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Abstract

We consider a vertically related market in which two symmetric upstream firms provide perfectly complementary inputs for two downstream manufacturers, one of which has a non-controlling interest in its rival. Each upstream firm can choose between two pricing regimes: discriminatory or uniform. This study shows that although uniform pricing limits the firm's flexibility, one upstream firm voluntarily chooses uniform pricing, and the other chooses discriminatory pricing in equilibrium. To our knowledge, this study first demonstrates such an asymmetric pricing equilibrium.

JEL Classification: D43; L11; L40; G34

Keywords: uniform price; input price discrimination; complementary inputs; horizontal shareholding; self-regulation

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1 Introduction

The literature on input price discrimination has focused mainly on welfare analysis and the policy implication of a ban on input price discrimination. In their seminal papers, DeGraba (1990), Katz (1987), and Yoshida (2000) show that input price discrimination has ambiguous effects on social welfare. The recent papers (Chen, 2022; Choi et al., 2022; Hu et al., 2022; Li & Shuai, 2022; Lestage, 2022; Lømo 2023; Matsuoka, 2022) report that input price discrimination is desirable for society in many situations.

In practice, some upstream firms choose uniform pricing for various goods, such as groceries, professional services, components, health supplies, equipment, motor vehicles, and so on (Shang & Cai, 2022). Yet, previous studies on input price discrimination implicitly assume that discriminatory pricing is better for an upstream firm than (self-regulatory) uniform pricing.

We focus on the automotive industry: most products are combined with various complementary components (Asanuma, 1989; Cusumano & Takeishi, 1991; Laussel, 2008), and horizontally competitive firms often have a small share of their rivals (Alley, 1997; Elhauge, 2016; Gilo et al., 2006). Our analysis shows that when a downstream firm has a non-controlling share of its rival, even if this share rate is sufficiently small, upstream uniform pricing increases the average input price more than discriminatory pricing. Thus, although uniform pricing sacrifices price flexibility, an upstream firm producing a perfectly complementary input has an incentive to choose uniform pricing in equilibrium.

Formally, we consider a vertically related market in which two monopolistic suppliers provide each perfectly complementary input to two downstream manufacturers with symmetric marginal production costs. One manufacturer holds the non-controlling share of the other manufacturer. At the initial stage, each supplier can choose its own pricing regime: discriminatory or uniform. As an extension, we consider the model with asymmetric downstream marginal production costs.

We find that due to the downstream asymmetric ownership structure, self-regulatory uniform pricing raises the average input prices more than discriminatory pricing does. The rationale for this is as follows. When the horizontal shareholdings exist in the downstream market, the holder's rival is more aggressive than the holder. Thus, upstream firm with discriminatory pricing sets the higher input price for the holder's rival. If the input price for the holder's rival increases, the holder increases its quantity. This implies that upstream firm with discriminatory pricing is forced to use the less aggressive channel, which is inefficient for the upstream firm. Therefore, since upstream firm with discriminatory pricing becomes a little reluctant to increase the input price for the holder's rival, the average input price in discriminatory pricing is lower than that in uniform pricing.

Our main result is that when the horizontal shareholding exists, an upstream firm voluntarily chooses uniform pricing in equilibrium. This results reverses our conventional wisdom that input price discrimination is better for an upstream firm. Intuitively, since the average input price is higher under uniform pricing than under discriminatory pricing, an upstream firm voluntarily chooses uniform pricing in equilibrium. Note that in our baseline model, uniform pricing by both upstream firms raises the input prices so much that both uniform pricing case does not occur in equilibrium.

From a consumer perspective, we find that this self-regulatory uniform pricing always undermines consumer surplus. If the input price increases, the price of the final goods also increases. Thus, since uniform pricing is a higher price commitment, uniform pricing is undesirable for consumer. This analysis first demonstrates the anticompetitive effect of voluntary compliance with a ban on price discrimination.

We analyze the model with asymmetric downstream marginal cost in the extension section. When the holder's marginal cost is sufficiently low, the holder becomes more aggressive than the holder's rival. Thus, in contrast to the baseline model, the average input price in discriminatory pricing may be higher than that in uniform pricing. Furthermore, since the aggressiveness of the holder is consistent with the difference in downstream marginal costs, both upstream firms may choose uniform pricing or discriminatory pricing in equilibrium.

We provide a policy implication of a ban on input price discrimination. Much of the previous literature on input price discrimination implicitly assume that discriminatory pricing is desirable for an upstream firm and show that input price discrimination has ambiguous welfare effects. Thus, even if the upstream firm chooses discriminatory pricing, policymakers cannot judge whether this pricing is desirable for society, because policymakers cannot easily observe costs and demand. In contrast, our results suggest that if both upstream producers of the perfectly complementary inputs choose discriminatory pricing, a ban on input price discrimination is desirable for society when the downstream firm has the shares of its rival. Therefore, without estimating costs and demand, policymakers may be able to decide whether to enforce antitrust laws based solely on the pricing regimes of the upstream firms of the perfectly complementary inputs.

1.1 Literature Review

Our study builds on the previous research on input price discrimination. The initial literature on input price discrimination (DeGraba, 1990; Katz, 1987; Yoshida,

2000) focused on the anticompetitive effects of discriminatory pricing. These analyses suggest that the production reallocation from inefficient to efficient firms through discriminatory pricing has an ambiguous effect on social welfare. Recent literature shows that this reallocation may be socially desirable in some situations: vertical differentiation (Chen, 2017), upstream R&D (Pinopoulos, 2020), price discrimination by resale markets (Miklòs-Thal & Shaffer, 2021), increasing marginal costs of manufacturers (Chen, 2022), the sequence of contracts with retailers (Kim & Sim, 2015; Choi et al., 2022), strategic inventory (Matsuoka, 2022) and vertical shareholding (Lestage, 2021). Hence, the antitrust legislation of the Robinson-Patman Act became controversial and is not strictly enforced (Luchs et al., 2010; Yonezawa et al., 2020).

The most relevant studies on input price discrimination are those by Li and Shuai (2022) and Hu et al. (2022). They suggest that input price discrimination mitigates the anticompetitive effect of horizontal shareholding and is socially desirable. We obtain the same result qualitatively. However, the "non-discriminatory" aspect has received relatively less attention in the literature on input price discrimination. By introducing perfectly complementary inputs in the analyses of Li and Shuai (2022) and Hu et al. (2022), we examine this aspect of input price discrimination and fill this gap in the literature.

We also contribute to the growing body of literature on perfectly complementary inputs in vertical markets. Laussel (2008) analyzes vertical integration by a downstream assembler under a Nash bargaining between the assembler and each supplier (subcontractor). Matsushima and Mizuno (2013) analyze a downstream firm's strategic incentive for a vertical separation to reduce external suppliers' market power. Reisinger and Tarantino (2019) analyze the effect on competition of a patent pool with nonlinear tariffs and vertical integration. The analysis of perfectly complementary inputs in a vertical market has also been applied to a variety of other topics, including conglomerate mergers (Etro, 2019; Kadner-Graziano, 2023; Spulber, 2017), vertical foreclosure (Kitamura et al., 2018), sequential bargaining with labor unions (Chongvilaivan et al., 2013), make-or-buy decisions (Sim & Kim, 2021), and downstream entry (Nariu et al., 2021).

