Articulated Vehicle Stability Control Using Brake-Based Torque Vectoring on Trailer using Non-Linear Model Predictive Control

ABSTRACT

Unstable articulated vehicles pose a serious threat to the occupants driving them as well as the occupants of the vehicles around them. Articulated vehicles typically experience three types of instability: snaking, jack-knifing and rollover. An articulated vehicle subjected to any of these instabilities can result in major accidents. In this study a Nonlinear Model Predictive Control (NMPC) that applies brake-based torque vectoring on the trailer is developed to improve the articulated vehicle stability. The NMPC formulation includes tire saturation and applies constraints to prevent rollover. The controller output is a left and right brake force, allowing the longitudinal velocity change to be incorporated into the model. Simulations were conducted to instigate snaking and jack-knifing and shows the NMPC controller result compared to a simple proportional controller. The NMPC controller can prevent these instabilities and improves the overall handling and safety of the articulated vehicle.

Keywords: model predictive control, torque vectoring, non-linear, articulated vehicle

1. INTRODUCTION

The stability of articulated vehicle has become a growing concern in more recent years. This is because transport is moving towards longer and larger articulated vehicles for increased efficiency and productivity. This could result in significant increases the number of possible safety risks on roads [1]. An articulated vehicle typically experiences three different types of instabilities: snaking, jack-knifing and rollover. Statistics have shown that articulated vehicles are at risk of causing major accidents. From the South African Road Safety Report for January-March 2018, 3.6% of all major road accidents in South Africa were due to jack-knifing [2]. A survey done in the UK over a 12-month period proved that jack-knifing occurs in over half of articulated vehicle handling incidents with a smaller percentage of accidents occurring due to snaking [3]. Most other handling-related accidents that are not due to jack-knifing or snaking occur due to the inability to negotiate corners, most likely due to excessive speed but also because of high loads leading to a higher center of gravity. The risk of suffering injuries or fatalities are also ten times higher for other road users than that of the driver of the articulated vehicle [3]. In Australia the risk posed by articulated vehicles is also projected to increase [4]. This indicates just how dangerous an unstable articulated vehicle is. Thus, by ensuring the yaw-plane and roll dynamics are stable will result in a significant reduction in incidents and fatalities on roads. The instabilities that occur in the yaw-plane include snaking and jack-knifing. Snaking causes the oscillation of the hitch angle to increase progressively until the articulated vehicle results in an accident [5]. Jack-knifing, on the other hand occurs when the towing vehicle reaches the friction limit, but the trailer does not. The momentum of the trailer pushes the towing vehicle ultimately causing the vehicle to spin out [5]. Jack-knifing is more likely to occur when the payload is closer to the hitch point while snaking occurs when the Centre of Gravity (CG) lies towards the rear of the trailer [6]. While these instabilities can be reduced by passive means, the use of active control strategies have a much higher potential in eliminating these instabilities especially with changing environmental and vehicle conditions.

The use of active control strategies to improve articulated vehicle stability has been explored before. Anti-jack-knifing controllers have been developed using combined feedback and a MPC controller in [7] and using back-stepping controller in [8]. The works focus on preventing jack-knifing especially during reversing with alluding that [7] can also be used in the forward direction. The controllers use kinematic models and velocity control in their development as the speed investigated is under 3km/h. The applicability at higher speeds and more dynamic manoeuvres as well as the ability to prevent snaking is not addressed. Stabilisation by means of using yaw moment control is performed by controlling the vehicle yaw rate [9]. Yaw moment control can be implemented in various ways but there are three main strategies that have been the focus for the past few years. These strategies include active steering control, active brake or torque vectoring and adjusting the swing torque. Active steering is implemented by adding an additional steering angle to the front or rear wheels of the towing vehicle, trailer, or both [9]. Torque vectoring, also known as differential braking, is achieved by generating a yaw moment using either a braking force or a driving force on both sides of the vehicle. Adjusting the swing torque is implemented using a Variable Geometry Approach (VGA). VGA is used to control the lateral displacement of the hitch. A comparison of the three strategies is performed in [10] and highlight the advantages and disadvantages of each strategy. A Linear Quadratic Regulator (LQR) was used as the controller to implement the strategies. The authors found that VGA showed the worst controller performance with the largest overshoots and longest settling time [10]. The active trailer braking had smaller dynamic responses in all aspects except the hitch angle in comparison to the active steering control, but the active steering control had shorter settling times in all aspects. Overall, it was shown that the active trailer braking has the best capability of rejecting external disturbance to maintain stable operating of the articulated vehicle at high speeds. There, is also a move to using ABS based brake systems on trailers which enables brake-based torque vectoring to be implemented much easier than steering on the trailer.

Several control strategies have been used in the development of an articulated vehicle stability control system. The four main types of controllers that have been explored in literature include: feedback controllers, sliding mode controllers, fuzzy logic controllers and Model Predictive Control (MPC). In [5] a torque vectoring formulation using the hitch angle and yaw rate of an articulated vehicle by creating a yaw moment on the towing vehicle is developed. The controller used a single-input-single-output system where the yaw rate of the towing vehicle is altered when instability is detected using a hitch angle sensor. The results show that a torque vectoring controller, which includes the hitch angle, provides safe trailer behaviour during the manoeuvres therefore justifying the hitch angle measurement [5]. Sliding Mode Control (SMC) is based on a variable structure system that have been developed with independent structures that have different properties with a switching logic between them [11]. The control structure is switched when the system state trajectory has crossed a particular hypersurface in state space. In [12] controller for an optimum distribution of longitudinal and lateral forces of the four tires of the towing vehicle is proposed. This controller is based on sliding control law using planar equations of motion. The sliding control law is used to derive the required total lateral force and yaw moment required for the vehicle to follow the desired response. The required forces are then split into desired vehicle front and rear steering, as well as yaw moment using a tire load distribution algorithm. An additional controller uses optimal control to derive independent wheel steering angles. The two controllers were able to stabilise the articulated vehicle motion. In a more severe situation, the combined rear and front steering and yaw moment controller failed to achieve a desirable response. On the other hand, the proposed optimum control successively achieves smooth and reasonable responses [12]. A combined LQR and sliding mode controller in [13] is used to obtain roll-over and jack-knifing stability. Once instability thresholds are reached the LQR is used to obtain the required yaw moment and applied using a sliding mode controller to prevent lock-up of a wheel.

A logic-based controller is used in[14] which activates brakes, limits drive based on thresholds placed on the vehicle slip angle. The amount of braking or derive reduction is not defined and is evaluated using simple simulations. In [15] a fuzzy controller is implemented that only takes the state of the vehicle into account and therefore does not make use of a reference model. It makes use of the fact that there is a time delay between the actions of the trailer and the towing vehicle which provides the controller with enough time to predict a potential trailer instability. It was found that the fuzzy logic controller performed relatively well for different loads, road conditions and driving manoeuvres. The only downside is that the tuning of the controller is quite demanding since the set groupings, rule structure and bound selection had to satisfy competing needs for a variety of different stability risk [15]. In [6]an MPC to prevent instability in an articulated vehicle equipped with yaw moment control on both vehicle and trailer is developed. A 3DOF single-track linear model is used as the MPC predictive model to track steady state cornering vehicle yaw rate and hitch angle. Constraints are applied on the control moment by defining a maximum and minimum moment derived using the friction circle. The results that were found with this controller showed that it effectively prevents instability [6].

The above studies have implemented the active control on the towing vehicle mainly due to the vehicle already being equipped with ABS brake modulators and steer by wire systems. However, due to the advancement trailers becoming more advanced and ABS brakes becoming standard these control strategies can be implemented on the trailer. The control of the trailer has been explored before but not as in depth as the control of the towing vehicle. In some studies yaw moment control is applied to both the vehicle and trailer [6], [16], [17]. The methods all use linear vehicle models to determine the control inputs.

The contribution of this study is the application of braking control on the trailer of the articulated vehicle implemented using a NMPC. The NMPC formulation allows the use of non-linear tyre dynamics to be included that allows for better prediction of vehicle states in high slip angles common during snaking. The controller improves the yaw-plane dynamics whilst having a constraint on the roll dynamics to prevent roll-over. The proposed formulation also includes the effect that braking has on the vehicle velocity as a reduction in speed generally improves articulated vehicle stability.

