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Multinational corporations' capital allocation decisions across asymmetric risk locations: intertemporal equilibrium and optimal transitional adjustment paths[†]

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Multinational corporations operate across locations with different risk profiles. We examine how multinational corporations address the optimal allocation of capital across multiple locations and analyse the transition path to the intertemporal equilibrium. Our model considers returns, risks and adjustment costs to reflect the dynamics of allocating capital assets across locations over time, as well as the mix of assets across locations in equilibrium. Variational calculus is employed to show that the model confirms standard expectations that where a location's rates of return on assets increase, or adjustment costs decrease, equilibrium capital allocation and transitional capital flows to that location will increase. Symmetrically, rising (falling) risk increases (decreases) the proportion of the capital asset holdings of a location. The crucial insight is that for the transitional dynamics to intertemporal equilibrium, the optimal relative capital flow response to changes in risk can generate *relative* portfolio allocations that may initially move in the opposite direction to that implied by the stock equilibrium. Specifically, an increase in risk for the high-risk location may initially result in an increase in the relative capital asset flow to the high-risk location *relative* to the low-risk location. Empirical research must account for the possibility of non-monotonicity in asset allocation flows to avoid misspecification. Moreover, policy makers will have to anticipate possible pressure for reversal resulting from short-term worsening capital flows. These reflections are mirrored in recent research calls for separating structural and transition effects of institutional change on the investment decisions by multinational corporations.

Keywords: multinational corporations; institutional risk; locational choice; intertemporal dynamic optimization; intertemporal equilibrium; transitional dynamics; multinational strategy.

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1. Introduction

Multinational corporations (MNCs) by definition operate in multiple locations. An immediate corollary is that a fundamental decision MNCs face is how to allocate capital assets across locations. The associated analytical challenge that we examine in this paper is to determine the potential drivers of the capital allocation decisions. The solution we provide has two dimensions. The first is to provide the terminal equilibrium point for the MNC's capital asset holdings, a state of rest in which there is no further incentive to adjust the capital asset holding in that location, which we term intertemporal equilibrium. The second is to derive the optimal time paths in the adjustment of capital asset holdings of MNCs from any arbitrary initial value to the intertemporal equilibrium value, which we denote the optimal transitional dynamics of MNC capital asset holdings.

It is well established that MNC capital allocation will favour locations with high rates of return, and low risk (Bekaert *et al.*, 2014; Buckley *et al.*, 2018; Giambona *et al.*, 2017; Holburn & Zelner, 2010; Miller, 1998; Makhija & Stewart, 2002). However, much of the prior analysis has focused on MNC entry decisions as individual independent events, rather than as part of an allocation decision across multiple locations with distinct return and risk profiles (Belderbos *et al.*, 2020; Contractor *et al.*, 2020). The result is a failure to recognize that the capital allocation decision of the MNC fundamentally reflects multiple locations, whose relative returns, costs and risks drive the capital allocation decision of the MNC (Ghemawat, 1991; Nachum & Song, 2011).

A further limitation of much prior theory regarding locational capital allocation of MNCs has been that the analytics have concentrated on comparative statics rather than dynamics (Belderbos *et al.*, 2019; Brouthers *et al.*, 2008; Chi *et al.*, 2019; Rugman, 1979). Yet since risk is subject to change over time, and such change may be differential across locations (Luiz & Barnard, 2022), for firms that operate in international contexts, this inherently requires dynamic analysis (Aguilera *et al.*, 2019; Buckley *et al.*, 2018).

A number of responses to such limitations have emerged in the literature. Thus international business scholars (Kogut & Kulatilaka, 1994; Rugman, 1979) used portfolio theory to explain that a key reason why firms internationalized was in fact to diversify risk. MNC's geographic allocation decisions can mitigate the capriciousness of revenue streams because variances in demand conditions can average out idiosyncratic risks (Belderbos *et al.*, 2020). By way of extension, real options theory (Belderbos *et al.*, 2019; 2020; Chi *et al.*, 2019; Kogut & Kulatilaka, 1994; Rodrigo, 2022; Wu & Lin, 2005) suggests that MNCs possess a portfolio of switching options not available to purely domestic firms, allowing the MNC to shift investments across locations as it responds to risk and return within its portfolio of diversified global investments (Luehrman, 1998). The crucial additional insight that real options theory provides is that the irreversibility of investment decisions renders the adjustment of capital allocations 'sticky' (Dixit & Pindyck, 1994). It is harder to change the locational capital allocation of an MNC than in a pure financial portfolio of assets (Fontanari *et al.*, 2021; Gaillardet & Hachem, 2022; Li *et al.*, 2020; Ortobelli Lozza *et al.*, 2017). The result is that MNCs in the face of uncertainty defer investment decisions (even if mandated by net return considerations) until uncertainty reduces, reducing the responsiveness of capital flows to their determinants—MNCs effectively exercise the option to wait (Brouthers *et al.*, 2008; Tong & Reuer, 2007). Yet extant literature generally considers the 'start' point and the 'end' point, without much attention to the path for getting there.

Our approach considers a capital allocation decision of an MNC across two locations. The two locations are distinguished by a risk asymmetry, such that one location has negligible risk, the other faces non-negligible risk. By risk we allow both for the possibility that the 'raw' return on an asset may not be fully realized, and for the probability value of such a loss occurring. For the sake of generality, we also allow returns net of operating cost, and the costs of adjustment of capital asset holding to be distinct across the two locations. Importantly, for our purposes risk is not represented by the volatility of returns, but by a loss of a proportion of asset returns. We posit that this representation is useful for the context of MNC capital allocation decisions, since the risk associated with a significant proportion of international jurisdictions results from institutional dispensations that allow for the predation of special interest on the raw returns that MNC are able to generate from their capital asset holdings (Barnard & Luiz, 2018; Bernard *et al*., 2017; Henisz & Delios, 2004; Liesch *et al*., 2011). The explicit consideration of adjustment costs in the intertemporal allocation decision of the MNC is mandated by the established findings that entry involves the costs associated with acquiring knowledge about business and institutional conditions in the host location (Eriksson *et al*., 2015), as does exit (Dai *et al*., 2017; Norbäck, *et al*., 2015; Tan & Sousa, 2019).

An infinitely lived MNC then maximizes its net present value of rates of return adjusted for risk and adjustment costs in the two state variables provided by capital allocation to the two location types (low and high risk) faced by the firm, subject to a time rate of discount (by specifying a time rate of discount we are simply rendering explicit that the discount rate applies across the time dimension, rather than across values specified in a static context). Variational calculus allows for the derivation of intertemporal equilibrium values and optimal transitional time paths of adjustment of both capital assets simultaneously. The optimal behaviour of the MNC thus reflects simultaneous optimal capital allocations across multiple locations, distinguished by returns net of operating costs, adjustment costs and risk profile, both in the optimal time path of adjustment and in intertemporal equilibrium. This allows reflection both on the absolute capital asset allocation, and the relative capital asset allocation in intertemporal equilibrium and in transitional dynamics.

Importantly, our model confirms standard expectations surrounding absolute and relative stock allocations of capital assets in intertemporal equilibrium. The important novel insight of the model relates to the transitional dynamics to intertemporal equilibrium, in which relative portfolio allocation may initially move in the opposite direction to that implied by the intertemporal stock equilibrium.

The implication of our research is that empirical research on capital allocation decisions must account for possible countervailing consequences of changing risk conditions over shorter and longer time frames. At a macro level it implies that policy interventions that improve risk conditions, through for example institutional advances, might not immediately bear fruit and why pressure often builds for policy reversals in the face of a short-term deterioration in capital inflows. Understanding how this translates at the micro managerial decision-making level in the face of uncertainty raises interesting possibilities (Buckley & Casson, 2019). A concrete example illustrates that these theoretical insights are more than mere conjecture. The French company Danone first invested in the Chinese market in 1996 through a series of joint ventures with the Chinese Wahaha group. The investment was successful and saw revenues rising from \$100 million in 1996 to \$2.25 billion in 2006. Due to its success in the Chinese market, by 2006 Danone sought to increase its investment in the country and buy out Wahaha's share of the joint ventures. Things went awry and various risks attached to the commercial operations and the institutional environment in China became apparent and Danone was forced to sell its 51% share in the joint ventures instead. Thus, its desire to expand its investment resulted in a (temporary) divestment as various risks attached to its ventures in the country. Danone subsequently then re-entered China, with the Chinese market presently its second largest regional market (Peng & Meyer, 2019).

The paper is structured as follows. Section 2 sets up the model and presents the decision problem of the MNC. Section 3.1 derives the intertemporally optimal locational asset holdings and optimal adjustment paths of locational asset holdings, while Section 3.2 characterizes optimal intertemporal asset mixes, and optimal capital flow mixes. Section 4 concludes and draws implications for empirical research surrounding capital asset allocation and managerial behaviour.

2. The model

Infinitely lived MNCs confront an intertemporal locational choice in the allocation of capital assets in the face of risk. Capital assets generate returns net of operating costs. MNCs may reallocate their capital holdings between locations, subject to adjustment costs. Locations are distinguished by differential risk, defined as a loss on net returns (e.g. due to expropriation). The objective of MNCs is to maximize the net present value of expected net returns, subject to adjustment costs and risk. We approach the modelling by means of the calculus of variations. Fedderke (2002) provides a precursor.

