Robust Car-Following Control of Connected and Autonomous Vehicles: A Stochastic Model Predictive Control Approach

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***Abstract*—Vehicle platooning has attracted growing attention for its potential to enhance traffic capacity and road safety. This paper proposes an innovative distributed Stochastic Model Predictive Control (SMPC) for a vehicle platoon system to enhance the robustness and safety of the vehicles in uncertain traffic environments. In particular, considering the similarity between the acceleration or deceleration behaviour of neighbouring vehicles and the spring-scale properties, we use a two-mass spring system for the first time to construct an uncertain dynamic model of a formation system. In the presence of uncertain perturbations with known distributional attributes (expectation, variance), we propose an objective function in the form of expectation along with probabilistic chance constraints. Subsequently, a state feedback control mechanism is devised accordingly. Under the cumulative probability distribution function of stochastic perturbations, we theoretically derive a computationally tractable equivalent of the SMPC model. Finally, simulation experiments are designed to validate the control performance of the SMPC platoon controllers, along with an analysis of the stability performance under varying probabilities. The experimental findings demonstrate that the model can be efficiently solved in real-time with appropriately chosen prediction horizon lengths, ensuring robust and safe longitudinal vehicle formation control.**

***Index Terms*—vehicle platooning, stochastic optimisation, model predictive control, chance constraints, uncertain perturbation**

I. Introduction

Over the past two decades, there has been a substantial escalation in vehicular presence on roadways due to rapid urbanization and heightened demands for mobility. This surge has posed significant implications for congestion, safety protocols, parking availability, and emissions within the domain of road transport systems, warranting scholarly attention and investigation [1], [2]. Vehicle platooning has garnered significant interest due to its advantages of optimizing road space and enhancing safety [3]–[5]. The key objectives in designing the formation controller are to coordinate and maintain a close formation of vehicles, aiming to ensure improved safety [6], facilitate smoother traffic flow [7], and maintain a consistent distance and speed [8]. The design of the controller is crucial for vehicle platooning. A well-crafted controller design enables adaptability for vehicle platooning across various road conditions and environments, ensuring consistent and efficient performance in diverse situations. Therefore, Model Predictive Control (MPC) has become a widely adopted approach [9], [10]. MPC constitutes a control methodology founded upon the mathematical modelling of a system. Its core principle resides in the anticipation of the future behavior of the system, coupled with the resolution of an optimization problem aimed at producing an optimal sequence of control inputs.

MPC is tailored for systems characterized by multiple variables, constraints, and dynamic behavior, as exemplified in scenarios such as vehicle platooning. However, uncertainty in platooning is inherent due to the challenge of obtaining precise data for numerous vehicles and the constantly changing environmental factors. This uncertainty not only degrades the performance of individual vehicles but also propagates along the vehicle chain, potentially leading to accidents [11]. Accounting for uncertainty is crucial in designing controllers to ensure guaranteed performance in successful platooning [12]. In addressing the challenge posed by external uncertainties in platoon control systems, we introduce an SMPC method to improve the robustness of the system. Our focus is on countering uncertain disturbance parameters within the formation control system, particularly those characterized by randomly distributed information. The importance of such advancements lies in not only enhancing platooning performance but also ensuring adherence to physical constraints despite the presence of external uncertainties. By introducing the SMPC method, we aimed to fortify the platoon control system, providing resilience against disturbances with unpredictable and randomly distributed characteristics.

II. Related Work

Several studies have shown that MPC is a suitable option for platoon control. For example, [13] addressed a platoon

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control problem in a discrete-time system through an MPC approach. Additionally, the proposed approach is applicable to various topology structures. In the context of multi-lane roads and highways, a distributed MPC strategy is developed in [14] to address the longitudinal platoon control and a classical PI lateral control algorithm is designed for each vehicle. Another work by [15] proposed an MPC strategy to facilitate robust control in the face of external disturbances within a vehicle platooning system. In [16], a distributed MPC algorithm for connected vehicle platoons under abnormal communication conditions is introduced. [17] introduced an MPC method that can adapt to various mixed traffic conditions in a platooning system. Considering the complex nonlinear dynamics of vehicle platooning systems, Nonlinear MPC emerges as a more effective strategy [18]. In [19], a secure distributed nonlinear MPC algorithm is proposed, comprising detection and mitigation phases, capable of ensuring formation control performance despite diverse communication topologies. [20] presented an ecological MPC control strategy for CAV platoon under V2V topological communication, this proposed scheme minimizes energy consumption by optimizing the speed error, distance deviation, and fuel consumption in the cost function.

In the investigation of vehicle platooning systems, Deterministic Model Predictive Control (DMPC) or nominal models are commonly employed. Nonetheless, these models often neglect uncertainties, including measurement errors, sensor errors, and external noise. Such uncertainties are prevalent in real-world scenarios and have the potential to significantly influence the stability and safety of the system [21], [22]. In particular, random perturbations may lead to the system state surpassing safety thresholds, thereby increasing the potential risk of chain collisions among formation vehicles. Hence, the development of robust formation controllers is crucial to ensuring the stability and safety of the system. In recent years, many scholars have explored the vehicle formation planning and control problem based on reinforcement learning-based control methods, robust bounded control methods, robust tube control methods, and stochastic optimization methods. [23] presented a Deep Neural Network (DNN) control scheme for vehicle parking manoeuvres. They designed a multi-layer optimised structure for this solution. The first layer uses a desensitised trajectory optimisation method to establish the optimal parking trajectory. The second layer uses pre-planned trajectory data to train DNNs to learn the relationship between system states and control actions. [24] introduced a fusion of deep reinforcement learning and genetic algorithms for intelligent formation control, addressing the dynamically evolving challenges in cooperative driving for self-driving vehicles. [25] presented an optimization algorithm for communication proximity policies, targeting the complex multi-agent body problem in fleet control. The algorithm incorporates a parameter sharing structure to accommodate diverse vehicle dynamics, mitigating collision risks through the implementation of communication protocols and course learning. Although reinforcement learning-based vehicle formation control methods have potential advantages in achieving collaborative behaviour, they also have some limitations, such as the lack of visual embodiment of the model, and possible training and computational complexity when dealing with large-scale vehicle platoons.

In addition, the Robust Bounded MPC (RBMPC) method focuses on addressing parameter uncertainties or deterministic disturbances within known bounds. It aims to design controllers that ensure system robustness against these specified uncertainties [26]. Typically employing deterministic models, RBMPC designs robust controllers to handle known variations or disturbances within established ranges, ensuring system stability and performance [27]–[29]. For example, [30] introduced an RBMPC algorithm for cooperative control of connected vehicle platoons under parameter uncertainty. The algorithm, tolerant to certain parameter uncertainties, establishes stability conditions in the form of linear matrix inequalities. Another work [31] introduced a centralized robust model predictive control algorithm for reentry vehicles, ensuring robust constraint satisfaction amid uncertainties. In summary, RBMPC is commonly employed to deal with deterministic uncertainties within known ranges [32]. However, if the considered probability distribution of uncertain disturbance has infinite support, such as Gaussian distribution or Uniform distribution, there is no fixed upper bound on the disturbance realizations. This limitation restricts the applicability of the RBMPC method [33]–[35].

In contrast to RBMPC, Robust Tube MPC (RTMPC) integrates point parameter estimation and a method based on pipe structure to address parameter uncertainties and additional disturbances [36]. By employing feedback and feedforward control laws, nominal and error states are obtained separately, dynamically adjusting the actual vehicle tracking error to ensure it stays within the boundaries set by the tube. Feng et al. introduced a robust formation control framework based on a tube-based MPC model, specifically addressing the collaborative adaptive cruise control problem within mixed traffic flow [37]. This method dynamically mitigates prediction uncertainties by constraining them within a defined range, enabling the planning of a tube sequence. Through the activation of feedforward control, the actual trajectories of CAVs are restricted. In a related study, Luo et al. investigated the impact of disturbances and modelling errors on the control system [38]. They employed an unknown input proportional multiple integral observers to estimate centralized disturbances, concurrently estimating the vehicle tracking error state. Subsequently, a tube-based RMPC method was implemented, demonstrating that the deviation between the actual system and the nominal system is confined within a robust positively invariant set. Using the RTMPC, certain potential advantages, such as improved robustness and guaranteed stability, are likely to be achieved. However, applying this strategy directly to the tracking control problem under consideration may pose challenges due to the asymmetrical nature of actual disturbances. The method may assume symmetry in the effects of disturbances in positive and negative directions, but real-world disturbances might exhibit non-symmetrical characteristics. Consequently, this discrepancy between the assumed and actual disturbance properties can impact the performance of the RTMPC method, especially when dealing with disturbances that deviate significantly from symmetrical behaviour.

Compared to the above methods, the SMPC method mainly copes with random perturbations in the system for which characteristic information about the probability distribution (expectation and variance) of these perturbations is known. Therefore, SMPC is a promising method for solving the vehicle platoon control problem in uncertain environments due to its ability to handle unbounded or asymmetric disturbances [39], [40]. Based on the characteristics of stochastic disturbances, the corresponding probabilistic chance constraints and expected objective functions are constructed, and then the prediction model is proposed to predict future control behaviour with robustness. In each control period, the proposed SMPC model minimises the expected objective of the platoon system under the system dynamic update and chance constraints [41]. Several scholars have applied stochastic optimisation ideas to vehicle control systems [42], [43]. For example, [44] designed an economic traffic signal control method under a speed management framework, this model includes fuel consumption and emission objectives and is implemented through the MPC framework. [45] designed SMPC control for a hybrid vehicle platoon system consisting of human-driven and autonomous trucks, aiming to ensure platoon feasibility, robustness and safety. [46] proposed a discrete hybrid stochastic model for efficient traffic management through connected and automated vehicle platooning, with a focus on ensuring safety through an emergency braking system. [47] introduced an efficient trajectory planning framework for automated vehicles, combining SMPC for optimized trajectories with a safety-oriented backup trajectory planning using reachable sets. Thus, SMPC is an effective method to handle the vehicle control problem in uncertain traffic environments [48].

