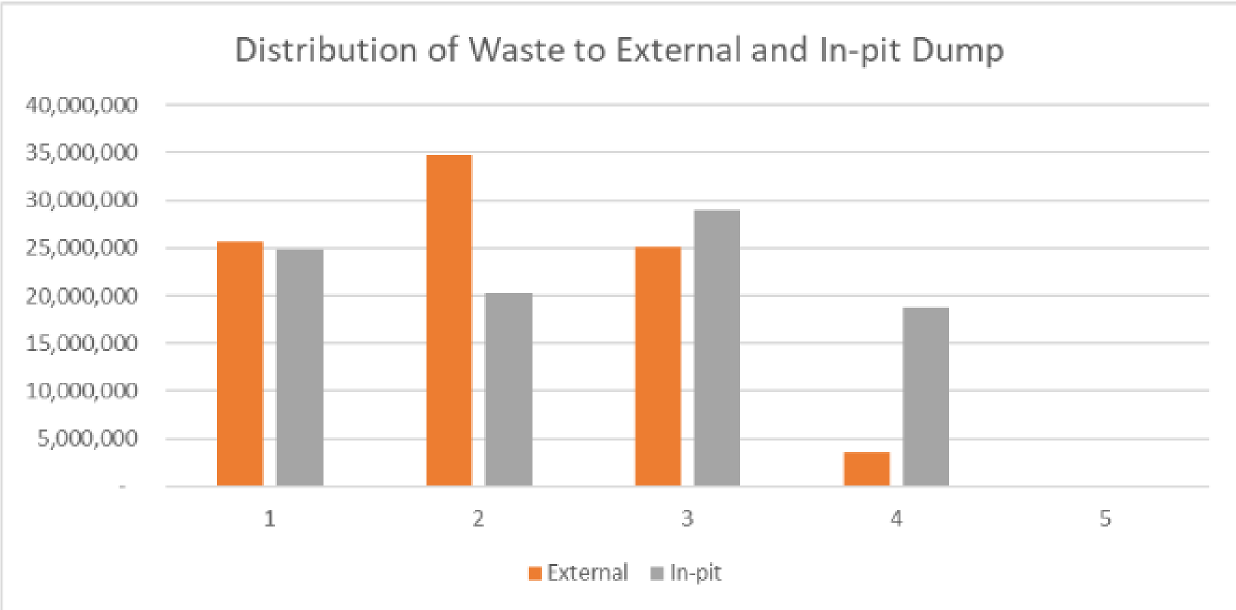


Figure 5.8: Distribution of waste and ore for Case study 7

In order to compare the results obtained from Case study 7 using the meta-heuristic approach, the same data was scheduled manually using an interactive mine planning and scheduling software. The capacity and conditions were kept similar for a like-to-like comparison. This schedule was done manually hence there could have been several options possible for selecting blocks to be mined interactively. The results of the schedule in terms of quantities achieved are shown in Table 5.8.

Table 5.8: Volume and tonnes for Case study 7 scheduled in a mine planning software

Year	Waste volume	Dump vol. (cu.m)		Coal To plant	Coal to Stockpile	Coal from Stockpile	Total to Plant
	(cu.m)	External	In-pit	from Mine(t)	from Mine(t)	to Plant (t)	(t)
1	59,965,396	59,965,396	-	7,000,436	-	-	7,000,436
2	44,843,436	33,489,303	11,354,352	7,001,168	-	-	7,000,168
3	44,842,664	15,584,789	29,257,873	7,000,738	-	-	7,000,738
4	32,306,428	-	32,306,428	7,001,197	-		7,000,197
5	-	-	-	-	-	6,735,465	6,735,465
Total	181,957,924	109,089,483	72,918,456	34,739,004	-	6,735,465	34,739,004

As seen in Table 5.8, the schedule meets the annual coal requirement the same as the schedule done using the meta- heuristic approach in Table 5.6. However, the waste handled into in-pit dump is less and starts only in Year-2, although both runs have been made using a 150m lag distance. This mine planning software did not have the capability of considering stockpiles.

Figure 5.8 and Figure 5.9 can be compared to see the difference in handling of waste into internal dumps. The total material going to internal dumps in the five years using the software is about 73 million BCM where as using the meta heuristic approach is about 130 million BCM, which is an improvement of 44% over the software approach. This is owing to the fact that the model developed tries to expedite in-pit dumping allowing for quicker rehabilitation of mined out areas and returning them for land use.

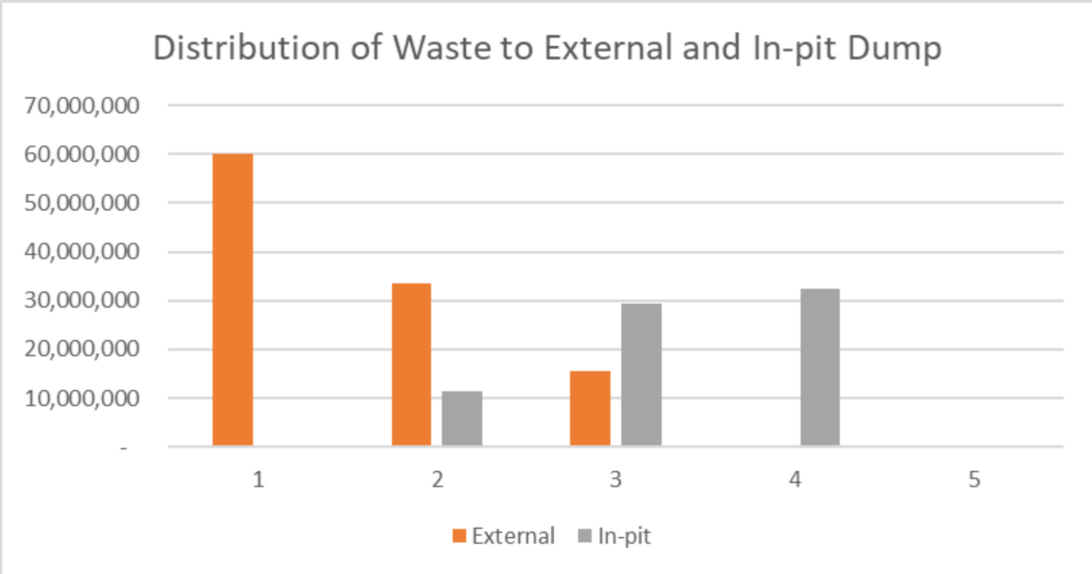


Figure 5.9: Distribution of Waste and Ore for case study 7, Scheduled in a Mine Planning Software