2. CONTROL SYSTEM DEVELOPMENT

An NMPC system is developed which incorporates tire saturation and places constraints on the controller. The constraints are used to prevent roll-over rather than directly incorporating continuous roll control. Thus, roll-prevention actions are only applied if the control and vehicle inputs would induce roll-over. In this way the planar dynamics of the vehicle is not affected by any roll prevention controller until required to prevent roll over. The NMPC controller comes at additional computational power over a linear MPC, however the better non-linear modelling should provide better control specifically at high tire sideslip angles. The NMPC control action focuses on placing braking on the trailer rather than the vehicle. The NMPC was developed and implemented in the ACADO toolkit [18]. The ACADO toolkit is a software environment and a collection of algorithms that has been written in C++. It is designed specifically for the use of automatic control and dynamic optimisation. The full problem is formulated within ACADO which can then generate C++ code which can be run on an embedded system such as dSpace. The controller outputs $\boldsymbol{u}$ are the brake forces instead of the yaw moment applied to the trailer to allow the model to capture the effect of speed reduction due to braking, while this can be done using a yaw moment as well, it results in a more ill-posed non-linear model due to the sign functions required. Hence, the left F_{xl} and right F_{xr} brake forces are used as control inputs and limits placed to ensure they are only brake forces and not tractive forces. The NMPC problem minimised the cost function:

$$J(\boldsymbol{x},\boldsymbol{u}) \triangleq \min_{\boldsymbol{U_0}} \sum_{\boldsymbol{k=0}}^{\boldsymbol{N-1}} [\boldsymbol{z_k^T Q z_k} + \boldsymbol{u_k^T R u_k}] + \boldsymbol{z_N^T P z_N} \tag{1}$$

subject to:

$$\boldsymbol{z} = \boldsymbol{y}_{ref} - \boldsymbol{y}$$

$$\boldsymbol{y} = [\dot{\psi}_1 \quad \dot{\psi}_2 \quad \theta]$$

$$\boldsymbol{u} = [F_{xl} \ F_{xr}]$$

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}, \delta)$$

$$\boldsymbol{x_o} = \boldsymbol{x}(\boldsymbol{0})$$

Where $\boldsymbol{y}_{ref}$ are desired reference trajectories to track, $\boldsymbol{y}$ is a subset of the NPMC model predicted states $\boldsymbol{x}$. The subset of the NMPC states used in the cost function include the towing vehicle and trailer yaw rates as well as the hitch angle. Thus, the states used in the cost are $\boldsymbol{y} = [\dot{\psi}_1 \quad \dot{\psi}_2 \quad \theta]$. These states have been included in the cost function as they provide the most information regarding whether the articulated vehicle is stable or not.

The desired state trajectories are generated using a vehicle reference model in the form of a linear Single-Track Model (STM). The actual states are predicted by the NMPC using a more complex model that is a nonlinear Extended Single-Track Model (ESTM).

2.1. Vehicle Reference Model

The desired state trajectory of the vehicle is obtained using a stable reference model. The reference model is a conventional linear 3DOF single-track model that only takes the steering angle and velocity of the towing vehicle as input [19]. The following model was proposed by [19] for the main aim of studying the yaw plane dynamics of an articulated vehicle. The schematic of an articulated vehicle in the yaw-plane is portrayed in Figure 1. The definition of variables and parameters and their values for an unloaded trailer used in the model are provided in the nomenclature and parameter values at the end of the paper. The following assumptions are made:

- Negligible aerodynamic forces
- Only planar motion
- Left and right tires can be approximated to single equivalent tire at the centre of the axle.
- Small angle assumption
- Constant longitudinal velocity where the velocity of the towing vehicle V_{x1} and the trailer V_{x2} are equal therefore, $V_{x1} = V_{x2} = V_x$
- Tire force is determined using a cornering stiffness C_i and sideslip angle α_i of each tire. For example, the vehicle rear tire force is $F_{yr} = C_r \alpha_r$

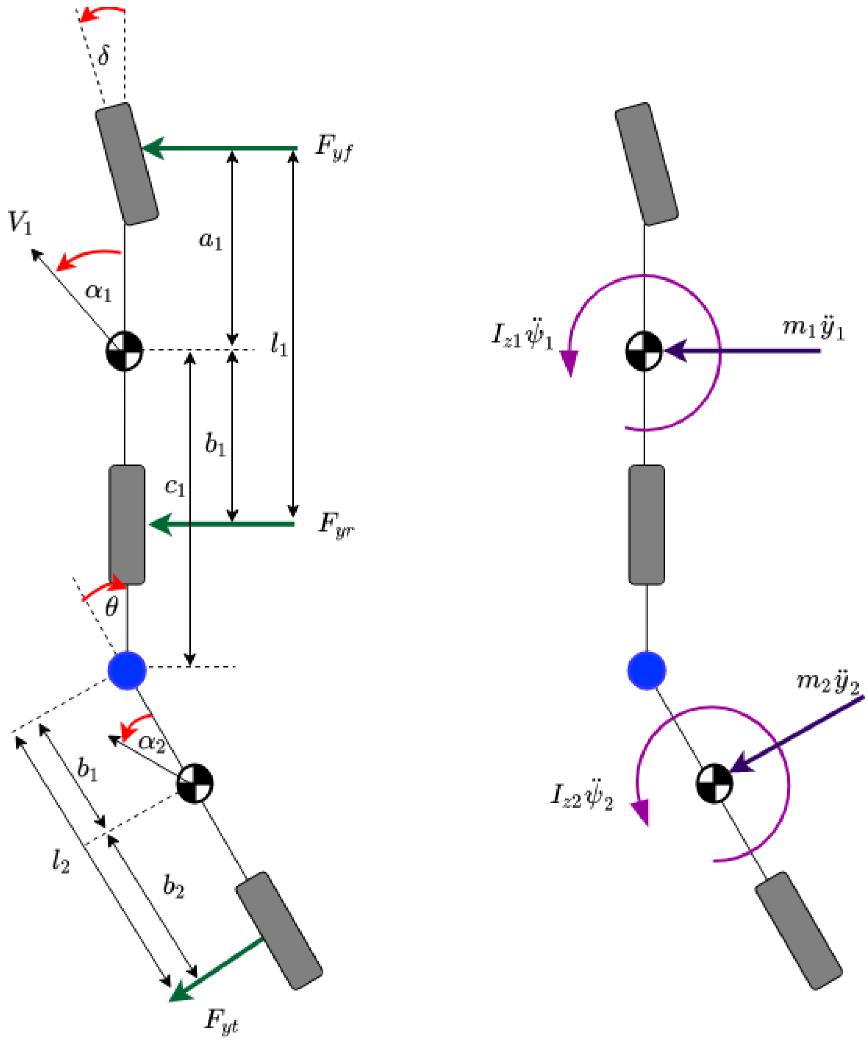


Figure 1 Simplified model of an articulated vehicle in the yaw plane

2.1.1. Equations of Motion

Splitting the articulated vehicle at the hitch point and taking the equations of motions of each body (vehicle denoted as subscript 1 and trailer as subscript 2) a vehicle dynamics model can be obtained. It is assumed that the hitch angle is small and thus the lateral force at the hitch point on the vehicle is also in the lateral direction of the trailer. The linear vehicle model can be obtained in the form:

$$\boldsymbol{M}\dot{\boldsymbol{x}}_{\boldsymbol{ref}} = \boldsymbol{D}\boldsymbol{x}_{\boldsymbol{ref}} + \boldsymbol{E}u \tag{2}$$

With states $\boldsymbol{x}_{\boldsymbol{ref}} = \left[v_{y1}\ \dot{\psi}_1\ \dot{\theta}\ \theta\right]^T$ and input $u = [\delta]$ and where,

$$\boldsymbol{M} = \begin{bmatrix} m_1 + m_2 & -m_2(c_1 + a_2) & -m_2 a_2 & 0 \\ m_1 c_1 & I_{z1} & 0 & 0 \\ -m_2 a_2 & I_{z2} + m_2 a_2 (c_1 + a_2) & I_{z2} + m_2 a_2^2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$\boldsymbol{D}=\begin{bmatrix} -\frac{C_{yf}+C_{yr}+C_{yt}}{V_x} & -\frac{C_{yf}a_1+C_{yr}b_1+C_{yt}(c_1+l_2)-(m_1+m_2)V_x^2}{V_x} & \frac{C_{yt}l_2}{V_x} & C_{yt} \\ m_1c_1 & I_{z1} & 0 & 0 \\ -m_2a_2 & I_{z2}+m_2a_2(c_1+a_2) & -\frac{C_{yt}l_2^2}{V_x} & -C_{yt}l_2 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad (4)$$

$$\boldsymbol{E}=\begin{bmatrix} C_{yf} \\ C_{yf}(a_1+c_1) \\ 0 \\ 0 \end{bmatrix} \quad (5)$$

(2) can then further be written in state space form as:

$$\dot{\boldsymbol{x}}_{ref}=\boldsymbol{M^{-1}D}\boldsymbol{x}_{ref}+\boldsymbol{M^{-1}E}u \quad (6)$$

The linear model can be developed to always provide a stable vehicle behavior by specifying the position and mass of the trailer. It can therefore be used to generate stable reference trajectories for the lateral slip velocity, vehicle yaw and hitch angular rate, the hitch angle or any combination of the states. For example, the trailer yaw rate which is merely the combination of the vehicle yaw rate and hitch angular rate.