Inspired by the stochastic optimization theory, this paper formulates a distributed SMPC model tailored for vehicle platooning systems in uncertain environments. The primary objective is to ensure vehicle safety and enhance the system's robustness to disturbances. In contrast to conventional models, this study pioneers the construction of kinematic equations for the platoon system using a two-mass spring structure. Within this SMPC model, vehicles in a platoon are represented as mass blocks interconnected by springs (Figure 2). Consequently, attractive or repulsive forces, stemming from these springs between adjacent vehicles, ensure velocity consistency and maintain an ideal spacing between the vehicles. Building upon this structural innovation, the paper proposes a distributed SMPC platoon control method grounded in a two-mass spring structure, incorporating probabilistic chance constraints and a specified objective function. Utilizing the distributional information of random perturbations, the study derives a computable counterpart model for the SMPC model under the cumulative distribution function. The proposed model's effectiveness in terms of platoon safety and resistance to perturbations is then rigorously evaluated through simulation tests, including parametric analysis and comparative tests. In summary, the key contributions of this paper are:

- We introduce a novel distributed SMPC framework for platooning systems, with a primary focus on ensuring safety and robustness. Our key innovation involves incorporating a two-mass spring structure to accurately represent the dynamic state of each vehicle in the platoon. This distinctive modelling approach enables the formulation of an error equation for the platoon system, offering a unique perspective on the interaction dynamics among vehicles.
- We derive a counterpart model for the SMPC model, leveraging the cumulative probability distribution function. Through the utilization of information on the probability distribution of the random variable, encompassing its expectation and a fixed distribution, we theoretically formulate a deterministic representation for both the expectation objective and the associated probability constraint. This formulation is computationally feasible.
- To validate the practical efficacy of our proposed SMPC method, we conduct simulation experiments to demonstrate the effectiveness of the SMPC method in real-world applications. Additionally, we perform some comparative experiments to illustrate that our method excels over other approaches in terms of platoon safety, anti-interference and computational efficiency.

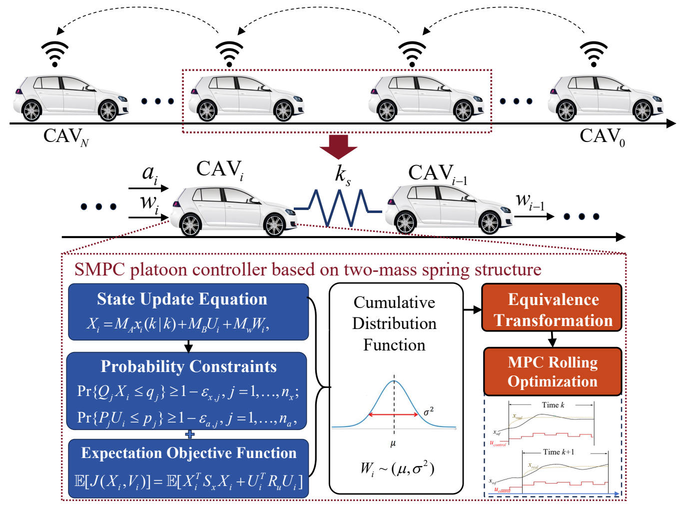


Fig. 1: The system scenario and implementation framework of the SMPC method.

The organization of the remaining sections in this paper is as follows. In Section III, we propose the SMPC platoon control model under stochastic disturbances. In Section IV, we derive the deterministic equivalent form of the SMPC model, and in Section V, we conduct experimental validation and comparative experiments, illustrating the computational real-time performance of the method. In Section VI, we summarize the full paper and elaborate on future research directions.

III. OVERVIEW OF SMPC PLATOON SYSTEM

A. Vehicle Platoon Dynamics based on Two-mass Spring Structure

In this section, we present an SMPC model designed for vehicular platoon systems to mitigate the impact of random perturbations and ensure the resilience of the system. The interaction dynamics between two adjacent vehicles are encapsulated through a two-mass spring structure, depicted in Figure 2. Each vehicle is interconnected with its neighbouring

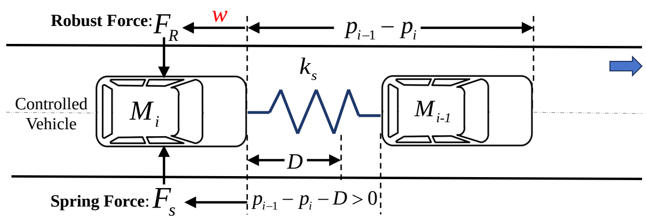


Fig. 2: The sketch of two-mass spring system.

counterparts through a spring structure, and the attractive or repulsive forces arising from these springs aim to uphold the desired distance and speed between the vehicles. The stiffness of the spring, denoted as k_s, reflects the spring's rigidity [49]. This coefficient is tailored based on the disparity between the current vehicle spacing and the ideal vehicle spacing: the greater the difference, the higher the coefficient, resulting in a more robust corrective force (control signal) being generated. This approach enhances the system's ability to resist disturbances and ensures its stability.

Figure 2 shows the sketch of the two-mass spring system. Inspired by literature [50], we construct the dynamic equations for the controlled vehicle i:

$$\begin{aligned} \dot{p}_{i-1} &= v_{i-1}, \\ \dot{p}_i &= v_i, \dot{v}_i = a_i, \\ F_{i-1} = M_{i-1}\ddot{p}_{i-1} &= -k_s\left(p_{i-1} - p_i - D\right) + w_1, \\ F_i = M_i\ddot{p}_i &= k_s\left(p_{i-1} - p_i - D\right) + M_i u_i + w_2, \end{aligned} \tag{1}$$

among that M_{i-1} and M_i are the masses of the vehicles $i-1$ and i. p_{i-1} and v_{i-1} are the position and speed of vehicle $i-1$, p_i and v_i are the position and speed of vehicle i. a_i is the real acceleration of the controlled vehicle i. D is the ideal spacing between two vehicles. F_{i-1} simulates the spring force on the vehicle $i-1$ and F_i is the real force on the controlled vehicle i, which is composed of spring force F_s and robust force F_R. Thus, u_i is the control signal generated by the robust force, and the true acceleration of the vehicle is determined by F_i, that is $F_i = M_i a_i$. In addition, k_s is a constant of the two-mass spring structure. vehicle $i-1$ is not a controlled object in this system, and its acceleration is obtained from the previous pair of spring systems [51]. w_1 and w_2 are stochastic disturbances of state system.

Remark 1: 1) In the dynamic equation (1), the stochastic disturbances mainly refer to measurement errors caused by sensor non-coordination and the influence of vehicle dynamic characteristics on distance perception. These uncertain disturbances are randomly generated, and they have certain characteristic information [52], [53]. 2) In this paper, we explore a robust car-following control method with the two-mass spring structure and build an SMPC model for the uncertain traffic environment. We assume that inter-vehicle communication adopts the leader-follower topology, thus we consider the force exerted by the spring between the two vehicles.

The state error vector can be defined as $\boldsymbol{x} = [e_{1,i}, e_{2,i}]^{\mathrm{T}}$, where $e_{1,i} = p_{i-1} - p_i - D$, $e_{2,i} = v_{i-1} - v_i$. Thus, the i−th platooning error system is

$$\begin{bmatrix} \dot{e}_{1,i} \\ \dot{e}_{2,i} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k_s}{M_{i-1}} - \frac{k_s}{M_i} & 0 \end{bmatrix} \begin{bmatrix} e_{1,i} \\ e_{2,i} \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} u_i + \begin{bmatrix} 0 & 0 \\ \frac{1}{M_{i-1}} & -\frac{1}{M_i} \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}. \tag{2}$$

In a proper sampling time slot τ, we can obtain the following state equation based on Euler's approximation approach:

$$\boldsymbol{x}_i(k+1) = \boldsymbol{A}\boldsymbol{x}_i(k) + \boldsymbol{B}u_i(k) + \boldsymbol{G}\boldsymbol{w}_i(k), \tag{3}$$

where $\boldsymbol{A} = \begin{bmatrix} 1 & \tau \\ k_s(-\frac{\tau}{M_{i-1}} - \frac{\tau}{M_i}) & 1 \end{bmatrix}$; $\boldsymbol{B} = \begin{bmatrix} 0 \\ -\tau \end{bmatrix}$; $\boldsymbol{G} = \begin{bmatrix} 0 & 0 \\ \frac{\tau}{M_{i-1}} & -\frac{\tau}{M_i} \end{bmatrix}$. In Equation (3), $\boldsymbol{w}_i = [w_1, w_2]^{\mathrm{T}}$ is stochastic perturbation caused by the measurement error of the sensor and uncertainty noise. In our settings, the characteristic information of stochastic perturbation $\boldsymbol{w}_i$ are known, i.e., $\mathbb{E}[\boldsymbol{w}_i] = \boldsymbol{0}$. Thus, the nominal state equation is:

$$\bar{\boldsymbol{x}}_i(k+1) = \boldsymbol{A}\bar{\boldsymbol{x}}_i(k) + \boldsymbol{B}u_i(k). \tag{4}$$

Next, we will conduct the probability chance constraint and the expectation objective of the platoon control model.

