OPEN ACCESS
IOP Publishing
Smart Materials and Structures
Smart Mater. Struct. **33** (2024) 083001 (21pp)
https://doi.org/10.1088/1361-665X/ad5bcc

Topical Review

Mastering the art of designing mechanical metamaterials with quasi-zero stiffness for passive vibration isolation: a review

Ramin Hamzehei[1], **Mahdi Bodaghi**[2,*] **and Nan Wu**[1,*]

[1] Department of Mechanical Engineering, University of Manitoba Winnipeg, Manitoba R3T 5V6, Canada
[2] Department of Engineering School of Science and Technology, Nottingham Trent University, Nottingham NG11 8NS, United Kingdom

E-mail: Mahdi.Bodaghi@ntu.ac.uk and Nan.wu@umanitoba.ca

Received 4 October 2023, revised 19 April 2024
Accepted for publication 25 June 2024
Published 11 July 2024

Abstract
This review serves as a comprehensive design strategy for designing quasi-zero stiffness (QZS) mechanical metamaterials (MMs). It discusses their underlying deformation mechanisms that enable the attainment of QZS behavior under both compressive and tensile loadings. While the QZS characteristic of metamaterials has garnered considerable attention, further research is essential to unlock their potential fully. Numerous QZS metamaterials have been meticulously reviewed. They comprise various elements and mechanisms, including positive and negative stiffness elements (PS and NS), PS elements with variable stiffness, bending mechanisms employing stiff joints/areas, buckling, buckling-rotating, and bending/buckling deformation mechanisms leading to a QZS feature. Furthermore, the capability of multi-material, adaptive, smart metamaterials, origami (bending around the hinge of the folded joints), and kirigami lattices (out-of-plane buckling via cutting patterns) are weighted. These diverse mechanisms contribute to achieving QZS behavior in metamaterials under both compression and tension loads, which is paramount for various mechanical applications such as passive vibration isolation. This review effectively categorizes QZS metamaterials based on their underlying mechanisms, providing scholars with valuable insights to identify suitable mechanisms for the desired QZS feature.

Keywords: Smart metamaterials, Adaptive metamaterials, quasi-zero stiffness, vibration isolation, bending, buckling, 3D printing

* Authors to whom any correspondence should be addressed.

Nomenclature

MMs	Mechanical metamaterials
PCs	Phononic crystals
QZS	Quasi-zero stiffness
PS	Positive stiffness
NS	Negative stiffness
PPRs	Positive Poisson's ratio structures
ZPRs	Zero Poisson's ratio structures
NPRs	Negative Poisson's ratio structures

1. Introduction

MMs refer to synthetic materials that are typically not naturally occurring. The term originates from the Greek word 'meta,' signifying beyond. To elaborate, MMs are composite materials designed to possess a specific combination of properties that cannot be achieved by merely combining the individual properties of their components [1–4]. By arranging the architectural design of the lattice structures in specific patterns, MMs exhibit a variety of mechanical responses, including unique electromagnetic [5–13], acoustic [14–24], energy absorption/dissipation [25–29], sound absorption [30–34], impact resistance [35, 36], blast resistance [37, 38] and vibration isolation [39–50] features. They could exist as mechanical structures with negative parameters [51, 52], and can be divided into several main groups, including PPRs [28, 53, 54], ZPRs [55–57], NPRs [58–76], NS [77–83], negative thermal expansion [84–89], negative compressibility [90–92], permittivity [93], and negative mass density [94] structures. These lattice structures derive their extraordinary properties from their microstructure design rather than the chemical composition of the parent material, enabling tailored functionalities and extraordinary mechanical properties such as the variety in structural stiffness [59, 74, 75, 94–99]. The design of lattice structures could be carried out by analytical methods like Monte Carlo [100, 101], genetic algorithm-based optimization [102], and topology optimization methods such as SIMP [71, 103], BESO [104], and level set [105, 106]. It is worth mentioning that apart from the lattice structures, NPR behavior can be seen in composites and foams [25, 44, 107–109].

The manipulation of stiffness in MMs provides intricate control over mechanical characteristics. Stiffness is a fundamental mechanical property that measures the resistance of a material or structure to deformation under an applied force [110]. It quantifies the relationship between the force exerted on an object and the resulting displacement or deformation it undergoes. Stiffness is often represented by the ratio of force to displacement and is typically described in terms of the material's elasticity. In most cases, conventional materials exhibit PS [111], where an increase in applied force leads to a proportional increase in deformation. This behavior is intuitive and can be observed in everyday materials such as metals, plastics, and composites. However, NS or zero stiffness can be advantageous in certain scenarios.

Zero stiffness refers to a mechanical system or material that exhibits no resistance to deformation under an applied force [112]. It implies that any displacement applied to the system does not increase force. This behavior can be achieved by using specific designs or materials that incorporate zero stiffness elements. Zero stiffness elements act as highly compliant (flexible) components that allow for the absorption and dissipation of vibrational energy.

Mechanical vibration, the oscillatory motion of an object around a stable equilibrium position, is a phenomenon encountered across various fields and applications [111]. It is usually divided into two main groups, active and passive vibration [113]. Passive vibration isolation relies on mechanical components like springs and dampers to absorb vibrations without external power, offering simplicity but limited adaptability. Active vibration isolation employs sensors and actuators in a closed-loop system, actively countering vibrations in real-time, providing adaptability and performance across a broader range of frequencies but requiring external power and increased complexity. Understanding the characteristics and control of mechanical vibration is crucial in ensuring the optimal performance and reliability of structures and systems. One fundamental aspect that greatly influences mechanical vibrations is the stiffness of the system.

When it comes to vibration isolation, PS is conventionally employed to attenuate vibrations [113]. The presence of PS in a system provides resistance to external forces, thereby reducing the transmission of vibrations from one part of the system to another. This mechanism is commonly employed in traditional vibration isolation techniques, where materials with high PS, such as rubber or springs, are used to absorb or dampen vibrations and prevent their propagation [113].

In contrast, the concept of low or even QZS leads to ultra-low natural frequencies and presents a fascinating alternative for vibration isolation and free boundary conditions [114]. To enhance clarity, a typical frequency response curve of a second-order differential equation system is considered [115], see figure 1. When stiffness goes to zero, the natural frequency ($Wn = \sqrt{\frac{k}{m}}$, where k is stiffness, and m is the system's mass respectively) goes to zero. Then, the frequency ratio (W/Wn) goes to infinity, and the magnitude response becomes very low. This implies that the system is highly effective at isolating vibrations at low frequencies.

Consequently, systems with zero stiffness behave effectively at vibration isolation to enhance the system's ability to isolate and dampen vibrations. By carefully designing the system to incorporate zero stiffness elements, it is possible to achieve effective vibration isolation by allowing controlled displacements and reducing the transmission of vibrations [114].

One intriguing aspect of MMs is their ability to exhibit properties not found in natural materials. This includes the concept of tailored or NS, which can be realized in metamaterial designs [116]. Unlike conventional materials with PS resisting deformation, MMs can be engineered to exhibit unconventional behaviors under specific conditions

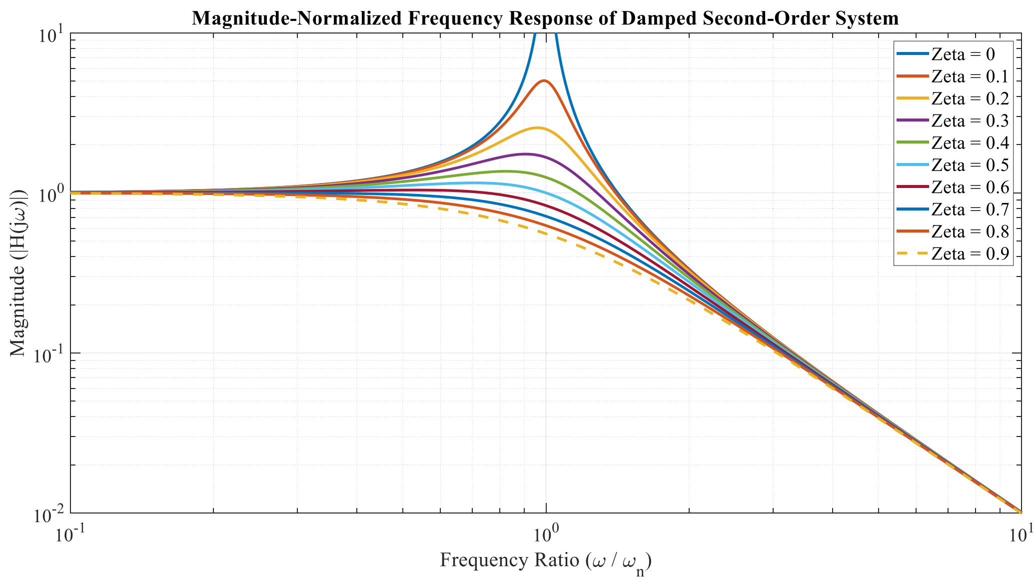


Figure 1. Typical frequency response curve of a second-order differential equation system at different damping ratios, Zeta [115].

such as zero or near-zero stiffness. These MMs can exhibit unique vibrational characteristics, providing opportunities for novel vibration control, shock, and isolation approaches [117–124].

Given the substantial impact of MMs on QZS feature and vibration isolation performance, this review serves as an inquiry into the domain of MMs, centering its focus on lattice structures endowed with the QZS attribute, elucidating the effective deformation mechanisms to pave the way for the MMs' designers to take the effective items into account to achieve a QZS behavior under compressive/tensile loads. Figure 2 (a) and (b) illustrate a discernible upward trajectory, obtained from Scopus, in the field of research concerning vibration isolation and QZS feature. This notable trend underscores the imperative for a concurrent and comprehensive evaluation. This review begins with periodic structures concept, including MMs, and PCs, and provides a brief introduction of the effective factors which can be considered within MMs designs to provide a QZS feature. In the following, section 3 profoundly elucidates the effective items which must be taken into account for designing a lattice structure to achieve a QZS feature. Ultimately, a brief indication of mechanical vibration is provided in section 4, demonstrating the effectiveness of QZS metamaterials in passive vibration isolation applications.

2. Periodicity

Periodicity refers to the regular and repeated arrangement of elements in a system, where each unit is identical to the previous one. In a periodic structure, there is a recurring pattern that can be extended infinitely. This concept is fundamental in understanding the properties of periodic structures, which are divided into two main groups, including PCs, and MMs, see figure 3. MMs encompass various engineered structures designed to exhibit unique mechanical properties, achieved through careful structural designs at the macro, micro, or even nanoscale. Their focus is on manipulating mechanical properties like changing the local stiffness within the lattice structure, changing flexibility, and deformation response for such an important engineering application as vibration control. PCs, on the other hand, are a subset of MMs specifically designed to control the propagation of mechanical waves, known as phonons [125]. Phonons are vibrational modes that transmit energy through a material. PCs manipulate these phonons by incorporating carefully designed structures that can alter the transmission, reflection, or absorption of mechanical waves. While both PCs and MMs for vibration isolation involve the manipulation of mechanical waves, they serve distinct purposes. PCs are tailored for controlling the propagation of phonons in specific frequency ranges and are used in applications such as acoustics. On the other hand, MMs for vibration isolation are designed to absorb, dampen, or prevent the transmission of mechanical vibrations, often employing features like QZS or NS. The focus of this review is on the QZS feature of MMs. The geometry of MMs in this review revolves around cylindrical structures [26], two-dimensional (2D) structures [39, 62, 71, 85, 126, 127], three-dimensional (3D) [69, 120, 128], Origami [129] and Kirigami [130] lattices. In addition, the effective items that must be incorporated within the lattice design to provide the QZS feature are illuminated. Figures 4 and 5 show the geometries of MMs considered in this study, and the mechanisms leading to a QZS feature in lattice structures respectively. They contain consideration of PS and NS elements [39, 118], variable PS elements [50], bending/buckling [56, 62, 63, 68, 131–133], smart materials in local areas within the lattice structure [134], and multi-material printing [96, 135].