The reference trajectory is generated by assuming the steering input of the vehicle is constant during the prediction period. This is simply due to any lack of additional information such as the desired path the vehicle needs to follow. The current vehicle states are used as the initial states in the reference model. The model can also be used to determine the steady state values of the state vector for a given input. This is often used in simple proportional yaw moment control systems, as the desired vehicle states, for stability control and will be used as a comparative controller in this study.

2.2. Extended Single-Track Model

To accurately predict the actual vehicle, motion a non-linear vehicle model is developed. The model is based on the model developed by [16] for the main purpose of analysing the dynamic stability of articulated vehicles with nonlinear suspension damper properties [16]. The model developed by [16] was used as a baseline and this model was adapted for a single axle trailer. The yaw dynamics are similar to the reference model shown in Figure 1, modifications mainly include adding right and left brake forces on the trailer which constitute the controller output. The roll dynamics are added and the tire model is changed from a linear tire model to that of a non- Magic Formula tire model [20]. The roll dynamics are added and shown in Figure 2.

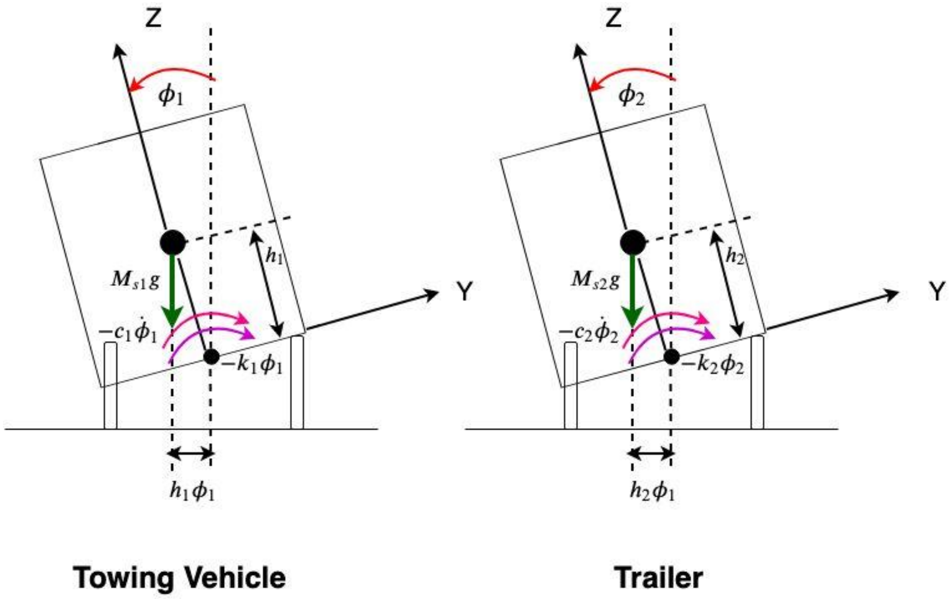


Figure 2 Articulated vehicle schematic in the roll plane.

2.2.1. Equations of Motion

The articulated vehicle is split at the articulations point and it is assumed that the lateral coupling force F_H at the articulation point is lateral on both vehicle and trailer i.e., small hitch angles. The equations of motion for the roll and yaw dynamics for the towing vehicle and the trailer are defined in Equations 7 to 13. Equation 11 includes the controller output which is a left and a right brake force. In this approach a constraint can be placed on the brake forces to only allow braking forces to be applied, and the controller can also apply both left and right braking at the same time as opposed to the standard yaw moment control.

$$m_1 a_{y1} + m_{s1} h_1 \ddot{\phi}_1 = F_{yf} + F_{yr} - F_H \tag{7}$$

$$I_{z1} \ddot{\psi}_1 = F_{yf} a_1 - F_{yr} b_1 + F_H c_1 \tag{8}$$

$$I_{xs1} \ddot{\phi}_1 + m_{s1} h_1 a_{y1} = -C_{\phi 1} \dot{\phi}_1 + (-K_{\phi 1} + m_{s1} g h_1) \phi_1 \tag{9}$$

$$m_2 a_{y2} + m_{s2} h_2 \ddot{\phi}_2 = F_{yt} + F_H \tag{10}$$

$$I_{z2} \ddot{\psi}_2 = -F_{yt} b_2 + F_H a_2 + \left(F_{xl} \left(\frac{t}{2} \right) - F_{xr} \left(\frac{t}{2} \right) \right) \tag{11}$$

$$I_{xs2} \ddot{\phi}_2 + m_{s2} h_2 a_{y2} = -C_{\phi 2} \dot{\phi}_2 + (-K_{\phi 2} + m_{s2} g h_2) \phi_2 \tag{12}$$

$$\dot{V}_x = \frac{(-F_{xl} - F_{xr}) \cos \theta}{m_1 + m_2} \tag{13}$$

2.2.2. Kinematic Relationships

With the combination of the towing vehicle and the single axle trailer it was found that certain kinematic relationships hold. These relationships are defined in Equations 14-16.

$$\dot{\psi}_2 = \dot{\psi}_1 + \dot{\theta} \tag{14}$$

$$a_{y1} = \dot{V}_{y1} + V_x\dot{\psi}_1 \tag{15}$$

$$a_{y2} = \dot{V}_{y1} + V_x\dot{\psi}_1 - c_1\ddot{\psi}_1 - a_2(\ddot{\psi}_1 + \ddot{\theta}) \tag{16}$$

2.2.3. Lateral Tire Forces

The tire forces are obtained from the nonlinear Magic Formula tire model also known as the Pacejka tire model. The Magic Formula tire model is used to determine the required lateral forces. the Pacejka tire model is a function of the tire slip angle and current vertical force. The slip angles for the front and rear of the SUV as well as the trailer are defined in Equations 17-20, respectively. The Pacejka tire model is represented in Equation 20 and is multiplied by 2 to account for the left and right tire on each axle. The inputs to the Magic Formula are defined in degrees and thus the slip-angles are converted from radians to degrees.

$$\alpha_f = \left(\frac{V_{y1} + a_1\dot{\psi}_1}{V_x}\right)\frac{180}{\pi} - \delta \tag{17}$$

$$\alpha_r = \left(\frac{V_{y1} - b_1\dot{\psi}_1}{V_x}\right)\frac{180}{\pi} \tag{18}$$

$$\alpha_t = \left(\frac{V_{y1} - (c_1 + l_2)\dot{\psi}_1 - l_2\dot{\theta}}{v_x} - \theta\right)\frac{180}{\pi} \tag{19}$$

$$F_{yi} = 2f(\alpha_i, F_{zi}) \text{ for } i \; = \; f, r, t \tag{20}$$

2.2.4. Nonlinear System of Equations

Equations 7 to 20 consist of a set of implicit non-linear equations and are combined, using MATALB's symbolic toolbox, to a set of explicit equations in state space form:

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}, \delta, \boldsymbol{u}) \tag{21}$$

The input to the model is steering angle and left and right brake force. The state vector $\boldsymbol{x}$ is defined in as:

$$\boldsymbol{x} = \begin{bmatrix} x_{y1} & \psi_1 & \theta & \phi_1 & \phi_2 & V_{y1} & \dot{\psi}_1 & \dot{\theta} & \dot{\phi}_1 & \dot{\phi}_2 \end{bmatrix}^T \tag{22}$$

The steering input similarly to the reference model is considered constant during the prediction horizon. [21]has shown that during severe manoeuvres or constantly changing manoeuvres the assumption of a constant steering input loses accuracy beyond a prediction horizon of 200ms. Thus, a 200ms prediction horizon is used. The NPMC equations are integrated using a 4th order Runge-Kutta integrator and the control input is solved using the quadratic programming solver QPOASES [22].

To enforce that the control inputs are braking forces and not acceleration forces, constraints are placed on the control inputs. A maximum brake force is placed to ensure that the braking is not too intrusive, this can be relaxed to allow higher levels of intervention. The amount of brake force results in a maximum 0.1g deceleration on the fully loaded vehicle and trailer if one wheel is braked.

$$0\,N \leq F_{xl} \leq 3500\,N \tag{23}$$

$$0\,N \leq F_{xr} \leq 3500\,N \tag{24}$$

The wheel force constraints are the total left and right forces applied to the trailer, since the trailer consist of a single axle this equates to the left and right tyre forces. In the event where the trailer has multiple axles a force distribution model should be used to split the forces on the wheels based on their respective vertical force and slip angle to ensure that no single wheel exceeds its maximum friction potential.