Matsushima and Mizuno (2012) and Kopel et al. (2016) only analyze input price discrimination with perfectly complementary inputs. These studies consider two types of suppliers, common and specific. Their extension section shows that a common input supplier may choose uniform pricing endogenously. In contrast, we consider a situation where two common input suppliers can endogenously choose uniform pricing, and obtain the asymmetric pricing regime equilibrium.

The analysis most similar to ours is the patent pool analysis by Li and Shuai (2019). Li and Shuai (2019) show that upstream firms' uniform pricing encourages manufacturers' cost-reducing investment, allowing upstream firms to set higher prices than under discriminatory pricing. Thus, uniform pricing is always the dominant strategy. In contrast, uniform pricing is not always the dominant strategy in our model. Since Li and Shuai (2019) and ours analyze the upstream firms' incentive to choose uniform pricing, our analysis complements Li and Shuai (2019).

The remainder of this paper is organized as follows. Section 2 describes the proposed model. After deriving the equilibrium outcomes in Section 3, we compare these outcomes and show the results for strategic interaction, consumer surplus, social welfare, and policy implications in Section 4. In Section 5, we conclude the paper.

2 Baseline Model

We consider a vertically related market with two monopolistic upstream firms and duopolistic downstream manufacturers. Each monopolistic upstream firm $k = A, B$ produces a perfectly complementary input k and sells it to manufacturer $i = 1, 2$. Manufacturer i produces homogeneous final goods with Leontief production technology (Etro, 2019; Laussel, 2008; Matsushima & Mizuno, 2013). For simplicity, using one unit of each input, manufacturers produce one unit of the final product. We denote the inverse demand function $p = 1 - q_1 - q_2$, where p is the price of the final goods, and q_i is the output of manufacturer i.

Upstream firm k sells the inputs to manufacturer i at an input price w_{ki}. We assume that the marginal cost of upstream firm k is zero. Then, these firms' profits are as follows:

$$\pi_A = w_{A1}q_1 + w_{A2}q_2, \quad \pi_B = w_{B1}q_1 + w_{B2}q_2. \tag{1}$$

Each upstream firm can commit to employing uniform pricing for the input.[1] With this commitment, upstream firm k charges an equal input price w_{kU} to both manufacturers. Without this commitment, it charges w_{k1} to manufacturer 1 and w_{k2} to manufacturer 2. Note that when upstream firm k does not choose uniform pricing, it can charge $\bar{w} = w_{k1} = w_{k2}$ to each downstream firm.

The operating profit of the manufacturer i is $\pi_i = (p - w_{Ai} - w_{Bi})q_i$. We assume that their marginal production cost is zero. We consider that manufacturer 2 owns $r \times 100\%$ of the non-controlling share of firm 1, where r is the degree of horizontal

[1] We can interpret this commitment as a patent holder's declaration that its patent is one of the standard essential patents (SEPs) on, for example, connected cars or autonomous driving technology. Standard-setting organizations generally require SEP holders to offer licenses on fair, reasonable, and non-discriminatory (FRAND) terms (Bourreau et al., 2023). Li and Shuai (2019) and Bourreau et al. (2023) consider this "non-discriminatory" aspect as a uniform pricing commitment (obligation) of the licensee.

shareholding ($0 < r < 1/2$). Then, the total value function for each manufacturer is:

$$V_1 = (1-r)\pi_1, \quad V_2 = r\pi_1 + \pi_2. \tag{2}$$

We assume that the manufacturers compete on quantity to maximize their total values.[2] If r converges to $1/2$, the downstream shareholder (firm 2) has a greater incentive to decrease its quantity to increase the profit of the downstream rival (firm 1). Thus, the downstream competition is alleviated. If r converges to 0, there is no such incentive. Hence, the downstream competition becomes fierce as the standard Cournot competition. We denote consumer surplus and social welfare by $CS \equiv (q_1+q_2)^2/2$ and $SW \equiv CS + \pi_1 + \pi_2 + \pi_A + \pi_B$, respectively.

The timing of the game is as follows: In stage 1, upstream firm k chooses their pricing regime: discriminatory (D) or uniform (U). In stage 2, firm k sets the input price w_{ki}. In stage 3, the downstream firm i chooses its output to maximize its total value. We solve the game using backward induction.

3 Analysis

3.1 Downstream Quantity Competition

First, we derive the outcomes of the third stage. From the first-order conditions, $\partial V_i/\partial q_i = 0$, we obtain the following outputs:

$$\begin{aligned} q_1(w_{A1}, w_{A2}, w_{B1}, w_{B2}) &= \frac{1-c-2w_{A1}-2w_{B1}+w_{A2}+w_{B2}}{3-r}, \\ q_2(w_{A1}, w_{A2}, w_{B1}, w_{B2}) &= \frac{(1-r)(1-c)-2w_{A2}-2w_{B2}+(1+r)(w_{A1}+w_{B1})}{3-r}. \end{aligned} \tag{3}$$

[2] As our interest is in the strategic interaction between monopolistic upstream firms, we treat r as exogenous.

Focusing on $q_1(w_{A1}, w_{A2}, w_{B1}, w_{B2})$ and $q_2(w_{A1}, w_{A2}, w_{B1}, w_{B2})$, we confirm the following two effects. First, horizontal shareholding makes the holder less aggressive in producing. We can find this effect at $(1-r)(1-c)$ in the numerator of $q_2(w_{A1}, w_{A2}, w_{B1}, w_{B2})$. This effect, called *competition effect*, is well-known in the previous literature. Second, if the input price for the holder's rival w_{k1} increases, the shared profit $r\pi_1$ will decrease, and thus the holder will focus on the operating profit π_2, thereby increasing its own quantity. We can confirm this effect at $(1+r)(w_{A1}+w_{B1})$ in the numerator of $q_2(w_{A1}, w_{A2}, w_{B1}, w_{B2})$. This effect, called *production reallocation effect*, is a new effect derived from perfectly complementary inputs.

3.2 Input Price Decision

Based on the decision in the first stage, we have three subgames: (i) both upstream firms perform input price discrimination (case D), (ii) both upstream firms employ uniform pricing (case U), and (iii) one upstream firm takes a uniform price commitment, and the other does not (case M). In the following, we provide outcomes in each subgame.

