368. Analysis holographic interferogram of links of vibrating systems

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(Received 30 April 2008; accepted 13 June 2008)

Abstract

A procedure for calculating the amplitudes of the normal components of the displacement vector in case of three-dimensional vibrations of any point on the surface of mechanical vibrating system is proposed on the basis of experimental holographic interfrometry data and the theory of vibrations of mechanical systems. The analysis of fringe in holographic interferogram is based on several holographic interferogram of the vibrating links of wave mechanical system which are obtained using Time-Average holographic interferometry method.

Keywords: holographic interferometry, time-average method, interferogram, vibrating links, mechanical system

Introduction

The problem of determination of the amplitude – frequency response characteristics of a vibrating surface, which vibrates in three dimensions in the majority of cases, is encountered in the analysis of vibrations of mechanical system. The development of methods for calculating the characteristics of three – dimensional (3D) vibrations contributes to the solution of vital problems in the investigation, design, testing, and diagnostics of systems.

The publications dedicated for evaluation of the deformation vector, quantitive analysis of holographic data presented in [1-9]. The papers gives an instrument for interpretation and calculation of components of the vector of deformation of the surface of solid bodies of the simple shapes and the use for complex forms of the links is not directly available and require to modify it. The problem is that procedure and algorithms must include into the calculation relations between the type of vibrations and the geometry of the shapes of links. The 3-D vibrations for wave mechanical system shapes of links take part in the process of analysis and it enables to find algorithms and common methods of investigation.

In the present chapter a method of calculating the amplitudes of the normal and tangential components of the displacement vector of 3D vibrations of the surface of deformable elements on the basis of experimental holographic-interferometry data and the theory of vibrations of mechanical systems is described. In contrast with previous publication on this topic, the proposed method permits a severalfold reduction in the quantity of input data for analysis of vibrations from holographic interferograms.

When analysing the links of mechanical vibrating systems it is necessary to estimate the value of the amplitude of vibrations at each point on the surface of the link precisely. The method is presented which enables to calculate the values of amplitudes at each point on the surface on the basis of the holograms of the vibrations of the links.

Theoretical Considerations

The normal vibrations of the surface of the rigid body may be expressed in the following way [10, 11].

$$W_i = \sum_{j=1}^{n} A_j F_{ij}, \quad i = 1, 2, \ldots m \qquad (1)$$

where W_i - the value of the amplitude of the rigid body, which is obtained from the holographic interferogram; F_{ij} - the numerical value of the amplitude of the j -th vibrational eigenmode at point i, which is calculated according to fastening; A_j - the influence coefficient of the j -th eigenmode.

The values of the amplitude of the vibrations of the rigid body from the holographic interferogram are calculated from the formula [12]:

$$W_i = \frac{\lambda\Omega_k}{2\pi(\cos\theta_1 + \cos\theta_2)}, \quad i = 1,2, \quad m, \tag{2}$$

λ - the wave length of the laser which was used when performing the measurements; k - the sequential number of the interferention band. Ω - the argument of the Bessel function, which is calculated according to the formula:

$$\Omega_k = (k-0{,}25)\pi + 0{,}125/\left[\pi(k-0{,}25)\right] \tag{3}$$

The essence of the method is the following: because the link of the mechanical system has strictly defined conditions of fastening, when calculating the eigenmodes of the vibration of the link F_{ij} we use the analytical relationships of the theory of mechanical vibrations. As we have W_i from the holograms, we calculate F_{ij} - according to the analytical relationships, then using (1) we can calculate the influence coefficients of the eigenmode. As A_j is independent from the surface coordinates, using the formula (1) we can find the amplitude of normal vibration at each point on the surface of the rigid body.

We present the analytical relationships for the calculation of F_{ij} for the eigenmodes of the links of vibrational systems, taking into account the conditions of fastening and the geometry of the link [14].