To enforce roll over prevention a constraint is placed on the yaw rates of the trailer and vehicle. The rollover prevention constraints are derived using the measure for rollover propensity, which is the inverse of the rollover threshold. The rollover threshold is the maximum lateral acceleration that a vehicle driving in steady state can resist to prevent rollover from occurring [23]. The lateral acceleration can be assumed to be the product of the longitudinal velocity and yaw rate as $\ddot{y} = V_x\dot{\psi}$ The rollover constraints are derived for the towing vehicle and trailer as:

$$-g \leq \frac{2h_{CG}V_x\dot{\psi}_1}{t} \leq g \tag{25}$$

$$-g \leq \frac{2h_{CG}V_x\dot{\psi}_2}{t} \leq g \tag{26}$$

Additional constraints can be further placed on the roll-angle if it is desired to limit the roll angle of the trailer or vehicle. The weights of the controller are used as tuning parameters. The initial weights were based on the maximum values expected during a severe manoeuvre. From these initial values an iterative process was applied to get the desired results. The input weight $\boldsymbol{Q} = diag[Q_{11}, Q_{22}, Q_{33}]$, state weight $\boldsymbol{P} = diag[P_{11}, P_{22}, P_{33}]$ and terminal weight $\boldsymbol{R} = diag[R_{11}, R_{22}]$, time step and preview horizon have been recorded in Table 1.

Table 1 NMPC parameters

	State Weight	**Terminal Weight**
Vehicle Yaw Rate	$Q_{11} = \left(\frac{1}{0.6°/s}\right)^2 = \left(\frac{1}{0.01rad/s}\right)^2$	$P_{11} = \left(\frac{1}{0.4°/s}\right)^2 = \left(\frac{1}{0.007rad/s}\right)^2$
Trailer Yaw Rate	$Q_{22} = \left(\frac{1}{1.7°/s}\right)^2 = \left(\frac{1}{0.03rad/s}\right)^2$	$P_{22} = \left(\frac{1}{1.15°/s}\right)^2 = \left(\frac{1}{0.02rad/s}\right)^2$
Hitch Angle	$Q_{33} = \left(\frac{1}{0.6°/s}\right)^2 = \left(\frac{1}{0.01rad/s}\right)^2$	$P_{33} = \left(\frac{1}{0.4°/s}\right)^2 = \left(\frac{1}{0.007rad/s}\right)^2$
Input Weight		
Left and Right Brake Force	$R_{11} = R_{22} = \left(\frac{1}{60\,N}\right)^2$	
Integration and control Step Size	0.01s	
Preview Horizon	20steps	

2.3. Force Distribution Model

An optimal trailer yaw moment obtained by the controller is applied via the brakes of the trailer. On standard brake-based yaw moment control a yaw enhancing moment that is in the direction of the yaw rate will act on the tire that is closest to the radius of the turn, whereas a yaw opposing moment that is opposite to the yaw rate will act on the opposite wheel. The NMPC in this study has been designed such that it determines the optimal brake forces of the left and right trailer tires. These optimal brake forces are converted to individual wheel brake torques by means of a force distribution algorithm. A friction circle is determined for each wheel based on its vertical loading and estimated tire side-slip angle. The tire side-slip angle is obtained from the ESTM. The lateral tire force is calculated using the Magic Formula tire model using the articulated vehicles static vertical forces. This tire model was chosen due to its real time implementation and low computation requirements. Each tire's maximum longitudinal brake force is determined based on the friction circle and the road surface friction coefficient. The equations used to define the maximum brake force is defined in Equation 10.

$$F_{x,max} = \sqrt{\mu F_z^2 - F_y^2} \tag{27}$$

Where μ is the road friction coefficient, F_z is the vertical load of the tire and F_y is the lateral tire force as determined by the Magic Formula tire model. Since the roll dynamics are not fully incorporated, the vertical force on the tire does not increase during cornering, this aspect could be addressed to improve the brake distribution model. The reason the maximum brake force is calculated is to prevent the over saturation of the tire, which would reduce the lateral force potential of the tire. If the optimal brake force from the NMPC is greater than the maximum force, then the maximum brake force is used instead. Finally, the desired brake torque is calculated as a function of the optimal brake force and the tires rolling radius, defined in Equation 11.

$$T_i = F_{x,i}R \tag{28}$$

where R is the rolling radius and *i* is *l* for the left tire and *r* for the right tire.

3. SIMULATION MODEL

The articulated vehicle used for this study is made up of an SUV, a Land Rover Defender 110 Tdi, and an in-house built testing trailer. The model is constructed using MSC ADAMS in co-simulation with Matlab/Simulink. The SUV ADAMS model is a fully validated nonlinear model with 16 degrees of unconstrained freedom of the vehicle. The body of the vehicle is represented by two rigid bodies that are connected with a torsional spring in order to model the torsional stiffness of the vehicle chassis. The vehicle can be interchanged between a soft and hard suspension due to the semi-active hydro-pneumatic suspension known as the $4S_4$ system [24]. The $4S_4$ is a four state semi-active suspension with two discrete spring and damper settings. The trailer consists of a frame with two separate weights and a single axle. These weights can be removed or moved to change the trailer characteristics. Each wheel in the model can be individual driven with a desired torque. A combination of different tire models (an FTire and Magic Formula), suspension settings and weight configurations were used to induce the different instabilities of the uncontrolled vehicle during simulations. In all simulations the NMPC model was unchanged and modelled as the standard fully laden configuration. The overall inertial properties and dimensions of the vehicle and trailer are the same as that of the NMPC model and can be found in the nomenclature.

4. CONTROLLER EVALUATION IN SIMULATION

The simulation is a co-simulation between Simulink and ADAMS. The controller is implemented within Simulink with a S-function interface to the C++ code generated by ACADO. As C++ code is generated the controller can be implemented on any embedded system with sufficient computational power. The vehicle dynamic equations are solved in ADAMS using a variable step solver. The vehicle states as measured in ADAMS is passed to the controller in Simulink to determine the reference trajectories and controller outputs. A first order delay is implemented on the control signal to realistically simulate the transient response of the actual brake system. The amount of delay was found from previous experimental studies on the same ABS modulator. These delayed forces are then passed to the torque vectoring algorithm and sent back to the ADAMS model where they act on the trailer tires therefore stabilising the articulated vehicle. The layout of the controller is portrayed in Figure 3. The NPMC with 4^{th} order integrator, 20 preview steps with 10ms timestep was solved on an Intel i5-9700F running at 2.9GHz. Using constraints or NMPC generally results in the problem becoming non-convex, as opposed to the convexity of linear MPC. The convexity of the linear MPC problem guarantees a global solution every time instant. However, the non-convexity of NMPC prevents such guarantees generally resulting in longer solutions times. ACADO uses a real-time iteration (RTI) scheme to solve non-linear problems. The authors of [25] argue that under certain assumptions the RTI scheme will follow the global solution of the NMPC problem by performing a single full Newton step. This results in a fast solution which should follow the global optimum. During simulations the solving time was on average 7.5ms per iteration with a maximum of around 10ms. Thus, the controller is solvable in real time at 100-125Hz on a mid-range desktop processor. The computation times can be reduced with additional optimisation and with using different integrator and time-step configurations. The solution times can be depended on initial conditions and active constraint violations and thus difficult to guarantee solution times for all possible situations.

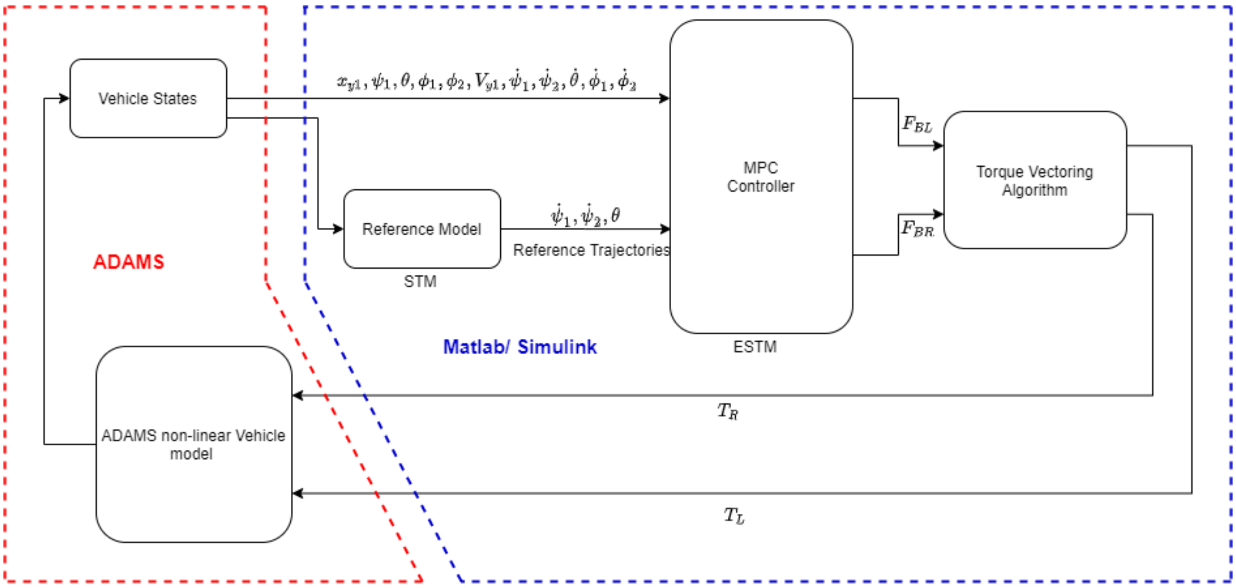


Figure 3 Simulation co-simulation structure between ADAMS and Matlab/Simulink

4.1. Proportional Controller Design

The NMPC controller is compared to a simple proportional controller acting to reduce the trailer yaw rate error. The same vehicle reference model as in the NMPC model is used as a desired trajectory. The yaw rate error and proportional controller is used to generate a yaw moment applied to the trailer centre of gravity. A constraint is placed on this controller to also limit the amount of braking applied such as was done for the NMPC.