MNCs hold capital assets, denoted K, across two locations, L and H, with L denoting low-risk, and H a high-risk location (risk is defined below). Both capital holdings, $K_i, i \in \{L, H\}$, map into returns net of operating costs, $R_i, i \in \{L, H\}$, respectively. The $K_L \to R_L, K_H \to R_H$ mappings are then given by

$$R_L = \gamma K_L - \delta K_L^2, \;\; R_H = \alpha K_H - \beta K_H^2, \quad \{\alpha, \beta, \gamma, \delta\} \in \mathbb{R}, \quad \alpha, \beta, \gamma, \delta > 0 \tag{1}$$

The real positive number constraint on the parameter set, $\{\alpha, \beta, \gamma, \delta\}$, imposes concavity and diminishing marginal returns.[1] The first motivation for the imposition of diminishing returns is the standard argument from economic production theory, that equilibrium and price determination require all efficient subsets of the output sets be bounded (i.e. returns cannot be raised indefinitely by ever more intensive use of given inputs). In general, this would hold both where the asset holdings of the MNC reflects real investment (in plant and equipment) and where the holdings reflect equity ownership of underlying real assets (provided returns on financial assets are linked to those of the underlying real assets).[2] A second motivation for the concavity assumption is that together with convex adjustment costs it ensures that the integrand of the decision problem specified in equation (8) below is bounded, a sufficient condition for convergence to an optimum. Further, in general we also assume that returns across L, H are distinctly responsive to capital holdings, such that $\alpha \neq \gamma, \beta \neq \delta$. Total returns, R, net of operating costs from the capital asset holdings of the MNC are then

$$R = R_L + R_H = \gamma K_L - \delta K_L^2 + \alpha K_H - \beta K_H^2 \tag{2}$$

Risk allows for the possibility that the ‘raw’ return on an asset, R_i, as specified under equation (1) may not be fully realized. Risk then entails both that a proportion of the raw return may not be realized, and a probability value of such a loss occurring. Let λ_i denote the proportion of the asset return that may

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[1]Choice of the second-order polynomial concavity representation rests on the Weierstrass approximation theorem (Weierstrass, 1885, Stone, 1937, 1948) result that every continuous function defined on a closed interval can be uniformly approximated as closely as desired by a polynomial function. Given a suitably specified closed interval, a second-order polynomial captures the possibility of diminishing returns in asset holdings, such that an upper bound defined by the first-order conditions, $\partial R_H/\partial K_H = 0$, $\partial R_L/\partial K_L = 0$, is present for returns on high- and low-risk location assets, given the decreasing rate of return to both classes of assets, $\partial^2 R_H/\partial K_H^2 < 0$, $\partial^2 R_L/\partial K_L^2 < 0$.

[2]The ‘law of diminishing returns’ is standard to microeconomic textbooks. For a formal derivation of the mathematical requirements for the law to hold, see Shephard (1970: 42ff.) For interested readers, Shephard traces the theoretical treatment of the law through the contributions of Eichhorn, K.Menger, Wicksell, Boehm-Bawerk, to the 18th century physiocrat Turgot. See also the discussion in Abel & Eberly (1994).

be lost, and p_i the probability that the loss will *not* occur. Then the standard expected return to an asset holding can be represented as

$$\begin{aligned} E\left(R_i\right) &= R_i p_i + R_i\left(1-\lambda_i\right)\left(1-p_i\right), \quad 0 \le p_i \le 1, \quad 0 \le \lambda_i \le 1, \quad i \in \{L, H\} \\ &= R_i\left(1-\lambda_i\left(1-p_i\right)\right), \end{aligned} \tag{3}$$

where E denotes the mathematical expectation operator. Now define $\pi_i \equiv \lambda_i\left(1-p_i\right)$, such that π_i incorporates *both* the magnitude of the potential loss (λ_i) *and* the probability of the loss occurring $\left(1-p_i\right)$. Note that given $0 \le p_i \le 1, 0 \le \lambda_i \le 1$, it follows that $0 \le \pi_i \le 1$. By assumption there is risk asymmetry between assets across the L, H, locations.[3] For the sake of analytical convenience risk asymmetry across locations is represented by assigning $\pi_L = 0$ and $0 < \pi_H < 1$.[4,5,6]

Returns corrected for risk, which henceforth we denote as effective returns,[7] across the two locations are now

$$\begin{aligned} R_H^e &= R_H\left(1-\pi_H\right) = \left[\alpha K_H - \beta K_H^2\right]\left(1-\pi_H\right), \quad 0 < \pi_H < 1 \\ R_L^e &= R_L\left(1-\pi_L\right) = \left[\gamma K_L - \delta K_L^2\right]\left(1-\pi_L\right) = \gamma K_L - \delta K_L^2, \quad \pi_L = 0 \end{aligned} \tag{4}$$

under the notation and parameter sign restrictions already defined, and where R_L^e, R_H^e denotes that asset returns have been adjusted for risk. Effective total returns, R^e, then are

$$R^e = R_L^e + R_H^e = \gamma K_L - \delta K_L^2 + \left[\alpha K_H - \beta K_H^2\right]\left(1-\pi_H\right), \quad 0 < \pi_H < 1 \tag{5}$$

It is worth noting that where $\left[\alpha K_H - \beta K_H^2\right] \le \left[\gamma K_L - \delta K_L^2\right]$, given the assumptions that $\pi_L = 0$, $0 < \pi_H < 1$, it would follow that $R_H^e < R_L^e$, with the trivial $K_H = 0$ result. The problem statement and solution applies to cases in which $\left[\alpha K_H - \beta K_H^2\right] > \left[\gamma K_L - \delta K_L^2\right]$.

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[3] 'Low' risk locations may be thought of as developed economies, with well-functioning institutions in which the rule of law and due process applies, so as to allow for the enforcement of contracts. 'High' risk locations may be less developed economies, with incomplete institutions, which may function sporadically or with corruption such that contract enforcement may be incomplete.

[4] There is no loss of generality by setting $\pi_L = 0$. The alternative of specifying $0 < \pi_L < \pi_H < 1$ yields the same inferences as those derived below, but with greater notational complexity.

[5] Note that the assumption of zero risk for the low-risk location does not render the decision problem non-covergent due to infinite returns, given the assumption of concave returns (see the assumptions regarding the real positive number constraint on the parameter set, $\{\alpha, \beta, \gamma, \delta\}$), as well as the convexity of adjustment costs and the presence of a time rate of discount - on which see below.

[6] We suppress any consideration of principal–agent considerations dealing with differential risk evaluations between holders of equity (principals) and managers (agents) of firms in the interest of analytical tractability.

[7] Since $(1-\pi_i)$ is defined across the unit interval, effective returns have an analogue in expected returns (returns corrected for the probability of realizing the raw return). Returns corrected for risk are conventionally represented in an expected return format such as $E(R) = R_{high}p + R_{low}(1-p_i)$, in which the unfavourable return outcome subsumes our explicit λ_i discount value, thus placing emphasis on the probability of the 'high' or 'low' value outcome. Our representation, while a strict analogue, is more compact by weighting raw returns by the single π_i parameter which incorporates *both* the unfavourable outcome discount λ_i and the probability of the unfavourable outcome $(1-p_i)$. The advantage is both greater compactness of representation, and that two distinct elements of risk (discount, probability) are rendered explicit. Despite the strict association with expected returns, we explicitly acknowledge the distinct representation of risk and its impact on returns by the use of the alternative label of 'effective' returns.

Adjustment costs are incurred by MNCs due to decisions to change the level of capital holdings in either location, such that for the time rates of change $K_L^{'} = dK_L/dt > 0$, and/or $K_H^{'} = dK_H/dt > 0$, with t denoting a time unit metric. Let $C_i, i \in \{L, H\}$, denote adjustment costs across the low- and high-risk locations, respectively. The $K_L^{'} \to C_L, K_H^{'} \to C_H$ mappings are then

$$C_L = cK_L^{'} + dK_L^{'2}, \quad C_H = aK_H^{'} + bK_H^{'2}, \quad \{a, b, c, d\} \in \mathbb{R}, \quad a, b, c, d > 0 \tag{6}$$

The positive real number restriction on the parameter set, $\{a, b, c, d\}$, imposes convexity and hence increasing marginal adjustment costs.[8] The first motivation for the convexity assumption is the one standard to economic theory, that greater intensity of any activity increasingly draws on resources that have ever greater opportunity cost, thus raising the marginal cost of its use. In general this would hold both where the adjustment of asset holdings is in real real assets (where accelerated changes in plant and equipment may force use of either less productive factors of production or higher prices in bidding more productive factors of production from their alternative uses), or financial assets (where accelerated changes in financial asset holding may generate greater price premia or discounts in purchases or sales, respectively, higher underwriting costs, etc.).[9] The second motivation for the convexity assumption is that together with concave returns it ensures that the integrand of the decision problem specified in equation (8) below is bounded, a sufficient condition for convergence to an optimum.[10] Again, in general we also assume that adjustment costs are distinct across L, H, in the sense that $a \neq c, b \neq d$. Total adjustment costs, C, of the MNC are then

$$C = C_L + C_H = cK_L^{'} + dK_L^{'2} + aK_H^{'} + bK_H^{'2} \tag{7}$$

The net present value of the effective return on a portfolio of capital assets held across the two locations over an infinite time horizon for the MNC is then

$$N\left[K_H, K_L\right] = \int_0^{\infty} \left(R^e - C\right) e^{-\rho t} dt, \tag{8}$$

where ρ denotes the MNC time rate of discount, t denotes a time unit metric, and in which the assumed functional forms ensure that $(R^e - C)$ is bounded, so that present value is rendered convergent.

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[8] This thus gives positive marginal adjustment costs, $\partial C_H/\partial K_H^{'} > 0$, $\partial C_L/\partial K_L^{'} > 0$, increasing at an increasing rate, $\partial^2 C_H/\partial K_H^{'2} > 0$, $\partial^2 C_L/\partial K_L^{'2} > 0$.

[9] Note that the assumption is thus the standard Marshallian theory of cost, as formalized at least since Schultz (1929) and Viner (1931), and standard to microeconomic textbooks. See further the discussion in Hayashi (1982), and note that the assumption is supported by a range of empirical evidence—see Gilchrist & Himmelberg (1995), Cooper & Ejarque (2001), Whited (1998)—though circumstances in which convexity may not apply have also been found—see Caballero *et al.,* (1995), Cooper *et al* (1999), Cooper & Haltiwanger (1993, 2000) and Abel & Eberly (1994).

[10] Concave net returns and convex adjustment costs have the added analytical advantage of providing an upper bound value to the intertemporal allocation problem of the investor—see equation (8) below. In contrast, under concave adjustment costs, for which adjustment costs increase at a decreasing rate in the intensity of activity, the prospect of unbounded returns to the activity such as investment would mandate infinitely large allocations to the activity under consideration. This is empirically implausible. Similarly, linear adjustment costs would render the intertemporal optimization problem trivial, with simple first-order conditions specifying capital allocation at each point in time. The implication would be that investment is simply not intertemporal in nature, an inference that is again empirically implausible both from a consideration of the accounts of practitioners and of the academic literature.