The link shaped as a bar. a) the ends of the bar are free

$$F_{i1} = 1; \quad F_{i2} = 3(1 - 2x_i/l),$$
$$F_{ij} = K_1(p_j x_i/l) - \left[K_2(p_j)/K_3(p_j)\right]K_2(p_j x_i/l)$$
$$(i = 1,2,\ldots\ldots, m; \quad j = 1,2, \quad .\ n) \tag{4}$$

x_i- the coordinates, l- the length of the bar; K_1, K_2, K_3 - the functions of Krylov; $p_1=0$; $p_2=0$;$p_3=4{,}730$;$p_u=7{,}83$.

$$p_j = \frac{2(j-2)+1}{2}\pi \qquad (j = 5,6, \quad ,n) \tag{5}$$

b) the ends of the bar are fastened by rotational links

$$F_{ij} = \sin(p_j x_i/l) \qquad (i = 1,2\ldots,m;\ j = 1,2,\ldots n) \tag{6}$$

where $p_j = j\pi$

c) one end of the bar fastened, another end free

$$F_{ij} = K_3(p_j x_i/l) - \left[K_1(p_j)/K_2(p_j)\right]K_4(p_j x_i/l) \tag{7}$$
$$(i = 1,2,\ldots,m; \quad j = 1,2,\ldots,n$$

where K_1, K_2, K_3, K_4 - the functions of Krylov; $p_1=1{,}875$; $p_2=4{,}694$;

$$p_j = \frac{2j-1}{2}\pi, \quad j = 3,4,\ldots\ldots,n, \tag{8}$$

d) one end of the bar rigidly fastened, and another fastened with a rotational link

$$F_{ij} = K_3(p_j x_i/l) - \left[K_1(p_j)/K_2(p_j)\right]K_4(p_j x_i/l)$$
$$(i = 1,2,\ldots\ldots m,; \quad j = 3,4,\ldots\ldots,n) \tag{9}$$

Where $p_1 = 3{,}927$; $p_2 = 7{,}069$;

and $p_j = \frac{4j+1}{4}\pi, \quad (j = 3,4,\ldots,n)$

e) both ends of the bar rigidly fastened

$$F_{ij} = K_3(p_j x_i/l) - \left[K_3(p_j)/K_4(p_j)\right]K_4(p_j x_i/l)$$
$$(i = 1,2,\ldots\ldots m,; \quad j = 3,4,\ldots\ldots,n) \tag{10}$$

where $p_1 = 4{,}730$; $p_2 = 7{,}853$; $p_j = \frac{2j+1}{2}\pi, \quad (j = 3,4,\ldots,n)$

By using the relationships (4) - (10) we can calculate the relative values of the vibrational modes, taking into account the various conditions of fastening of the bar.

The link shaped as a plate. We have the case of fastening, when the vibrational eigenmodes are expressed [3]

$$F_{ij} = \Phi_j(\gamma_i)\sin(j\pi x_i/a_x) \quad i = 1,2,\ldots,m; \quad j = 1,2,\ldots,n. \tag{11}$$

x_i, y_i - the coordinates; a_x - the length of the plate in the x direction;

Φ_j is expressed as

$$\Phi_j = C_{1j}\cos(\gamma_{1j}\gamma_i) + C_{2j}\sin(\gamma_{1j}\gamma_i) +$$
$$+ C_{3j}\operatorname{ch}(\gamma_{2j}\gamma_i) + C_{4j}\operatorname{ch}(\gamma_{2j}/\gamma_i) \tag{12}$$

where C_{1j}, C_{2j}, C_{3j}, C_{4j} - depend on the conditions of fastening,

$$\gamma_{1j} = \sqrt{(\rho h\omega_j/D)^{1/2}} - j^2\pi^2/a_x,$$

$$\gamma_{2j} = \sqrt{(\rho h\omega_j/D)^{1/2}} - j^2\pi^2/a_y, \tag{13}$$

where $D = Eh^3/[12(1-v^2)]$, E- the modulus of elasticity; v - the Poisson coefficient; ρ - the density of the plate; h - thickness of the plate; ω_j - the eigenfrequency.