3.2.1 Case D: Discrimination by Both Upstream Firms

First, we consider the case D in which both upstream firms adopt discriminatory pricing. We obtain the following input price by solving the first-order condition for w_{ki}.

$$\begin{aligned} w_{A1}^D = w_{B1}^D &= \frac{(1-c)(9-2r-r^2)}{27-9r-2r^2}, \\ w_{A2}^D = w_{B2}^D &= \frac{(1-c)(9-4r)}{27-9r-2r^2}, \end{aligned} \tag{4}$$

where the superscript D represents price discrimination by both upstream firms. Thus, the equilibrium profits of the upstream firms, consumer surplus, and social welfare are

$$\begin{aligned} \pi_A^D = \pi_B^D &= \frac{(1-c)^2((2-r))}{27-9r-2r^2}, \\ CS^D &= \frac{9(1-c)^2(2-r)^2)^2}{2(27-9r-2r^2)^2}, \\ SW^D &= \frac{3(1-c)^2(2-r)(48-15r-4r^2)}{2(27-9r-2r^2)^2}, \end{aligned} \tag{5}$$

3.2.2 Case U: No Discrimination

Next, we analyze the case in which each upstream firm k makes a uniform price commitment; we impose conditions $w_{A1} = w_{A2} \equiv w_A$ and $w_{B1} = w_{B2} \equiv w_B$. Substituting $q_i(w_A, w_A, w_B, w_B)$ into π_k and solving the first-order conditions for w_{kU}, we obtain the following input price:

$$w_{kU}^U = \frac{1-c}{3}, k = A, B, \tag{6}$$

where the superscript U represents the non-discriminatory pricing case. Based on these results, we obtain the following outcomes:

$$\begin{aligned} \pi_A^U = \pi_B^U &= \frac{(1-c)^2(2-r)^2}{9(3-r)^2}, \\ CS^U &= \frac{(1-c)^2(2-r)^2}{18(3-r)^2}, \\ SW^U &= \frac{(1-c)^2(2-r)(16-5r)}{18(3-r)^2}. \end{aligned} \tag{7}$$

3.2.3 Case M: Partial Discrimination

Finally, we consider the case in which firm $k = A, B$ chooses discriminatory pricing, and firm $l = A, B$ $(l \neq k)$ commits to choosing uniform pricing in stage 1. By solving the first-order conditions for w_{ki} and w_{lU}, we obtain the following input price:

$$\begin{aligned} w_{k1}^M &= \frac{(1-c)(2-r)(3+r)}{18-6r-r^2}, \\ w_{k2}^M &= \frac{3(1-c)(2-r))}{18-6r-r^2}, \\ w_{lU}^M &= \frac{2(1-c)(3-r)}{18-6r-r^2}, \quad k, l = A, B, k \neq l, \end{aligned} \tag{8}$$

where the superscript M represents the case of partial discrimination. The equilibrium profits of the upstream firms, consumer surplus, and social welfare are

$$\begin{aligned} \pi_k^M &= \frac{(1-c)^2(2-r)^2(6+r)}{(18-6r-r^2)^2}, \\ \pi_l^M &= \frac{4(1-c)^2(2-r)(3-r)}{(18-6r-r^2)^2}, \\ CS^M &= \frac{2(1-c)^2(2-r)^2}{(18-6r-r^2)^2}, \\ SW^M &= \frac{2(1-c)^2(2-r)(16-5r-r^2)}{(18-6r-r^2)^2} \end{aligned} \tag{9}$$

4 Main Results

We compare the equilibrium outcomes and analyze the strategy sets realized in Stage 1. All proofs are relegated to the Appendix.

4.1 Effect of Uniform Pricing

Comparing the average input prices, we obtain the following Lemma:

Lemma 1. *When a downstream firm holds its rival's shares, if an upstream firm switches the pricing regime from discriminatory to uniform, its average input price will increase:*

$$w_{lU}^M > \frac{w_{k1}^D + w_{k2}^D}{2}, \quad w_{kU}^U > \frac{w_{k1}^M + w_{k2}^M}{2}.$$

Proof. See Appendix.

The intuition for the inequality in Lemma 1 is as follows. Since the holder's rival (firm 1) is more aggressive than the holder (firm 2), upstream firm k with discriminatory pricing sets the higher input price for the holder's rival. Due to *production reallocation effect* $(1+r)(w_{A1}+w_{B1})$ in the numerator of $q_2(w_{A1}, w_{A2}, w_{B1}, w_{B2})$, if the input price for the holder's rival w_{k1} increases, the holder focuses on its operating profit π_2, thereby increasing its quantity q_2. Thus, *production reallocation effect* prevents upstream firm k from increasing w_{k1}. Conversely, *production reallocation effect* does not directly influence w_{k2}; thus, adjusting w_{k2} is still neutral for upstream firm k. Therefore, the average input price under uniform pricing is higher than under discriminatory pricing.[3]

4.2 Equilibrium Pricing Regime

Here, we discuss the equilibrium pricing in stage 1. Comparing the profits of the upstream firms in each case, we obtain the following result:

Proposition 1. *When a downstream firm holds its rival's shares (for any $r > 0$), one upstream firm chooses uniform pricing, and the other chooses discriminatory pricing in equilibrium.*

[3]Note that at $r = 0$, upstream firm k's average input price is equivalent in the discriminatory pricing and uniform pricing.

Proof. See Appendix.

This proposition suggests that when horizontal shareholding exists, the asymmetric equilibrium of the pricing regime is always realized. Sacrificing pricing flexibility, one of the symmetric upstream firms chooses uniform pricing. This result contrasts with Li & Shuai (2019), where all upstream firms commit to choosing uniform pricing in equilibrium.

An intuition behind this result is as follows. Lemma 1 implies that switching from discriminatory pricing to uniform pricing increases the switcher's average input price. Furthermore, input complementarity decreases the other's average input price. Thus, case D is not an equilibrium outcome.

In our model, uniform pricing by both upstream firms raises the input prices too much; if both firms choose uniform pricing, their profits will be lower than those in the asymmetric pricing equilibrium. Therefore, case U is not an equilibrium, and case M is always realized in equilibrium.

4.3 Welfare

Finally, we summarize the results of the welfare analysis as follows:

Proposition 2. *When a downstream firm holds its rival's shares, case D is the first-best for the consumer and society. (Formally, $CS^D > CS^M > CS^U$ and $SW^D > SW^M > SW^U$).*

Proof. See Appendix.

This result suggests that the self-regulatory uniform pricing harms consumers and society. Intuitively, as Lemma 1 suggested, switching to uniform pricing increases the average input price. Thus, this switching increases manufacturers' marginal cost, and reduces consumer surplus and social welfare. This is in stark contrast to Li & Shuai (2019): Self-regulatory uniform pricing always benefits consumers and society.

5 Extension: Asymmetric Downstream costs

Hereafter, we analyze the model with asymmetric marginal costs of downstream manufacturers. In this model, the manufacturers' costs consist of production costs and input payment costs. We assume that the marginal production costs of the manufacturers 1 and 2 are c and $c + c_2$, respectively. We allow c_2 to take a negative value; if $c_2 < 0$, downstream firm 2 is more efficient than downstream firm 1. Then, the operating profits of the manufacturers 1 and 2 are

$$\pi_1 = (p - c - w_{A1} - w_{B1})q_1, \quad \pi_2 = (p - c - c_2 - w_{A2} - w_{B2})q_2.$$

First, we derive the outcomes of the third stage. From the first-order conditions, $\partial V_i/\partial q_i = 0$, we obtain the following outputs:

$$q_1(w_{A1}, w_{A2}, w_{B1}, w_{B2}) = \frac{1 - c + c_2 - 2w_{A1} - 2w_{B1} + w_{A2} + w_{B2}}{3 - r},$$
$$q_2(w_{A1}, w_{A2}, w_{B1}, w_{B2}) = \frac{(1 - r)(1 - c) - 2c_2 - 2w_{A2} - 2w_{B2} + (1 + r)(w_{A1} + w_{B1})}{3 - r}.$$

One can confirm that as the baseline model, this extension model also has the following two effects: *competition effect* $(1 - r)(1 - c)$ and *production reallocation effect* $(1 + r)(w_{A1} + w_{B1})$ in the numerator of $q_2(w_{A1}, w_{A2}, w_{B1}, w_{B2})$.