The decision problem of the MNC

$$\underset{K_H,K_L}{\arg\max}\left(N\left[K_H,K_L\right]\right) \tag{9}$$

is to arrive at the optimal intertemporal and time path values of the choice variables, K_H, K_L, which determine the allocation of capital assets of the MNC across the L, H, locations with distinct risk profiles.[11]

3. Model solution

The interest of the analysis lies in the allocation of capital across multiple locations characterized by different risk (by assumption), and potentially also by distinct returns and adjustment costs. Analysis is in continuous time over infinite time horizons subject to time discounting and convex adjustment costs. Our interest lies not only in the comparative statics between the initial location of the decision maker, and the terminal point provided by intertemporal equilibrium, but in confronting the decision maker's choice between alternative time paths from the initial location to the intertemporal equilibrium. As such, the choice of the decision maker is between entire functional mappings (in time), not between points contained within the mapping. For this the differential calculus does not serve. Instead variational calculus techniques offer a straightforward solution framework. Focus of the analytics that follow is the derivation of two core solutions, for both of the asset allocations. The first is the terminal equilibrium point for the MNC's asset holding, a state of rest in which there is no further incentive to adjust the capital asset holding in that location. To reflect that this is not a comparative static equilibrium point, but the result of an optimal adjustment process in asset holding over time, we term the equilibrium the intertemporal equilibrium. The second important solution we derive is the optimal adjustment path in asset holdings in any location from an initial holding of capital assets in the location, to the intertemporal equilibrium holding of that asset (trivially, if the initial capital asset holdings correspond to the intertemporal equilibrium value, there is only a zero-valued adjustment; in general there is a non-zero adjustment value). We term these adjustment paths the transitional dynamics of the solution (strictly, all adjustment paths from initial to terminal state provide transitional dynamics, of which the optimal adjustment path(s) are a subset). Thus intertemporal equilibrium will specify an equilibrium state value for the capital asset holding (simply a *level* of $K_i, i \in \{L.H\}$), while the transitional dynamics specify an optimal *flow* of the asset value given by a time rate of change, $K_i^{'}, i \in \{L, H\}$.

[11] An important literature has begun to explore the impact of uncertainty on asset allocation through the channel of parameter uncertainty (see Belderbos *et al*., 2019; Buckley *et al*., 2018; Chi *et al*., 2019; Ioulianou *et al*., 2021). While our model does not treat parameter uncertainty explicitly, implicitly one could think of the π_i parameters as encapsulating a 'net' parameter uncertainty effect. Alternatively, Lemmas 1 through 3 below specify the impact of changes in the parameters of our model on asset allocations in our model. Provided that the decision maker knows the sign of the parameter change, the impact on the optimal asset holdings and the optimal asset mix can be determined directly from these results.

3.1 *Intertemporal portfolio equilibrium and optimal adjustment paths*

The intertemporal equilibrium value for the K_H state variable, denoted $\overline{K_H}$, and its optimal time path of adjustment, $I_H^*(t)$, can be shown to be (Appendix A.1 provides the derivation):

$$\overline{K_H} = \frac{(1-\pi_H)\,\alpha - a\rho}{2\beta\,(1-\pi_H)} \tag{10}$$

$$I_H^*(t) = K_H^{*'}(t) = \frac{1}{2}\left(\rho - \left(\rho^2 + \frac{4\beta\,(1-\pi_H)}{b}\right)^{\frac{1}{2}}\right)\left(K_{H,0} - \overline{K_H}\right) e^{\frac{1}{2}\left(\rho - \left(\rho^2 + \frac{4\beta(1-\pi_H)}{b}\right)^{\frac{1}{2}}\right)t}$$

$$\textit{with } I_H^*(t) \gtreqless 0 \textit{ as } \left(K_{H,0} - \overline{K_H}\right) \lesseqgtr 0,$$

where $K_{H,0}$ denotes the K_H-state in time period 0. Symmetrically for the K_L state variable:

$$\overline{K_L} = \frac{\gamma - c\rho}{2\delta} \tag{11}$$

$$I_L^*(t) = K_L^{*'}(t) = \frac{1}{2}\left(\rho - \left(\rho^2 + \frac{4\delta}{d}\right)^{\frac{1}{2}}\right)\left(K_{L,0} - \overline{K_L}\right) e^{\frac{1}{2}\left(\rho - \left(\rho^2 + \frac{4\delta}{d}\right)^{\frac{1}{2}}\right)t}$$

$$\textit{with } I_L^*(t) \gtreqless 0 \textit{ as } \left(K_{L,0} - \overline{K_L}\right) \lesseqgtr 0$$

To aid intuition in the discussion that follows, see the associated phase diagram in Fig. 1. The phase diagram presents both state (here capital asset holdings in high- and low-risk locations) and adjustment (investment flows in high- and low-risk assets) in a two-space, to represent both intertemporal equilibrium and adjustment paths to equilibrium. For both asset types (high- and low-risk), from the Euler equation of each state variable we obtain the associated demarcation curves (Appendix A.2 provides the derivation):

$$K_H = \frac{\alpha\,(1-\pi_H) - a\rho}{2\beta\,(1-\pi_H)} - \left(\frac{\rho b}{\beta\,(1-\pi_H)}\right) I_H; \quad \text{and} \quad I_H = 0 \tag{12}$$

$$K_L = \frac{\gamma - c\rho}{2\delta} - \left(\frac{\rho d}{\delta}\right) I_L; \quad \text{and} \quad I_L = 0, \tag{13}$$

which, for each of the two asset classes, provides the phase diagram. Intertemporal equilibrium for the two asset classes, $\overline{K_H}, \overline{K_L}$, is defined by the confluence of the steady state conditions that $K_H^{'} = 0 = I_H^{'}$, $K_L^{'} = 0 = I_L^{'}$. Note that since in general across locations parameters determining the rate of return, adjustment cost and risk are not equivalent, $\alpha \neq \gamma$, $\beta \neq \delta$, $a \neq c$, $b \neq d$, $\pi_H \neq 0$, in general intertemporal equilibrium holdings of the two asset classes will differ, and the responsiveness of asset holdings to investment flows will also be distinct.

Detailed economic intuitions associated with these results are rendered explicit in Section 3.2 below. Nonetheless, we note the following:

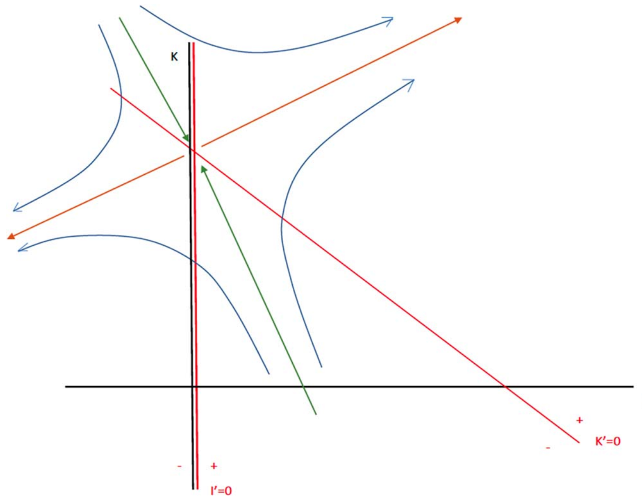

FIG. 1. Generic phase diagram for each of the asset classes (low or high risk). The yellow brick road of stable adjustment to intertemporal equilibrium (the green adjustment paths) identifies the unique interaction between investment and its rate of adjustment.

- An increase in the rate of return on an asset increases the holdings of the asset in intertemporal equilibrium (note the impact of an increase of α on $\overline{K_H}$, and an increase of γ on $\overline{K_L}$). Increased concavity in returns (in the sense of an increase in β or δ) reduces the intertemporal equilibrium holdings of the associated capital asset. See also Lemma 2.
- An increase in adjustment costs in asset holdings lowers the intertemporal equilibrium holdings of that asset (note the impact of an increase in a on $\overline{K_H}$, and an increase of c on $\overline{K_L}$). However, the severity of convexity in the adjustment costs (b, d) does not affect intertemporal equilibrium holdings of the assets. However, increased convexity of adjustment costs *does* affect capital asset holdings along the transitional dynamics. Note that an increase in b increases $I_H^*(t)$ in equation (10), while increase in d increases $I_L^*(t)$ in equation (11). Then from equations (12) and (13) the optimal capital asset holding at each time point of adjustment to intertemporal equilibrium will be lower. The intuition is immediate: the more costly rapid adjustment, is, the slower optimal adjustment will prove.
- Both sets of results are immediately intuitive: higher returns result in increased asset holdings, higher adjustment costs lower asset holdings.
- An increase in the time rate of discount, ρ, lowers the equilibrium levels of all capital asset classes (see the impact of an increase in ρ on both $\overline{K_H}$ and $\overline{K_L}$). Again, the result accords with standard economic intuition: higher rates of time discount lowers capital accumulation due to the implied higher cost of capital accumulation.
- An increase in risk lowers the holdings of the associated capital asset in intertemporal equilibrium (note the impact of an increase in π_H on $\overline{K_H}$). The intuition in this instance is standard and immediate. See also Lemma 3.
- As already rendered explicit in the model set-up, convergence to an intertemporal equilibrium is ensured by the fact that diminishing returns govern returns to capital asset holdings, combined with

adjustment costs that increase at an increasing rate, and the presence of a discount rate over future returns net of adjustment costs.

- The transitional dynamics are not simply a set of comparative static solutions for optimal capital asset holdings, but instead link capital asset holdings across time periods due to the presence of nonlinear adjustment costs.

