Letter

Cosmological double-copy relations

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We present differential double-copy relations between gluon and graviton three-point functions in AdS_{d+1}. We introduce a set of differential operators in anti–de Sitter (AdS) that naturally generalize on shell kinematics of scattering amplitudes in flat space. This provides a way to construct AdS correlators by replacing the kinematic variables of amplitudes with the corresponding differential operators and suitably ordering them. By construction, the resulting correlators are manifestly conformally invariant, with the correct flat-space limit and exhibit a differential double-copy structure.

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I. INTRODUCTION

Correlation functions in an approximate de Sitter (dS) space are the fundamental observables of inflationary cosmology. The past several years have seen an intensive focus on the study of cosmological correlators from a boundary perspective. In this framework, basic physical principles such as symmetry and unitarity are used as fundamental inputs to determine the final observables, rather than arising as nontrivial outputs of a calculation [1–27]. The ongoing program of the *cosmological bootstrap* (see [28,29], for reviews) has revealed the underlying analytic structure of cosmological correlator, and powerful new ways of computing them that are highly obscure from the Lagrangian formalism.

Both the philosophy and technology of the cosmological bootstrap are heavily inspired by the modern on shell program of scattering amplitudes [30–33]. In momentum space, a direct connection between cosmological correlators and amplitudes in flat space is furnished by the *total energy singularity* [1,2]. Essentially, cosmological correlators arising from local bulk dynamics must reduce to amplitudes in the limit when the sum of external energies goes to zero in the complex energy plane [7,9–11], which allows us to think of cosmological correlators as a particular deformation of amplitudes away from the singular locus. This raises a tantalizing prospect that many of the remarkable properties of amplitudes can be generalized to cosmological correlators.

One of the most striking features of amplitudes is the double-copy relation between gauge and gravity theories, which expresses graviton amplitudes as two copies of gluon amplitudes. After its original discovery in string theory [34], this relation has been extended to amplitudes at higher multiplicities and multiple loops [35,36], to scalar and supersymmetric theories [37,38], and has also found applications in gravitational-wave physics (see [39–41], for reviews). A natural question is then whether there exists a generalized notion of double copy in curved backgrounds. It remains technically challenging to compute graviton correlators in anti–de Sitter (AdS) beyond three points [3,42,43], and therefore, an extension of double copy beyond flat space will be highly valuable.

A nontrivial insight from the Bern-Carrasco-Johansson double copy [35,36] is that the right objects to be double copied are the special combinations of kinematic variables that obey the Jacobi relation. This motivates a similar strategy in AdS. That is, to first identify the right kinematic building blocks for correlators. In [10,14,16], so-called *weight-shifting operators*—differential operators that shift quantum numbers in conformal field theories [44,45]—were developed in the context of cosmology. This approach highlighted the fact that differential operators can be used as basic building blocks to generate spinning correlators from simpler scalar correlators. A similar approach was used in [46–54], showing that exchange diagrams in AdS can be expressed as differential operators acting on a scalar contact diagram. In particular, these recent developments have uncovered the curved-space generalization of the double copy of scalar theories. Yet, a double-copy formulation of spinning correlators has so far remained elusive, even at the three-point level (see [48–51,55–63] for recent investigations).

In this paper, we present new differential representations of the gluon and graviton three-point functions in AdS_{d+1}.[1] We first determine conformally invariant differential

[1]Specifically, we consider Euclidean AdS correlators and dS wave function coefficients on the respective boundaries, which have the same kinematic structure up to overall normalization factors that we drop.

operators that serve as kinematic building blocks for spinning conformal correlators. We find that these operators, when suitably ordered, become natural generalizations of the kinematic variables of amplitudes to AdS. This mapping between the basic kinematic structures allows us to promote flat-space amplitudes to the corresponding AdS correlators in a straightforward fashion. We construct the three-point functions of gauge and gravity theories in this way and show that their kinematic building blocks exhibit a manifest double-copy structure.

II. CORRELATOR BUILDING BLOCKS

We will consider correlators of conserved currents on the boundary, which are dual to massless spinning particles in the bulk. Two important physical criteria for these correlators are conformal invariance and current conservation, which are the analogs of Lorentz invariance and on shell gauge invariance for amplitudes. To solve the symmetry constraint, we will use the weight-shifting operators developed in [14,44,45,64]. These operators are naturally constructed using the embedding space formalism [65,66], where conformal transformations in $\mathbb{R}^d$ are realized as Lorentz transformations on a higher-dimensional light cone embedded in $\mathbb{R}^{1,d+1}$.