B. Probabilistic Chance Constraint

In this section, we subsequently incorporate chance constraints for the platoon system. The body of the chance constraints are:

$$e_{p,\min} \leq e_{1,i}(k) \leq e_{p,\max}, \tag{5}$$

where $e_{p,\min}$ and $e_{p,\max}$ are the minimum and maximum values of the position error.

$$e_{v,\min} \leq e_{2,i}(k) \leq e_{v,\max}, \tag{6}$$

where $e_{v,\min}$ and $e_{v,\max}$ are the minimum and maximum values of the speed error.

$$u_{\min} \leq u_i(k) \leq u_{\max}, \tag{7}$$

where $u_{\min}$ and $u_{\max}$ are the minimum and maximum values of the control signal for the vehicle i.

Combing the spacing error (6) and velocity error (7) between vehicle $i-1$ and i, we can obtain the error state constraint:

$$\boldsymbol{x}_{\min} \leq \boldsymbol{x}_i(k) \leq \boldsymbol{x}_{\max}, \tag{8}$$

where $\boldsymbol{x}_{\min} = [e_{p,\min}, e_{p,\min}]^{\mathrm{T}}$ and $\boldsymbol{x}_{\max} = [e_{v,\max}, e_{v,\max}]^{\mathrm{T}}$.

Furthermore, all constraints of the vehicle platoon system at time k are summarised as:

$$\begin{cases} \boldsymbol{H}\boldsymbol{x}_i(k) \leq \boldsymbol{h}; \\ \boldsymbol{D}u_i(k) \leq \boldsymbol{d}, \end{cases} \tag{9}$$

where

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{I} & 0 \\ 0 & -\boldsymbol{I} \end{bmatrix}; \boldsymbol{h} = \begin{bmatrix} \boldsymbol{x}_{\min} \\ \boldsymbol{x}_{\max} \end{bmatrix}; \boldsymbol{D} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}; \boldsymbol{d} = \begin{bmatrix} u_{\min} \\ u_{\max} \end{bmatrix}.$$

The above constraint is impacted by random perturbations, allowing it to be formulated as a probabilistic chance constraint

$$\begin{cases} \mathrm{Pr}_k\{\boldsymbol{H}_j\boldsymbol{x}_i(k) \leq \boldsymbol{h}_j\} \geq 1-\varepsilon_{x,j}, j=1,\ldots,n_x; & (10)\\ \mathrm{Pr}_k\{\boldsymbol{D}_j u_i(k) \leq \boldsymbol{d}_j\} \geq 1-\varepsilon_{u,j}, j=1,\ldots,n_u, & (11)\end{cases}$$

where the value of j represents the jth row of the overall constraint. Based on the characteristic information about the probability distribution of the stochastic perturbations, we construct chance constraints in the form of constraints (10) and (11). In constraint (10), the probability that body $\boldsymbol{H}_j\boldsymbol{x}_i(k) \leq \boldsymbol{h}_j$ holds must be greater than/equal to $1-\varepsilon_{x,j}$, where $\varepsilon_{x,j}$ is the violation probability and $\varepsilon_{x,j} \in [0,1]$. In constraint (11), the probability that body $\boldsymbol{D}_j u_i(k) \leq \boldsymbol{d}_j$ holds must be greater than/equal to $1-\varepsilon_{u,j}$, where $\varepsilon_{u,j}$ is the violation probability and $\varepsilon_{u,j} \in [0,1]$. The construction of these chance constraints is based on the stochastic optimization concept, which is the key to dealing with stochastic perturbations.

C. Objective Function in Expectation Form

In this section, we propose the objective function of the distributed SMPC platoon controllers, aiming to achieve the objectives $\lim_{k\to\infty} |\boldsymbol{x}_i(k)| = \boldsymbol{0}$ in an uncertain traffic environment. According to Literature [54], we use model prediction for this process-based control approach, which performs optimal control only in the current prediction intervals, then transitions to the next prediction time interval and applies real-time control. Based on the constructed probabilistic stochastic constraints (10) and (11) and the expectation objective, we propose the SMPC model for vehicle platoon systems. Assuming the prediction horizon is N_p, $\boldsymbol{x}_i(k+h|k)$, $\boldsymbol{u}_i(k+h|k)$, and $\boldsymbol{w}_i(k+h|k)$ signify the predicted state, predictive control, and random perturbation at the time $k+h$. The initial error state is $\boldsymbol{x}_i(k|k)$, respectively. Consequently, the objective function at time k is:

$$\begin{aligned} J(k) = \Bigg[&\sum_{h=0}^{N_p-1} \left(\boldsymbol{x}_i^{\mathrm{T}}(k+h|k)\boldsymbol{S}\boldsymbol{x}_i(k+h|k)\right) \\ &+ u_i(k+h|k)Ru_i(k+h|k) + \Phi(\boldsymbol{x}_i(k+N_p|k))\Bigg], \end{aligned} \tag{12}$$

where $\boldsymbol{S}$ and R are the positive definite diagonal matrices. $\Phi(\boldsymbol{x}_i(k+N_p|k))$ is the terminal function and $\Phi(\boldsymbol{x}_i(k+N_p|k)) = \boldsymbol{x}_i^{\mathrm{T}}(k+N_p|k)\boldsymbol{Q}_{N_p}\boldsymbol{x}_i(k+N_p|k)$. Based on stochastic optimisation concepts, we proposed the expected objective of cost function (12):

$$\begin{aligned} \mathbb{E}[J(k)] = \Bigg[&\sum_{h=0}^{N_p-1} \left(\overline{\boldsymbol{x}}_i^{\mathrm{T}}(k+h|k)\boldsymbol{Q}\overline{\boldsymbol{x}}_i(k+h|k)\right. \\ &\left.+u_i(k+h|k)Ru_i(k+h|k)\right) + \Phi(\overline{\boldsymbol{x}}_i(k+N_p|k)\Bigg]. \end{aligned} \tag{13}$$

D. Closed Loop SMPC Model for Vehicle Platoon

In this section, we derive a compact form of constraints (10) and (11) under stochastic perturbations in the predicted time horizon. In addition, we incorporate a feedback control strategy into the construction of the SMPC model. Thus, in all predicted time horizon N_p, we define the following vector:

$$\begin{cases} \boldsymbol{X}_i = \mathrm{col}\{\boldsymbol{x}_i(k+1|k), \boldsymbol{x}_i(k+2|k), \ldots, \boldsymbol{x}_i(k+N_p|k)\}; \\ \boldsymbol{U}_i = \mathrm{col}\{u_i(k|k), u_i(k+1|k), \ldots, u_i(k+N_p-1|k)\}; \\ \boldsymbol{W}_i = \mathrm{col}\{\boldsymbol{w}_i(k|k), \boldsymbol{w}_i(k+1|k), \ldots, \boldsymbol{w}_i(k+N_p-1|k)\}. \end{cases} \tag{14}$$

In addition, the state error equation (3) in all prediction horizon N_p has the following form:

$$\boldsymbol{X}_i = \boldsymbol{M}_A\boldsymbol{x}_i(k|k) + \boldsymbol{M}_B\boldsymbol{U}_i + \boldsymbol{M}_w\boldsymbol{W}_i, \tag{15}$$

where $\boldsymbol{M}_A$, $\boldsymbol{M}_B$ and $\boldsymbol{M}_w$ are given by

$$\begin{aligned} \boldsymbol{M}_A &= \mathrm{diag}\left\{\boldsymbol{A}^1, \boldsymbol{A}^2, \ldots, \boldsymbol{A}^{N_p}\right\}, \\ \boldsymbol{M}_B &= \begin{bmatrix} \boldsymbol{B} & 0 & 0 & \cdots & 0 & 0 \\ \boldsymbol{AB} & \boldsymbol{B} & 0 & \cdots & 0 & 0 \\ \boldsymbol{A}^2\boldsymbol{B} & \boldsymbol{AB} & \boldsymbol{B} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \boldsymbol{A}^{N_p-1}\boldsymbol{B} & \boldsymbol{A}^{N_p-2}\boldsymbol{B} & \boldsymbol{A}^{N_p-3}\boldsymbol{B} & \cdots & \boldsymbol{AB} & \boldsymbol{B} \end{bmatrix}, \\ \boldsymbol{M}_w &= \begin{bmatrix} \boldsymbol{G} & 0 & 0 & \cdots & 0 & 0 \\ \boldsymbol{AG} & \boldsymbol{G} & 0 & \cdots & 0 & 0 \\ \boldsymbol{A}^2\boldsymbol{G} & \boldsymbol{AG} & \boldsymbol{G} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \boldsymbol{A}^{N_p-1}\boldsymbol{G} & \boldsymbol{A}^{N_p-2}\boldsymbol{G} & \boldsymbol{A}^{N_p-3}\boldsymbol{G} & \cdots & \boldsymbol{AG} & \boldsymbol{G} \end{bmatrix}. \end{aligned} \tag{16}$$

Since the expected information of the random perturbation is $\mathbb{E}[\boldsymbol{W}_i] = \boldsymbol{0}$, the nominal state equation is

$$\overline{\boldsymbol{X}}_i = \boldsymbol{M}_A\boldsymbol{x}_i(k|k) + \boldsymbol{M}_B\boldsymbol{U}_i, \tag{17}$$