The core interest of our analysis lies in the *mix* of capital assets held by MNCs across high- and low-risk locations. For this reason, we introduce two ratios that capture the capital asset mix for the MNC, both in intertemporal equilibrium, and in transitional dynamics. Define ϖ_K as the ratio of the *stock* of low- and high-risk capital holdings after agents have adjusted to optimal capital holdings, from (A1) and (A4) (see Appendix A.1):

$$\varpi_K \equiv \frac{\overline{K_L}}{\overline{K_H}} = \frac{\beta\left(\gamma - c\rho\right)\left(1-\pi_H\right)}{\delta\left[\left(1-\pi_H\right)\alpha - a\rho\right]} \tag{14}$$

Note that the optimal intertemporal asset mix can thus assume all of $\varpi_K \gtreqless 1$, conditional on $\frac{\gamma - c\rho}{2\delta} \gtreqless \frac{\alpha(1-\pi_H)-a\rho}{2\beta(1-\pi_H)}$, i.e. whether the low-risk intertemporal equilibrium asset holding value is greater, equal or less than the high-risk intertemporal equilibrium asset holding value.

Further define ϖ_I as the ratio of the *flow* of funds to low-risk assets, to the flow of funds to high-risk assets,from (A3) and (A5) (see Appendix A.1):

$$\varpi_I = \frac{I_L^*(t)}{I_H^*(t)} = \left(\frac{\rho - \left(\rho^2 + \frac{4\delta}{d}\right)^{\frac{1}{2}}}{\rho - \left(\rho^2 + \frac{4\beta(1-\pi_{dH})}{b}\right)^{\frac{1}{2}}}\right)\left(\frac{K_{L,0} - \overline{K_L}}{K_{H,0} - \overline{K_H}}\right) e^{\frac{1}{2}t\left[\left(\rho^2 + \frac{4\beta(1-\pi_H)}{b}\right)^{\frac{1}{2}} - \left(\rho^2 + \frac{4\delta}{d}\right)^{\frac{1}{2}}\right]}, \tag{15}$$

where again all of $\varpi_I \gtreqless 1$ is feasible, conditional on whether $I_L^*(t) \gtreqless I_H^*(t)$.

This characterizes the intertemporal equilibrium and the optimal time paths to intertemporal equilibrium for assets in both low- and high-risk locations in the MNC portfolio.

3.2 *Optimal asset mixes*

Our concern is with the mix of the low- and high-risk locations in the MNC capital allocation, both when that mix achieves equilibrium, and the process of adjustment to intertemporal equilibrium. Effectively therefore our focus is on the behaviour of (14) and (15). A number of core results are germane.

3.2.1 *Reassuring baseline results.* We begin by confirming that our model generates standard results regarding capital allocation accepted in the literature. These reflect asymmetric capital distribution across locations with distinct risk profiles, intuitive responses to net returns and adjustment costs. The results are reassuring since they demonstrate that the results of Section 3.2.2 are not simply a reflection of a non-standard set-up of our analytical framework.

The first of these implications is that MNCs in their choice of an optimal capital allocation across locations differentiate between assets held in the distinct (low versus high risk) locations. We state this formally as

LEMMA 1. Both optimal asset holdings in intertemporal equilibrium, and optimal time paths in asset holdings are asymmetrical between assets holdings in low- and high-risk locations in the MNC portfolio of capital assets.

Proof. For (a.) follows trivially from comparison of (A2) and (A4), for (b.) from comparison of (A3) and (A5). □

The asymmetry arises due to the possibility of distinct parameterization of the return and adjustment cost functions, which raises the possibility that any or all of $\alpha \neq \gamma$, $\beta \neq \delta$, $a \neq c$, $b \neq d$, hold. But note that even where return and adjustment cost structures are identical across locations, such that $\alpha = \gamma$, $\beta = \delta$, $a = c$, $b = d$, asymmetry also arises due to our assumption of risk differentials, across locations, with $\pi_H > \pi_L = 0$. The implication is that in general an optimizing MNC will clearly differentiate between assets held in different locations, likely both because of distinct return and cost performance, but at a minimum due to distinct risk profiles of assets held in different locations.

The second is simply that MNCs respond positively in their capital allocation decisions to factors that raise the return on capital in a location.

LEMMA 2. An increase in expected returns net of adjustment costs on any asset class not due to changes in risk increases the holdings of that asset class in intertemporal equilibrium.

Proof. From (4) the marginal rate of return on the high- and low-risk asset classes is given by $\frac{\partial E(R_H)}{\partial K_H} = \left[\alpha - 2\beta K_H\right]\left(1-\pi_H\right)$, $\frac{\partial E(R_L)}{\partial K_L} = \left[\gamma - 2\delta K_L\right]$, while from (6) the symmetrical marginal adjustment costs are $\frac{\partial C_L}{\partial K_L'} = c + 2dK_L'$, $\frac{\partial C_H}{\partial K_H'} = a + 2bK_H'$. Thus an increase in expected returns net of adjustment costs on high-risk assets not due to changes in risk at the margin follows from $d\alpha > 0$, $d\beta < 0$, $da < 0$, $db < 0$, and for low-risk assets symmetrically from $d\gamma > 0$, $d\delta < 0$, $dc < 0$, $dd < 0$.

Given intertemporal equilibrium holdings of low- and high-risk location capital holdings (10, 11):

$$\overline{K_H} = \frac{\left(1-\pi_H\right)\alpha - a\rho}{2\beta\left(1-\pi_H\right)}$$

$$\overline{K_L} = \frac{\gamma - c\rho}{2\delta}$$

it follows immediately that

$$\begin{array}{ll}
\frac{\partial\overline{K_H}}{\partial\alpha} > 0 & \frac{\partial\overline{K_L}}{\partial\gamma} > 0 \\
\frac{\partial\overline{K_H}}{\partial\beta} \leq 0 \; if \; \left(1-\pi_H\right)\alpha - a\rho \geq 0 & \frac{\partial\overline{K_L}}{\partial\delta} \leq 0 \; if \; \gamma - c\rho \geq 0 \\
\frac{\partial\overline{K_H}}{\partial a} < 0 & \frac{\partial\overline{K_L}}{\partial c} < 0 \\
\frac{\partial\overline{K_H}}{\partial b} = 0 & \frac{\partial\overline{K_L}}{\partial d} = 0
\end{array}$$

in which the $\left(1-\pi_H\right)\alpha - a\rho \geq 0$, $\gamma - c\rho \geq 0$ constraints are simply non-negativity constraints on $\overline{K_H}$, $\overline{K_L}$, respectively. Thus determinants of increases unexpected returns net of adjustment costs on high-risk capital either raise $\overline{K_H}$ or are neutral, symmetrically for $\overline{K_L}$. □

Third, increased risk in a location lowers the intertemporal equilibrium capital allocation to that location. Since in our model, $\pi_H > \pi_L = 0$, this manifests itself for the high-risk location, in response to $d\pi_H > 0$, by extension this implies that the mix of capital assets in intertemporal equilibrium shifts from high- to low-risk locations, thus raising ϖ_K. The intuition here is immediate, since an increase in risk lowers the effective return on the associated asset.

LEMMA 3. An increase in risk associated with an asset lowers (a.) the intertemporal equilibrium holdings of the asset, and (b.) the proportion of that asset held in intertemporal equilibrium.

Proof. For proof of Lemma 3 (a.): From $\overline{K_H} = \frac{(1-\pi_H)\alpha - a\rho}{2\beta(1-\pi_H)}$, $\frac{\partial \overline{K_H}}{\partial \pi_H} = \frac{-2a\beta\rho}{[2\beta(1-\pi_H)]^2} < 0$ follows directly. For proof of Lemma 3 (b.): From $\varpi_K \equiv \frac{\overline{K_L}}{\overline{K_H}}$, $\frac{\partial \varpi_K}{\partial \pi_H} = \frac{(\partial \overline{K_L}/\partial \pi_H)\overline{K_H} - \overline{K_L}(\partial \overline{K_H}/\partial \pi_H)}{\overline{K_H}^2} = \frac{-\overline{K_L}(\partial \overline{K_H}/\partial \pi_H)}{\overline{K_H}^2} > 0$, given Lemma 3 (a.). □

3.2.2 *New insight: optimal asset mix in transitional dynamics.* The novel result to emerge from our model relates to the transitional dynamics: the optimal adjustment path of the MNC in adjusting its initial capital holdings in low- and high-risk locations, $K_{L,0}, K_{H,0}$, to their intertemporal equilibrium values, $\overline{K_L}$, $\overline{K_H}$, in response to an increase in risk. Specifically, an increase in risk may result in either an increase or a decrease in the ratio of capital flows to the two locations, $\varpi_I = \frac{I_L^*(t)}{I_H^*(t)}$.[12]

PROPOSITION 1. For $d\pi_d > 0$, the mix of optimal capital asset flows to high and low risk locations, $\varpi_I = \frac{I_L^*(t)}{I_H^*(t)}$, will show one of four responses:

$$Case\ 1 : \frac{\partial \varpi_I}{\partial \pi_H} \gtrless 0 \ if \left\{ \begin{array}{l} |\frac{\partial A_3}{\partial \pi_H}A_1A_2| < \frac{\partial A_1}{\partial \pi_H}A_2A_3 + \frac{\partial A_2}{\partial \pi_H}A_1A_3 \\ |\frac{\partial A_3}{\partial \pi_H}A_1A_2| > \frac{\partial A_1}{\partial \pi_H}A_2A_3 + \frac{\partial A_2}{\partial \pi_H}A_1A_3 \end{array} \right. \text{and if } K_{L,0} < \overline{K_L},\ K_{H,0} < \overline{K_H}$$

$$Case\ 2 : \frac{\partial \varpi_I}{\partial \pi_H} \gtrless 0 \ if \left\{ \begin{array}{l} \frac{\partial A_3}{\partial \pi_H}A_1A_2 > |\frac{\partial A_1}{\partial \pi_H}A_2A_3 + \frac{\partial A_2}{\partial \pi_H}A_1A_3| \\ \frac{\partial A_3}{\partial \pi_H}A_1A_2 < |\frac{\partial A_1}{\partial \pi_H}A_2A_3 + \frac{\partial A_2}{\partial \pi_H}A_1A_3| \end{array} \right. \text{and if } K_{L,0} > \overline{K_L},\ K_{H,0} < \overline{K_H}$$