We omit the remaining derivation of the equilibrium outcomes and summarize them in Table 1. For simplicity, we denote $Z \equiv c_2/(1 - c)$, and we relegate the values of Ω in SW to the appendix.

	case D	case M	case U
w_{k1}	$\frac{(1-c)(9-2r-r^2-2rZ)}{27-9r-2r^2}$	$\frac{(1-c)(12-8r-r^2+r^3+(3-8r+3r^2)Z)}{(2-r)(18-6r+r^2)}$	$N.A.$
w_{k2}	$\frac{(1-c)(9-4r-9Z+2rZ)}{27-9r-2r^2}$	$\frac{(1-c)(12-12r+3r^2-(15-9r+r^2))Z}{(2-r)(18-6r+r^2)}$	$N.A.$
w_U	$N.A.$	$\frac{(1-c)(3-r)(2-Z)}{(18-6r+r^2)}$	$\frac{(1-c)(2-r-Z)}{3(2-r)}$
π_k	$\frac{(1-c)^2((2-r)(1-Z)+2Z^2)}{27-9r-2r^2}$	$\frac{(1-c)^2(2-r)(3-r)(2-Z)^2}{(18-6r-r^2)^2}$	$N.A.$
π_U	$N.A.$	$\frac{(1-c)^2(2-r)^3(6+r)(1-Z)+(93-70r+6r^2-3r^3)Z^2}{(2-r)(18-6r-r^2)^2}$	$\frac{(1-c)^2(2-r-Z)^2}{9(3-r)^2(2-r)^2}$
CS	$\frac{(1-c)^2(6-3r-(3-2r)Z)^2}{2(27-9r-2r^2)^2}$	$\frac{(1-c)^2(2-r)^2(2-Z)^2}{2(18-6r-r^2)^2}$	$\frac{(1-c)^2(2-r-Z)^2}{18(3-r)^2}$
SW	$\frac{(1-c)^2(\Omega_{D0}-\Omega_{D1}Z+\Omega_{D2}Z^2)}{2(27-9r-2r^2)^2}$	$\frac{(1-c)^2(\Omega_{M0}-\Omega_{M1}Z+\Omega_{M2}Z^2)}{2(2-r)^2(18-6r-r^2)^2}$	$\frac{(1-c)^2(\Omega_{U0}-\Omega_{U1}Z+\Omega_{U2}Z^2)}{18(3-r)^2(2-r)^2}$

Table 1: Equilibrium Outcomes with Asymmetric Downstream Costs

5.1 Effect of Uniform Pricing

From Table 1, we rank the average input price set by the upstream firms in each case as follows.

Lemma 2. *When a downstream firm holds its rival's shares and downstream firms have different marginal costs, if upstream firm k switches the pricing regime from discriminatory to uniform, its average input price may decrease. Formally,*

(i) $w_{lU}^M \geq \frac{w_{k1}^D+w_{k2}^D}{2}, \quad w_{kU}^U \geq \frac{w_{k1}^M+w_{k2}^M}{2}$, *if* $Z \geq \hat{Z}$,

(ii) $w_{lU}^M < \frac{w_{k1}^D+w_{k2}^D}{2}, \quad w_{kU}^U < \frac{w_{k1}^M+w_{k2}^M}{2}$, *if* $Z < \hat{Z}$,

where $\hat{Z} \equiv -\frac{(2-r)r}{9-4r}(< 0)$.

Proof. See Appendix.

The key to this result is the relative aggressiveness of the downstream firms. In the baseline model with symmetric downstream marginal cost, the holder's rival (firm 1) is always more aggressive than the holder (firm 2). In the extension model, the relative aggressiveness depends on the ownership structure and downstream marginal costs. If Z is non-negative, the holder's rival is more efficient and aggressive than the downstream holder. In contrast, if Z is negative and sufficiently small, the downstream holder becomes more aggressive.

Intuitively, if Z is large, *production reallocation effect* is undesirable for the upstream firm with discriminatory pricing. Thus, the intuition of (i) is the same as the model with symmetric downstream marginal cost. Conversely, if Z is negative and sufficiently small, *production reallocation effect* becomes a positive effect for upstream firm k. Thus, upstream firm k can easily increase the input price for the holder's rival w_{k1}. Therefore, the average input price of uniform pricing is lower than that of discriminatory pricing, which leads to the intuition of (ii).

Next, we focus on the magnitude of the input price increase due to the switch to uniform pricing. It can be presented as follows;

Lemma 3. *When a downstream firm holds its rival's shares and the holder is not sufficiently efficient ($Z > \hat{Z}$), the less efficient the holder is, the greater the difference in the average input prices between uniform and discriminatory pricing.*

Proof. See Appendix.

This lemma suggests that when $Z > \hat{Z}$, as c_2 increases, the average input price of uniform pricing is much higher than that of discriminatory pricing. The relative aggressiveness of downstream firms explains this result straightforwardly. As Z increases, the holder's rival becomes much more efficient and aggressive than the downstream holder, and *production reallocation effect* becomes more undesirable for the upstream firm with discriminatory pricing.

5.2 Equilibrium Pricing Regime

Here, we discuss the equilibrium outcomes in stage 1. By comparing the profits of the upstream firms in each case, we obtain the following result:

Proposition 3. *When a downstream firm holds its rival's shares, one or both upstream firms choose uniform pricing in equilibrium depending on the efficiency difference between the downstream firms. Formally, each equilibrium is as follows:*

(i) If $\bar{Z} > Z > A_1$, both upstream firms choose discriminatory pricing;

(ii) If $A_1 > Z > A_2$, one upstream firm chooses discriminatory pricing and the other chooses uniform pricing;

(iii) If $A_2 > Z > \hat{Z}$, both upstream firms choose uniform pricing;

(iv) If $\hat{Z} > Z > \underline{Z}$, both upstream firms choose discriminatory pricing.

where $A_1 \equiv \frac{(2-r)r}{9-2r-r^2}(> 0)$, $A_2 \equiv -\frac{(2-r)r(9-3r-r^2)}{243-171r+12r^2+7r^3}(< 0)$, $\bar{Z} = \frac{\bar{c_2}}{(1-c)}$, *and* $\underline{Z} = \frac{c_2}{(1-c)}$

Proof. See Appendix.