To make a direct connection with scattering amplitudes, we consider the momentum-space version of the weight-shifting operators. For three-point functions, we find it most useful to consider the following set of operators [14],

$$S_{ab} \equiv \rho_a\rho_b(\vec{z}_a\cdot\vec{z}_b) + (\vec{z}_b\cdot\vec{k}_b)D_{ab} + (\vec{z}_a\cdot\vec{k}_a)D_{ba} + (\vec{z}_a\cdot\vec{k}_a)(\vec{z}_b\cdot\vec{k}_b)W_{ab}, \tag{1}$$

$$D_{ab} \equiv \rho_a(\vec{z}_a\cdot\vec{K}_{ab}) - (\vec{z}_a\cdot\vec{k}_a)W_{ab}, \tag{2}$$

$$F_{ab} \equiv (\vec{k}_b\cdot\vec{K}_{ab} + \Delta_b - d)\vec{z}_a\cdot\vec{K}_{ab} - (\vec{z}_b\cdot\vec{K}_{ab})(\vec{z}_a\cdot\partial_{\vec{z}_b}) + (\vec{z}_a\cdot\vec{z}_b)\partial_{\vec{z}_b}\cdot\vec{K}_{ab} - (\vec{z}_a\cdot\vec{k}_b)W_{ab}, \tag{3}$$

$$W_{ab} \equiv \frac{1}{2}\vec{K}_{ab}\cdot\vec{K}_{ab}, \qquad \vec{K}_{ab} \equiv \partial_{\vec{k}_a} - \partial_{\vec{k}_b}, \tag{4}$$

where $\vec{k}_a$ is the momentum, $\vec{z}_a$ is an auxiliary null vector, and $\rho_a \equiv \Delta_a + \ell_a - 1$, with Δ_a denoting the weight, and ℓ_a the spin of a conformal primary $\mathcal{O}_{\Delta_a,\ell_a}$. The subscripts a, $b = 1$, 2, 3 are field labels, the arrow over a variable denotes a vector in $\mathbb{R}^d$, and a dot product indicates the Euclidean inner product of two vectors with the metric $\delta_{\mu\nu}$, with μ,ν labeling spatial indices in $\mathbb{R}^d$. The operators above have the following action: the *spin operator* S_{ab} raises the spin at points a and b by one unit, whereas the *weight operator* W_{ab} lowers the weights at points a and b by one unit. The *spin-weight operator* D_{ab} raises the spin at point a by one unit, while lowering the weight at point b. Finally, F_{ab} raises the spin and lowers the weight at point a by one unit. The corresponding embedding-space expressions of these operators can be found in [14,45].

In this paper, we aim to address the following question: what is the flat-space limit of the weight-shifting operators? As we will see, this knowledge will enable us to directly construct AdS correlators given the corresponding amplitudes in flat space via appropriate replacements of kinematic building blocks. Seeing this requires a proper normalization and ordering of these operators, which we discuss next.

III. AMPLITUDE-CORRELATOR DICTIONARY

In the weight-shifting approach, boundary spinning three-point functions in AdS_{d+1} are represented as

$$\langle J_\ell J_\ell J_\ell\rangle = \hat{n}_\ell\langle\Phi\Phi\Phi\rangle, \tag{5}$$

where $\hat{n}_\ell$ represents a combination of weight-shifting operators, J_ℓ is a spin-ℓ conserved tensor with $\Delta_{J_\ell} = d+\ell-2$, and Φ is an integer-weight scalar dual to a shift-symmetric bulk scalar ϕ [67,68]. We use the index-free notation $J_\ell \equiv \epsilon_{\mu_1}\cdots\epsilon_{\mu_\ell}J_\ell^{\mu_1\cdots\mu_\ell}$ with the indices contracted with the polarization vector $\vec{\epsilon}$. The weight-shifting operators are nonsingular, whereas the three-point function of Φ diverges as $K^{\frac{3-d}{2}}$ for $d>3$ (or $-\log K$ for $d=3$),