$$Case\ 3 : \frac{\partial \varpi_I}{\partial \pi_H} \gtrless 0 \ if \left\{ \begin{array}{l} |\frac{\partial A_1}{\partial \pi_H}A_2A_3| < \frac{\partial A_2}{\partial \pi_H}A_1A_3 + \frac{\partial A_3}{\partial \pi_H}A_1A_2 \\ |\frac{\partial A_1}{\partial \pi_H}A_2A_3| > \frac{\partial A_2}{\partial \pi_H}A_1A_3 + \frac{\partial A_3}{\partial \pi_H}A_1A_2 \end{array} \right. \text{and if } K_{L,0} < \overline{K_L},\ K_{H,0} > \overline{K_H}$$

$$Case\ 4 : \frac{\partial \varpi_I}{\partial \pi_H} \gtrless 0 \ if \left\{ \begin{array}{l} \frac{\partial A_1}{\partial \pi_H}A_2A_3 > |\frac{\partial A_2}{\partial \pi_H}A_1A_3 + \frac{\partial A_3}{\partial \pi_H}A_1A_2| \\ \frac{\partial A_1}{\partial \pi_H}A_2A_3 < |\frac{\partial A_2}{\partial \pi_H}A_1A_3 + \frac{\partial A_3}{\partial \pi_H}A_1A_2| \end{array} \right. \text{and if } K_{L,0} > \overline{K_L},\ K_{H,0} > \overline{K_H}$$

where

$$A_1 \equiv \frac{\rho - \left(\rho^2 + \frac{4\delta}{d}\right)^{\frac{1}{2}}}{\rho - \left(\rho^2 + \frac{4\beta(1-\pi_H)}{b}\right)^{\frac{1}{2}}}$$

$$A_2 \equiv \frac{K_{L,0} - \overline{K_L}}{K_{H,0} - \overline{K_H}}$$

$$A_3 \equiv e^{\frac{1}{2}t\left[\left(\rho^2 + \frac{4\beta(1-\pi_H)}{b}\right)^{\frac{1}{2}} - \left(\rho^2 + \frac{4\delta}{d}\right)^{\frac{1}{2}}\right]}$$

[12]We note that similar novel implications can be derived for changes in the marginal rate of return to capital and marginal cost of adjustment of capital holdings parameters. For the sake of compactness, we focus on the response to changes in risk on the grounds that these are the most germane in practical application.

Proof. Consider

$$\varpi_I = \frac{I_L^*(t)}{I_H^*(t)} = \underbrace{\left(\frac{\rho - \left(\rho^2 + \frac{4\delta}{d}\right)^{\frac{1}{2}}}{\rho - \left(\rho^2 + \frac{4\beta(1-\pi_H)}{b}\right)^{\frac{1}{2}}}\right)}_{A_1}\underbrace{\left(\frac{K_{L,0} - \overline{K_L}}{K_{H,0} - \overline{K_H}}\right)}_{A_2}\underbrace{e^{\frac{1}{2}t\left[\left(\rho^2 + \frac{4\beta(1-\pi_H)}{b}\right)^{\frac{1}{2}} - \left(\rho^2 + \frac{4\delta}{d}\right)^{\frac{1}{2}}\right]}}_{A_3}$$

By contradiction, $\rho - \left(\rho^2 + \frac{4\delta}{d}\right)^{\frac{1}{2}} < 0$ given $\delta, d > 0$, since $\rho - \left(\rho^2 + \frac{4\delta}{d}\right)^{\frac{1}{2}} > 0$ requires $\rho > \left(\rho^2 + \frac{4\delta}{d}\right)^{\frac{1}{2}}$, hence $\rho^2 > \rho^2 + \frac{4\delta}{d}$. Symmetrically by contradiction $\rho - \left(\rho^2 + \frac{4\beta(1-\pi_H)}{b}\right)^{\frac{1}{2}} < 0$ given $\beta, b, (1-\pi_H) > 0$, since $\rho - \left(\rho^2 + \frac{4\beta(1-\pi_H)}{b}\right)^{\frac{1}{2}} > 0$ requires $\rho^2 > \rho^2 + \frac{4\beta(1-\pi_H)}{b}$. Hence necessarily $A_1 > 0$, and

$$\frac{\partial A_1}{\partial \pi_H} = \frac{\left(\rho - \left(\rho^2 + \frac{4\delta}{d}\right)^{\frac{1}{2}}\right)\left[\left(\frac{2\beta}{b}\right)\left(\rho^2 + \frac{4\beta(1-\pi_H)}{b}\right)^{-\frac{1}{2}}\right]}{2\left[\rho - \left(\rho^2 + \frac{4\beta(1-\pi_H)}{b}\right)^{\frac{1}{2}}\right]^2} > 0$$

$$\textit{Next}, A_2 \gtrless 0 \textit{ if } \begin{cases} \left(K_{L,0} - \overline{K_L}\right) > 0 \textit{ and } \left(K_{H,0} - \overline{K_H}\right) > 0, \textit{ or } \left(K_{L,0} - \overline{K_L}\right) < 0 \;\; \left(K_{H,0} - \overline{K_H}\right) < 0 \\ \left(K_{L,0} - \overline{K_L}\right) > 0 \textit{ and } \left(K_{H,0} - \overline{K_H}\right) < 0, \textit{ or } \left(K_{L,0} - \overline{K_L}\right) < 0 \textit{ and } \left(K_{H,0} - \overline{K_H}\right) > 0 \end{cases}$$

$$\Longrightarrow \frac{\partial A_2}{\partial \pi_H} = \frac{\left(K_{L,0} - \overline{K_L}\right)\frac{\partial \overline{K_H}}{\partial \pi_H}}{\left(K_{H,0} - \overline{K_H}\right)^2} \gtrless 0 \textit{ if } \begin{cases} K_{L,0} - \overline{K_L} < 0 \\ K_{L,0} - \overline{K_L} > 0 \end{cases}, \textit{ since } \frac{\partial \overline{K_H}}{\partial \pi_H} < 0 \textit{ from Lemma 3}$$

Further, $A_3 > 0$, *and*

$$\frac{\partial A_3}{\partial \pi_H} = \left[\frac{-\beta t}{b}\left(\left(\rho^2 + \frac{4\beta(1-\pi_H)}{b}\right)^{-\frac{1}{2}}\right)\right] e^{\frac{1}{2}t\left[\left(\rho^2 + \frac{4\beta(1-\pi_H)}{b}\right)^{\frac{1}{2}} - \left(\rho^2 + \frac{4\delta}{d}\right)^{\frac{1}{2}}\right]} < 0$$

Since

$$\frac{\partial \varpi_I}{\partial \pi_H} = \frac{\partial A_1}{\partial \pi_H}A_2A_3 + \frac{\partial A_2}{\partial \pi_H}A_1A_3 + \frac{\partial A_3}{\partial \pi_H}A_1A_2,$$

where

$$\frac{\partial A_1}{\partial \pi_H}A_2A_3 \gtrless 0 \textit{ if } \begin{cases} A_2 > 0 \\ A_2 < 0 \end{cases}, \textit{ since } \frac{\partial A_1}{\partial \pi_H} > 0, \;\; A_3 > 0$$

$$\frac{\partial A_2}{\partial \pi_H}A_1A_3 \gtrless 0 \textit{ if } \begin{cases} K_{L,0} - \overline{K_L} < 0 \\ K_{L,0} - \overline{K_L} > 0 \end{cases}, \textit{ since } A_1 > 0, \;\; A_3 > 0$$

$$\frac{\partial A_3}{\partial \pi_H}A_1A_2 \gtrless 0 \textit{ if } \begin{cases} A_2 < 0 \\ A_2 > 0 \end{cases}, \textit{ since } \frac{\partial A_3}{\partial \pi_H} < 0, \;\; A_1 > 0, \;\; A_3 > 0$$

given

	$K_{L,0} < \overline{K_L}$	$K_{L,0} > \overline{K_L}$
$K_{H,0} < \overline{K_H}$	$A_2 > 0$ $\frac{\partial A_2}{\partial \pi_H} > 0$	$A_2 < 0$ $\frac{\partial A_2}{\partial \pi_H} < 0$
$K_{H,0} > \overline{K_H}$	$A_2 < 0$ $\frac{\partial A_2}{\partial \pi_H} > 0$	$A_2 > 0$ $\frac{\partial A_2}{\partial \pi_H} < 0$

$\frac{\partial \varpi_I}{\partial \pi_H} \gtrless 0$ will be determined by the net effect of the sign restrictions on $\left(K_{L,0} - \overline{K_L}\right)$ and A_2, hence implicitly by the net sign restriction that follows from a comparison of $\left(K_{L,0} - \overline{K_L}\right)$ and $\left(K_{H,0} - \overline{K_H}\right)$. Specifically,

	$K_{L,0} < \overline{K_L}$	$K_{L,0} > \overline{K_L}$
$K_{H,0} < \overline{K_H}$	$\frac{\partial A_1}{\partial \pi_H} A_2 A_3 > 0$ $\frac{\partial A_2}{\partial \pi_H} A_1 A_3 > 0$ $\frac{\partial A_3}{\partial \pi_H} A_1 A_2 < 0$	$\frac{\partial A_1}{\partial \pi_H} A_2 A_3 < 0$ $\frac{\partial A_2}{\partial \pi_H} A_1 A_3 < 0$ $\frac{\partial A_3}{\partial \pi_H} A_1 A_2 > 0$
$K_{H,0} > \overline{K_H}$	$\frac{\partial A_1}{\partial \pi_H} A_2 A_3 < 0$ $\frac{\partial A_2}{\partial \pi_H} A_1 A_3 > 0$ $\frac{\partial A_3}{\partial \pi_H} A_1 A_2 > 0$	$\frac{\partial A_1}{\partial \pi_H} A_2 A_3 > 0$ $\frac{\partial A_2}{\partial \pi_H} A_1 A_3 < 0$ $\frac{\partial A_3}{\partial \pi_H} A_1 A_2 < 0$

This provides four distinct cases that determine the sign restriction on $\frac{\partial \varpi_I}{\partial \pi_H}$.