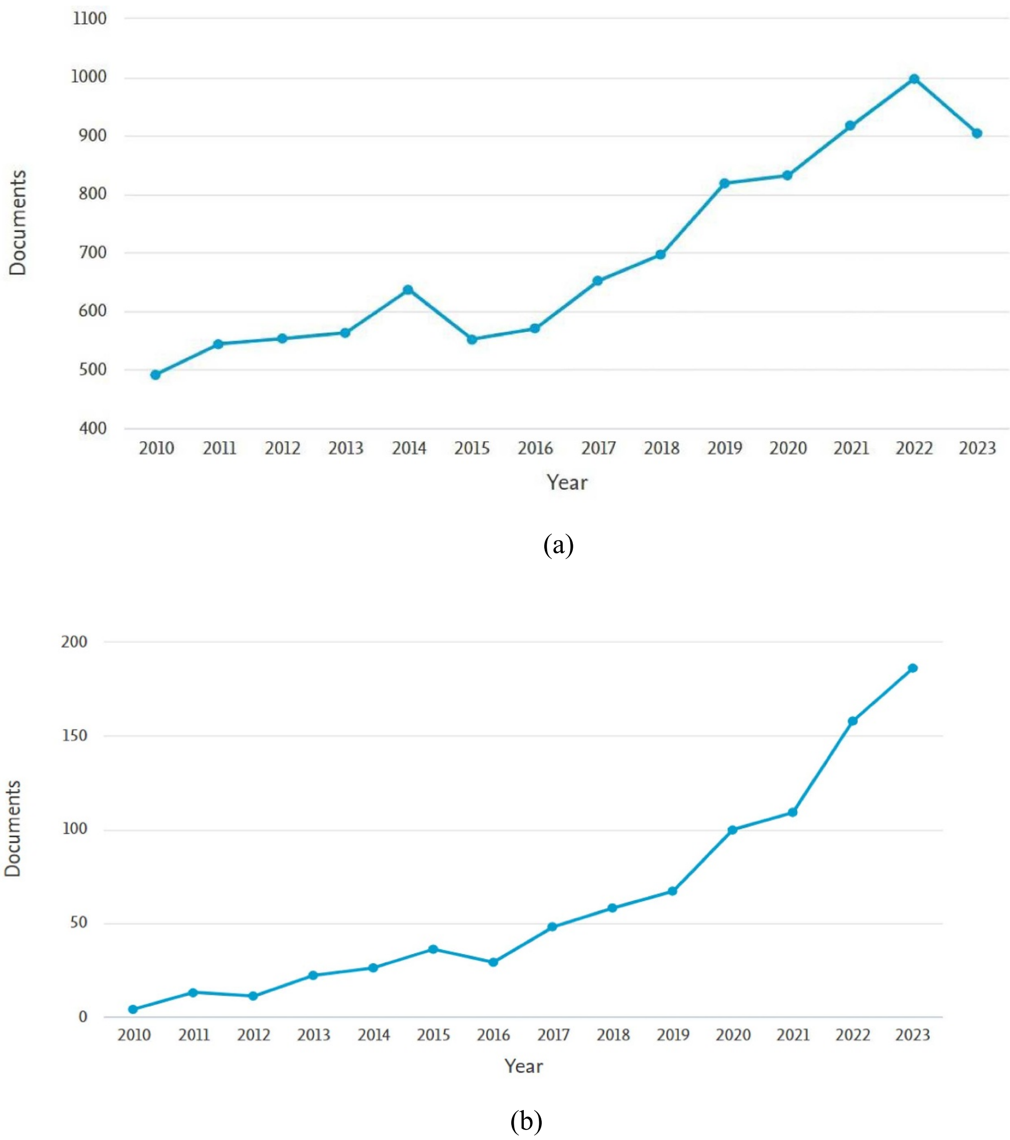


Figure 2. Statistics obtained from the database of Scopus for (a) vibration isolation, and (b) QZS metamaterials research.

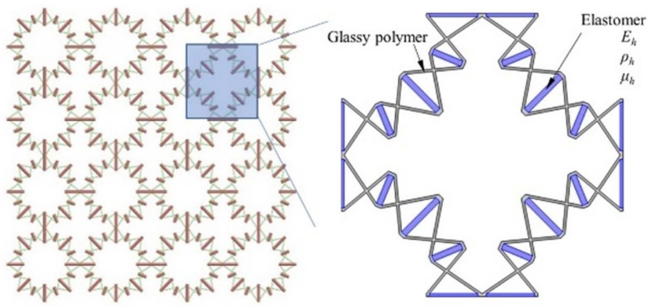


Figure 3. 2D structure of phononic crystals, Reprinted from [125], Copyright (2019), with permission from Elsevier.

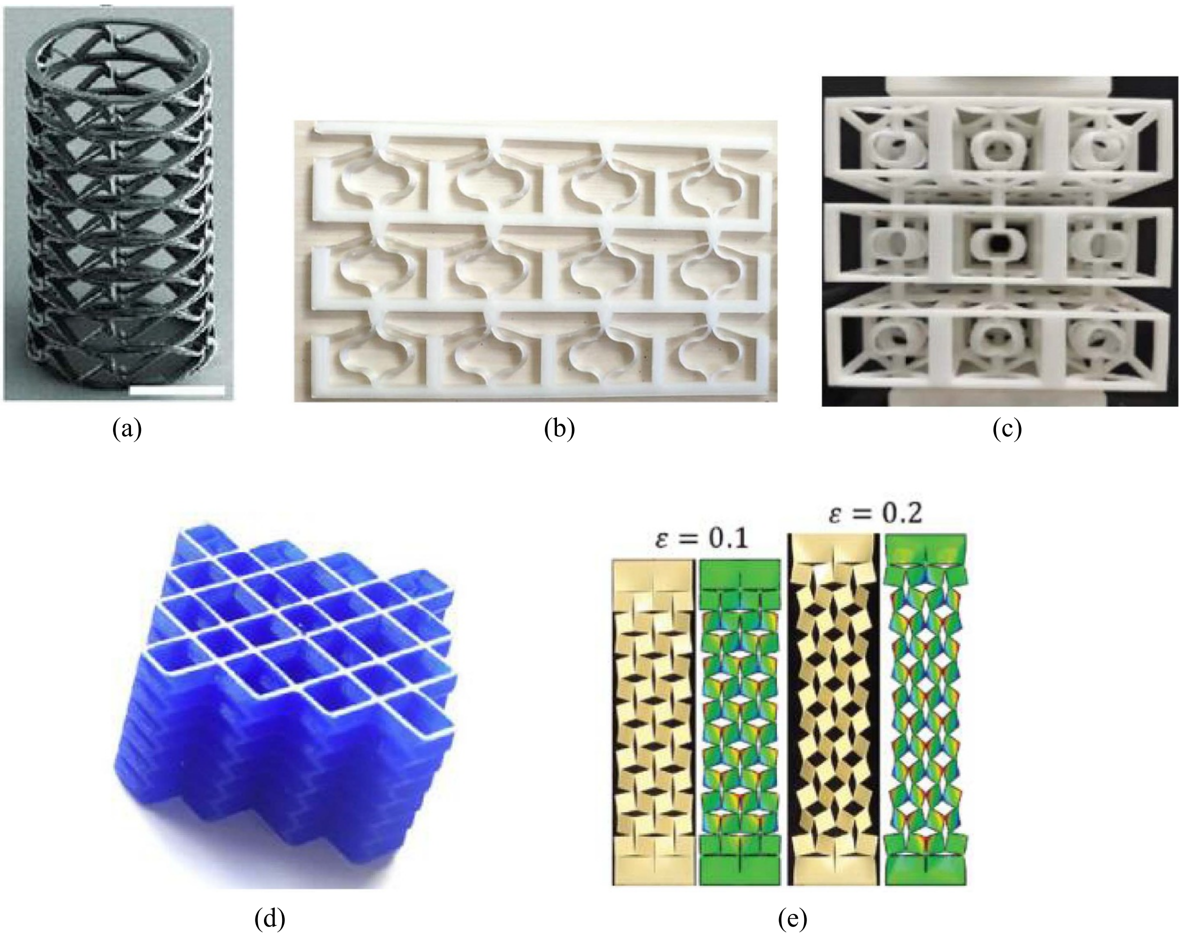


Figure 4. (a) cylindrical, Reproduced from [26]. CC BY 4.0. (b) 2D, Reprinted from [39], Copyright (2020), with permission from Elsevier, (c) 3D, Reprinted from [120], Copyright (2023), with permission from Elsevier, (d) origami structure, Reproduced from [129]. CC BY 4.0 and (e) Kirigami [130], John Wiley & Sons. © 2019 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim.

3. Deformation mechanisms leading to a QZS feature

This section sheds light on the various deformation mechanisms leading to a QZS phenomenon in metamaterials under compressive/tensile mechanical loads.

3.1. Combination of PS and NS elements

In metamaterials, the combination of PS and NS elements can lead to the emergence of QZS behavior [39, 49, 118, 120, 137, 138], see figure 6. Take the individual elements into account to understand how this is achieved. PS elements resist deformation and exhibit a linear relationship between applied force and resulting displacement. On the other hand, NS elements behave oppositely. They exhibit a relationship where the applied force and displacement have opposing signs, causing a reduction in stiffness when combined with PS elements. These PS and NS elements are arranged in a specific configuration to achieve QZS behavior in a metamaterial. The PS elements dominate at small deformations, providing a linear response. However, as the applied force or displacement increases, the NS elements/mechanisms become activated, reducing the overall effective stiffness of the metamaterial. This reduction in stiffness occurs due to the energy exchange between PS and NS elements. As the PS elements are compressed or extended, they store elastic energy. When the NS element is activated, it releases this stored energy, effectively cancelling out the PS effects and reducing the net stiffness of the metamaterial. Herein, some unit cell designs leading to a QZS behavior are presented, see figure 6. This arrangement allows for distinct mechanical behaviors under different loading conditions. By strategically arranging these elements, the unit cell and the overall meta structures can achieve a state of QZS. The mechanical response of the sinusoidal beam can be estimated by the research conducted by Qiu *et al* [139]. It suggests that when the parameters of the sinusoidal beam meet the condition of $h/t \geqslant 6$, where parameters 'h' and 't' indicate the curvature and the thickness of the beam respectively, the force-displacement relationship contains a plateau region. Consequently, the force-displacement curve of the metamaterials containing the simultaneous NS and PS elements can represent a plateau region under compression.

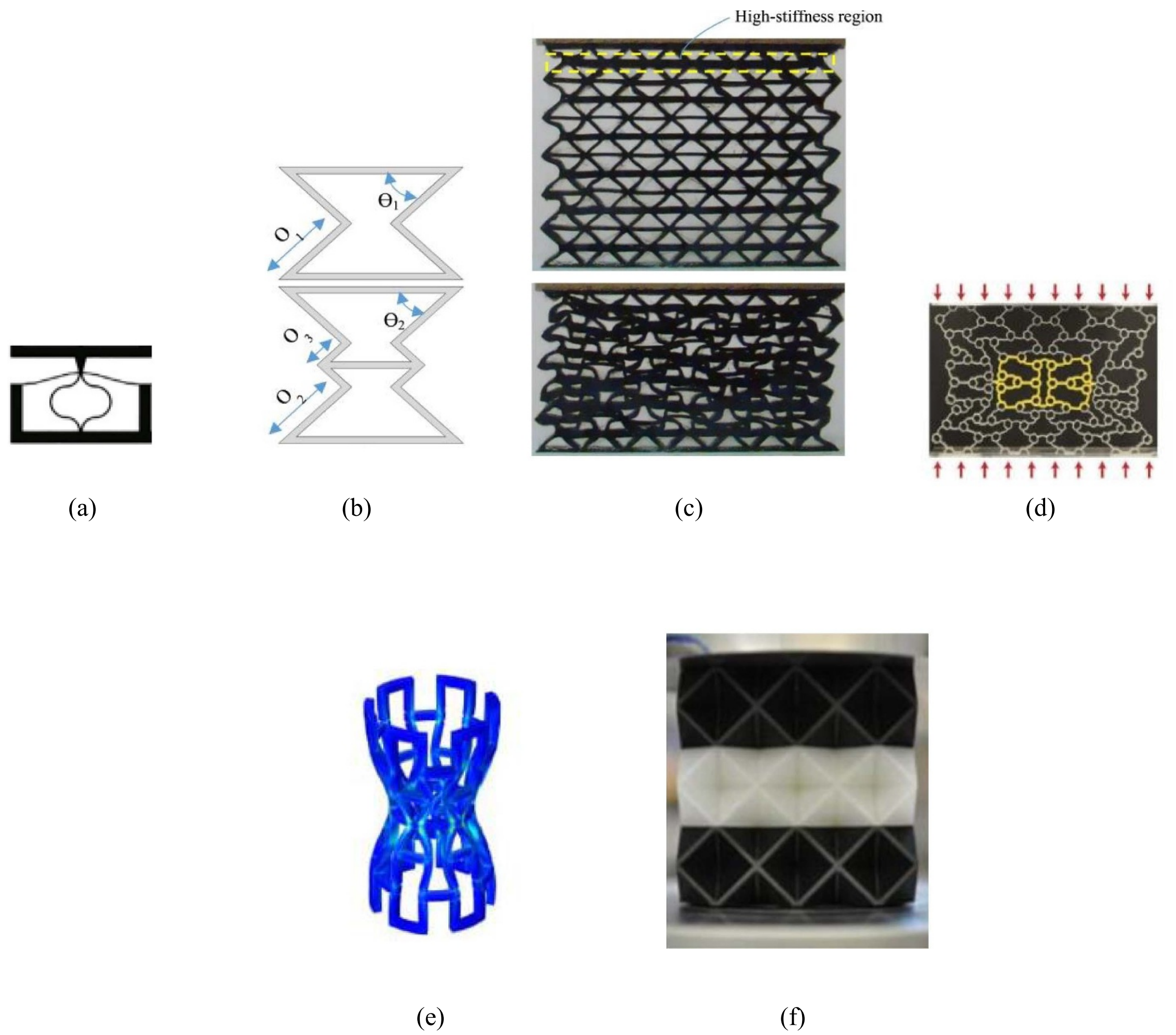


Figure 5. The effective mechanisms leading to the QZS feature via (a) a combination of PS and NS elements, Reprinted from [39], Copyright (2020), with permission from Elsevier, (b) variable PS elements, (c) bending and buckling caused by high-stiffness regions, (b) and (c) Reproduced from [56]. © The Author(s). Published by IOP Publishing Ltd. CC BY 4.0, (d) buckling, Reproduced from [68]. CC BY 4.0, (e) localized smart materials, Reproduced from [136]. CC BY 4.0 and (f) multi-material printing, Reproduced from [135]. CC BY 4.0.

3.2. Combination of variable PS elements

In specific configurations and arrangements, the combination of low and high PS elements can create metamaterials with nonlinear mechanical properties that exhibit QZS behavior [50, 55, 56]. The nonlinear behavior arises from the interaction between different elastic elements and the overall system response. By carefully designing the structure and geometry, and interconnection of the low and high-stiffness elements, it is possible to achieve a nonlinear mechanical response where the effective stiffness approaches zero over a specific range of applied forces or displacements. The interplay between different PS elements can lead to QZS behavior due to the nonlinearity of each element. In the following, the nonlinearity of combined elastic elements and their roles in realizing QZS feature will be discussed.

