Hyperbolic Adversarial Learning for Personalized Item Recommendation

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Abstract. Personalized recommendation systems are indispensable intelligent components for social media and e-commerce. Traditional personalized item recommendation models are vulnerable to adversarial perturbations, resulting in poor robustness. Although adversarial learning-based recommendation models are able to improve the robustness, they inherently model the interaction relationships between users and items in Euclidean space, where it is difficult for them to capture the hierarchical relationships among entities. To address the above issues, we propose a hyperbolic adversarial learning based personalized item recommendation model, called HALRec. Specifically, HALRec models the interactions in hyperbolic space and utilizes hyperbolic distances to measure the similarities among entities. Moreover, instead of in Euclidean space, HALRec exploits the adversarial learning technique in hyperbolic space, i.e., HALRec maximizes the hyperbolic adversarial perturbations loss while minimizing the hyperbolic based Bayesian personalized ranking loss. Hence, HALRec inherits the advantages of hyperbolic representation learning in capturing hierarchical relationships and adversarial learning in enhancing the robustness of the recommendation model. In addition, we utilize tangent space optimization to simplify the learning of model parameters. Experimental results on real-world datasets show that our proposed hyperbolic adversarial learning-based personalized item recommendation method outperforms the state-of-the-art personalized recommendation algorithms.

Keywords: Recommendation systems · Adversarial learning · Hyperbolic spaces.

1 Introduction

Personalized recommendation systems are indispensable intelligent components for social media and e-commerce. Personalized recommendation systems mine users' preferences from their historical interaction behaviors and provide user with some items that user may be interested in.

As reported by [1], most machine learning models are fragile and susceptible to adversarial examples. Adversarial learning is one of the most effective approaches to enhance the robustness of machine learning models when facing with adversarial examples. Mathematically, adversarial learning is formulated as a mini-max problem, which searches for the best solution to the worst-case optimum. Due to the capability to enhance the robustness of various machine learning models, some researchers explore the application of adversarial learning in the field of recommendation systems [2–4]. For example, APR [2] enhances the robustness of BPR [5] via adversarial learning. In order to improve the robustness of the neural network-based recommendation models, Yuan et al. [4] proposed adversarial collaborative auto-encoder. In addition, for multimedia recommendation tasks, Tang et al. [3] proposed adversarial multimedia recommendation, namely AMR. It should be noted that the above adversarial learning based recommendation methods model the interaction behaviors between users and items and learn the representations of users and items in Euclidean space. Some researches [6, 7] have demonstrated that representation learning methods built in Euclidean space are not able to effectively model implicit hierarchical relationships among entities. However, some works [8–10] indicate that capturing the hierarchical relationships hidden in interaction behaviors is crucial for improving the performance of recommendation systems.

Recently, the hyperbolic representation learning technique has received wide attention in computer vision, natural language processing and computational biology due to the high capacity of hyperbolic space and the capability of effectively capturing implicit hierarchical relationships. Recently, some researchers have attempted to integrate the hyperbolic representation learning technique into traditional recommendation models. For instance, Vinh et al. [8] proposed the hyperbolic Bayesian personalized ranking, namely H-BPR, which utilizes hyperbolic distance to characterize the similarities of entities. Tran et al. [9] explored the metric learning technique in hyperbolic space and proposed the hyperbolic metric learning recommendation model. The above-mentioned work indicates that designing recommendation models in hyperbolic space is able to effectively capture the implicit hierarchical structure among entities.

In this paper, in order to simultaneously empower the recommendation model with robustness and the capability of capturing implicit hierarchical relationships among entities, we propose a hyperbolic adversarial learning based personalized recommendation algorithm, namely HALRec. Specifically, HALRec models the interaction behaviors between users and items in the hyperbolic space rather than in the Euclidean space. And HALRec measures the similarity between entities by utilizing hyperbolic distance. Then, we integrate the adversarial learning technique into the hyperbolic based recommendation models by injecting adversarial perturbations into the hyperbolic embeddings of the entities. Moreover, HALRec learns model parameters via a mini-max game, which maximizes the hyperbolic adversarial perturbations loss while minimizing the hyperbolic Bayesian personalized ranking loss. In this way, HALRec not only effectively captures the hierarchical relationships between entities, but also enhances the robustness of the recommendation model. In addition, we utilize the tangent space optimization method to simplify the process of learning model parameters. The main contributions of this paper are summarized as follows:

- In order to empower the recommendation model with both robustness and the capability of capturing implicit hierarchical relationships among entities, we propose a hyperbolic adversarial learning based personalized recommendation algorithm.
- We simplify the hyperbolic parameter optimization problem as the Euclidean parameter optimization problem, avoiding the complicated process of calculating Riemannian gradients.
- We conducted extensive experiments on three real-world datasets to evaluate our proposed models. The experimental results demonstrate that HALRec outperforms SOTA models.

2 Related Work

In this section, we reviewed the relevant studies on adversarial learning techniques and hyperbolic representation learning.

2.1 Adversarial learning

In the recent years, the adversarial learning technique is widely utilized in computer vision and nature language processing since it is able to enhance the robustness of machine learning models. Some researchers have also utilized the adversarial learning technique to boost the robustness of recommendation models. For instance, Wang et al. [11] proposed IRGAN, which unifies the generative and discriminative models via adversarial training in a minimax game. He et al. [2] proposed APR, which improves the robustness of the model by adversarial learning. Tang et al. [3] developed AMR for multimedia recommendation tasks. Chae et al. [12] studied a generic GAN-based CF framework, which is based on vector-wise adversarial learning. Yuan et al. [4] exploited the ACAE model. ACAE not only nonlinearly captures features via deep neural networks, but also effectively enhances the model's robustness against noise by adversarial learning. It should be noted that the above works model the interaction behaviors between users and items in the Euclidean space. Although they are able to improve the robustness of recommendation models in a certain extent, but they are limited in capturing hierarchical relationships among entities.

2.2 Hyperbolic representation learning

Recently, researchers have proposed several hyperbolic machine learning models because hyperbolic representation learning is able to capture implicit hierarchical relationships from structural data. Some researchers have explored how to utilize hyperbolic representation learning to improve the performance of recommendation models. For instance, Trans et al. [9] proposed HyperML, which measures the similarity between entities via hyperbolic distance. Similar to HyperML, Feng et al. [13] mapped check-in data into the hyperbolic space and proposed HME for point-of-interest recommendation. Since the above work ignores high-order collaborative signals among entities, Sun et al. [10] proposed HGCF, which integrated the graph convolution module into hyperbolic space. To utilize the implicit hierarchical structure of edge information in user interaction graphs, Zhang et al. [14] proposed HGCC. Noted that the above hyperbolic recommendation models do not take the robustness of the recommendation models into account. Different from the above methods, we perform adversarial learning in the hyperbolic space, which maximizes the hyperbolic adversarial perturbation loss while minimizing the hyperbolic Bayesian personalized ranking loss.

3 Preliminaries

3.1 Problem Description

Generally, personalized item recommendation systems include two entities: the set of users $U = \left\{u_1, u_2, \cdots, u_{|U|}\right\}$ and the set of items $V = \left\{v_1, v_2, \cdots, v_{|V|}\right\}$. The historical interaction information between users and items can be represented as a set of two-tuple $I \subseteq U \times V$. Usually, recommendation systems convert I into a two-dimensional matrix $Y \in \mathbb{R}^{|U| \times |V|}$, where the element $y_{u,i}$ is defined as

$$y_{u,i} = \begin{cases} 1, (u,i) \in I \\ 0, otherwise. \end{cases} \tag{1}$$

Given each user u and the corresponding set of visited items, a personalized item recommendation system aims to compute the probabilities of users clicking on candidate items, and recommend the top-N items with the highest probabilities to users.

3.2 Poincaré embedding

As one of most popular hyperbolic models, the Poincaré model $\mathbb{B}$ is a manifold with a Riemannian metric $g^{\mathbb{B}}(x) = \left(\frac{2}{1-\|x\|^2}\right)^2 g^{\mathbb{E}}$, where $g^{\mathbb{E}}$ denotes the Euclidean metric tensor. $\mathbb{B}^n = \{x \in \mathbb{R}^n : \|x\| < 1\}$ is the *open* n-dimensional unit ball, where $\|\cdot\|$ denotes the Euclidean norm.