Case 1: Where $K_{L,0} < \overline{K_L}$ and $K_{H,0} < \overline{K_H}$:

$$\frac{\partial \varpi_I}{\partial \pi_H} = \underbrace{\frac{\partial A_1}{\partial \pi_H} A_2 A_3}_{>0} + \underbrace{\frac{\partial A_2}{\partial \pi_H} A_1 A_3}_{>0} + \underbrace{\frac{\partial A_3}{\partial \pi_H} A_1 A_2}_{<0}$$

$$\implies \frac{\partial \varpi_I}{\partial \pi_H} \gtrless 0 \;\; if \begin{cases} |\frac{\partial A_3}{\partial \pi_H} A_1 A_2| < \frac{\partial A_1}{\partial \pi_H} A_2 A_3 + \frac{\partial A_2}{\partial \pi_H} A_1 A_3 \\ |\frac{\partial A_3}{\partial \pi_H} A_1 A_2| > \frac{\partial A_1}{\partial \pi_H} A_2 A_3 + \frac{\partial A_2}{\partial \pi_H} A_1 A_3 \end{cases}$$

Case 2: Where $K_{L,0} > \overline{K_L}$ and $K_{H,0} < \overline{K_H}$:

$$\frac{\partial \varpi_I}{\partial \pi_H} = \underbrace{\frac{\partial A_1}{\partial \pi_H} A_2 A_3}_{<0} + \underbrace{\frac{\partial A_2}{\partial \pi_H} A_1 A_3}_{<0} + \underbrace{\frac{\partial A_3}{\partial \pi_H} A_1 A_2}_{>0}$$

$$\implies \frac{\partial \varpi_I}{\partial \pi_H} \gtrless 0 \;\; if \begin{cases} \frac{\partial A_3}{\partial \pi_H} A_1 A_2 > |\frac{\partial A_1}{\partial \pi_H} A_2 A_3 + \frac{\partial A_2}{\partial \pi_H} A_1 A_3| \\ \frac{\partial A_3}{\partial \pi_H} A_1 A_2 < |\frac{\partial A_1}{\partial \pi_H} A_2 A_3 + \frac{\partial A_2}{\partial \pi_H} A_1 A_3| \end{cases}$$

Case 3: Where $K_{L,0} < \overline{K_L}$ and $K_{H,0} > \overline{K_H}$:

$$\frac{\partial \varpi_I}{\partial \pi_H} = \underbrace{\frac{\partial A_1}{\partial \pi_H} A_2 A_3}_{<0} + \underbrace{\frac{\partial A_2}{\partial \pi_H} A_1 A_3}_{>0} + \underbrace{\frac{\partial A_3}{\partial \pi_H} A_1 A_2}_{>0}$$

$$\Longrightarrow \frac{\partial \varpi_I}{\partial \pi_H} \gtrless 0 \;\; if \begin{cases} |\frac{\partial A_1}{\partial \pi_H} A_2 A_3| < \frac{\partial A_2}{\partial \pi_H} A_1 A_3 + \frac{\partial A_3}{\partial \pi_H} A_1 A_2 \\ |\frac{\partial A_1}{\partial \pi_H} A_2 A_3| > \frac{\partial A_2}{\partial \pi_H} A_1 A_3 + \frac{\partial A_3}{\partial \pi_H} A_1 A_2 \end{cases}$$

Case 4: Where $K_{L,0} > \overline{K_L}$ and $K_{H,0} > \overline{K_H}$:

$$\frac{\partial \varpi_I}{\partial \pi_H} = \underbrace{\frac{\partial A_1}{\partial \pi_H} A_2 A_3}_{>0} + \underbrace{\frac{\partial A_2}{\partial \pi_H} A_1 A_3}_{<0} + \underbrace{\frac{\partial A_3}{\partial \pi_H} A_1 A_2}_{<0}$$

$$\Longrightarrow \frac{\partial \varpi_I}{\partial \pi_H} \gtrless 0 \;\; if \begin{cases} \frac{\partial A_1}{\partial \pi_H} A_2 A_3 > |\frac{\partial A_2}{\partial \pi_H} A_1 A_3 + \frac{\partial A_3}{\partial \pi_H} A_1 A_2| \\ \frac{\partial A_1}{\partial \pi_H} A_2 A_3 < |\frac{\partial A_2}{\partial \pi_H} A_1 A_3 + \frac{\partial A_3}{\partial \pi_H} A_1 A_2| \end{cases}$$

$\square$

Note therefore that irrespective of the initial capital asset holdings relative to intertemporal equilibrium capital asset holdings for both high- and low-risk locations (i.e. irrespective of whether $K_{L,0} \gtrless \overline{K_L},\ and/or\ K_{H,0} \gtrless \overline{K_H}$), an increase in risk ($d\pi_d > 0$) can result in either an increase or a decrease in relative capital asset flows to the high-risk location.

Table 1 illustrates all four cases noted in Proposition 1. We consider the four cases as defined by the $K_{L,0} \gtrless \overline{K_L}, K_{H,0} \gtrless \overline{K_H}$ associations, and consider an increase in risk, from $\pi_H = 0.2$ to $\pi_H = 0.4$. Note that in all cases, consistent with Lemma 3, the implication is that in intertemporal equilibrium the MNC will favour the low-risk location, as is evident from $\varpi_K > 1$, and that $\varpi_{K,\pi_H=0.4} > \varpi_{K\pi_H=0.2}$ in all four cases. For all four cases we then consider the impact of the increase in risk, under the conditions for both $\frac{\partial \varpi_I}{\partial \pi_H} > 0$ and $\frac{\partial \varpi_I}{\partial \pi_H} < 0$, as specified by Proposition 1. Where $\frac{\partial \varpi_I}{\partial \pi_H} > 0$, the expectation is that $\varpi_{I,\pi_H=0.4}/\varpi_{I,\pi_H=0.2} < 1$, since the implication is that $\varpi_{I,\pi_H=0.2} < \varpi_{I,\pi_H=0.4}$. Conversely, where $\frac{\partial \varpi_I}{\partial \pi_H} < 0$, the expectation is that $\varpi_{I,\pi_H=0.4}/\varpi_{I,\pi_H=0.2} > 1$, since the implication is that $\varpi_{I,\pi_H=0.2} > \varpi_{I,\pi_H=0.4}$. For all four cases this is confirmed by the numerical simulations of Table 1.[13]

Thus, irrespective of the initial conditions in terms of capital asset holdings relative to intertemporal equilibrium, for both the low- and the high-risk location, an increase in risk for the high-risk location may initially result either in an *increase* in the capital asset flow to the high-risk location relative to the low-risk location, or in a *decrease*.

This fundamental insight implies that the optimal mix of capital flows towards intertemporal equilibrium in response to changes in risk is difficult to predict empirically. It reflects the combinatorial possibilities implicit in the $\left(K_{L,0} - \overline{K_L}\right) \gtrless 0$ and $\left(K_{H,0} - \overline{K_H}\right) \gtrless 0$ conditions the firm may face, and

[13]Note that for $\frac{\partial \varpi_I}{\partial \pi_H} > 0$ we also specify the time point at which the $\frac{\partial \varpi_I}{\partial \pi_H} = 1$ value is realized, such that the intertemporal equilibrium increase in low-risk location capital asset values can be realized. Table 1 also specifies the parameter values for which the numerical simulations provide the results mandated by Proposition 1.

TABLE 1 *Numeric Illustration—Four Cases*

	Case 1		Case 2		Case 3		Case 4	
	$K_{L,0} < \overline{K_L}, K_{H,0} < \overline{K_H}$		$K_{L,0} > \overline{K_L}, K_{H,0} < \overline{K_H}$		$K_{L,0} < \overline{K_L}, K_{H,0} > \overline{K_H}$		$K_{L,0} > \overline{K_L}, K_{H,0} > \overline{K_H}$	
$\pi_H = 0.4:$	$\varpi_K = 1.35$	$\varpi_K = 3.97$	$\varpi_K = 2.85$	$\varpi_K = 1.35$	$\varpi_K = 1.49$	$\varpi_K = 1.35$	$\varpi_K = 1.35$	$\varpi_K = 3.60$
$\pi_H = 0.2:$	$\varpi_K = 1.29$	$\varpi_K = 3.46$	$\varpi_K = 2.71$	$\varpi_K = 1.29$	$\varpi_K = 1.41$	$\varpi_K = 1.29$	$\varpi_K = 1.29$	$\varpi_K = 3.43$
Time	$\frac{\partial \varpi_I}{\partial \pi_H} > 0$	$\frac{\partial \varpi_I}{\partial \pi_H} < 0$	$\frac{\partial \varpi_I}{\partial \pi_H} > 0$	$\frac{\partial \varpi_I}{\partial \pi_H} < 0$	$\frac{\partial \varpi_I}{\partial \pi_H} > 0$	$\frac{\partial \varpi_I}{\partial \pi_H} < 0$	$\frac{\partial \varpi_I}{\partial \pi_H} > 0$	$\frac{\partial \varpi_I}{\partial \pi_H} < 0$
1	0.07	4.25	0.60	2.98	0.19	2.40	0.41	1.12
2	0.09	4.96	0.70	3.47	0.19	2.80	0.48	1.31
3	0.10	5.79	0.81	4.05	0.20	3.27	0.55	1.52
4	0.12	6.76	0.95	4.73	0.21	3.82	0.658	1.78
5	0.14	7.89	1.11	5.52	0.22	4.45	0.76	2.08
6	0.16	9.21	1.29	6.44	0.23	5.20	0.88	2.42
7	0.19	10.75	1.51	7.52	0.24	6.07	1.03	2.83
8	0.22	12.55	1.76	8.78	0.25	7.08	1.20	3.30
9	0.26	14.645	2.05	10.24	0.27	8.268	1.40	3.85
10	0.30	17.09	2.39	11.96	0.29	9.648	1.64	4.50
11	0.35	19.95	2.79	13.95	0.30	11.258	1.91	5.25
12	0.41	23.28	3.26	16.29	0.32	13.14	2.23	6.13
13	0.47	27.17	3.80	19.01	0.33	15.33	2.60	7.15
14	0.55	31.71	4.44	22.18	0.35	17.89	3.03	8.34
15	0.65	37.01	5.18	25.89	0.36	20.88	3.54	9.74
16	0.75	43.19	6.05	30.22	0.38	24.37	4.13	11.37
=1 at T=	16	-	5	-	36	-	8	-
$\alpha:$	1	0.45	1	1	1	1	1	1
$\beta:$	0.5	0.5	0.5	0.5	0.05	0.5	0.5	0.5
$\gamma:$	1	1	2	1	1	1	1	2
$\delta:$	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4
$a:$	1	1	1	1	1	1	1	1
$b:$	0.3	0.3	0.3	0.3	0.3	0.3	0.3	0.3
$c:$	1	1	1	1	1	1	1	1
$d:$	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$\rho:$	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

may trigger either a decreased or an increased relative flow of capital to the location experiencing a risk increase during the transitional dynamics of adjustment to the intertemporal equilibrium.