From the structural standpoint, a metamaterial design could contain the simultaneous heterogeneous arrangement of low and high-stiffness elements, see figure 7. This arrangement introduces nonlinearity into the system, which alters the overall stiffness characteristics during deformation. When an external force is applied to the metamaterial, the load is distributed among the stiff and soft unit cells. The stiff unit cell bears a significant portion of the applied load due to its higher stiffness. This leads to a transfer of forces between the stiff and soft unit cells, resulting in bending of the soft unit cells. In fact, the deformation of the soft unit cells can exhibit a nonlinear behavior, resulting in a reduction in the effective stiffness of the metamaterial as the applied force or displacement increases. This causes a QZS feature in metamaterials under compression.

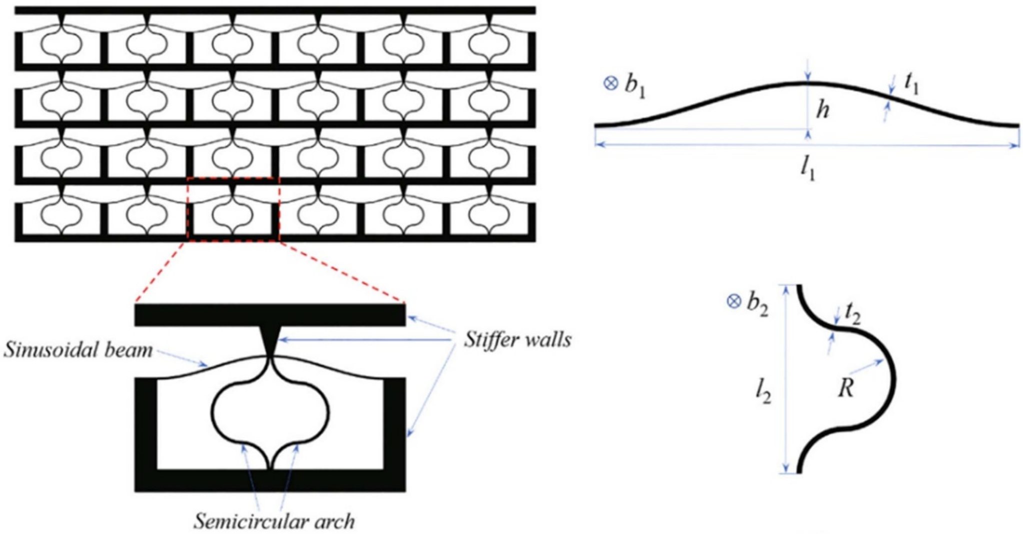


Figure 6. The combination of NS and PS elements effect on QZS feature, Reprinted from [39], Copyright (2020), with permission from Elsevier.

3.3. Rotating mechanism via stiff joints

In those lattice structures containing stiff joints within their designs, the QZS behavior can be achieved [47, 62, 122, 124, 140]. When a compressive force is applied to a lattice metamaterial, the stiff joints experience a rotational motion leading to change/re-arrangement of structural design and unit cells. This rotation is a consequence of the stiff joints arrangement, see figure 8. As a result, the cell walls, or beams in the vicinity of the stiff joints undergo bending. The rotation of the stiff joints and the simultaneous bending deformation of neighboring elements contribute to a nonlinear mechanical response of the structure, leading to a QZS feature on force-displacement relation under compression.

3.4. Bending mechanism via localized stiff area/region contacts

Following the discussion of rotating motion, another similar mechanism for a QZS feature is introduced here by taking advantage of the bending motions of the elements located close to the high-stiffness regions in metamaterials. The lattice metamaterial contains specific regions that are designed to be stiff, meaning that they offer high resistance to deformation. These regions can be composed of rigid materials or containing specific structural arrangements that enhance stiffness. When an external compression force is applied to the lattice metamaterial, it experiences overall deformation. The force is transmitted throughout the structure, causing the unit cells and their constituent cell walls to deform, see figure 9. Due to the presence of the high-stiffness regions (either originally existing or from contacts during deformation), the cell walls in their vicinity experience localized bending. Indeed, as the high-stiffness regions resist deformation more than the surrounding elastic elements, bending occurs, resulting in stress concentrations and deformation redistribution. The bending of cell walls near the high-stiffness regions creates a unique mechanical response. In these regions, the effective stiffness of the material is greatly reduced, approaching zero [26, 56, 141]. This behavior is termed QZS because, although the material still offers some resistance to compression, it is significantly lower than the stiffness exhibited by the rest of the lattice. In other words, high-stiffness regions can induce localized bending or a kind of structural failure in nearby elastic elements under loading, resulting in a QZS feature. This behavior opens up opportunities for applications in shock absorption, shape-changing structures, and other areas that benefit from enhanced mechanical properties and unique responses to external forces.

3.5. Metamaterials with structural instability

3.5.1. Buckling via support elements. In metamaterial design, structural failure or instability can be used to realize the QZS behavior. Buckling is a common type of instability with the sudden stiffness drop and lateral deflection increment of a slender structural element under compression [142] and can be considered as an effective mechanism leading to a QZS feature [41, 63, 126, 133]. The structure can undergo a buckling deformation when a compressive force exceeds a critical threshold. Buckling introduces nonlinear behavior into the system, which cannot provide effective support for the external loads. Instead, the relationship between the applied force and resulting displacement becomes highly nonlinear, often exhibiting a softening behavior, see figure 10. In the buckled state, the structure becomes more flexible, meaning that it can undergo larger deformations for a given change in applied force. This reduced effective stiffness is a key characteristic associated with QZS behavior. In addition, buckling redistributes the applied load

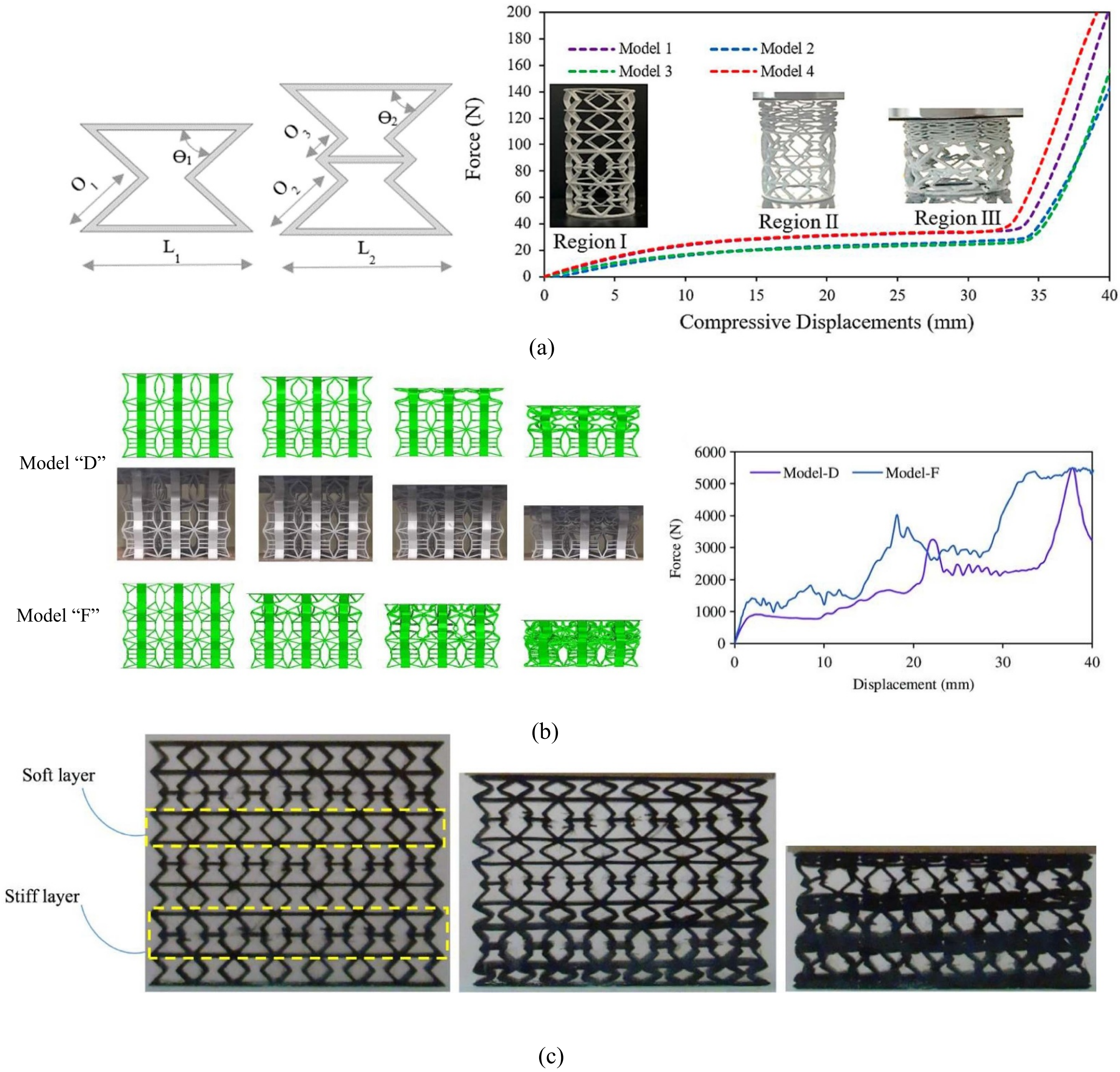


Figure 7. The emergence of QZS feature due to high and low positive stiffness unit cells in (a) cylindrical, Reproduced from [50]. CC BY 4.0, (b) 3D, Reproduced from [55]. CC BY 4.0, and (c) 2D metamaterials, Reproduced from [56]. CC BY 4.0.

within the structure, leading to changes in stress distribution. This redistribution of internal stiffness and forces can help to mitigate the impact of external loads or vibrations and provide enhanced isolation or damping characteristics. Overall, buckling-induced QZS behavior arises from the interplay of geometric constraints, instability, nonlinearity, and energy dissipation. By leveraging these phenomena, designers can create structures that exhibit reduced effective stiffness within specific operating strain ranges.

3.5.2. Buckling-induced rotating mechanism via stiff joints. As mentioned in section 3.5.1, buckling can contribute to changing the overall mechanical response of a structure under mechanical loads. It may influence the emergence of a QZS feature. Buckling typically introduces nonlinear behavior into the system. The response of the structure may no longer be linearly proportional to the applied force or deformation. Nonlinearities can arise from the changes in stiffness, damping, or geometrical configurations associated with buckling