Table 5.9: Comparison of haulage distances for case study 7, between the model and a mining Software

In Km	Waste Haulage to External Dump		Coal Haulage to Plant from Pit	
	Model	Mining Software	Model	Mining Software
Year 1	2.19	3.82	2.07	4.17
Year 2	2.45	4.18	2.24	4.28
Year 3	3.36	4.30	2.56	4.20
Year 4	3.19	4.30	2.74	4.40
Year 5				4.64
Average	2.80	4.15	2.40	4.34

The improvement in haulage distance as seen in Table 5.9 leads to lower cost as well as lower carbon footprint by way of less diesel burnt in trucks.

5.6 Chapter summary

A mathematical model has been developed and demonstrated that achieved concurrent ore and waste production scheduling along with haulage selection. The model also considers in-pit and external dumping while maintaining a lag distance from the working face with the in-pit dump which provides less mining footprint and cost of mining. Since the proposed model is NP-hard, a meta-heuristic method has also been developed using a weighted topological sort algorithm and simulated annealing for solving realistic datasets. Several case studies were implemented to establish the efficiency of the proposed meta-heuristic method vs. the exact method showing acceptable tolerances of the difference in solutions achieved but a major improvement in solution time. One case study has been further detailed in this paper which was solved using the meta-heuristic method as the exact method did not provide any solution.

The chapter presents a unique and pioneering approach in the research area to include optimal haulage road selection along with optimizing a pit and dump schedule, along with in-pit dumping. Furthermore, it presents a new meta-heuristic method which is a combination of several strategies. It is expected to generate benefits for several mining projects across the world in optimizing their mining and dumping schedules, selecting the right haulage paths, and optimally expediting back-fill dumping and rehabilitation. This chapter is particularly focused on stratified deposits where back-filling is possible to a great extent, however, for future versions a minor change in the method of determining predecessor blocks is possible to use the model for non-stratified deposits.

Chapter 6

Conclusions and future research

This chapter briefly summarises the research that has been conducted in this thesis, as well as its findings and conclusions. Possible directions for future research are also presented.

6.1 Main Findings and Contribution to Knowledge

The primary objective of the research was to develop a mathematical model that can concurrently generate an optimal schedule for ore and waste production along with internal dumping as well as external dumping option, along with optimal haulage selection for stratified deposits. Mine planning for stratified deposits is different in more than one way. The cost of waste removal and handling in open pit mines accounts for some of the largest operational expenses, as prescribed by stripping ratios and can be about 5 to 10 times or ore mining costs. With the inevitable depletion of world class ore bodies, such ratios can only increase and with them, the importance of waste management optimisation and environmental footprint minimisation.

While most mine planning studies so far were focused around non-stratified or metallic deposits, the mathematical model developed presents an integration of pit and dump scheduling with a focus on stratified deposits and includes an in-pit dumping strategy while considering the selection of optimal haulage options from available shortest paths. The ability to decide the optimal destination of waste blocks into in-pit and external dumps along with the optimal mining sequence can add significant value to mining operations. Not only does it allow maximising the value of the project but it also allows for quicker back-filling of the in-pit dumps and making them available for rehabilitation, thereby decreasing the footprint of external dumps. The model shall be very useful to the mining industry to achieve less carbon footprint of mining with potential reduction in haulage and related emissions while maximizing the NPV. With a gradual replacement of separate ore and waste scheduling by inclusive, multi-purpose models, improved economic and environmental outcomes are assured.

The model has been successfully applied to a coal mine dataset with 118 pit blocks and 104 in-pit dump blocks. The results were encouraging with an early and increased in-pit dumping to be able to do progressive rehabilitation earlier in the life of the project, as well as have an improved NPV by 27% compared to a schedule done in a mine planning software using similar parameters. However, with larger data like 549 blocks the model was found to be NP Hard, meaning a solution could not be obtained within a desired time-frame.

In the second phase of the research the mathematical model was further updated in order to include the selection of optimal haul roads from a database of shortest paths between pit blocks and all possible destinations including all dump blocks, stockpiles and process plants. The Dijkstra algorithm has been used to determine the shortest path between a source and destination. The methodology considers connections to neighbouring blocks from each block as edges. To be considered as a valid edge the slope of the connector has to be within a maximum limit. The same applies for adjacent dump blocks to create valid edges. An array of paths have been considered as options in the optimisation model for carrying the ore and waste from a block within a pit to destinations like dump locations, stockpile or process plant. The path with the least cost is chosen by the model, while considering the other constraints used in the first part of the research. This model was implemented in OPL, CPLEX, where it was possible to solve cases with a limited number of pit and dump blocks, thereafter the model appeared to be NP Hard.

Since the proposed model has been found to be NP-hard, a meta-heuristic methodology has also been developed using weighted topological sort algorithm and simulated annealing for solving realistic datasets. The results showed acceptable tolerances of the difference in solutions achieved but a major improvement in solution time was achieved. The study included several case studies where both the proposed meta-heuristic method and the exact method were applied. The proposed model provides significant NPV improvement for a case by 26% increase over a mine planning software while also in-pit dumping increased from 53% to 83%.

With the meta-heuristic solution approach for a case with 549 pit blocks and 458 dump blocks including 338 in-pit dump blocks, and 253,638 path options, a solution was obtained in 12 hours and 6 min. This case was also solved using a mine planning software which does not optimize. The solution from the software had a 39% higher average haulage distance and a 44% lower back-filling of waste compared to the solution from the model.

6.2 Future research & recommendation

The research considered blocks as a whole, where blocks could contain ore and waste. However, stratified deposits are modeled as seams and a block could contain one or more seams along with inter-burden waste in between seams. In order to schedule more accurately the physical location of the layers of seams and waste within a block could be considered so that the seams and inter-burden waste are mined in a sequence in which they occur instead of just considering the total coal and total waste in the block.