Thus, the open-loop SMPC model $\mathcal{M}_1$ for the vehicle platoon system has the following form

$$\begin{aligned} \mathcal{M}_1 : \min_{\boldsymbol{U}_i}\ & \mathbb{E}[J(k)] \\ \text{s.t. } & \boldsymbol{X}_i = \boldsymbol{M}_A\boldsymbol{x}_i(k|k) + \boldsymbol{M}_B\boldsymbol{U}_i + \boldsymbol{M}_w\boldsymbol{W}_i, \\ & \overline{\boldsymbol{X}}_i = \boldsymbol{M}_A\boldsymbol{x}_i(k|k) + \boldsymbol{M}_B\boldsymbol{U}_i, \\ & \mathrm{Pr}\{\boldsymbol{Q}_j\boldsymbol{X}_i \leq \boldsymbol{q}_j\} \geq 1-\varepsilon_{x,j}, j=1,\ldots,n_x, \\ & \mathrm{Pr}\{\boldsymbol{P}_j\boldsymbol{U}_i \leq \boldsymbol{p}_j\} \geq 1-\varepsilon_{u,j}, j=1,\ldots,n_u, \\ & \overline{\boldsymbol{x}}_i(k+N_p|k) \in \Psi_{N_p}. \end{aligned} \tag{18}$$

To guarantee system stability, we present a linear feedback control strategy that calculates future controls by linearly combining the system state [55], [56], that is

$$u_i(k+h|k) = \boldsymbol{k}\boldsymbol{x}_i(k+h|k) + u_i^{new}(k+h|k), \tag{19}$$

where $u_i^{new}(k+h|k)$ is the new control signal and $\boldsymbol{k}$ is control gain which can be obtained offline. Based on the state equation

(3) and the feedback control strategy (19), a new state update equation can be derived as

$$\begin{aligned}
&\boldsymbol{x}_i(k+h+1|k)\\
&= \boldsymbol{A}\boldsymbol{x}_i(k+h|k) + \boldsymbol{B}u_i(k+h|k) + \boldsymbol{G}\boldsymbol{w}_i(k+h|k)\\
&= (\boldsymbol{A}+\boldsymbol{B}\boldsymbol{k})\boldsymbol{x}_i(k+h|k) + \boldsymbol{B}u_i^{new}(k+h|k) + \boldsymbol{G}\boldsymbol{w}_i(k+h|k)\\
&= \boldsymbol{\varphi}\boldsymbol{x}_i(k+h|k) + \boldsymbol{B}u_i^{new}(k+h|k) + \boldsymbol{G}\boldsymbol{w}_i(k+h|k).
\end{aligned} \tag{20}$$

Thus, we can derive the compact form of the state equation in all prediction horizons N_p

$$\boldsymbol{X}_i = \boldsymbol{G}_\varphi \boldsymbol{x}_i(k|k) + \boldsymbol{G}_B \boldsymbol{U}_i^{new} + \boldsymbol{M}_w \boldsymbol{W}_i, \tag{21}$$

where $\boldsymbol{G}_\varphi$ and $\boldsymbol{G}_B$ are given by

$$\boldsymbol{G}_\varphi = \mathrm{diag}\left\{\boldsymbol{\varphi}^1, \boldsymbol{\varphi}^2, \ldots, \boldsymbol{\varphi}^{N_p}\right\},$$

$$\boldsymbol{G}_B = \begin{bmatrix} \boldsymbol{B} & 0 & 0 & \cdots & 0\\ \boldsymbol{\varphi}\boldsymbol{B} & \boldsymbol{B} & 0 & \cdots & 0\\ \boldsymbol{\varphi}^2\boldsymbol{B} & \boldsymbol{\varphi}\boldsymbol{B} & \boldsymbol{B} & \cdots & 0\\ \vdots & \vdots & \vdots & \cdots & \vdots\\ \boldsymbol{\varphi}^{N_p-1}\boldsymbol{B} & \boldsymbol{\varphi}^{N_p-2}\boldsymbol{B} & \boldsymbol{\varphi}^{N_p-3}\boldsymbol{B} & \cdots & \boldsymbol{B} \end{bmatrix}. \tag{22}$$

In addition, the compact form of (19) in all prediction horizons has the following form:

$$\boldsymbol{U}_i = \boldsymbol{K}\boldsymbol{X}_i + \boldsymbol{V}_i. \tag{23}$$

where the new decision $\boldsymbol{V}_i$ in all prediction horizon N_p is $\boldsymbol{V}_i = \mathrm{col}\{u_i^{new}(k|k), u_i^{new}(k+1|k), \ldots, u_i^{new}(k+N_p-1|k)\}$. Similarly, we can deduce the compact form of probability constraints (10) and (11) under all prediction horizons:

$$\begin{cases} \Pr\{\boldsymbol{Q}_j\boldsymbol{X}_i \le \boldsymbol{q}_j\} \ge 1-\varepsilon_{x,j}, j=1,\ldots,n_x; & (24)\\ \Pr\{\boldsymbol{P}_j\boldsymbol{U}_i \le \boldsymbol{p}_j\} \ge 1-\varepsilon_{u,j}, j=1,\ldots,n_u, & (25)\end{cases}$$

where $\boldsymbol{Q} = \mathrm{diag}\{\boldsymbol{H}, h = 1,\ldots,N_p\}$, $\boldsymbol{q} = \mathrm{diag}\{\boldsymbol{h}, h = 1,\ldots,N_p\}$, $\boldsymbol{P} = \mathrm{diag}\{\boldsymbol{D}, h = 1,\ldots,N_p\}$ and $\boldsymbol{p} = \mathrm{diag}\{\boldsymbol{d}, h = 1,\ldots,N_p\}$. Besides, j is the row index of all matrices, n_x is the row index to the matrices $\boldsymbol{Q}$ and n_u is the row index to the matrices $\boldsymbol{p}$. In all predicted time domains, the objective function (13) can be rewritten as:

$$\mathbb{E}[J(\boldsymbol{X}_i, \boldsymbol{V}_i)] = \mathbb{E}[\boldsymbol{X}_i^{\mathrm{T}}\boldsymbol{S}_x\boldsymbol{X}_i + \boldsymbol{U}_i^{\mathrm{T}}\boldsymbol{R}_u\boldsymbol{U}_i], \tag{26}$$

where

$$\begin{cases} \boldsymbol{S}_x = \mathrm{diag}\{\underbrace{\boldsymbol{S},\ldots,\boldsymbol{S}}_{N_p-1}, \boldsymbol{S}_{N_p}\},\\ \boldsymbol{R}_u = \mathrm{diag}\{\underbrace{\boldsymbol{R},\ldots,\boldsymbol{R}}_{N_p}\}, \end{cases} \tag{27}$$

Combining the state equation (15), constraints (24), (25) and objective (26) of the platoon system, we propose the closed loop SMPC model $\mathcal{M}_2$:

$$\begin{aligned}
\mathcal{M}_2 : \min_{\boldsymbol{V}_i, \boldsymbol{K}}\ & \mathbb{E}[J(\boldsymbol{X}_i, \boldsymbol{V}_i)]\\
\text{s.t.}\ & \boldsymbol{X}_i = \boldsymbol{G}_\varphi\boldsymbol{x}_i(k|k) + \boldsymbol{G}_B\boldsymbol{V}_i + \boldsymbol{M}_w\boldsymbol{W}_i,\\
& \overline{\boldsymbol{X}}_i = \boldsymbol{G}_\varphi\boldsymbol{x}_i(k|k) + \boldsymbol{G}_B\boldsymbol{V}_i,\\
& \boldsymbol{U}_i = \boldsymbol{K}\boldsymbol{X}_i + \boldsymbol{V}_i,\\
& \Pr\{\boldsymbol{Q}_j\boldsymbol{X}_i \le \boldsymbol{q}_j\} \ge 1-\varepsilon_{x,j}, j=1,\ldots,n_x,\\
& \Pr\{\boldsymbol{P}_j\boldsymbol{U}_i \le \boldsymbol{p}_j\} \ge 1-\varepsilon_{u,j}, j=1,\ldots,n_u,\\
& \overline{\boldsymbol{x}}_i(k+N_p|k) \in \Psi_{N_p}.
\end{aligned} \tag{28}$$

where Ψ_{N_p} is terminal constraint set.

IV. Model Reformulation

In this section, we theoretically derive the computationally tractable equivalent form of the SMPC with the cumulative distribution function of the stochastic perturbation. Based on the mean and variance information of the stochastic perturbation, we obtain a deterministic form of the expectation objective (26):

$$\mathbb{E}[J(\boldsymbol{X}_i, \boldsymbol{V}_i)] = \overline{\boldsymbol{X}}_i^{\mathrm{T}}(\boldsymbol{S}_x + \boldsymbol{K}^{\mathrm{T}}\boldsymbol{R}_u\boldsymbol{K})\overline{\boldsymbol{X}}_i + \boldsymbol{V}_i^{\mathrm{T}}\boldsymbol{R}_u\boldsymbol{V}_i + 2(\boldsymbol{K}\overline{\boldsymbol{X}}_i)^{\mathrm{T}}\boldsymbol{R}_u\boldsymbol{V}_i. \tag{29}$$

In the following, we obtain the equivalent forms of state chance constraints and control chance constraints by using Theorem 1 and Theorem 2.

Theorem 1: For the close loop SMPC model, the state probability constraint $\Pr\{\boldsymbol{Q}_j\boldsymbol{X}_i \le \boldsymbol{q}_j\} \ge 1-\varepsilon_{x,j}, j=1,\ldots,n_x$ has the following equivalent form:

$$\boldsymbol{Q}_j\overline{\boldsymbol{X}}_i \le -F^{-1}_{\boldsymbol{Q}\widetilde{\boldsymbol{X}}-\boldsymbol{q}}(1-\varepsilon_{x,j}), j=1,\ldots,n_x, \tag{30}$$

among that $F^{-1}_{\boldsymbol{Q}\widetilde{\boldsymbol{X}}-\boldsymbol{q}}$ is cumulative distribution function (CDF) of the disturbance term $\boldsymbol{Q}\widetilde{\boldsymbol{X}} - \boldsymbol{q}$.