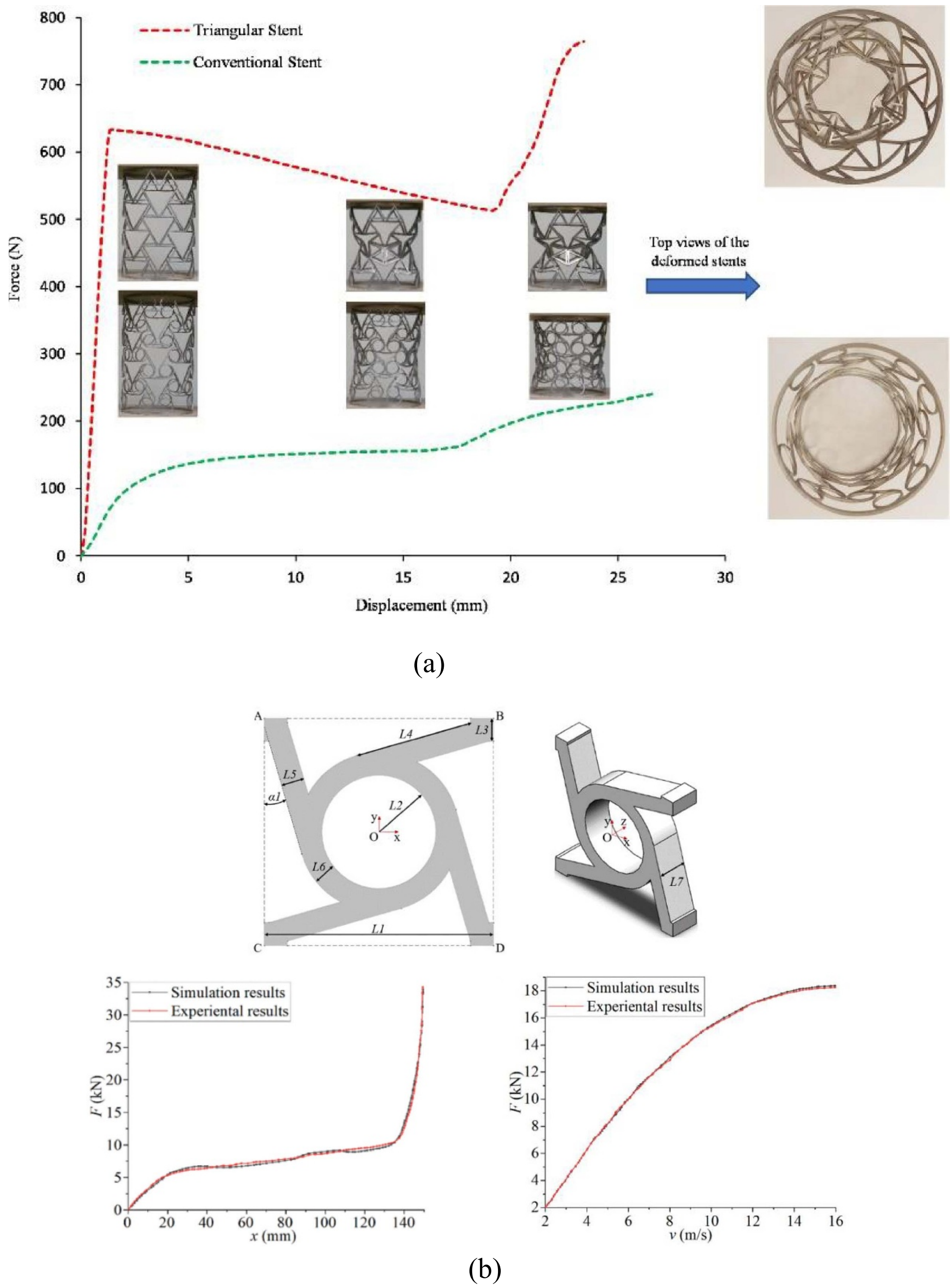
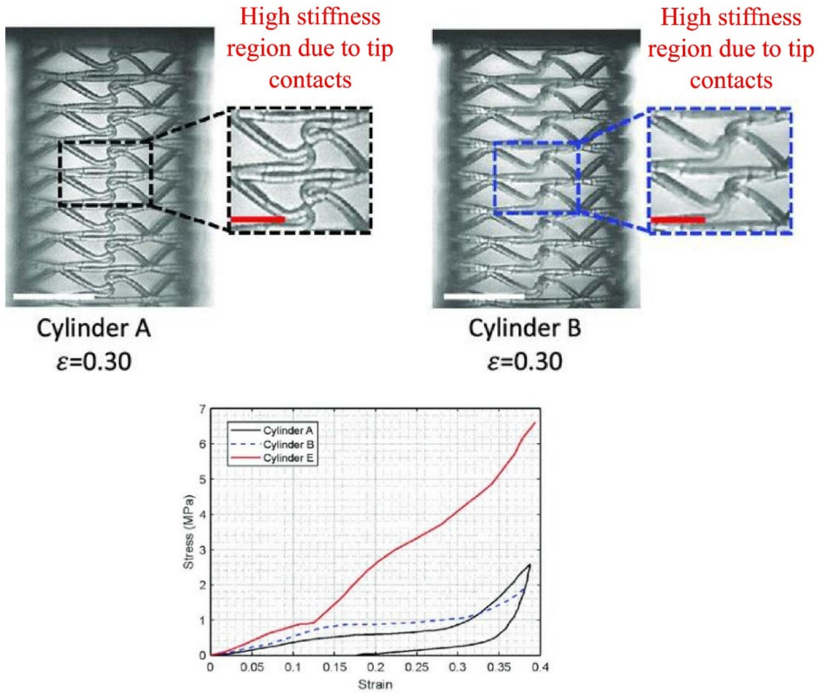
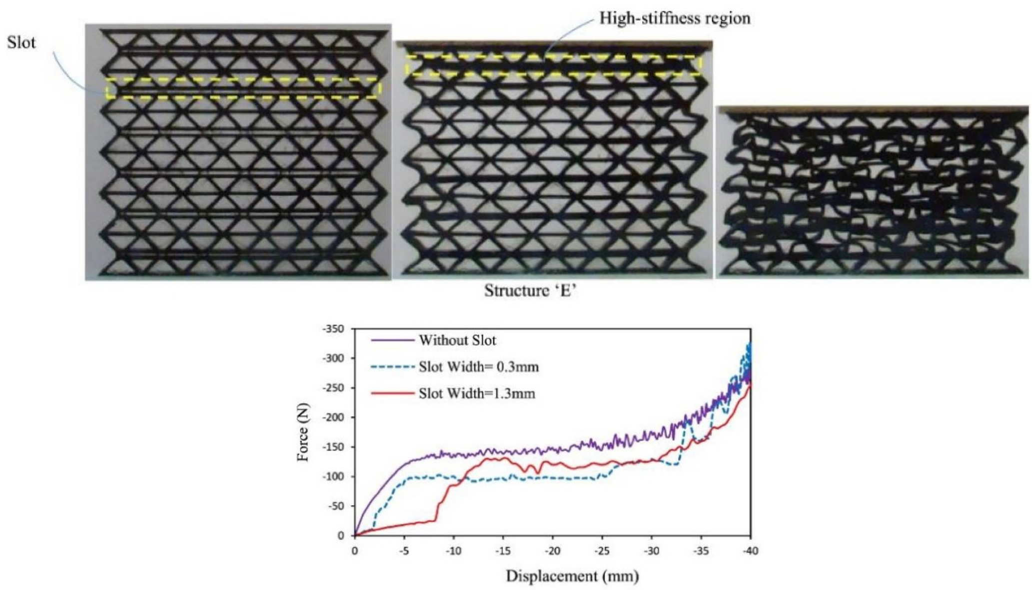


Figure 8. The effect of high-stiffness rotating joints on QZS feature of (a) triangular, Reprinted from [62], Copyright (2020), with permission from Elsevier and (b) common chiral structures, Reproduced from [140]. CC BY 4.0.

deformation. In some cases, the combined effects of buckling and other design factors (such as the arrangement of elements or materials) can contribute to the emergence of QZS behavior. For example, in certain metamaterials, the interplay between rotation joints, or geometric configurations can lead to a reduction of effective stiffness within a specific strain range, resulting in a QZS behavior and possible vibration isolation performance [61, 68]. Figure 11 illustrates the effective role of the rotational motion of stiff joints under compression and the corresponding buckling of the cell walls in their vicinitieson the emergence of QZS feature. The mechanical loads induce the stiff joints to rotate first due to their high stiffness, which consequently results in buckling of the cell walls. This phenomenon introduces instability into the structure, giving rise to negative Poisson's ratio behavior and the emergence of a prolonged QZS characteristic.



(a)

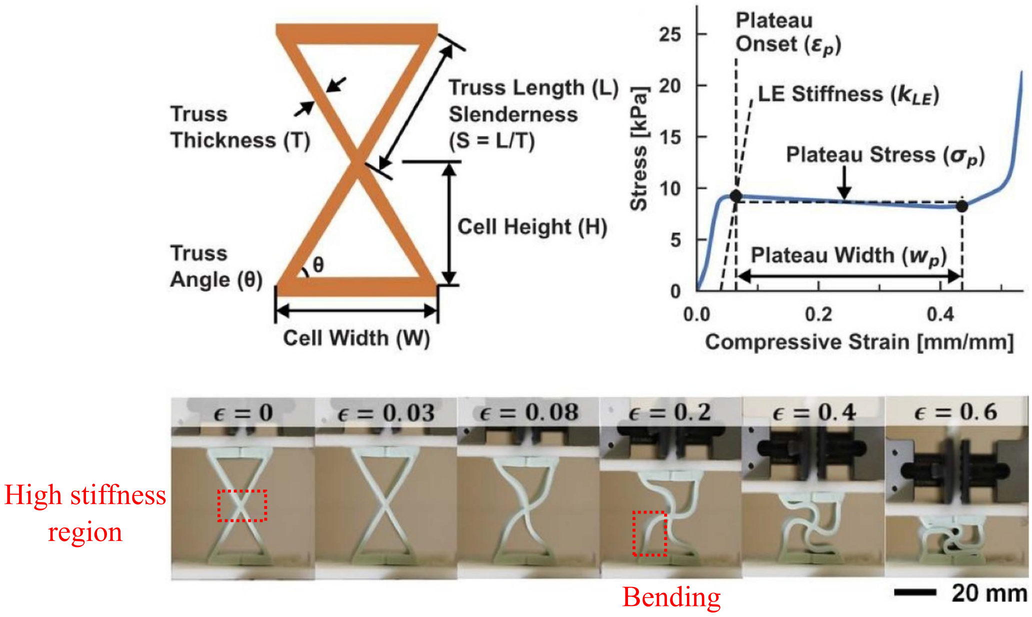


(b)

Figure 9. High-stiffness regions because of (a) tip contacts of the unit cells, Reproduced from [26]. CC BY 4.0, (b) contact of the walls in the slots, Reproduced from [56]. CC BY 4.0, and **(c)** triangles positions, Reproduced from [141]. CC BY 4.0.

3.6. Bending/buckling around hinges of origami metamaterials

Origami, as an art form, traditionally involves folding materials to create intricate and visually appealing designs [129]. In the realm of engineering, researchers have explored incorporating specific folding patterns and principles into the design of structures to achieve zero or extremely low stiffness feature [143–145]. Origami metamaterials typically consist of repeated folding units arranged in a periodic or geometric pattern. These folding units can be designed with flexible and rigid regions, creating hinges at the points of folding. When a mechanical load is applied, the flexible regions of the origami metamaterial undergo bending around



(c)

Figure 9. (Continued.)

these hinges, allowing the lattice to deform while the rigid regions provide stability. Initially, the deformation is relatively small, and the response of the lattice can be approximately linear. However, as the applied force increases, the bending around the hinges becomes more significant, resulting in nonlinearity in the system. This nonlinearity arises from various factors, including the change in force distribution as the lattice deforms. The bending/buckling at the local flexible regions redistributes forces, transferring a portion of the load to the previously unengaged parts of the lattice. This redistribution alters the stiffness and response of the structure, leading to nonlinear behavior. The ability to bend/buckle around high-stiffness hinges and the redistribution of forces enables the origami lattice to exhibit unique mechanical behaviors such as QZS feature. This characteristic allows for easy deformation of the lattice under mechanical loads, making it highly suitable for applications such as shock absorption, precise force control, and adaptable structures. An alternative method for introducing nonlinearity and achieving tunable stiffness within origami lattice structures is through the implementation of embedded tension springs [144]. By integrating tension springs into the structure, it becomes possible to manipulate the stiffness of the creases and consequently achieve adjustable stiffness for the entire system, see figure 12. This approach offers a versatile means of tailoring the mechanical properties of origami-based structures, allowing for enhanced control over their behavior and response to external forces. The integration of tension springs represents a valuable technique for expanding the design possibilities and optimizing the functionality of origami lattice structures in various engineering and architectural applications.

Furthermore, Sadeghi and Li [145] conducted research demonstrating a potential method for incorporating the QZS characteristic into origami metamaterials by implementing a sealed pressurized structure. The approach takes advantage of the nonlinear relationship between folding and internal volume change. By pressurizing the structure, the origami metamaterial exhibits the desired QZS feature, referring to extremely low stiffness or high flexibility in specific configurations.

3.7. Buckling/bending of kirigami metamaterials

Kirigami metamaterials are indeed based on sheet-cutting and folding techniques [130]. They involve patterns of cuts and folds within the structure to create unique mechanical properties. While origami primarily focuses on folding, kirigami introduces cuts, allowing for more complex elastic element layouts and deformation behaviors. In a kirigami lattice structure, when a tensile load is applied, the lattice structure tends to buckle and deform, see figure 13. The cuts in the lattice act as hinges that enable the material to fold in specific ways, facilitating the lattice's response to mechanical loads. The interaction between the cuts and folds in the lattice is indeed a source of nonlinearity. As the lattice deforms, the cuts open, modifying the effective stiffness of the structure. This change in stiffness contributes to the nonlinearity observed in the mechanical behavior and can lead to a phenomenon like QZS [130, 131]. By strategically designing the pattern of cuts, engineers can precisely control the resulting folding behavior and the response of the lattice to mechanical loads. This control over the deformation behavior enables the creation of kirigami structures with desired mechanical properties.

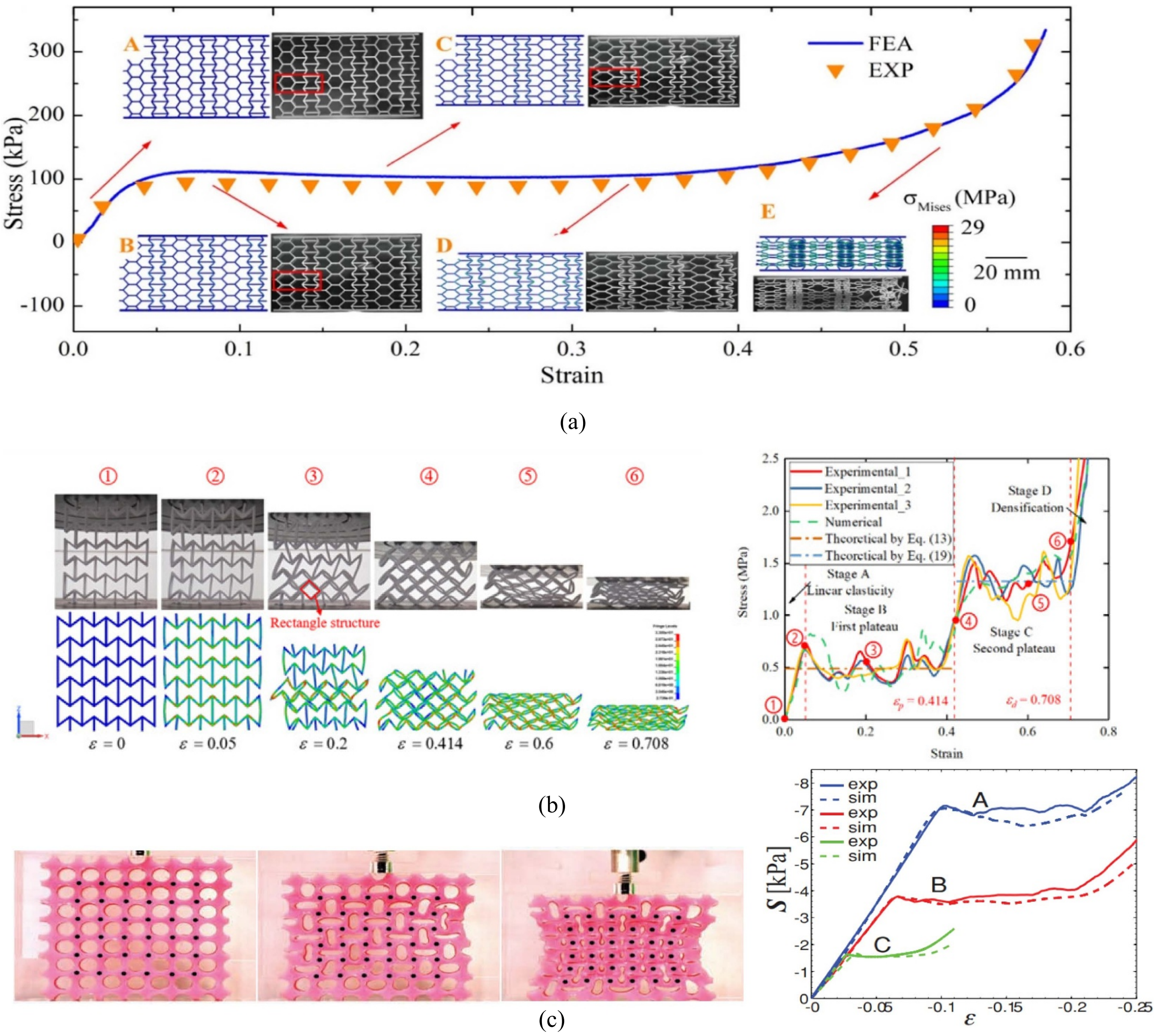


Figure 10. The QZS feature caused by (a) buckling distortion, Reprinted from [41], Copyright (2020), with permission from Elsevier, (b) buckling of vertical struts, Reprinted from [63], Copyright (2022), with permission from Elsevier, and **(c)** sudden transformation of structure [126], John Wiley & Sons. Copyright © 2012 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim.