Given two points $\mathbf{x}_a \in \mathbb{B}^n$ and $\mathbf{x}_b \in \mathbb{B}^n$ in hyperbolic space, the poincaré distance between $\mathbf{x}_a$ and $\mathbf{x}_b$ is computed as follows:

$$d_{\mathbb{B}}\left(\mathbf{x}_a, \mathbf{x}_b\right) = arcosh\left(1 + 2\frac{\|\mathbf{x}_a - \mathbf{x}_b\|^2}{\sqrt{\left(1 - \|\mathbf{x}_a\|^2\right)\left(1 - \|\mathbf{x}_b\|^2\right)}}\right), \tag{2}$$

where $arcosh\left(x\right) = \ln\left(x + \sqrt{x^2 - 1}\right)$ is the inverse function of hyperbolic cosine. For each point $\mathbf{x} \in \mathbb{B}^n$, there is a corresponding tangent space $\mathcal{T}_x\mathbb{B}^n$. Since the map function at the origin point $\mathbf{o}$ is simple and symmetric, we set the target point x as the origin $\mathbf{o}$. For $\mathbf{v} \in \mathcal{T}_{\mathbf{o}}\mathbb{B}^n$, the Poincaré model defines the exponential map function $exp_{\mathbf{o}} : \mathcal{T}_{\mathbf{o}}\mathbb{B}^n \rightarrow \mathbb{B}^n$, which is used to map point $\mathbf{v}$ into hyperbolic space. Formally,

$$exp_{\mathbf{o}}\left(\mathbf{v}\right) = \tanh\left(\|\mathbf{v}\|\right)\frac{\mathbf{v}}{\|\mathbf{v}\|}. \tag{3}$$

4 Hyperbolic Adversarial Learning based Personalized Recommendation

We propose a hyperbolic adversarial learning based recommendation model, namely HALRec. HALRec aims to empower the recommendation model with both robustness and the capability of capturing implicit hierarchical relationships among entities. Firstly, we integrate adversarial learning technique into the hyperbolic based recommendation models by injecting adversarial perturbations into the hyperbolic embeddings of the entities. Then, HALRec learns the model parameters by playing a mini-max game, which maximizes the hyperbolic adversarial perturbations loss, at the same time minimizes the hyperbolic Bayesian personalized ranking loss.

4.1 Model equtation

Different from MF-style methods that utilize the inner product to model interaction behaviors between users and items in Euclidean space, HALRec measures the degree of user preferences over items via hyperbolic distances. Similar to BPR and H-BPR, we utilize $P\left(i >_u j|\Theta\right)$ to denote the probability of user u preferring item i over other item j,

$$P\left(i >_u j|\Theta\right) = \sigma\left(\alpha_{ui} - \alpha_{uj}\right), \tag{4}$$

where $\sigma\left(x\right) = \frac{1}{1+e^{-x}}$ is the logistic regression function, and $\alpha_{ui} = f\left(d_{\mathbb{B}}\left(u, i\right)\right)$. $f\left(\mathbf{x}\right) = \beta\mathbf{x} + c$ is a linear function, $\beta \in \mathbb{R}, c \in \mathbb{R}$ are learnable scalar parameters. $\Theta = \{P, Q, \beta, c\}$ is the set of model parameters, P and Q denote the embedded feature matrix of users and items, respectively.

Supposing that all partial order relationships are independent, the objective function of hyperbolic item recommendation model is formalized as,

$$L_H\left(\Theta\right) = \sum_{(u,i,j)\in D_s} - \ln\sigma(\alpha_{ui} - \alpha_{uj}) + \lambda_\Theta\|\Theta\|_F^2, \tag{5}$$

where λ_Θ is the regularization coefficient, $\|\cdot\|_F^2$ represents the L_2 regularization norm, and $D_s = \{(u,i,j) \,|\, (u,i) \in I \wedge (u,\jmath) \notin I\}$ denotes the training data.