This result suggests that every pair of pricing regimes can be realized in the model of asymmetric downstream marginal costs. That is, this model explains the practices. We can confirm this result from Figure 1 which illustrates how the degree of horizontal shareholding (r) affects the threshold value in Proposition 3. The vertical and horizontal axes are $Z = c_2/(1 - c)$ and r, respectively. It can be seen that at $r = 0$, the threshold values are the same.[4]

[4] We also confirm that when the horizontal shareholding is like acquisition ($0.492 < r$), the production condition of the downstream rival (firm 1) in no discrimination case ($\bar{Z}$) is lower than A_1. Note that upstream firms have no incentive to monopolize the downstream market because this only reduces input demand.

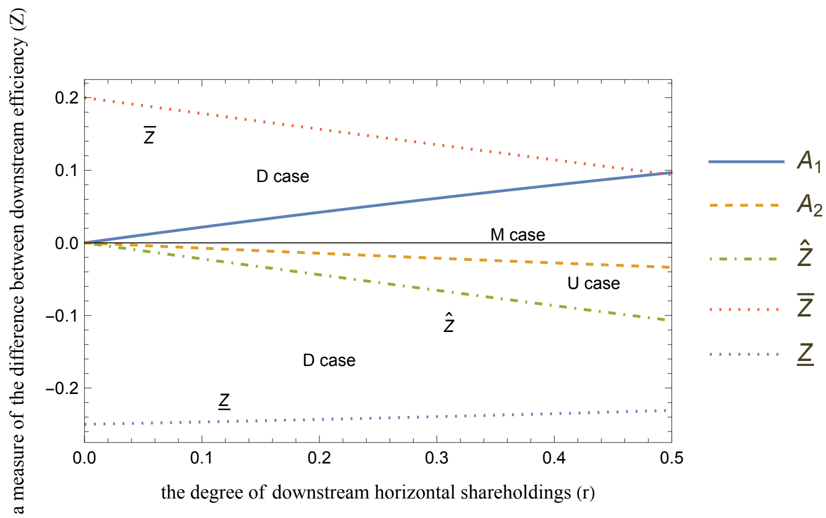

Figure 1: Realized Equilibria and Production Condition (: Proposition 3)

We explain an intuition by Lemma 2 and 3. Lemma 2 suggests that when $r > 0$ and $Z < \hat{Z}$, the average input price of uniform pricing is lower than that of discriminatory pricing. Thus, both upstream firms choose discriminatory pricing in equilibrium. Lemma 3 suggests that when Z is larger than and close to $\hat{Z}$, uniform pricing slightly increases the average input price, which relaxes upstream competition. Thus, both upstream firms choose uniform pricing in equilibrium. As Z increases, uniform pricing increases the average input price enough that either upstream firm chooses uniform pricing in equilibrium. When Z is large, uniform pricing significantly increases the average input price. Therefore, no upstream firm chooses uniform pricing in equilibrium.

5.3 Welfare

Finally, we summarize the results of welfare analysis and derive the policy implications of a ban on input price discrimination. From Table 1, the ranking of consumer surplus is as follows:

Proposition 4. *When horizontal shareholding does not exist, the consumer surplus is the same in each case. When it does exist, the ranking of the consumer surplus has the following three patterns:*

(i) $CS^D > CS^M > CS^U$, *if* $\bar{Z} > Z > \hat{Z}$,

(ii) $CS^D = CS^M = CS^U$, *if* $Z = \hat{Z}$,

(iii) $CS^U > CS^M > CS^D$, *if* $\hat{Z} > Z > \underline{Z}$.

Proof. See Appendix.

This result suggests that the self-regulatory uniform pricing also harms consumers in the extension model. Proposition 3 shows that when $A_1 > Z > \hat{Z}$, one or both upstream firms choose the self-regulatory uniform pricing. However, according to Proposition 4, when $A_1 > Z > \hat{Z}$, no discriminatory case is the worst one for consumers. Therefore, self-regulatory uniform pricing always undermines the consumer surplus.

We provide an intuition of Proposition 4. As Lemma 2 suggested, switching to uniform pricing has an ambiguous effect on the average input price, depending on the difference between downstream firms' efficiency. Thus, if this switching increases (decreases) the average input price, this also increases (decreases) manufacturers' marginal cost, and decreases (increases) consumer surplus, respectively.

Then, the ranking of social welfare can be presented as follows;

Proposition 5. *When horizontal shareholding does not exist, social welfare remains the same in all cases. When it does exist, the ranking of social welfare has the following six patterns:*

(i) $SW^U > SW^M > SW^D$, *if* $\bar{Z} > Z > B_1$,

(ii) $SW^U > SW^D \geq SW^M$, *if* $B_1 \geq Z > B_2$,

(iii) $SW^D \geq SW^U > SW^M$, *if* $B_2 \geq Z > B_3$,

(iv) $SW^D > SW^M \geq SW^U$, *if* $B_3 \geq Z > \hat{Z}$,

(v) $SW^D = SW^M = SW^U$, *if* $Z = \hat{Z}$,

(vi) $SW^U > SW^M > SW^D$, *if* $\hat{Z} > Z > \underline{Z}$,

where

$$B_1 \equiv -\frac{(2-r)r(432-954r+656r^2-150r^3-5r^4+4r^5)}{17496-19386r+4608r^2+826r^3+53r^4-172r^5+6r^6+4r^7} \ (<0),$$
$$B_2 \equiv -\frac{(2-r)r(108-117r+59r^2-24r^3+5r^4)}{4374-3753r-90r^2+685r^3-66r^4-41r^5+6r^6} \ (<0),$$
$$B_3 \equiv -\frac{(2-r)r(144-102r+32r^2-18r^3+5r^4)}{5832-4518r-672r^2+1198r^3-189r^4-32r^5+6r^6} \ (<0).$$

Proof. See Appendix.

Figure 2 illustrates how the degree of horizontal shareholding (r) affects the threshold values in Proposition 5. As before, we consider the vertical and horizontal axes to be $Z = c_2/(1-c)$ and r, respectively. We can confirm that case (ii) and (iii) are very narrow. We can also confirm that when both upstream firms chooses discriminatory pricing in equilibrium (Figure 1), no discriminatory case is the most desirable for society (Figure 2)

Partial discrimination cases never become the first-best outcome for consumers and society. In these cases, when uniform pricing is a higher price commitment ($Z > \hat{Z}$), one of the upstream firms can expropriate most of the social welfare. By

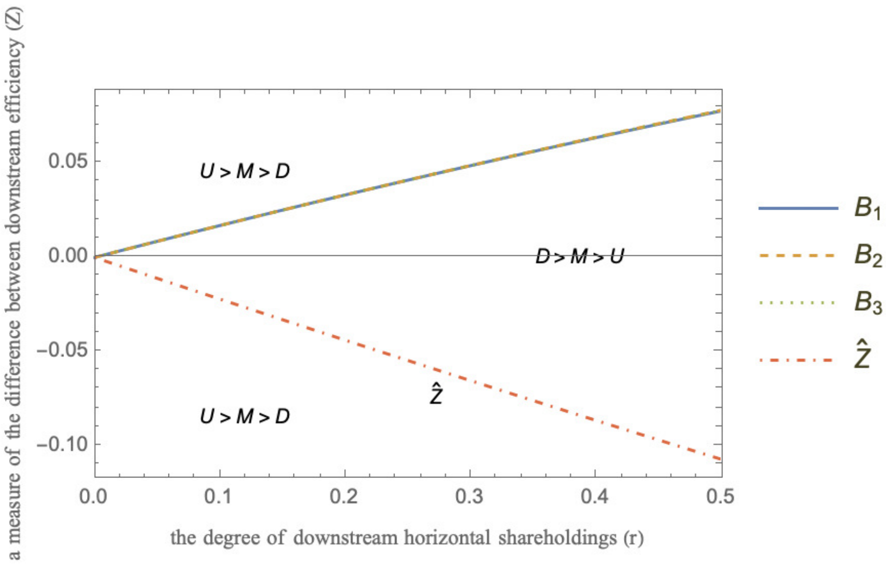

Figure 2: The Ranking of Social Welfare (: Proposition 5)

contrast, when uniform pricing is a lower price commitment ($Z < \hat{Z}$), the increase in demand through partially uniform pricing is always smaller than that in the case of uniform pricing by both upstream firms.