The performance of the NMPC controller is investigated through simulations. Three scenarios are used to test the controller's capabilities. Manoeuvres, which induces snaking, jack-knifing and the controller output under less severe driving conditions were developed. The two instabilities are used to test the controller's ability to prevent or at least reduce the instabilities. The less severe driving condition is used to show that the controller is not too intrusive when the vehicle remains stable. All the simulations are performed using open loop steering to isolate the stability controller on the vehicle response and exclude the added effect of a driver. The speed of the vehicle is kept constant for the snaking and jack-knifing scenarios, by means of a driving force applied to the vehicle centre of gravity. This was done to investigate the torque vectoring capabilities of the controller independent of speed, as a reduction in vehicle speed would further improve vehicle and trailer stability.

4.2. Snaking

Snaking occurs when the tires of the trailer saturate which causes the trailer to move from side to side divergently. The longitudinal position of the trailer CG plays an especially important role in the stability of the articulated vehicle which is why it is imperative to load a trailer properly. If the CG of a trailer lies towards the rear of the trailer, it makes the trailer vulnerable to snaking [6]. Snaking causes the oscillation of the hitch angle to increase progressively until the articulated vehicle can no longer be recovered[5]. The vehicle under normal driving conditions remains stable even when fully loaded. Thus, snaking is instigated by reducing the road surface coefficient to 0.7 and performing an open loop steering input on the front wheels, Figure 4., consistent with a Double Lane Change (DLC) performed at 55km/h. The trailer is fully loaded with most of the weight slightly behind the trailer axis. The reference model is of an unloaded trailer which is stable under almost all conditions. The NMPC model is of the fully loaded trailer. The ideal reference trajectory though the entire manoeuvre was generated as reference using the same initial conditions at the start of the simulation and supplying the vehicle speed and steering angle throughout the entire simulation. This effectively shows the trajectories of the linear reference model. This reference trajectory may not be the same reference used in the NMPC model at all instances as deviation occurs the ideal trajectory may change and is therefore generated using the current vehicle states as initial conditions. The road friction coefficient of the NMPC model is the same as the simulation model so 0.7. The results shown for the hard suspension setting are depicted in Figure 5.

The snaking for the trailer can clearly be observed when looking at the oscillatory behaviour of the hitch angle and the trailer yaw rate of the uncontrolled vehicle. It can be seen from Figure 5 that the proportional controller was unable to prevent the instability, even though the force magnitudes are similar to those of the NMPC. The NMPC, on the other hand, was able to successfully prevent the snaking instability and further reduced the maximum peaks in the manoeuvre. The controller allowed good tracking of the vehicle yaw rate with acceptable tracking of the other states. When comparing the amount of braking between the controllers, it can be seen that the NMPC brake forces are a lot smoother and less erratic than that of the gain controller. It must also be noted that the NMPC was able to reduce the trailer roll angle quite significantly. Overall, the vehicle response was significantly improved by reducing yaw rate oscillation by more than 50% compared to the uncontrolled vehicle. The maximum braking force applied also never reached the constraint of 3500N.

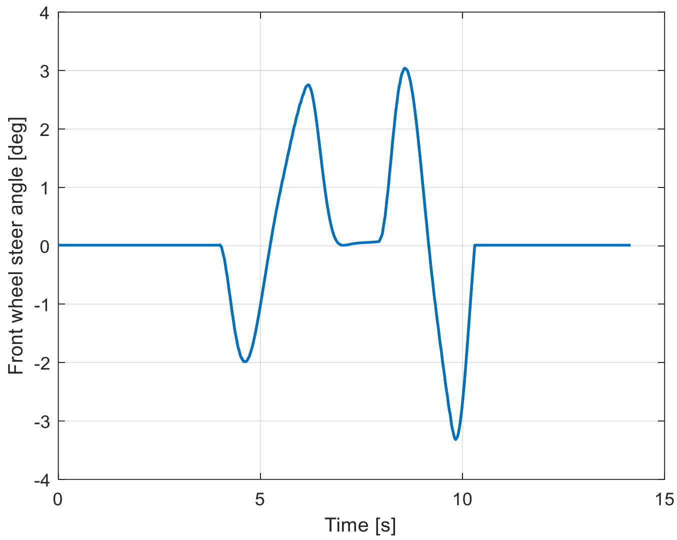
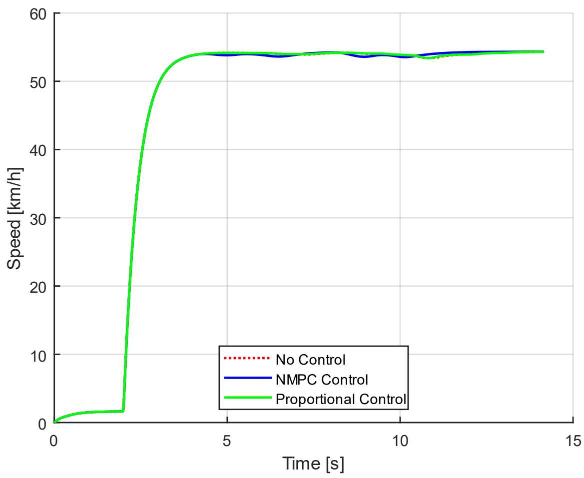
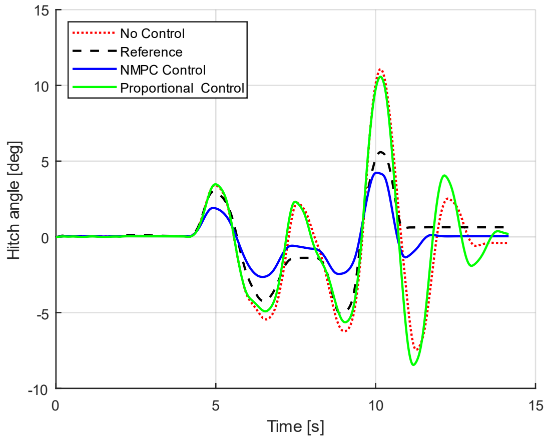


Figure 4 Open loop steering input for severe DLC manoeuvre to induce snaking



(a)



(b)