Inspired by [2, 15], we integrate the hyperbolic adversarial learning module into the hyperbolic item recommendation model to enhance the model robustness. Formally, the objective function of hyperbolic adversarial learning based recommendation is defined as follows,

$$L_{HALRec}(\Theta) = L_H(\Theta) + \lambda L_H(\Theta + \triangle_{adv}), \tag{6}$$

where

$$\triangle_{adv} = \underset{\triangle, \|\triangle\| \le \varepsilon}{\arg\max}\ L_H\left(\hat{\Theta} + \triangle\right). \tag{7}$$

$\hat{\Theta}$ denotes the parameters of the current model, $\triangle$ denotes adversarial perturbations to model parameters, and ε controls the magnitude of adversarial perturbations. $L_H(\Theta + \triangle_{adv})$ is an adversarial term and the coefficient λ controls the strength of the adversarial term. HALRec learns the model parameters by playing a mini-max game, which takes minimizing $L_H(\Theta)$ as a minimized player and maximizing $L_H(\Theta + \triangle)$ as a maximized player. Hence, the objective function of HALRec can be rewritten as,

$$\Theta^*,\ \triangle^* = \arg\min_{\Theta}\ \underset{\triangle, \|\triangle\| \le \varepsilon}{max} L_H(\Theta) + \lambda L_H(\Theta + \triangle). \tag{8}$$

4.2 Parameter Optimization

HALRec defines model parameters in the tangent space and optimizes model parameters via Euclidean based optimizers. Specifically, for one training sample (u,i,j), the hyperbolic adversarial perturbation loss is formalized as,

$$l_{adv}((u,i,j) \mid \triangle) = -\ln\sigma\left(\alpha_{ui}(\widehat{\Theta} + \Delta) - \alpha_{uj}(\widehat{\Theta} + \triangle)\right). \tag{9}$$

Noted that, when calculating $\alpha_{ui}\left(\hat{\Theta} + \triangle\right)$, we convert the embeddings of user p_u and the embedding of item q_i from tangent space to hyperbolic space via exponential mapping exp_o^1,

$$p_u^H = exp_o^1(p_u + \triangle_u),\ q_i^H = exp_o^1(q_i + \triangle_i), \tag{10}$$

where $\triangle_u$ and $\triangle_i$ represent the adversarial perturbations injected into the embedding of user u and item i, respectively. Following the reference [15], we utilize the fast gradient method to compute $\triangle_{adv}$,

$$\triangle_{adv} = \varepsilon\frac{\Gamma}{\|\Gamma\|},\ \text{where}\ \Gamma = \frac{\partial l_{adv}((u,i,j) \mid \triangle)}{\partial \triangle}. \tag{11}$$

After obtaining $\triangle_{adv}$, the loss of HALRec for a single training sample is

$$\begin{aligned} l_{\text{HALRec}}((u,i,j) \mid \Theta) = -\ln\sigma(\alpha_{ui}(\Theta) - \alpha_{uj}(\Theta)) + \lambda_\Theta\|\Theta\|_F^2 \\ -\lambda\ln\sigma(\alpha_{ui}(\Theta + \triangle_{adv}) - \alpha_{uj}(\Theta + \triangle_{adv})). \end{aligned} \tag{12}$$

Then, we can utilize Euclidean based optimizers, such as Adam, to learn model parameter Θ. With the above operations, we simplify the hyperbolic parameters optimization problem to a Euclidean parameters optimization problem.

5 Experiments

5.1 Datasets

We choose three widely used datasets, i.e. Amazon book, Yelp2018 and Pinterest, to evaluate our proposed method. Following the common strategy [16, 17], we transform explicit ratings into implicit feedback. Statistics of datasets are summarized in Table 1.

Table 1: Statistics of datasets

DataSet	$\#users$	$\#items$	$\#interactions$	$Density$
Pinterest	55,187	9,916	1,500,809	0.99725746
Yelp2018	31,668	38,048	1,561,406	0.99870412
Amazon-book	52,643	91,599	2,984,108	0.99938115

5.2 Evaluation metrics and experimental settings

Since personalized recommendation essentially is a ranking problem, we choose two widely used rank-oriented metrics, i.e. $Recall$@20 and $NDCG$@20, to evaluate the performance of all compared methods.

