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Improvement of teaching methods and student learning status in teacher training course II – Topics in teaching Relative Value

We would like to explain our concrete plans on the pre-service of mathematics education methodology in teacher training course of university. It is difficult for many pupils at elementary school to understand on Relative Value in Japan. Students at university, who want to be teachers, don't understand Relative Value, sufficiently, too. We have taught the new teaching methods to students. They understood the meaning of Relative Value and learned how to teach it to pupils.

Introduction

In Japan, pupils learn on Relative Value in only the fifth grade of elementary school (Fig.1). There are some key features:

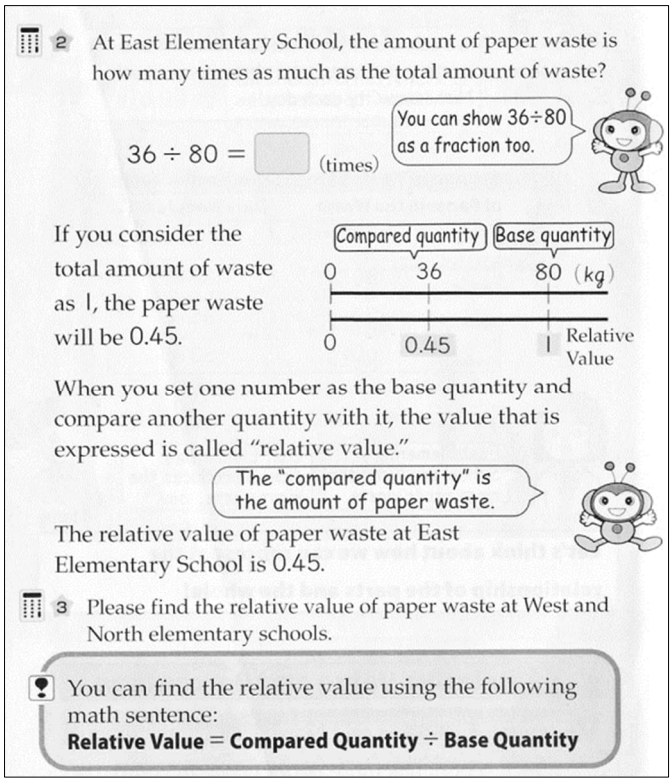

Fig.1: Example of school text book (Hironaka et al., 2006) in English

- Definition of Relative Value is "Relative Value = Compared Quantity / Base Quantity". We call "The first type formula".
- Number lines are used to explain mean of problem and hint about solution.
- Pupils memorize and use three types of formulas. The first type formula is definition. The second type formula is "Compared Quantity = Base Quantity ・ Relative Value". The third type formula is "Base Quantity = Compared Quantity / Relative Value".
- Pupils learn them by logical explanation of the teacher with number lines, without real activity by hand work.

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The conditions for solving problems on Relative Value
Procedure

The first purpose of this study was to elucidate students' understanding of Relative Value. The problems of Relative Values were classified into the following three types, and the data were analyzed for each type. The first type: Compared value and base quantity are already known, and relative value is to be answered. The second type: Base quantity and relative value are already known, and compared quantity is to be answered. The third type: Compared quantity and relative value are already known, and base quantity is to be answered.

Participants were 24 students of university who want to be teachers of elementary school. A Question sheet that contains four problems on relative value was prepared. The participants were asked to solve all problems. The problems presented to participants are as follows:

Problem 1: There is a 60 cm stick. You cut out 36 cm for work. The length you cut is how many times the length of the original stick? (The First Type)

Problem 2: A school consists of a total of 400 students. The number of boys in the class was 0.6 times the total number of the school. How many boys are there in this school? (The Second Type)

Problem 3: A boy gave 18 marbles to his sister. The number of marbles that the boy gave to his sister was 0.6 times the marbles that he originally had. How many marbles did the boy originally have? (The Third Type)

Results

The purpose of this study was to examine students' understanding of the three problem types. Therefore, responses to the problems were evaluated based on the adequacy of the formula written by students. If the correct formula was used, but a wrong answer was written on the answer sheet due to a miscalculation, it was regarded as a correct answer (Tab. 1).