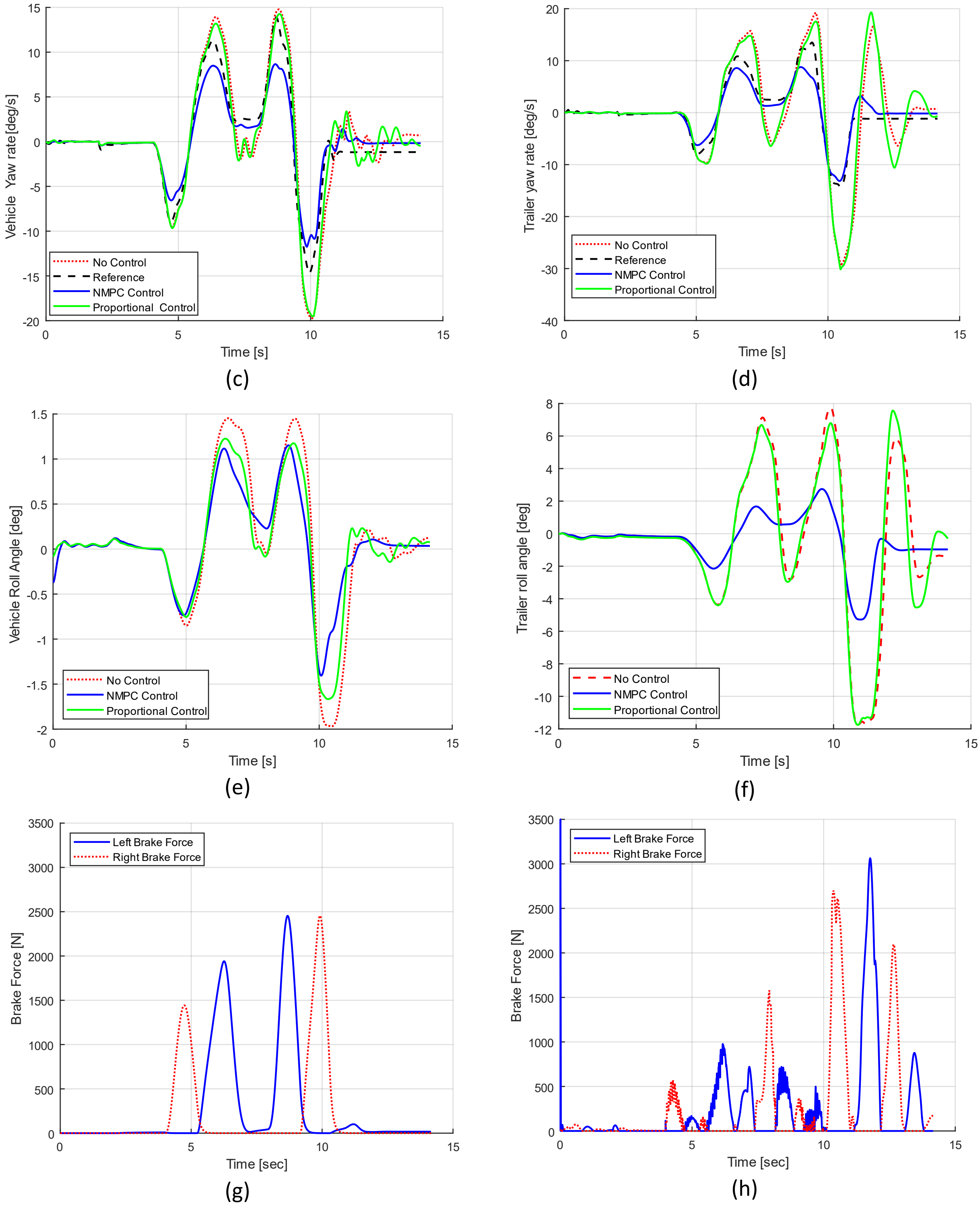


Figure 5 Vehicle snaking response with and without controllers with open loop DLC steer input (a) Vehicle speed (b) Hitch angle (c) Vehicle yaw rate (d) Trailer yaw rate (e) Vehicle roll angle (f) Trailer roll angle (g) NMPC brake forces (h) Proportional controller brake forces

4.3. Jack-knifing

Jack-knifing occurs when the tires of the towing vehicle saturate[6]. The momentum generated by the trailer pushes the towing vehicle, causing it to spin. The articulated vehicle ultimately ends up in a "folded" position [5]. The jack-knifing instability is generated using a step steer manoeuvre at 55 km/h. In the simulation model the mass of each trailer weight is increased to 800 kg and is moved closer to the hitch by 3m with weight one being 1m from the hitch point. The road friction coefficient is once again set to 0.7. The reference model is of an unloaded trailer and the NMPC model is of a loaded trailer but with the weights in their normal positions. Open loop steering with a constant velocity is used once again and the reference trajectories were generated using the same manner as in snaking. It should be noted that since the NMPC model was not updated to compensate for the change in the simulation model, this will also test the robustness of the controller to a change in parameters which are not directly modelled. The steering input is depicted in Figure 6 and the results are portrayed in Figure 7.

From Figure 7, both controllers can prevent the jack-knifing instability. The jack-knife itself can be seen when analysing the trailer yaw rate as the articulated vehicle without control spins out. Although both controllers prevent the instability, the NMPC clearly performs better. The desired trailer yaw rate is tracked very well and overall, the vehicle handling is much more stable. The trailer roll angle is also reduced significantly, by about 50% compared to the proportional controller, proving that the controller can reduce both yaw and roll instabilities.

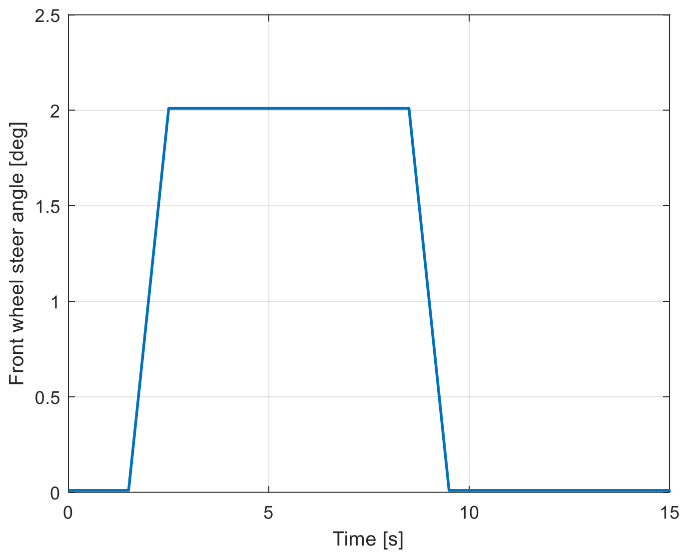
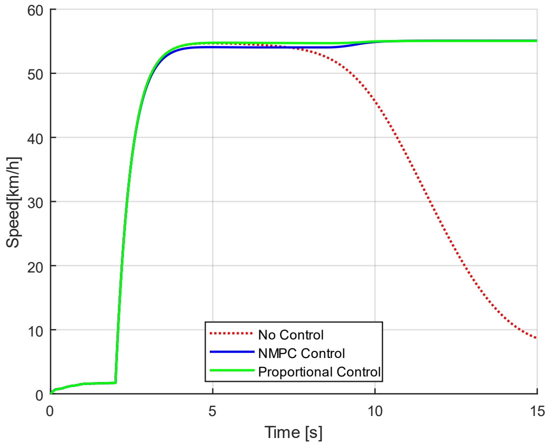
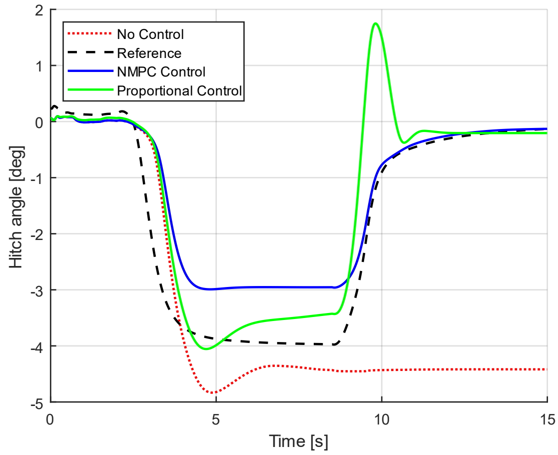


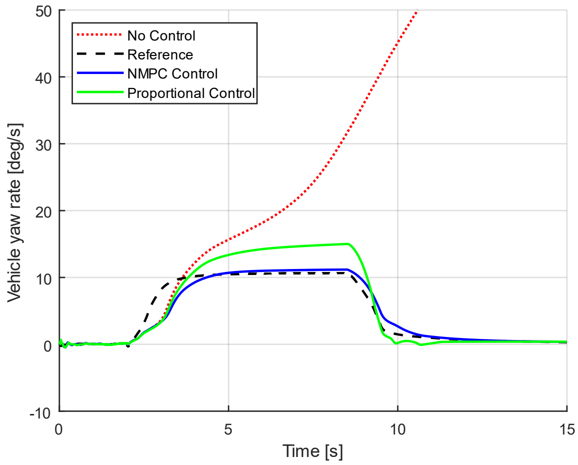
Figure 6 Open loop step steer to induce jack-knifing.


No Control
NMPC Control
Proportional Control
Speed[km/h]
Time [s]
0
10
20
30
40
50
60
0
5
10
15

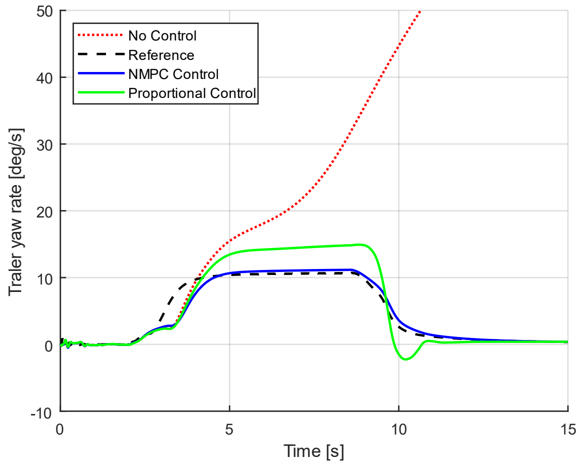
(a)


No Control
Reference
NMPC Control
Proportional Control
Hitch angle [deg]
Time [s]
2
1
0
-1
-2
-3
-4
-5
0
5
10
15

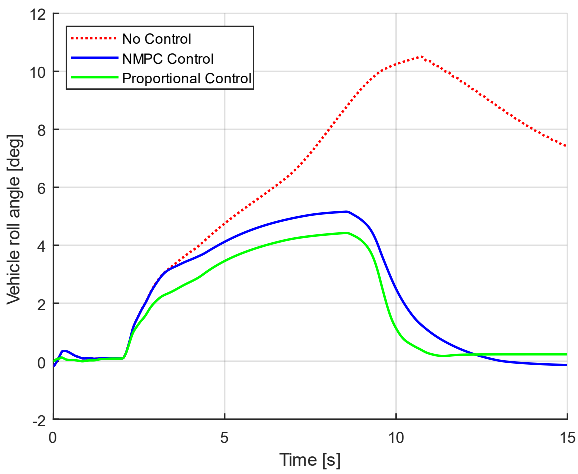
(b)