$$\lim_{K\to 0}\langle\Phi\Phi\Phi\rangle = A_{\phi^3}\times(k_1k_2k_3)^{\Delta_\Phi-\frac{d+1}{2}}K^{\frac{3-d}{2}}, \tag{6}$$

where $k_a \equiv |\vec{k}_a|$ denotes the energy at point a, $K \equiv k_1+k_2+k_3$ is the total energy, and we have suppressed the delta function that enforces spatial momentum conservation. The coefficient A_{ϕ^3} is the corresponding constant amplitude, which we will set to unity. The specifics regarding the calculation of scalar seeds in momentum space can be found in the Supplemental Material [69].

Due to the inherent noncommutativity of weight-shifting operators, the differential representation (5) is far from unique. A widely used strategy is to enumerate all possible combinations of operators and fix their coefficients by other dynamical constraints such as imposing the correct behavior in the flat-space limit. However, naively applying this procedure generically leads to representations of correlators that are both algebraically cumbersome and physically unintuitive, obscuring their connection to scattering amplitudes.

In fact, there is a canonical normalization and ordering of operators that most directly reveals the flat-space limit. First of all, it turns out that it is most natural to have all the weight operators to act on the scalar correlator first. This is due to the special property of W_{ab} that it does not change the degree of singularity in K when acting on a function that goes as $K^{\frac{3-d}{2}}$, which is precisely the behavior of the scalar seed function in (6). In other words,

$$\lim_{K\to 0} W_{ab}(fK^{\frac{3-d}{2}}) = \lim_{K\to 0}(W_{ab}f)K^{\frac{3-d}{2}}, \tag{7}$$

with f some function of momenta. Let us define the normalized version of the operator as

$$\hat{W}_{ab} \equiv -\frac{2W_{ab}}{(\Delta_a + \Delta_b - \Delta_c - 2)(\Delta_a + \Delta_b - \tilde{\Delta}_c - 2)}, \tag{8}$$

where a, b, c are field labels with $c \neq a$, b and $\tilde{\Delta}_c = d - \Delta_c$, and the weights appearing on the right-hand side are those before acting with the weight-shifting operator. This choice ensures unit normalization in the flat-space limit and takes into account the weights for which the singularity in K vanishes after acting with W_{ab}. Another advantage of acting first with W_{ab} is that this avoids acting on the longitudinal factors $\vec{z}_a \cdot \vec{k}_a$ in the other weight-shifting operators. These factors vanish when we evaluate the correlator *on shell*, by which we mean computing the transverse-traceless part of the correlator with $\vec{z}_a$ replaced by the physical polarization vectors $\vec{\epsilon}_a$.

Next, consider the spin operator S_{ab}. This has a nonderivative term that becomes $\vec{\epsilon}_a \cdot \vec{\epsilon}_b$ on shell, while all of its derivative terms get multiplied by longitudinal factors. Consequently, they have a very simple on shell action,

$$\hat{S}_{a_1b_1} \cdots \hat{S}_{a_nb_n}|_{z\to\epsilon} = (\vec{\epsilon}_{a_1} \cdot \vec{\epsilon}_{b_1}) \cdots (\vec{\epsilon}_{a_n} \cdot \vec{\epsilon}_{b_n}), \tag{9}$$

when no other operators act on them, where we have normalized the operator as

$$\hat{S}_{ab} \equiv \frac{1}{\rho_a \rho_b} S_{ab}. \tag{10}$$

It is thus most natural to act with the spin operators last, in which case they simply turn into a product of polarization factors.

It remains to discuss the spin-weight operators D_{ab} and F_{ab}. While their on shell actions are less trivial, it turns out that they both turn into $\vec{\epsilon}_a \cdot \vec{k}_b$ in the flat-space limit. To see this, consider the on shell action of two D_{ab} operators, which can be expressed in terms of energy derivatives as

$$\begin{aligned}\hat{D}_{ab}\hat{D}_{cd}|_{z\to\epsilon} &= \frac{(\vec{\epsilon}_a \cdot \vec{k}_b)(\vec{\epsilon}_c \cdot \vec{k}_d)}{k_b k_d}\left(\partial_{k_b}\partial_{k_d} - \frac{\delta_{bd}}{k_d}\partial_{k_d}\right) \\ &\quad + (\vec{\epsilon}_a \cdot \vec{\epsilon}_c)\left[\frac{\delta_{bd} - \delta_{ad}}{k_d}\partial_{k_d} - \delta_{bc}\left(\frac{\partial_{k_c}}{k_c} + \frac{W_{cd}}{\rho_c}\right)\right],\end{aligned} \tag{11}$$