We choose the following classic personalized recommendation algorithms as baselines:

- BPR [5]: BPR assumes that users prefer clicking items over those that they have not been interacted with, and utilize personalized Bayesian optimization criteria to learn model parameters.
- CML [17]: CML utilizes Euclidean distance to measure the similarity between user-item, user-user, and item-item, which effectively improves the model's generalization.
- H-BPR [8]: H-BPR maps hyperbolic distances into scores via a linear mapping layer and employs pairwise optimization to learn model parameters.
- HyperML [9]: HyperML projects the embeddings of users and items into hyperbolic space, and encodes the similarities between entities by hyperbolic distance.
- APR [2]: APR integrates adversarial learning modules into BPR, which enhances the performance and robustness of the recommendation model.

In order to make a fair comparison, we set the parameters of each method according to the respective references or based on our experiments. For all compared models, the learning rate is selected from {0.001, 0.005, 0.01, 0.05, 0.1}. For BPR, CML, H-BPR and HyperML, the regularization coefficient is chosen among {0.0001, 0.001, 0.01, 0.1, 1, 3, 5}. For CML and HyperML, the margin is tuned within {0.1, 0.3, 0.5, 1.0}. For APR and HALRec, ε varies in {0.1, 0.3, 0.5, 0.8, 1.0, 1.5, 2.0}, and λ is selected from {0.001, 0.01, 0.1, 1, 10, 100}. For H-BPR, HyperML and HALRec, the target point $\mathbf{x}$ is set to the origin of hyperbolic space. In addition, we utilize Adam to optimize all model parameters. Moreover, we divide each dataset into two parts, i.e. the training set and testing set, with a ratio of 8:2.

5.3 Performance analysis

Table 2: Performance Comparison

Dataset	Embed	Metric	BPR	CML	H-BPR	HyperML	APR	HALRec	Improve
pinterest	$K = 16$	Rec@20	0.05927	0.08208	0.08955	0.09453	0.08305	0.10705	13.25%
		$NDCG$@20	0.02392	0.03148	0.03459	0.03647	0.03343	0.04198	15.12%
	$K = 64$	Rec@20	0.06010	0.08228	0.09247	0.09504	0.08539	0.11241	18.27%
		$NDCG$@20	0.02417	0.03155	0.03528	0.03677	0.03447	0.04408	19.89%
yelp2018	$K = 16$	Rec@20	0.03830	0.03649	0.04120	0.04257	0.03686	0.05043	18.47%
		$NDCG$@20	0.02764	0.02685	0.03257	0.03276	0.03092	0.04116	25.65%
	$K = 64$	Rec@20	0.03950	0.04205	0.04237	0.04408	0.03698	0.05259	19.32%
		$NDCG$@20	0.02881	0.03106	0.03467	0.03563	0.03116	0.04342	21.85%
amazonbook	$K = 16$	Rec@20	0.01898	0.02022	0.02041	0.02099	0.02290	0.02436	6.37%
		$NDCG$@20	0.01463	0.01536	0.01576	0.01639	0.01809	0.01914	5.79%
	$K = 64$	Rec@20	0.01997	0.02458	0.02366	0.02417	0.02470	0.02919	18.18%
		$NDCG$@20	0.01565	0.01900	0.01892	0.01918	0.01954	0.02348	20.18%

The performance result comparison is presented in Table 2. The last column represents the improvements of HALRec over the strongest baseline. The main observations are summarized as follows.