Proof: The error equation (15) can be divided into nominal term $\overline{\boldsymbol{X}}_i$ and perturbation term $\widetilde{\boldsymbol{X}}_i$, i.e.,

$$\boldsymbol{X}_i = \underbrace{\boldsymbol{G}_\varphi\boldsymbol{x}_i(k|k) + \boldsymbol{G}_B\boldsymbol{U}_i}_{\overline{\boldsymbol{X}}_i} + \underbrace{\boldsymbol{M}_w\boldsymbol{W}_i}_{\widetilde{\boldsymbol{X}}_i}. \tag{31}$$

Thus, the state chance constraints can be written as

$$\begin{aligned}
&\Pr\left\{\boldsymbol{Q}_j(\overline{\boldsymbol{X}}_i + \widetilde{\boldsymbol{X}}_i) \le \boldsymbol{q}_j\right\} \ge 1-\varepsilon_{x,j}, j=1,\ldots,n_x,\\
&\Rightarrow \Pr\left\{\boldsymbol{Q}_j\overline{\boldsymbol{X}}_i \le \boldsymbol{q}_j - \boldsymbol{Q}_j\widetilde{\boldsymbol{X}}_i\right\} \ge 1-\varepsilon_{x,j}, j=1,\ldots,n_x.
\end{aligned} \tag{32}$$

By introducing a new upper boundary $\boldsymbol{Y}_i$, we can compute the nominal term $\boldsymbol{Q}_j\overline{\boldsymbol{X}}_i \le \boldsymbol{Y}_i$, and the (32) can be rewritten as:

$$\Pr\{\boldsymbol{Y}_i \le \boldsymbol{q}_j - \boldsymbol{Q}_j\widetilde{\boldsymbol{X}}_i\} \ge 1-\varepsilon_{x,j}, j=1,\ldots,n_x. \tag{33}$$

Rearrangement Equation (33), we can obtain

$$\Pr\{\boldsymbol{Q}_j\widetilde{\boldsymbol{X}}_i - \boldsymbol{q}_j \le -\boldsymbol{Y}_i\} \ge 1-\varepsilon_{x,j}, j=1,\ldots,n_x. \tag{34}$$

Assumed that $\boldsymbol{W}_i$ belongs to a known probability distribution, thus the CDF of perturbation term $\boldsymbol{Q}_j\widetilde{\boldsymbol{X}}_i - \boldsymbol{q}_j$ can be obtained, and the equivalent equation of state chance constraint (34) is

$$F_{\boldsymbol{Q}\widetilde{\boldsymbol{X}}-\boldsymbol{q}}(-\boldsymbol{Y}_i) \ge (1-\varepsilon_{x,j}). \tag{35}$$

We can calculate the value of $\boldsymbol{Y}_i$ by computing $F^{-1}_{\boldsymbol{Q}\widetilde{\boldsymbol{X}}-\boldsymbol{q}}$ with probability $1-\varepsilon_{x,j}$

$$\boldsymbol{Y}_i = -F^{-1}_{\boldsymbol{Q}\widetilde{\boldsymbol{X}}-\boldsymbol{q}}(1-\varepsilon_{x,j}). \tag{36}$$

Thus, the equivalent form of state chance constraint (34) is

$$\boldsymbol{Q}_j\overline{\boldsymbol{X}}_i \le -F^{-1}_{\boldsymbol{Q}\widetilde{\boldsymbol{X}}-\boldsymbol{q}}(1-\varepsilon_{x,j}). \tag{37}$$

■

Theorem 2: For the close loop SMPC model of vehicle i, the control chance constraints $\Pr\{\boldsymbol{P}_j \boldsymbol{U}_i \leq \boldsymbol{p}_j\} \geq 1-\varepsilon_{u,j}, j=1,\ldots,n_u$ has the following deterministic equivalent:

$$\boldsymbol{P}_j(\boldsymbol{K}\overline{\boldsymbol{X}}_i + \boldsymbol{V}_i) \leq -F^{-1}_{\boldsymbol{P}\widetilde{\boldsymbol{X}}-\boldsymbol{p}}(1-\varepsilon_{u,j}), j=1,\ldots,n_u, \quad (38)$$

where $F^{-1}_{\boldsymbol{P}\widetilde{\boldsymbol{X}}-\boldsymbol{p}}$ is the cumulative distribution function of the disturbance term $\boldsymbol{P}\widetilde{\boldsymbol{X}}-\boldsymbol{p}$, different from the one used previously.

Proof: For the body of control chance constraint $\boldsymbol{P}_j \boldsymbol{U}_i \leq \boldsymbol{p}_j$, we have

$$\begin{aligned} &\boldsymbol{P}_j \boldsymbol{U}_i \leq \boldsymbol{p}_j \\ \Rightarrow &\boldsymbol{P}_j(\boldsymbol{K}\boldsymbol{X}_i + \boldsymbol{V}_i) \leq \boldsymbol{p}_j \\ \Rightarrow &\boldsymbol{P}_j[\boldsymbol{K}(\overline{\boldsymbol{X}}_i + \widetilde{\boldsymbol{X}}_i) + \boldsymbol{V}_i] \leq \boldsymbol{p}_j \\ \Rightarrow &\boldsymbol{P}_j(\boldsymbol{K}\overline{\boldsymbol{X}}_i + \boldsymbol{V}_i) \leq \boldsymbol{p}_j - \boldsymbol{P}_j\widetilde{\boldsymbol{X}}_i. \end{aligned} \quad (39)$$

Therefore, the control probability constraint can be reformulated as

$$\Pr\{\boldsymbol{P}_j(\boldsymbol{K}\overline{\boldsymbol{X}}_i + \boldsymbol{V}_i) \leq \boldsymbol{p}_j - \boldsymbol{P}_j\widetilde{\boldsymbol{X}}_i\} \geq 1-\varepsilon_{u,j}, j=1,\ldots,n_u \quad (40)$$

Similarly, a new up bound $\boldsymbol{Z}_i$ is introduced, which can be computed from $\boldsymbol{P}_j(\boldsymbol{K}\overline{\boldsymbol{X}}_i + \boldsymbol{V}_i) \leq \boldsymbol{Z}_i$, we can have

$$\Pr\{\boldsymbol{Z}_i \leq \boldsymbol{p}_j - \boldsymbol{P}_j\widetilde{\boldsymbol{X}}_i\} \geq 1-\varepsilon_{u,j}, j=1,\ldots,n_u. \quad (41)$$

Equation (41) can be rearranged as follows

$$\Pr\{\boldsymbol{P}_j\widetilde{\boldsymbol{X}}_i - \boldsymbol{p}_j \leq -\boldsymbol{Z}_i\} \geq 1-\varepsilon_{u,j}, j=1,\ldots,n_u. \quad (42)$$

Under the same assumption, the CDF of perturbation term $\boldsymbol{P}_j\widetilde{\boldsymbol{X}}_i - \boldsymbol{p}_j$ can be obtained. Accordingly, the equivalent expression of Equation (42) is

$$F_{\boldsymbol{P}\widetilde{\boldsymbol{X}}-\boldsymbol{p}}(-\boldsymbol{Z}_i) \geq (1-\varepsilon_{u,j}). \quad (43)$$

Likewise, $\boldsymbol{Z}_i$ can be calculated by computing $F^{-1}_{\boldsymbol{P}\widetilde{\boldsymbol{X}}-\boldsymbol{p}}$ with probability $1-\varepsilon_{u,j}$

$$\boldsymbol{Z}_i = -F^{-1}_{\boldsymbol{P}\widetilde{\boldsymbol{X}}-\boldsymbol{p}}(1-\varepsilon_{u,j}). \quad (44)$$

Thus, the equivalent inequality of the constraints (42) is

$$\boldsymbol{P}_j(\boldsymbol{K}\overline{\boldsymbol{X}}_i + \boldsymbol{V}_i) \leq -F^{-1}_{\boldsymbol{P}\widetilde{\boldsymbol{X}}-\boldsymbol{p}}(1-\varepsilon_{u,j}). \quad (45)$$