Since the research is particularly focused on stratified deposits where back-filling is possible to a great extent, however, future versions a minor change in the method of determining predecessor blocks is possible to use the model for non-stratified deposits as well.

In case of projects which have simultaneous back-filling opportunity of underground mines, the same could be considered in the mathematical model as well.

There is also scope to extend the model for stochastic optimisation to quantify the uncertainty in input variables.

Further, The proposed model is for Life of Mine planning and can be updated for short-term mine planning which will need to include layers or seams within a block for instance.

The research work could bring immense value to the mining industry specially the stratified deposit mines such as coal, phosphate etc. It is recommended to use the model in such deposits to reap the benefit of this research.

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Appendices

Appendix A

Code for mathematical model

The following code has been used for finding solutions to the mathematical model developed in order to find an optimised pit and dump schedule considering in-pit dumping and haul road selection from a network of road options.

```
/*******************************************
  * Creation Date: 14 Feb 2019 at 10:01:33 am
 * Version : 2.0
 *******************************************/
//Declaration of the indices
{int} TimePeriods =...;
{string} PitBlocks =...;
{string} DumpBlocks=...;
{string} Stockpiles=...;
{string} Plants=...;
{string} Pathid =...;
// Declaration of the parameters
float grade[PitBlocks] = ...; //grade g_b
float oreTons[PitBlocks] = ...; //
float density[PitBlocks] = ...; // d[b]
float wasteVolume[PitBlocks] = ...; //
float totalVolume[PitBlocks] = ...;//q[b]
float dumpVolume[DumpBlocks] =...;
float resourceMaxCap[TimePeriods] =...;
```

```
float resourceMinCap[TimePeriods] =...;
float processMinCap[Plants][TimePeriods]=...;
float processMaxCap[Plants][TimePeriods]=...;
float GradeMin[Plants][TimePeriods] =...;
float SellPrice[TimePeriods] =...;
float wasteMiningCost[TimePeriods] =...;
float coalMiningCost[TimePeriods] =...;
float washCost[TimePeriods] =...;
float HaulageCost[TimePeriods] =...;
float StockPileRehandlingCost[TimePeriods] =...;
float AverageGrade[Stockpiles] =...; //g_ns
float DensityGradeBins[Stockpiles] = ...;
float SwellFactor =...;
float StockPileVol[Stockpiles][TimePeriods];
float StockPileMinCap[Stockpiles][TimePeriods]=...;
float StockPileMaxCap[Stockpiles][TimePeriods]=...;
float DisountRate =...;
float DumpCapacity =...;
// Location Tuple to Capture the Locations of Different pit/dump Block, stockpile etc with x,y,z
tuple xyz {
  string id;
  float x;
  float y;
  float z;
};
xyz PitXYZ[PitBlocks]=...; // location of Pit block b
xyz DumpXYZ[DumpBlocks]=...; // location of Dump block d
xyz PlantXYZ[Plants]=...;     // location of Plant m
xyz StockpilesXYZ[Stockpiles]=...; // location of Stockpile s
// Lag constraints modelling steps
{xyz} DumpLagInfoXYB=...; // consist of Dump ID, X, Y, Bench information
{xyz} PitLagInfoXYB=...; // consist of Pit ID, X, Y, Bench information
float D = 160; // lag distance
float BottomPitBenNo = 9; //last bench of pit where lag is to be maintained
{xyz} OntopDumpLag[d in DumpLagInfoXYB] =
```

```
     {b | b in PitLagInfoXYB: (sqrt(pow((d.x-b.x),2)+pow((d.y-b.y),2)) <= D) &&
                        (d.z  == 1 ) &&
                         (b.z  == BottomPitBenNo )};
// to check dump lag info
execute{
for(var i in DumpLagInfoXYB){
writeln(i, OntopDumpLag[i]);
}
}
tuple sourceDestination{
 string source;
 string dest;
}
{sourceDestination} sourceDestD=...;
{sourceDestination} sourceDestM=...;
{sourceDestination} sourceDestS=...;
//since the Shortest path all_paths cannot be read as a tuple
//it is first read as a string. Then a tuple Path is created
//which is set to empty sets
//these empty sets are filled based on the 3 for loops in the execute below
//the split and ad fnction converts it from string to set data for pit block set and dump bl
tuple Raw {
  string id;
  string source;
  string dest;
  string pitblockSet;
  string roadPoints;
  string dumpblockSet;
  string others;
  float dist;
}
{Raw} rawPbd = ...; //
{Raw} rawPbm = ...; //
{Raw} rawPbs = ...; //
tuple Path {
```

```
string id;
string source;
string dest;
{string} pitblockSet;
{string} roadPoints;
{string} dumpblockSet;
{string} others;
float dist;
};
{Path} Pbd={}; // set of paths from block b to dump d
{Path} Pbm ={}; //set of paths from block b to plant m
{Path} Pbs={}; // set of paths from block b to stock pile s
{string} emptysetd = {}; // Helper to create new empty sets in scripting.
{string} emptysetm = {};