3.8. Multi-material lattices

Multi-stiffness metamaterials are a class of engineered materials that possess multiple regions or elements with distinct stiffness properties [96, 134, 135, 146, 147]. These materials exhibit a spatial variation in their mechanical response, allowing for the control and manipulation of stiffness at different locations within the structure, see figure 14. This capability opens up a wide range of applications in areas such as soft robotics, energy absorption, and tunable mechanical systems. The design and fabrication of multi-stiffness metamaterials often involve the use of additive manufacturing techniques. These processes enable the precise arrangement of different materials, thereby achieving the desired variations in stiffness at different structural locations. Multi-material lattice structures can introduce nonlinearity in their mechanical response. Different materials within the lattice may exhibit nonlinear stress-strain behavior. For example, materials with elastoplastic or viscoelastic properties can undergo nonlinear deformation under mechanical loads. When combined in a lattice structure, the overall response becomes nonlinear due to the collective behavior of the constituent materials. Multi-material lattice structures can achieve QZS behavior through introducing softer materials with lower stiffness within the lattice structure. Indeed, the softer materials located at critical structural locations contribute to the overall compliance of the structure, effectively reducing its stiffness under compression and making an easier realization of the QZS feature during deformation.

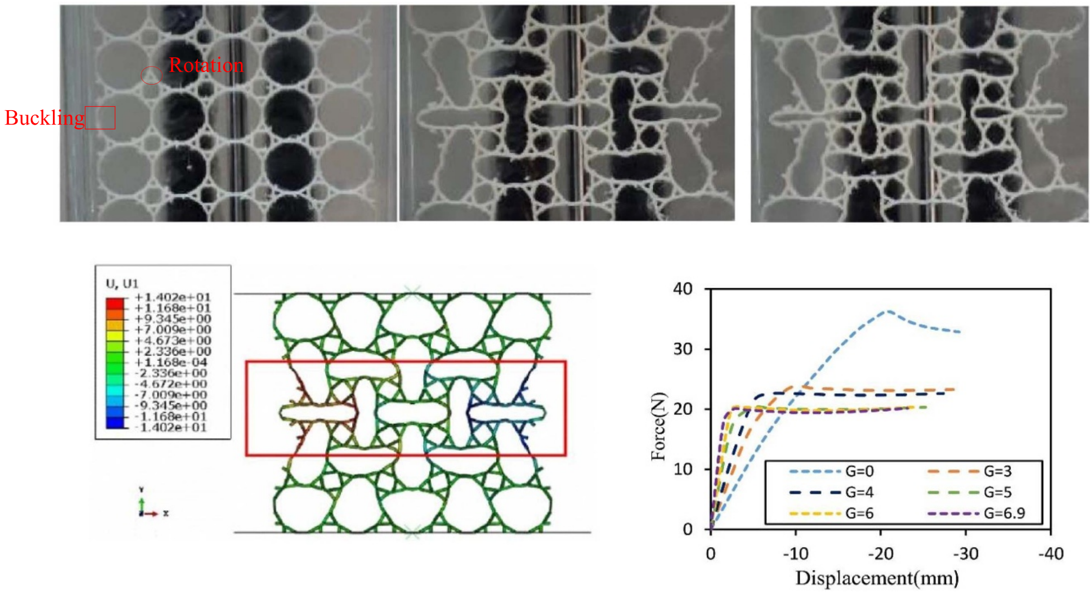


Figure 11. The effect of buckling-induced rotating mechanism on QZS feature, Reprinted from [61], Copyright (2018), with permission from Elsevier.

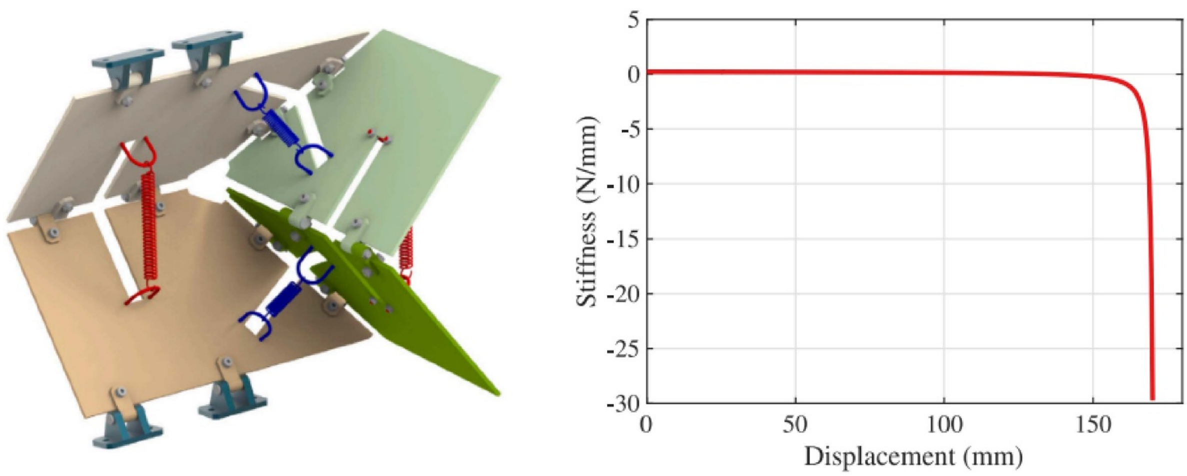


Figure 12. The effect of linear springs to provide QZS feature in origami lattices, Reprinted from [144], Copyright (2023), with permission from Elsevier.

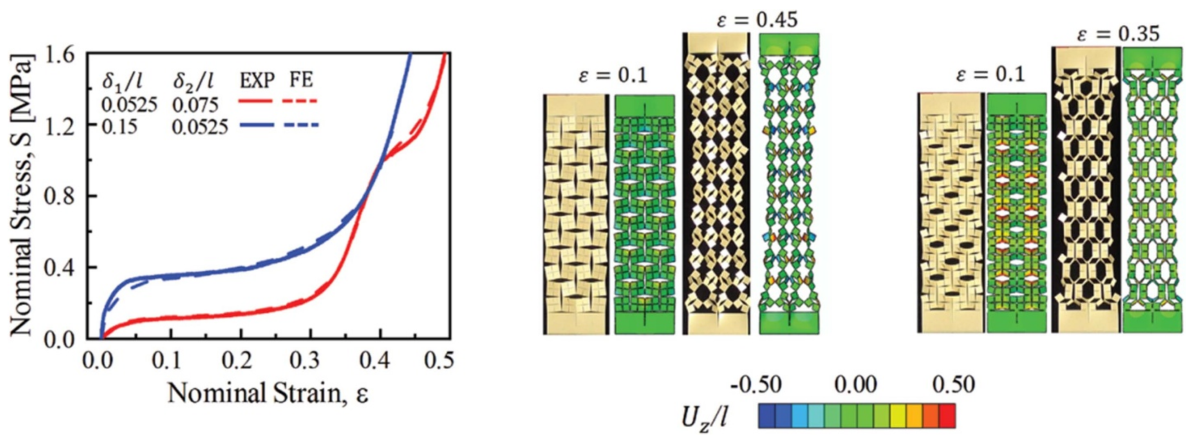
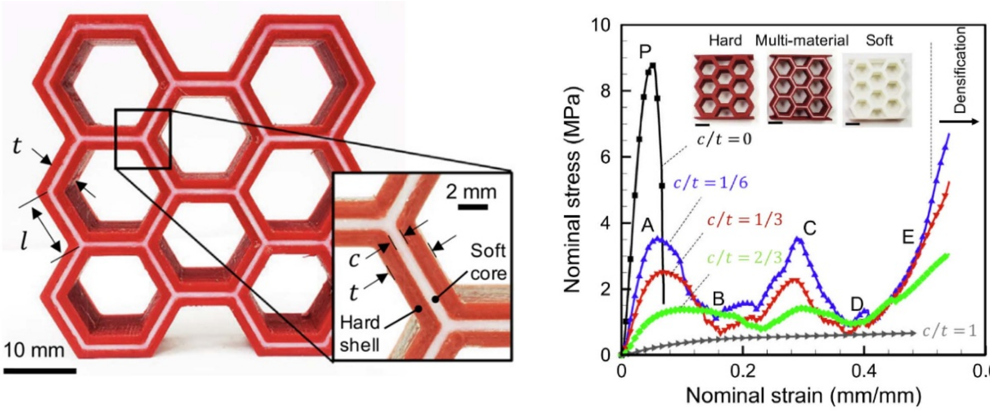
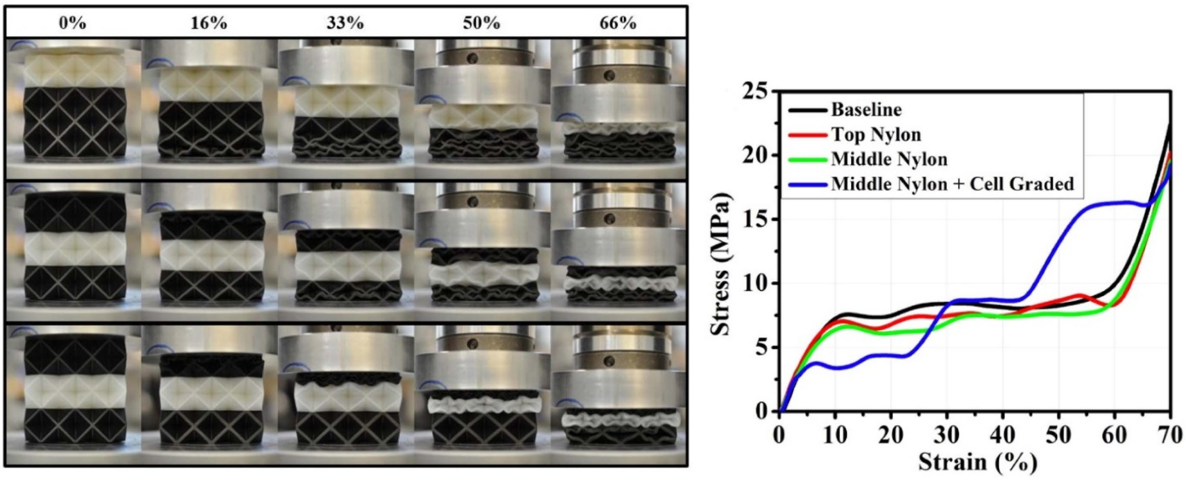


Figure 13. The effect of cutting pattern on the QZS feature in Kirigami lattices [130], John Wiley & Sons. © 2019 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim.



(a)



(b)

Figure 14. The multi-material printing effect on the QZS feature (a) within the cell walls, Reprinted from [96], Copyright (2022), with permission from Elsevier and (b) a separated combination of two materials, Reproduced from [135]. CC BY 4.0.

3.9. Smart and adaptive metamaterials

Smart materials are a category of materials that exhibit dynamic and adaptive behavior in response to various stimuli [147, 148]. These materials possess the ability to sense external changes, respond to them, alter their properties to adapt accordingly, and even activate actuation mechanisms. One example of smart metamaterials is light-responsive lattice structures [136]. Light-responsive smart materials, such as photo-responsive polymers, can indeed be utilized to create soft regions within a material [136]. When combined with passive materials, this arrangement can lead to the emergence of QZS feature under compression in specific regions, see figure 15. Light-responsive smart materials undergo reversible changes in their mechanical properties when exposed to specific wavelengths or intensities of light. These materials typically exhibit a transition between a stiff state and a soft state, allowing for controlled modulation of their mechanical response. By incorporating light-responsive smart materials in certain regions of a structure, those regions can be designed to exhibit softness when subjected to light activation. This softness arises from a reduction in the material's effective stiffness, allowing for enhanced flexibility and adaptability in response to compression. To achieve QZS in combination with passive materials, a strategic integration of the soft light-responsive regions and passive materials is required. The passive materials possess a relatively high stiffness and contribute to the overall structural integrity of the system. When compression is applied to the combined structure, the passive materials initially provide resistance, contributing to the overall stiffness. However, in the regions where the light-responsive smart materials are incorporated, the application of light can induce a transition to a soft state, leading to a significant reduction in stiffness. This reduction in stiffness effectively results in zero stiffness in those specific regions under compression.

