Title
Droplet Model Electric Dipole Moments

Permalink

Authors
Myers, W D
Swiatecki, W J

Publication Date
1990-07-01

Copyright Information

UC-410
LBL-29246
Preprint

Lawrence Berkeley Laboratory

UNIVERSITY OF CALIFORNIA

Submitted to Nuclear Physics B

Droplet Model Electric Dipole Moments

W.D. Myers and W.J. Swiatecki

July 1990

NUCLEAR SCIENCE DIVISION

Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098.

LBL - 29246

Droplet Model Electric Dipole Moments*

W.D. Myers and W.J. Swiatecki

Nuclear Science Division

Lawrence Berkeley Laboratory

1 Cyclotron Road

Berkeley, California 94720

Abstract

V.Yu. Denisov's recent criticism of the Droplet Model formula for the dipole moment of a deformed nucleus as derived by C.D. Dorso et al., is shown to be invalid. This helps to clarify the relation of theory to the measured dipole moments, as discussed in the review article by S. Aberg et al.

*This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Nuclear Physics Division of the US Department of Energy under Contract DE - AC03 - 76SF00098.

Contents

1 Introduction

In ref. [1]) the following formula was derived for the electric dipole moment D of a nucleus idealized according to the Droplet Model [2–4]):

$$\begin{aligned} D/\mathrm{e} &= C_r \sum_{\ell=2}^{\infty} \frac{12(\ell-1)(\ell+1)(8\ell+9)}{5(2\ell+1)^2(2\ell+3)^2} \alpha_\ell \alpha_{\ell+1} \\ & - C_s \sum_{\ell=2}^{\infty} \frac{(\ell-1)(\ell+1)(\ell+3)}{(2\ell+1)(2\ell+3)} \alpha_\ell \alpha_{\ell+1} \\ & + \text{ terms of higher order in the } \alpha_\ell\text{'s.} \end{aligned} \tag{1}$$

In the above, e is the electric charge on the proton, C_r, C_s are constants given in terms of the parameters specifying the nucleus in question, and α_ℓ are the coefficients in the expansion of the radius vector $R(\theta)$ (specifying the surface of the nucleus) in terms of Legendre polynomials $P_\ell(\cos\theta)$:

$$R(\theta) = R_0 \left[1 + \sum_{\ell=2}^{\infty} \alpha_\ell P_\ell(\cos\theta) \right] . \tag{2}$$

A recent publication [5]) implies that formula (1) is in error because the second term ought to be identically zero. The vanishing of this term is supposed to be the result of taking account of the fixity of the center of mass of the nucleus with respect to the origin of coordinates, a constraint which was not respected in ref. [1]).

In this note we argue that: a) the position of the center of mass with respect to the origin of coordinates is irrelevant to the formula for the dipole moment, eq. (1), which does not have to be modified in any way on that account; b) the formula for the dipole moment in ref.[5]) (i.e. eq. (1) with the second term missing), is demonstrably wrong in a certain limiting case; and c) the erroneous conclusions in ref.[5]) are probably due to assuming that a shape of the type given by eq. (2) and a shape obtained from it by adding a surface layer of uniform thickness, both have the same center of mass locations.

2 The Irrelevance of the Center of Mass Location

Specifying a set of coefficients α_ℓ in eq. (2) specifies a certain geometrical shape together with its location with respect to the origin of coordinates. The center of mass of the volume inside this shape is not, in general, at the origin if odd values of ℓ are present. In ref. [1]) such a shape was taken to specify the effective sharp surface Σ of the nuclear matter distribution (i.e. of the sum of the neutron and proton densities.) With the shape of the matter distribution fixed, minimization of the nuclear energy calculated according to the Droplet Model determines the spacial distributions of the neutron and proton densities, $\rho_n(\mathbf{r})$, $\rho_p(\mathbf{r})$, and their respective effective sharp surfaces Σ_n and Σ_p. The Droplet Model formulae can then be used to obtain the positions $\mathbf{r}_p$ and $\mathbf{r}_A$ of the centers of mass of the proton and of the matter distributions. Again, neither $\mathbf{r}_p$ nor $\mathbf{r}_A$ remains, in general, at the origin of coordinates.

The dipole moment relevant in the nuclear context and calculated in ref. [1]) is not, of course, $Ze\mathbf{r}_p$ (the dipole moment of the charge Ze with respect to the origin of coordinates) but $Ze(\mathbf{r}_p - \mathbf{r}_A)$, the dipole moment of the charge distribution with respect to the center of mass at $\mathbf{r}_A$. The dipole moment is thus determined by the difference $\mathbf{r}_p - \mathbf{r}_A$, the absolute locations of $\mathbf{r}_p$ and $\mathbf{r}_A$ with respect to the origin being irrelevant. Thus, provided the quantity

Ze$(\mathbf{r}_p - \mathbf{r}_A)$ is calculated correctly in terms of the coefficients α_ℓ, the resulting expression is the correct dipole moment, independently of whether the center of mass $\mathbf{r}_A$ is at the origin or not.

If in a dynamical problem (e.g. when the α_ℓ's oscillate periodically in time) it is necessary to insist that the center of mass should stay fixed (e.g. at the origin) then, for each instantaneous set of α_ℓ's, one must apply an appropriate overall translation to the nuclear shape described by eq. (2) so as to move the center of mass back to the origin. This translation is $-\mathbf{r}_A$, for which quantity a Droplet Model formula is available. But since the dipole moment Ze$(\mathbf{r}_p - \mathbf{r}_A)$ is strictly independent of overall translations, such a shift has no effect whatever on the formula for Ze$(\mathbf{r}_p - \mathbf{r}_A)$ in terms of the α_ℓ's.