{string} emptysets = {};
// Populate the path set.
execute {
  // Split a string by a separator and add all the tokens to a set.
  function splitAndAdd(set, data, sep) {
    var fields = data.split(sep);
    for (var i = 0; i < fields.length; ++i) {
      set.add(fields[i]);
    }
  }
  for (var r in rawPbd) {
    var s = Opl.operatorUNION(emptysetd, emptysetd);
    // Add a new tuple. The fields of type set are empty sets for now.
    // We cannot just pass 'emptyset' because then all fields would
    // reference the same set. So we create a new emptyset as the union
    // of two empty sets.
    var t = Pbd.add(r.id, r.source, r.dest,
                    Opl.operatorUNION(emptysetd, emptysetd),
                    Opl.operatorUNION(emptysetd, emptysetd),
                    Opl.operatorUNION(emptysetd, emptysetd),
                    Opl.operatorUNION(emptysetd, emptysetd),
```

```
                  r.dist);
   // Now populate the fields of type set in the newly created tuple.
   splitAndAdd(t.pitblockSet, r.pitblockSet, " ");
   splitAndAdd(t.roadPoints, r.roadPoints, " ");
   splitAndAdd(t.dumpblockSet, r.dumpblockSet, " ");
 }
// writeln(Pbd);
 for (var r in rawPbm) {
   var s = Opl.operatorUNION(emptysetm, emptysetm);
   // Add a new tuple. The fields of type set are empty sets for now.
   // We cannot just pass 'emptyset' because then all fields would
   // reference the same set. So we create a new emptyset as the union
   // of two empty sets.
   var t = Pbm.add(r.id, r.source, r.dest,
                  Opl.operatorUNION(emptysetm, emptysetm),
                  Opl.operatorUNION(emptysetm, emptysetm),
                  Opl.operatorUNION(emptysetm, emptysetm),
                  Opl.operatorUNION(emptysetm, emptysetm),
                  r.dist);
   // Now populate the fields of type set in the newly created tuple.
   splitAndAdd(t.pitblockSet, r.pitblockSet, " ");
   splitAndAdd(t.roadPoints, r.roadPoints, " ");
   splitAndAdd(t.dumpblockSet, r.dumpblockSet, " ");
   splitAndAdd(t.others, r.others, " ");
 }
// writeln(Pbm);
 for (var r in rawPbs) {
   var s = Opl.operatorUNION(emptysets, emptysets);
   // Add a new tuple. The fields of type set are empty sets for now.
   // We cannot just pass 'emptyset' because then all fields would
   // reference the same set. So we create a new emptyset as the union
   // of two empty sets.
   var t = Pbs.add(r.id, r.source, r.dest,
                  Opl.operatorUNION(emptysets, emptysets),
                  Opl.operatorUNION(emptysets, emptysets),
```

```
                     Opl.operatorUNION(emptysets, emptysets),
                     Opl.operatorUNION(emptysets, emptysets),
                     r.dist);
    // Now populate the fields of type set in the newly created tuple.
    splitAndAdd(t.pitblockSet, r.pitblockSet, " ");
    splitAndAdd(t.roadPoints, r.roadPoints, " ");
    splitAndAdd(t.dumpblockSet, r.dumpblockSet, " ");
    splitAndAdd(t.others, r.others, " ");
  }
 // writeln(Pbs);
}
float hc[Pathid][TimePeriods];
//determine haulage cost for each path
execute {
//distances to plant
for (var i in Pbm) {
for (var t in TimePeriods){
  hc[i.id][t] = i.dist*HaulageCost[t];
} }
//distances to stockpile
for (var i in Pbs) {
for (var t in TimePeriods){
  hc[i.id][t] = i.dist*HaulageCost[t];
} }
//distances to dumps
for (var i in Pbd) {
for (var t in TimePeriods){
  hc[i.id][t] = i.dist*HaulageCost[t];
} } }
tuple blockType {
    string id;
    int i;
    int j;
    int k;
 };
```

```
//{blockType} PitBlocksType = ...;
{blockType} DumpBlocksType = ...;
tuple ijk {
  string id;
  int i;
  int j;
  int k;
  int min_i;
}
{ijk} PitBlocksType = ...;
{ijk} OntopPit[b1 in PitBlocksType] =
      {b2 | b2 in PitBlocksType: (b2.id != last(PitBlocksType).id  &&
       b1.id != b2.id) &&
       b1.min_i<=b2.i &&
         b1.i >= b2.i &&
                      ((b1.k  == b2.k-1 ) ||
                       (b1.k  == b2.k+1 ) ||
                       (b1.k  == b2.k )  ) &&
                      (((b1.j  == b2.j+1 )) ||
                       (b1.j  == b2.j && b1.i > b2.i)   )
                        };
{blockType} OnBelowDump[d1 in DumpBlocksType] =
     {d | d in DumpBlocksType: d1.i == d.i+1 &&
                      ((d1.k  == d.k-1 ) ||
                       (d1.k  == d.k+1 ) ||
                       (d1.k  == d.k )  ) &&
                      ((d1.j  == d.j-1 ) ||
                       (d1.j  == d.j ) ) ||
                       (d1.i == d.i &&
                        ((d1.k  == d.k-1 ) ||
                       (d1.k  == d.k+1 ) ||
                       (d1.k  == d.k )  ) &&
                      (d1.j  == d.j+1 )  ) };

//one block below - for keeping roads on surface
```

```
{ijk} BlockBelow[b1 in PitBlocksType] =
     {b | b in PitBlocksType: b1.i -1 == b.i  &&