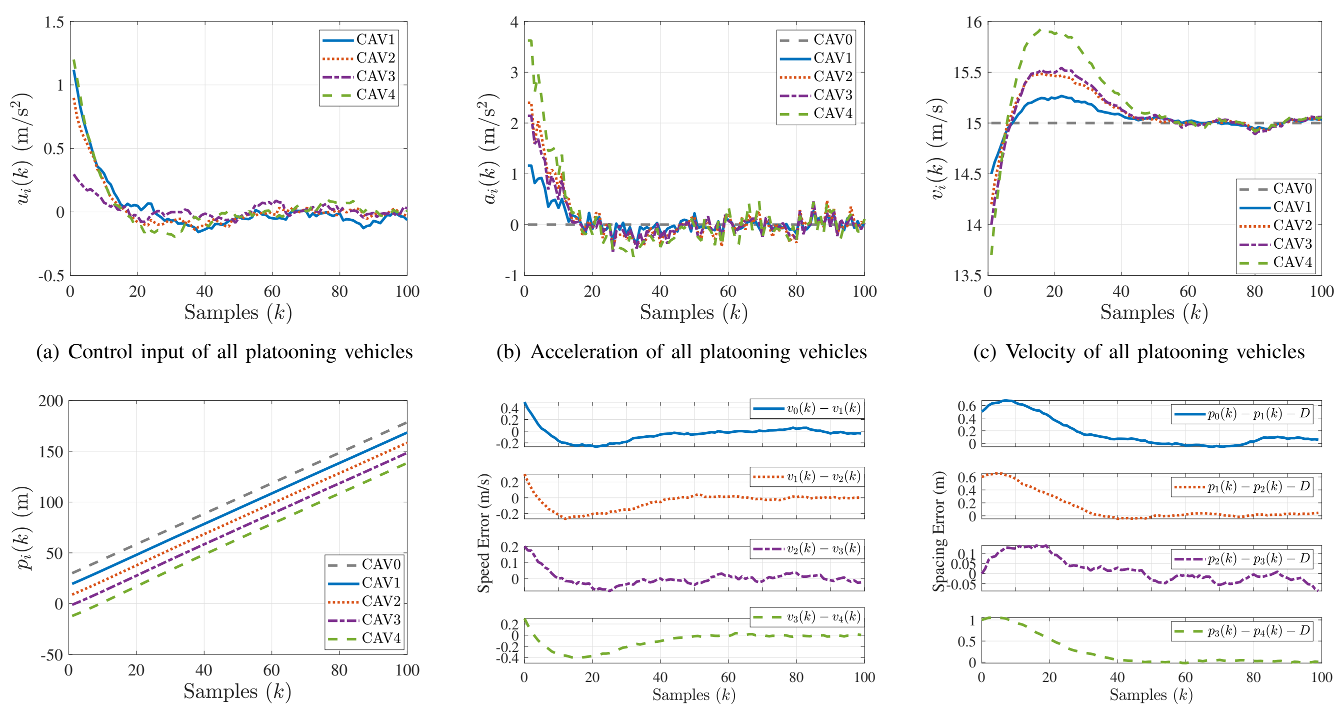
■

Combining Theorem 1 and Theorem 2, the equivalent model $\mathcal{M}_3$ of the SMPC model $\mathcal{M}_2$ can be deduced as:

$$\begin{aligned} \mathcal{M}_3 : \min_{\boldsymbol{V}_i} \ & \mathbb{E}[J(\boldsymbol{X}_i, \boldsymbol{U}_i)] \\ \text{s.t.} \ & \boldsymbol{X}_i = \boldsymbol{M}_A \boldsymbol{x}_i(k|k) + \boldsymbol{M}_B \boldsymbol{U}_i + \boldsymbol{M}_w \boldsymbol{W}_i, \\ & \overline{\boldsymbol{X}}_i = \boldsymbol{M}_A \boldsymbol{x}_i(k|k) + \boldsymbol{M}_B \boldsymbol{U}_i, \\ & \boldsymbol{U}_i = \boldsymbol{K}\boldsymbol{X}_i + \boldsymbol{V}_i, \\ & \boldsymbol{Q}_j \overline{\boldsymbol{X}}_i \leq -F^{-1}_{\boldsymbol{Q}\widetilde{\boldsymbol{X}}-\boldsymbol{q}}(1-\varepsilon_{x,j}), j=1,\ldots,n_x, \\ & \boldsymbol{P}_j(\boldsymbol{K}\overline{\boldsymbol{X}}_i + \boldsymbol{V}_i) \leq -F^{-1}_{\boldsymbol{P}\widetilde{\boldsymbol{X}}-\boldsymbol{p}}(1-\varepsilon_{u,j}), j=1,\ldots,n_u, \\ & \bar{\boldsymbol{x}}_i(k+N_p|k) \in \Psi_{N_p}. \end{aligned} \quad (46)$$

V. Performance Evaluation

In this section, simulation experiments are conducted in MATLAB to confirm the effectiveness of our proposed SMPC model for designated platoon systems. We explore the setup details of the simulation simulation experiment and showcase the safety, control performance and robustness of the SMPC method. In the experimental setting, a formation system of five



(a) Control input of all platooning vehicles
(b) Acceleration of all platooning vehicles
(c) Velocity of all platooning vehicles
(d) Position of all platooning vehicles
(e) Velocity error of all platooning vehicles
(f) Position error of all platooning vehicles

Fig. 3: The control signal, acceleration, velocity, position, velocity error and position error of all vehicles in the platoon.

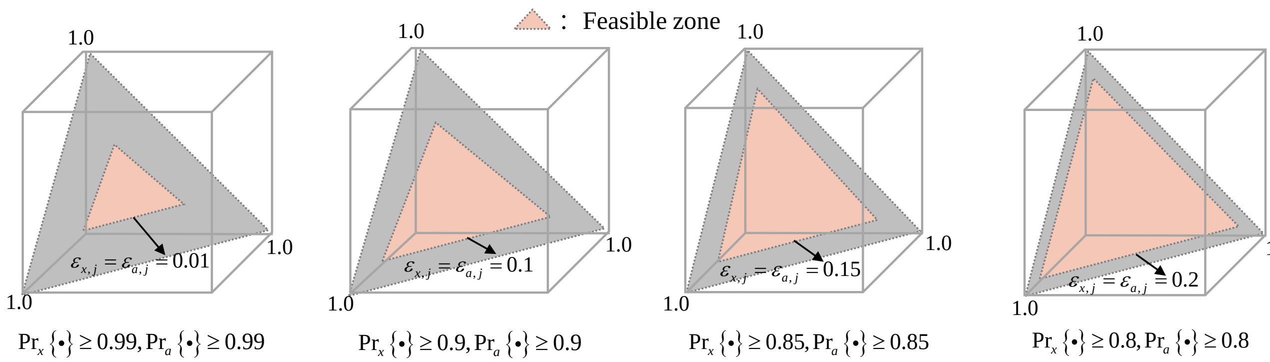
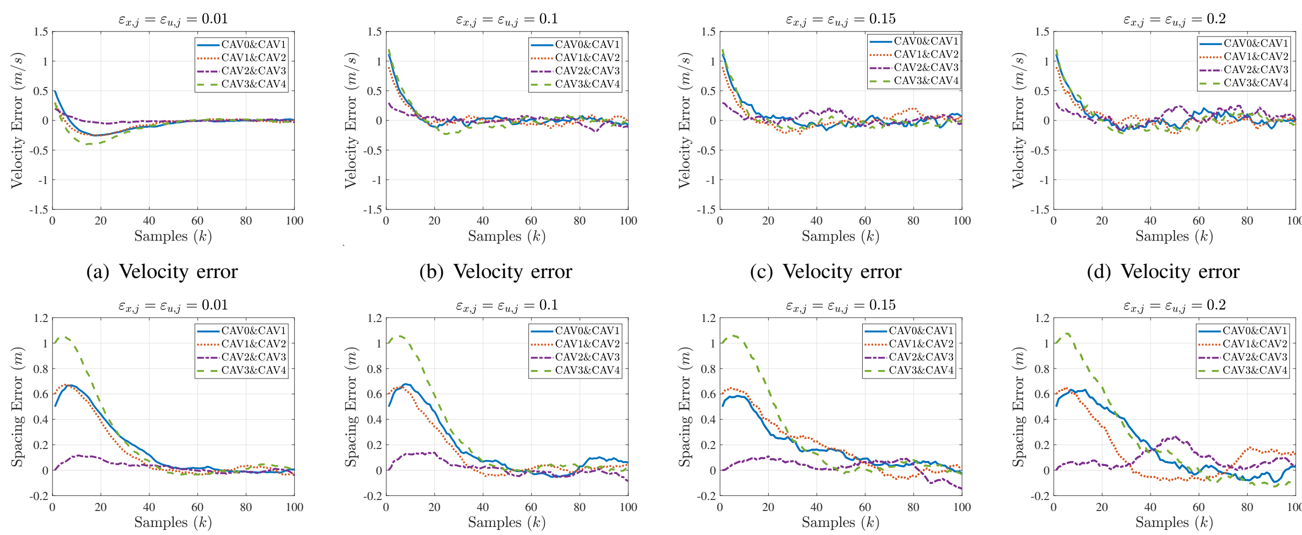


Fig. 4: The feasible zone under different probabilities.



(a) Velocity error (b) Velocity error (c) Velocity error (d) Velocity error

(e) Spacing error (f) Spacing error (g) Spacing error (h) Spacing error

Fig. 5: The acceleration, velocity error and position error of platooning CAVs under different probabilities.

vehicles is defined, where the lead vehicle has a predetermined trajectory travelling at a constant speed. All vehicles are indexed as $0, 1, \ldots, 4$. Table I shows the fundamental parameter configurations. Moreover, the probability distribution information of uncertain perturbation $\boldsymbol{W}_i$ are $\boldsymbol{W}_i \sim (0, \Sigma_w)$ where $\Sigma_w = 0.1$. All perturbations are randomly generated within each prediction time domain in the Matlab function toolbox. In addition, the elastic constant k_s between two adjacent vehicles is set to $k_s = 1.25$. The initial state are set to $\boldsymbol{x}_1(0|0) = [0.5, 0.5]$, $\boldsymbol{x}_2(0|0) = [0.6, 0.3]$, $\boldsymbol{x}_3(0|0) = [0, 0.2]$ and $\boldsymbol{x}_4(0|0) = [1, 0.3]$. To ensure the real-time performance of the proposed model, we define the prediction horizon as $N_p = 6$ and the simulation time is $N_{T_s} = 100$ sampling. Additionally, all simulations are executed using the Matlab/Simulink-based solver version R2020 on a 2.8-GHz 64-bit Core i7-8400U CPU machine running Windows 10 Professional.