To understand the associated intuitions for the four cases, note that the four cases fall into two distinct categories, viz. where the disequilibrium state in asset holdings are symmetrical (Cases 1 and 4), and where the disequilibrium state in asset holdings is divergent (Cases 2 and 3). Specific intuitions then follow two associated broad patterns:

- Cases 1 and 4:
  - For Case 1: the increase in risk lowers $\overline{K_H}$, and given that the MNC wishes to increase its holdings in K_L (i.e. $\left(K_{L,0} - \overline{K_L}\right) < 0$), hence $\frac{\partial A_2}{\partial \pi_H} A_1 A_3 > 0$.
  - For Case 4, the increase in risk lowers $\overline{K_H}$, and given that the MNC wishes to decrease its holdings in K_L (i.e. $\left(K_{L,0} - \overline{K_L}\right) > 0$), hence $\frac{\partial A_2}{\partial \pi_H} A_1 A_3 < 0$.
  - Then:
    * Both Cases 1 and 4 share a common relative growth rate effect: the increase in risk lowers the growth rate in K_H holdings relative to the growth rate in K_L holdings, providing the $\frac{\partial A_3}{\partial \pi_H} A_1 A_2 < 0$ and $\frac{\partial A_1}{\partial \pi_H} A_2 A_3 > 0$ results.
    * For Case 1 the stock of capital reallocation effect is as follows: the increase in risk lowers $\overline{K_H}$, and given that the MNC wishes to increase its holdings in K_L ($\left(K_{L,0} - \overline{K_L}\right) < 0$), hence $\frac{\partial A_2}{\partial \pi_H} A_1 A_3 > 0$. Then:
      · Where the *negative* relative growth rate effect ($\frac{\partial A_3}{\partial \pi_H} A_1 A_2 < 0$) dominates the *positive* relative growth rate effect ($\frac{\partial A_1}{\partial \pi_H} A_2 A_3 > 0$) and asset stock reallocation effects ($\frac{\partial A_2}{\partial \pi_H} A_1 A_3 > 0$), the relative allocation of capital assets favours the high-risk location, such that $\frac{\partial \varpi_I}{\partial \pi_H} < 0$.
      · Where the *negative* relative growth rate effect ($\frac{\partial A_3}{\partial \pi_H} A_1 A_2 < 0$) is dominated by the *positive* relative growth rate effect ($\frac{\partial A_1}{\partial \pi_H} A_2 A_3 > 0$) and asset stock reallocation effects ($\frac{\partial A_2}{\partial \pi_H} A_1 A_3 > 0$), the relative allocation of capital assets favours the low-risk location, such that $\frac{\partial \varpi_I}{\partial \pi_H} > 0$.
    * For Case 4 the stock of capital reallocation effect is as follows: the increase in risk lowers $\overline{K_H}$, and given that the MNC wishes to decrease its holdings in K_L ($\left(K_{L,0} - \overline{K_L}\right) > 0$), hence $\frac{\partial A_2}{\partial \pi_H} A_1 A_3 < 0$. Then,
      · Where the *negative* relative growth rate effect ($\frac{\partial A_3}{\partial \pi_H} A_1 A_2 < 0$) and asset stock reallocation effects ($\frac{\partial A_2}{\partial \pi_H} A_1 A_3 > 0$) dominate the *positive* relative growth rate effect ($\frac{\partial A_1}{\partial \pi_H} A_2 A_3 > 0$), the relative allocation of capital assets favours the high-risk location, such that $\frac{\partial \varpi_I}{\partial \pi_H} < 0$.
      · Where the *negative* relative growth rate effect ($\frac{\partial A_3}{\partial \pi_H} A_1 A_2 < 0$) and asset stock reallocation effects ($\frac{\partial A_2}{\partial \pi_H} A_1 A_3 > 0$) are dominated by the *positive* relative growth rate

effect ($\frac{\partial A_1}{\partial \pi_H} A_2 A_3 > 0$), the relative allocation of capital assets favours the high-risk location, such that $\frac{\partial \varpi_I}{\partial \pi_H} > 0$.

- Cases 2 and 3:
  - For Case 2: the increase in risk lowers $\overline{K_H}$, and given that the MNC wishes to decrease its holdings in K_L (i.e. $\left(K_{L,0} - \overline{K_L}\right) > 0$), hence $\frac{\partial A_2}{\partial \pi_H} A_1 A_3 < 0$.
  - For Case 3: the increase in risk lowers $\overline{K_H}$, and given that the MNC wishes to increase its holdings in K_L (i.e. $\left(K_{L,0} - \overline{K_L}\right) < 0$), hence $\frac{\partial A_2}{\partial \pi_H} A_1 A_3 > 0$.
  - Then,
    * Both Cases 2 and 3 share a common relative growth rate effect: the increase in risk raises the growth rate in K_H holdings relative to the growth rate in K_L holdings, providing the $\frac{\partial A_3}{\partial \pi_H} A_1 A_2 > 0$ and $\frac{\partial A_1}{\partial \pi_H} A_2 A_3 < 0$ results.
    * For Case 2 the stock of capital reallocation effect is as follows: the increase in risk lowers $\overline{K_H}$, and given that the MNC wishes to decrease its holdings in K_L ($\left(K_{L,0} - \overline{K_L}\right) > 0$), hence $\frac{\partial A_2}{\partial \pi_H} A_1 A_3 < 0$. Then,
      · Where the *negative* relative growth rate effect ($\frac{\partial A_1}{\partial \pi_H} A_2 A_3 < 0$) and asset stock reallocation effects ($\frac{\partial A_2}{\partial \pi_H} A_1 A_3 < 0$) dominate the *positive* relative growth rate effect ($\frac{\partial A_3}{\partial \pi_H} A_1 A_2 > 0$), the relative allocation of capital assets favours the high-risk location, such that $\frac{\partial \varpi_I}{\partial \pi_H} < 0$.
      · Where the *negative* relative growth rate effect ($\frac{\partial A_1}{\partial \pi_H} A_2 A_3 < 0$) is dominated by the *positive* relative growth rate effect ($\frac{\partial A_3}{\partial \pi_H} A_1 A_2 > 0$) and asset stock reallocation effects ($\frac{\partial A_2}{\partial \pi_H} A_1 A_3 > 0$), the relative allocation of capital assets favours the low-risk location, such that $\frac{\partial \varpi_I}{\partial \pi_H} > 0$.
    * For Case 3 the stock of capital reallocation effect is as follows: the increase in risk lowers $\overline{K_H}$, and given that the MNC wishes to increase its holdings in K_L ($\left(K_{L,0} - \overline{K_L}\right) < 0$), hence $\frac{\partial A_2}{\partial \pi_H} A_1 A_3 > 0$. Then,
      · Where the *negative* relative growth rate effect ($\frac{\partial A_3}{\partial \pi_H} A_1 A_2 < 0$) dominates the *positive* relative growth rate effect ($\frac{\partial A_1}{\partial \pi_H} A_2 A_3 > 0$) and asset stock reallocation effects ($\frac{\partial A_2}{\partial \pi_H} A_1 A_3 > 0$), the relative allocation of capital assets favours the high-risk location, such that $\frac{\partial \varpi_I}{\partial \pi_H} < 0$.
      · Where the *negative* relative growth rate effect ($\frac{\partial A_3}{\partial \pi_H} A_1 A_2 < 0$) is dominated by the *positive* relative growth rate effect ($\frac{\partial A_1}{\partial \pi_H} A_2 A_3 > 0$) and asset stock reallocation effects ($\frac{\partial A_2}{\partial \pi_H} A_1 A_3 > 0$), the relative allocation of capital assets favours the high-risk location, such that $\frac{\partial \varpi_I}{\partial \pi_H} > 0$.

The fundamental insight is that the optimal mix of capital flows towards intertemporal equilibrium in response to changes in risk is difficult to predict empirically. It reflects the combinatorial possibilities implicit in the $\left(K_{L,0} - \overline{K_L}\right) \gtrless 0$ and $\left(K_{H,0} - \overline{K_H}\right) \gtrless 0$ conditions the firm may face, and may trigger either a decreased or an increased flow of capital to the location experiencing a risk increase during the transitional dynamics of adjustment to the intertemporal equilibrium.

4. Conclusion and managerial implications

It has generally been assumed that MNCs' increased relative investment flows to a location can be interpreted as evidence of the MNCs' endorsement of the location, and its desire to increase its exposure there—and vice versa where an MNC exits a location. The implication of our model is that it may not necessarily be the case.

We model the decision of MNCs in allocating capital assets across high- and low-risk locations. Modelling accounts for returns on capital assets net of operating costs, the possibility of asymmetric risks across locations, and adjustment costs associated with capital flows between locations. Maximizing the net present value of infinitely lived MNCs presents an intertemporal optimization problem to the MNCs.