                       (b1.k  == b.k ) &&
                        (b1.j  == b.j) };
{ijk} BlockBelow[b1 in PitBlocksType] =
     {b | b in PitBlocksType: (b.id != first(PitBlocksType).id  &&
       b1.id != b.id) &&
       b1.i ==b.min_i &&
         (b1.k  == b.k ) &&
                        (b1.j  == b.j)
                         };
{blockType} DumpBlockBelow[d1 in DumpBlocksType] =
     {d | d in DumpBlocksType: d1.i == d.i +1 &&
                       (d1.k  == d.k ) &&
                        (d1.j  == d.j) };
{string} splitPitBlocksPath[Pbd];
{string} splitPitBlocksPathM[Pbm];
{string} splitPitBlocksPathS[Pbs];
{string} splitDumpBlocksPath[Pbd];
int MaxS=10;
execute {
for(var p in Pbd) {
var stringSetP = Opl.item(p.pitblockSet,0);
var splitP= new Array(MaxS);
splitP=stringSetP.split(",") ;
for(var i=0;i<=MaxS;i++) if ((splitP[i]!='null') && (splitP[i]!='')) splitPitBlocksPath[p].add(split
}
for(var p in Pbm) {
var stringSetM = Opl.item(p.pitblockSet,0);
var splitM= new Array(MaxS);
splitM=stringSetM.split(",") ;
for(var i=0;i<=MaxS;i++) if ((splitM[i]!='null') && (splitM[i]!='')) splitPitBlocksPathM[p].add(spli
}
for(var p in Pbs) {
var stringSetS = Opl.item(p.pitblockSet,0);
```

```
var splitS= new Array(MaxS);
splitS=stringSetS.split(",") ;
for(var i=0;i<=MaxS;i++) if ((splitS[i]!='null') && (splitS[i]!='')) splitPitBlocksPathS[p].
}
for(var p in Pbd) {
var stringSetD = Opl.item(p.dumpblockSet,0);
var splitD= new Array(MaxS);
splitD=stringSetD.split(",") ;
for(var i=0;i<=MaxS;i++) if ((splitD[i]!='null') && (splitD[i]!='')) splitDumpBlocksPath[p].
} }
{ijk} PitBlocksInPathD[p in Pbd] = union(b in splitPitBlocksPath[p]) {b2 | b2 in PitBlocksTy
{ijk} PitBlocksInPathM[p in Pbm] = union(b in splitPitBlocksPathM[p]) {b2 | b2 in PitBlocksT
{ijk} PitBlocksInPathS[p in Pbs] = union(b in splitPitBlocksPathS[p]) {b2 | b2 in PitBlocksT
{blockType} DumpBlocksInPathD[p in Pbd] = union(d in splitDumpBlocksPath[p]) {d2 | d2 in Dum
/*ESS Implementation*/
tuple nullVariables {
  string block_id;
  int time_period;
};
{nullVariables} NullVariablesSet = ...;
float capBMT[PitBlocks][Plants][TimePeriods];
float capBDT[PitBlocks][DumpBlocks][TimePeriods];
float capBST[PitBlocks][Stockpiles][TimePeriods];
int capBT[PitBlocks][TimePeriods];
int capschedulePit[PitBlocks][TimePeriods];
execute{
 // For continuous variables
//Inititialize all cap to 1 or 1000000
for (var p in PitBlocks){
for (var t in TimePeriods){
 for (var pl in Plants){
       capBMT[p][pl][t] = 1000000.00;         }
for (var d in DumpBlocks){
         capBDT[p][d][t] = 1000000000.00; }
for (var s in Stockpiles){
```

```
          capBST[p][s][t] = 10000000.00;
         }
          capBT[p][t] = 1;
          capschedulePit[p][t] = 1;   } }
//Check nullVariables and populate
for (var p in PitBlocks){
for (var t in TimePeriods){
for (var nv in NullVariablesSet){
     // for XBMT
     for (var pl in Plants){
     if (p==nv.block_id && t==nv.time_period){
        capBMT[p][pl][t] = 0;  }         }
     // for XBDT
     for (var d in DumpBlocks){
     if (p==nv.block_id && t==nv.time_period){
        capBDT[p][d][t] = 0;    }    }
     // for XBST
     for (var s in Stockpiles){
                  if (p==nv.block_id && t==nv.time_period){
                 capBST[p][s][t] = 0;       }  }
     // for ZBT and schedulePitBT
                  if (p==nv.block_id && t==nv.time_period ){
                    capBT[p][t] = 0;
                      capschedulePit[p][t] = 0;
                       // writeln("######------", p,"  ", t,"   ",capschedulePit[p][t]);
                      // flagBT[p][t] = 1
        // writeln("####", p," ",nv.block_id,":: ",t," ",nv.time_period ," ", capschedulePit[p][t]);
                   }  }    }   }
for (var p in PitBlocks){
for (var t in TimePeriods){
 writeln("######", p,"  ", t,"   ",capschedulePit[p][t]);
} }
//end of ESS initialize execute
}
 /*
```

```
dvar float+ Xbmt[p in PitBlocks][pl in Plants][t in TimePeriods] in 1..capBMT[p][pl][t];
dvar float+ Xbdt[p in PitBlocks][d in DumpBlocks][t in TimePeriods] in 0..capBDT[p][d][t];
dvar float+ Xbst[p in PitBlocks][s in Stockpiles][t in TimePeriods] in 0..capBST[p][s][t];
dvar boolean zbt[p in PitBlocks][t in TimePeriods] in 0..capBT[p][t];
dvar boolean schedulePit[p in PitBlocks][t in TimePeriods] in 0..capschedulePit[p][t];
dvar float+ ypt[path in Pathid][TimePeriods] in 0..capPT[path];
dvar float+ Xsmt[Stockpiles][Plants][TimePeriods];
dvar boolean scheduleDump[DumpBlocks][TimePeriods];
*/
/////// ESS Implementation end of variable declaration
//Decision variables & expressions
//dvar float+ Xbmt[PitBlocks][Plants][TimePeriods];
//dvar float+ Xbmt[xBMTCapX];
//dvar float+ Xbdt[PitBlocks][DumpBlocks][TimePeriods];
//dvar float+ Xbst[PitBlocks][Stockpiles][TimePeriods];