where δ is the Kronecker delta, and we have normalized the operator as

$$\hat{D}_{ab} \equiv \frac{1}{\rho_a} D_{ab}. \tag{12}$$

The two-derivative term in the first line of (11) gives the most singular term in K and reduces to the aforementioned kinematic structure in the flat-space limit. The other terms in (11) have different consequences depending on the index permutations of the operators. To see why, consider correlators of conserved currents in odd d. These are rational functions of energies, whereas the scalar seeds always have a logarithmic singularity. The spin-weight operators must then combine to remove this logarithmic singularity, which implies a set of selection rules for index permutations that can appear. For instance, the one-derivative term in the first line of (11) gives a logarithmic singularity that cannot be canceled against other terms due to its polarization structure, which forbids the operator combinations such as $\hat{D}_{13}\hat{D}_{23}$, for which $b = d$. Similarly, F_{ab} has the same kinematic structure as D_{ab} in the flat-space limit due to the fact that $\vec{k}_b \cdot \vec{K}_{ab}$ in (3) does not increase the degree of singularity in K. Its normalized version is given by

$$\hat{F}_{ab} \equiv \frac{1}{\Delta_a + \ell_b + \ell_c - 2} F_{ab}. \tag{13}$$

Similar to (8), this takes into account the spin and weight combinations for which correlators become trivial.

We will refer to the ordering $\hat{S}\cdots\hat{S}\,\hat{X}\cdots\hat{X}\,\hat{W}\cdots\hat{W}$ of the weight-shifting operators as *normal ordering*, where $\hat{X} \in \{\hat{D}, \hat{F}\}$. These properly normalized, normal-ordered, weight-shifting operators then serve a dual purpose: they trivialize both conformal symmetry and the flat-space limit. In particular, we have the following dictionary between the kinematic variables for amplitudes and the normalized weight-shifting operators in the flat-space limit:

$$\vec{\epsilon}_a \cdot \vec{\epsilon}_b \leftrightarrow \hat{S}_{ab}, \qquad \vec{\epsilon}_a \cdot \vec{k}_b \leftrightarrow \hat{D}_{ab}, \qquad \hat{F}_{ab}, 1 \leftrightarrow \hat{W}_{ab}, \tag{14}$$

when normal ordered. While the weight operators reduce to unity in the flat-space limit, they need to be suitably inserted in correlators to give the correct scaling weights. As we describe below, the choice between $\hat{D}_{ab}$ and $\hat{F}_{ab}$ depends on the type of interactions under consideration. Note that for $\vec{\epsilon}_a \cdot \vec{k}_b$, this is in fact a one-to-two mapping; $\vec{\epsilon}_1 \cdot \vec{k}_2 = -\vec{\epsilon}_1 \cdot \vec{k}_3$ but $\hat{D}_{12} \neq -\hat{D}_{13}$ away from $K = 0$. These two permutations typically only differ by a local term that has a delta-function support in position space, which is the boundary manifestation of the field redefinition freedom in the bulk.

IV. THREE-POINT DOUBLE COPY

The three-particle amplitudes for Yang-Mills (YM) theory and general relativity (GR) take the form,[2]

[2]To make a direct comparison with correlators in the flat-space limit, we have shown the amplitudes computed in axial gauge and also suppressed the coupling constants and color factors.

$$A_{\rm YM} = (\vec{\epsilon}_1 \cdot \vec{\epsilon}_2)(\vec{\epsilon}_3 \cdot \vec{k}_1) + {\rm cyc}, \qquad A_{\rm GR} = A_{\rm YM}^2, \quad (15)$$

$$A_{F^3} = (\vec{\epsilon}_1 \cdot \vec{k}_2)(\vec{\epsilon}_2 \cdot \vec{k}_3)(\vec{\epsilon}_3 \cdot \vec{k}_1), \qquad A_{W^3} = A_{F^3}^2, \quad (16)$$

where the first line shows the pure YM and GR amplitudes, while the second line shows the amplitudes from the higher-derivative interactions F^3 and W^3, with F the YM field-strength tensor and W the Weyl tensor. We see that the three-point amplitudes exhibit manifest double-copy relations between gauge and gravity theories. In this section, we present similar differential double-copy relations for spinning three-point functions in AdS space.