Finally, we summarize two policy implications. First, when uniform pricing is a *lower price commitment* ($Z < \hat{Z}$), a ban on input price discrimination is desirable for both consumers and society. This result is consistent with those of Hu et al. (2022) and Li and Shuai (2022). However, our analysis has implications for how strictly policymakers should implement this ban. Since the optimal strategy for upstream firms in this situation is discriminatory pricing, policymakers should strictly ban it. Therefore, if policymakers want to ban input price discrimination seriously, the current regulations may be too lax.

Second, we obtain a new policy implication: when both upstream firms choose discriminatory pricing ($\bar{Z} > Z > A_1$ or $\hat{Z} > Z > \underline{Z}$), policymakers should ban on input price discrimination for social welfare. We confirm this from the third property of the result that the change in the worst case in Proposition 3 from the case (i) to (vi) corresponds to the change in the realized equilibrium in Proposition 1 from the case (i) to (iv). Intuitively, a ban on input price discrimination may suppress multiple upstream exploitations of surplus. This result also suggests that policymakers may decide whether to ban input price discrimination based only on the pricing of the upstream firms. In other words, they may be able to decide this without estimating costs or demand.

6 Conclusion

The literature on input price discrimination typically focuses on single-input situations. To shed light on the strategic desirability of uniform pricing, we build a model based on perfectly complementary inputs. Using a linear inverse demand function under downstream asymmetries of efficiency and horizontal ownership structure, we find that uniform pricing may render the total input price higher than discriminatory pricing. Thus, uniform pricing may become the optimal strategy for one or both upstream firms. In addition, we find that this self-regulatory uniform pricing always undermines the consumer surplus. This analysis first demonstrates the anticompetitive effect of voluntary compliance with a ban on price discrimination, such as the Robinson-Patman Act in the US and Article 102(c) of the Treaty on the Functioning of the European Union.

Furthermore, we provide a new policy implication of a ban on input price discrimination: when both two upstream firms engage in discriminatory pricing, it is desirable for the society to enforce a ban on input price discrimination because a ban on input price discrimination can suppress multiple upstream exploitations of the surplus. This result suggests that policymakers may be able to judge whether to ban input price discrimination based only on the pricing of the upstream firms rather than on the manufacturers' marginal costs, which are difficult to estimate.

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Appendix

Proof

In this section, when we consider the partial discrimination case (case M), we denote l as the upstream firm that chooses discriminatory pricing, and k as the one that chooses uniform pricing.

Proof of Lemma 1. By comparing (4), (6), and (8) for any $k, l = \{A, B\}, k \neq l$, we have

$$2w_{kU}^{M} - (w_{k1}^{D} + w_{k2}^{D}) = \frac{(1-c)(6+r)(2-r)r^2}{(18-6r-r^2)(27-9r-2r^2)} > 0,$$

$$2w_{kU}^{U} - (w_{l1}^{M} + w_{l2}^{M}) = \frac{(1-c)r^2}{54+18r+3r^2} > 0.$$

□

Proof of Proposition 1. To analyze the incentive to deviate, we calculate the upstream firm's best response to the other firm's pricing regime. First, we investigate the incentive to change its pricing regime and deviate from case D to case M:

$$\pi_l^M - \pi_k^D = \frac{(1-c)^2(2-r)^2(6+r)r^2}{(18-6r-r^2)^2(27-9r-2r^2)} \geq 0.$$

Next, we investigate the incentive to change its pricing regime and deviate from case U to case M:

$$\pi_k^M - \pi_l^U = \frac{\left[\begin{array}{c}(1-c)^2((2-r)r+(9-4r)Z) \\ (18r-15r^2+r^3+r^4+(243-171r+12r^2+7r^3)Z)\end{array}\right]}{9(3-r)(18-6r-r^2)^2} \leq 0.$$

Therefore, we obtain the following results.

When a downstream firm holds its rival's shares (for any $r > 0$), one upstream firm chooses uniform pricing, and the other chooses discriminatory pricing in equilibrium.

(i) If $\bar{Z} > Z > A_1$, case of price discrimination by both upstream firms is realized,

(ii) If $A_1 > Z > A_2$, case of a partial price discrimination is realized,

(iii) If $A_2 > Z > \hat{Z}$, case of uniform pricing is realized,

(iv) If $\hat{Z} > Z > \underline{Z}$, case of price discrimination by both upstream firms is realized,

where $\bar{Z} = \frac{\bar{c_2}}{(1-c)}$, and $\underline{Z} = \frac{c_2}{(1-c)}$

$\square$

Proof of Proposition 2. Comparing the consumer surplus in the cases of price discrimination by both upstream firms and partial discrimination, we have

$$CS^D - CS^M = \frac{\begin{bmatrix} (1-c)^2 r((2-r)r + (9-4r)Z) \\ (216 - 180r + 22r^2 + 7r^3 - (108 - 99r + 14r^2 + 4r^3)Z) \end{bmatrix}}{2(18 - 6r - r^2)^2(27 - 9r - 2r^2)^2} > 0.$$

Solving this, we obtain $Z > \hat{Z}$ under the assumption that both manufacturers produce.

Comparing the consumer surplus in the cases of partial discrimination and no discrimination, we have

$$CS^M - CS^U = \frac{\begin{bmatrix} (1-c)^2 r((2-r)r + (9-4r)Z) \\ (72 - 60r + 10r^2 + r^3 - (36 - 21r + 2r^2)Z) \end{bmatrix}}{18(3-r)^2(18 - 6r - r^2)^2} > 0.$$

Solving this, we obtain $Z > \hat{Z}$ under the assumption that both manufacturers produce.

Thus, we obtain the following results.

(i) $CS^D > CS^M > CS^U$, if $\bar{Z} > Z > \hat{Z}$,

(ii) $CS^D = CS^M = CS^U$, if $Z = \hat{Z}$,

(iii) $CS^U > CS^M > CS^D$, if $\hat{Z} > Z > \underline{Z}$.