No Control
Reference
NMPC Control
Proportional Control
Vehicle yaw rate [deg/s]
Time [s]
50
40
30
20
10
0
-10
0
5
10
15

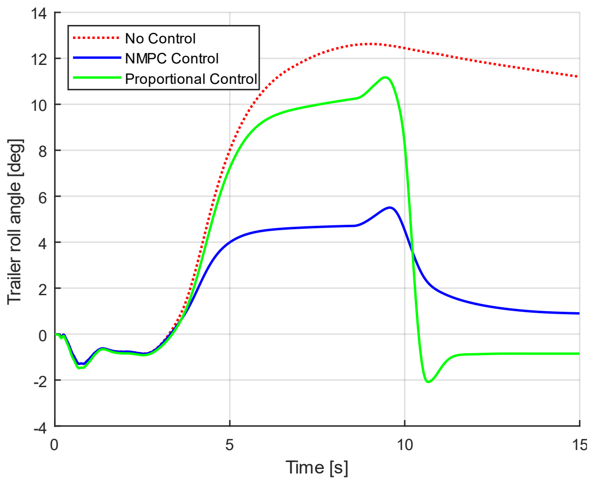
(c)


No Control
Reference
NMPC Control
Proportional Control
Traler yaw rate [deg/s]
Time [s]
50
40
30
20
10
0
-10
0
5
10
15

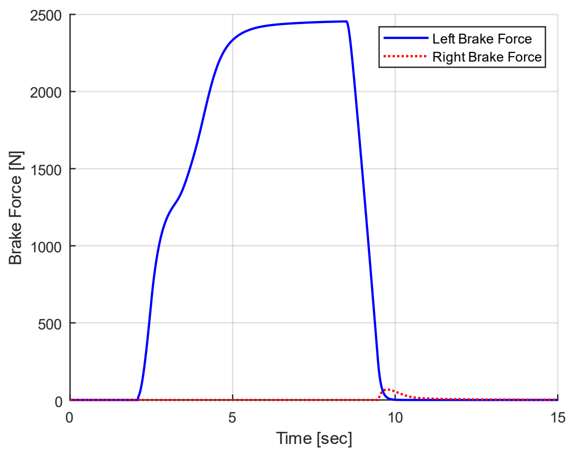
(d)


No Control
NMPC Control
Proportional Control
Vehicle roll angle [deg]
Time [s]
12
10
8
6
4
2
0
-2
0
5
10
15

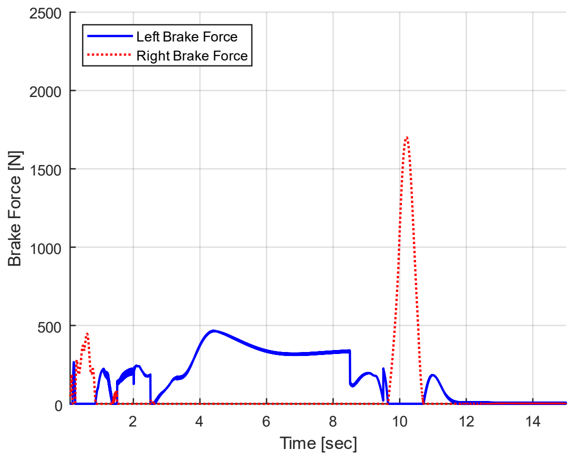
(e)


No Control
NMPC Control
Proportional Control
Trailer roll angle [deg]
Time [s]
14
12
10
8
6
4
2
0
-2
-4
0
5
10
15

(f)



(g)



(h)

Figure 7 Vehicle jack-knifing response with and without controllers with open loop step steer input (a) Vehicle speed (b) Hitch angle (c) Vehicle yaw rate (d) Trailer yaw rate (e) Vehicle roll angle (f) Trailer roll angle (g) NMPC brake forces (h) Proportional controller brake forces

4.4. Double Lane Change under less severe conditions

The purpose of running a simulation with the articulated vehicle under less severe driving conditions is to show that the controller is not too intrusive while the vehicle is stable. This is evaluated based on the loss of speed through a DLC manoeuvre. Since the system imposes control by braking the more intrusive the system is the larger the loss of speed will be. This was simulated by turning the drive force off after the vehicle reached 55 km/h. The road friction was set to 1 and the trailer weights were placed over the trailer axle. The manoeuvre is still considered severe but does not induce any vehicle instability. The open loop steering angle input, for a typical DLC, is portrayed in Figure 8. The results are depicted in Figure 9.

The speed of the vehicle, even without control, drops throughout the simulation due to the scrubbing of the tires. The NMPC does alter the dynamics of the articulated vehicle and still improves the vehicle handling by reducing some of the largest peaks in the trailer motion. The effect of speed reduction can be seen in the longitudinal speed plot. There is a clear decrease in speed, as mentioned this is due to tire scrubbing. There is only a 3 km/h speed difference between the NMPC and uncontrolled response. This clearly shows that the braking due to the NMPC is not too intrusive. There is relatively no difference between the proportional controller and the articulated vehicle without control. This is since a relatively low amount of braking is applied as seen in the gain controller brake plot. The NMPC brake forces reaches a maximum of around 1400 N which is a relatively low amount of braking which again shows that the system is not too intrusive.

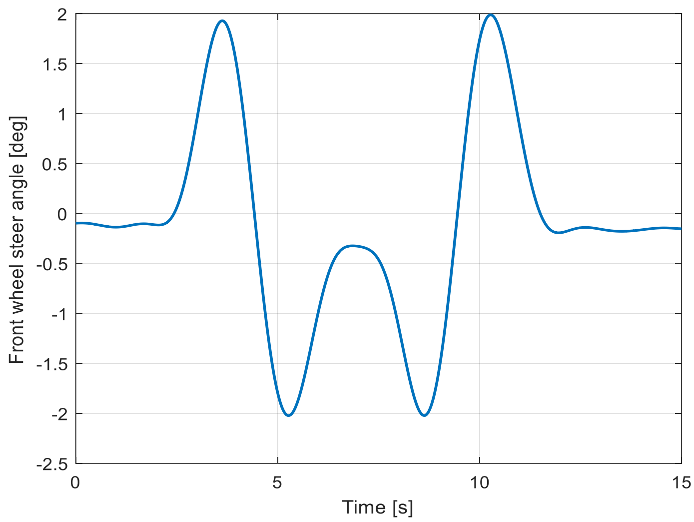
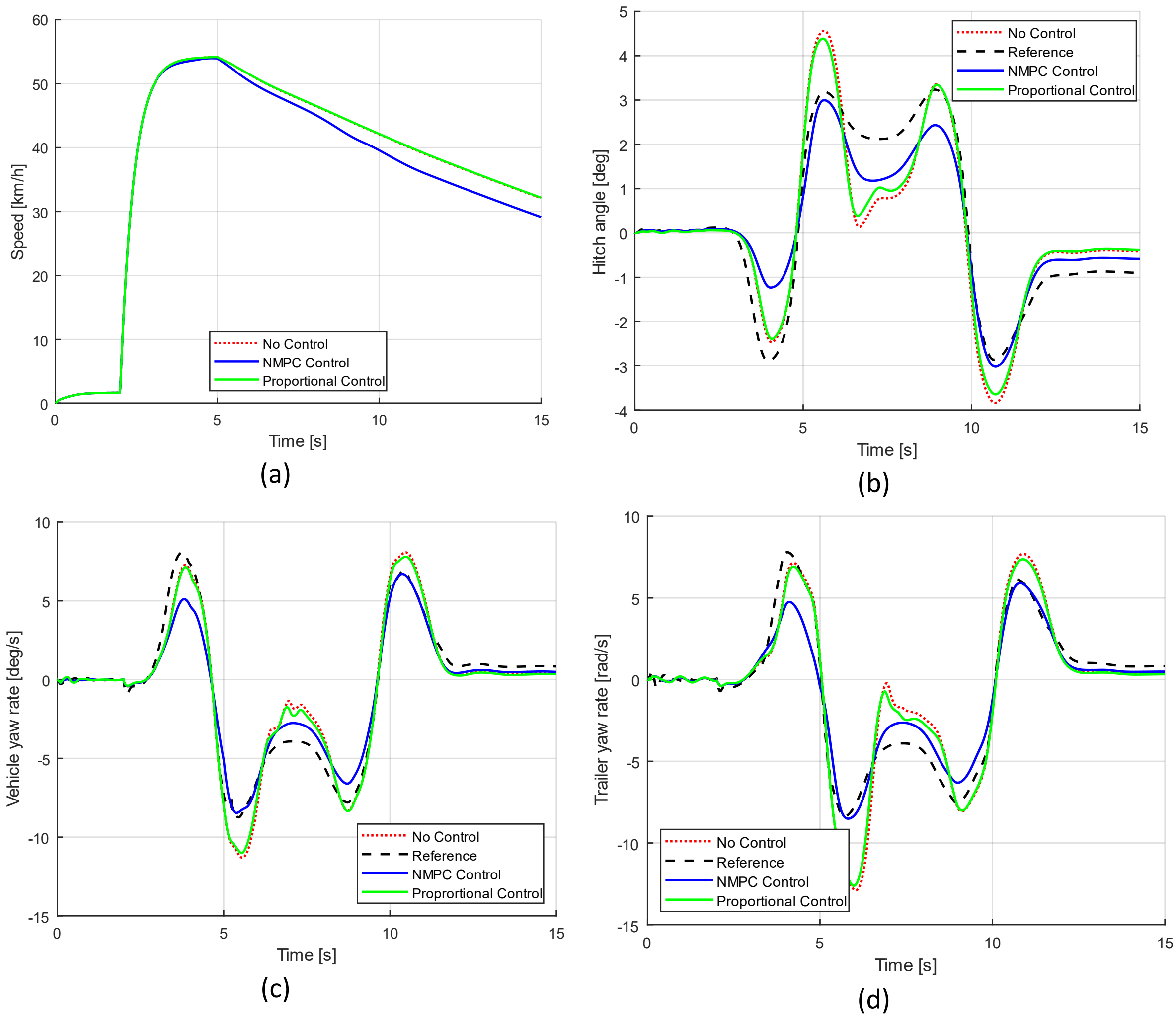
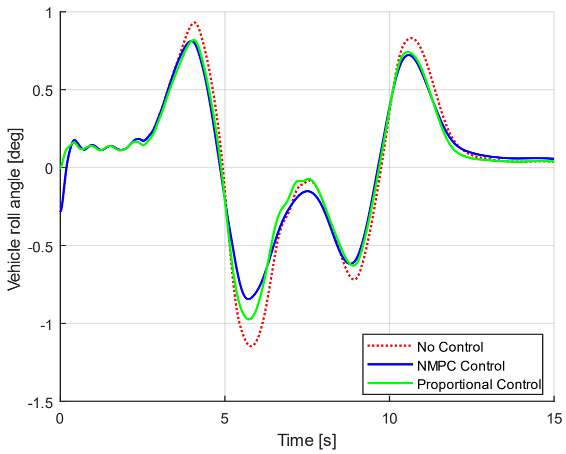
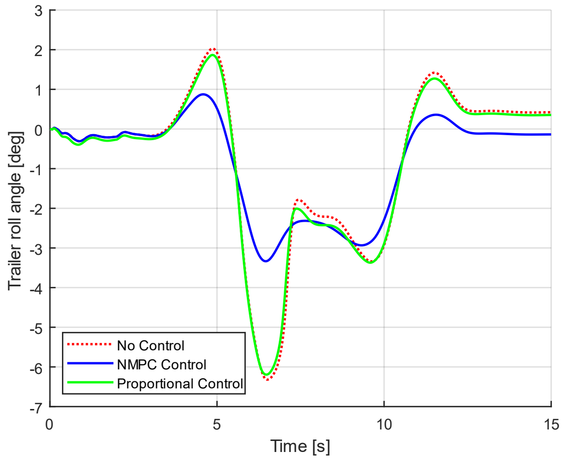