A. YM and GR

Let us first consider the three-point function of conserved spin-1 currents dual to bulk gluons. The idea is to promote the amplitude building blocks in (15) to differential operators via the dictionary (14). This turns $(\vec{\epsilon}_1 \cdot \vec{\epsilon}_2)(\vec{\epsilon}_3 \cdot \vec{k}_1)$ into, e.g., the spin-raising combination $\hat{S}_{12}\hat{D}_{31}$, which lowers the weight at point 1 by one unit. To land on the correct weight for the conserved spin-1 current $\Delta_{J_1} = d - 1$ at all three points, a natural seed object to use is the massless scalar three-point function $\langle \Phi\Phi\Phi \rangle$ with $\Delta_\Phi = d$ accompanied by $\hat{W}_{23}$. This allows us to write

$$\langle J_1 J_1 J_1 \rangle = \underbrace{(\hat{S}_{12}\hat{D}_{31}\hat{W}_{23} + {\rm cyc})}_{\equiv \hat{n}_1} \langle \Phi\Phi\Phi \rangle_{\Delta_\Phi = d}. \quad (17)$$

By construction, this is conformally invariant and has the correct flat-space limit. One still needs to check the current conservation condition, which requires the correlator to be annihilated by the divergence operator in embedding space [66],

$$\mathrm{div}_a \equiv \partial_{X_a} \cdot T_{Z_a}, \quad (18)$$

$$T_{Z_a} \equiv \left(\frac{d}{2} - 1 + Z_a \cdot \partial_{Z_a}\right)\partial_{Z_a} - \frac{1}{2} Z_a \partial_{Z_a} \cdot \partial_{Z_a}, \quad (19)$$

where X_a is an embedding-space coordinate and Z_a is an auxiliary null vector in $\mathbb{R}^{1,d+1}$, which are related to $\vec{k}_a$ and $\vec{z}_a$ upon projection to $\mathbb{R}^d$. The equivalent condition in momentum space is the Ward-Takahashi (WT) identity [5,16,23,70], which relates the longitudinal part of a correlator to lower-point functions. It can be checked that (17) is indeed divergenceless in general dimensions.

We now come to our double-copy construction of the graviton three-point function. Note that the naive procedure of squaring the whole correlator would not work for the following reasons. First, since $\Delta_{J_2} = \Delta_{J_1} + 1$ and the operator $\hat{n}_1$ has an overall scaling weight of -1, we need to accordingly adjust the weight of the seed scalar from $\Delta_\Phi = d$ to $\Delta_\Phi = d + 2$. Another important subtlety is that conformal symmetry combined with the flat-space limit does not fully guarantee that the resulting correlator satisfies the WT identity. As we described before, only certain operator combinations cancel the undesired singularity of the scalar seed. To see this, note that $\hat{S}_{12}\hat{S}_{23}\hat{S}_{31}\langle \Phi\Phi\Phi \rangle|_{\Delta_\Phi = d}$ is conformally invariant and has the correct quantum numbers of a conserved spin-2 three-point function, and so it can in principle be part of the correlator. However, it has an unphysical, lower-order singularity, which is not constrained by the flat-space limit.

Taking these considerations into account, we have found that the graviton three-point function admits the following representation:

$$\langle J_2 J_2 J_2 \rangle = \underbrace{:\hat{n}_1^2:}_{\equiv \hat{n}_2} \langle \Phi\Phi\Phi \rangle_{\Delta_\Phi = d+2}, \quad (20)$$

where operators enclosed within colons are normal ordered, with $\hat{D}_{ab}\hat{D}_{cd}$ ordered such that $a \leq c$. This ordering of the operators ensures the cancellation of the undesired singularity of the scalar seed. Explicitly, we have

$$\hat{n}_2|_{\hat{W}=1} = \hat{S}_{12}^2\hat{D}_{31}^2 + \hat{S}_{23}^2\hat{D}_{12}^2 + \hat{S}_{31}^2\hat{D}_{23}^2 + 2(\hat{S}_{12}\hat{S}_{23}\hat{D}_{12}\hat{D}_{31} + \hat{S}_{12}\hat{S}_{31}\hat{D}_{23}\hat{D}_{31} + \hat{S}_{23}\hat{S}_{31}\hat{D}_{12}\hat{D}_{23}). \quad (21)$$