dvar float+ Xsmt[Stockpiles][Plants][TimePeriods];
//dvar float+ ypt[Pathid][TimePeriods]; //a continuous variable to measure the quantity of m
//dvar boolean schedulePit[PitBlocks][TimePeriods] ;
dvar boolean scheduleDump[DumpBlocks][TimePeriods];
//dvar boolean zbt[PitBlocks][TimePeriods] ;
dvar float+ Xbmt[p in PitBlocks][pl in Plants][t in TimePeriods] in 0..capBMT[p][pl][t];
dvar float+ Xbdt[p in PitBlocks][d in DumpBlocks][t in TimePeriods] in 0..capBDT[p][d][t];
dvar float+ Xbst[p in PitBlocks][s in Stockpiles][t in TimePeriods] in 0..capBST[p][s][t];
dvar boolean zbt[p in PitBlocks][t in TimePeriods] in 0..capBT[p][t];
dvar boolean schedulePit[p in PitBlocks][t in TimePeriods] in 0..capschedulePit[p][t];
dvar float+ ypt[path in Pathid][TimePeriods] ;
// Asuming recovery is 0.9
//Dfbimt
dexpr float Dfbmt[b in PitBlocks][m in Plants][t in TimePeriods] =
( SellPrice[t] * grade[b] * 0.9  - coalMiningCost[t] - washCost[t]  ) *  1/(1+DisountRate)^t
//Dfbiedt
dexpr float Dfbdt[b in PitBlocks][d in DumpBlocks][t in TimePeriods] =
( - wasteMiningCost[t]  ) *  1/(1+DisountRate)^t;
//Dfbisnst
dexpr float Dfbst[b in PitBlocks][s in Stockpiles][t in TimePeriods] =
```

```
( - coalMiningCost[t]  ) *  1/(1+DisountRate)^t;
//Dfsnsmt
dexpr float Dfsmt[s in Stockpiles][m in Plants][t in TimePeriods] =
(SellPrice[t] * AverageGrade[s]* 0.9 - washCost[t]-StockPileRehandlingCost[t]  ) *  1/(1+DisountRate
//equation 01
dexpr float npv = sum( b in PitBlocks,  m in Plants, t in TimePeriods) (Dfbmt[b][m][t]* Xbmt[b][m][t
   +sum( b in PitBlocks,  d in DumpBlocks, t in TimePeriods) (Dfbdt[b][d][t]* Xbdt[b][d][t])
+sum( b in PitBlocks,  s in Stockpiles,  t in TimePeriods) (Dfbst[b][s][t]* Xbst[b][s][t])
 + sum( s in Stockpiles,  m in Plants, t in TimePeriods) (Dfsmt[s][m][t]* Xsmt[s][m][t] )
 -sum(b in PitBlocks, t in TimePeriods,p in Pathid) ( hc[p][t]*ypt[p][t] ) *  1/(1+DisountRate)^t ;
maximize
npv;
subject to{
//Cons02 :: Block ore tonnes
 //OreInBlockTonnes:
 //Equation 2
forall(b in PitBlocks)
  {
  //oreTons
sum(  m in Plants, t in TimePeriods) ( Xbmt[b][m][t])
  + sum(  s in Stockpiles, t in TimePeriods)(Xbst[b][s][t]) <= oreTons[b];
  }
//Cons03 :: Block total Volume
//Equation 3
forall(b in PitBlocks)
  {
  //blocktotalVolume :
sum(  d in DumpBlocks, t in TimePeriods)(Xbdt[b][d][t])
  + sum( m in Plants, t in TimePeriods) ( Xbmt[b][m][t])
  + sum(  s in Stockpiles, t in TimePeriods)(Xbst[b][s][t]) <= totalVolume[b];
  }
//Cons04 : Total Mining capacity
// Reference : Const (4) of Z.Fu et al. Cons (5) not modelled
//Equation 4
forall(t in TimePeriods)
```

```
  {
        // maxmining:
         sum(b in PitBlocks,  m in Plants) ( Xbmt[b][m][t])
           +sum(b in PitBlocks,  d in DumpBlocks)(Xbdt[b][d][t])
           +sum( b in PitBlocks,  s in Stockpiles)(Xbst[b][s][t])
           <= resourceMaxCap[t];          }
//Eqution 5
forall(t in TimePeriods)
  {
        // minmining:
         sum(b in PitBlocks,  m in Plants) ( Xbmt[b][m][t])
           +sum(b in PitBlocks,  d in DumpBlocks)(Xbdt[b][d][t])
           +sum( b in PitBlocks,  s in Stockpiles)(Xbst[b][s][t])
           >= resourceMinCap[t];
         }
//Cons06 :: Total Process Capacity - Max
//Equation 6
forall(m in Plants, t in TimePeriods)
  {
        // maxplant:
         sum(b in PitBlocks ) ( Xbmt[b][m][t])
           +sum(s in Stockpiles)(Xsmt[s][m][t])
           >= processMinCap[m][t];          }
//Cons05 :: Total Process Capacity - Min
// Reference : Const (6) and (7) of Z.Fu et al
//Equation 7
forall(m in Plants, t in TimePeriods)
  {
        // minplant:
         sum(b in PitBlocks) ( Xbmt[b][m][t])
           +sum(s in Stockpiles)(Xsmt[s][m][t])
           <= processMaxCap[m][t];          }
//Cons07: Grade Min
// Reference : Const (8) of Z.Fu et al, cons (9) not modelled
//Equation 8
```

```
forall(m in Plants, t in TimePeriods)
  {
        // grademin:
         sum(b in PitBlocks ) ((grade[b]-GradeMin[m][t])* Xbmt[b][m][t])
           +sum(s in Stockpiles)((AverageGrade[s]-GradeMin[m][t])*Xsmt[s][m][t])
           >= 0;         }
//Cons08: Dump capacity
// Reference : Const (10) of Z.Fu et al, cons (11) not required
// In this case modelled as <= not as === as this dump has a location and coordinate unlike Z.Fu's du
// which is based on just a pile with levels
//Equation 9
forall(t in TimePeriods)
  {
        // totaldumpcapacity:
         sum( b in PitBlocks, d in DumpBlocks,  r in TimePeriods: r<=t) (SwellFactor* Xbdt[b][d][r])
           <= DumpCapacity;          }
//Cons09 : Dump waste volume [Not in paper of Z.Fu]
//modelled here as individual dump blocks are important here
//Equation 10
forall(d in DumpBlocks,t in TimePeriods)
  {
             //dumpblockvol