When both ends of the plate are free we may write:

$$\left[\frac{\partial F_{xy}}{\partial y} + v\frac{\partial^2 F_{xy}}{\partial x^2}\right]_{y=0} = 0,$$

$$\left[\frac{\partial^3 F_{xy}}{\partial y^3} + 2(1-v)\frac{\partial^3 F_{xy}}{\partial x^2 \partial y}\right]_{y=0} = 0,$$

$$\left[\frac{\partial F_{xy}}{\partial y} + v\frac{\partial^2 F_{xy}}{\partial x^2}\right]_{y=a_x} = 0,$$

$$\left[\frac{\partial^3 F_{xy}}{\partial y^3} + 2(1-v)\frac{\partial^3 F_{xy}}{\partial x^2 \partial y}\right]_{y=a_x} = 0, \qquad (14)$$

where x,y - the coordinates of the plate; F_{xy}- the eigenmode.

We calculate the frequency of eigenforms

$$\omega_j = \frac{k^3 rl}{a_x^2}\sqrt{Dl\rho h}\text{, (where } k^2 rl = \pi^2 \lambda\text{)} \qquad (15)$$

$$\lambda^2 = G_y^4(r) + \left(\frac{a_x}{a_y}\right)^4 G_x(l) + 2\left(\frac{a_x}{a_y}\right)^2 (vH_y(r)H_x(l) + 1(-v)I_y(r)I_x(l)) \qquad (16)$$

when r = 0.12:

$$G_y(0)=0;\ H_y(0)=0;\ I_y(0)=0$$
$$G_y'(0)=0;\ H_y'(0)=0;\ I_y'(0)=0$$
$$G_y(0)=1{,}506;\ H_y(0)=1{,}248;\ I_y(0)=4{,}017$$

when r >2

$$G_y(r) = \left(r-\frac{1}{2}\right); \quad H_y(r) = (r-\frac{1}{2})\left[1-\frac{2}{(r-\frac{1}{2})\pi}\right];$$
$$I_y(r) = \left(r-\frac{1}{2}\right)^2\left[1+\frac{b}{\left(r-\frac{1}{2}\right)\pi}\right] \qquad (17)$$

when I=2,3,….

$$G_x(l) = (l-1); \quad H_x(l) = (l-1)^2; \quad I_x(l) = (l-1)^2$$

In order to find $C_{1j}, C_{2j}, C_{3j}, C_{4j}$ - we substitute w_j into the equality (12), which we calculate using formulas (15)-(17) in the equality (14). We obtain a system of linear equations with respect to $C_{1j}, C_{2j}, C_{3j}, C_{4j}$. After solving the system of equations (14) according to formula (11) taking into account (12) and (13) we calculate the j-th mode of eigenvibrations for the points of the surface of the plate that is analysed.

In case of a cylinder or sphere - the relative values of the amplitudes we can find from the [14].

Applications

Example 1. By using the presented method the calculation of vibrations of the flat body was performed according to the holographic interferogram (see fig. 1-fig. 3).

