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Shape optimised geometries for ductile damaging materials

Fabian Guhr[1,*] and **Franz-Joseph Barthold**[1]

[1] Structural Mechanics, TU Dortmund University, August-Schmidt-Str. 8, D-44227 Dortmund, Germany

Shape optimisation is utilised to generate damage resistant structures. By means of a variational approach, the analytical gradients for an elasto-plastic material model with regularised damage properties are derived. Due to the complexity of the underlying material model, the application of the variational approach requires additional handling of the history field. The gradients are then used for Sequential Quadratic Programming (SQP) which is applied to shape optimisation and thus generation of damage optimised geometries.

1 Sensitivities for ductile damage

The model used for the description of the ductile material behaviour here is derived in [1], wherein the focus is placed on parameter identification. The basic idea of the model is to couple the local damage d to a global field variable ϕ to then regularise the global damage without the need to solve the underlying Karush-Kuhn Tucker conditions at global level, see eg. [2] and [3] where this is done. This allows an easy implementation in the finite-element context and is known as the micromorphic approach. On the local material model, damage is strongly coupled to plasticity by means of effective quantities within their respective dissipation potentials.

To calculate the gradients for the application in shape optimisation, a variational approach, see [4], is used. This allows the separation of the geometry and thus the distinct calculation of their respective analytical derivatives. The total variation of the residual then reads

$$\delta R = \delta_w R + \delta_s R + \delta_{h_n} R = \frac{\partial R}{\partial w}\delta w + \frac{\partial R}{\partial s}\delta s + \frac{\partial R}{\partial h_n}\delta h_n = 0, \tag{1}$$

which can be rewritten in matrix form based on the finite element discretisation as

$$\delta \mathbf{R} = \mathbf{K}\,\delta\mathbf{w} + \mathbf{P}\,\delta\mathbf{s} + \mathbf{H}\,\delta\mathbf{h}_n = \mathbf{0}. \tag{2}$$

The quantities here are the residual $\mathbf{R}$, the stiffness matrix $\mathbf{K}$ with the field variables $\mathbf{w} = \mathbf{u}, \boldsymbol{\phi}$, the pseudo-load matrix $\mathbf{P}$ and design variables $\mathbf{s}$, as well as the history sensitivity matrix $\mathbf{H}$ and the history variables $\mathbf{h}_n = \{\boldsymbol{C}_n^{\mathrm{p}-1}, \alpha_n, d_n\}$ at the last (pseudo-)time step n. Rewriting the above equations then allows the definition of the sensitivity matrix $\mathbf{S}$, which describes the change of deformation $\mathbf{u}$ and global damage $\boldsymbol{\phi}$ due to a change in design $\mathbf{s}$

$$\begin{aligned}\delta\mathbf{w} &= -\mathbf{K}^{-1}\left[\mathbf{P}\,\delta\boldsymbol{s} + \mathbf{H}\,\delta\mathbf{h}_n\right] \\ &= -\mathbf{K}^{-1}\left[\mathbf{P} + \mathbf{H}\,\mathbf{Z}_n\right]\delta\mathbf{s} = \mathbf{S}\,\delta\mathbf{s}.\end{aligned} \tag{3}$$

Here the matrix $\mathbf{Z}_n$ is introduced, which is the total design derivative of the history variables

$$\delta\mathbf{h}_n = \mathbf{Z}_n\,\delta\mathbf{s}. \tag{4}$$

Since the calculation of the sensitivity matrix is dependant on the (pseudo-)time increments within the finite-element framework, the sensitivity matrix and the history variation have to be updated at the converged global newton step. The calculation of the history variation for the update then reads

$$\mathbf{h} = \frac{\partial\mathbf{h}}{\partial\mathbf{w}}\delta\mathbf{w} + \frac{\partial\mathbf{h}}{\partial\mathbf{s}}\delta\mathbf{s} + \frac{\partial\mathbf{h}}{\partial\mathbf{h}_n}\delta\mathbf{h}_n \tag{5}$$

$$= \left[\frac{\partial\mathbf{h}}{\partial\mathbf{w}}\,\mathbf{S} + \frac{\partial\mathbf{h}}{\partial\mathbf{s}} + \frac{\partial\mathbf{h}}{\partial\mathbf{h}_n}\mathbf{Z}_n\right]\delta\mathbf{s} = \mathbf{Z}\,\delta\mathbf{s}. \tag{6}$$

For readability, the $n+1$ subscript is omitted for all respective values in the above equation. Note that, in the first time-step the quantities for $t_0 = 0$ are required, and thus assuming $\mathbf{h}_0 = \mathbf{0}$ and $\delta\mathbf{h}_0 = \mathbf{0}$, the sensitivity matrix reads

$$\mathbf{S}_1 = -\mathbf{K}^{-1}\mathbf{P} \quad \text{with} \quad \mathbf{Z}_0 = \mathbf{0}. \tag{7}$$

For more details on the derivation and aspects of implementation, see eg. [5].