In parallel with smart metamaterials, adaptive metamaterials are also capable of exhibiting QZS features [149]. The terms 'adaptive metamaterials' and 'smart metamaterials' are sometimes used interchangeably due to their similarities in functionality. The key distinction between adaptive and smart metamaterials lies in the level of autonomous and intelligent

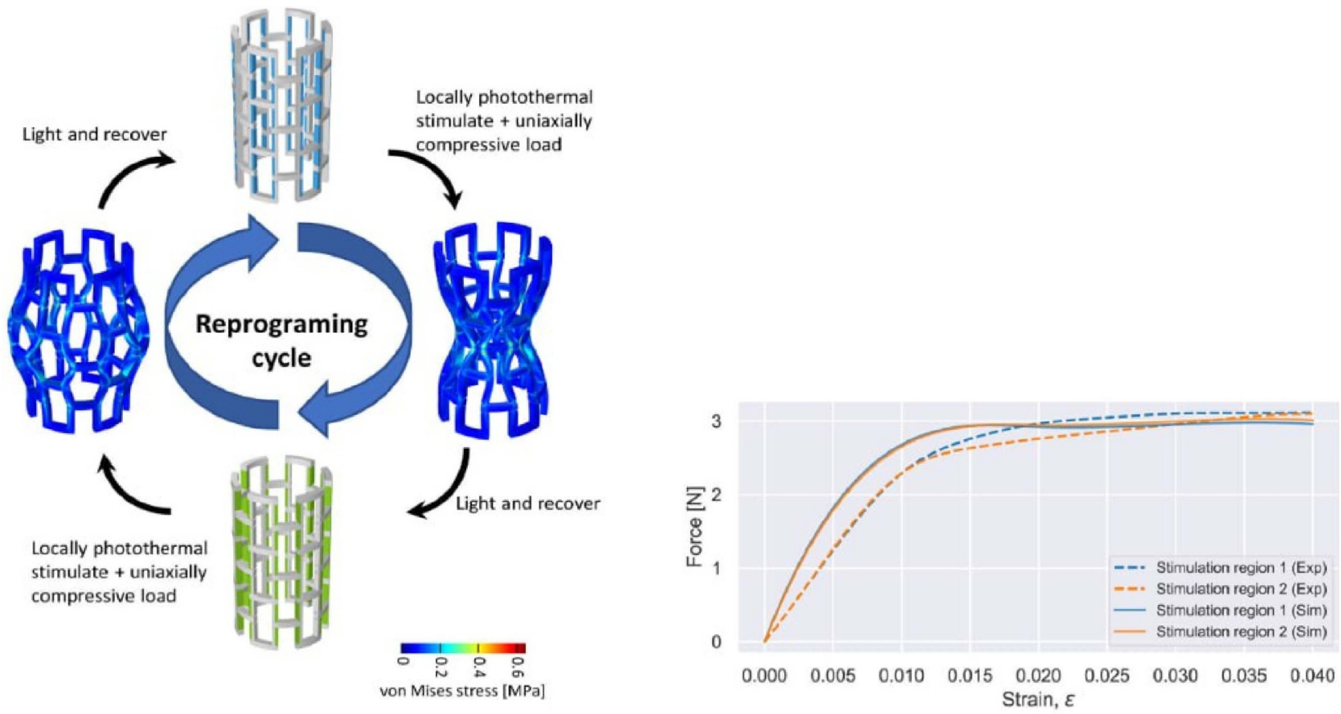


Figure 15. The effect of smart light-responsive materials on the QZS feature, Reprinted from [136], Copyright (2022), with permission from Elsevier.

behavior exhibited by smart metamaterials. While adaptive metamaterials focus on active adaptation, smart metamaterials take it a step further by incorporating actuation capabilities [150]. One example of adaptive metamaterials is thermally programmable lattice architectures [149], see figure 16. These structures consist of a network of unit cells arranged in a lattice pattern, where the unit cells are designed to undergo controlled shape changes upon thermal activation. In the scenario where the active material weakens upon heating, the concept of QZS in lattice structures under compression can still be achieved. When the active material weakens upon heating, it undergoes a reduction in its mechanical properties such as stiffness or strength. By combining this active material with a passive material in a lattice structure, it is possible to leverage their differential mechanical responses to achieve the desired QZS feature. Under compression, the lattice structure experiences deformation. Initially, the passive material provides the primary stiffness to resist the applied compressive forces. As the structure is heated, the active material weakens, reducing the overall structural stiffness. This weakening effect allows the lattice to exhibit lower effective stiffnessunder compression, leading to the QZS behavior. The combination of the active material's weakening response and the passive material's stiffness creates a scenario where the structure adapts its mechanical properties under the influence of temperature. The advantage of smart and adaptive metamaterials is that their unique mechanical behaviors, like QZS properties, are controllable by excitations from different physical fields other than just mechanical inputs.

4. Mechanical vibration control

Understanding and controlling mechanical vibration is of utmost importance in engineering and design. Excessive vibrations can lead to a variety of issues, including structural damage, reduced performance, increased noise levels, and even safety hazards. Engineers and designers strive to minimize vibrations to ensure the longevity, reliability, and efficiency of structures, machines, and systems. Mechanical vibration has significant implications in numerous industries and applications. Some notable examples include:

1. Automotive Industry [151]: Vibration control is critical for vehicle performance, passenger comfort, and safety. It involves optimizing engine mounts, suspension systems, and tires to reduce unwanted vibrations.
2. Manufacturing and Industrial Machinery [152]: Vibrations in industrial machinery can lead to increased wear, decreased productivity, and safety concerns. Effective vibration control measures are essential to maintain operational efficiency and worker well-being.
3. Construction and Infrastructure [153]: Vibrations from construction activities such as pile driving or heavy machinery, can impact nearby structures, requiring careful monitoring and mitigation to prevent damage.
4. Structural Engineering [154]: Vibration analysis is crucial for assessing the dynamic behavior of structures like bridges, buildings, and stadiums. It helps to ensure their stability, comfort, and safety under normal and extreme conditions.

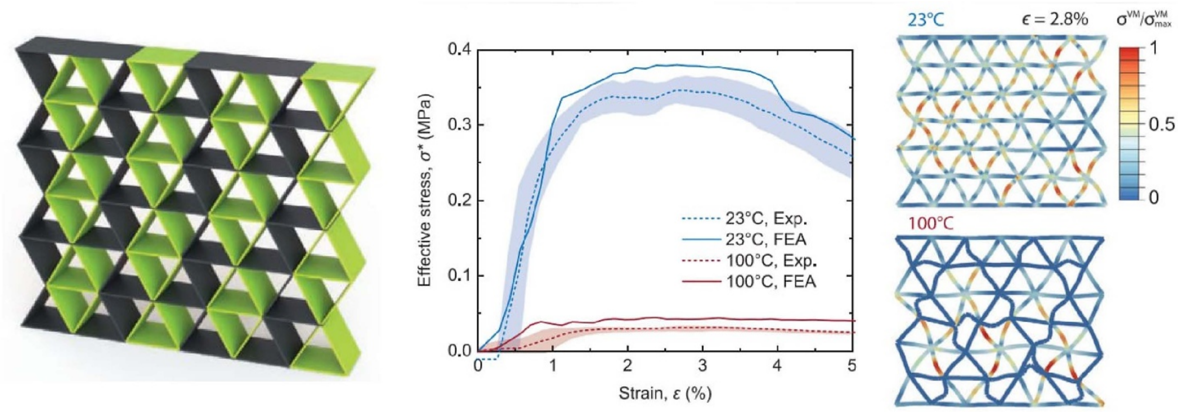


Figure 16. The effect of adaptive thermally programmable materials on the QZS feature [149], John Wiley & Sons. © 2021 Wiley-VCH GmbH.

Table 1. QZS metamaterials recent works.

Research group	The effective mechanism leading to QZS	Frequency ranges (high/low)	Year
Liu *et al* [138]	Combination of PS & NS	Low	2024
Zhao *et al* [123]	Bending via stiff joints	Low	2023
Guo *et al* [120]	Combination of PS & NS	Low	2023
Lin *et al* [121]	Bending via stiff joints	Low	2023
Zheng *et al* [49]	Combination of PS & NS	Low	2023
Zolfagharian *et al* [50]	Variable PS elements	Low	2022
Cai *et al* [117]	Bending via stiff joints	Low	2022
Lin *et al* [122]	Bending via stiff joints	Low	2022
Zhang *et al* [47]	Bending via stiff joints	Low	2021
Zhou *et al* [124]	Bending via stiff joints	Low	2021
Cai *et al* [118]	Combination of PS & NS	Low	2020
Fan *et al* [39]	Combination of PS & NS	Low	2020
Liu *et al* [41]	Buckling	Low	2020

Understanding and managing mechanical vibrations is crucial for optimizing performance, ensuring safety, and improving the overall quality of various systems and structures in the real world. By studying and controlling vibrations, engineers strive to enhance efficiency, reduce downtime, improve comfort, and extend the lifespan of machinery and infrastructure.

4.1. The significant effects of metamaterials for vibration isolation

Using vibration isolator metamaterials offers several advantages over common vibration isolator mechanisms. Unlike traditional isolator mechanisms, which often have fixed characteristics or require complex controlling process, metamaterials can be designed to exhibit extraordinary variable properties. This allows for customization and optimization of isolator performance based on the specific application. QZS metamaterial-based isolators have the potential to provide effective vibration isolation over a wide frequency range. By carefully designing the structural elements and arrangements within the metamaterial design, it is possible to control and manipulate the transmission of vibrations across a broad spectrum of frequencies. This adaptability is particularly advantageous in applications where vibrations occur at varying frequencies. It is worth noting that QZS metamaterials do not inherently offer benefits in vibration isolation. Generally, they are employed as integral parts of high static and low dynamic stiffness systems. These systems are specifically crafted for vibration isolation applications, leveraging the effectiveness of QZS within a restricted range of displacements. Besides, QZS metamaterials can be used as shock and impact absorbers [120, 137], especially where a high strength-to-weight ratio is required. This is particularly beneficial in industries where weight and space considerations are critical such as aerospace and automotive applications [155]. Several scholarly publications have reported on the remarkable vibration isolation capabilities exhibited by specific QZS metamaterials. For a detailed overview of recent scholarly endeavors in the field of QZS metamaterials, please refer to table 1.

5. Conclusions

This review provided a discussion of the design strategy of lattice metamaterials exhibiting a QZS feature. The key observations are summarized as follows.

- One effective way to provide a QZS feature in lattice metamaterials under mechanical loads is to introduce nonlinearity into the system.
- The simultaneous consideration of PS and NS elements within the metamaterial designs can introduce nonlinearity into the system. This can lead to the emergence of zero stiffness. When PS and NS elements are combined in a system, their interaction can lead to nonlinearity and the creation of zero-stiffness regions. This occurs due to the balance between PS and NS elements by cancellation of forces provided by PS elements.
- Another approach for introducing nonlinearity can be the coexistence of variable PS elements. Under compression, the stiffer elements lead to the densification of the soft unit cells, resulting in a nonlinear behavior and a QZS feature.
- The other effective approach could be buckling caused by rotational motion and instability into metamaterials. When the lattice structure is under compressive loads, the buckled beams undergo large deformations, resulting in an increase in flexibility (a decrease in stiffness). As a result, the effective stiffness of the structure decreases significantly, allowing for large displacements with minimal resistance.
- By varying material properties of the lattice structure, using multi-material printing, it is possible to achieve regions with very low effective stiffness or even zero stiffness. This can also be obtained by designing unit cells with specific composite configurations with smart and adaptative materials that provide QZS behavior under mechanical loads.

As future works, QZS metamaterials can provide stability to sensitive equipment, ensuring the protection of structures from seismic waves, and limiting ride harshness in automotive and aerospace industries. The use of QZS metamaterials in seismic-resistant building foundations and structural components can be further explored and refined. These lattice structures could contribute to improving the resilience of infrastructures in earthquake-prone regions, minimizing damage during seismic events, and enhancing the safety of occupants.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Acknowledgments

This work was supported by the University of Manitoba, Natural Sciences and Engineering Research Council of Canada (NSERC), and the UK Engineering and Physical Sciences Research Council (EPSRC) [Grant Number EP/Y011457/1].

ORCID iDs

Ramin Hamzehei https://orcid.org/0009-0005-0401-8069
Mahdi Bodaghi https://orcid.org/0000-0002-0707-944X
Nan Wu https://orcid.org/0000-0001-8768-1328

References

[1] Álvarez-Trejo A, Cuan-Urquizo E, Bhate D and Roman-Flores A 2023 Mechanical metamaterials with topologies based on curved elements: an overview of design, additive manufacturing and mechanical properties *Mater. Des.* **233** 112190
[2] Grima J N, Mizzi L, Azzopardi K M and Gatt R 2016 Auxetic perforated mechanical metamaterials with randomly oriented cuts *Adv. Mater.* **28** 385–9
[3] Walser R M (ed) 2000 Metamaterials: what are they? What are they good for? *APS March Meeting Abstracts*
[4] Lakes R 2020 *Composites and Metamaterials* (World Scientific)
[5] An H, Liu G, Li Y, Song J, Zhang C and Liu M 2019 Inhomogeneous electromagnetic metamaterial design method for improving efficiency and range of wireless power transfer *IET Microw. Antennas Propag.* **13** 2110–8
[6] Banerjee S, Pal B P and Roy Chowdhury D 2020 Resonance phenomena in electromagnetic metamaterials for the terahertz domain: a review *J. Electromagn. Waves Appl.* **34** 1314–37
[7] De Miguel-Hernández J, Hoyland R J, Sosa-Cabrera D, Deviaene S, Fuerte-Rodríguez P A, González-Carretero E D and Vega-Moreno A 2019 Manufacturing of 3D-metallic electromagnetic metamaterials for feedhorns used in radioastronomy and satellite communications *Mech. Mater.* **139** 103195
[8] Khatib O, Ren S, Malof J and Padilla W J 2021 Deep learning the electromagnetic properties of metamaterials—a comprehensive review *Adv. Funct. Mater.* **31** 2101748
[9] Kumar P, Ali T and Pai M M 2021 Electromagnetic metamaterials: a new paradigm of antenna design *IEEE Access* **9** 18722–51
[10] Liu C, Wang W, Olivier D N, Wang Z, Ding B and Feng J 2022 Target driven design of electromagnetic metamaterial for dual-band Wi-Fi energy harvester *Sens. Actuators* A **345** 113815
[11] Sakoda K 2019 *Electromagnetic Metamaterials: Modern Insights into Macroscopic Electromagnetic Fields* (Springer)
[12] Xu J, Yang R, Fan Y, Fu Q and Zhang F 2021 A review of tunable electromagnetic metamaterials with anisotropic liquid crystals *Front. Phys.* **9** 633104
[13] Zahertar S, Wang Y, Tao R, Xie J, Fu Y and Torun H 2019 A fully integrated biosensing platform combining acoustofluidics and electromagnetic metamaterials *J. Phys. D: Appl. Phys.* **52** 485004
[14] Brunet T, Merlin A, Mascaro B, Zimny K, Leng J, Poncelet O, Aristégui C and Mondain-Monval O 2015 Soft 3D acoustic metamaterial with negative index *Nat. Mater.* **14** 384–8
[15] Cummer S A, Christensen J and Alù A 2016 Controlling sound with acoustic metamaterials *Nat. Rev. Mater.* **1** 1–13
[16] Fok L, Ambati M and Zhang X 2008 Acoustic metamaterials *MRS Bull.* **33** 931–4
[17] Gao N, Zhang Z, Deng J, Guo X, Cheng B and Hou H 2022 Acoustic metamaterials for noise reduction: a review *Adv. Mater. Technol.* **7** 2100698
[18] Haberman M R and Guild M D 2016 Acoustic metamaterials *Phys. Today* **69** 42–48
[19] Lee S H, Park C M, Seo Y M, Wang Z G and Kim C K 2009 Acoustic metamaterial with negative modulus *J. Phys.: Condens. Matter* **21** 175704
[20] Li J and Chan C T 2004 Double-negative acoustic metamaterial *Phys. Rev.* E **70** 055602
[21] Ma G and Sheng P 2016 Acoustic metamaterials: from local resonances to broad horizons *Sci. Adv.* **2** e1501595