First, we compare the cases of price discrimination by both upstream firms and partial discrimination.

$$SW^M - SW^D = \frac{\left[\begin{array}{c}(1-c)^2(2r - r^2 + (9-4r)Z) \\ (864r - 2340r^2 + 2266r^3 - 956r^4 + 140r^5 + 13r^6 - 4r^7 \\ +(17496 - 19386r + 4608r^2 + 826r^3 + 53r^4 - 172r^5 + 6r^6 + 4r^7)Z)\end{array}\right]}{(2(2-r)^2(18 - r(6+r))^2(27 - r(9+2r))^2)} \geq 0.$$

Solving this, we obtain $-\frac{(2-r)r(432-954r+656r^2-150r^3-5r^4+4r^5)}{17496-19386r+4608r^2+826r^3+53r^4-172r^5+6r^6+4r^7} (\equiv B_1) \geq c_2 \geq \hat{Z}$.

Next, we compare the cases of price discrimination by both upstream firms and of no discrimination.

$$SW^U - SW^D = \frac{\left[\begin{array}{c}2(1-c)^2(2r - r^2 + (9-4r)Z) \\ (216r - 342r^2 + 235r^3 - 107r^4 + 34r^5 - 5r^6 \\ +(4374 - 3753r - 90r^2 + 685r^3 - 66r^4 - 41r^5 + 6r^6)Z)\end{array}\right]}{9(3-r)^2(2-r)^2(27 - r(9+2r))^2} \geq 0.$$

Solving this, we obtain $-\frac{(2-r)r(108-117r+59r^2-24r^3+5r^4)}{4374-3753r-90r^2+685r^3-66r^4-41r^5+6r^6} (\equiv B_2) \geq c_2 \geq \hat{Z}$.

Finally, we compare the no discrimination and partial discrimination cases.

$$SW^U - SW^M = \frac{\begin{bmatrix} (1-c)^2(2r-r^2-(9-4r)Z) \\ (288r-348r^2+166r^3-68r^4+28r^5-5r^6 \\ +(5832-4518r-672r^2+1198r^3-189r^4-32r^5+6r^6)Z) \end{bmatrix}}{18(3-r)^2(2-r)^2(18-r(6+r))^2} \geq 0.$$

Solving this, we obtain $-\frac{(2-r)r(144-102r+32r^2-18r^3+5r^4)}{5832-4518r-672r^2+1198r^3-189r^4-32r^5+6r^6}(\equiv B_3) \geq c_2 \geq \hat{Z}$.

As $B_1 \geq B_2 \geq B_3 \geq \hat{Z}$, we obtain the following results;

(i) $SW^U > SW^M > SW^D$, if $\bar{Z} > Z > B_1$,

(ii) $SW^U > SW^D \geq SW^M$, if $B_1 \geq Z > B_2$,

(iii) $SW^D \geq SW^U > SW^M$, if $B_2 \geq Z > B_3$,

(iv) $SW^D > SW^M \geq SW^U$, if $B_3 \geq Z > \hat{Z}$,

(v) $SW^D = SW^M = SW^U$, if $Z = \hat{Z}$,

(vi) $SW^U > SW^M > SW^D$, if $\hat{Z} > Z > \underline{Z}$,

where

$$B_1 \equiv -\frac{(2-r)r(432-954r+656r^2-150r^3-5r^4+4r^5)}{17496-19386r+4608r^2+826r^3+53r^4-172r^5+6r^6+4r^7} \ (<0),$$
$$B_2 \equiv -\frac{(2-r)r(108-117r+59r^2-24r^3+5r^4)}{4374-3753r-90r^2+685r^3-66r^4-41r^5+6r^6} \ (<0),$$
$$B_3 \equiv -\frac{(2-r)r(144-102r+32r^2-18r^3+5r^4)}{5832-4518r-672r^2+1198r^3-189r^4-32r^5+6r^6} \ (<0).$$

□

Proof of Lemma 2. From Table 1, comparing the input prices in equilibrium, we have

$$2w^M_{kU} - (w^D_{k1} + w^D_{k2}) = \frac{(1-c)r(6+r)((2-r)r + (9-4r)Z)}{(18-6r-r^2)(27-9r-2r^2)} > 0 \quad \text{if} \quad Z \geq \hat{Z}, \tag{10}$$

$$2w^U_{kU} - (w^M_{l1} + w^M_{l2}) = \frac{(1-c)r((2-r)r + (9-4r)Z)}{3(2-r)(18-6r-r^2)} > 0 \quad \text{if} \quad Z \geq \hat{Z}, \tag{11}$$

where $\hat{Z} = -(2-r)r/(9-4r) < 0$. Note that in case M, k is the upstream firm that chooses uniform pricing, and l is the firm that chooses discriminatory pricing.

□

Proof of Lemma 3. Differenciating (10) and (11) with c_2, we obtain

$$\frac{\partial(2w^M_{kU} - (w^D_{k1} + w^D_{k2}))}{\partial c_2} = \frac{\partial(2w^M_{kU} - (w^D_{k1} + w^D_{k2}))}{\partial Z} = \frac{(9-4r)r}{3(2-r)(18-r(6+r))} > 0,$$

$$\frac{\partial(2w^U_{kU} - (w^M_{k1} + w^M_{k2}))}{\partial c_2} = \frac{\partial(2w^U_{kU} - (w^M_{k1} + w^M_{k2}))}{\partial Z} = \frac{(9-4r)r(6+r)}{(18-r(6+r))(27-r(9+2r))} > 0.$$

Thus, as c_2 increases, the difference of the average input price between uniform and discriminatory pricing also increases.

□

Proof of Proposition 3. To analyze the incentive to deviate, we calculate the best response to whether the other input supplier makes a uniform price commitment. If we consider the partial discrimination case, firm $l = A, B$ commits to choosing a uniform price, and firm $k = A, B (k \neq l)$ commits to choosing a discriminatory price in stage 1.

First, we investigate the incentive to deviate from discrimination by both upstream firms to partial discrimination.

$$\pi_l^M - \pi_l^D = \frac{\left[\begin{array}{c}(1-c)^2((2-r)r+(9-4r)Z) \\ (6+r)((2-r)r-(9-2r-r^2)Z)\end{array}\right]}{(18-6r-r^2)^2(27-9r-2r^2)} \geq 0.$$

Solving this, we obtain $\frac{(2-r)r}{9-2r-r^2)}(\equiv A_1) \geq c_2 \geq -\frac{(2-r)r}{9-4r}(\equiv \hat{Z})$.

Next, we investigate the incentive to deviate from no discrimination to partial discrimination.

$$\pi_k^M - \pi_k^U = \frac{\left[\begin{array}{c}(1-c)^2((2-r)r+(9-4r)Z) \\ (18r-15r^2+r^3+r^4+(243-171r+12r^2+7r^3)Z)\end{array}\right]}{9(3-r)(2-r)(18-6r-r^2)^2} \geq 0.$$

Solving it, we obtain $-\frac{(2-r)r(9-3r-r^2)}{243-171r+12r^2+7r^3}(\equiv A_2) \geq c_2 \geq \hat{Z}$.

Because $A_1 \geq A_2 \geq \hat{Z}$, we obtain the following results.