Question (Type)	Pre-test Correct answer rate (%)
Q1 (1st type)	54.2
Q2 (2nd type)	95.8
Q3 (3rd type)	70.8

Tab. 1: Pre-test results (n=24)

The percentages of question answered correctly was 54.2%, 95.8 %, and 70.8% for question 1, 2 and 3, respectively. In first type, students were confused by the word "cut out" in the question sentence and answered 24/60 = 0.4. (8.3%). And they simply divided the large number by the small number. 60 / 24 = 2.5 (12.5%). In third type, they thought that Q3 was question of second type. They answered 18・0.6 = 10.8 (20.8%) . We instructed them to

think using mathematical figures, but few students drew the number line in their answers.

Discussion

The above results indicated the following four points. Firstly, many students don't understand the definition of relative Value. Secondly, it is particularly difficult for them to identify the difference between Base Quantity and Compared Quantity in question sentence. Thirdly, it is easy to use the second type formula. Fourthly, number lines which they had learned in elementary school was not used to reach a solution for the problem. They solve the problems in trend to directly use one of formulas to solve questions.

Lecture how to teach on Relative Value

Teaching premise

- Students were asked to fully complete activities to draw a line that was *p* times a given arbitrary length. Next, they were asked to calculate the values using actual measured values by three type's formulas.
- The structure of the number line indicating the relationship between an actual measured value and a relative value was carefully taught to students.
- The Second Type, obtaining a compared quantity, was introduced first to students.
- The First Type and Third Type were introduced by expression transformations using the unknown as □ in the formula of the second type.

Five Teaching Steps and Teaching Process

Step 1. Teaching Number Lines. Students were instructed on how to draw a number line marked origin 0 and a unit 1, and how to measure *p* times length.

Step 2. Making a *My Ruler* by Themselves. Students were asked to make a *My Ruler* by using a paper tape, the height of each student was regarded as 1C (in Mr. C's case; the unitary name is the name of each student). Students were instructed that 1C was divided into 10 or 100 equal parts represented as 0.1 or 0.01, respectively.

Step 3. Application to the Second Type Formula. They form a pair and measure each other's height with *My Ruler*. They calculate the height of the other person by using their own height and Relative Value. They learn how to draw number lines, calculated as 157cm×1.10=172.7cm (Height of other person was 1.10 times of my height). The formula of the second type was derived.

Step 4. Application to the First Type for deriving Relative Value. Students were asked to calculate the ratio Mr. K's height (i.e., 172 cm) would be based on Miss. H's height (i.e., 157 cm). First, the Relative Value was set as □ , and students were asked to express it on a number line. H wrote and transformed H‘s formula in the following order: 157・□＝172, □＝172/152, □＝1.10, and Relative Value is 1.10. The formula of the first type was derived.

Step 5. Application to the Third Type for deriving Base Quantity. They worked to find my height from the height of the other person and the Relative Value of the other person measured by *My Ruler*. The answer was □ ・1.10 ＝172.7, 172.7/1.10 ＝□, □=157. The formula of the third type was derived.

In teaching the First and Third Type, we had them draw a number line and then taught the merits of a teaching method that transforms the formula of the Second Type.

Results

A Post-test that was same as the Per-test was carried out for 20 students. We compare results of post-test with results of pre-test (Tab. 2). Increase in correct answer's rate on Q1, Q2 and Q3 is significant results. But all students are not perfect answer for Q1. Therefore, we must research this cause.

Question (Type)	Pre-test Correct answer rate (%) N=24	Post-test Correct answer rate (%) N=20
Q1 (1st type)	54.2	85.0
Q2 (2nd type)	95.8	100
Q3 (3rd type)	70.8	100

Tab. 2: Pre-test and post-test results compared

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