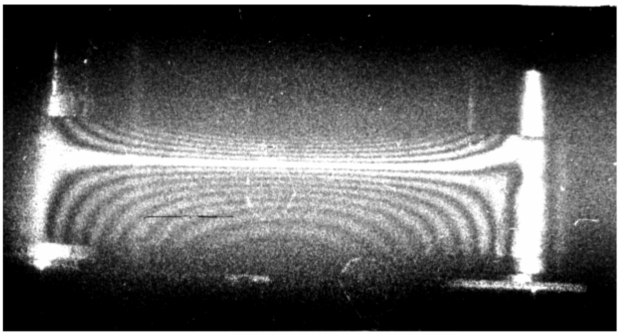
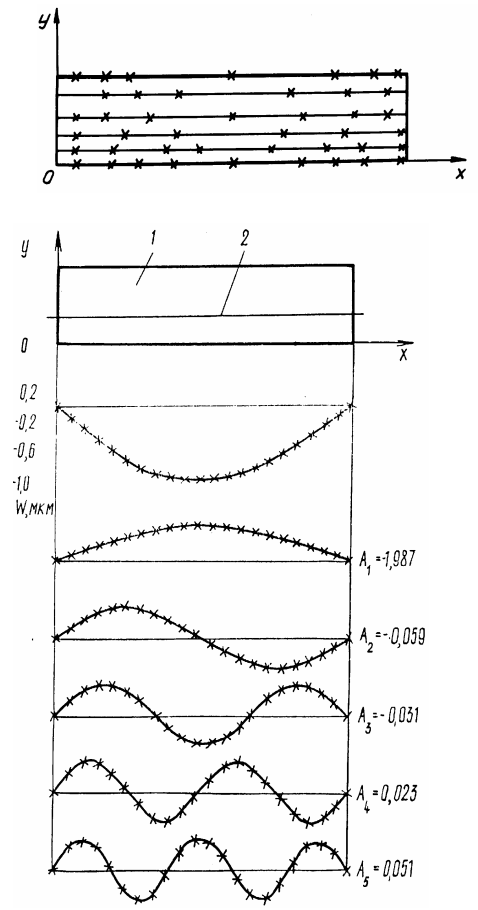


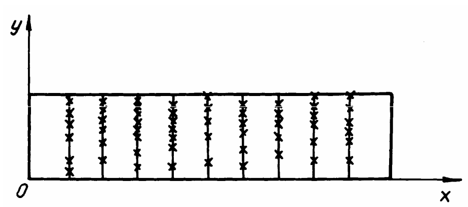
Fig. 1. The holographic interferogram of the magnetic tape

1)

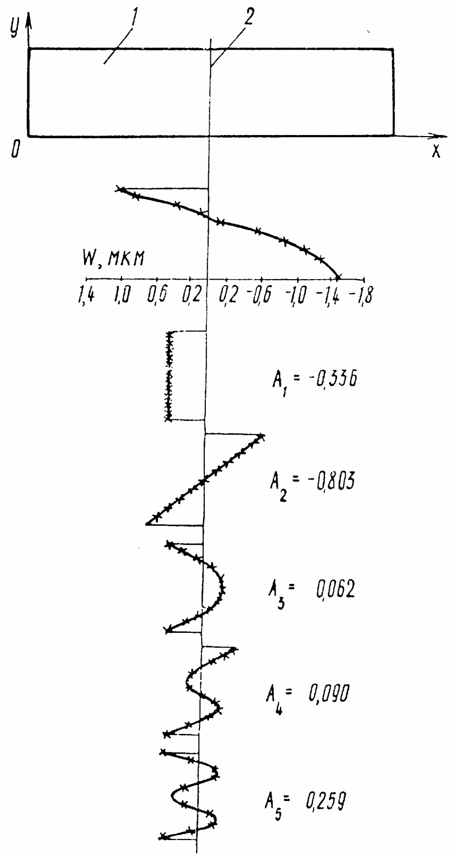


2)

Fig. 2. (1) - the coordinates of the points in which the amplitudes are calculated according to the interferogram; (2) - the influence coefficients of the eigenmode in the direction of the axis. 1 - the tape; 2 - the coordinates in which the amplitudes of vibrations are calculated from the holograms



1)



2)

Fig. 3. (1) - the coordinates of the points in which the amplitudes are calculated according to the interferogram; (2) - the numerical influence values of the coefficients of the eigenmode

Example 2. The object of analysis is contacting cylinder shell body, performing high frequency harmonic vibrations in the contact zone. The parameters of the contact zone are the following: R - 7,5 mm, L - 10,5 mm; coefficient of elasticity E - 0,49 x 10^7 N/m^2; coefficient of Poisson υ - 0,48. The coefficient of friction between the bodies in the contact zone μ - 0,25. The force of pressing the contacting bodies together P - 25 N.