Due to the different cases for the material model that can occur in the simulation at different load increments, the above equations have to be derived for all possible cases, ie. purely plastic, purely damaging or coupled plastic-damage respectively, and as such the distinct cases have to be taken into account within the numerical implementation.

* Corresponding author: e-mail fabian.guhr@tu-dortmund.de, phone (+49)231 755 7244, fax (+49)231 755 7260

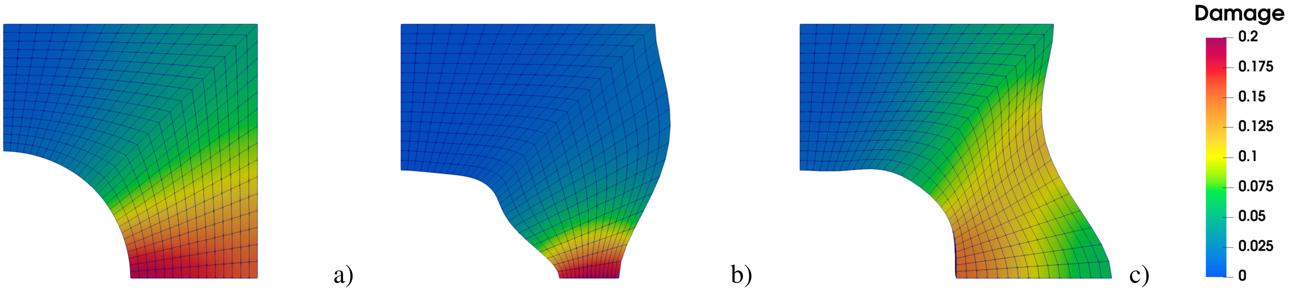

Fig. 1: Presented are the reference damage distributions and shapes for the initial reference design (a), the unconstrained damage problem (b) and the problem with the additional global damage constraints (c).

2 Optimisation problem

The main application for the derived gradients is to generate structures which are more damage resistant, i.e. under the same load they show less overall damage accumulation. This general problem can be easily stated by directly minimising the total damage accumulation in the structure calculated by the least square of the global damage variable ϕ_i of each node. To compare the results, the final volume shall remain constant. Additionally, another problem is stated where, in addition to the general damage minimisation, the exact value at each node is furthermore constrained to a maximum value ϕ_{crit} that may not be exceeded. The two optimisation problem thus read

$$\begin{aligned} \underset{\boldsymbol{s}_l \leq \boldsymbol{s} \leq \boldsymbol{s}_u}{\text{minimise}} \quad & \mathfrak{J}_1 = \|\boldsymbol{\phi}(\boldsymbol{s})\|^2 \\ \text{subject to} \quad & V(\boldsymbol{s}) = V_0 \end{aligned} \tag{8}$$

$$\begin{aligned} \underset{\boldsymbol{s}_l \leq \boldsymbol{s} \leq \boldsymbol{s}_u}{\text{minimise}} \quad & \mathfrak{J}_2 = \|\boldsymbol{\phi}(\boldsymbol{s})\|^2 \\ \text{subject to} \quad & V(\boldsymbol{s}) = V_0 \\ & \phi_i(\boldsymbol{s}) \leq \phi_{\text{crit}} \end{aligned} \tag{9}$$

3 Optimisation results

For the application of the previously calculated gradients, a plate with a hole is analysed. Due to its shape, symmetric boundary conditions can be applied, resulting in the mesh and calculation of only 1/8th of the whole plate. The plate is loaded by a prescribed deformation at the top and bottom. Additionally, the shape is defined by control points of a computer aided geometric design (CAGD) which act as the design variables within the optimisation.

The results are presented in Fig. 1 with the damage distribution of the original design as a reference. The optimised design with problem definition Eq. (8) leads to a localisation of the damage accumulation in the cross-section at the lower boundary. This is achieved due to a reshaping of the plate such that the cross-section area is minimised w.r.t. the allowed design space $\mathbf{s}_l$ and $\mathbf{s}_u$ of the control points. While this leads to the best minimisation of the overall damage accumulation ($\mathfrak{J}_1 = 7.2174$) within the allowed design space, the maximum damage value at this section reaches a value of $\phi^1_{\text{max}} = 0.1930$ and as such exceeds the maximum value of the reference design ($\phi^{\text{init}}_{\text{max}} = 0.1893$). Additionally constraining the maximum damage value in problem Eq. (9) to a value of $\phi_{\text{crit}} = 0.15$ then bounds the admissible space for the optimal design, such that the optimiser leads to drastically different final shape where the damage is more evenly distributed to allow for an overall reduced damage accumulation. Due to this constraint the objective function only reaches a value of $\mathfrak{J}_2 = 11.6341$, which is still lower than the initial value of $\mathfrak{J}_{\text{init}} = 14.3404$.

As a conclusion regarding the results generated, one can make the statement, that a simple reduction of the overall damage accumulation by means of numerical optimisation may not always be the best approach, as this can lead to results where damage starts to localise and a more controlled approach by constraining the damage values may lead more desirable results.

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