- In most cases, CML performs better than BPR. Unlike CML that utilizes distance to measure the partial order among items, BPR ranks items by using inner product, which does not meet the triangle inequality, resulting in insufficient generalization of the recommendation model.
- BPR is inferior to H-BPR. Meanwhile, CML performs worse than HyperML. Those results once again confirm the necessary of capturing the hierarchical relationships among entities in recommendation systems. Moreover, HyperML outperforms H-BPR, which verifies that metric learning based recommendation models have stronger generalization than recommendation models that utilize inner product to predict user preferences.
- Compared to BPR and CML, APR achieves better performance. This observation indicates that integrating adversarial learning into the recommendation model is not only able to enhance the model robustness, but also improve the recommendation performance.
- On all datasets, HALRec consistently outperforms other compared methods, which verifies the effectiveness of our proposed model. This observation confirms that simultaneously capturing the hierarchical relationships via hyperbolic embedding and enhancing the robustness utilizing adversarial learning are both beneficial for recommendation models.

5.4 Hyper-parameter Sensitivity

Since ε and λ are important parameters of our proposed method in this study, we perform two groups of experiments to analyse the impact of ε and λ on recommendation performance.

For the sensitivity of parameter ε, we set $\lambda = 1$, the dimension of embedding $K = 16$, and ε varying in $\{0.1, 0.3, 0.5, 0.8, 1.0, 1.5, 2.0\}$. As shown in Fig 1, the improvement of HALRec over H-BPR is not remarkable when ε is significantly small. On the contrary, when ε is large enough, the performance of HALRec

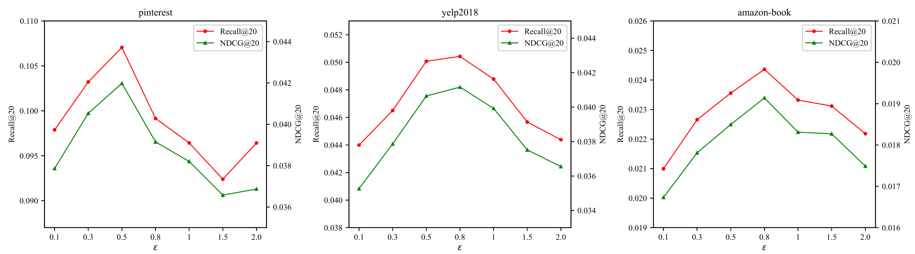

Fig. 1: The impact of ε

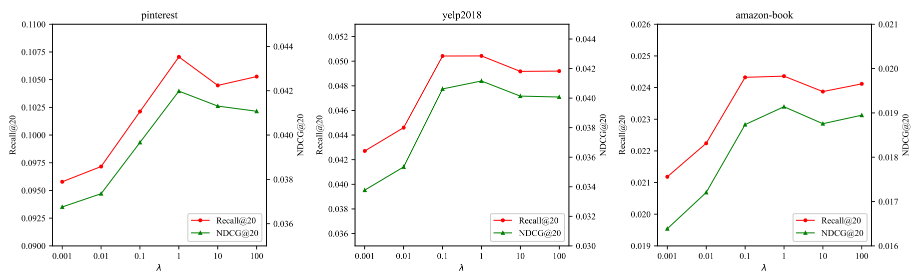

Fig. 2: The impact of λ

is influenced greatly by adversarial perturbations, leading to considerable performance deterioration. This indicates that adversarial perturbations with suitable magnitude are helpful for recommendation model to learn user preferences, while large values of ε will hinder the learning of model parameters. On Amazon-book,Yelp2018 and Pinterest, HALRec achieves its best performance when ε is around 0.5, 0.8 and 0.8, respectively.

For the sensitivity of parameter λ, we fix ε and change λ within $\{0.001, 0.01, 0.1, 1, 10, 100\}$. From Fig. 2, when $\lambda < 1$, we observe that the performance of HALRec continuously improves with the increase of λ. Under the condition of $\lambda \geq 1$, the performance of our proposed model begins to slightly oscillate, which means that HALRec is not sensitive to large λ.

6 Conclusion

In this paper, we propose a hyperbolic adversarial learning based personalized item recommendation, namely HALRec. HALRec integrates hyperbolic adversarial learning module into personalized item recommendation models, which simultaneously empowers the recommendation model with robustness and the capability of capturing implicit hierarchical relationships among entities. Experimental results on real-world datasets show that our proposed hyperbolic adversarial learning based personalized item recommendation model outperforms the state-of-the-art personalized recommendation algorithms.

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