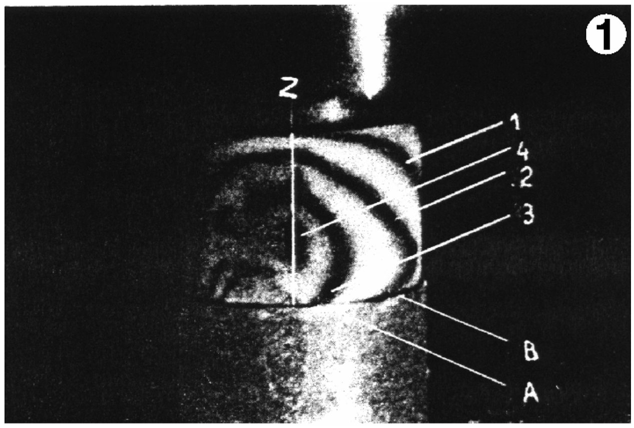
The foundation of the contacting body is rigidly fixed on the piezoelectric actuator which is connected to high frequency amplifier (1 - 20 kHz) and a power source. The amplitude of vibrations is about 0,1 μm in the axial direction of the cylinder. Also, the stiffness of foundation (E – 2 x10^{11} N/m^2) is much higher than the stiffness of the contacting cylinder (E - 0,49 x 10^7 N/m). Thus the deformations of the foundation may be neglected. Thus the longitudinal vibrations of the surface of analysed body will be of the same range than the ones of the foundation, and the magnitude of axial, radial and tangential displacement vectors of the points on the surface may be expressed using the following relations:

$$\varepsilon_z = \partial W/\partial z, \ \ \varepsilon_r \approx \varepsilon_t = -\nu\varepsilon_z$$

Here

$$\varepsilon_z = a_z / L = 1\cdot 10^{-4}; \quad |\varepsilon_r| \approx |\varepsilon_t| = 0.48\cdot|\varepsilon_z| = 0.48\cdot 10^{-4}.$$

The experimental investigation (fig. 4) of the surface deformations of the output member in the contacting zone is performed by means of the method holographic time averaging interferometry [15].



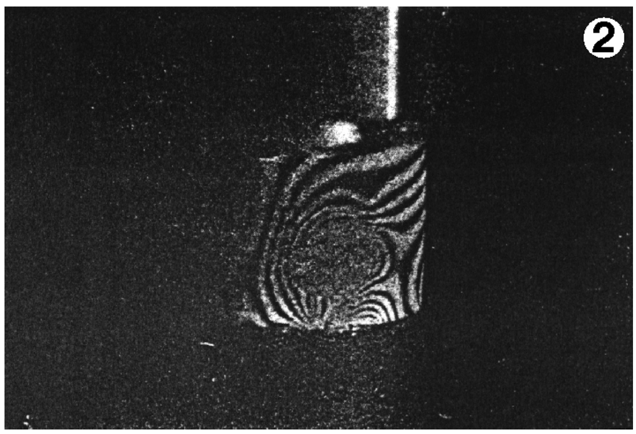


Fig. 4. Holographic interferogram of vibrating contact zone. (1) – f=1800 Hz; (2) – f=2600 Hz

The forced oscillations of the vibration of the foundation in the contact zone excite the cylinder vibrations, and the distribution of the holographic interference bands over the surface of the cylinder on the resonance frequencies enables the calculation of vibration vectors of the surface points of the cylinder.

The experiments were performed using the following parameters of the experimental stand (see fig. 5): PP_i = 411,2 mm, A = 240,2 mm, R_V = 21,5 mm, α_1 = 0,752 rad, α_1 = 0,9774 rad, α_3 = 1,222 rad.