sum( b in PitBlocks,  r in TimePeriods: r<=t)(SwellFactor * Xbdt[b][d][r]) <= dumpVolume[d];
}
//Cons10: Pit blocks on top getting mined first
// Reference : Const (12) of Z.Fu et al
//Equation 11
forall(b in PitBlocksType, t in TimePeriods) {
       // PitBlocksOnTopTotal :
       sum(j in OntopPit[b],  m in Plants, r in TimePeriods: r<=t)(1)*Xbmt[j.id][m][r]
 + sum(j in OntopPit[b],  s in Stockpiles,  r in TimePeriods: r<=t)Xbst[j.id][s][r]
 + sum(j in OntopPit[b],  d in DumpBlocks, r in TimePeriods: r<=t)Xbdt[j.id][d][r]
 - schedulePit[b.id][t]* sum(j in OntopPit[b])totalVolume[j.id] >=0;         }
//Equation 12
forall(b in PitBlocks,  t in TimePeriods)   {
```

```
 //wasteandOre
sum(  m in Plants,r in TimePeriods: r<=t )Xbmt[b][m][r]
+ sum(  s in Stockpiles,r in TimePeriods: r<=t )Xbst[b][s][r]
+ sum( d in DumpBlocks,r in TimePeriods: r<=t )Xbdt[b][d][r]
- schedulePit[b][t] * totalVolume[b] <= 0; }
// Cons12: Dump Blocks below get dumped first
// Reference : Const (15) of Z.Fu et al
//Equation 13
 forall(d in DumpBlocksType, t in TimePeriods) {
  //DumpblocksBelow
  sum( b in PitBlocks,j in OnBelowDump[d],  r in TimePeriods: r<=t)(Xbdt[b][j.id][r]*SwellFa
 - scheduleDump[d.id][t]* sum(j in OnBelowDump[d])(dumpVolume[j.id]) >= 0;}
//cons 13
// Reference : Const (16) of Z.Fu et al
//Equation 14
 forall(d in DumpBlocks,  t in TimePeriods) {
  //DumpblocksVolume
  sum( b in PitBlocks, r in TimePeriods: r<=t)(SwellFactor*Xbdt[b][d][r])
 - scheduleDump[d][t]* (dumpVolume[d]) <= 0;}
//Cons14 : Lag constraint [Not in paper of Z.Fu et al]
//Equstion 15
forall( i in DumpLagInfoXYB,t in TimePeriods) {
       // Lag constraint :
        sum(j in OntopDumpLag[i])(1) * scheduleDump[i.id][t] * totalVolume[j.id] <=
        (sum(j in OntopDumpLag[i],r in TimePeriods : r <= t,d in DumpBlocks)(Xbdt[j.id][d][r
        + sum(j in OntopDumpLag[i],r in TimePeriods : r <= t, s in Stockpiles)(Xbst[j.id][s]
        +sum(j in OntopDumpLag[i],r in TimePeriods : r <= t, m in Plants)(Xbmt[j.id][m][r]))
// /density[j.id]
//StockPile constrains
//Equation 16
forall(s in Stockpiles,t in TimePeriods: t >=2) {
   //StockPile
  StockPileVol[s][t-1] + sum(b in PitBlocks )Xbst[b][s][t] - sum(m in Plants)Xsmt[s][m][t] =
//Equation 17
forall(s in Stockpiles) {
```

```
//Firstyear
sum(b in PitBlocks)Xbst[b][s][1] - sum(m in Plants)Xsmt[s][m][1] == StockPileVol[s][1];}
//Equation 18
//Stockpile capacity within Max Min limits
forall(s in Stockpiles,t in TimePeriods) {
StockPileMinCap[s][t] <= StockPileVol[s][t];}
forall(s in Stockpiles,t in TimePeriods) {
StockPileVol[s][t] <= StockPileMaxCap[s][t] ;}
//Equstion 19
forall(b in PitBlocks, t in TimePeriods )
{
sum(d in DumpBlocks, r in TimePeriods : r <= t )(Xbdt[b][d][r]) >= (wasteVolume[b] - 9999999999 * (1-
//Equation 20 :
forall(b in PitBlocks, t in TimePeriods )
//
{
zbt[b][t] * oreTons[b] >= (sum(m in Plants) Xbmt[b][m][t]+ sum( s in Stockpiles) Xbst[b][s][t] );}
//Equation 21
//...not documented
forall(b in PitBlocks, t in TimePeriods )
//
{
zbt[b][t] <= (sum(m in Plants) Xbmt[b][m][t]+ sum(s in Stockpiles) Xbst[b][s][t] );}
//############### HAULAGE CONSTRAINTS ##########################
//These constraints allow the flow to multile paths
forall( t in TimePeriods )
{
sum(p in Pbm,r in TimePeriods : r <= t) ypt[p.id][r] == sum(b in PitBlocks,m in Plants,r in TimePeri
}
forall(t in TimePeriods )
{
sum(p in Pbs,r in TimePeriods : r <= t) ypt[p.id][r] == sum(b in PitBlocks, s in Stockpiles,r in Tim
forall(t in TimePeriods )
{
sum(p in Pbd,r in TimePeriods : r <= t) ypt[p.id][r] == sum(b in PitBlocks, d in DumpBlocks,r in Tim
```

```
}
// each block should follow an unique path
forall(b in sourceDestD,t in TimePeriods )
   {
    Xbdt[b.source][b.dest][t] == sum(p in Pbd: p.source == b.source && b.dest == p.dest) ypt
forall(b in sourceDestM,t in TimePeriods )
 {
    Xbmt[b.source][b.dest][t] == sum(p in Pbm: p.source == b.source && b.dest == p.dest) ypt
  forall(b in sourceDestS,t in TimePeriods )
 {
    Xbst[b.source][b.dest][t] == sum(p in Pbs: p.source == b.source && b.dest == p.dest) ypt
forall(p in Pbm,q in Pbs,t in TimePeriods )
 {
 sum(r in TimePeriods : r <= t)ypt[p.id][r] +sum(r in TimePeriods : r <= t)ypt[q.id][r] ==
 sum(r in TimePeriods : r <= t) Xbmt[p.source][p.dest][r] +sum(r in TimePeriods : r <= t) Xb
//This could not run even in 6.17 hrs
forall(pd in Pbd,pm in Pbm,ps in Pbs,b in PitBlocks,t in TimePeriods )
  //
 {
 sum(r in TimePeriods : r <= t)ypt[pd.id][r]+ sum(r in TimePeriods : r <= t)ypt[pm.id][r]+su
 sum(r in TimePeriods : r <= t) Xbdt[b][pd.dest][r] + sum(r in TimePeriods : r <= t) Xbmt[b]
 + sum(r in TimePeriods : r <= t) Xbst[b][ps.dest][r]  ; }
*/
//to ensure whatever has been mined has been carried by ypt
forall(t in TimePeriods )
  //ypt
  {
sum(i in Pathid, r in TimePeriods : r <= t ) ypt[i][r]  ==