TABLE I: Parameters settings

Parameter	Value	Parameter	Value
$\varepsilon_{x,j}$	0.1	$\varepsilon_{u,j}$	0.1
M_i	1500 Kg	T_s	0.1 s
$e_{p,\min}$	$-1\,\mathrm{m}$	$e_{p,\max}$	$+3\,\mathrm{m}$
$e_{v,\min}$	$-3\,\mathrm{m/s}$	$e_{v,\min}$	$+3\,\mathrm{m/s}$
$a_{\min}$	$-4\,\mathrm{m/s^2}$	$a_{\max}$	$+4\,\mathrm{m/s^2}$
$\boldsymbol{S}$	$\mathrm{diag}\{1, 1\}$	R	1

A. Experimental results

In this section, we analyse the control performance of the proposed SMPC model. Specifically, the control input, acceleration, velocity, position, velocity error and spacing error of the vehicles in the platoon system are shown in Figure 3. The subfigure 3(c) demonstrates that even if all vehicles have different initial states all following vehicles can achieve the same velocity as the lead vehicle and travel at a steady velocity of 15 m/s. In addition, all the vehicles forming a longitudinal platoon travelling at a stable spacing $D = 10\,m$, and there is no risk of chain collision [Subfigure 3(d)]. The experimental results show that even in a traffic environment with random disturbances, each following vehicle is guaranteed to have consistent velocity and to travel in formation with stable spacing under the regulation of the proposed SMPC model, which guarantees the safety of the vehicles and improves the

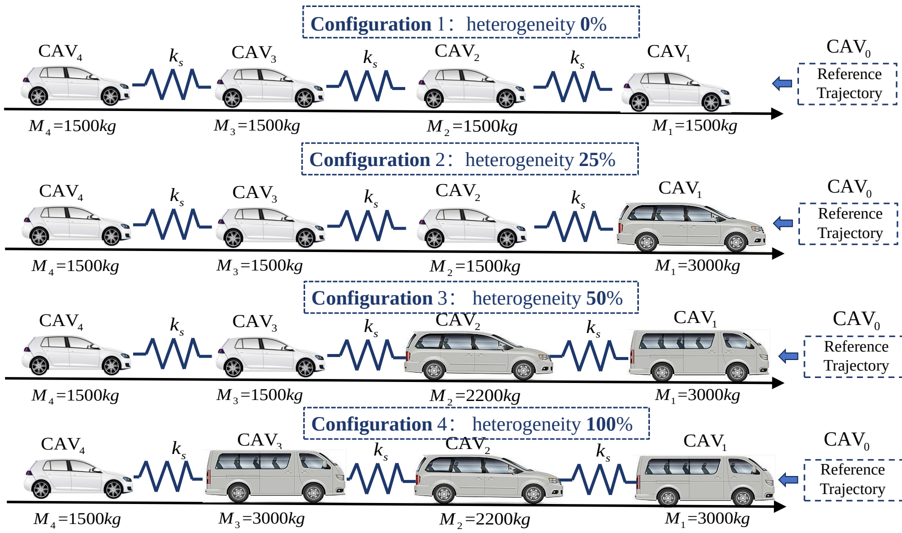
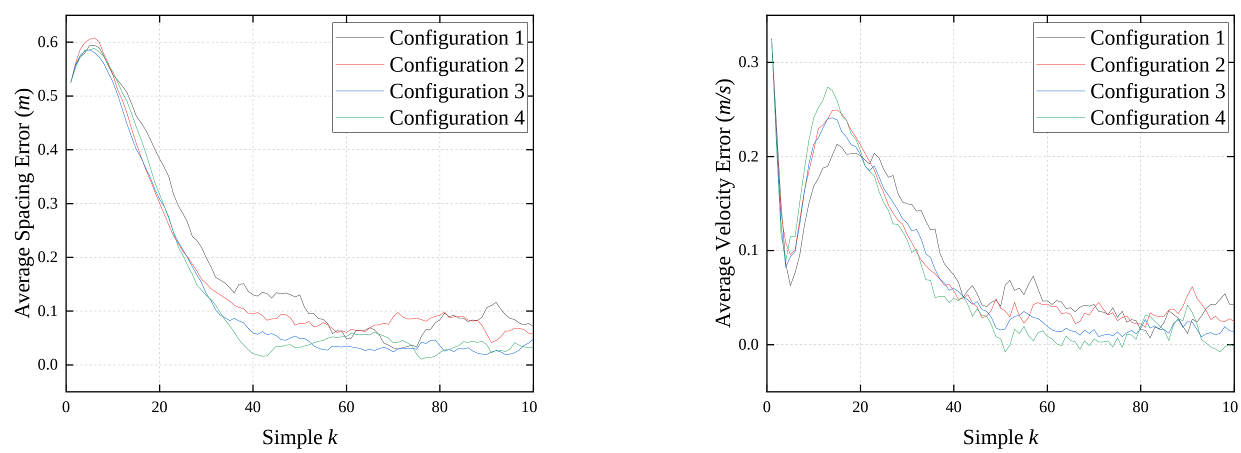


Fig. 6: The four different platoon configurations.



(a) The average spacing error under different platoon configuration

(b) The average velocity error under different platoon configuration

Fig. 7: The average spacing error and velocity error under four platoon configurations.

robustness of the controller.

Furthermore, Figures 3(e) and 3(f) depict the velocity error and position error between adjacent vehicles in the platoon, respectively. In Figure 3(c), it is evident that the velocity error of the vehicle undergoes fluctuations within a narrow range due to initial conditions and stochastic perturbations, which is reasonable. However, the sustained application of the SMPC consistently diminishes the velocity error over time. This convergence signifies the accurate alignment of the followers with the velocity of the leading vehicle, underscoring the controller's efficacy in maintaining a uniform velocity across the platoon. Similarly, Figure 3(f) illustrates the evolution of spacing errors between consecutive vehicles. Despite the initial state of every vehicle is different, the SMPC model effectively regulates the acceleration and velocity of each vehicle, ensuring the necessary inter-vehicle distance. Ultimately, vehicles can uphold uniform distances from one another, thereby minimizing collision risks and ensuring the compactness and stability of the platoon.

B. Parametric Analysis

1) The control performance under different violation probability: We explore the control performance of the proposed SMPC model under varying violation probabilities $\varepsilon_{x,j}$ and $\varepsilon_{u,j}$ in the chance constraints. It can be observed that the values of $\varepsilon_{x,j}$ and $\varepsilon_{u,j}$ determine the feasible region for the solution of model $\mathcal{M}_3$. Specifically, as $\varepsilon_{x,j}$ and $\varepsilon_{u,j}$ increase, the probability of constraints $\Pr_x\{\cdot\} \geq 1-\varepsilon_{x,j}$ and $\Pr_a\{\cdot\} \geq 1-\varepsilon_{u,j}$ being satisfied decreases. Consequently, this relaxation of constraints places a less stringent demand on the system, leading to a greater number of feasible solutions and an expansion of the feasible region. For instance, under a probability of 0.9, the feasible region is smaller than that under a probability of 0.8, given the former's more stringent limitation on the potential solution space.

To verify the reasonability of the theoretical analyses, we perform simulation experiments for four cases:

Case 1: $\varepsilon_{x,j} = \varepsilon_{u,j} = 0.01$; Case 2: $\varepsilon_{x,j} = \varepsilon_{u,j} = 0.10$;
Case 3: $\varepsilon_{x,j} = \varepsilon_{u,j} = 0.15$; Case 4: $\varepsilon_{x,j} = \varepsilon_{u,j} = 0.20$.

Figure 4 illustrates the feasible zone in four cases with different probabilities. Meanwhile, Figure 5 presents the spacing error and velocity error of platooning vehicles under varying probabilities. The figure reveals that a higher probability of the chance constraint holding (smaller $\varepsilon_{x,j}$ and $\varepsilon_{u,j}$) corresponds to increased stability of the controlled vehicle. Despite the velocity errors and spacing errors satisfying boundary constraints in the four scenarios, an elevated probability of violation diminishes the vehicles' resilience to stochastic perturbations. Consequently, significant fluctuations in vehicle spacing error and velocity errors are observed (Subfigures 5(a)-5(h)). Hence, the experimental results confirm the alignment with theoretical analysis. The choice of the violation probability significantly influences the stability and safety of the platoon controller. These experimental findings offer practical insights for decision-makers in selecting an appropriate probability of chance constraint violation. In the following experiment, $\varepsilon_{x,j}$ and $\varepsilon_{u,j}$ take the value of 0.01.

2) The control performance under different Spring systems: In the SMPC model, we use a two-mass spring structure for the kinematic states of neighbouring vehicles. Therefore, in this section, we explore the adaptability of the proposed SMPC method to heterogeneous platoons. Figure 6 shows the four different platoon configurations, with the isomerization rate increasing from the top to the bottom vehicle. For instance, Configuration 1 exhibits 0% heterogeneity, while Configuration 4 demonstrates 100% heterogeneity.

Figure 7 illustrates the average spacing and velocity error across the four platoon configurations. In subfigure 7(a), a sequential decrease in average spacing error is observed for configurations 1-4 at 40 s, suggesting that platoon configurations with low heterogeneity exhibit higher safety and robustness. Numerical results in Table II provide details on the average spacing and velocity error for each platoon configuration. Configuration 1 records the highest average spacing error at 0.194788 m, representing improvements of 10.99%, 33.06%, and 31.25% compared to configurations 2-4, respectively. These findings show that the proposed SMPC platoon control method based on a two-mass spring system is also applicable to heterogeneous platoons. Notably, platoon configurations with low heterogeneity showcase enhanced robustness and safety within the formation system.