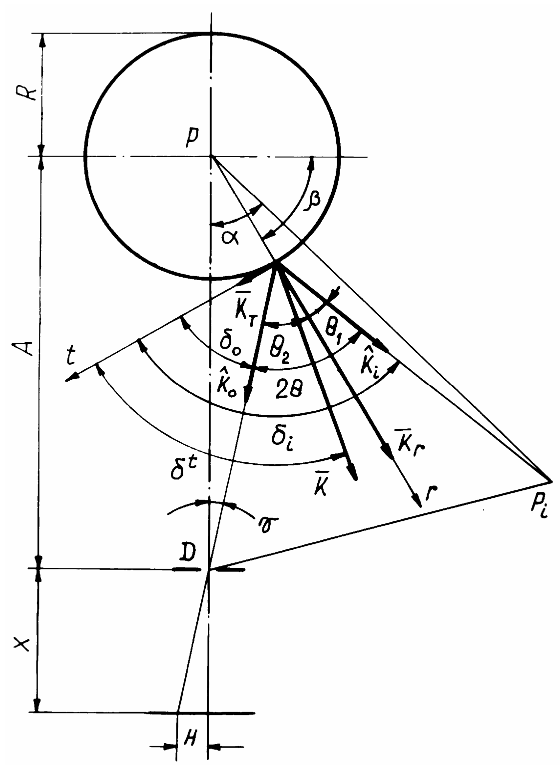


Fig. 5. The interpretation diagram of the holographic measurement of the point lying on the surface of the cylinder for the case when the cylinder axis is perpendicular to the plane of unit illumination and observation vectors; $\hat{K}$ is the sensitivity vector; $\hat{K}_r, \hat{K}_t$ are the projections of sensitivity vectors on axis *r, t; R* is the radius of the cylinder; *A* is the distance from the center of the cylinder *P* to the aperture; *x* is the distance from aperture *D* to screen; γ is the aperture angle

The distance PP_i in figure 5 is the distance from the point of illumination P_i to the center of rotation of the cylinder *P*; β is the angle defining the coordinate of the investigated point on the surface; α is the central angle between the directions of observation and illumination determinable experimentally from the optical holographic train. The calculation of the angle of illumination and observation according to this measurement scheme is presented in paper [15].

The wave length of He-Ne continuous laser λ - 632,8 nm.

Holographic interferograms are produced at vibration frequencies f_1 - 1800 Hz and f_2 - 2600 Hz and α_1 - 0,752 rad are presented in figures 4. It can be seen that there are points with nonzero vibration amplitudes on the surface of contacting structure.

The elastic oscillations of contacting points take place in the result of sliding of contacting zones, and reach their maximum amplitudes at the resonant frequencies.

The components of vectors U and V on the contour of the contact are calculated from 3 holograms using methodology presented in paper [15]. Thus U_a -0,4773 μm V_a - 0,2162 μm at f_1 - 1800 Hz at point A which is located at the distance H_a - 4,2 mm from the symmetry axis of the cylinder and placed in the center of the third dark band(Fig. 4 (1)). Similarly, U_b - 0,3684 μm V_b - 0,2707 μm at point B, H_b - 17,2 mm, and located in the center of second dark interference band. (Fig. .4 (1)). The uncertainty of experiment is e < 0,10 U.

For the other resonance frequency f_2 - 2600 Hz (Fig. 4 (2)) the number of vibrating points on the contour of the contact is increased, also their amplitudes are increased.

Example 3. Other object of experimental investigations is a deformable output member - piece-wise portion of tape between two tape transporting rollers which have the geometric shape of convex cylinder surface.

The tape piece has the form of cylindrical shell with the following parameters: radius of bending R - 24 mm, length of section L - 60 mm, thickness of tape h - 0,026 mm, width of tape S - 20,5 mm, coefficient of elasticity E - 4,8 x 10^9 N/m^2. The tension of tape is 4 N, the frequency of harmonic acoustical excitation f - 950 Hz. The produced photos by the holographic stand in case when the axis of cylindrical shell coincides with the direction of the lighting vector direction and the to the viewer are presented in Fig (6).