We emphasize that our model generates standard results consistent with prior findings for capital allocation responses to net rates of return, adjustment costs and risk. Novel implications of the model are thus not a result of non-standard representations of the decision problem of the MNC. Reassuringly, thus, our model confirms the standard expectation that any factor that raises the rate of return on capital assets or lowers adjustment costs in a location will raise both the desirability of that location for an MNC (i.e. the intertemporal equilibrium capital allocation to that location in both absolute and relative terms) as well as the rate at which the MNC will extend its investment in that location. Equally reassuringly, rising (falling) risk (here modelled as a proportional reduction in the return on capital in high-risk locations) increases (decreases) the proportion of the capital asset holdings that is held in low-risk locations.[14]

The important (and novel) insight of the model relates to the transitional dynamics to intertemporal equilibrium. Irrespective of the initial conditions in terms of capital asset holdings, for both the low- and the high-risk location, an increase in risk for the high-risk location may initially result in an increase in the capital asset flow to the high-risk location *relative* to the low-risk location (Proposition 1). Intertemporal equilibrium, in contrast, will lower the stock of capital assets held in the high-risk location relative to the low-risk location, in response to the increase in risk for the high-risk location (Lemma 3). Thus Proposition 1 shows that relative portfolio allocation may initially move in the opposite direction to that implied by the intertemporal stock equilibrium established by Lemma 3. In effect, the dynamics of capital asset allocation will not necessarily match the comparative statics of capital asset allocation in response to changes in risk.

This carries important implications for empirical research on capital allocation flows (such as FDI). While increases in risk in a location have a predictable long-run effect of suppressing capital allocation to that location, in the short run the counterintuitive result of higher flows of capital to the high-risk location relative to low-risk locations is possible. Empirical testing needs to accommodate the possible countervailing consequences over shorter and longer time frames. The potentially counterintuitive nature

[14]Under our functional form assumptions, note that the impact of increases in risk is potentially strongly non-linear. Increases in risk would lead to a flight to safe institutional environments at an increasing rate.

of optimal transition paths between intertemporal equilibria can serve as one potential explanation of the contradictory empirical evidence associated with locational risk and capital allocation (see Jiménez, 2011; Jiménez *et al*., 2015; Skovoroda *et al*., 2019) that has accumulated in the literature.

The challenge for empirical research compounds given that in practice MNCs operate in environments that are inherently dynamic and subject to shocks, such that the steady state of intertemporal equilibrium is seldom actually realized, with MNCs instead constantly negotiating optimal transitional dynamic approach paths to steady state. Understanding the MNCs locational capital allocation decisions in terms of the transition paths we derive may allow a number of divergent findings in the international business and strategy research to be resolved.

Our findings connect to recent scholarship examining the effects of institutional reforms in host countries and drawing a distinction between short-term and long-term effects. Leymann & Lundan (2023) argue that institutional dynamism requires separating the structural and transition effects when analysing the relationship between institutional quality and change and the investment decisions by MNCs. They empirically find that institutional reforms (even when they are positive) can have negative transition effects and they raise the concept of transition uncertainty and transition costs to explain this. More research is required to examine firm responses to institutional change and the co-evolution of these and this requires multifaceted complex models that can account for these dynamics (Dau *et al*., 2020). This means moving between micro and macro levels of analyses and accounting for short- and long-term effects, recognising that uncertainty associated with institutional reforms (and possible reversals) are affected by the overall locational portfolio of an MNC as well as managerial perceptions of risk and capabilities (Cuervo-Cazurra *et al*., 2019; Luiz & Barnard, 2022). The interplay between country characteristics and MNC characteristics and managerial decision-making in the face of uncertainty and risk requires a more integrative approach in empirical studies that accounts for risks being interconnected and asymmetric (Dang *et al*., 2020; Eduardsen & Marinova, 2020; Kotler *et al*., 2019). Lastly, a recent review study on mergers and acquisitions (M&A) finds that institutional theory has been curiously absent from M&A research (Hossain, 2021), and we provide further justification for why accounting for institutions matters to international investment decisions, and additionally demonstrate the nuanced effects by differentiating between the short- and long-term effects.

An interesting possibility for empirical research therefore would be to identify policy interventions that unambiguously improved the risk conditions in their jurisdictions, in order to establish whether in some instances FDI flows initially worsened, before improving. Ideally, under conditions that would satisfy the requirements of a natural experiment.

There is a fundamental policy implication that flows from our results. Given that the core result we establish is that FDI flows in response to an 'improvement' in the policy environment (i.e. lowering risk) may be non-monotonic (FDI flows may initially worsen in response to improved risk conditions, before generating a net positive FDI response), it is vital that policy makers factor in the possibility of initial FDI flows that appear to invalidate the (correct) policy intervention. To ensure that positive policy interventions are not prematurely reversed, it may be necessary to resort to some sort of pre-commitment strategy (e.g. creating an independent policy making institution) to insulate the policy from populist pressure in favour of reversal.

Data availability

There are no new data associated with this article.

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A. Appendix 1: Mathematical Derivations and Proofs

A.1 *Intertemporal Equilibrium and Optimal Adjustment Paths*

Substitution of (1) through (4) into (5), and (5) into (8) and (9) specifies the decision problem. The general solution to the Euler equation for the K_H state variable is given by

$$K_H^*(t) = A_1 e^{r_1 t} + A_2 e^{r_2 t} + \overline{K_H}, \tag{A.1}$$

where $r_1, r_2 = \frac{1}{2}\left[\rho \pm \left(\rho^2 + \frac{4\beta(1-\pi_H)}{b}\right)^{\frac{1}{2}}\right]$, $r_1 > \rho > 0 > r_2$, and $\overline{K_H} = \frac{(1-\pi_H)\alpha - a\rho}{2\beta(1-\pi_H)}$, such that meaningful (non-negative) solutions require $\left(1-\pi_H\right)\alpha - a\rho \geq 0$. Given the boundedness of R_H for profit maximizing agents, the holding of high-risk capital assets cannot exceed $K_H = \frac{\alpha}{2\beta}$, which follows immediately from the relevant first-order condition. The general solution to the K^H Euler can then satisfy the boundedness implication only under the assumption that $A_1 = 0$ given $r_1 > 0$. Hence, given an initial holding of high-risk capital assets of $K_{H,0}$, the specific solution is given by

$$K_H^*(t) = \left(K_{H,0} - \overline{K_H}\right) e^{\frac{1}{2}\left(\rho - \left(\rho^2 + \frac{4\beta(1-\pi_H)}{b}\right)^{\frac{1}{2}}\right)t} + \frac{\left(1-\pi_H\right)\alpha - a\rho}{2\beta\left(1-\pi_H\right)} \tag{A.2}$$

such that the optimal time path of investment in domestic assets is given by

$$I_H^*(t) = K_H^{*'}(t) = \frac{1}{2}\left(\rho - \left(\rho^2 + \frac{4\beta\left(1-\pi_H\right)}{b}\right)^{\frac{1}{2}}\right)\left(K_{H,0} - \overline{K_H}\right) e^{\frac{1}{2}\left(\rho - \left(\rho^2 + \frac{4\beta(1-\pi_H)}{b}\right)^{\frac{1}{2}}\right)t} \tag{A.3}$$

with $I_H^*(t) \gtreqless 0$ *as* $\left(K_{H,0} - \overline{K_H}\right) \lesseqgtr 0$

Symmetrical general solution to the Euler equation for the K_L state variable then provides

$$K_L^*(t) = \left(K_{L,0} - \overline{K_L}\right) e^{\frac{1}{2}\left(\rho - \left(\rho^2 + \frac{4\delta}{d}\right)^{\frac{1}{2}}\right)t} + \frac{\gamma - c\rho}{2\delta} \tag{A.4}$$

$$I_L^*(t) = K_L^{*'}(t) = \frac{1}{2}\left(\rho - \left(\rho^2 + \frac{4\delta}{d}\right)^{\frac{1}{2}}\right)\left(K_{L,0} - \overline{K_L}\right) e^{\frac{1}{2}\left(\rho - \left(\rho^2 + \frac{4\delta}{d}\right)^{\frac{1}{2}}\right)t} \tag{A.5}$$

$$\text{with } I_L^*(t) \gtreqless 0 \text{ as } \left(K_{L,0} - \overline{K_L}\right) \lesseqgtr 0$$

with $\overline{K_L} = \frac{\gamma - c\rho}{2\delta}$, $\gamma - c\rho \geq 0$.

A.2 *Derivation of Demarcation Curves for Phase Diagram*

Given (8), the Euler for the high-risk asset is given by

$$\begin{aligned}\left(-2be^{-\rho t}\right) K_H'' + 0K_H' + \rho\left(a + 2bK_H'\right) e^{-\rho t} - \left(\alpha - 2\beta K_H\right)\left(1 - \pi_H\right) e^{-\rho t} &= 0 \\ \Longrightarrow K_H'' - \rho K_H' - \left(\frac{\beta\left(1 - \pi_H\right)}{b}\right) K_H &= \frac{a\rho - \alpha\left(1 - \pi_H\right)}{2b}\end{aligned}$$

and since $I_H' = K_H''$, $I_H = K_H'$, the demarcation curve conditions that $I^{H'} = 0$, $I^H = 0$, are met where we have

$$\begin{aligned}K_H &= \frac{\alpha\left(1 - \pi_H\right) - a\rho}{2\beta\left(1 - \pi_H\right)} - \left(\frac{b\rho}{\beta\left(1 - \pi_H\right)}\right) I_H \\ I_H &= 0\end{aligned}$$

Symmetrically, the low risk asset Euler is given by

$$\begin{aligned}\left(-2de^{-\rho t}\right) K_L'' + 0K_L' + \rho\left(c + 2dK_L'\right) e^{-\rho t} - \left(\gamma - 2\delta K_L\right) e^{-\rho t} &= 0 \\ \Longrightarrow K_L'' - \rho K_L' - \left(\frac{\delta}{d}\right) K_L &= \frac{c\rho - \gamma}{2d}\end{aligned}$$

from which the demarcation curve conditions that $I_L' = 0$, $I_L = 0$, are met where we have

$$\begin{aligned}K_L &= \frac{\gamma - c\rho}{2\delta} - \left(\frac{d\rho}{\delta}\right) I_L \\ I_L &= 0\end{aligned}$$