Thus the criticism in ref.[5]) that eq. (1) is incorrect because of the disregard of the fixity of the center of mass with respect to the origin of coordinates appears to us without foundation.

3 A Formula without the Neutron Skin Term must be wrong.

Consider a certain especially simple idealized limiting case of a Droplet Model nucleus, for which the skin stiffness coefficient Q is taken to be very large and the symmetry energy coefficient J is taken to be even larger, so that the ratio J/Q is also large. For the sake of simplicity assume that the incompressibility coefficient K is also very large. As is readily verified from the equations for C_r and C_s (eqs. (3, 4) in ref. [1]), this case corresponds to C_r tending to zero with C_s remaining finite. Physically, this is the case where the neutron and proton densities are uniform and have essentially their standard nuclear matter values in the bulk. (The largeness of J and K prevents deviations from these values). This pedagogical limiting case corresponds to the situation where all excess neutrons have been pushed into a neutron skin region. Moreover, since Q is assumed to be large (compared to the electrostatic energy) the thickness of this skin is essentially uniform (eq. (A30) in ref. [1])). For such a configuration the center of mass of the protons is located at the center of mass of the bulk region, whereas the center of mass of the matter is a weighted mean of the center of mass of the bulk region and of the skin region, the weighting being in proportion to the amount of matter in the two regions. Now the center of mass of a skin of uniform thickness is not, in general, at the same location as the center of mass of the space it encloses. (Think, in this connection, of a wine bottle: the center of mass of the bottle does not coincide with the center of mass of the wine. In the case of a full decanter in the shape of, for example, a right-angled circular cone, the center of mass of the wine is at a distance from the base equal to one quarter of the decanter's height, whereas for the flask itself it is at a distance from the base equal to $(2-\sqrt{2})/3 = 0.1953$ times the height). It follows that the centers of mass of the protons and of the matter do not coincide for the idealized configurations with a uniform skin described above, and such configurations will, in general, have a dipole moment. This contradicts eq. (26) in ref. [5]) (i.e. our eq. (1) without the second term), which would predict a vanishing dipole moment when J, Q, and K tend to infinity and C_r tends to zero. By contrast, the full eq. (1) gives the correct result, since its second term, which remains finite, arises precisely from the non-coincidence of the centers of mass of a uniform skin and of the volume it encloses.

4 The Probable Reason for the Discrepancy

We have not analyzed in detail every relevant equation in ref.[5]) to be quite certain where an error has crept in, but it is likely to be in eqs.(II.5) and (II.6) on p. 651. Those equations state that the centers of mass of the skin and of the bulk regions coincide. This is obviously not generally true for a skin of uniform thickness (recall the example of the decanter in the previous section). The statement is true if the skin is generated by scaling up the original radius vector $R(\theta)$ by a fixed (angle-independent) constant λ, and defining the skin as the region between the scaled-up shape $\lambda R(\theta)$ and the original shape $R(\theta)$. Something like this seems indeed to have been the situation envisaged in ref.[5]), although the notation $\overline{R}$ and $\overline{R}+\overline{t}$ for the upper limits of the radial integrations does not imply this. Thus, the shapes with and without the skin are said, in the text between eqs. (II.5) and (II.6), to have "a dependence on angle of type (1)", i.e. both are presumably imagined to be proportional to the same function of angle, as given by $1+\sum \alpha_\ell P_\ell(\theta)$. Now such a similarity holds for a skin obtained by scaling, but not otherwise. On the other hand a scaling type skin does not have a constant thickness. (Neither does a skin obtained by adding a constant $\overline{t}$ to the radius vector $\overline{R}$ of a deformed shape. The thickness of such a skin is a rather complicated function of position, depending on the angle between the radius vector and the normal to the surface.) The center of mass of a thin skin of uniform thickness is derived on page 200 of ref.[1]) (eq. (A.39)) and it differs, as it should, from the center of mass of the bulk (eq. (A.35)).

5 Conclusion

In the recent review article by A. Aberg, H. Flocard and W. Nazarewicz, ref. [6]), experimental and theoretical information on nuclear dipole moments is discussed in Section 6.3 in terms of a sum of microscopic and macroscopic contributions. The above authors conclude as follows:

> "The macroscopic contribution to the dipole moment has been estimated within the droplet model by Dorso et al. and has turned out to be negligible. This conclusion, however, has recently been questioned by Denisov whose result is consistent with the previous estimate of Strutinsky. On the other hand, the values extracted from experiment are much smaller. From this point of view the question of the magnitude of the liquid drop dipole moment is still open. The shell-correction calculations provide an overall agreement with experimental data."

In the present paper we hope to have shown that Denisov's [5]) criticism of the result of Dorso et al. [1]) is not justified. This should help to clarify the relation of experimental and theoretical aspects of nuclear dipole moments.

References

[1] C.D. Dorso, W.D. Myers and W. J. Swiatecki, Nucl. Phys. **A451** (1986) 189.

[2] W.D. Myers and W. J. Swiatecki, Ann. Phys. (N. Y.) **55** (1969) 395.

[3] W.D. Myers and W.J. Swiatecki, Ann. Phys. (N. Y.) **84** (1974) 186.

[4] W.D. Myers, Droplet Model of Atomic Nuclei (IFI/Plenum, New York, 1977).

[5] V. Yu. Denisov,Yadernaya Fizika (Journal of Nucl. Phys.) **49** (1989) 644.

[6] S. Aberg, H. Flocard and W. Nazarewicz, to appear in Ann. Rev. Nucl. Part. Sci. Vol. 40 (Dec., 1990).