[22] Maier S A 2017 *World Scientific Handbook Of Metamaterials And Plasmonics vol 14* (World Scientific)
[23] Yang Z, Mei J, Yang M, Chan N and Sheng P 2008 Membrane-type acoustic metamaterial with negative dynamic mass *Phys. Rev. Lett.* **101** 204301
[24] Zhang S, Yin L and Fang N 2009 Focusing ultrasound with an acoustic metamaterial network *Phys. Rev. Lett.* **102** 194301
[25] Airoldi A, Novak N, Sgobba F, Gilardelli A and Borovinšek M 2020 Foam-filled energy absorbers with auxetic behaviour for localized impacts *Mater. Sci. Eng.* **788** 139500
[26] Hamzehei R, Bodaghi M, Iglesias Martinez J A, Ji Q, Ulliac G, Kadic M, Wang C, Zolfagharian A and Wu N 2023 Parrot Beak-Inspired metamaterials with friction and interlocking mechanisms 3D/4D printed in micro and macro scales for supreme energy absorption/dissipation *Adv. Eng. Mater.* **25** 2201842
[27] Mansoori H, Hamzehei R and Dariushi S 2022 Crashworthiness analysis of cylindrical tubes with coupling effects under quasi-static axial loading: an experimental and numerical study *Proc. Inst. Mech. Eng.* L **236** 647–62
[28] Bodaghi M, Namvar N, Yousefi A, Teymouri H, Demoly F and Zolfagharian A 2023 Metamaterial boat fenders with supreme shape recovery and energy absorption/dissipation via FFF 4D printing *Smart Mater. Struct.* **32** 095028
[29] Yousefi A, Jolaiy S, Lalegani Dezaki M, Zolfagharian A, Serjouei A and Bodaghi M 2023 3D-Printed Soft and Hard Meta-Structures with Supreme Energy Absorption and Dissipation Capacities in Cyclic Loading Conditions *Adv. Eng. Mater.* **25** 2201189
[30] Zhang X, Qu Z and Wang H 2020 Engineering acoustic metamaterials for sound absorption: from uniform to gradient structures *iScience* **23** 101110
[31] Bobrovnitskii Y I and Tomilina T 2018 Sound absorption and metamaterials: a review *Acoust. Phys.* **64** 519–26
[32] Fan J, Song B, Zhang L, Wang X, Zhang Z, Wei S, Xiang X, Zhu X and Shi Y 2023 Structural design and additive manufacturing of multifunctional metamaterials with low-frequency sound absorption and load-bearing performances *Int. J. Mech. Sci.* **238** 107848
[33] Zhang Y, Zhu Z, Sheng Z, He Y and Wang G 2024 Sound absorption properties of the metamaterial curved microperforated panel *Int. J. Mech. Sci.* **268** 109003
[34] Sun P, Xu S, Wang X, Gu L, Luo X, Zhao C and Huang Z 2024 Sound absorption of space-coiled metamaterials with soft walls *Int. J. Mech. Sci.* **261** 108696
[35] Novak N, Borovinšek M, Al-Ketan O, Ren Z and Vesenjak M 2022 Impact and blast resistance of uniform and graded sandwich panels with TPMS cellular structures *Compos. Struct.* **300** 116174
[36] Novak N, Vesenjak M, Kennedy G, Thadhani N and Ren Z 2020 Response of chiral auxetic composite sandwich panel to fragment simulating projectile impact *Phys. Status Solidi* b **257** 1900099
[37] Michalski J and Strek T 2020 Blast resistance of sandwich plate with auxetic anti-tetrachiral core *Vib. Phys. Syst.* **31** 2020317
[38] Novak N, Starčevič L, Vesenjak M and Ren Z 2019 Blast response study of the sandwich composite panels with 3D chiral auxetic core *Compos. Struct.* **210** 167–78
[39] Fan H, Yang L, Tian Y and Wang Z 2020 Design of metastructures with quasi-zero dynamic stiffness for vibration isolation *Compos. Struct.* **243** 112244
[40] Li Y, Baker E, Reissman T, Sun C and Liu W K 2017 Design of mechanical metamaterials for simultaneous vibration isolation and energy harvesting *Appl. Phys. Lett.* **111** 251903
[41] Liu K *et al* 2020 4D printed zero Poisson's ratio metamaterial with switching function of mechanical and vibration isolation performance *Mater. Des.* **196** 109153
[42] Mrozek A and Strek T 2022 Numerical analysis of dynamic properties of an auxetic structure with rotating squares with holes *Materials* **15** 8712
[43] Qian J, Cheng Y, Zhang A, Zhou Q and Zhang J 2021 Optimization design of metamaterial vibration isolator with honeycomb structure based on multi-fidelity surrogate model *Struct. Multidiscip. Optim.* **64** 423–39
[44] Strek T, Michalski J and Jopek H 2019 Computational analysis of the mechanical impedance of the sandwich beam with auxetic metal foam core *Phys. Status Solidi* b **256** 1800423
[45] Zhang L, Bai Z and Chen Y 2022 Dual-functional hierarchical mechanical metamaterial for vibration insulation and energy absorption *Eng. Struct.* **271** 114916
[46] Zhang L, Bai Z, Zhang Q, Jin Y and Chen Y 2023 On vibration isolation performance and crashworthiness of a three-dimensional lattice metamaterial *Eng. Struct.* **292** 116510
[47] Zhang Q, Guo D and Hu G 2021 Tailored mechanical metamaterials with programmable quasi-zero-stiffness features for full-band vibration isolation *Adv. Funct. Mater.* **31** 2101428
[48] Zhao P, Zhang K, Qi L and Deng Z 2022 3D chiral mechanical metamaterial for tailored band gap and manipulation of vibration isolation *Mech. Syst. Signal Process.* **180** 109430
[49] Zheng Y, Shangguan W-B, Yin Z and Liu X-A 2023 Design and modeling of a quasi-zero stiffness isolator for different loads *Mech. Syst. Signal Process.* **188** 110017
[50] Zolfagharian A, Bodaghi M, Hamzehei R, Parr L, Fard M and Rolfe B F 2022 3D-printed programmable mechanical metamaterials for vibration isolation and buckling control *Sustainability* **14** 6831
[51] Lim T-C 2020 *Mechanics of Metamaterials with Negative Parameters* (Springer Nature)
[52] Marqués R, Martin F and Sorolla M 2011 *Metamaterials with Negative Parameters: Theory, Design, and Microwave Applications* (Wiley)
[53] Chen Y, Zheng B-B, Fu M-H, Lan L-H and Zhang W-Z 2018 Doubly unusual 3D lattice honeycomb displaying simultaneous negative and zero Poisson's ratio properties *Smart Mater. Struct.* **27** 045003
[54] Mondal S, Katzschmann R and Clemens F 2023 Magnetorheological behavior of thermoplastic elastomeric honeycomb structures fabricated by additive manufacturing *Composites* B **252** 110498
[55] Hamzehei R, Serjouei A, Wu N, Zolfagharian A and Bodaghi M 2022 4D metamaterials with zero poisson's ratio, shape recovery, and energy absorption features *Adv. Eng. Mater.* **24** 2200656
[56] Hamzehei R, Zolfagharian A, Dariushi S and Bodaghi M 2022 3D-printed bio-inspired zero Poisson's ratio graded metamaterials with high energy absorption performance *Smart Mater. Struct.* **31** 035001
[57] Huang J, Zhang Q, Scarpa F, Liu Y and Leng J 2018 Multi-stiffness topology optimization of zero Poisson's ratio cellular structures *Composites* B **140** 35–43
[58] Bilski M, Wojciechowski K W, Stręk T, Kędziora P, Grima-Cornish J N and Dudek M R 2021 Extremely non-auxetic behavior of a typical auxetic microstructure due to its material properties *Materials* **14** 7837
[59] Cheng X, Zhang Y, Ren X, Han D, Jiang W, Zhang X G, Luo H C and Xie Y M 2022 Design and mechanical characteristics of auxetic metamaterial with tunable stiffness *Int. J. Mech. Sci.* **223** 107286

[60] Grima-Cornish J N, Attard D, Grima J N and Evans K E 2022 Auxetic behavior and other negative thermomechanical properties from rotating rigid units *Phys. Status Solidi* **16** 2100322
[61] Hamzehei R, Kadkhodapour J, Anaraki A P, Rezaei S, Dariushi S and Rezadoust A M 2018 Octagonal auxetic metamaterials with hyperelastic properties for large compressive deformation *Int. J. Mech. Sci.* **145** 96–105
[62] Hamzehei R, Rezaei S, Kadkhodapour J, Anaraki A P and Mahmoudi A 2020 2D triangular anti-trichiral structures and auxetic stents with symmetric shrinkage behavior and high energy absorption *Mech. Mater.* **142** 103291
[63] Jiang F, Yang S, Qi C and Liu H-T 2022 Two plateau characteristics of re-entrant auxetic honeycomb along concave direction *Thin-Walled Struct.* **179** 109665
[64] Lim T-C 2015 *Auxetic Materials and Structures* (Springer)
[65] Lim T C 2017 Analogies across auxetic models based on deformation mechanism *Phys. Status Solidi* **11** 1600440
[66] Pozniak A, Kaminski H, Kedziora P, Maruszewski B, Strek T and Wojciechowski K W 2010 Anomalous deformation of constrained auxetic square *Rev. Adv. Mater. Sci.* **23** 169–74
[67] Michalski J and Strek T 2022 Response of a sandwich plate with auxetic anti-tetrachiral core to puncture *Advances in MANUFACTURING III. MANUFACTURING 2022* (*Lecture Notes in Mechanical Engineering*) ed B Gapiński, O Ciszak and V Ivanov (Springer) pp 1–18
[68] Mousanezhad D, Babaee S, Ebrahimi H, Ghosh R, Hamouda A S, Bertoldi K and Vaziri A 2015 Hierarchical honeycomb auxetic metamaterials *Sci. Rep.* **5** 18306
[69] Novak N, Nowak M, Vesenjak M and Ren Z 2022 Structural optimization of the novel 3D graded axisymmetric chiral auxetic structure *Phys. Status Solidi* b **259** 2200409
[70] Novak N, Vesenjak M, Tanaka S, Hokamoto K and Ren Z 2020 Compressive behaviour of chiral auxetic cellular structures at different strain rates *Int. J. Impact Eng.* **141** 103566
[71] Rezaei S, Kadkhodapour J, Hamzehei R, Taherkhani B, Anaraki A P and Dariushi S 2021 Design and modeling of the 2D auxetic metamaterials with hyperelastic properties using topology optimization approach *Photon. Nanostruct.* **43** 100868
[72] Strek T, Maruszewski B, Narojczyk J W and Wojciechowski K 2008 Finite element analysis of auxetic plate deformation *J. Non-Cryst. Solids* **354** 4475–80
[73] Strek T, Matuszewska A and Jopek H 2017 Finite element analysis of the influence of the covering auxetic layer of plate on the contact pressure *Phys. Status Solidi* b **254** 1700103
[74] Zhang Y, Ren X, Han D, Cheng X, Jiang W, Zhang X G, Zhang X Y and Xie Y M 2022 Static and dynamic properties of a perforated metallic auxetic metamaterial with tunable stiffness and energy absorption *Int. J. Impact Eng.* **164** 104193
[75] Zhang Y, Sun L, Ren X, Zhang X Y, Tao Z and Xie Y M 2022 Design and analysis of an auxetic metamaterial with tuneable stiffness *Compos. Struct.* **281** 114997
[76] Evans K E 1991 Auxetic polymers: a new range of materials *Endeavour* **15** 170–4
[77] Chen B, Chen L, Du B, Liu H, Li W and Fang D 2021 Novel multifunctional negative stiffness mechanical metamaterial structure: tailored functions of multi-stable and compressive mono-stable *Composites* B **204** 108501
[78] Chen S, Tan X, Hu J, Wang B, Wang L, Zou Y and Wu L 2022 Continuous carbon fiber reinforced composite negative stiffness mechanical metamaterial for recoverable energy absorption *Compos. Struct.* **288** 115411
[79] Lakes R S, Lee T, Bersie A and Wang Y C 2001 Extreme damping in composite materials with negative-stiffness inclusions *Nature* **410** 565–7
[80] Li Q, Yang D, Ren C and Mao X 2022 A systematic group of multidirectional buckling-based negative stiffness metamaterials *Int. J. Mech. Sci.* **232** 107611
[81] Pan Y, Zhou Y, Wang M, Gao Q and Sun B 2023 A novel reinforced cylindrical negative stiffness metamaterial for shock isolation: analysis and application *Int. J. Solids Struct.* **279** 112391
[82] Tan X, Li Y, Wang L, Yao K, Ji Q, Wang B, Laude V and Kadic M 2023 Bioinspired flexible and programmable negative stiffness mechanical metamaterials *Adv. Intell. Syst.* **5** 2200400
[83] Sun M, Zhang K, Guo X, Zhang Z, Chen Y, Zhang G and Jiang S 2023 A novel negative stiffness metamaterials: discrete assembly and enhanced design capabilities *Smart Mater. Struct.* **32** 095036
[84] Grima J N, Gatt R, Zammit V, Cauchi R and Attard D 2011 Industrial applications of molecular simulations, C10: on the negative Poisson's ratios and thermal expansion in natrolite *Industrial Applications of Molecular Simulations* (Taylor & Francis)
[85] Lim T-C 2019 2D metamaterial with in-plane positive and negative thermal expansion and thermal shearing based on interconnected alternating bimaterials *Mater. Res. Express* **6** 115804
[86] N. Grima J, Farrugia P-S, Gatt R and Zammit V 2007 Connected triangles exhibiting negative Poisson's ratios and negative thermal expansion *J. Phys. Soc. Japan* **76** 025001
[87] Takezawa A and Kobashi M 2017 Design methodology for porous composites with tunable thermal expansion produced by multi-material topology optimization and additive manufacturing *Composites* B **131** 21–29
[88] Wang Q, Jackson J A, Ge Q, Hopkins J B, Spadaccini C M and Fang N X 2016 Lightweight mechanical metamaterials with tunable negative thermal expansion *Phys. Rev. Lett.* **117** 175901
[89] Hartwig G 1995 Support elements with extremely negative thermal expansion *Cryogenics* **35** 717–8
[90] Grima J N, Caruana-Gauci R, Attard D and Gatt R 2012 Three-dimensional cellular structures with negative Poisson's ratio and negative compressibility properties *Proc. R. Soc.* A **468** 3121–38
[91] Lakes R and Wojciechowski K 2008 Negative compressibility, negative Poisson's ratio, and stability *Phys. Status Solidi* b **245** 545–51
[92] Gatt R and Grima J N 2008 Negative compressibility *Phys. Status Solidi* **2** 236–8
[93] Ruppin R 2000 Extinction properties of a sphere with negative permittivity and permeability *Solid State Commun.* **116** 411–5
[94] Chen Y, Hu G and Huang G 2017 A hybrid elastic metamaterial with negative mass density and tunable bending stiffness *J. Mech. Phys. Solids* **105** 179–98
[95] Fleisch M, Thalhamer A, Meier G, Raguž I, Fuchs P, Pinter G, Schlögl S and Berer M 2021 Functional mechanical metamaterial with independently tunable stiffness in the three spatial directions *Mater. Today Adv.* **11** 100155
[96] Yavas D, Liu Q, Zhang Z and Wu D 2022 Design and fabrication of architected multi-material lattices with tunable stiffness, strength, and energy absorption *Mater. Des.* **217** 110613
[97] Poon R and Hopkins J B 2019 Phase-Changing Metamaterial Capable of Variable Stiffness and Shape Morphing *Adv. Eng. Mater.* **21** 1900802
[98] Qin Q and Dayyani I 2023 Large strain zero Poisson's ratio spring cell metamaterial with critical defect analysis and variable stiffness distributions *Compos. Struct.* **318** 117102