To avoid clutter, we have only shown part of the formula after stripping off various factors of the weight operators on the right. To reintroduce them, note that each $\hat{D}_{ab}$ in (17) is accompanied by $\hat{W}_{cd}$ with $b \neq c \neq d$. We see that the kinematic operator $\hat{n}_2 = :\hat{n}_1^2:$ exhibits a double-copy structure, akin to the amplitude (15). In the flat-space limit, the correlator directly reduces to the amplitude $A_{\rm GR}$, as implied by the dictionary (14). Again, it can be checked that (20) is divergenceless in general dimensions.

The differential representation is not unique, even when the operators are normal ordered. This is due to the nonvanishing commutator $[\hat{D}_{ab}, \hat{D}_{cd}] \neq 0$ for $a \neq c$. (In contrast, $\hat{S}_{ab}$ and $\hat{W}_{ab}$ have vanishing commutators among themselves.) We may also take two copies of $\hat{n}_1$ with different permutations, which gives the same nonlocal part of the correlator, but can differ by local terms. For instance, there exists a cyclic-symmetric representation of operators given by

$$\hat{n}_2^{\rm cyc}|_{\hat{W}=1} \equiv \hat{S}_{12}^2\hat{D}_{31}^2 + 2\hat{S}_{12}\hat{S}_{13}\frac{\hat{D}_{23}\hat{D}_{31} + \hat{D}_{21}\hat{D}_{32} - \hat{D}_{23}\hat{D}_{32}}{3} + {\rm cyc}, \quad (22)$$

which give the same correlator as (21) at separated points but differs from (21) by a local term.[3] In $d = 3$ momentum

[3]These local terms are relevant for renormalization of conformal correlators, which absorb the divergences of the scalar seed integrals [70,71]. However, they do not affect the dynamical, nonlocal part of the correlator.

space, representation (22) precisely reproduces the graviton three-point function computed in [1,72]. For concreteness, let us also provide the expressions for $d = 5$, 7 obtained from (22),[4]

$$\langle J_2 J_2 J_2 \rangle|_{d=5} = \frac{A_{\rm GR}}{K^3}[2e_3^2 + 3(e_2 + K^2)e_3 K + 3(e_2^2 - 3e_2K^2 + K^4)K^2], \tag{24}$$

$$\langle J_2 J_2 J_2 \rangle|_{d=7} = \frac{3A_{\rm GR}}{K^4}[2e_3^3 + (4e_2 - K^2)e_3^2 K + 5(e_2^2 + 3e_2K^2 - 3K^4)e_3K^2 + 5(e_2^3 - 6e_2^2K^2 + 5e_2K^4 - K^6)K^3], \tag{25}$$

where $e_2 \equiv k_1k_2 + k_2k_3 + k_3k_1$ and $e_3 \equiv k_1k_2k_3$. We derived these results using the scalar seed correlators of weight $\Delta_\Phi = 7, 9$ as the initial input. These seed functions can be computed using either the integral representation or the weight-shifting technique, both of which are elaborated in the Supplemental Material [69]. When using the former approach, the integrals are formally divergent due to the chosen weights and therefore renormalization is necessary (for details, see [71]). Interestingly, both the logarithms and the dependence on the renormalization scale are fully projected out in the representation (22).

B. Higher-derivative interactions

For correlators arising from higher-derivative interactions, it turns out that it is most useful to use the operator $\hat{F}_{ab}$ to replace $\vec{\epsilon}_a \cdot \vec{k}_b$ in (16). This is due to the property,

$$\mathrm{div}_a \hat{F}^\ell_{ab}\langle\Phi\Phi\Phi\rangle \propto (d + 2\ell - 2 - \Delta_\Phi) \times \cdots, \tag{26}$$

after acting on a scalar correlator and taking the divergence, where we have just shown the proportionality constant. This property also holds for $\hat{F}^\ell_{ab}\hat{F}^\ell_{cd}\hat{F}^\ell_{ef}$ with $a \neq c \neq e$, as long as the operators are grouped in this way. This means that the resulting correlator becomes automatically divergenceless if we use the scalar seed with $\Delta_\Phi = d + 2\ell - 2$. The seed function choice then agrees with that in (17) and (20) due to the fact that both $\hat{F}^\ell_{ab}\hat{F}^\ell_{cd}\hat{F}^\ell_{ef}$ and $\hat{n}_\ell$ have an overall weight of $-\ell$, so that they give the correct weight for the conserved spin-ℓ current, $\Delta_{J_\ell} = d + \ell - 2$.