C. Comparative Experiment

In this section, we contrast the safety, disturbance resistance and computational efficiency of our proposed SMPC control method with two other platoon control approaches under large-scale disturbance. These three methods include:

Deterministic MPC(DMPC): This method is based on a deterministic system modelling, which is not resistant to interference. It relies on accurate system dynamics to formulate control strategies.

TABLE II: The average spacing and velocity error of the four platoon configuration in all time samples

	Average Spacing Error	Average Velocity Error
Configuration 1	0.194788 m	0.088272 m/s
Configuration 2	0.175491 m	0.085134 m/s
Configuration 3	0.146445 m	0.076528 m/s
Configuration 4	0.148404 m	0.071568 m/s

TABLE III: The mean and standard deviation of computation time under three control methods

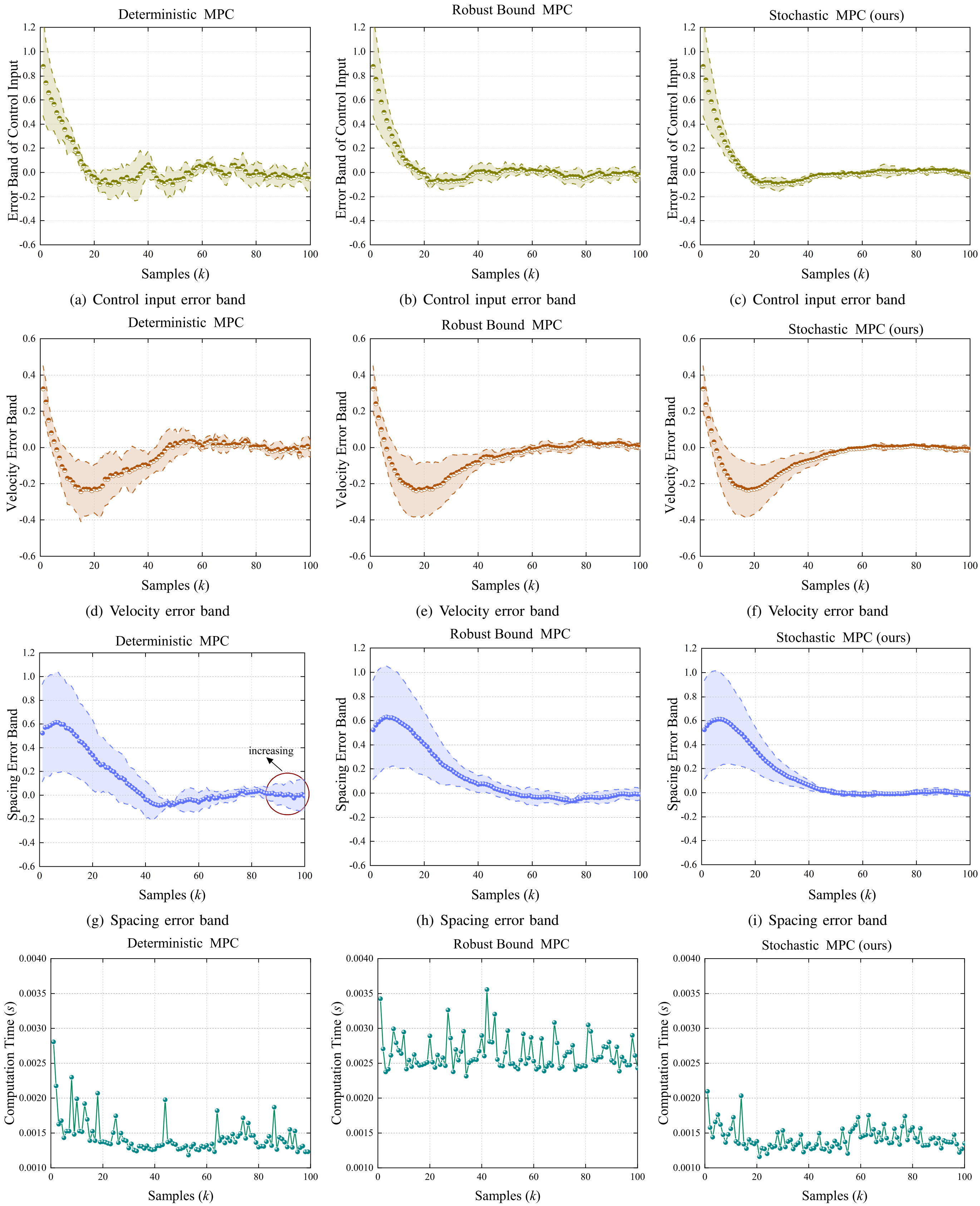
	Computation Time	
	Avg. [s]	Std. [s]
Deterministic MPC	1.344×10^{-3}	2.317×10^{-4}
Robust Bound MPC	2.632×10^{-3}	2.515×10^{-4}
Stochastic MPC(ours)	1.423×10^{-3}	1.603×10^{-4}

Robust Bound MPC(RBMPC): This method focuses on handling bounded interval disturbances within a vehicle platoon system and ensures safety and control performance under worst-case scenarios.

Stochastic MPC(SMPC): The proposed method introduces a probabilistic framework to accommodate randomness and uncertainties in real-world scenarios. It provides flexibility by considering the probabilistic nature of disturbances, enhancing adaptability.

In comparative experiments across the three control methods, we analyse the control performance and computation efficiency. In subfigures 8(a)-8(i), we present the error bands of control input, velocity and spacing for all vehicles in the platoon. It can be seen that the DMPC method lacks resilience against stochastic disturbances, leading to unstable margins for vehicle spacing errors. For example, the spacing error band of the whole formation system increases in the [80,100] s interval, which indicates the potential collision risk of vehicles under the DMPC method (See the first column in Figure 8). Although both RBMPC and SMPC have made efforts in disturbance rejection, it is clear that the proposed SMPC method outperforms in maintaining velocity consistency. For example, when k exceeds 61 s, the velocity error consistently hovers near zero, indicating that the convoy vehicles maintain a consistent travel velocity [subfigure 8(f)]. From the subfigures 8(h)-8(i), it is evident that the SMPC method excels in the consistency of inter-vehicle spacing compared to RBMPC and DMPC. For instance, when k exceeds 42 s, the spacing error consistently remains close to 0. This suggests that the platooning vehicles are maintaining uniform spacing, and the error bands of the SMPC method are fitting better than RBMPC.

In addition, we show the computation time under three control methods in subfigures 8(j)-8(l) and Table III computes the mean and standard deviation of computation time under the proposed SMPC and other control methods. In comparison, the DMPC method exhibits a lower average computation time per control execution compared to SMPC and RMPC.



(a) Control input error band (b) Control input error band (c) Control input error band

(d) Velocity error band (e) Velocity error band (f) Velocity error band

(g) Spacing error band (h) Spacing error band (i) Spacing error band

(j) Computation time (k) Computation time (l) Computation time

Fig. 8: The control performance and computational efficiency of different control methods: First column (Deterministic MPC); Second column (Robust Bound MPC); Third column (Stochastic MPC).

This is reasonable, given that the computational complexity in both SMPC and RBMPC is notably higher than that in DMPC. Additionally, the computation time for the SMPC method is lower than that for the RBMPC method, and the computational efficiency of the SMPC method is improved by 43.95% compared with that of the RBMPC. This indicates that the proposed control approach ensures algorithmic efficiency while resisting random disturbances. In summary, the NMPC method has limitations in effectively resisting random disturbances, emphasizing the necessity of considering the impact of such disturbances on platoon control. Although both RMPC and SMPC control methods exhibit the ability to resist uncertain disturbances, it is evident that under the SMPC method ($\varepsilon_{x,j} = \varepsilon_{u,j} = 0.01$), there is a more pronounced convergence of error margins. This clear observation indicates that the SMPC method excels in controlling and mitigating uncertainties, showcasing a superior performance in achieving significant convergence of error margins and computational efficiency.

VI. Conclusion and Future Work

This paper introduced a novel distributed SMPC method for a vehicle formation system, focusing on resisting random perturbations in the traffic environment and guaranteeing the safety of the platoon system. First of all, we established a vehicle motion state based on the two-mass spring principle. We then constructed probabilistic chance constraints under stochastic perturbations that are related to state and control and constructed the controller's objective in the form of the expectation of the cost function. Theoretically, we derived a computationally feasible deterministic version of the SMPC platoon controller. Finally, we conducted simulation experiments to validate the effectiveness of the proposed SMPC method and analysed the security and robustness of the SMPC model under different violation probabilities. Subsequently, we compared it with two other control methods (DMPC and RBMPC) in terms of perturbation resistance, safety, and computational efficiency. The simulation test results show that the control effect of the proposed SMPC is slightly higher than that of the RBMPC method under relatively tight probability, and the computational efficiency is improved by 43.95%. In the future, our research directions will expand to include more complex strategies with multiple mass units or other communication topology structures. In addition, we aim to address challenges related to malicious attacks in V2V communication and improve the fuel economy of the platoon. This extension endeavours to bolster the reliability of platoon formations, especially in complex environments.

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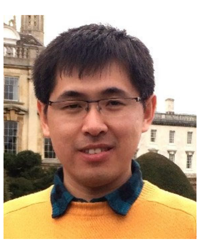

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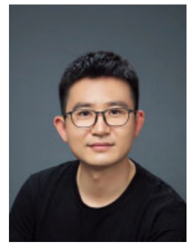

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