[99] Yang F 2022 *Variable Stiffness of Metamaterial Composite Structure* (The University of Waikato)
[100] Wojciechowski K 1987 Constant thermodynamic tension Monte Carlo studies of elastic properties of a two-dimensional system of hard cyclic hexamers *Mol. Phys.* **61** 1247–58
[101] Bilski M, Pigłowski P M and Wojciechowski K W 2021 Extreme Poisson's ratios of honeycomb, re-entrant, and zig-zag crystals of binary hard discs *Symmetry* **13** 1127
[102] Cerniauskas G and Alam P 2023 Cubically symmetric mechanical metamaterials projected from 4th dimensional geometries reveal high specific properties in shear *ACS Appl. Eng. Mater.* **1** 2472–86
[103] Diaz A R and Sigmund O 2010 A topology optimization method for design of negative permeability metamaterials *Struct. Multidiscip. Optim.* **41** 163–77
[104] Anaya-Jaimes L, Vicente W and Pavanello R 2022 Metamaterials design with a desired thermal expansion using a multi-material BESO method *Struct. Multidiscip. Optim.* **65** 355
[105] Wang Y, Luo Z, Zhang N and Kang Z 2014 Topological shape optimization of microstructural metamaterials using a level set method *Comput. Mater. Sci.* **87** 178–86
[106] Zhou S, Li W, Chen Y, Sun G and Li Q 2011 Topology optimization for negative permeability metamaterials using level-set algorithm *Acta Mater.* **59** 2624–36
[107] Lakes R 1987 Foam structures with a negative Poisson's ratio *Science* **235** 1038–40
[108] Alderson K, Simkins V, Coenen V, Davies P, Alderson A and Evans K 2005 How to make auxetic fibre reinforced composites *Phys. Status Solidi* b **242** 509–18
[109] Li T, Liu F and Wang L 2020 Enhancing indentation and impact resistance in auxetic composite materials *Composites* B **198** 108229
[110] Baumgart E 2000 Stiffness—an unknown world of mechanical science *Injury* **31**
[111] Den Hartog J P 1985 *Mechanical Vibrations* (Courier Corporation)
[112] Carrella A, Brennan M and Waters T 2007 Static analysis of a passive vibration isolator with quasi-zero-stiffness characteristic *J. Sound Vib.* **301** 678–89
[113] Baz A M 2019 *Active and Passive Vibration Damping* (Wiley)
[114] Qin Z, Pang X, Safaei B and Chu F 2019 Free vibration analysis of rotating functionally graded CNT reinforced composite cylindrical shells with arbitrary boundary conditions *Compos. Struct.* **220** 847–60
[115] Rao S S 2011 *Mechanical Vibrations Fifth Edition* (Pearson Education)
[116] Pan Y, Zhou Y, Gao Q and Sun B 2024 A novel 3D polygonal double-negative mechanical metamaterial with negative stiffness and negative Poisson's ratio *Compos. Struct.* **331** 117878
[117] Cai C, Zhou J, Wang K, Pan H, Tan D, Xu D and Wen G 2022 Flexural wave attenuation by metamaterial beam with compliant quasi-zero-stiffness resonators *Mech. Syst. Signal Process.* **174** 109119
[118] Cai C, Zhou J, Wu L, Wang K, Xu D and Ouyang H 2020 Design and numerical validation of quasi-zero-stiffness metamaterials for very low-frequency band gaps *Compos. Struct.* **236** 111862
[119] Gao W, Qin Z and Chu F 2022 Broadband vibration suppression of rainbow metamaterials with acoustic black hole *Int. J. Mech. Sci.* **228** 107485
[120] Guo S, Gao R, Tian X and Liu S 2023 A quasi-zero-stiffness elastic metamaterial for energy absorption and shock attenuation *Eng. Struct.* **280** 115687
[121] Lin Q, Zhou J, Wang K, Xu D, Wen G and Wang Q 2023 Three-dimensional quasi-zero-stiffness metamaterial for low-frequency and wide complete band gap *Compos. Struct.* **307** 116656
[122] Lin Q, Zhou J, Wang K, Xu D, Wen G, Wang Q and Cai C 2022 Low-frequency locally resonant band gap of the two-dimensional quasi-zero-stiffness metamaterials *Int. J. Mech. Sci.* **222** 107230
[123] Zhao J, Zhou G, Zhang D, Kovacic I, Zhu R and Hu H 2023 Integrated design of a lightweight metastructure for broadband vibration isolation *Int. J. Mech. Sci.* **244** 108069
[124] Zhou J, Pan H, Cai C and Xu D 2021 Tunable ultralow frequency wave attenuations in one-dimensional quasi-zero-stiffness metamaterial *Int. J. Mech. Mater. Des.* **17** 285–300
[125] Li Y, Cao S, Shen Y and Meng Y 2019 Phononic band-gaps of Hoberman spherical metamaterials in low frequencies *Mater. Des.* **181** 107935
[126] Overvelde J T B, Shan S and Bertoldi K 2012 Compaction through buckling in 2D periodic, soft and porous structures: effect of pore shape *Adv. Mater.* **24** 2337–42
[127] Gibson L J, Ashby M F, Schajer G and Robertson C 1982 The mechanics of two-dimensional cellular materials *Proc. R. Soc.* A **382** 25–42
[128] Almgren R F 1985 An isotropic three-dimensional structure with Poisson's ratio $= -1$ *J. Elast.* **15** 427–30
[129] Townsend S, Adams R, Robinson M, Hanna B and Theobald P 2020 3D printed origami honeycombs with tailored out-of-plane energy absorption behavior *Mater. Des.* **195** 108930
[130] An N, Domel A G, Zhou J, Rafsanjani A and Bertoldi K 2020 Programmable hierarchical kirigami *Adv. Funct. Mater.* **30** 1906711
[131] Rafsanjani A, Jin L, Deng B and Bertoldi K 2019 Propagation of pop ups in kirigami shells *Proc. Natl Acad. Sci.* **116** 8200–5
[132] Tao R, Ji L, Li Y, Wan Z, Hu W, Wu W, Liao B, Ma L and Fang D 2020 4D printed origami metamaterials with tunable compression twist behavior and stress-strain curves *Composites* B **201** 108344
[133] Wu X, Wang S, Ma Y and Deng Z 2024 Design of mechanical metamaterials with multiple stable stress plateaus *Mech. Adv. Mater. Struct.* **31** 1348–65
[134] Weeger O M, Kang Y S B, Yeung S-K and Dunn M L Multi-material optimization for 4d printing of active rod structures. *U.S. Patent Application* 16/348,505
[135] Nam R, Jakubinek M, Niknam H, Rahmat M, Ashrafi B and Naguib H E 2023 3D printed octet plate-lattices for tunable energy absorption *Mater. Des.* **228** 111835
[136] Zheng X, Uto K, Hu W-H, Chen T-T, Naito M and Watanabe I 2022 Reprogrammable flexible mechanical metamaterials *Appl. Mater. Today* **29** 101662
[137] Guo S, Liu S and Gao R 2024 A bidirectional quasi-zero stiffness metamaterial for impact attenuation *Int. J. Mech. Sci.* **268** 108998
[138] Liu J, Wang Y, Yang S, Sun T, Yang M and Niu W 2024 Customized quasi-zero-stiffness metamaterials for ultra-low frequency broadband vibration isolation *Int. J. Mech. Sci.* **269** 108958
[139] Qiu J, Lang J H and Slocum A H 2004 A curved-beam bistable mechanism *J. Microelectromech. Syst.* **13** 137–46
[140] Ye M, Gao L, Wang F and Li H 2021 A novel design method for energy absorption property of chiral mechanical metamaterials *Materials* **14** 5386
[141] Kim H, Tawfick S H and King W P 2022 Modeling and design of zero-stiffness elastomer springs using machine learning *Adv. Intell. Syst.* **4** 2200225
[142] Budiansky B 1974 Theory of buckling and post-buckling behavior of elastic structures *Adv. Appl. Mech.* **14** 1–65

[143] Zhai Z, Wang Y, Lin K, Wu L and Jiang H 2020 In situ stiffness manipulation using elegant curved origami *Sci. Adv.* **6** eabe2000
[144] Liu S, Peng G, Li Z, Li W and Jin K 2023 Nonlinear stiffness analysis and programming of a composite origami metamaterial with embedded joint-type metastructures *Compos. Struct.* **310** 116761
[145] Sadeghi S and Li S 2019 Fluidic origami cellular structure with asymmetric quasi-zero stiffness for low-frequency vibration isolation *Smart Mater. Struct.* **28** 065006
[146] Cheng Y, Li J, Qian X and Rudykh S 2021 3D printed recoverable honeycomb composites reinforced by continuous carbon fibers *Compos. Struct.* **268** 113974
[147] Bodaghi M, Serjouei A, Zolfagharian A, Fotouhi M, Rahman H and Durand D 2020 Reversible energy absorbing meta-sandwiches by FDM 4D printing *Int. J. Mech. Sci.* **173** 105451
[148] Dezaki M L and Bodaghi M 2023 Shape memory meta-laminar jamming actuators fabricated by 4D printing *Soft Matter* **19** 2186–203
[149] Mueller J, Lewis J A and Bertoldi K 2022 Architected multimaterial lattices with thermally programmable mechanical response *Adv. Funct. Mater.* **32** 2105128
[150] Zadpoor A A 2016 Mechanical meta-materials *Mater. Horiz.* **3** 371–81
[151] Panda K C 2016 Dealing with noise and vibration in automotive industry *Proc. Eng.* **144** 1167–74
[152] Altintas Y and Ber A 2001 Manufacturing automation: metal cutting mechanics, machine tool vibrations, and CNC design *Appl. Mech. Rev.* **54** B84
[153] Zona A 2020 Vision-based vibration monitoring of structures and infrastructures: an overview of recent applications *Infrastructures* **6** 4
[154] Beards C 1995 *Engineering Vibration Analysis with Application to Control Systems* (Elsevier)
[155] Yan B, Yu N, Wang Z, Wu C, Wang S and Zhang W 2022 Lever-type quasi-zero stiffness vibration isolator with magnetic spring *J. Sound Vib.* **527** 116865