(i) If $\bar{Z} > Z > A_1$, case of price discrimination by both upstream firms is realized,

(ii) If $A_1 > Z > A_2$, case of a partial price discrimination is realized,

(iii) If $A_2 > Z > \hat{Z}$, case of uniform pricing is realized,

(iv) If $\hat{Z} > Z > \underline{Z}$, case of price discrimination by both upstream firms is realized,

where $\bar{Z} = \frac{\bar{c_2}}{(1-c)}$, and $\underline{Z} = \frac{\underline{c_2}}{(1-c)}$

□

Proof of Proposition 4. Comparing the consumer surplus in the cases of price discrimination by both upstream firms and partial discrimination, we have

$$CS^D - CS^M = \frac{\begin{bmatrix} (1-c)^2 r((2-r)r + (9-4r)Z) \\ (216 - 180r + 22r^2 + 7r^3 - (108 - 99r + 14r^2 + 4r^3)Z) \end{bmatrix}}{2(18 - 6r - r^2)^2(27 - 9r - 2r^2)^2} > 0.$$

Solving this, we obtain $Z > \hat{Z}$ under the assumption that both manufacturers produce.

Comparing the consumer surplus in the cases of partial discrimination and no discrimination, we have

$$CS^M - CS^U = \frac{\begin{bmatrix} (1-c)^2 r((2-r)r + (9-4r)Z) \\ (72 - 60r + 10r^2 + r^3 - (36 - 21r + 2r^2)Z) \end{bmatrix}}{18(3-r)^2(18 - 6r - r^2)^2} > 0.$$

Solving this, we obtain $Z > \hat{Z}$ under the assumption that both manufacturers produce.

Thus, we obtain the following results.

(i) $CS^D > CS^M > CS^U$, if $\bar{Z} > Z > \hat{Z}$,

(ii) $CS^D = CS^M = CS^U$, if $Z = \hat{Z}$,

(iii) $CS^U > CS^M > CS^D$, if $\hat{Z} > Z > \underline{Z}$.

$\square$

Proof of Proposition 5. First, we compare the cases of price discrimination by both upstream firms and partial discrimination.

$$SW^M - SW^D = \frac{\left[\begin{array}{c} (1-c)^2(2r - r^2 + (9-4r)Z) \\ (864r - 2340r^2 + 2266r^3 - 956r^4 + 140r^5 + 13r^6 - 4r^7 \\ +(17496 - 19386r + 4608r^2 + 826r^3 + 53r^4 - 172r^5 + 6r^6 + 4r^7)Z) \end{array}\right]}{(2(2-r)^2(18 - r(6+r))^2(27 - r(9+2r))^2)} \geq 0.$$

Solving this, we obtain $-\frac{(2-r)r(432-954r+656r^2-150r^3-5r^4+4r^5)}{17496-19386r+4608r^2+826r^3+53r^4-172r^5+6r^6+4r^7}(\equiv B_1) \geq c_2 \geq \hat{Z}$.

Next, we compare the cases of price discrimination by both upstream firms and of no discrimination.

$$SW^U - SW^D = \frac{\left[\begin{array}{c} 2(1-c)^2(2r - r^2 + (9-4r)Z) \\ (216r - 342r^2 + 235r^3 - 107r^4 + 34r^5 - 5r^6 \\ +(4374 - 3753r - 90r^2 + 685r^3 - 66r^4 - 41r^5 + 6r^6)Z) \end{array}\right]}{9(3-r)^2(2-r)^2(27 - r(9+2r))^2} \geq 0.$$

Solving this, we obtain $-\frac{(2-r)r(108-117r+59r^2-24r^3+5r^4)}{4374-3753r-90r^2+685r^3-66r^4-41r^5+6r^6}(\equiv B_2) \geq c_2 \geq \hat{Z}$.

Finally, we compare the no discrimination and partial discrimination cases.

$$SW^U - SW^M = \frac{\left[\begin{array}{c} (1-c)^2(2r - r^2 - (9-4r)Z) \\ (288r - 348r^2 + 166r^3 - 68r^4 + 28r^5 - 5r^6 \\ +(5832 - 4518r - 672r^2 + 1198r^3 - 189r^4 - 32r^5 + 6r^6)Z) \end{array}\right]}{18(3-r)^2(2-r)^2(18 - r(6+r))^2} \geq 0.$$

Solving this, we obtain $-\frac{(2-r)r(144-102r+32r^2-18r^3+5r^4)}{5832-4518r-672r^2+1198r^3-189r^4-32r^5+6r^6}(\equiv B_3) \geq c_2 \geq \hat{Z}$.

As $B_1 \geq B_2 \geq B_3 \geq \hat{Z}$, we obtain the following results;

(i) $SW^U > SW^M > SW^D$, if $\bar{Z} > Z > B_1$,

(ii) $SW^U > SW^D \geq SW^M$, if $B_1 \geq Z > B_2$,

(iii) $SW^D \geq SW^U > SW^M$, if $B_2 \geq Z > B_3$,

(iv) $SW^D > SW^M \geq SW^U$, if $B_3 \geq Z > \hat{Z}$,

(v) $SW^D = SW^M = SW^U$, if $Z = \hat{Z}$,

(vi) $SW^U > SW^M > SW^D$, if $\hat{Z} > Z > \underline{Z}$,

where

$$B_1 \equiv -\frac{(2-r)r(432 - 954r + 656r^2 - 150r^3 - 5r^4 + 4r^5)}{17496 - 19386r + 4608r^2 + 826r^3 + 53r^4 - 172r^5 + 6r^6 + 4r^7} \ (< 0),$$
$$B_2 \equiv -\frac{(2-r)r(108 - 117r + 59r^2 - 24r^3 + 5r^4)}{4374 - 3753r - 90r^2 + 685r^3 - 66r^4 - 41r^5 + 6r^6} \ (< 0),$$
$$B_3 \equiv -\frac{(2-r)r(144 - 102r + 32r^2 - 18r^3 + 5r^4)}{5832 - 4518r - 672r^2 + 1198r^3 - 189r^4 - 32r^5 + 6r^6} \ (< 0).$$

□

The Value of Ω

$$\Omega_{D0} \equiv 288 - 234r + 21r^2 + 12r^3 (> 0),$$

$$\Omega_{D1} \equiv 288 - 228r + 18r^2 + 12r^3 (> 0),$$

$$\Omega_{D2} \equiv 315 - 96r - 28r^2 (> 0).$$

$$\Omega_{M0} \equiv 256 - 336r + 128r^2 - 4r^3 - 4r^4 (> 0),$$

$$\Omega_{M1} \equiv 256 - 336r + 128r^2 - 4r^3 - 4r^4 (> 0),$$

$$\Omega_{M2} \equiv 388 - 300r + 32r^2 + 11r^3 (> 0),$$

$$\Omega_{U0} \equiv 64 - 84r + 36r^2 - 5r^3 (> 0),$$

$$\Omega_{U1} \equiv 64 - 88r + 40r^2 - 6r^3 (> 0),$$

$$\Omega_{U2} \equiv 178 - 131r + 24r^2 (> 0).$$