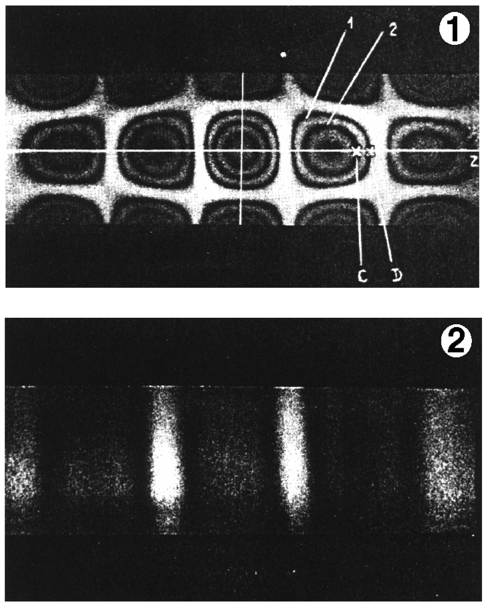


Fig. 6. Holographic interferograms of cylindrical shell of tape vibrating at resonance frequency 950 Hz. (1) – vector of sensitivity normal to the direction of the surface; (2) – angle between the vector of the sensitivity and the normal of the surface 30^0.

The components of the displacements of points on the cylinder shape of the surface are calculated using equations from paper [15] from two holograms (Fig. 6).

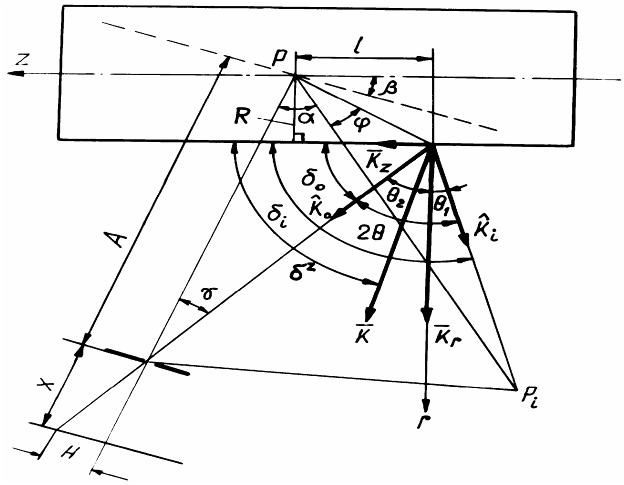


Fig. 7. The interpretation diagram of the holographic measurement of the point lying on the surface of the cylinder for the case when the cylinder axis is located in the plane of unit vectors of illumination $\hat{K}_i$ and observation $\hat{K}_{0r}, \hat{K}_{rt}, \hat{K}_z$ are projections of sensitivity vector on axis *r,z* respectively; *l* is the axial distance from the investigated point on the cylinder surface to the fixed point *P; x* is the distance from screen to aperture; *A* is the distance from the fixed point to aperture; *R* is the radius of the cylinder; α is the angle between illumination and observation directions

The PP_i in figure 7 is the distance from the fixed point *P* on the axis of the cylinder to the point of illumination P_t, which is determined experimentally from the optical circuit of the holographic rig; β is the angle defining the cylinder position relative to the observation direction and obtained experimentally from the optical train of the holographic rig; α is the central angle between the direction of observation and illumination, which is constant and is determinable experimentally from the holographic optical train. The calculation of the angles of illumination and observation according to this measurement scheme is presented in paper [15].

The reference frame of co-ordinates is longitudinal axis of symmetry. For a point C at distance H_c - 27,4 mm from the central axis of symmetry in the middle of the second dark interference band U_c - 2,254 μm, W_c - 0,4832 μm. Point D: H_d - 29,6 mm, located in the center of the first dark interference band, U_d - 2,076 μm, W_d - 0,4158 μm. The uncertainty of measurements δr < 0,08 U.

Conclutions

Algorithms and methodology for singling out the natural eigenforms from experimental holographic interferograms were developed. On top of that analysis the influence of eigenforms to the dynamics of the whole system has been performed.

Method is also applicable to the analysis of holographic interferograms of three-dimensional vibrations of deformable solids of any geometry of shapes.

Algorithm of calculation of components of the vector sensitivity can be used for design of an automated fringes analysis system.

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