Figure 8 Open loop steering input for less severe DLC manoeuvre

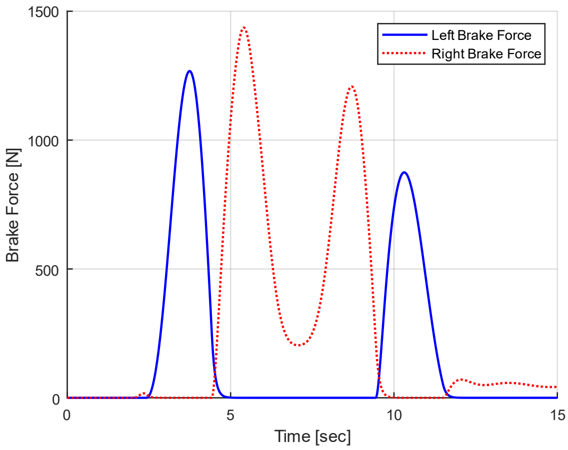




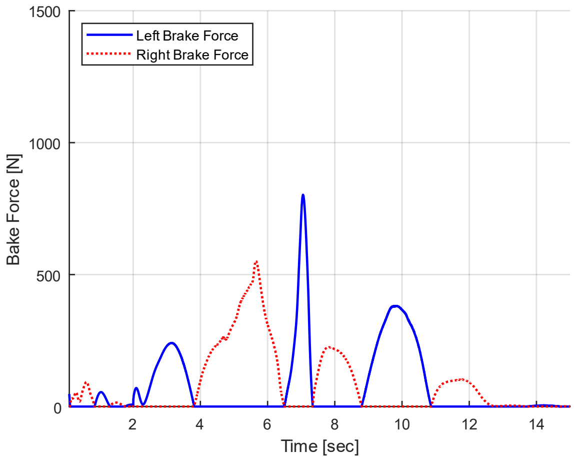
(e)



(f)



(g)



(h)

Figure 9 Vehicle response for lower severity DLC open loop steering with and without controllers with open loop DLC steer input (a) Vehicle speed (b) Hitch angle (c) Vehicle yaw rate (d) Trailer yaw rate (e) Vehicle roll angle (f) Trailer roll angle (g) NMPC brake forces (h) Proportional controller brake forces

5. CONCLUSION AND RECCOMMENDATIONS

This research study aimed to design and implement a control system for an articulated vehicle that can prevent instability. This aim was achieved through the development of a nonlinear model predictive controller. Two controller methods were developed, the first which is the main contribution to this research area, is a nonlinear model predictive controller. The second controller method is a simple proportional controller used to create a comparison between the two. Both controllers placed focus on yaw rate control by implementing torque vectoring on the trailer by means of braking. The controllers were analysed using the snaking and jack-knifing instabilities as well as normal driving conditions. The results proved that the gain controller based solely on trailer yaw rate was unable to prevent the instability of an articulated vehicle. On the other hand, the NMPC performed very well and is highly successful in altering the dynamics of the articulated vehicle to prevent instability from occurring. The results also show that the NMPC is not too intrusive under normal driving conditions. Ultimately the work provided in this study shows the NMPC can prevent instability within articulated vehicles and therefore the main objective of this study was achieved. Although, the finer details of the controller can be improved on. These include upgrading both the reference model and the predictive model from a single-track model to a full vehicle model and adding complexities to the model such as load transfer. The time delay of the braking actuation can also be incorporated in the NMPC modelling to further improve the performance.

NOMENCLATURE

Symbols		**Greek**		**Subscripts**	
F	Force, [N]	δ	Steering angle, [deg]	*1*	Vehicle parameters
V	Velocity, [m/s]	α	Slip angle, [deg]	*2*	Trailer parameters
a	Acceleration, [m/s^2]	ψ	Yaw angle, [deg]	*f*	Front
I	Moment of inertia, [kgm^2]	θ	Hitch angle, [deg]	*r*	Rear
M	Moment, [Nm]	$\dot{\psi}$	Yaw rate, [deg/s]	*H*	Hitch
a,b,c, e,l	Geometric Lengths, [m]	$\dot{\theta}$	Hitch rate, [deg/s]	*t*	Trailer
C	Cornering Stiffness, [N/rad] Damping, [Ns/m]	$\ddot{\psi}$	Yaw acceleration, [deg/s^2]	*x*	Longitudinal direction
K	Spring Stiffness, [N/m]	$\ddot{\theta}$	Hitch acceleration, [deg/s^2]	*y*	Lateral direction
m	Mass, [kg]	φ	Roll angle, [deg]	*z*	Vertical direction
R	Tire roll radius	$\dot{\varphi}$	Roll rate, [deg/s]	*s*	Sprung Distance between suspension struts
		$\ddot{\varphi}$	Roll acceleration [deg/s^2]		
		μ	Road friction coefficient		

PARAMETER VALUES

Parameter	**Value**	**Parameter**	**Value**
m_1	2047 kg	m_2	570 kg (unloaded)
m_{s1}	1576 kg	m_{s2}	404 kg (unloaded)
h_1	0.14 m	h_2	0.5 m (unloaded)
I_{z1}	2057 kgm^2	I_{z2}	911 kgm^2 (unloaded)
I_{xs1}	839 kgm^2	I_{xs2}	66.36 kgm^2 (unloaded)
a_1	1.3 m	a_2	3.66 m (unloaded)
b_1	1.5 m	b_2	0.82 m (unloaded)
c_1	2.74 m	$K_{\phi 2}$	30000 Nm/rad
$K_{\phi 1}$	13000 Nm/rad	$C_{\phi 2}$	4500 Nms/rad
$C_{\phi 1}$	5000 Nms/rad	C_{yt}	99 kN/rad
C_{yf}	122kN/rad		
C_{yr}	120kN/rad		

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