sum(b in PitBlocks, s in Stockpiles,r in TimePeriods : r <= t  ) Xbst[b][s][r] +
sum(b in PitBlocks, m in Plants,r in TimePeriods : r <= t  ) Xbmt[b][m][r] +
sum(b in PitBlocks, d in DumpBlocks,r in TimePeriods : r <= t  ) Xbdt[b][d][r] ;}
// To ensure that the roads are on surface
// ensure that the pitblockSet is on the surface - meaning blocks below and schedulePit[b][t
//following constraints ensure that the blocks below have not been mined and sum of all X is
forall (p in Pbd) {
```

```
  forall(i in PitBlocksInPathD[p] ,  t in TimePeriods ) {
    sum(j in BlockBelow[i],r in TimePeriods : r <= t, d in DumpBlocks) Xbdt[j.id][d][r] <= sum(r in T
  }  }
forall (p in Pbs) {
  forall(i in PitBlocksInPathS[p] , t in TimePeriods ) {
    sum(j in BlockBelow[i],s in Stockpiles, r in TimePeriods : r <= t   ) Xbst[j.id][s][r] <= sum(s i
forall (p in Pbm) {
  forall(i in PitBlocksInPathM[p] , t in TimePeriods ) {
    sum(j in BlockBelow[i], m in Plants, r in TimePeriods : r <= t   ) Xbmt[j.id][m][r] <= sum( m in
forall (p in Pbd) {
  forall(i in PitBlocksInPathD[p] ,  t in TimePeriods ) {
    sum(j in BlockBelow[i],r in TimePeriods : r <= t) schedulePit[j.id][r] <= sum(r in TimePeriods :
//for dump blocks
forall (p in Pbd) {
  forall(i in DumpBlocksInPathD[p], t in TimePeriods ) {
    sum(b in PitBlocks,j in DumpBlockBelow[i],r in TimePeriods : r <= t   ) Xbdt[b][j.id][r] >= 0;
//following constraints ensure that schedulePit[bb][t] = 1 for all blocks below
/*
forall( i in Pbd ,  t in TimePeriods) {
       // blockabove exposed Pbd:
        sum(j in PitBlocksInPathD[i]) schedulePit[j.id][t] * totalVolume[j.id] <=
        (sum(j in PitBlocksInPathD[i],r in TimePeriods : r <= t,d in DumpBlocks)(Xbdt[j.id][d][r])
        + sum(j in PitBlocksInPathD[i],r in TimePeriods : r <= t, s in Stockpiles)(Xbst[j.id][s][r]/d
        +sum(j in PitBlocksInPathD[i],r in TimePeriods : r <= t, m in Plants)(Xbmt[j.id][m][r]/densit
forall( i in Pbm ,  t in TimePeriods) {
       // blockabove exposed Pbd:
        sum(j in PitBlocksInPathM[i]) schedulePit[j.id][t] * totalVolume[j.id] <=
        (sum(j in PitBlocksInPathM[i],r in TimePeriods : r <= t,d in DumpBlocks)(Xbdt[j.id][d][r])
        + sum(j in PitBlocksInPathM[i],r in TimePeriods : r <= t, s in Stockpiles)(Xbst[j.id][s][r]/d
        +sum(j in PitBlocksInPathM[i],r in TimePeriods : r <= t, m in Plants)(Xbmt[j.id][m][r]/densit
forall( i in Pbs ,  t in TimePeriods) {
       // blockabove exposed Pbd:
        sum(j in PitBlocksInPathS[i]) schedulePit[j.id][t] * totalVolume[j.id] <=
        (sum(j in PitBlocksInPathS[i],r in TimePeriods : r <= t,d in DumpBlocks)(Xbdt[j.id][d][r])
        + sum(j in PitBlocksInPathS[i],r in TimePeriods : r <= t, s in Stockpiles)(Xbst[j.id][s][r]/d
```

```
        +sum(j in PitBlocksInPathS[i],r in TimePeriods : r <= t, m in Plants)(Xbmt[j.id][m][r
*/
// ensure that the dumpblockSet is also on the surface and has all ScheduleDump[d][t] = 1 fo
forall(d in Pbd, t in TimePeriods) {
        sum(j in DumpBlocksInPathD[d]) scheduleDump[j.id][t]  * dumpVolume[j.id] <= sum(j in
//################# END OF HAULAGE CONSTRAINTS ##################
}
// END OF CONSTRAINTS
// Creating Tuples for Writing to Solution Files
tuple SolXbmt{
string b;
string m;
int t;
float x_value;
}
{SolXbmt} solXbmt = {<b, m, t, Xbmt[b][m][t]> | b in PitBlocks, m in Plants, t in TimePeriod
tuple SolXbdt{
string b;
string d;
int t;
float x_value;
}
{SolXbdt} solXbdt    = {<b,d,t, Xbdt[b][d][t] > | b in PitBlocks,  d in DumpBlocks, t in Tim
tuple SolXbst{
string b;
string s;
int t;
float x_value;
}
{SolXbst} solXbst  = {<b,s,t, Xbst[b][s][t]> | b in PitBlocks,  s in Stockpiles,  t in TimeP
tuple SolXsmt{
string s;
string m;
int t;
float x_value;
```

```
}
{SolXsmt} solXsmt = {<s,m,t, Xsmt[s][m][t]> | s in Stockpiles,  m in Plants, t in TimePeriods:Xsmt[s]
// create output of the paths created for each pit block : SourcePitblock, Destination, Path id, Quan
// do sperately for pit to dump Pbd, pit to stockpile Pbs, pit to plant Pbm
tuple SolPath {
string p;
int t;
float y_value;
}
{SolPath} solPath = {<p,t, ypt[p][t]> | p in Pathid, t in TimePeriods:ypt[p][t]>0 };
execute
{
  var o=new IloOplOutputFile("optscheduleXbmt.dat");
  o.writeln(solXbmt);
  o.close();
  var o=new IloOplOutputFile("optscheduleXbdt.dat");
  o.writeln(solXbdt);
  o.close();
  var o=new IloOplOutputFile("optscheduleXbst.dat");
  o.writeln(solXbst);
  o.close();
  var o=new IloOplOutputFile("optscheduleXsmt.dat");
  o.writeln(solXsmt);
  o.close();
  var o=new IloOplOutputFile("optschedulePath.dat");
  o.writeln(solPath);
  o.close();
}
```

Appendix B

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The code provided in Appendix A should not be copied or reproduced in any form.