The above discussion implies the following spin-ℓ formula for three-point functions from higher-derivative interactions,

$$\langle J_\ell J_\ell J_\ell \rangle_{\rm h.d.} = \hat{F}^\ell_{12}\hat{F}^\ell_{23}\hat{F}^\ell_{31}\langle\Phi\Phi\Phi\rangle_{\Delta_\Phi = d+2\ell-2}, \tag{27}$$

where we have picked a particular permutation of the operators.[5] For $\ell = 1$, this agrees with [49,73]. For general spins, one should also check that (27) comes purely from higher-derivative interactions. This is not immediately obvious in embedding space, since the divergenceless condition does not distinguish between the types of interactions. In momentum space, however, these higher-derivative contributions solve the homogeneous WT identity and are thus identically conserved [16,23], as well as having higher-order singularities in K (see footnote 4). We have explicitly checked that (27) gives identically conserved momentum-space correlators, up to local terms, for $\ell = 2$, 3.

V. CONCLUSIONS

What are the right kinematic variables for cosmological correlators? Given an amplitude in flat space, can we directly reconstruct the corresponding correlator in curved backgrounds? In this paper, we have provided plausible answers to these questions for three-point functions in AdS_{d+1}. In particular, we used the weight-shifting operators developed in [14,44,45] as basic kinematic building blocks to construct AdS three-point functions. We introduced a normal ordering and the proper normalization of the weight-shifting operators, which allowed us to treat them as the AdS analogs of the kinematic variables of amplitudes. Remarkably, the final differential representation of the gluon and graviton three-point functions in general dimensions has exactly the same kinematic structure as the corresponding amplitude, and thus exhibits a manifest double-copy relation. Our final results for AdS_{d+1} three-point functions are valid in both embedding and momentum spaces.

The logical next step is to generalize our three-point double-copy construction to higher spins and higher multiplicities. In flat space, the spin-ℓ three-point amplitude is simply given by the ℓth power of the spin-1 amplitude, which motivates us to find a differential generalization of this in AdS. A generalization to higher multiplicities would involve enlarging the basis set of differential operators to include combinations of conformal generators, which are the AdS analogs of the Mandelstam variables [46–51]. In addition, it would be interesting to work out a supersymmetric generalization and make contact with the existing double-copy formulation in AdS Mellin space [61], although their analysis

[4]The flat-space limit of spinning three-point functions that is consistent with our normalization convention is

$$\lim_{K\to 0}\langle J_\ell J_\ell J_\ell\rangle = A_\ell \times \frac{(k_1k_2k_3)^{\frac{d+2\ell-5}{2}}}{\left(\delta_{d,3} + \frac{d-3}{2}\right)\left(\frac{d-1}{2}\right)_{\ell-1}} K^{\frac{3-d}{2}-\ell}, \tag{23}$$

where $(a)_n$ is the Pochhammer symbol and A_ℓ is the corresponding spin-ℓ amplitude in flat space. For three-point functions from higher-derivative interactions (27), the scaling instead becomes $K^{\frac{3-d}{2}-3\ell}$.

[5]As with (21) and (22), there are multiple possible orderings of (27) that can differ in local terms but still produce the same nonlocal part of the correlator. In the Supplemental Material [69], we provide a general proof of the formula (27) that is valid for any ordering.

was restricted to scalar components of super-multiplets. A similar differential technique has proven useful in recent generalizations of the scattering equations [74–76] to AdS [46,47,52,53,63,77], and it is worth exploring the synergy between related approaches. Finally, our findings may also have implications to analytic studies of spinning correlators in conformal field theories with a weakly coupled bulk dual [73,78,79].

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